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USING BISPECTRAL ANALYSIS TO DETECT THE ONSET OF FATIGUE DAMAGE IN RANDOMLY EXCITED STRUCTURES

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ABSTRACT

When structures are excited using random vibration, the input excitation is traditionally Gaussian. If the structure is reasonably linear, the output will also be Gaussian or nearly Gaussian. In contrast, the introduction of damage into the structure, such as fatigue cracks, will result in a shift away from a linear response to a more non-linear, and hence non-Gaussian response. The bispectrum is a third-order spectrum that has unique properties that make it a potentially useful tool to detect non-Gaussian signals buried within Gaussian random excitation. This paper presents the results of numerical simulations and experimental fatigue testing using random excitation along with bispectral analysis of the response data. The results show that the bispectrum can identify the onset and progression of fatigue damage in simple structures.

INTRODUCTION

Laboratory random vibration excitation of structures is typically accomplished using a Gaussian random distribution. Field data also are frequently Gaussian in nature, although non-Gaussian inputs do occur. If the structures being excited are reasonably linear, then the Gaussian random vibration excitation will also produce a Gaussian response. If damage is introduced in the structure, such as a fatigue crack, the resulting response will shift away from the original linear, or nearly-linear response, to a more non-linear response. The non-linearities are a result of the crack opening and closing as the part vibrates. The increased non-linearities in the system response are also non-Gaussian.

The bispectrum is from the class of higher-order spectra or poly-spectra and is defined as a third-order spectrum [1, 2]. The bispectrum has unique properties that lend themselves to the detection of non-Gaussian signals buried within Gaussian random excitation. Gaussian signals are symmetrically distributed about the mean whereas non-Gaussian signals are not necessarily symmetric. A symmetrically distributed signal will have a zero bispectrum with the bispectrum increasing as the asymmetry increases. Thus, a linear system subjected to Gaussian random excitation will respond with a very low or zero bispectrum and if non-linearities are introduced into the system, the bispectrum should increase proportionately.

Figure 1 shows an example of an increase in the bispectrum peak magnitude with the addition of non-linearities in a signal. The signal the left side of Figure 1 was produced using a uniform random signal that was then band-pass filtered to produce a narrow-band random acceleration time history. The same signal was then multiplied by a one-sided offset window to produce the altered signal shown in the center of Figure 1 and further amplified in the right-hand plot of Figure 1. The bispectrum maximum is shown in all three plots with the altered signals having a bispectrum peak magnitude 5.7 and 22 times greater than the magnitude of the original signal, respectively. Figure 2 shows a comparison of the auto-spectral density (ASD) function for each of the three signals shown in Figure 1. While magnitude of the ASD is larger for the signals with more non-linearities, there is nothing particularly characteristic in the ASD that would signal a shift from a linear to non-linear response. In this case, the ASD magnitude is increasing because the average amplitude in the time history is increasing, not because the response is more one-sided.

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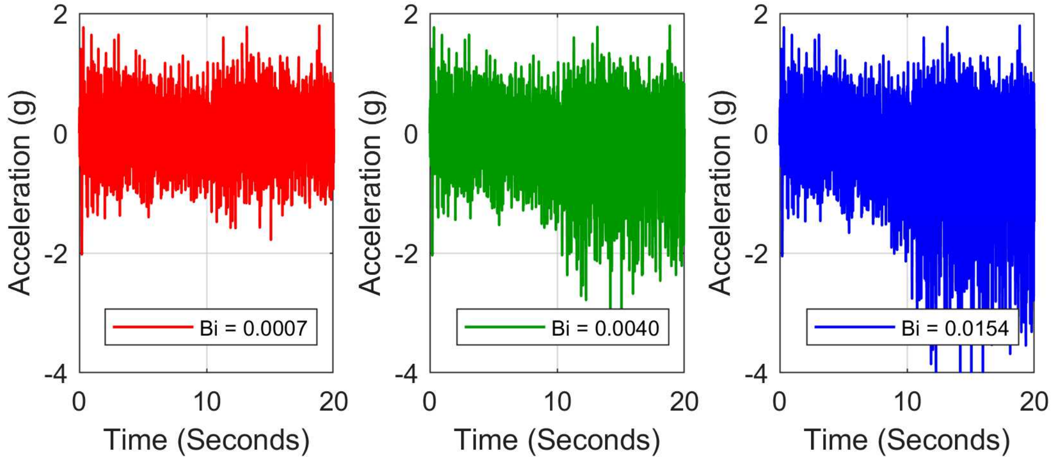

Figure 1. Random Acceleration Signal (Left) and Random Acceleration Signal with Late-Time Negative Acceleration Response Increases (Center and Right) Showing Increases in Bispectrum Magnitude

