

Tip-to-Tail System Modeling Using Reduced Order Modeling



PRESENTED BY

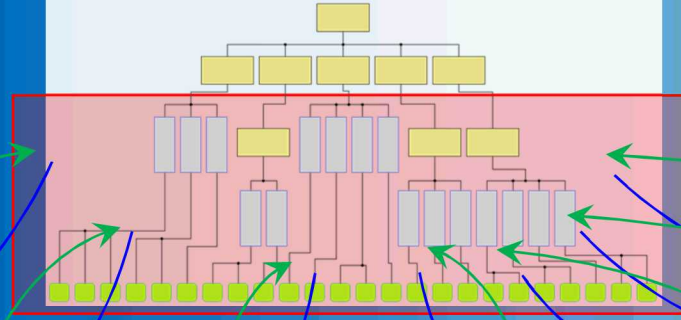
Erik Olson, 2496

December 10, 2018

The Role of MBSE and Reduced Order Modeling



System Architecture

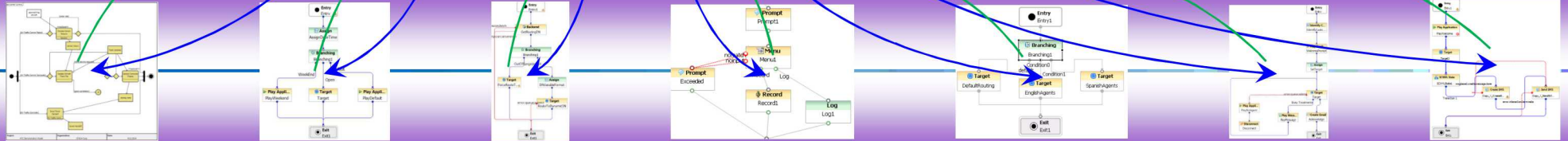


Derived Requirements



Validation

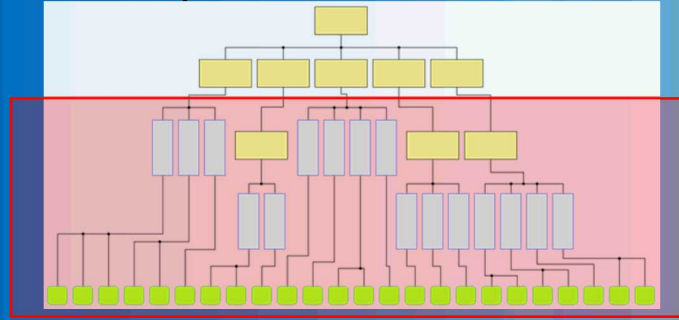
Function Models



The Role of MBSE and Reduced Order Modeling



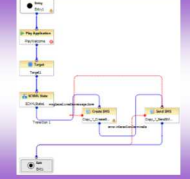
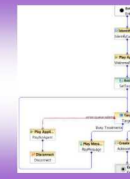
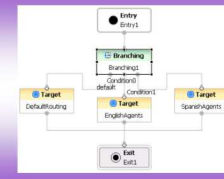
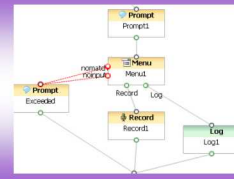
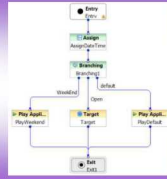
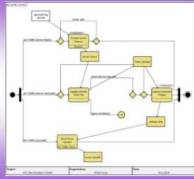
System Architecture



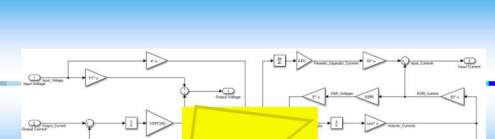
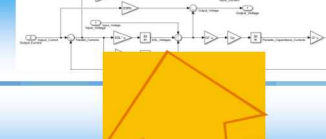
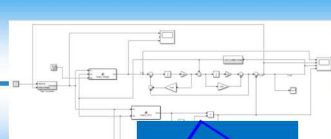
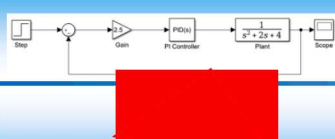
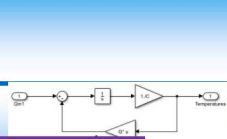
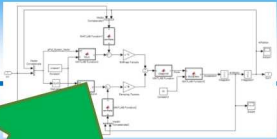
Derived Requirements



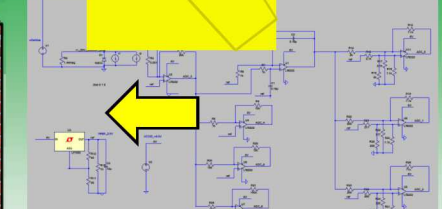
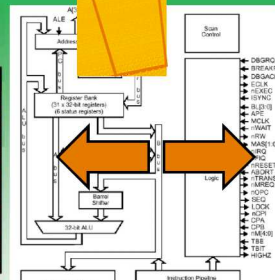
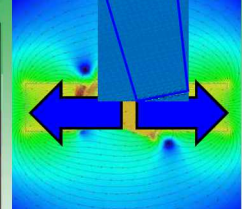
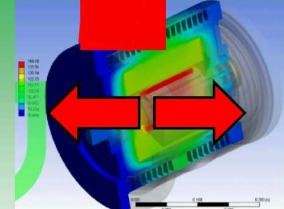
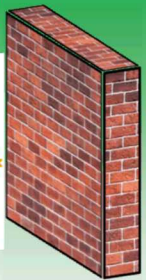
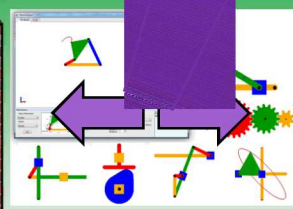
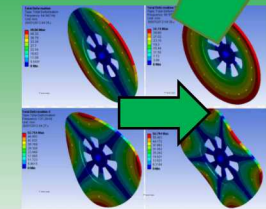
Function Models



Reduced Order Models



Component Designs



Circuit Modeling – Analog / Power

Mechanical Modeling - Vibration

Mechanical Modeling - Kinematics

Thermal Modeling – Conduction

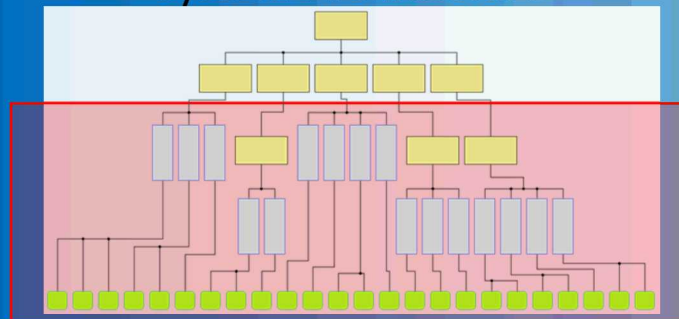
Electro-magnetics

Circuit Modeling – Digital

The Role of MBSE and Reduced Order Modeling

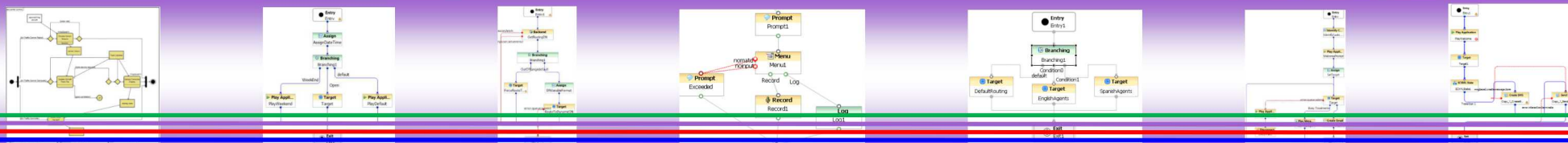


System Architecture

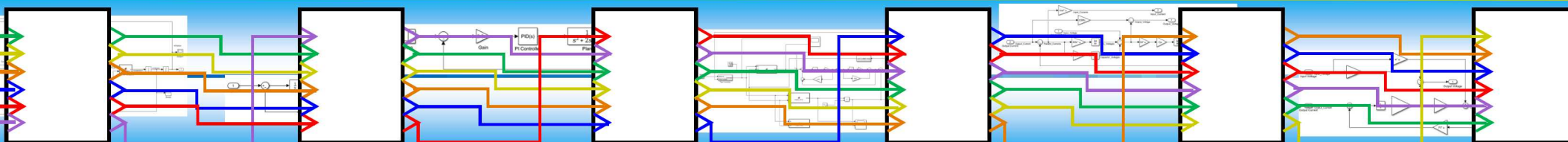


Derived Requirements

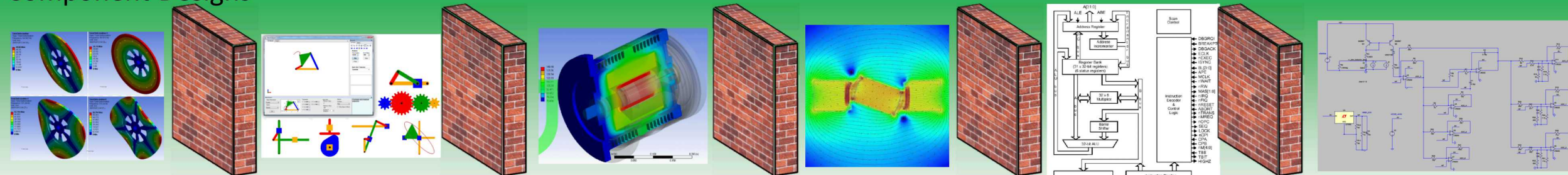
Function Models



Reduced Order Models



Component Designs



Mechanical Modeling - Vibration

Mechanical Modeling - Kinematics

Thermal Modeling - Conduction

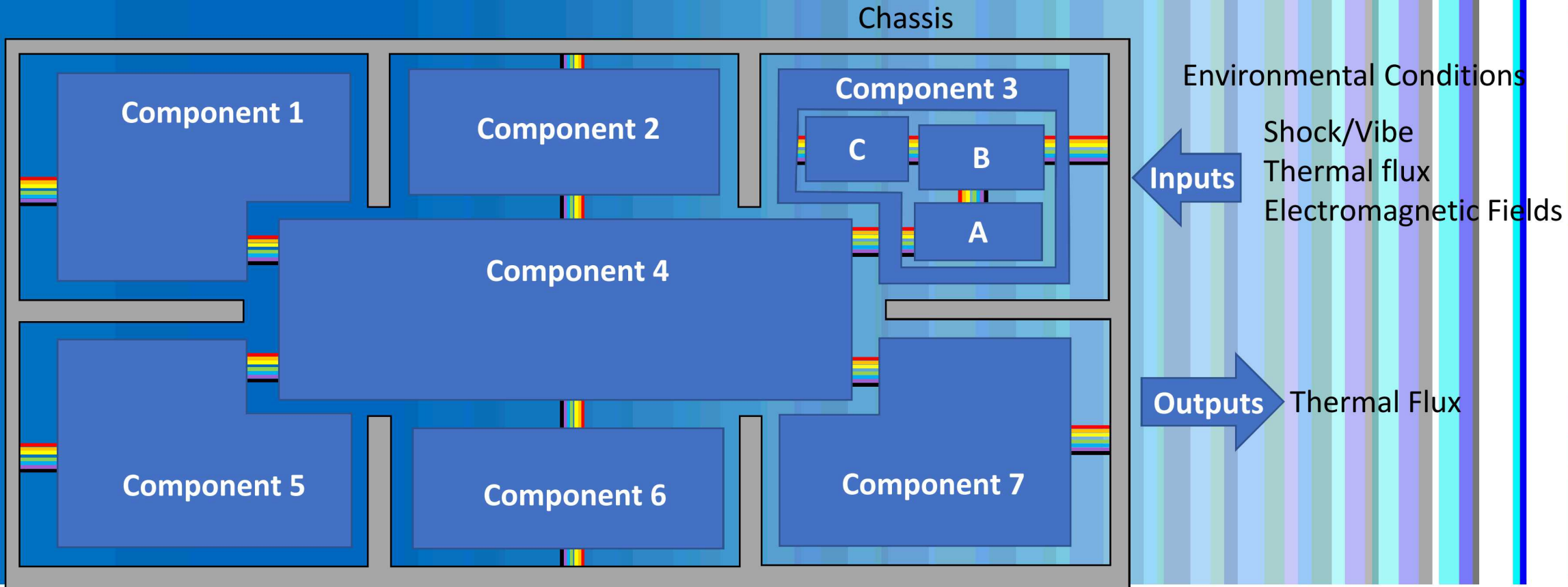
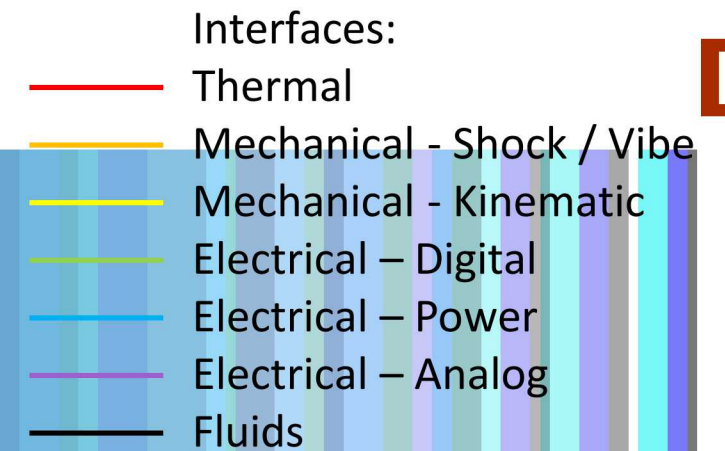
Electro-magnetics

Circuit Modeling - Digital

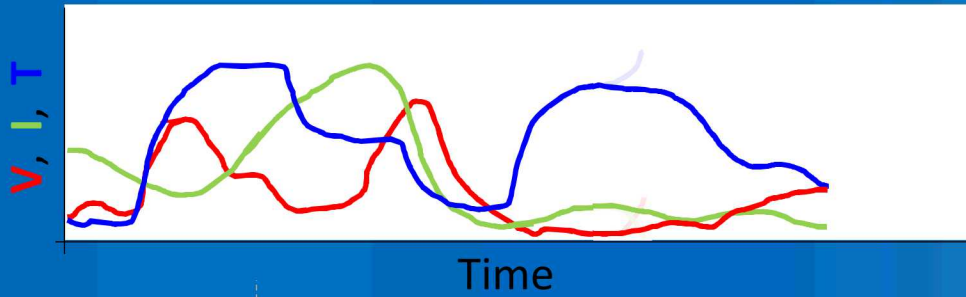
Circuit Modeling - Analog / Power

Tip to Tail System Level Model

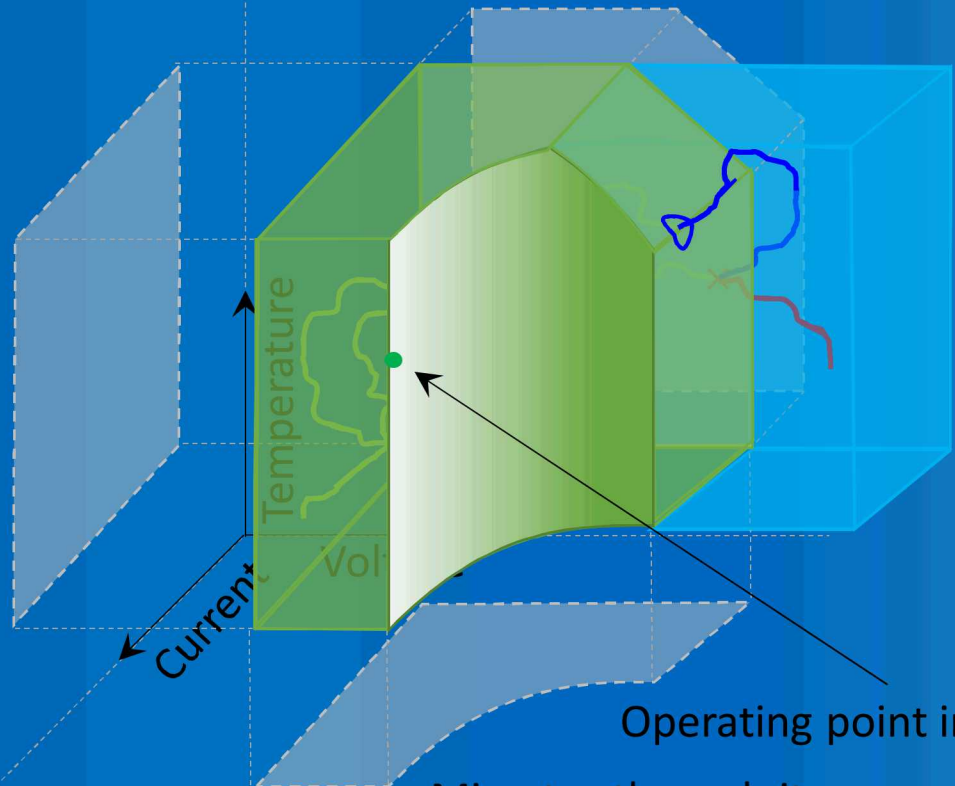
Modeling of Components and Interfaces



Physics Variable Trajectories



Time

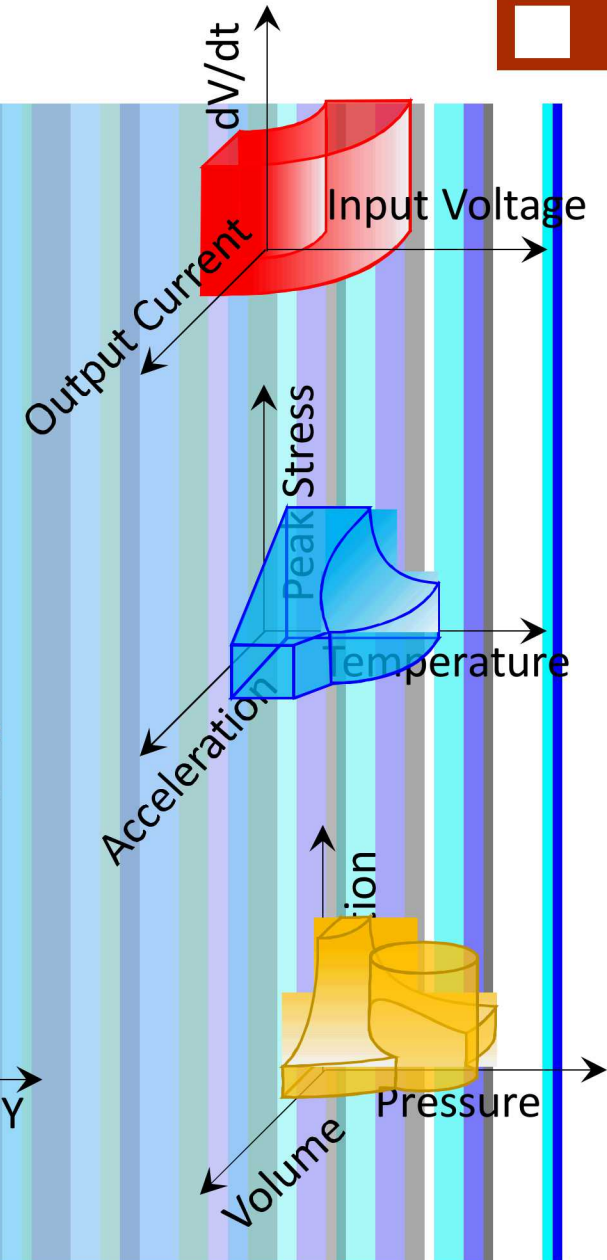
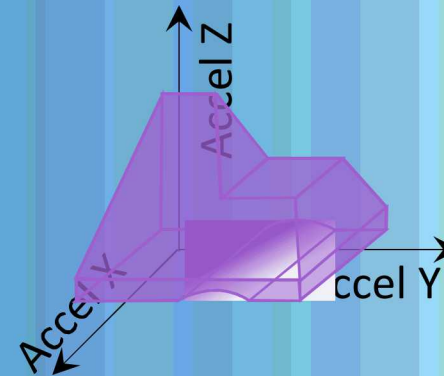