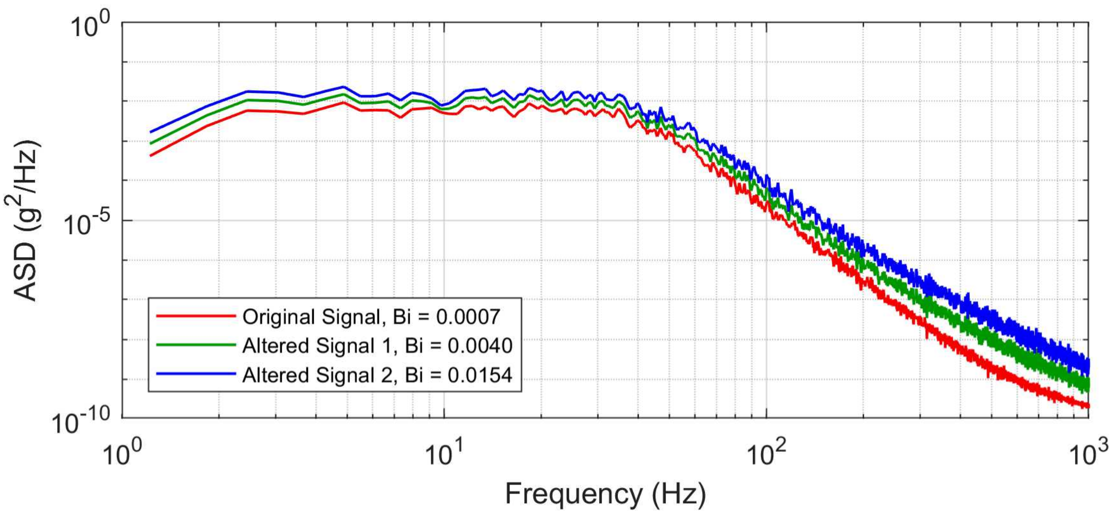

Figure 2. Auto-Spectral Density Function from the Original Random Acceleration Signal and the Two Altered Signals from Figure 1

A series of experiments was conducted on additively manufactured cantilever beams. In these experiments, the beams were exposed to a narrow-band random excitation centered about the beam's fundamental resonant mode until the beams failed due to fatigue. It was hypothesized that new beams would exhibit a relatively low bispectral response while they were undamaged and that the bispectrum would increase with increasing fatigue damage until failure occurred. This hypothesis was also tested using numerical simulations of a beam with a bi-linear stiffness and an applied random force excitation.

Figure 3 shows a sketch of a cantilever beam with a crack near the beam's fixed end. In the uncracked state, the beam stiffness is the same regardless of whether the applied force, ***F***, is oriented up or down. However, in the cracked condition, the stiffness for the beam bent downward is noticeably less than the stiffness for the beam bent upward. When the beam is curved downward, the crack opens because the upper portion is in tension, and the effective cross-sectional area and area moment of inertia are decreased. In contrast, when the beam is curved upwards, the crack is closed, and load can be carried across the crack since it is in compression. Thus, the stiffness is essentially the same as the uncracked beam when the crack is in compression.

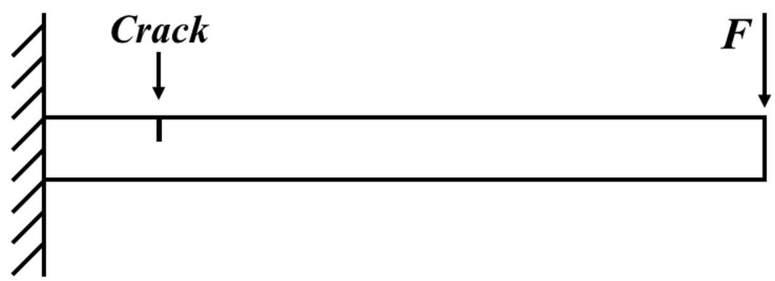

Figure 3. Sketch of a Cantilever Beam with a Crack Located Near the Base

BISPECTRUM THEORY

The most common spectral analysis performed today is the ASD, historically referred to as the power-spectral density (PSD). The ASD is defined as the finite Fourier transform of the second-order cumulant of the time history. The second-order cumulant is also known as the covariance, when calculated from a random variable with itself, for any time delay, τ, and is given by:

$$C_{2,x}(\tau) = E\{x(t)x(t+\tau)\}\,. \tag{1}$$

Or

$$C_{2,x}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)dt. \tag{2}$$

Eq. (2) is also known as the auto-correlation function of $x(t)$ and can also be written without the limit as:

$$C_{2,x}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt. \tag{3}$$

The auto-spectral density function is then given as the Fourier transform of the auto-correlation function as:

$$S_{xx}(f) = \mathcal{F}\left[C_{2,x}(\tau)\right] = \int_{-\infty}^{\infty} C_{2,x}(\tau)e^{-i2\pi f\tau}d\tau. \tag{4}$$

One requirement for Eq. (4) to work is that the integral of the auto-correlation must converge. Since the auto-correlation function tends to the signal mean, it is a requirement that the random vibration signal be zero mean [3].

As a matter of practicality, the ASD is not actually calculated using the auto-correlation integral as defined in Eq. (4). The reason is simply that the computational overhead is too high. Using the Wiener-Khinchin theorem, it can be shown that the ASD for a real-valued signal can be calculated as the product of the Fourier transform of the signal with its complex conjugate [4]. This method using the Finite Fourier Transform (FFT) is so much faster than solving the integral equation that most modern numerical methods for calculating the auto-correlation make use of the Wiener-Khinchin theorem to back into the auto-correlation rather than calculate it directly.

Likewise, the bispectrum is defined as the Fourier transform of the third-order cumulant of the time history. The third-order cumulant is the same as the third central moment and is defined with time delays, τ_1 and τ_2, as:

$$C_{3,x}(\tau_1\tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\}. \tag{5}$$

Or in integral terms as:

$$C_{3,x}(\tau_1,\tau_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t)x(t+\tau_1)x(t+\tau_2)dtdt. \tag{6}$$

Eq. (6) results in a two-dimensional matrix of values. The bispectrum is then calculated as the two-dimensional Fourier transform of the third-order cumulant function as:

$$B_{xx}(f) = \mathcal{F}[C_{3,x}(\tau_1,\tau_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{3,x}(\tau_1,\tau_2)e^{-i(2\pi f_1\tau_1+2\pi f_2\tau_2)}d\tau_1 d\tau_2. \tag{7}$$

Higher-order cumulant statistics possess an interesting property that will be exploited for this analysis. That is, for a real-valued, zero-mean Gaussian random process, $x(t)$, all cumulant statistics greater than second-order are zero. This property allows discrimination against additive Gaussian noise or detecting departures from a Gaussian signal. If the cumulant is zero, then the Fourier transform of that cumulant is also zero and the resulting higher-order spectra will also be zero.

Gaussian signals have zero third-order cumulant spectra because they are symmetrically distributed. Other non-Gaussian signals can have zero third-order cumulants but large higher-order cumulants. Exponential, Rayleigh, and k-distributions are some examples of processes with zero third-order cumulants and non-zero fourth-order cumulants [1].

NUMERICAL SIMULATION

To test the theory that the bispectrum can be an indicator of damage, a simple numerical example was studied. A cantilever beam, like the one shown in Figure 3, and with a lumped mass at the free end was modeled as a spring mass system. The stiffness of the cantilever beam is given by $k = 3EI/L^3$, where the stiffness and mass were chosen to match the 3D printed cantilever beams used in the experiment. The structural parameters are given in Table 1.

Table 1. Material Properties for 3D Printed Cantilever Beans

Modulus of Elasticity	2.94E5 psi
Cross-Sectional Area	4.91E-2 in^2
Area Moment of Inertia	1.918E-4 in^4
Beam Length	5 in
Density	9.29E-5 lb_f-sec^2/in^4
Mass at Free End	0.0289 lb_f

The equation of motion for this system is given by the usual expression:

$$\ddot{y}(t) + 2\zeta\omega\dot{y}(t) + \omega^2 y(t) = F(t). \tag{8}$$

For this example, the applied force, $F(t)$, was derived using a uniform random signal that was then band-pass filtered to produce a narrow-band random forcing function. From the parameters given in Table 1, the cantilever beam's fundamental natural frequency is about 20 Hz, so the band-pass filter frequencies were set at 2 Hz and 40Hz. It should be noted that a uniform random signal is not Gaussian; however, it should be symmetric about a mean of zero if sufficient time is analyzed; therefore, its bispectrum should be approximately zero. The randomly generated signal used as a forcing function for these simulations is shown in Figure 4.

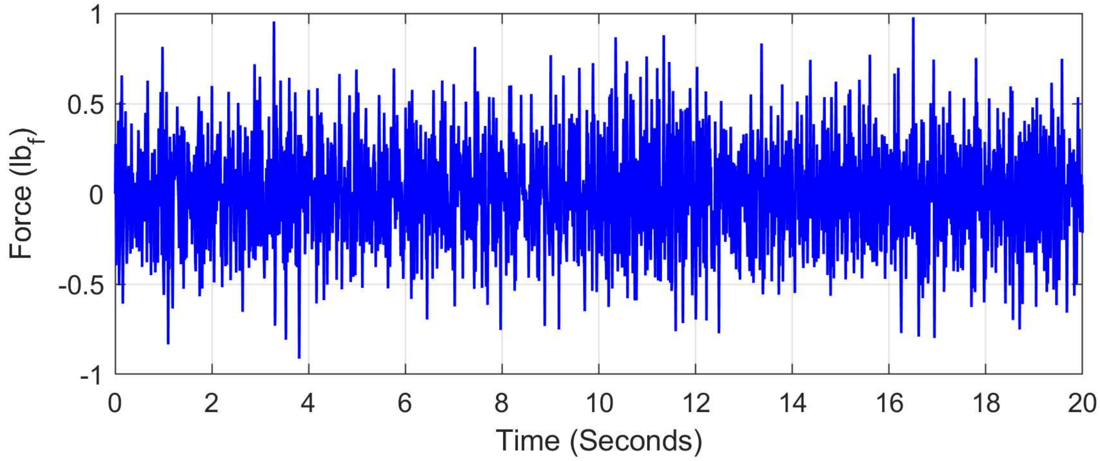

Figure 4. Band-Pass Random Force Input Used for Numerical Simulations

To simulate a crack in the cantilever beam, a bi-linear beam stiffness was assumed. The displacement at the tip was checked at every time-step and if the displacement was greater than zero (beam deflected upward and crack in compression) the full stiffness, $k = 3EI/L^3$, was used in the calculation. If the beam displacement was less than zero (beam deflected downward and crack opened) the stiffness was scaled by a constant, $k = \eta 3EI/L^3$. The constant, η, was varied between 0.6 and 1 to simulate a range of crack depths and the resulting beam tip responses were calculated. The $\eta = 1$ case represents an uncracked beam, or zero crack depth and was used as the baseline response. Figure 5 shows the calculated beam tip response from a simulation of the uncracked beam and a simulation of a cracked beam with $\eta = 0.7$ for the tensile bi-linear stiffness. While there are obvious differences between the two responses shown in Figure 5, the cracked beam case does not appear to be obviously skewed by the bi-linear stiffness.