Operating point in simulation space

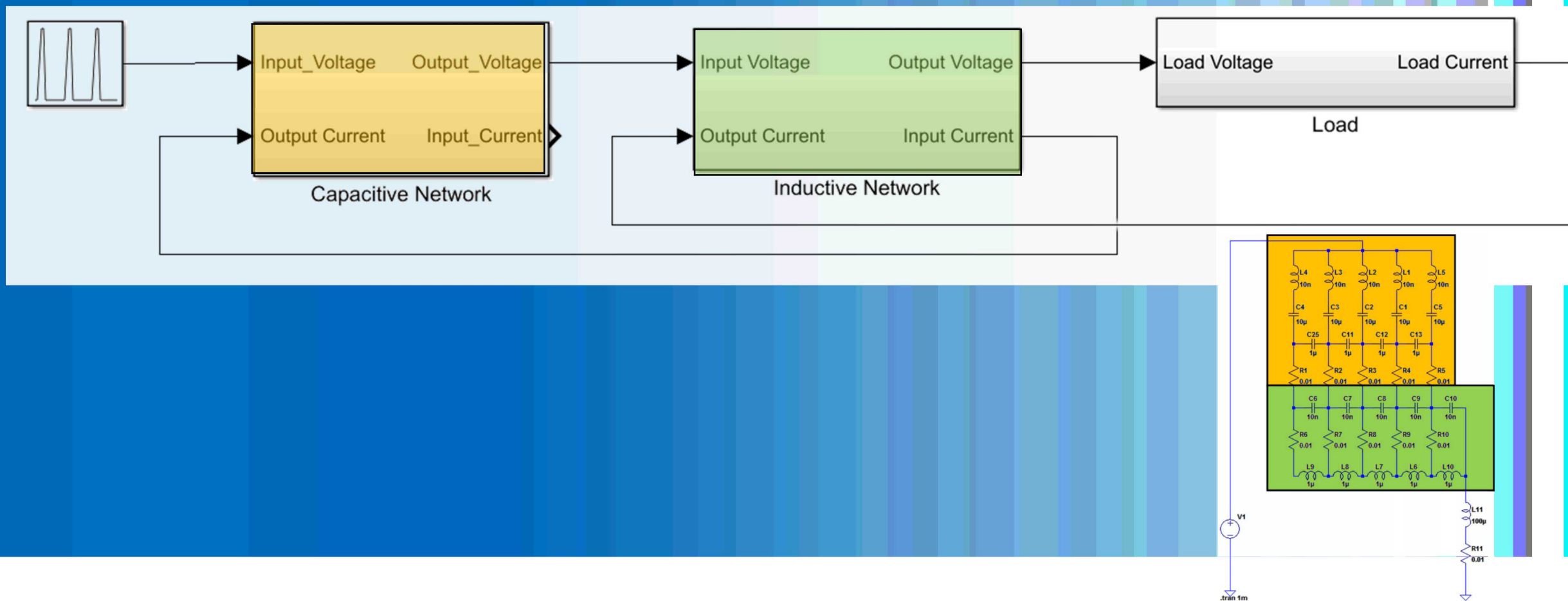
Migrates through its space during a simulation

Region of high accuracy, high confidence
Behavioral
Possibly Linear
“Ideal” performance
Verified against physics design model
(Finite Element, PSPICE)

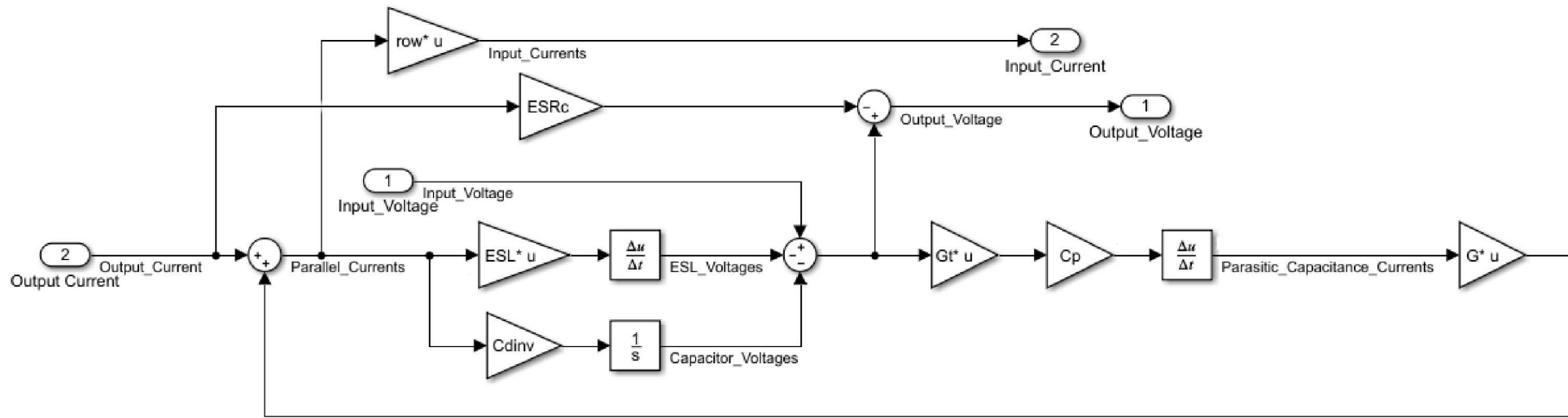
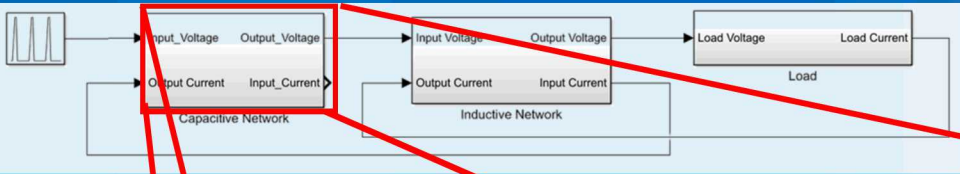
If external conditions force the modeled component beyond its limits of accuracy,
A piece-wise extended model can be made to extend the accuracy of the component’s model, if necessary.



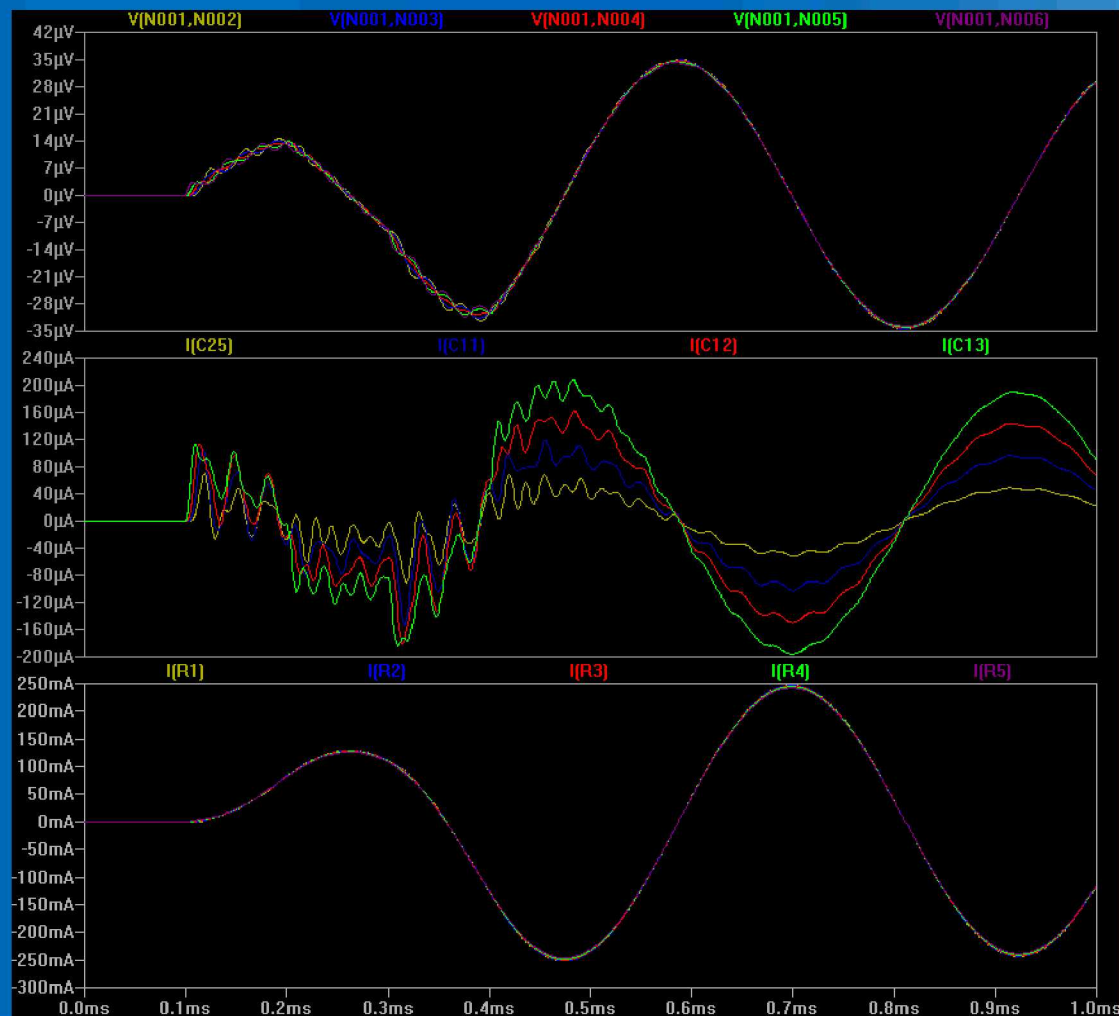
Connecting Passive Electrical Components