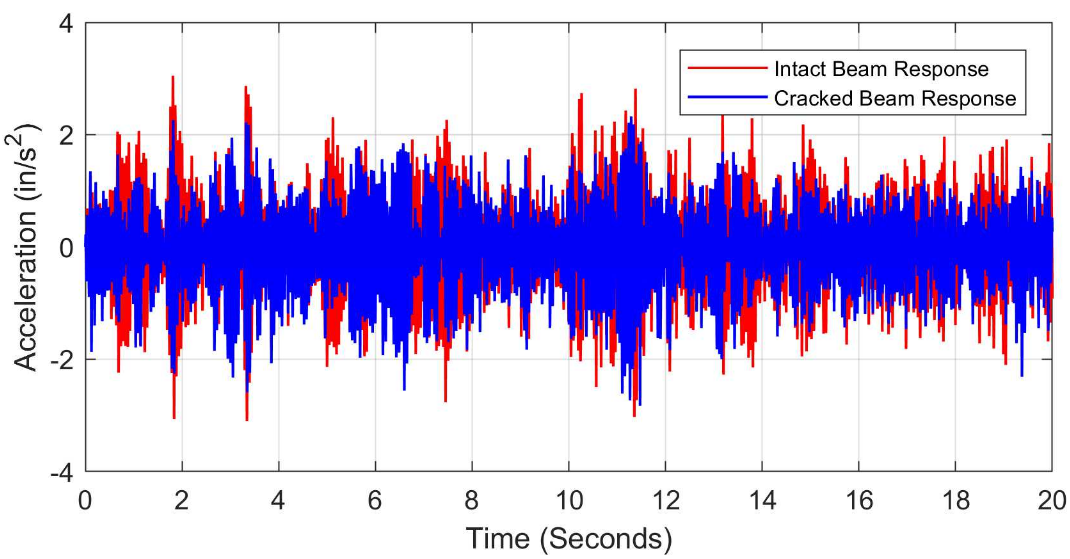

Figure 5. Calculated Beam Tip Response for an Intact Beam and a Cracked Beam with $\eta = 0.7$.

Figure 6 shows a plot of the variation in the maximum absolute bispectrum with the cracked beam bi-linear stiffness ratio. A line representing the bispectrum of the intact beam tip response is included as a reference. For very small cracks ($\eta \sim 1$) the maximum bispectrum converges to the intact beam bispectrum. As the crack size grows ($\eta \ll 1$) the maximum bispectrum of the beam tip response increases substantially. Figure 6 also shows that the while the beam tip response shown in Figure 5 is unremarkable for detecting damage, the maximum bispectrum indicates a change of over an order of magnitude.

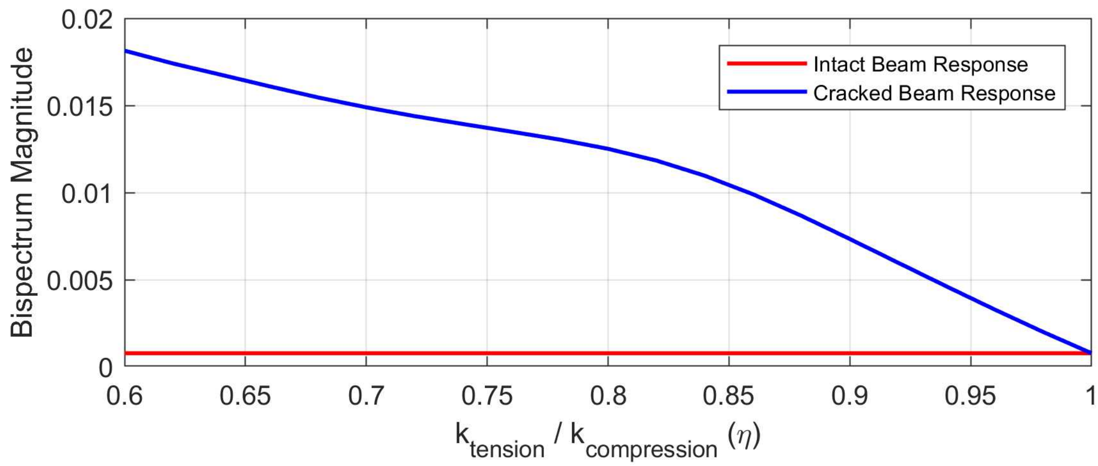

Figure 6. Variation in Maximum Bispectrum with Bi-Linear Stiffness Ratio

TEST SETUP

The experimental portion of this study used 3D printed cantilever beams tested in the Sandia National Laboratories Component Dynamics Laboratory on a small modal shaker system. The cantilever beams used for this investigation were made from ABS plastic and printed by the Sandia National Laboratories Additive Manufacturing group. A photograph of the test setup is shown in Figure 7. The tests were conducted with sets of four beams and a narrow-band random vibration input was applied until all four beams failed. The narrow-band random excitation covered the range from 10 – 30Hz, which was approximately centered around the first natural frequency of the cantilever beams. Since a relatively small shaker was used, a gravity off-load system was incorporated using soft springs to support the test fixture's weight independent of the shaker armature. This allowed for higher acceleration loads to be used for these experiments.

Figure 8 shows a close-up photograph of one cantilever beam used for testing. All beams were five inches long and 0.25 inches in diameter. The beams had a 0.025-inch notch near the base to create a stress concentration point. This feature helped ensure that all beams failed predictably at the same location. A clamp-on steel collar with an attached accelerometer was added at the free end to measure the response and to increase the bending moment in the beam. The added weight at the cantilever beam free end was about 0.0289 lb_f (13.1 grams), substantially more than the weight of the plastic beams.

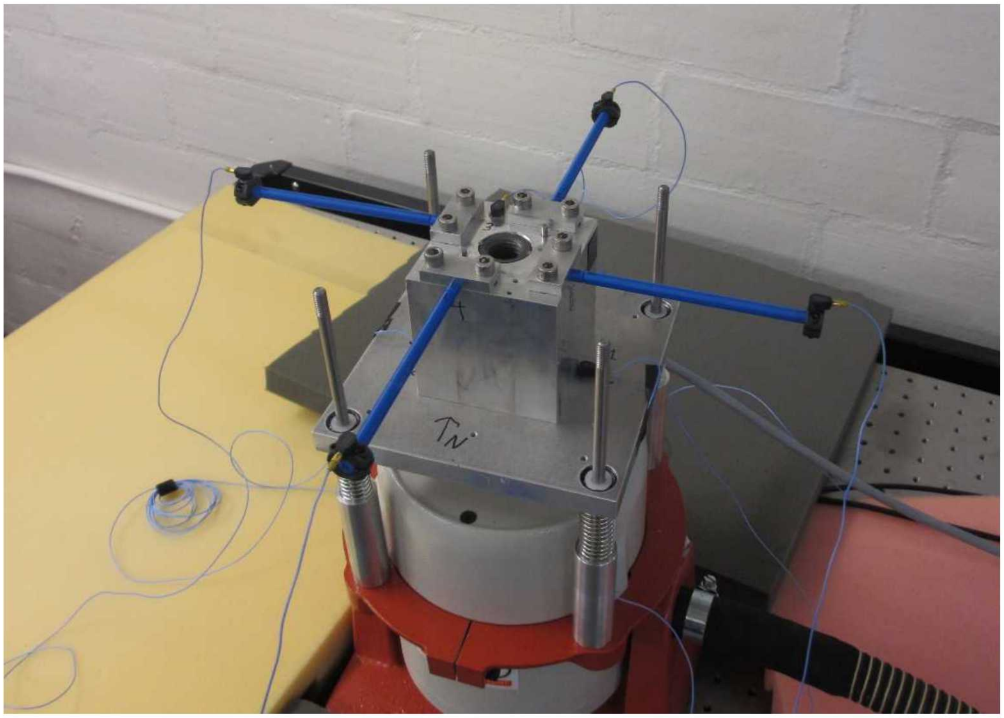

Figure 7. Shaker Test Setup with Four Cantilever Beams Installed

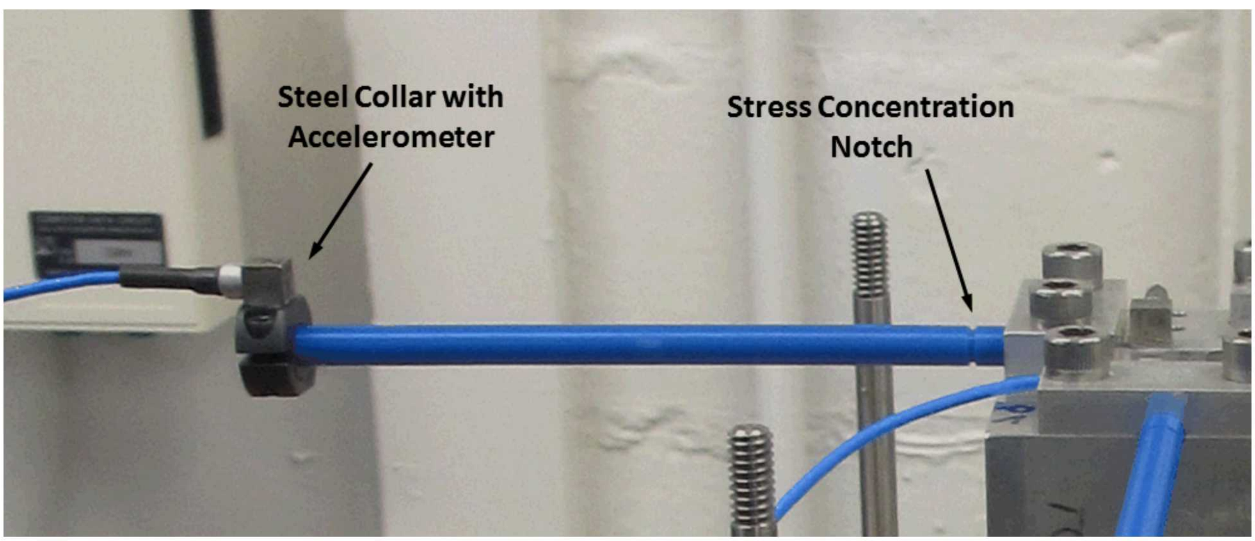

Figure 8. Cantilever Beam Test Specimen Showing Stress Concentration and Tip Weight

Figure 9 shows a plot of the base input acceleration time history and the resulting ASD from one test on four cantilever beams. The random vibration input was applied until all four cantilever beams had failed. In this example, the acceleration level was relatively low, at 0.76 GRMS. The test required 101.6 minutes to fail all four beams by fatigue. Other tests were conducted at different nominal acceleration magnitudes to gather fatigue data at different levels.