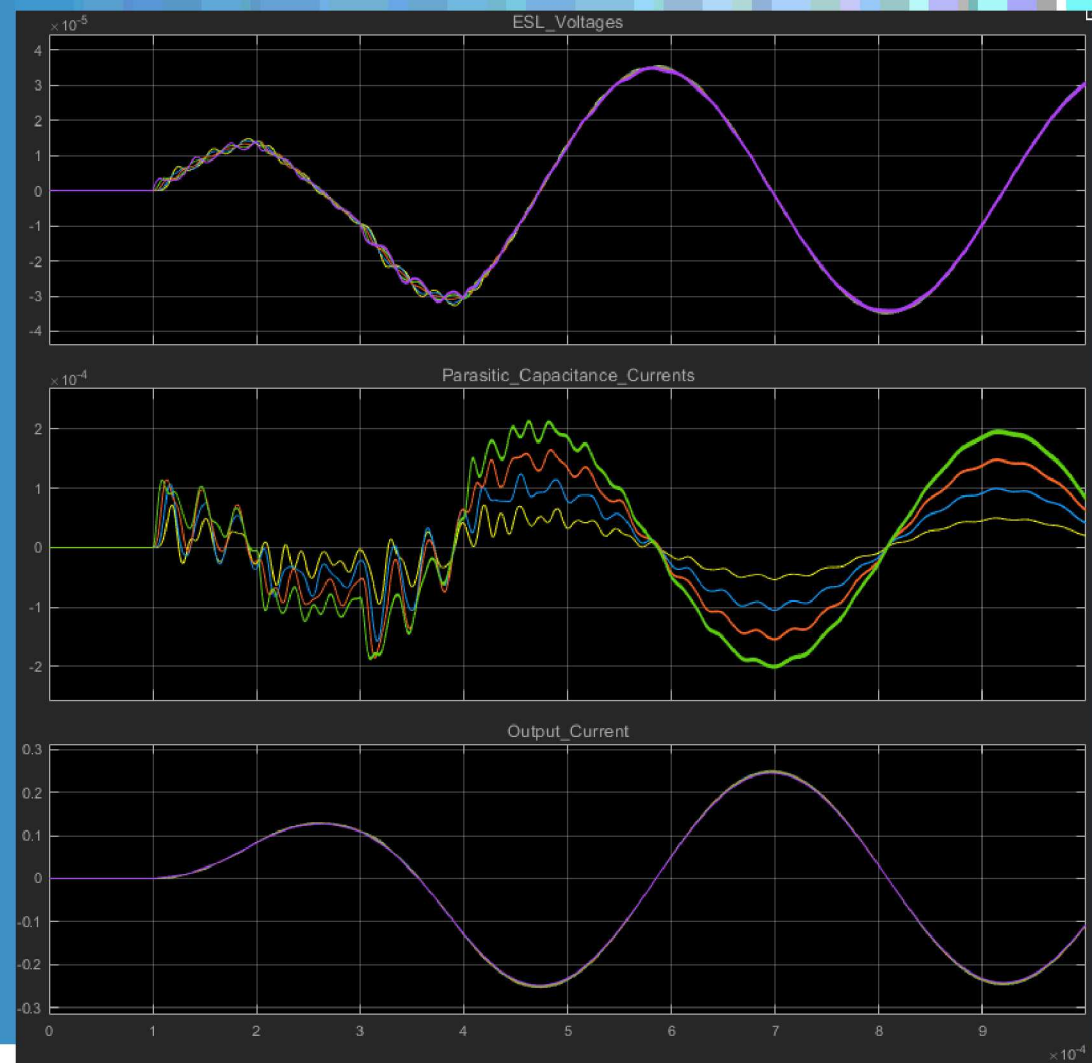
Electrical Modeling

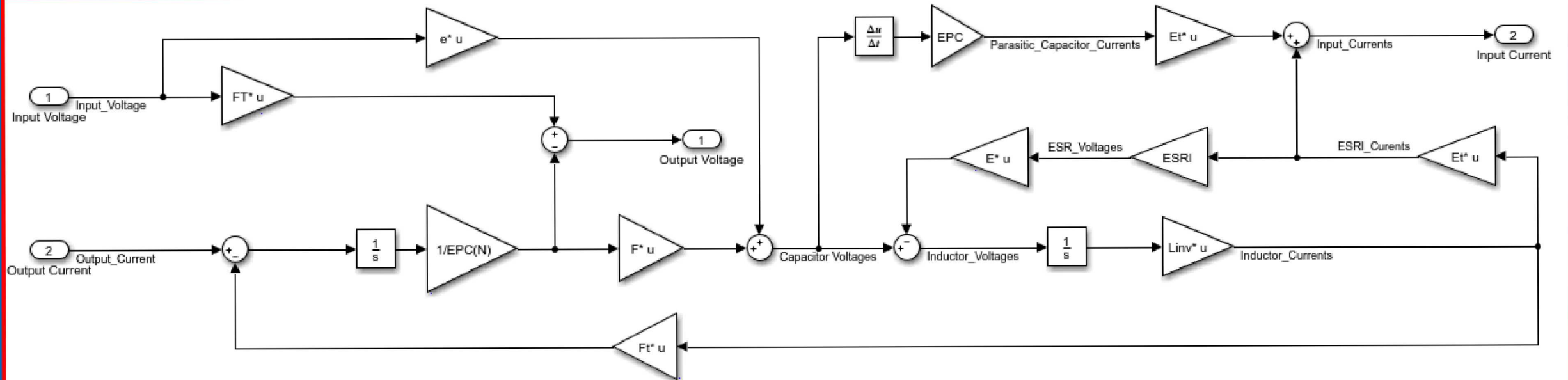


LTSpice



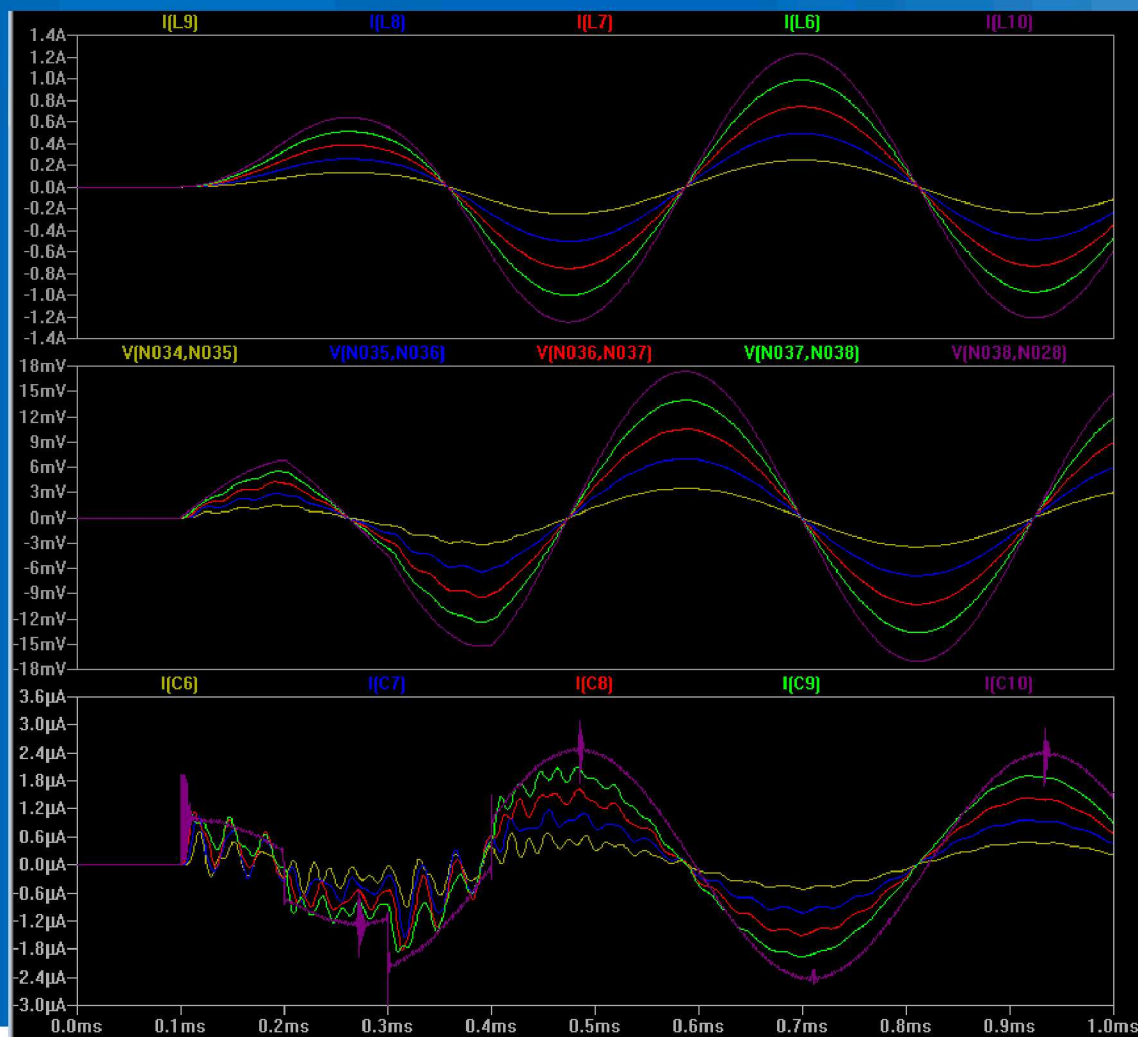
SIMULINK



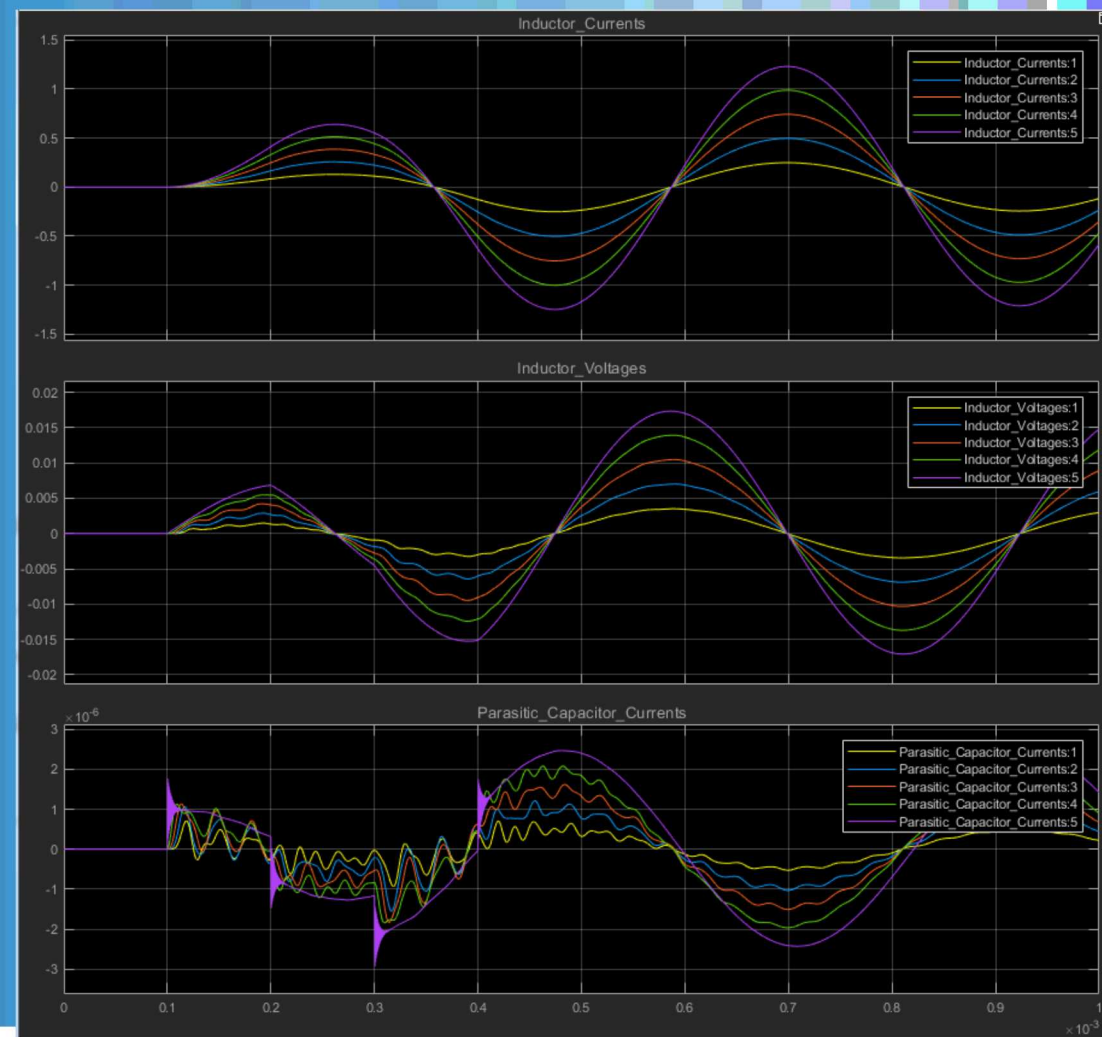


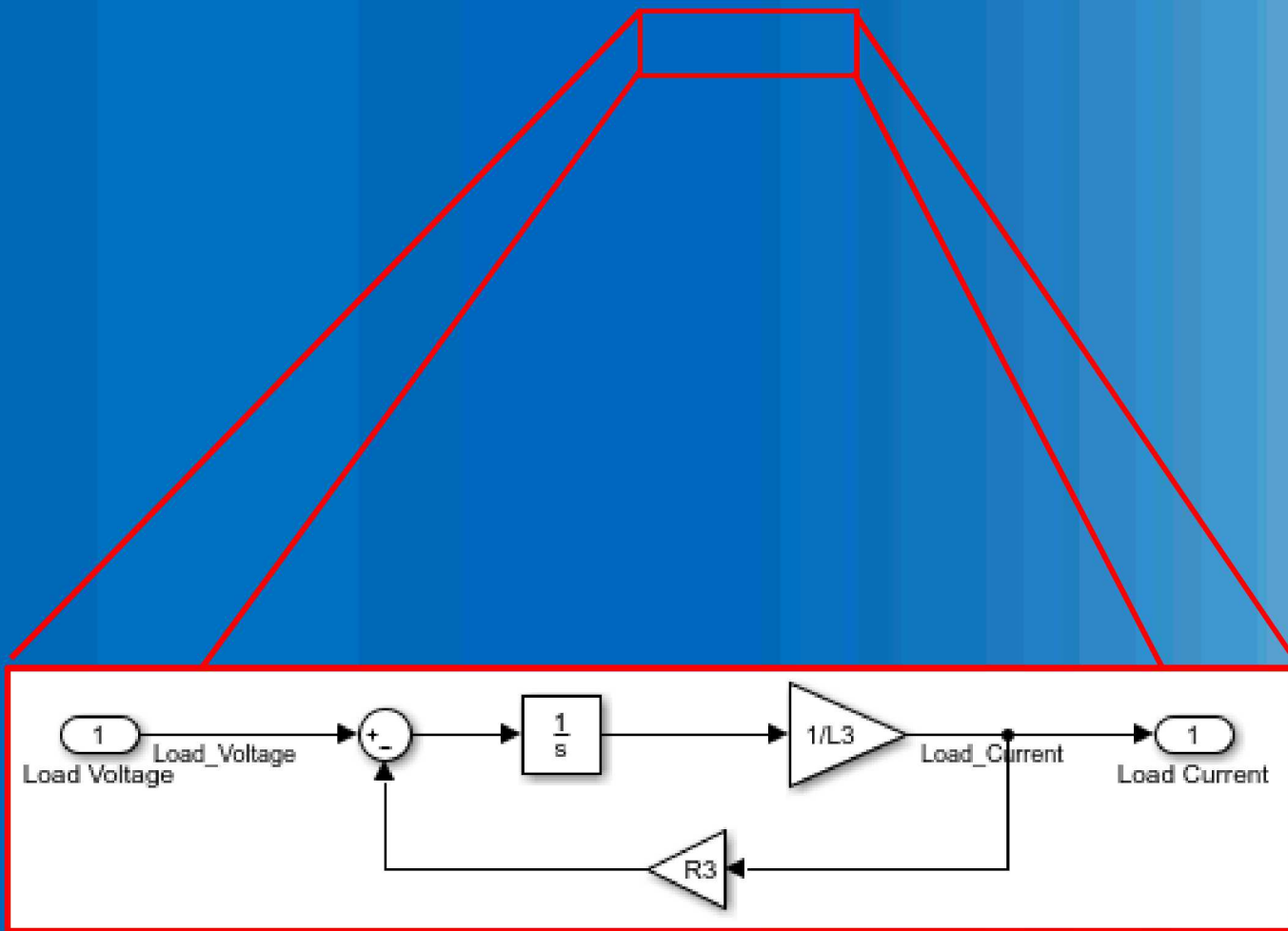
Electrical Modeling

LTSpice

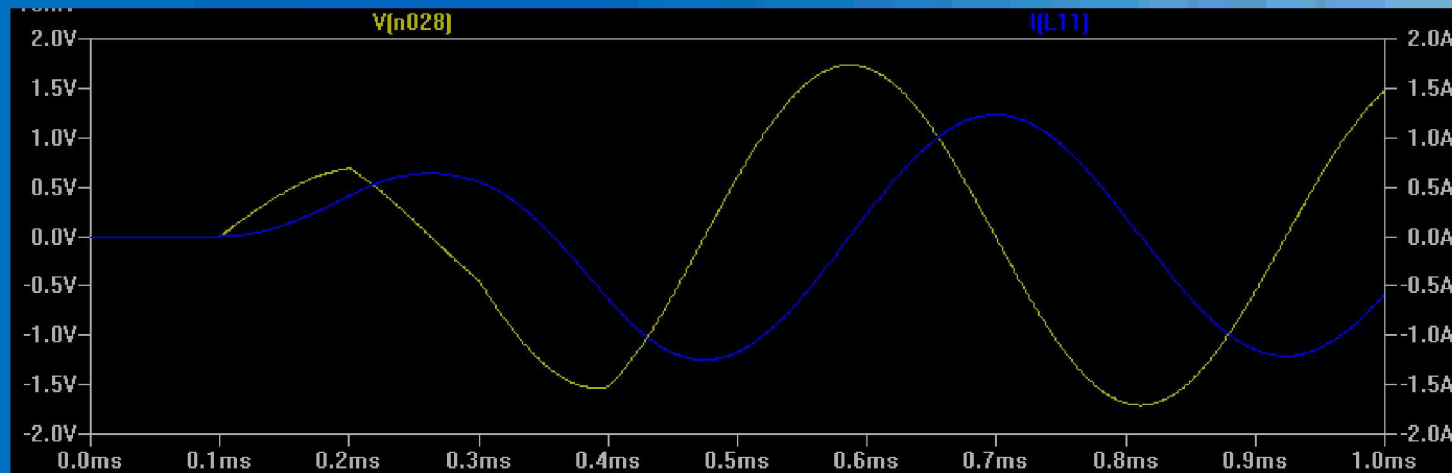


SIMULINK

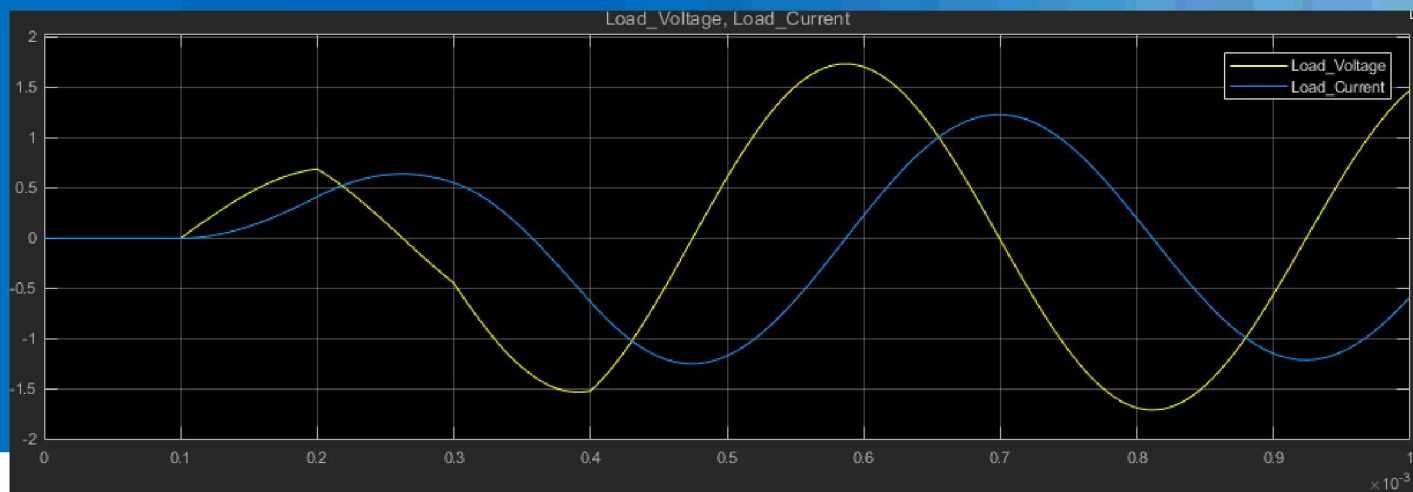




Electrical Modeling



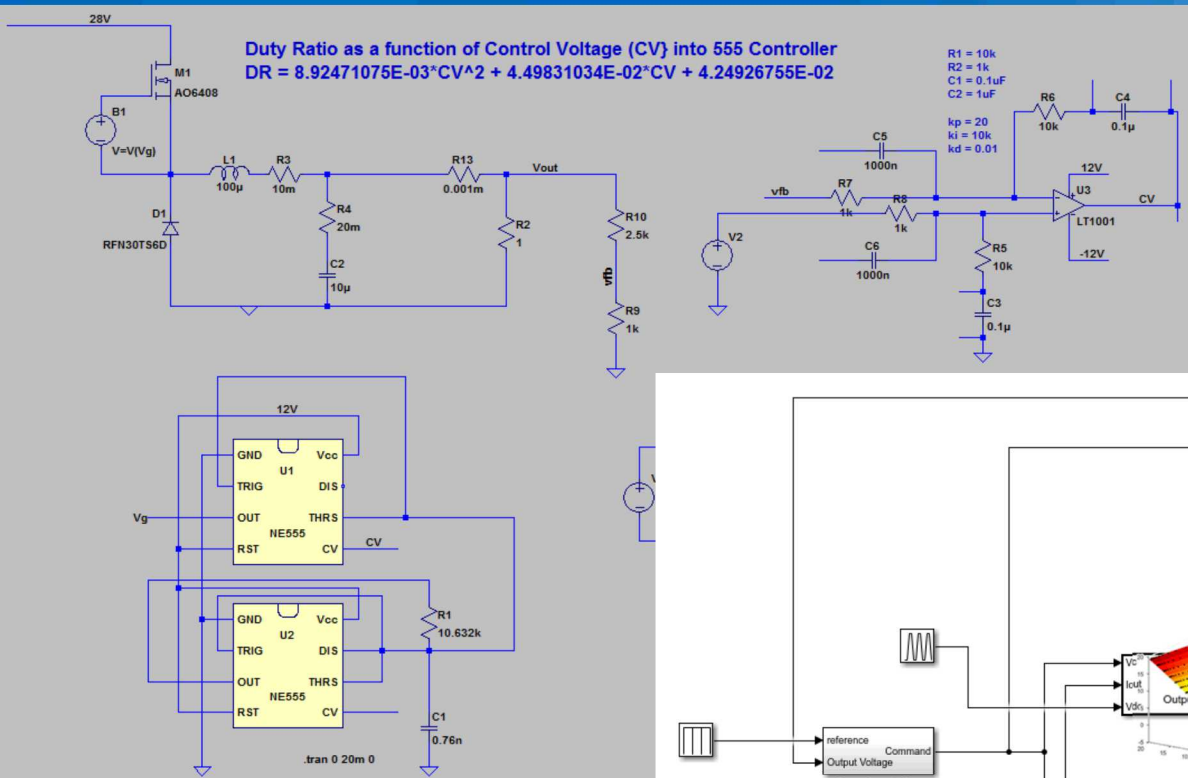
LTSpice



SIMULINK

Electrical Modeling – Power Converters

LTspice

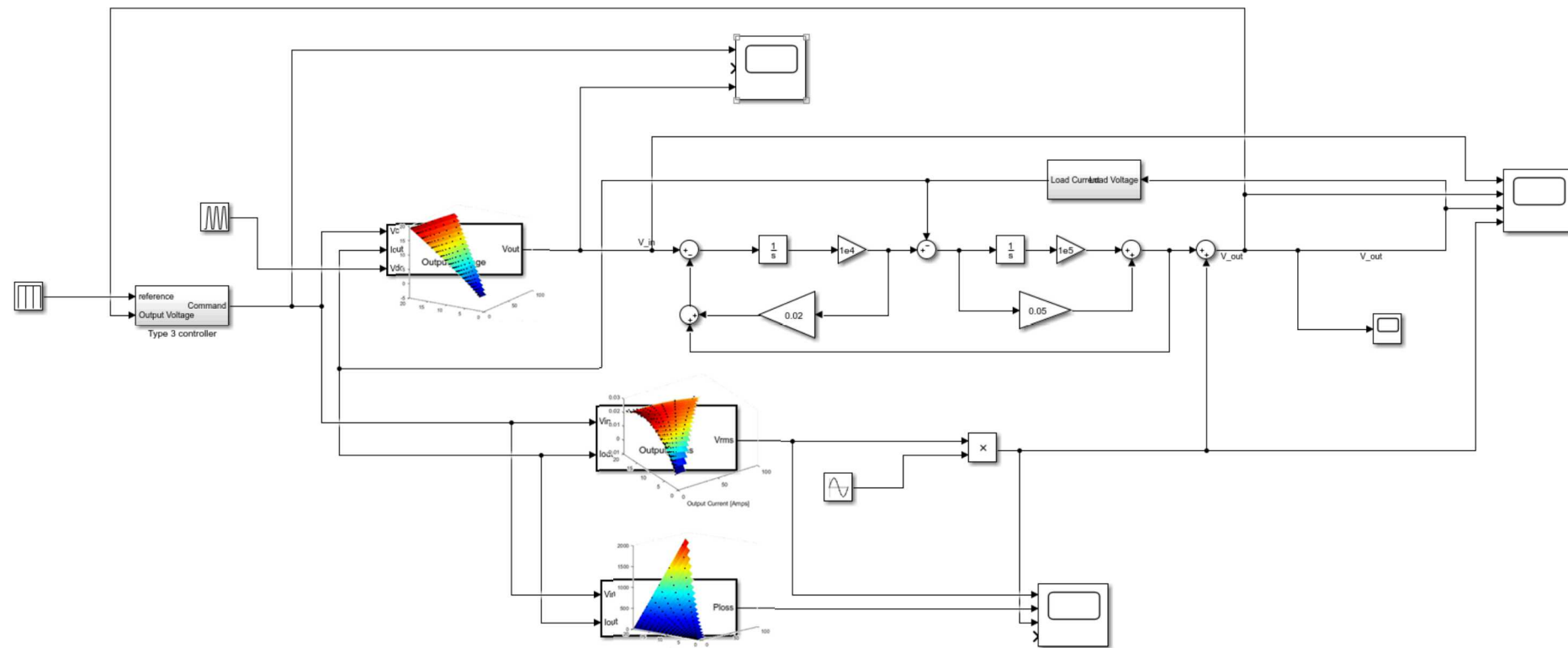


Output Voltage = f (Duty Ratio, Output Current)

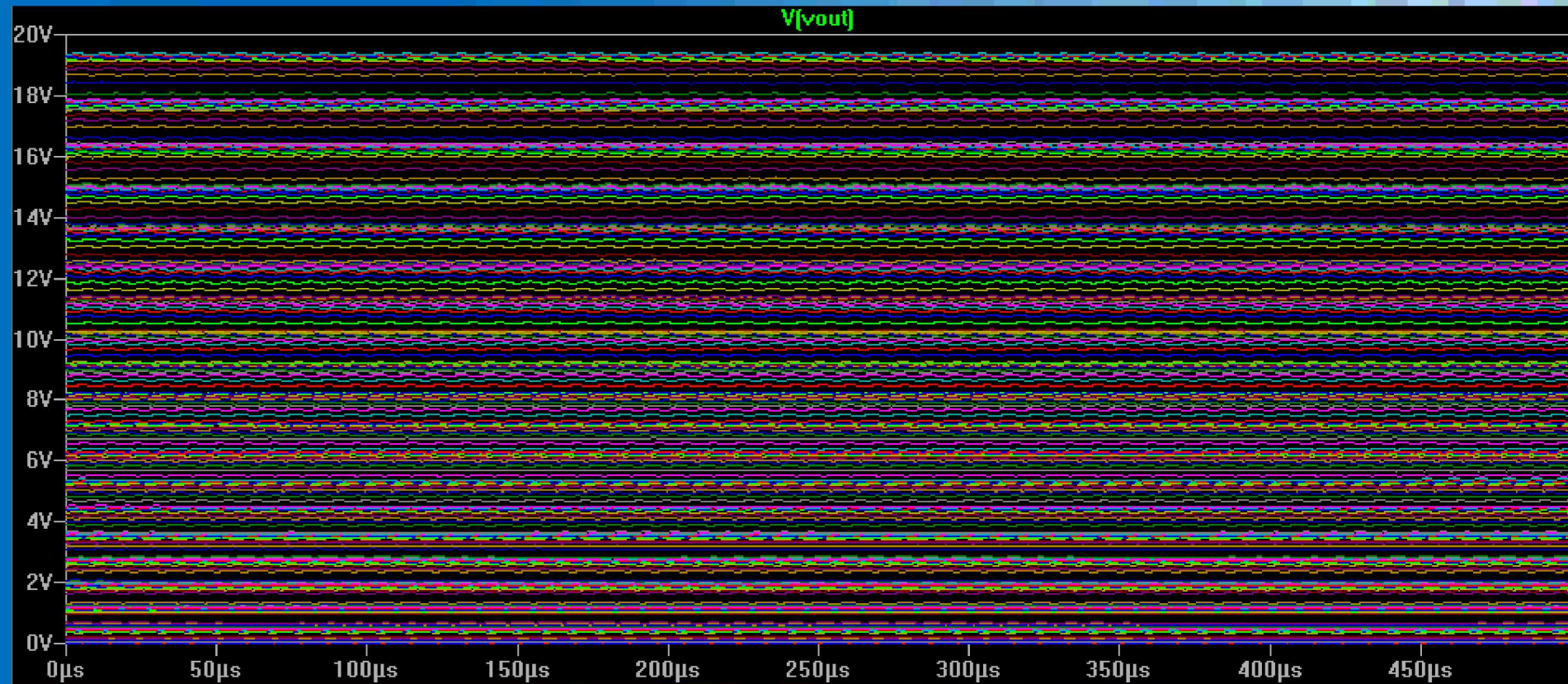
Output Voltage Ripple = f (Duty Ratio, Output Voltage)

Power Loss = f (Duty Ratio, Output Voltage)

SIMULINK

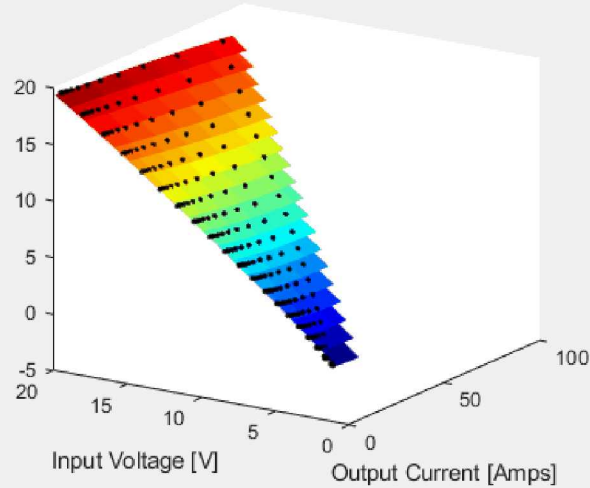


Sweeping Variables to extract Behavioral Parameters

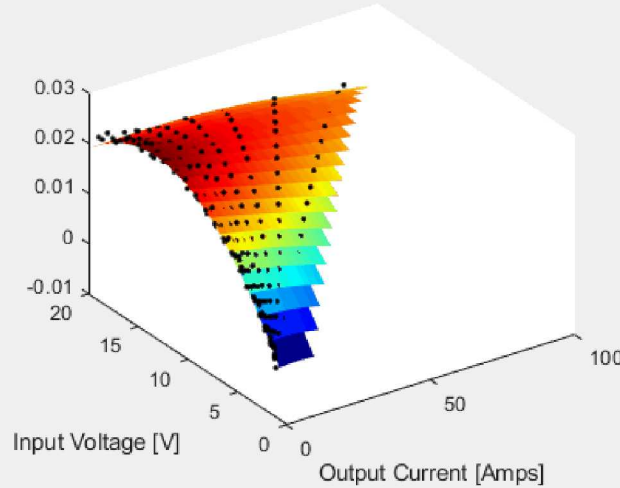


Modeling Parameters as Multi-Dimensional Polynomials

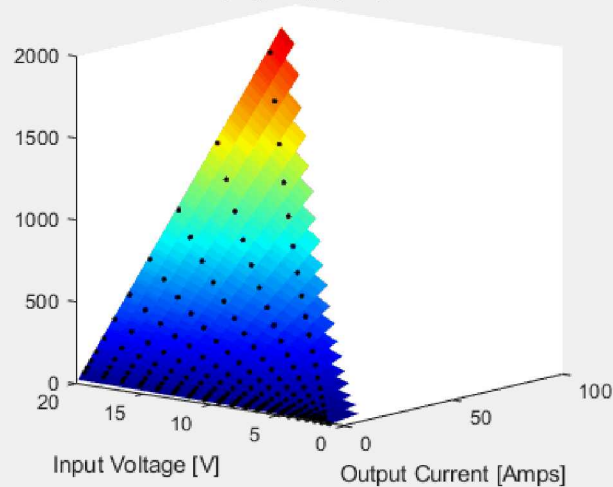
Output Voltage [V], Poly: [2x2], err = 1.9336