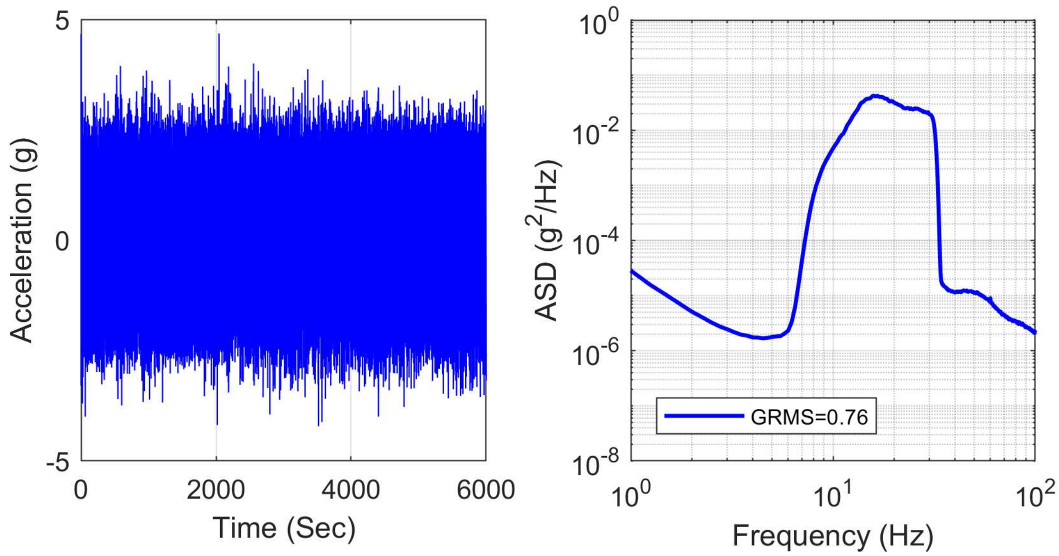

Figure 9. Base Input Acceleration Time History and the Resulting ASD

TEST RESULTS

Figure 10 shows a plot of a typical acceleration time history response at the cantilever beam's free end. The response is amplified from the input due to the motion of the cantilever beam. The output initially had a 2.06 GRMS compared to a 0.76 GRMS input. This beam failed after 64.6 minutes of testing. Figure 10 also shows a plot of the ASD calculated from the first minute of vibration data and the last minute of vibration data before the beam failed. The ASD plots show a downward shift in the beam's response frequency indicating that the beam is less stiff at the end of the exposure. It is assumed here that the softening is a direct result of crack growth and the resulting reduction in cross-sectional area. It is also possible to calculate ASD curves from time segments throughout the test to see the trend showing a reduction in the fundamental response frequency. Figure 11 shows a plot of the frequency at the ASD maximum calculated in sixteen-second time intervals across the entire data set. While the results show some significant variability, likely due to quantization of the ASD maximum, it is apparent that the frequency is relatively constant over the first 2500 seconds and then shows a decidedly downward trend from there until the beam fails when the frequency drops suddenly to zero. However, the ASD only shows a shift in the beam response but does not necessarily provide insight into the reason for the shift.

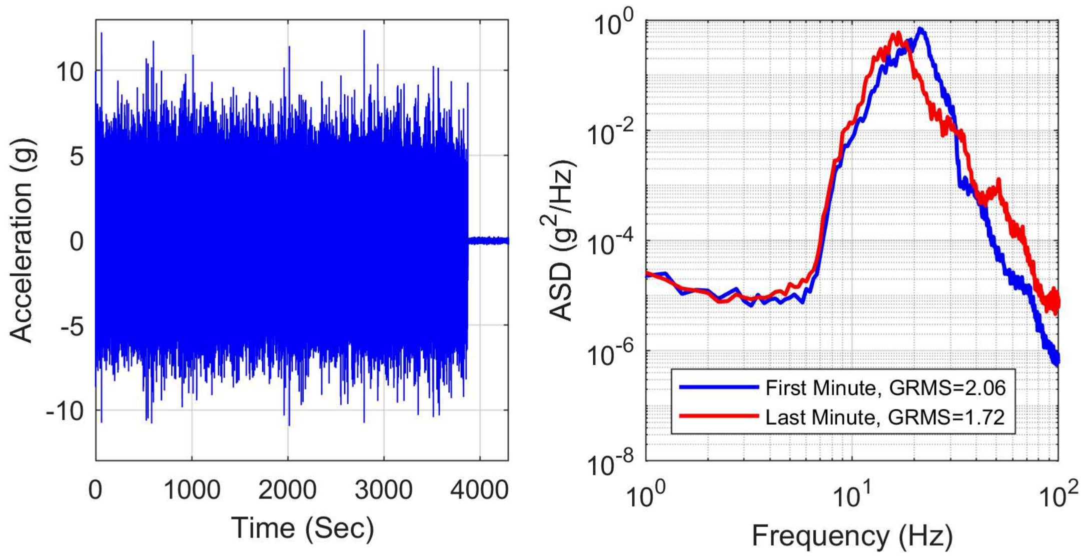

Figure 10. Cantilever Beam Tip Acceleration Response Time History Along with the ASD from the First Minute and Last Minute Before Failure

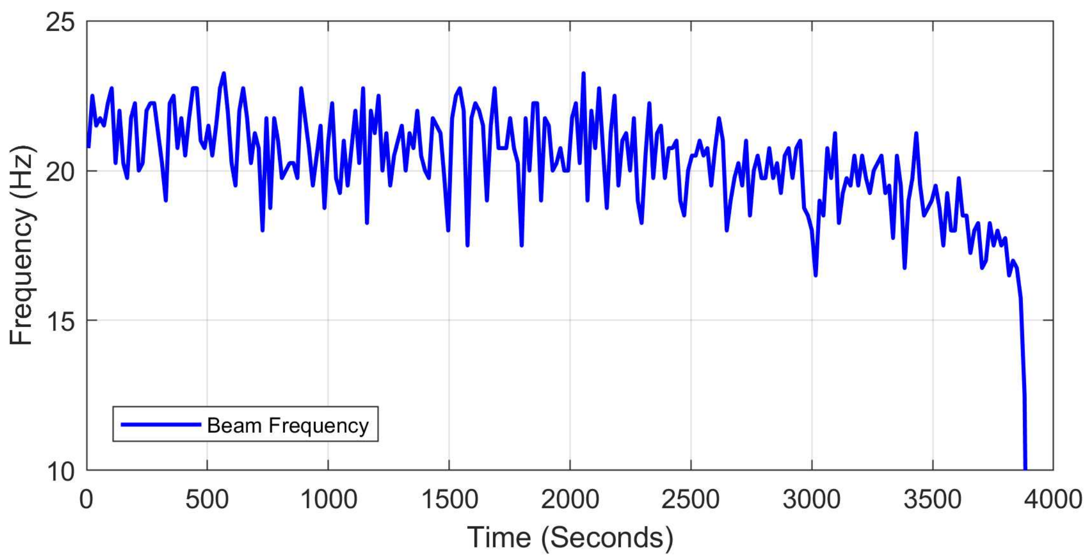

Figure 11. Plot of the ASD Maximum Frequency versus Exposure Time from the Cantilever Beam Tip Accelerometer

Figure 12 shows plots of the maximum absolute bispectrum from the input and the response accelerometers at the clamped and free end of the cantilever beam, respectively. For this plot, the bispectrum was calculated in eight-second time intervals across the entire data set. As discussed previously, the bispectrum for a Gaussian random signal should be identically zero, and Figure 12 shows that the maximum bispectrum from the input excitation is nearly zero for all time. In contrast, the bispectrum at the free end of the beam shows two interesting trends. First, even with a pristine cantilever beam, the bispectrum is nominally an order of magnitude greater at the beam's free end than at the input. The implication from this is that either the 3D printed cantilever beams or the test fixture setup is not perfectly linear, since the output is less Gaussian than the input. Second, the bispectrum magnitude shows a relatively flat trend up until around 2550 seconds and then a rapidly increasing amplitude until the beam fails at 3876 seconds. This is more easily seen when a trend-line is added through the calculated bispectrum at the beam tip. It is hypothesized that the onset of this upward trend in the bispectrum magnitude represents the onset of cracking in the cantilever beam. Cracks are known to generate a nonlinear stiffness condition as the crack opens under tensile loads and closes again under compressive load. The onset of the change in the bispectrum magnitude also corresponds to the onset of the downward trend in the beam's first natural frequency as obtained from the ASD. Thus, the increase in the bispectrum coupled with the decrease in stiffness associated with a decrease in the first natural frequency is indicative of crack formation and growth in the beam.

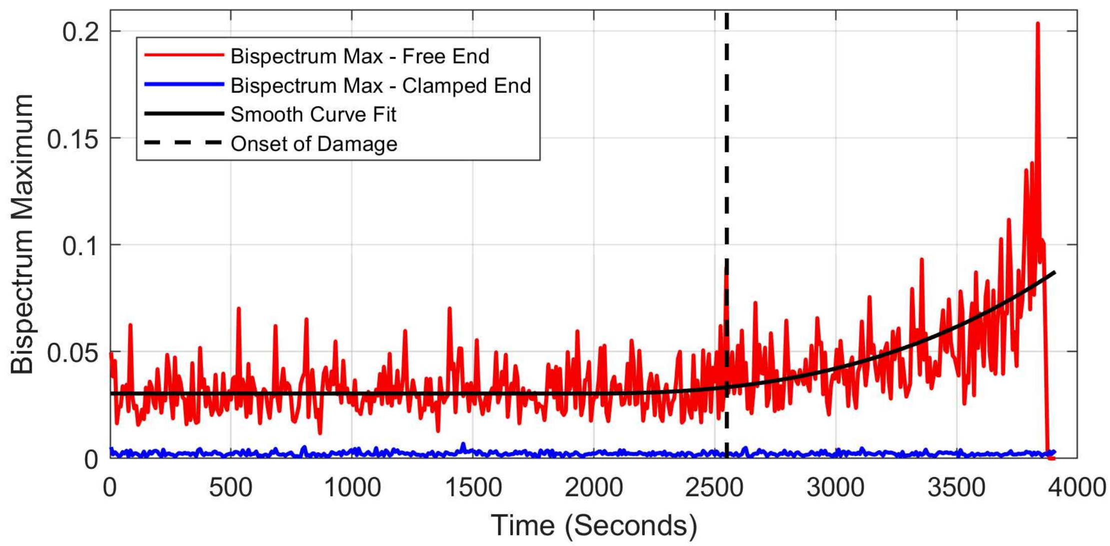

Figure 12. Maximum Absolute Bispectrum at the Cantilever Beam Base and the Cantilever Beam Tip with Trend Line Added