RMS Output Voltage [V], Poly: [2x2], err = 0.00016214



Power Loss [W], Poly: [1x1], err = 333.0655



$$\begin{bmatrix} 1.127\text{e-}06 & 2.411\text{e-}04 & -6.790\text{e-}03 \\ -7.693\text{e-}05 & -4.359\text{e-}03 & 1.162\text{e+}00 \\ 1.015\text{e-}03 & -2.069\text{e-}02 & -1.222\text{e+}00 \end{bmatrix} \begin{bmatrix} -3.219\text{e-}08 & 2.222\text{e-}06 & -1.667\text{e-}04 \\ 8.7346\text{e-}07 & -4.856\text{e-}05 & 4.504\text{e-}03 \\ -5.533\text{e-}06 & 1.205\text{e-}04 & -4.507\text{e-}03 \end{bmatrix}$$

$$\begin{bmatrix} 9.642\text{e-}01 & 4.549\text{e-}02 \\ 7.222\text{e-}01 & -9.500\text{e-}02 \end{bmatrix}$$

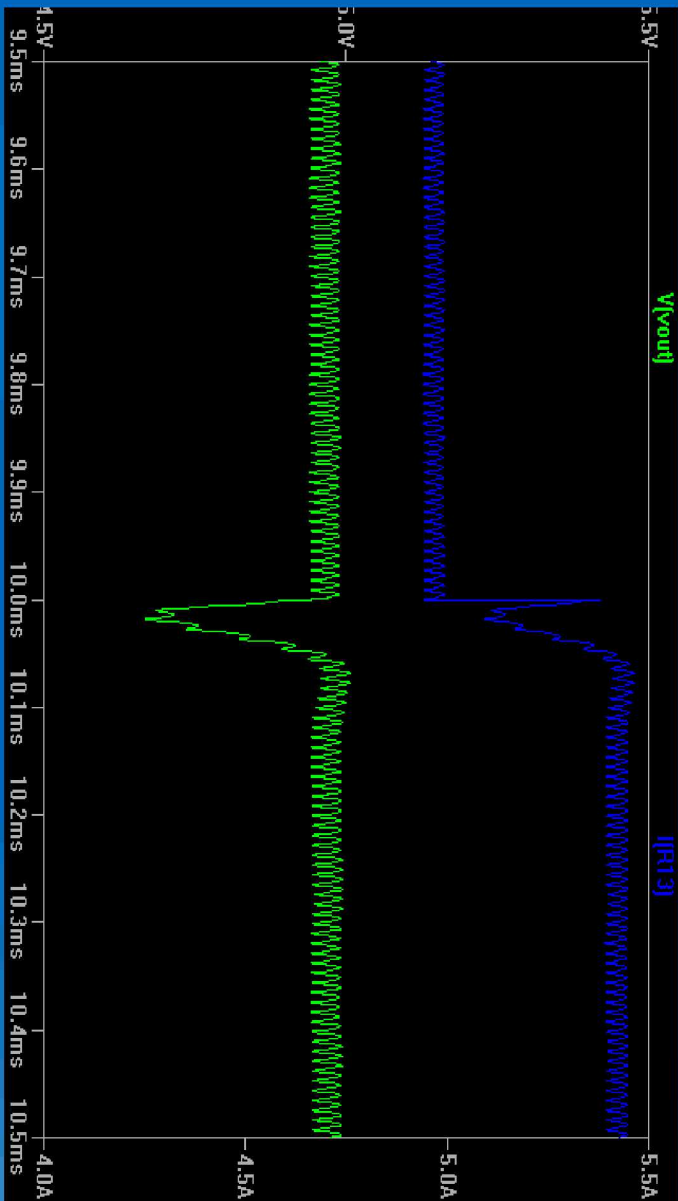
$$\begin{bmatrix} V_{in}^2 & V_{in} & 1 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} I_{out}^2 \\ I_{out} \\ 1 \end{bmatrix}$$

$$= A_{11} V_{in}^2 I_{out}^2 + A_{12} V_{in}^2 I_{out} + A_{13} V_{in}^2 + A_{21} V_{in} I_{out}^2 + A_{22} V_{in} I_{out} + A_{23} V_{in} + A_{31} I_{out}^2 + A_{32} I_{out} + A_{33}$$

Power Converter

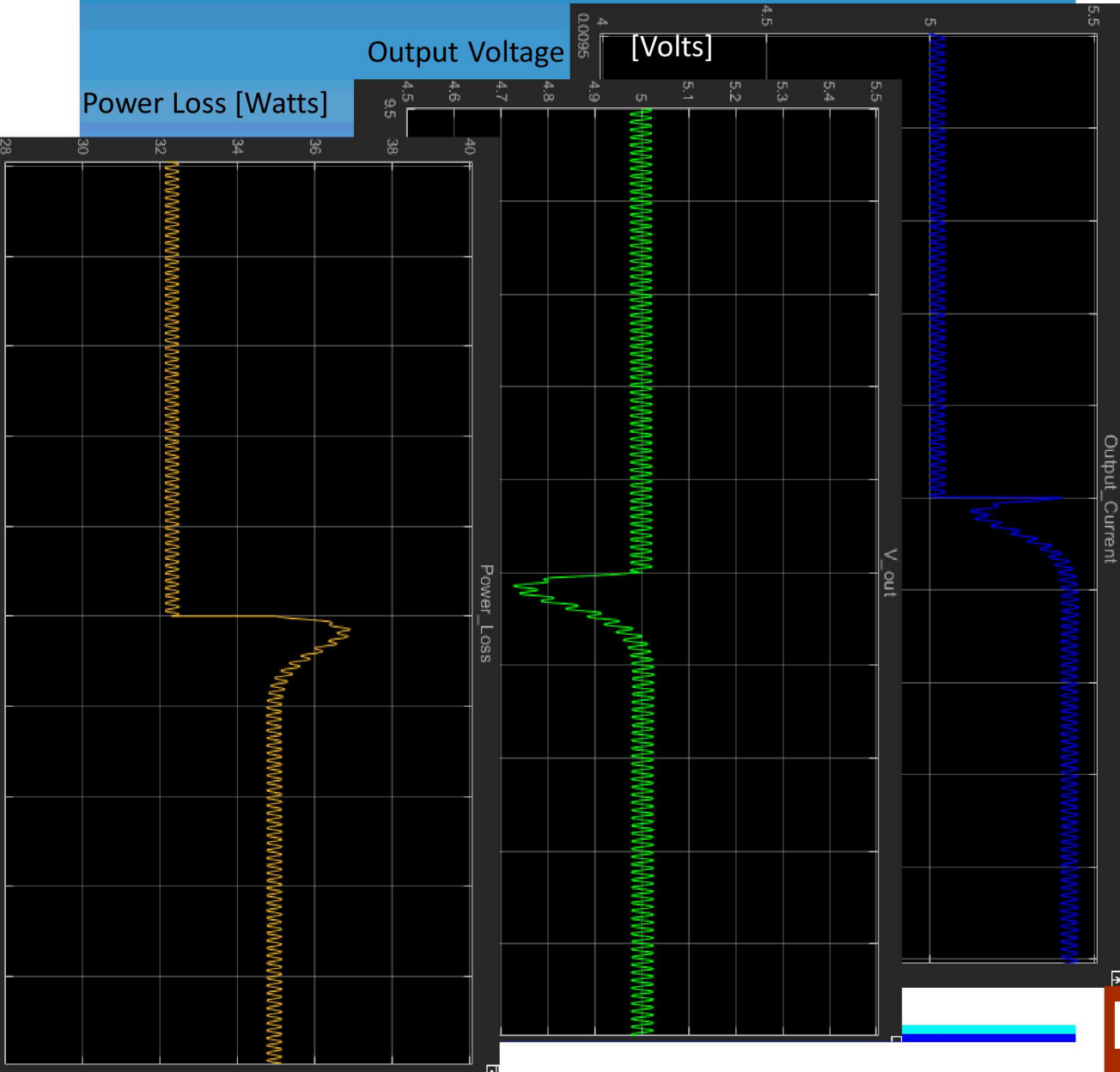
Modeling Results

LTspice



Output Current [Amps]

SIMULINK



Electrical Modeling - Charge Pump

Charge Pump

LTspice

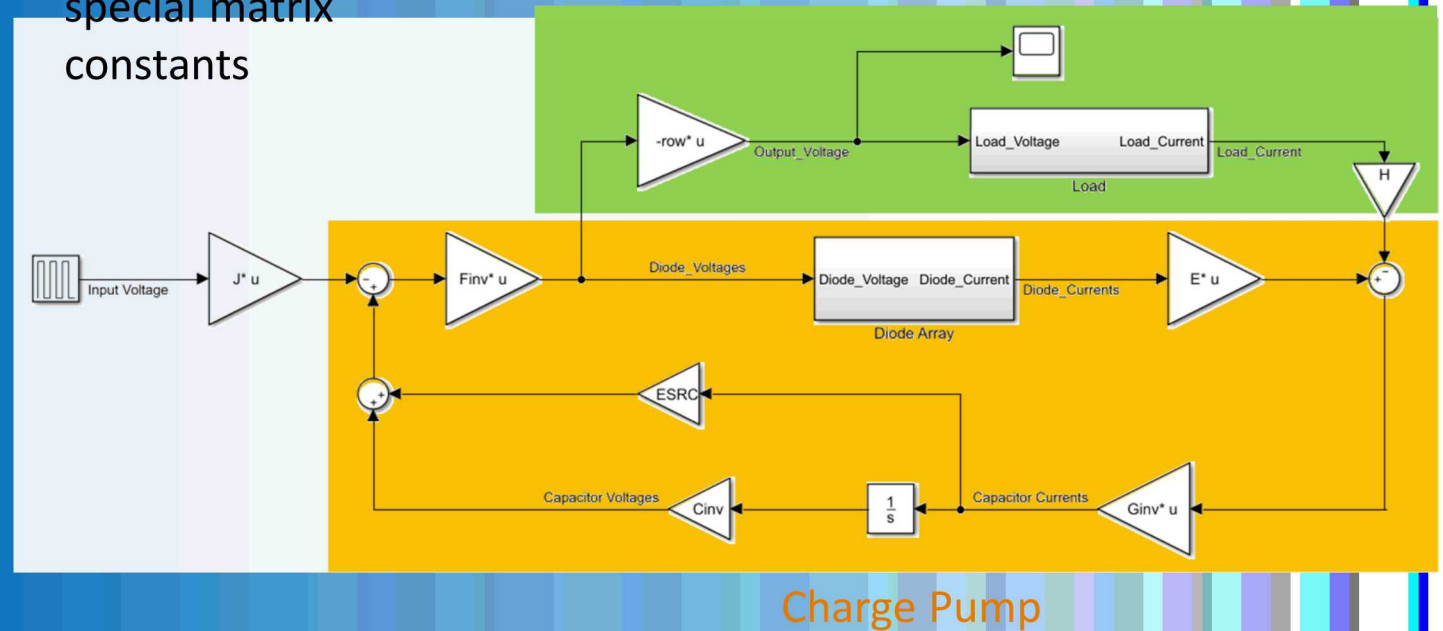
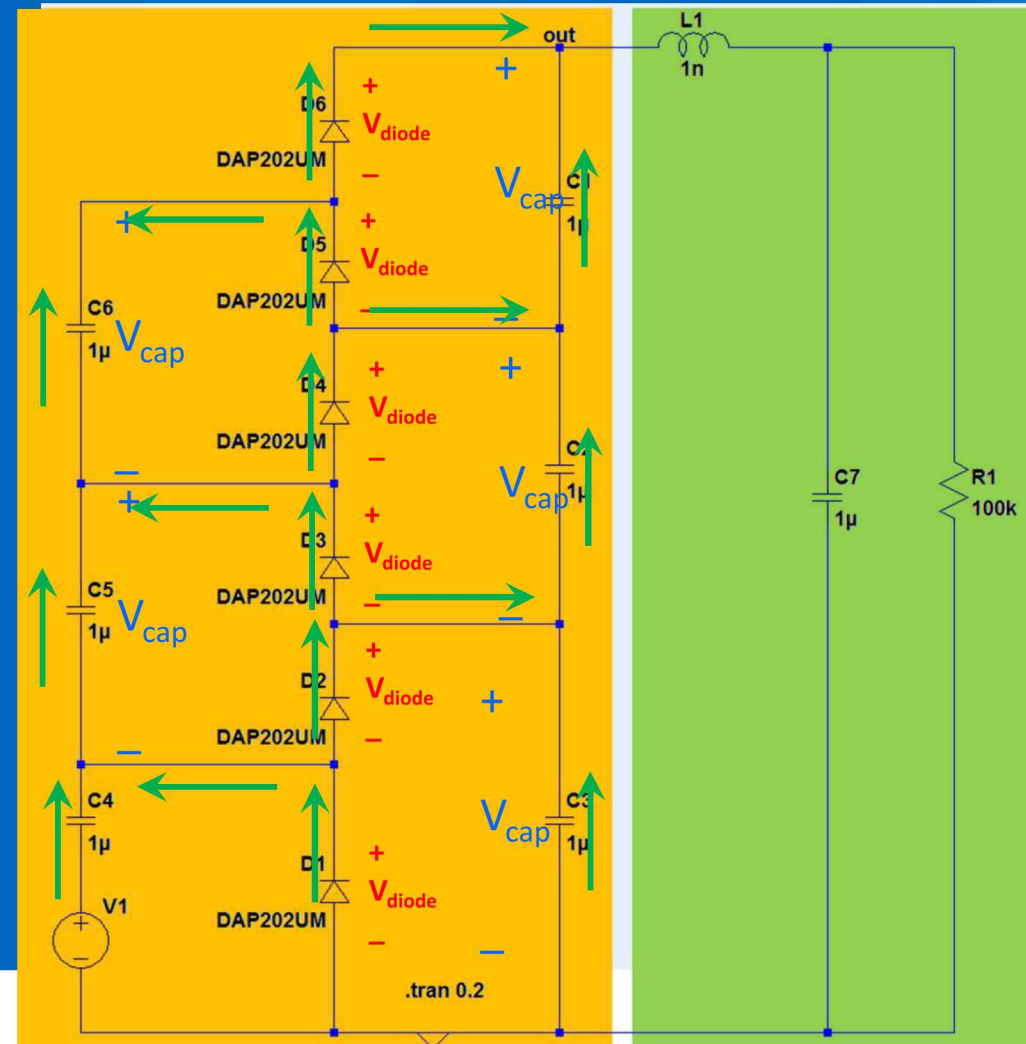
Load