The data presented in Figure 12 was obtained from one of the four beams in the test. Figure 13 shows bispectrum plots for the remaining three beams in the same test. Beam two, shown in the upper plot, failed at about 2550 seconds and shows a very flat, low-level bispectrum up until approximately 2340 seconds at which point the bispectrum magnitude begins to rise rapidly until the beam breaks. Beam three, shown in the center plot, has a much more varied bispectrum magnitude early on but does show the rapid rise trend beginning at about 1500 seconds and failing at about 1610 seconds. Beam three also failed earliest out of the three beams tested here, likely indicating that fatigue damage was accumulating more quickly in this test specimen or perhaps indicative of some other form of latent damage. The final beam, shown in the lower plot, is not as clear as the first three beams. The bispectrum magnitude shows several high spikes throughout the time record followed by a brief, but not obviously significant, run-up in the bispectrum magnitude in the last 100 seconds of the test. It should be noted that beam four survived considerably longer than the other three beams exposed to the same vibration input. This shows some of the inherent variability in fatigue data.

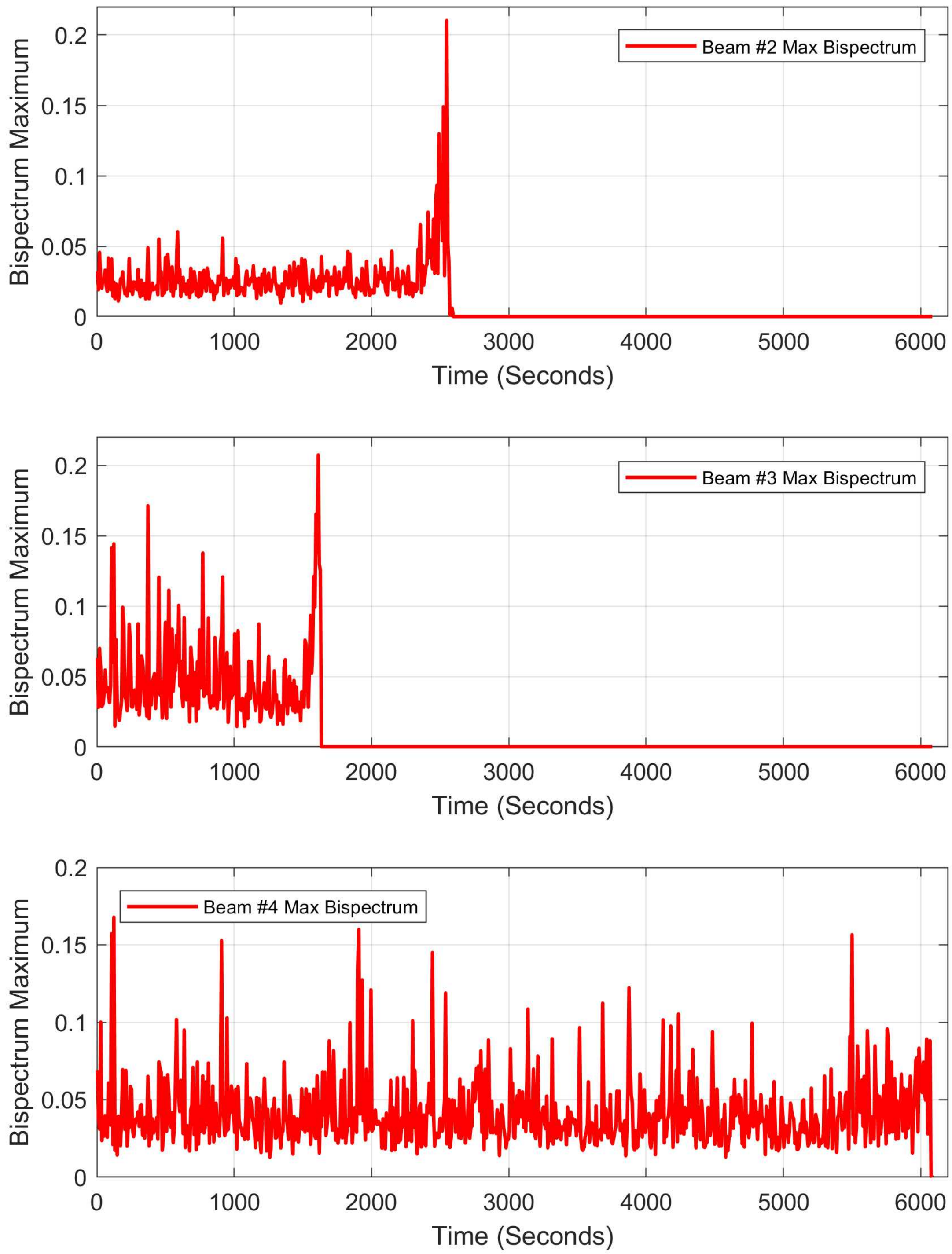

Figure 13. Maximum Absolute Bispectrum at the Cantilever Beam Tip for Beams 2, 3, and 4

CONCLUSIONS

In this paper, the bispectrum is presented as a possible indicator of fatigue damage. The bispectrum is zero for symmetric random vibration signals. If there is asymmetry in the signal, the bispectrum will increase with increasing magnitude as the asymmetry increases. This makes it a potentially useful indicator of damage in a structure, such as the fatigue cracks that create a bi-linear stiffness. The bispectrum appears to be more useful when crack growth is relatively slow.

The utility of the bispectrum was illustrated on random vibration tests of cantilever beams to failure. The change in the bispectrum as a function of time showed the onset and growth of a crack better than the change in the natural frequency.

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