Model Topologies

Topologies with repetitions use special matrix constants

SIMULINK

Load

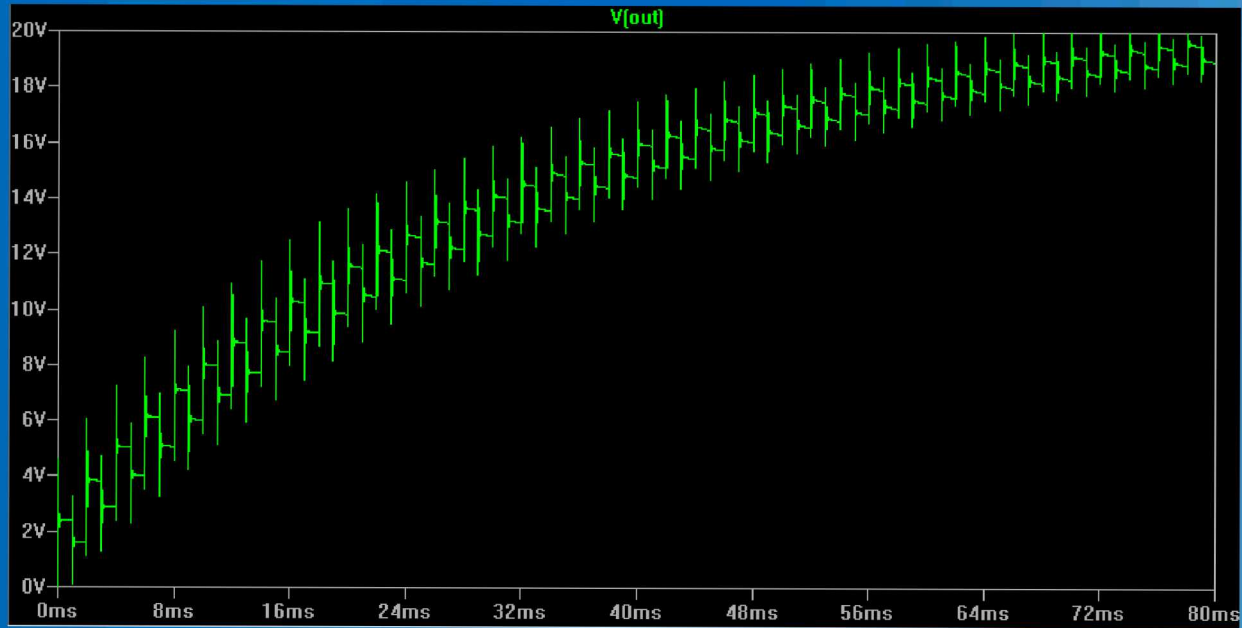


Charge Pump

Charge Pump Modeling Results

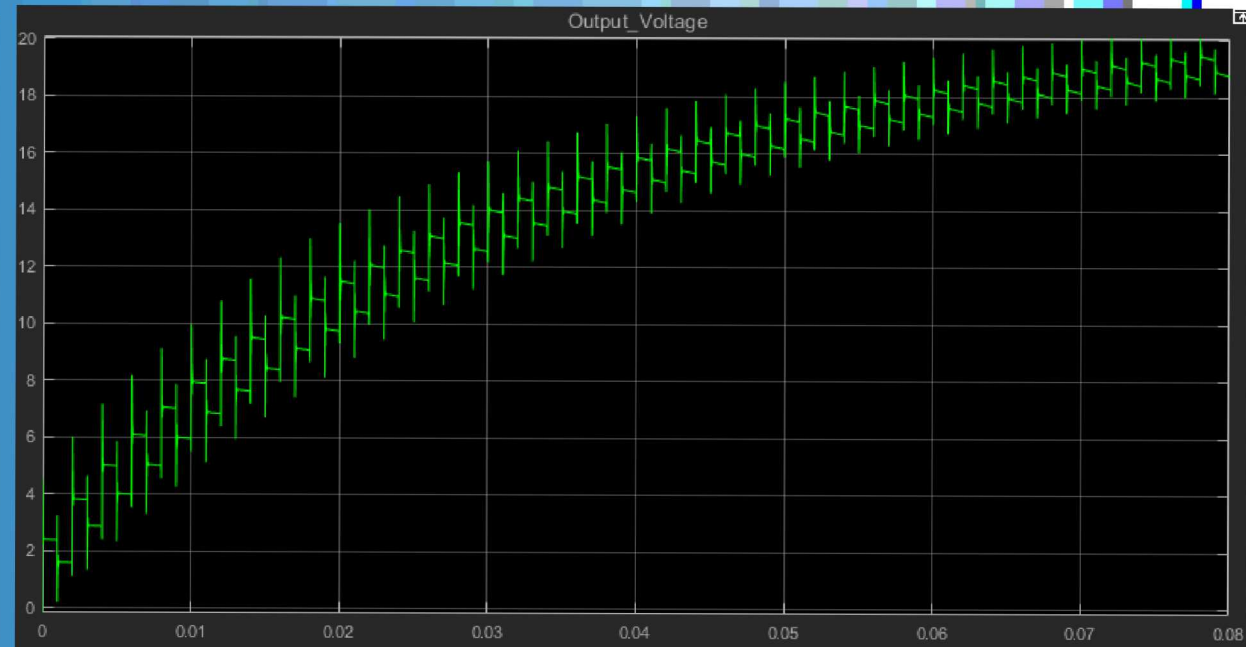


LTspice

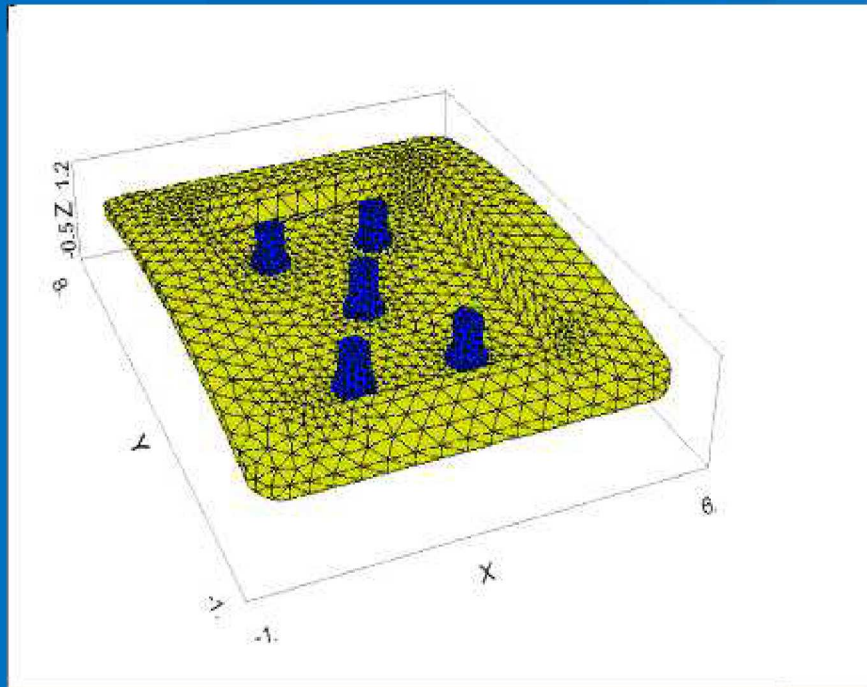


Output Voltage

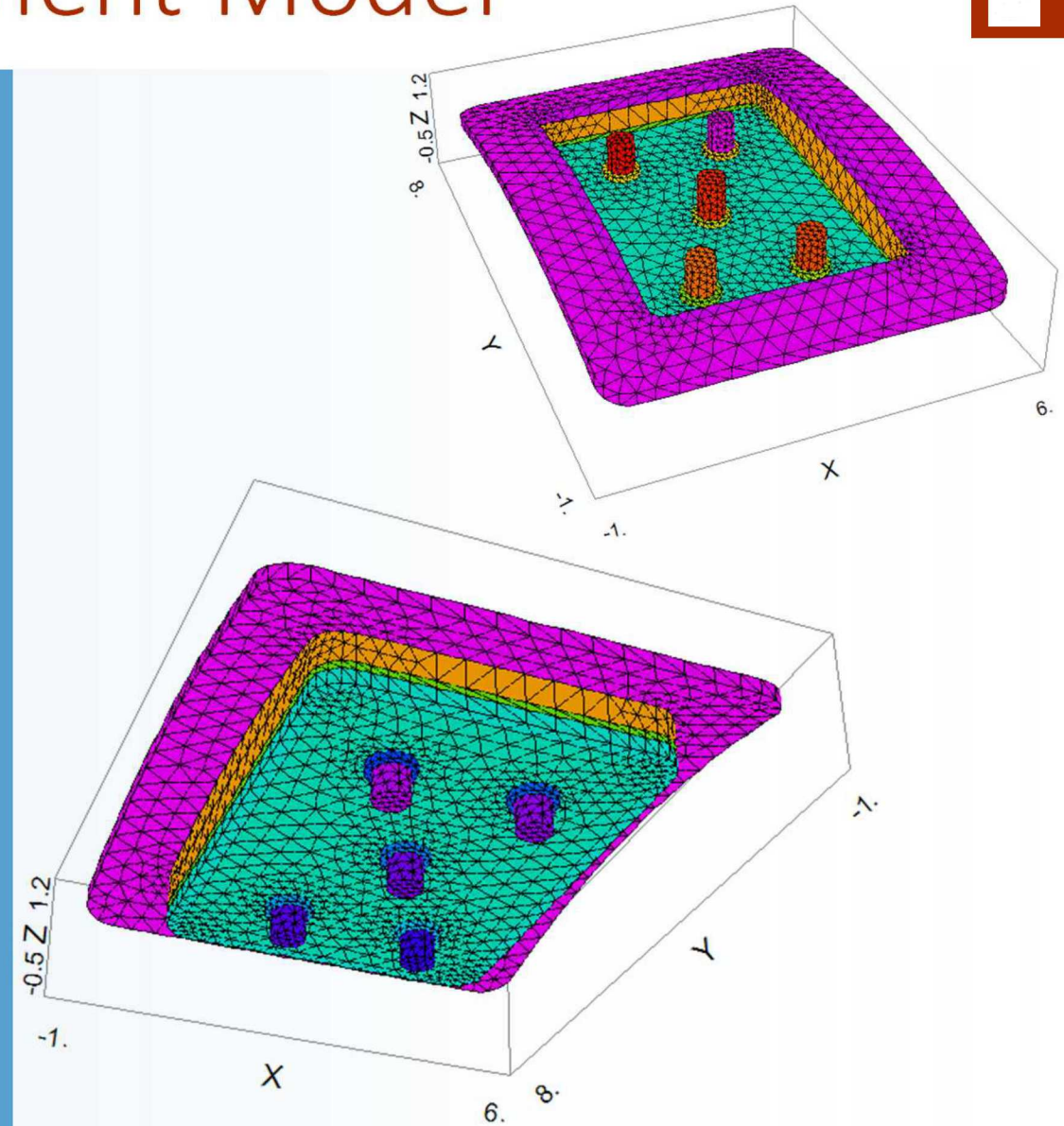
SIMULINK



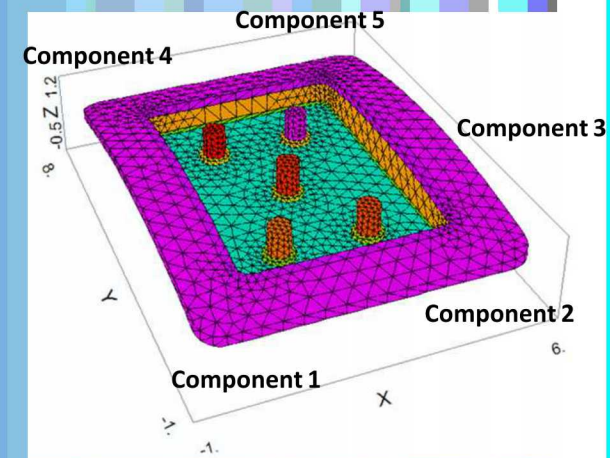
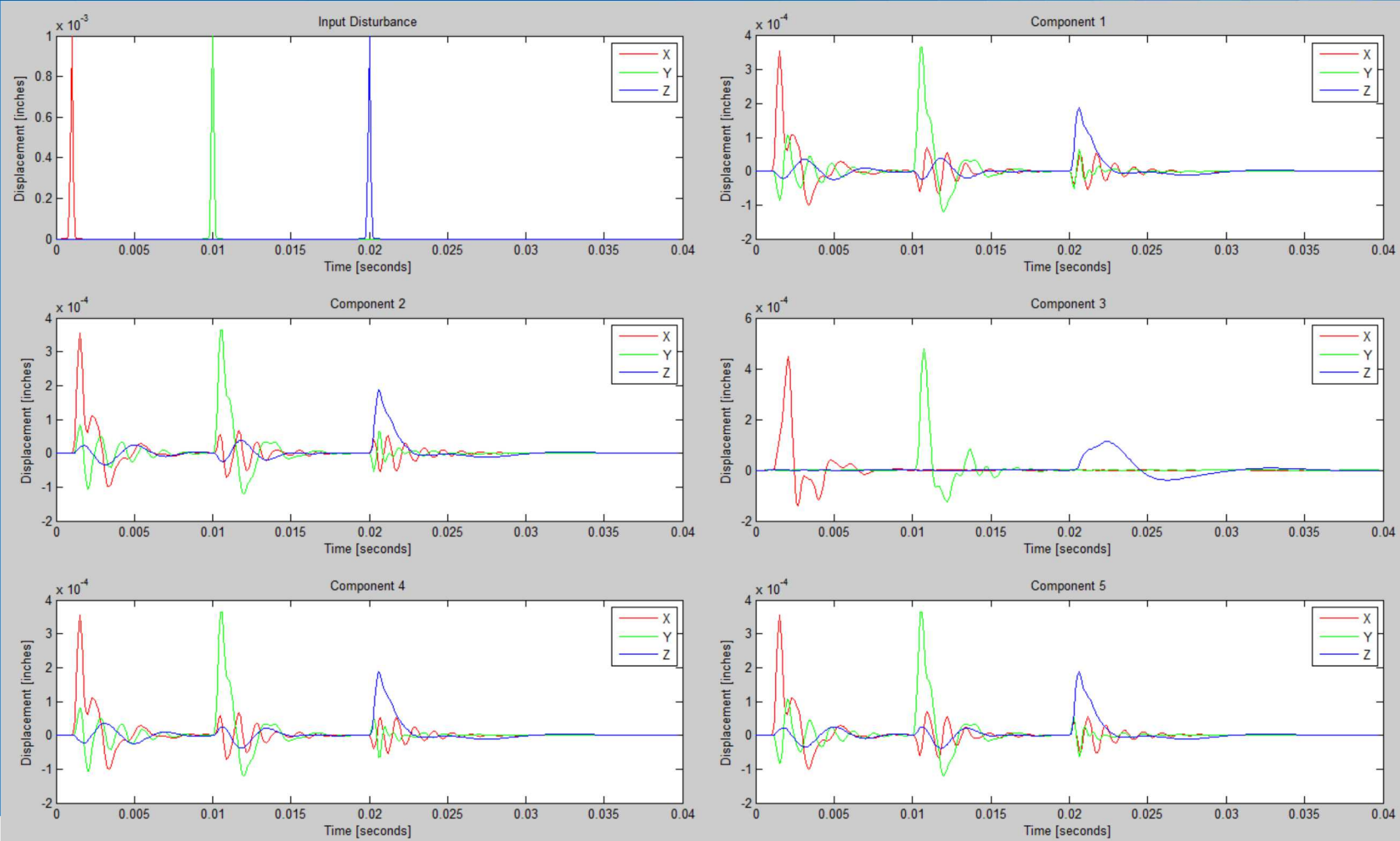
Physics Model - Finite Element Model



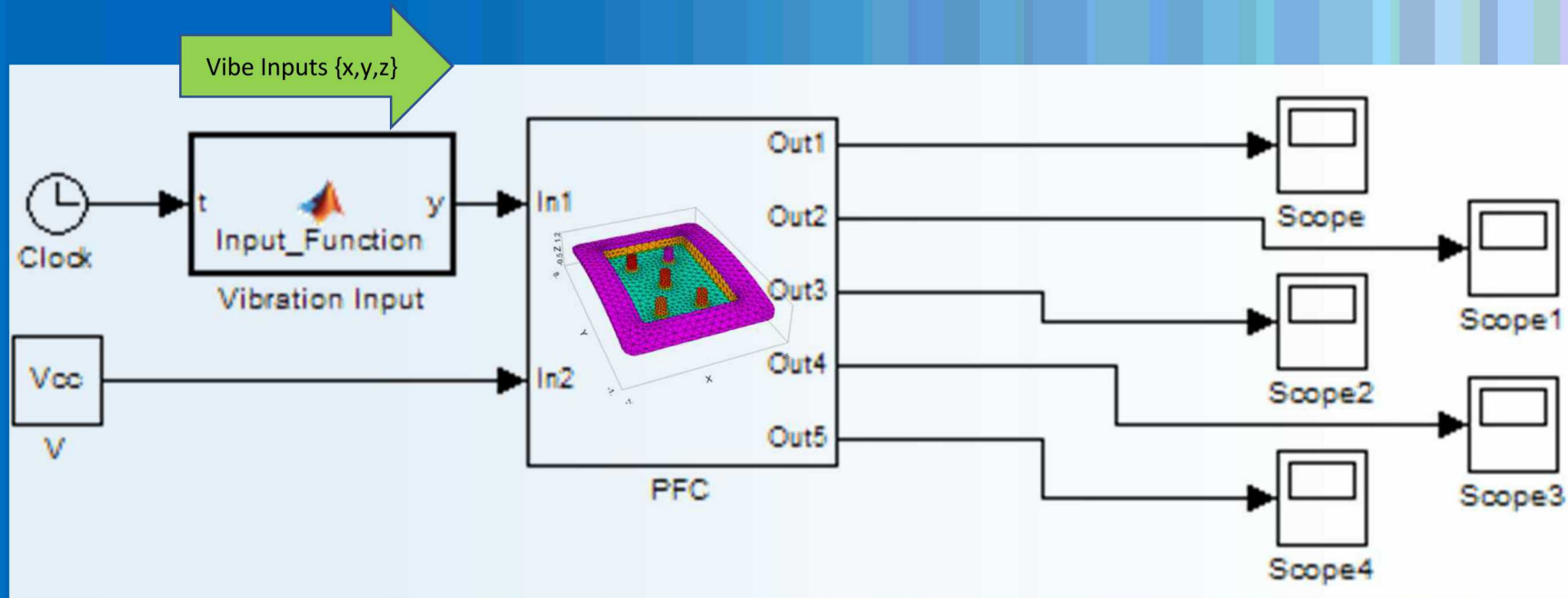
[Click on Video]



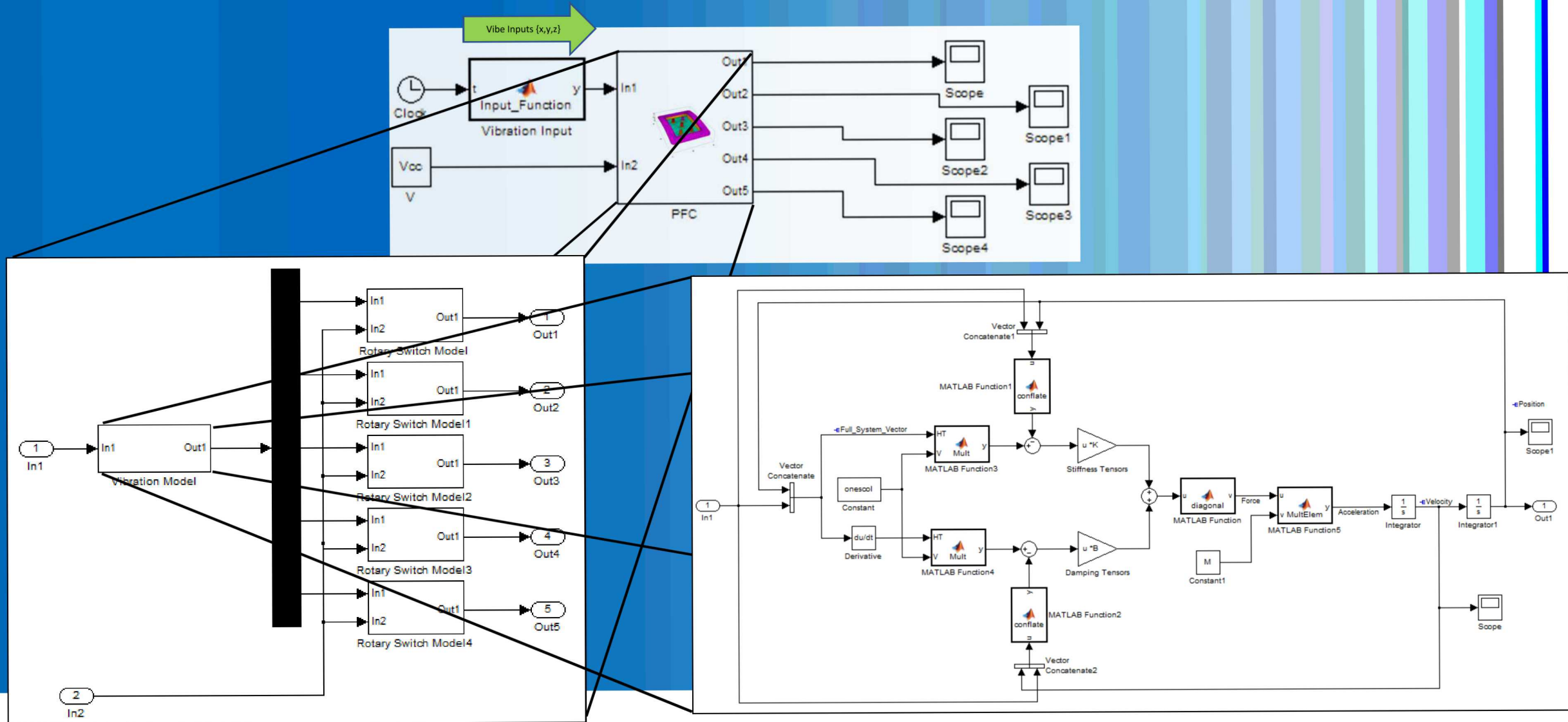
PFC Model – Shock/Vibe



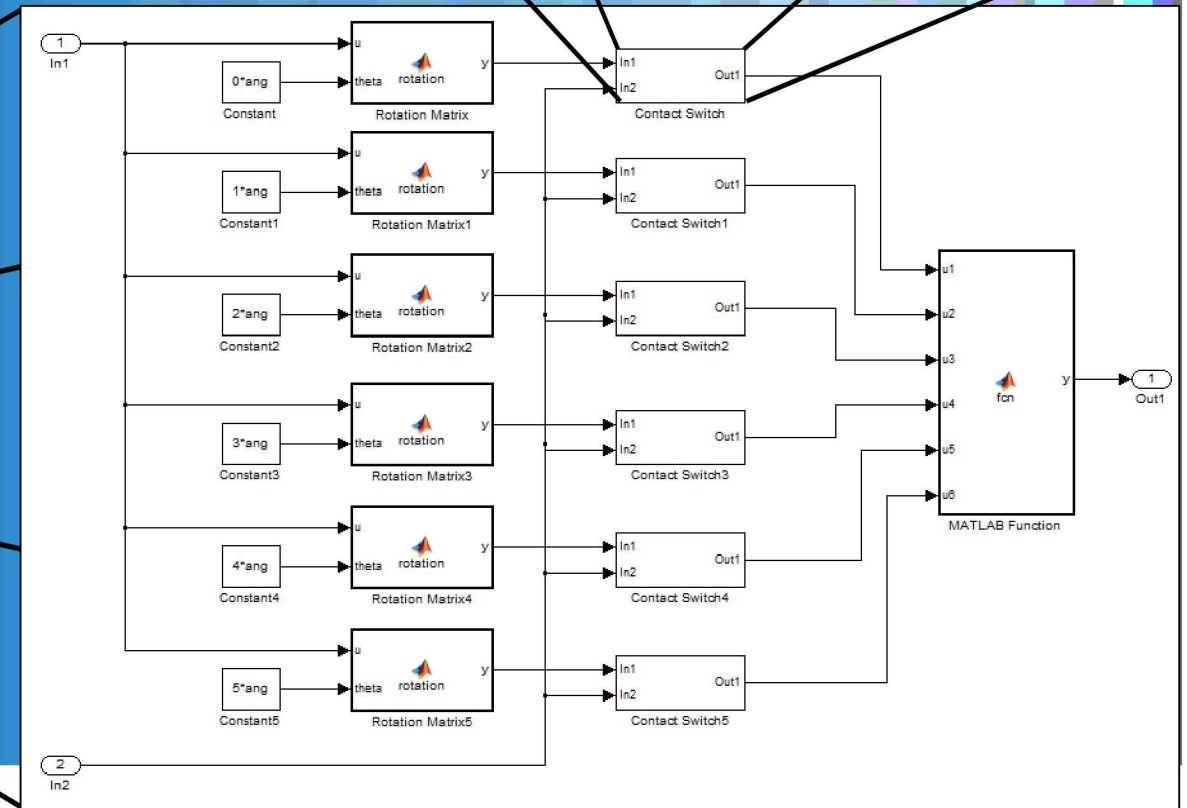
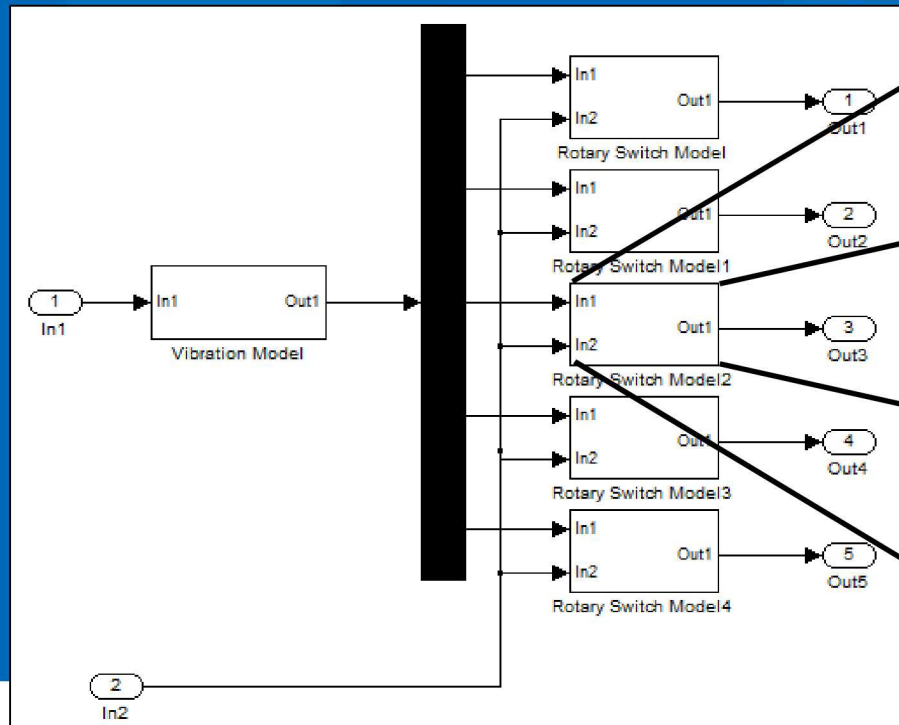
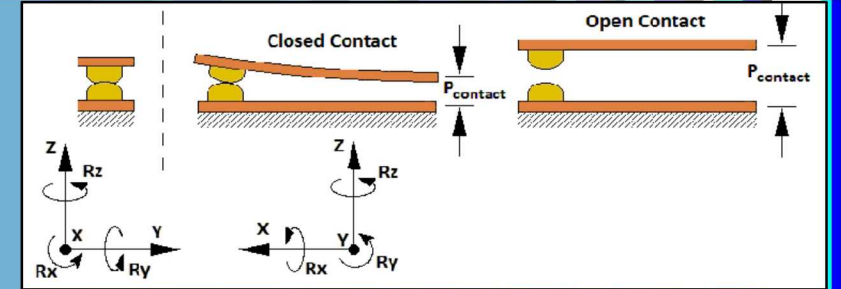
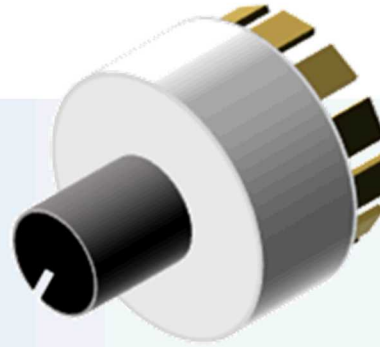
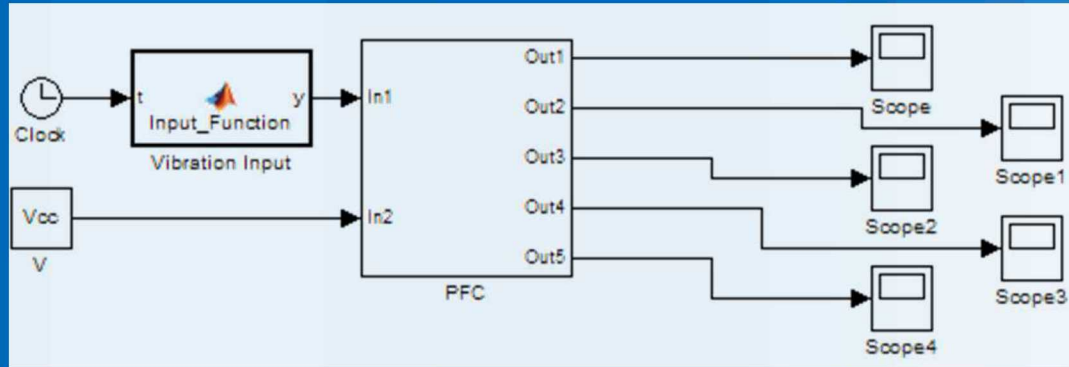
Top Level Simulink Model for Reduced Order



Simulink Model Reduced Order Shock/Vibration



PFC - Simulink Model





Acceleration at each connector on component 1
Each component has six contacts at 60° orientation

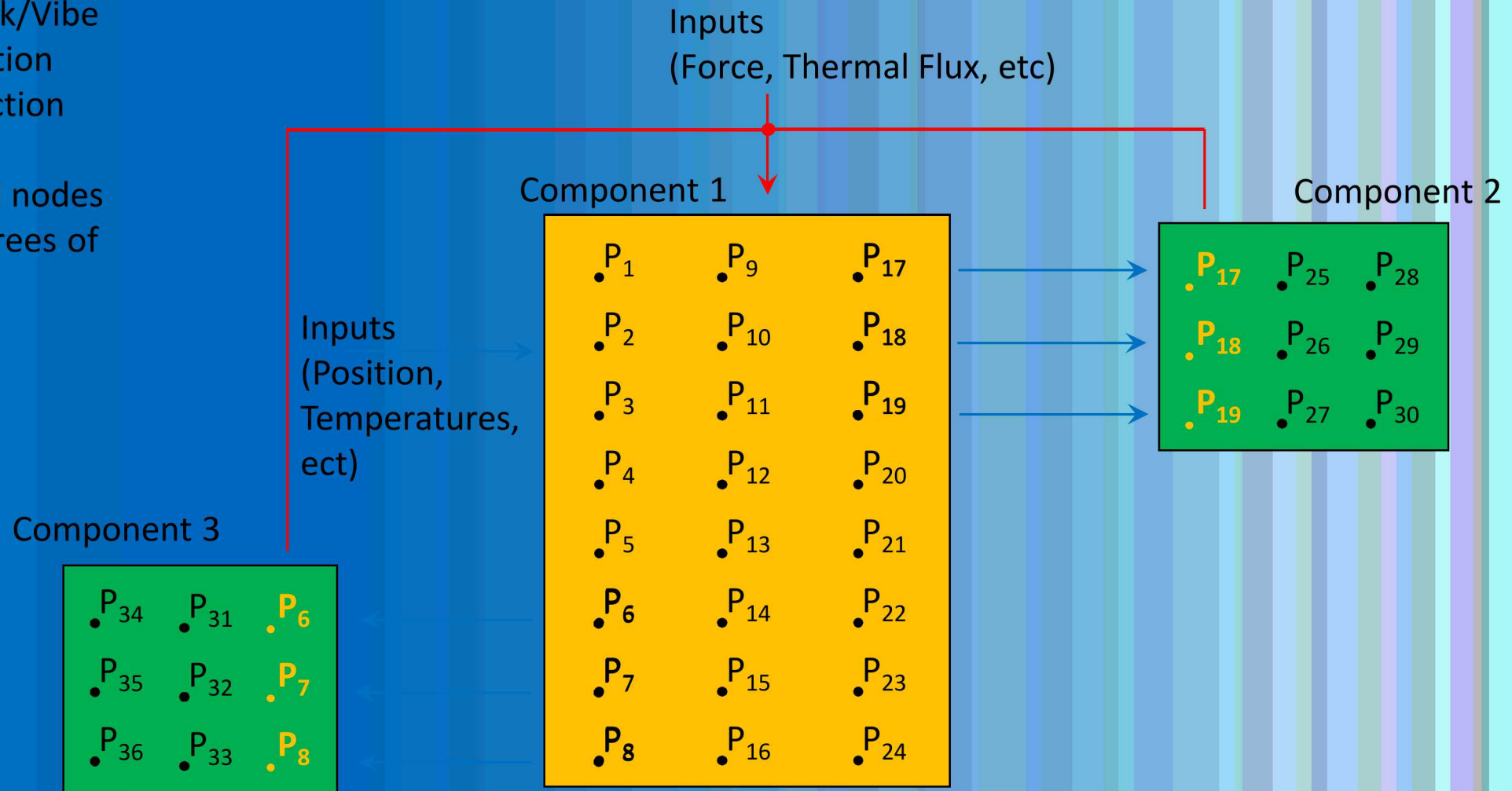


Connecting Linear Continuous Time Models

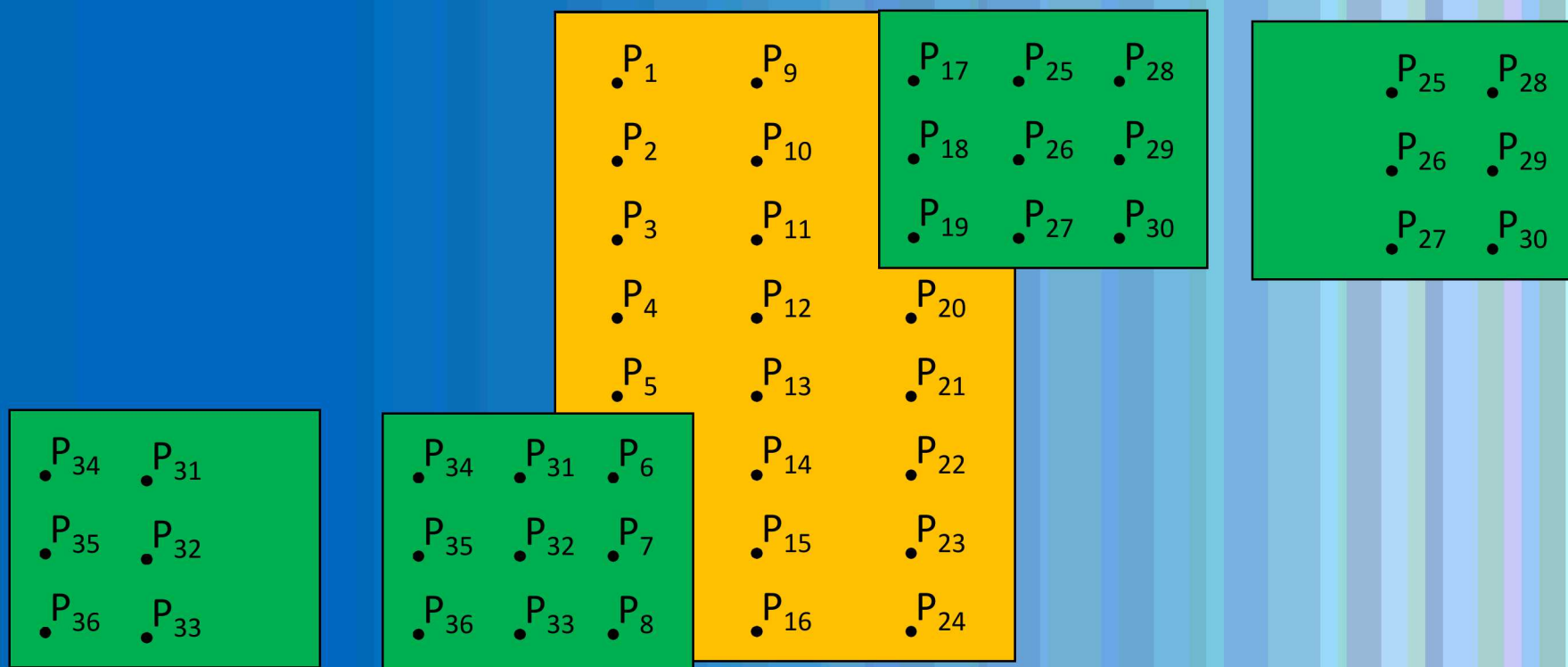
Mechanical Shock/Vibe
Thermal Conduction
Electrical Conduction

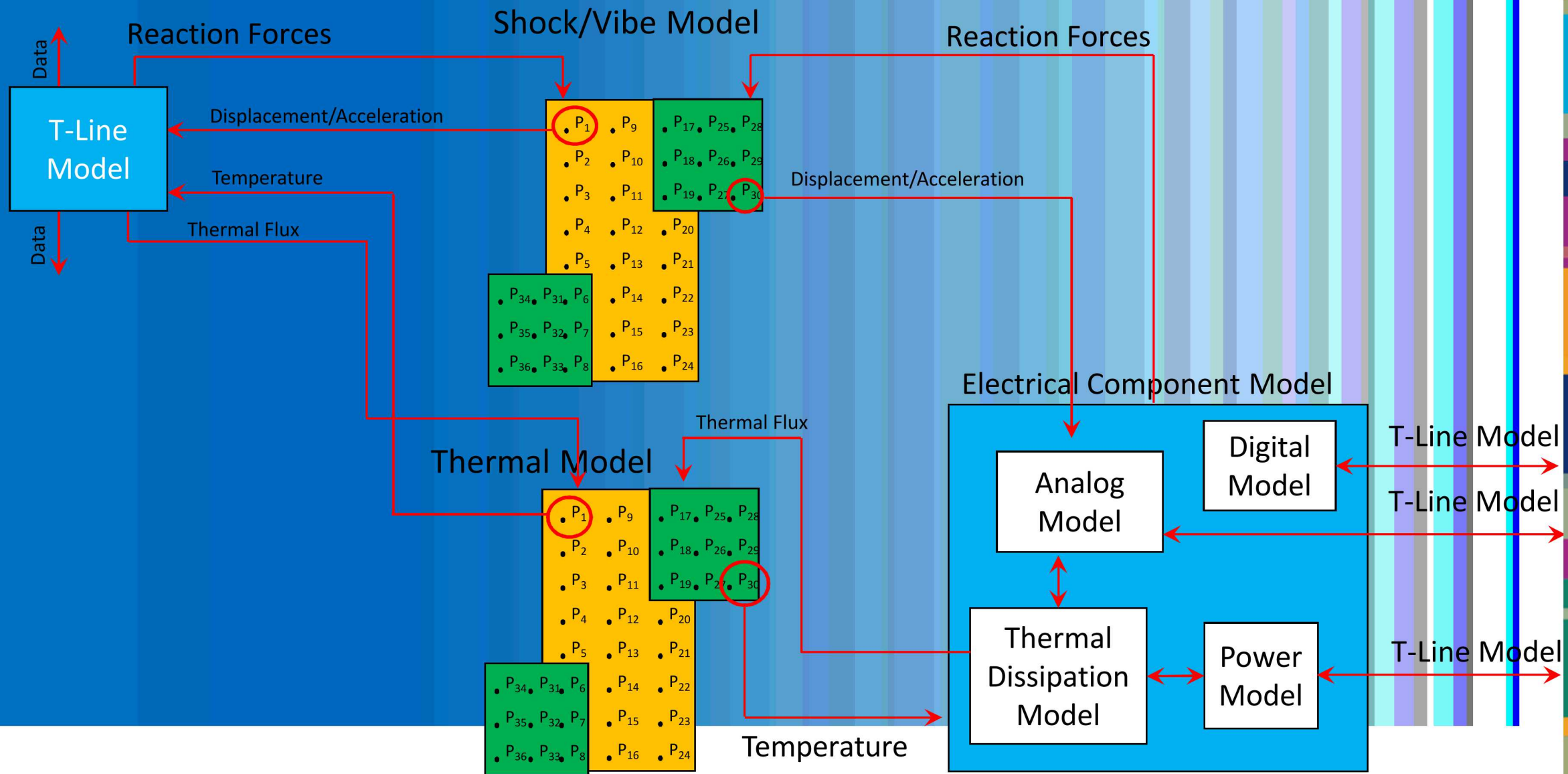
Points are spatial nodes
with physics degrees of
freedom:

- Temperature
- Displacement
- Voltage



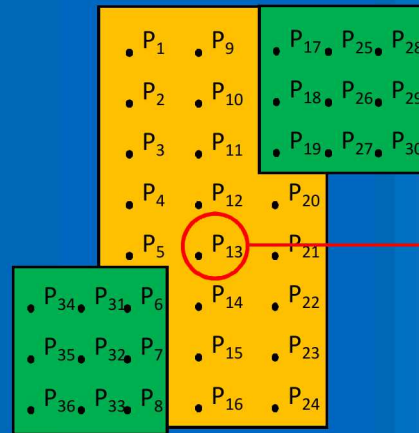
Connecting Linear Continuous Time Models



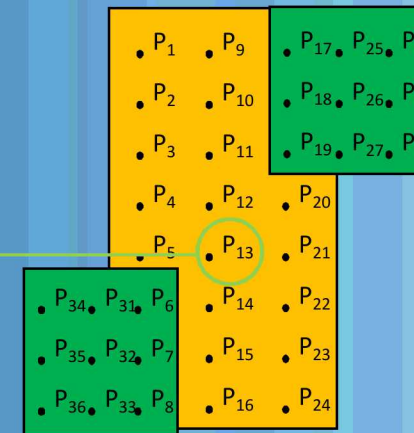


Modeling Interconnects in System Model

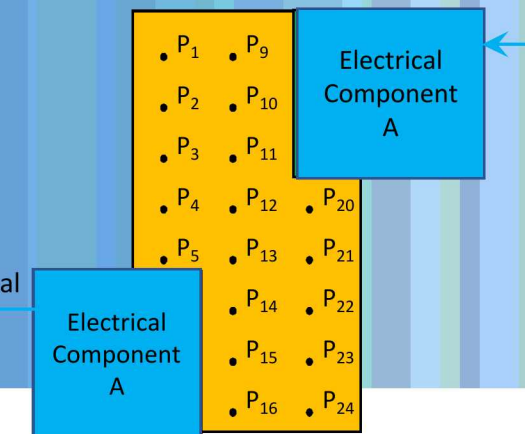
Thermal Model



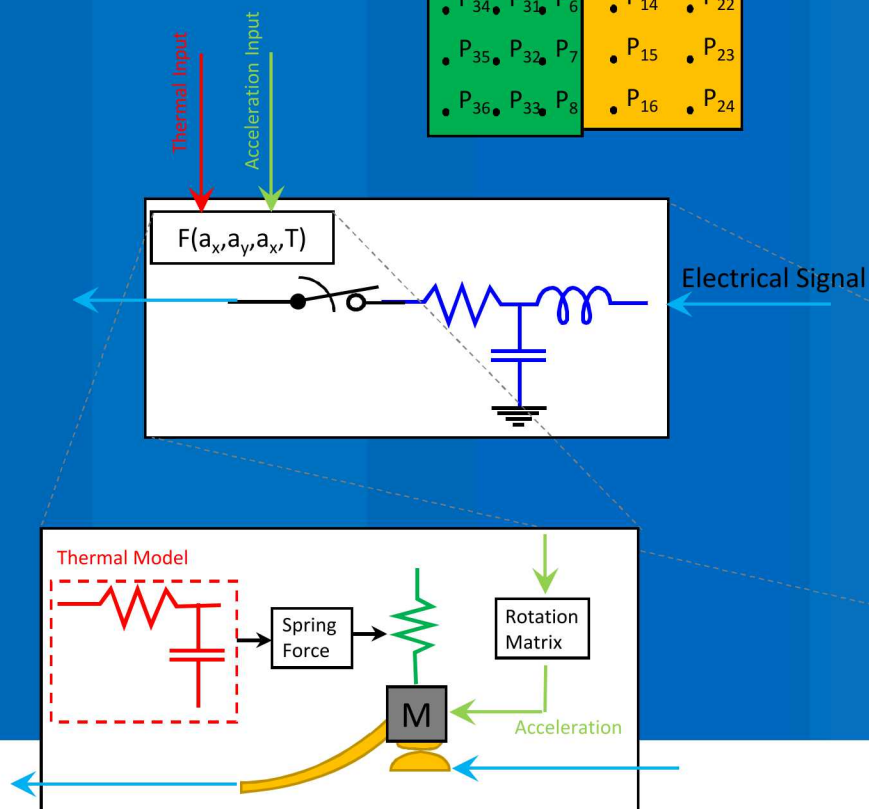
Shock/Vibe Model



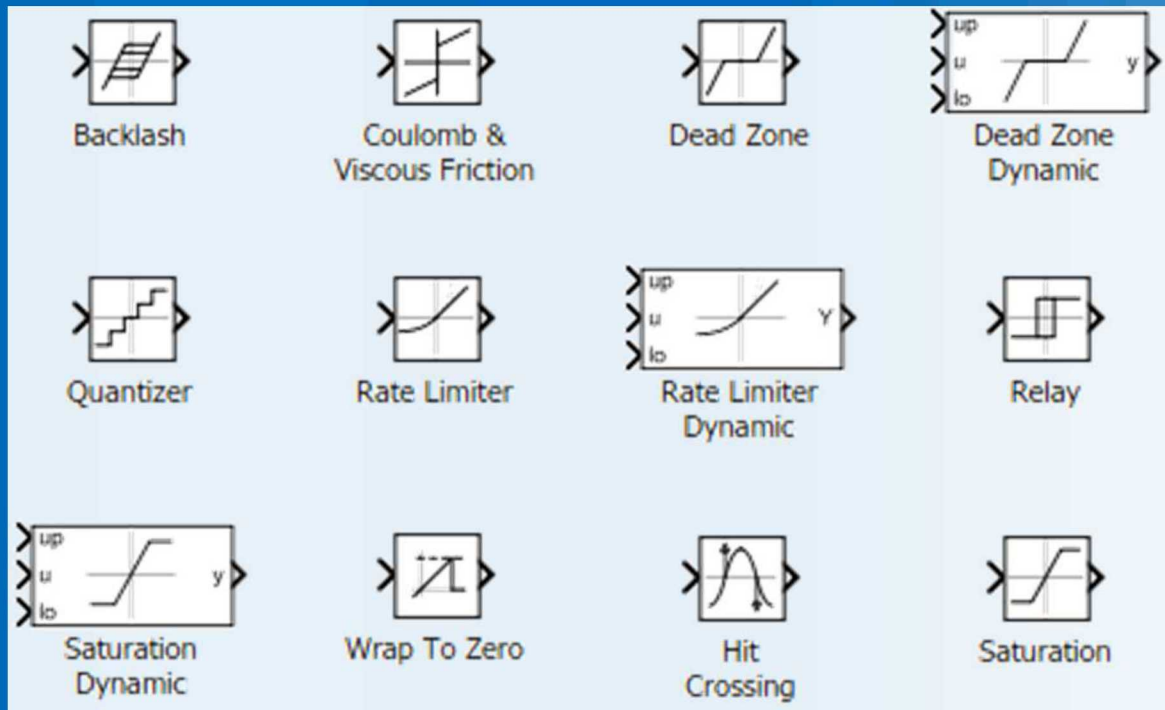
Electrical Model



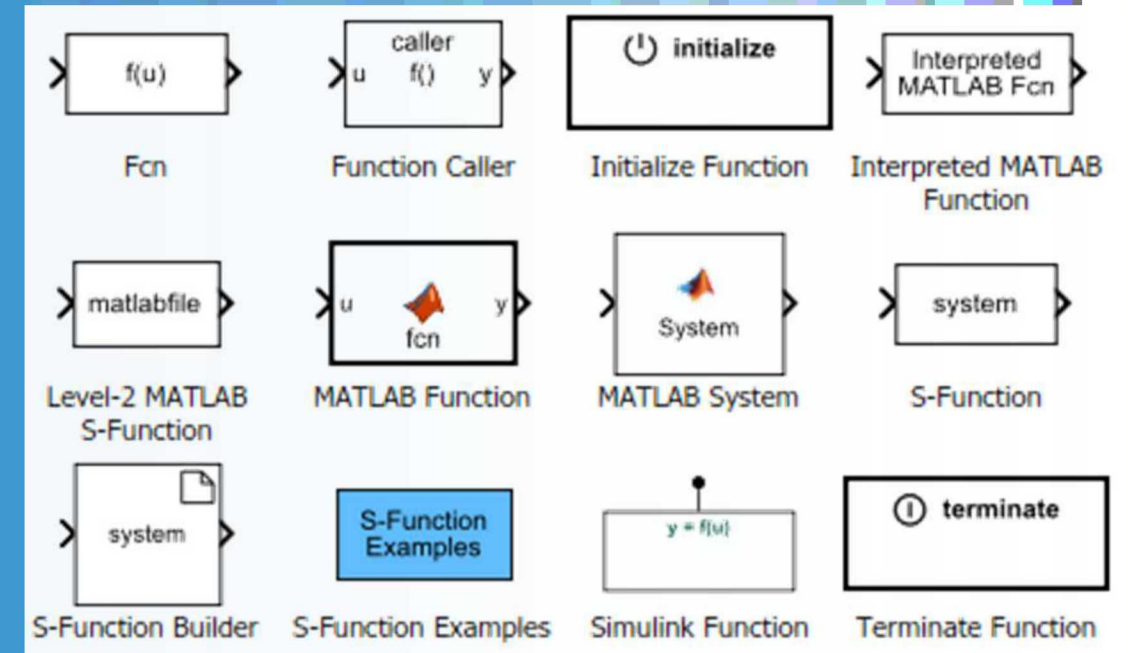
Interconnect Model



Modeling Nonlinearities



Can be added to existing linear models

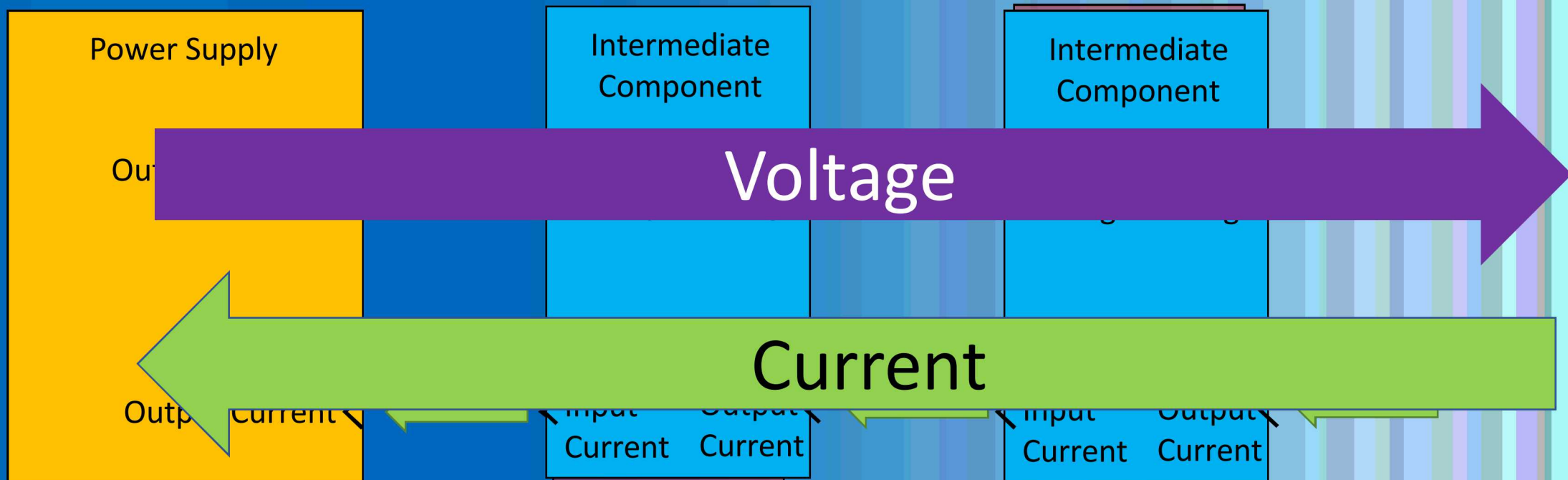


User-Defined, Custom models can be made



- Analog
- Digital
- Power
 - Electrical - Thermal

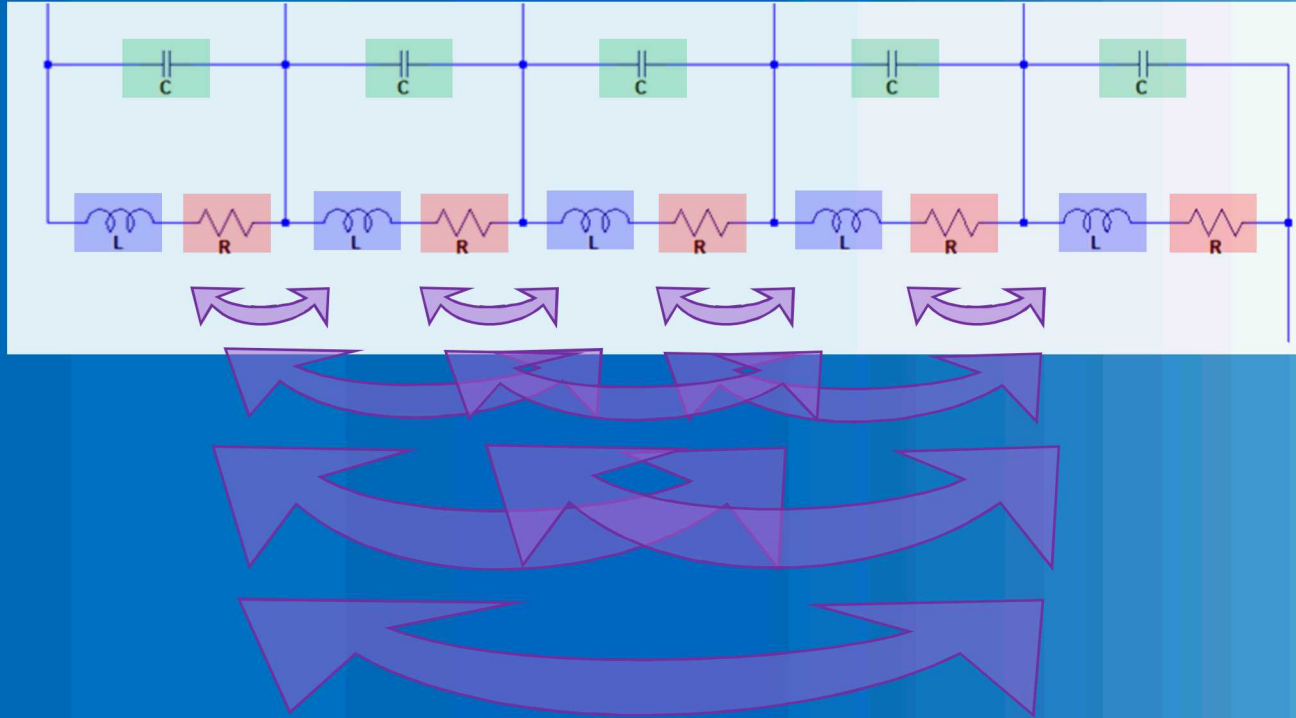
Electrical Modeling



Capacitive Networks



Inductive Networks



$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{bmatrix}$$

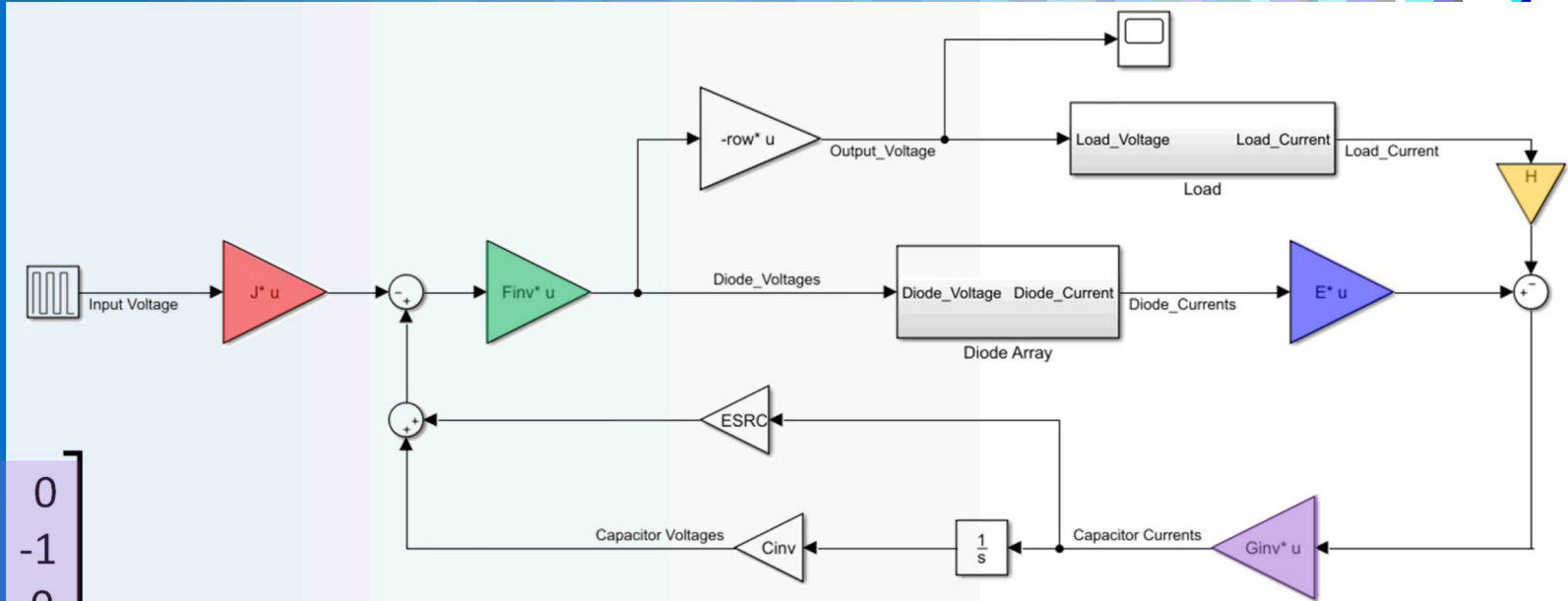
$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}$$

$$\begin{bmatrix} -C_1 & 0 & 0 & 0 & 0 \\ C_1 & -C_2 & 0 & 0 & 0 \\ 0 & C_2 & -C_3 & 0 & 0 \\ 0 & 0 & C_3 & -C_4 & 0 \\ 0 & 0 & 0 & C_4 & -C_5 \end{bmatrix}$$

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix}$$

Topologically Derived Charge Pump Constants

$$\begin{aligned}
 E &= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} & J &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & H &= \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \\
 F &= \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} & F^{-1} &= \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix} \\
 G &= \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} & G^{-1} &= \begin{bmatrix} -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}
 \end{aligned}$$



Electrical Modeling – Digital FPA Model Example

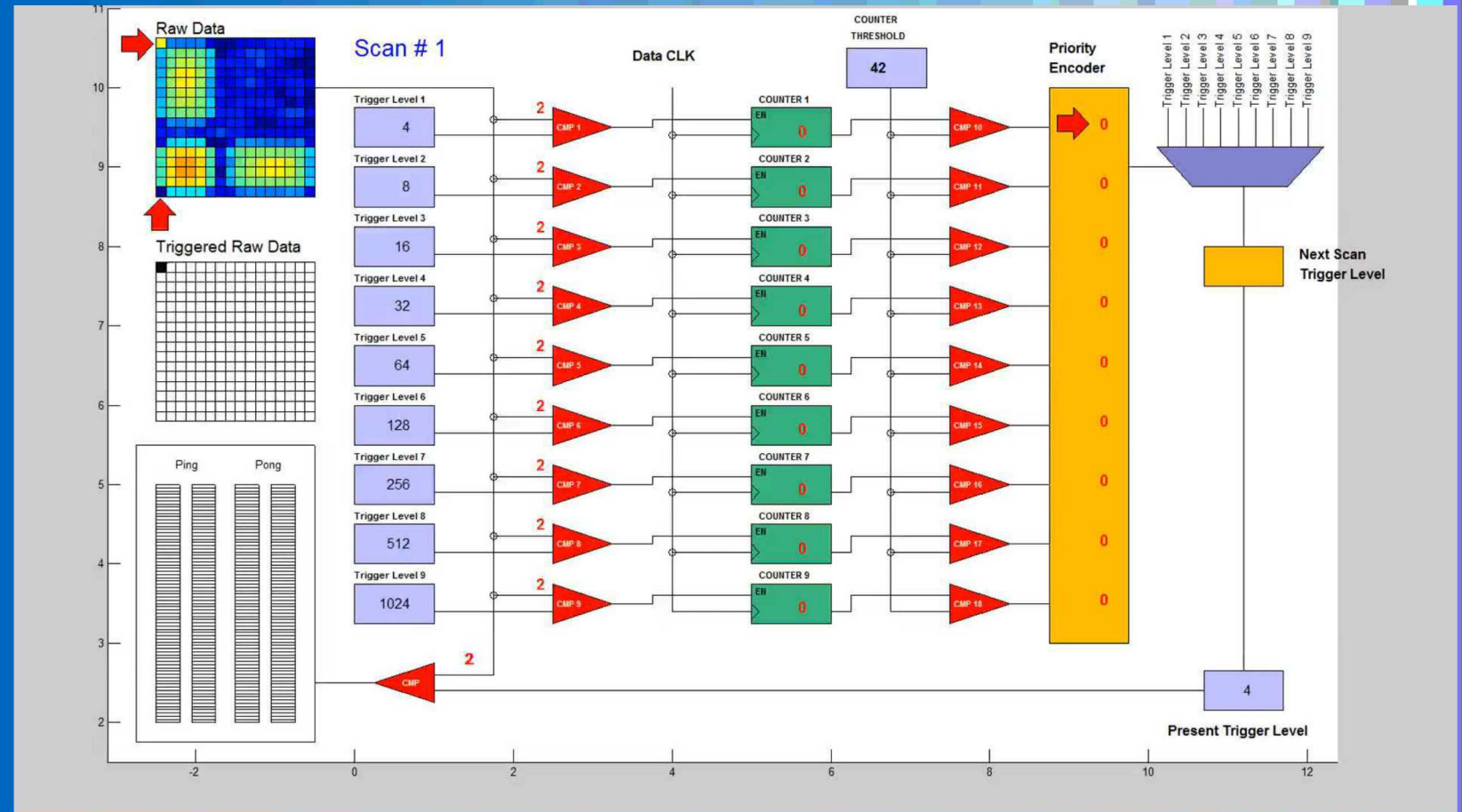
Modeled components:

- Configuration Registers
- Digital Signal Processors
- Ping-Pong SRAM Memories
- Counters
- Comparators
- Latches
- Other state machines

Functions:

- High-pass/low –pass image filtering
- Auto trigger threshold shifting
- Commandable windows
 - Force trigger
 - Mask
 - Return to normal
- Persistence latching
- Data Prioritization modes
- Noise threshold pointers

A Matlab model was made of a Sandia-designed High Speed Focal Plane Array (FPA)

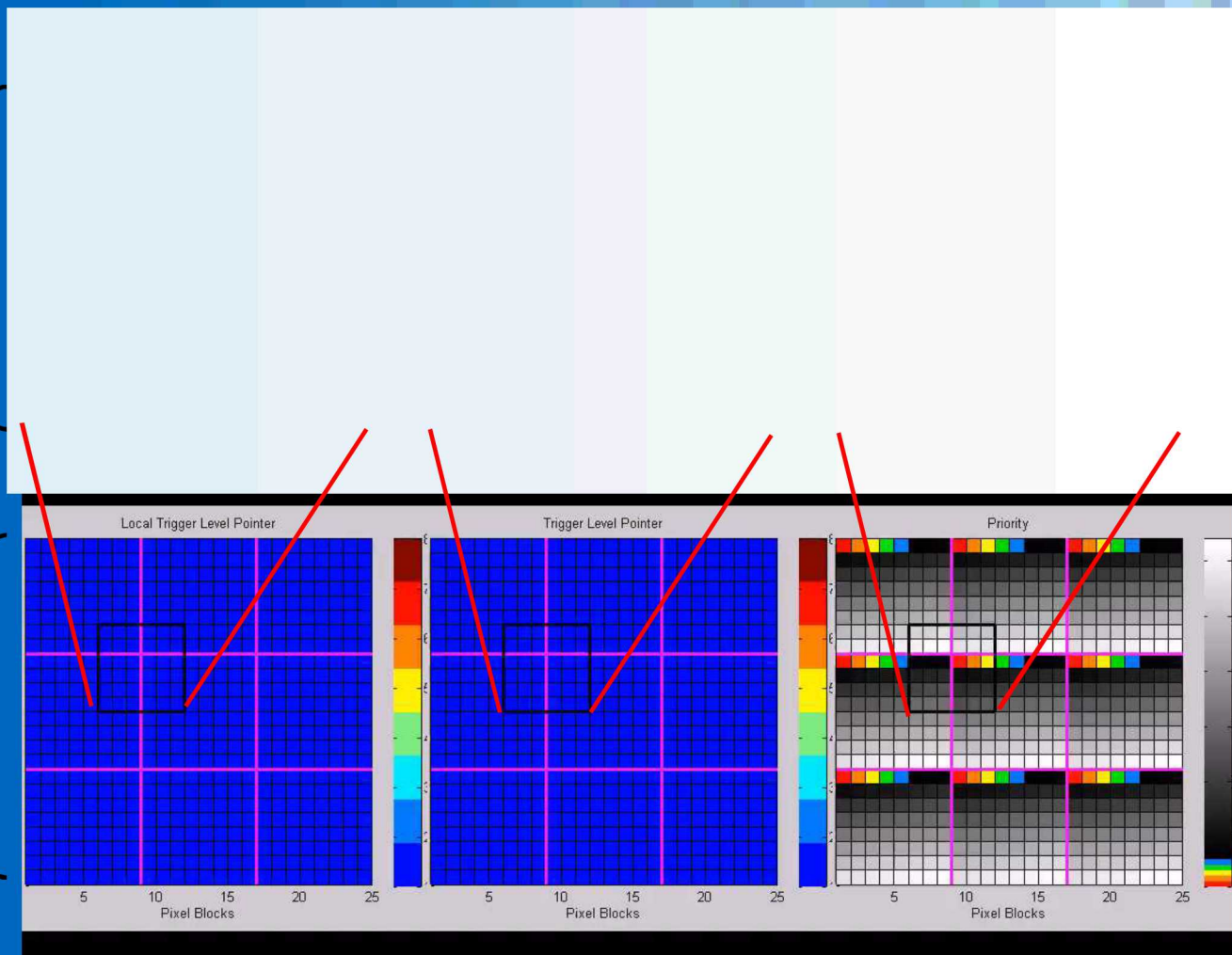


[Click on Video]

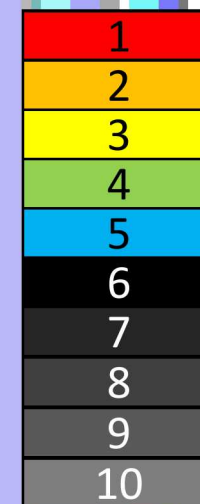
Electrical Modeling - Digital

384 x 384 Pixels,
2304 Digital Signal Processors

Entire Focal Plane array
1536 x 1536 Pixels



Top 5 Prioritized DSPs



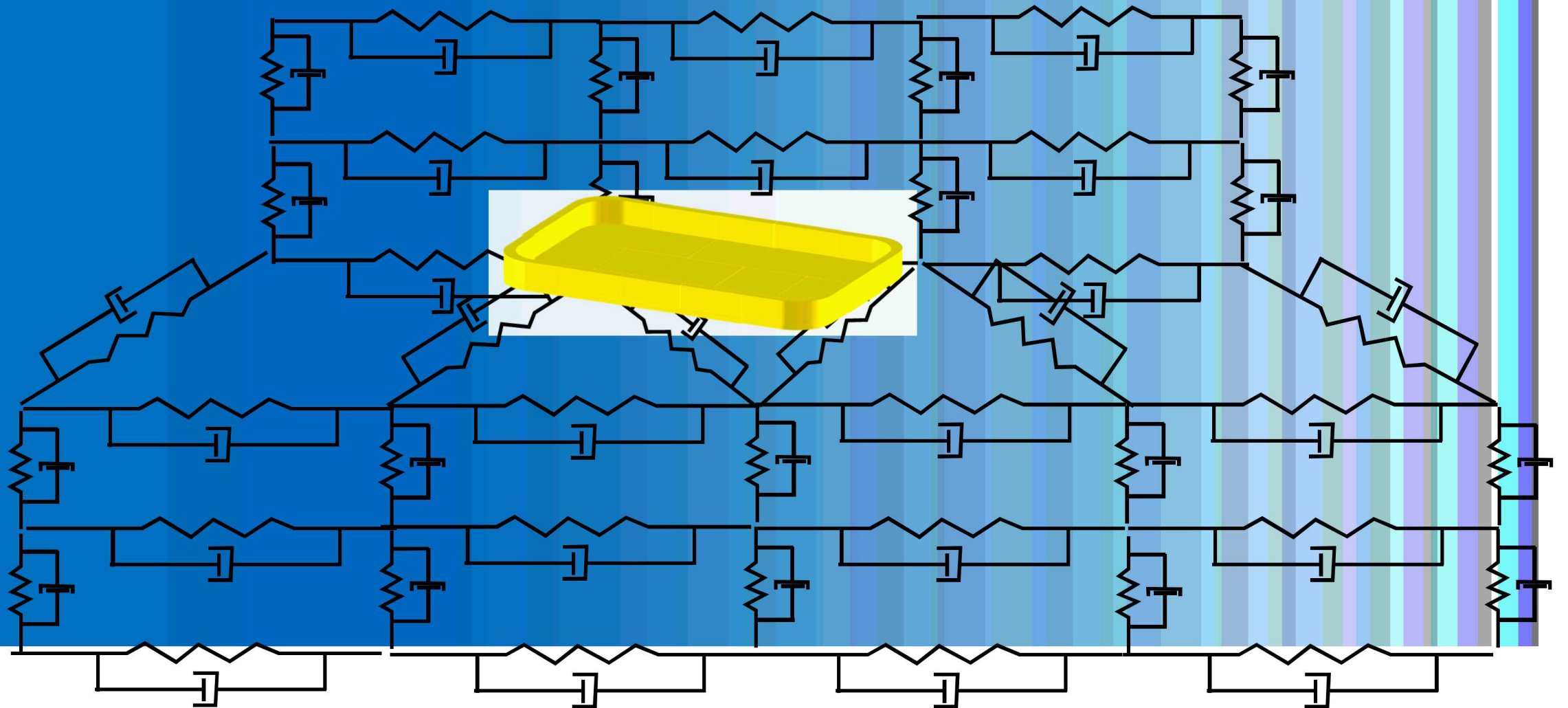
64

[Click on Video]



- Vibration / Shock
- Peak Stress
- Fatigue
- Kinematic Behavior
- Deformation

Lumped Mass Reduced Order Modeling



Coupling Elements as Tensors vs Constants

Stiffness Tensor

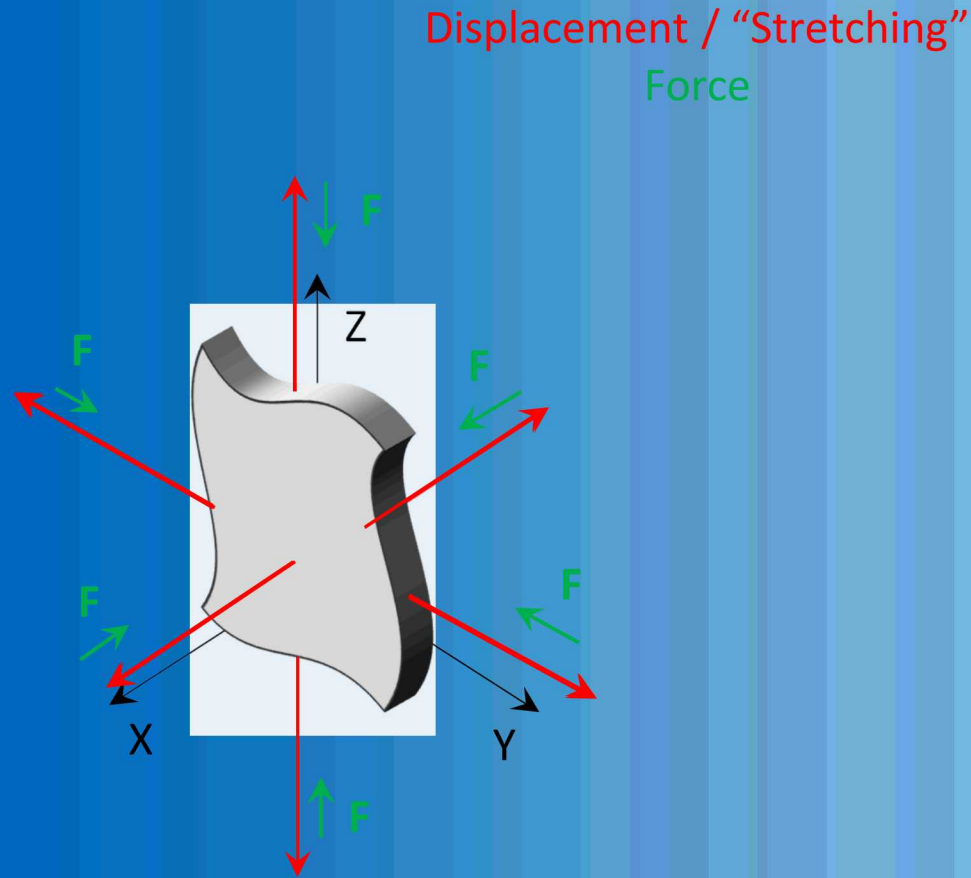
$$\begin{bmatrix} k_{xx23} & k_{xv23} & k_{xz23} \\ k_{yx23} & k_{yv23} & k_{yz23} \\ k_{zx23} & k_{zy23} & k_{zz23} \end{bmatrix}$$

Symmetry

Damping Tensor

$$\begin{bmatrix} b_{xx23} & b_{xv23} & b_{xz23} \\ b_{yx23} & b_{yv23} & b_{yz23} \\ b_{zx23} & b_{zy23} & b_{zz23} \end{bmatrix}$$

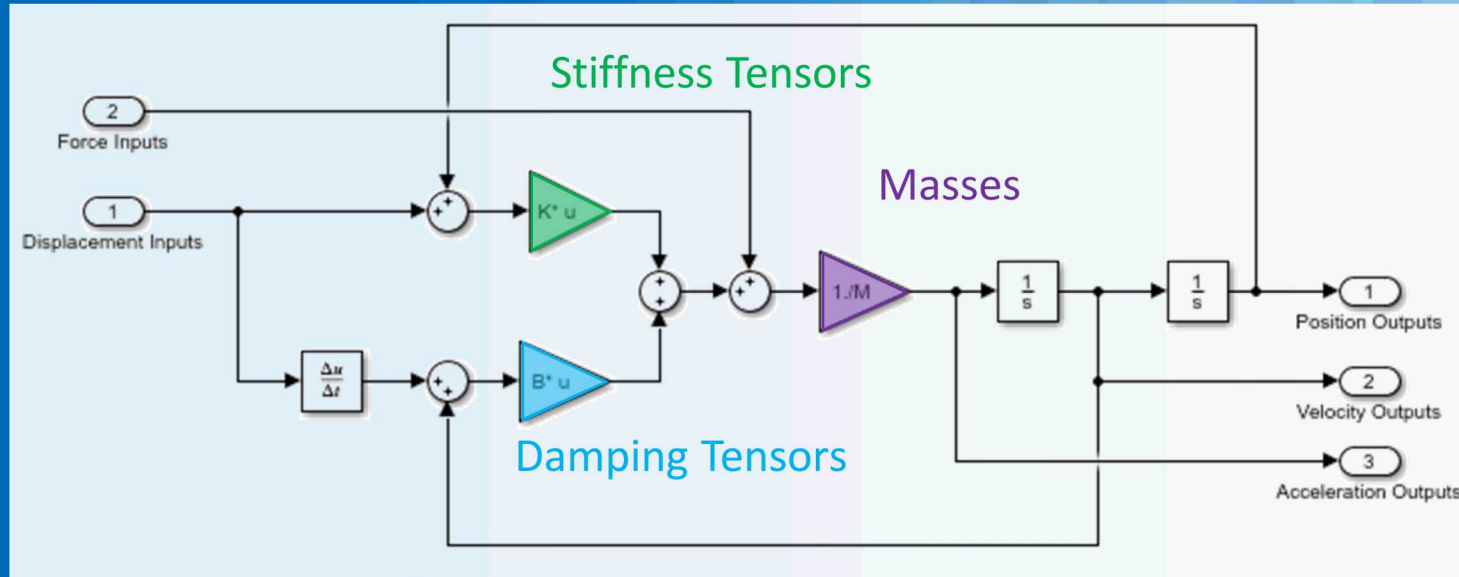
Symmetry

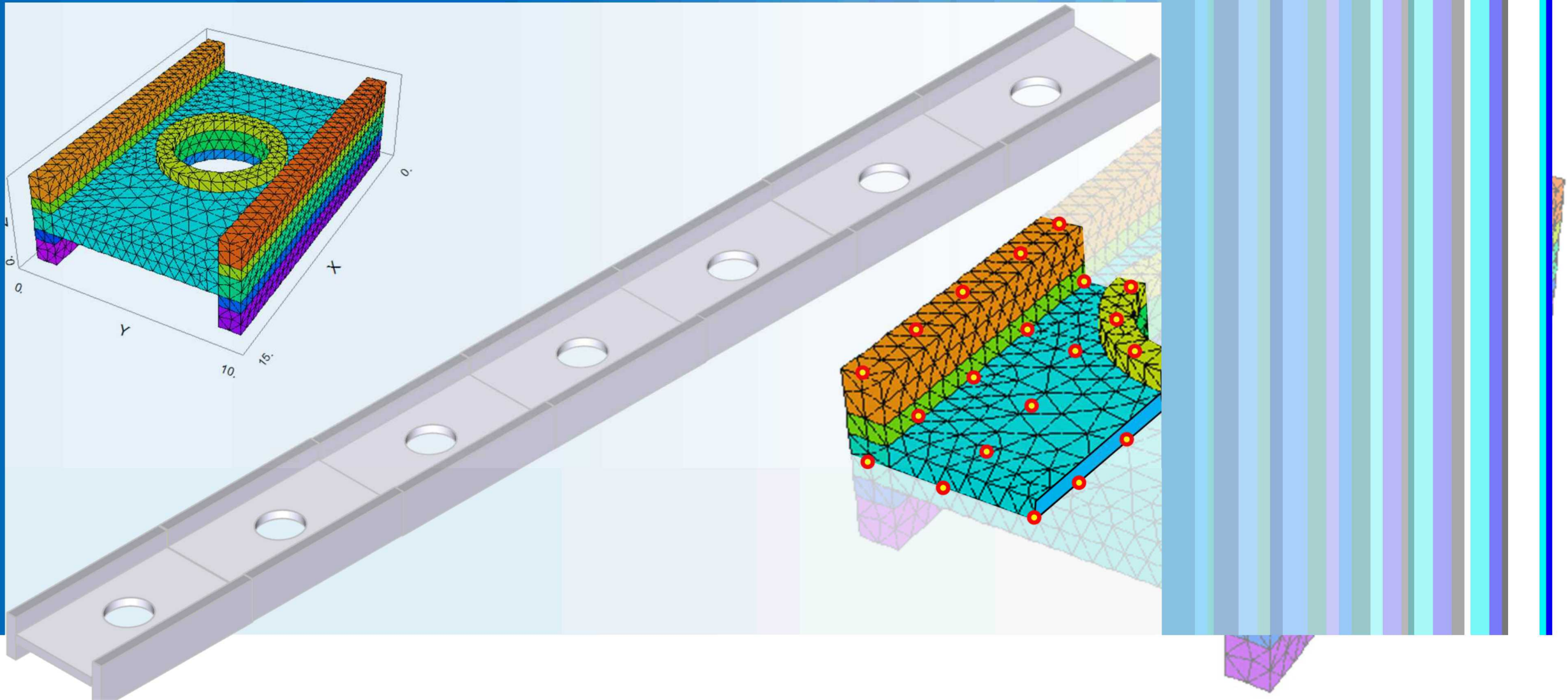


Stiffness Tensors in System Matrix

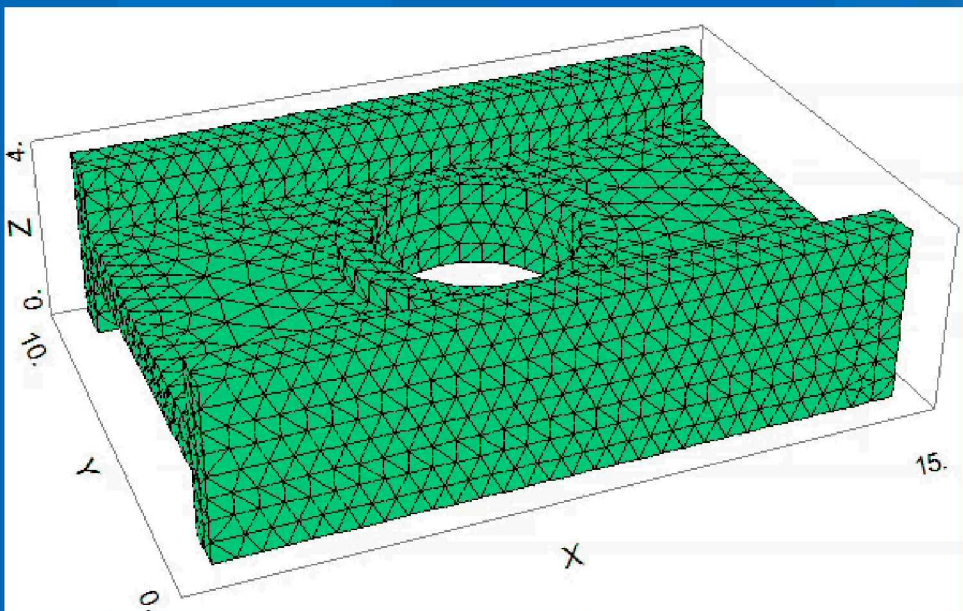
k_{xx11}	k_{xy11}	k_{xz11}	k_{xx12}	k_{xy12}	k_{xz12}	k_{xx13}	k_{xy13}	k_{xz13}	k_{xx14}	k_{xy14}	k_{xz14}
k_{yx11}	k_{yy11}	k_{yz11}	k_{yx12}	k_{yy12}	k_{yz12}	k_{yx13}	k_{yy13}	k_{yz13}	k_{yx14}	k_{yy14}	k_{yz14}
k_{zx11}	k_{zy11}	k_{zz11}	k_{zx12}	k_{zy12}	k_{zz12}	k_{zx13}	k_{zy13}	k_{zz13}	k_{zx14}	k_{zy14}	k_{zz14}
k_{xx21}	k_{xy21}	k_{xz21}	k_{xx22}	k_{xy22}	k_{xz22}	k_{xx23}	k_{xy23}	k_{xz23}	k_{xx24}	k_{xy24}	k_{xz24}
k_{yx21}	k_{yy21}	k_{yz21}	k_{yx22}	k_{yy22}	k_{yz22}	k_{yx23}	k_{yy23}	k_{yz23}	k_{yx24}	k_{yy24}	k_{yz24}
k_{zx21}	k_{zy21}	k_{zz21}	k_{zx22}	k_{zy22}	k_{zz22}	k_{zx23}	k_{zy23}	k_{zz23}	k_{zx24}	k_{zy24}	k_{zz24}
k_{xx31}	k_{xy31}	k_{xz31}	k_{xx32}	k_{xy32}	k_{xz32}	k_{xx33}	k_{xy33}	k_{xz33}	k_{xx34}	k_{xy34}	k_{xz34}
k_{yx31}	k_{yy31}	k_{yz31}	k_{yx32}	k_{yy32}	k_{yz32}	k_{yx33}	k_{yy33}	k_{yz33}	k_{yx34}	k_{yy34}	k_{yz34}
k_{zx31}	k_{zy31}	k_{zz31}	k_{zx32}	k_{zy32}	k_{zz32}	k_{zx33}	k_{zy33}	k_{zz33}	k_{zx34}	k_{zy34}	k_{zz34}
k_{xx41}	k_{xy41}	k_{xz41}	k_{xx42}	k_{xy42}	k_{xz42}	k_{xx43}	k_{xy43}	k_{xz43}	k_{xx44}	k_{xy44}	k_{xz44}
k_{yx41}	k_{yy41}	k_{yz41}	k_{yx42}	k_{yy42}	k_{yz42}	k_{yx43}	k_{yy43}	k_{yz43}	k_{yx44}	k_{yy44}	k_{yz44}
k_{zx41}	k_{zy41}	k_{zz41}	k_{zx42}	k_{zy42}	k_{zz42}	k_{zx43}	k_{zy43}	k_{zz43}	k_{zx44}	k_{zy44}	k_{zz44}

Modeling Mechanical Vibrations and Thermal Conduction in Simulink

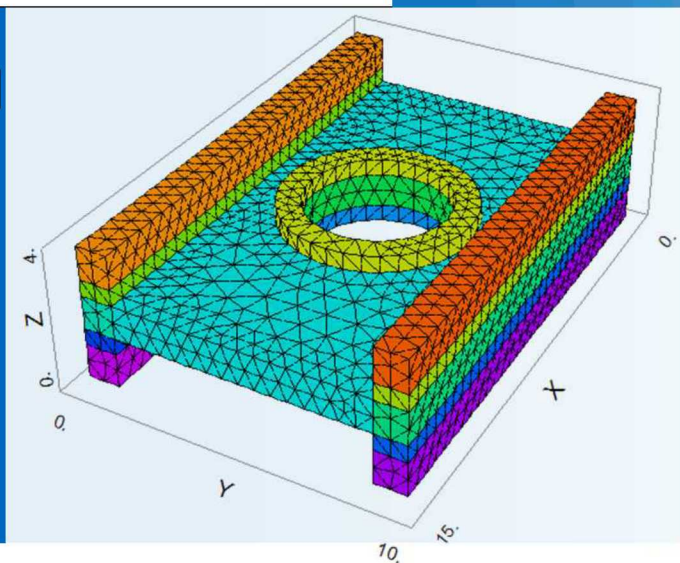




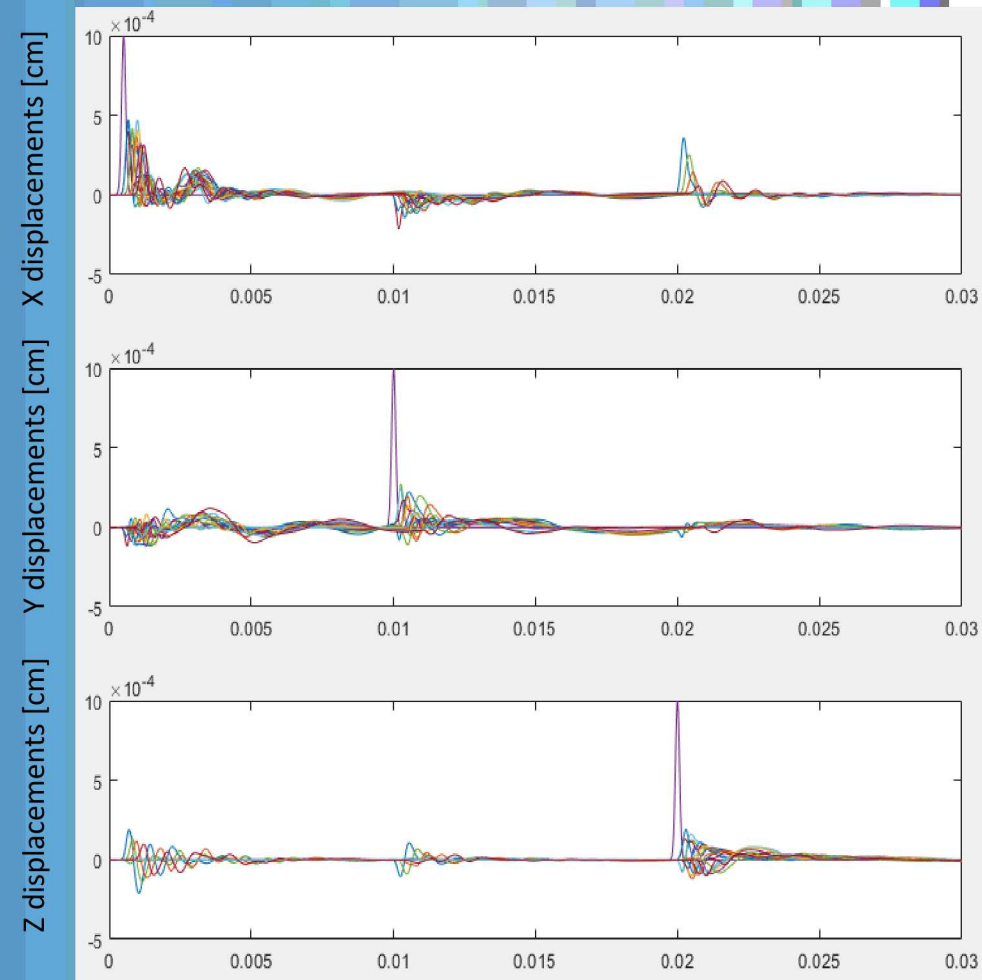
Vibration Example – Beam



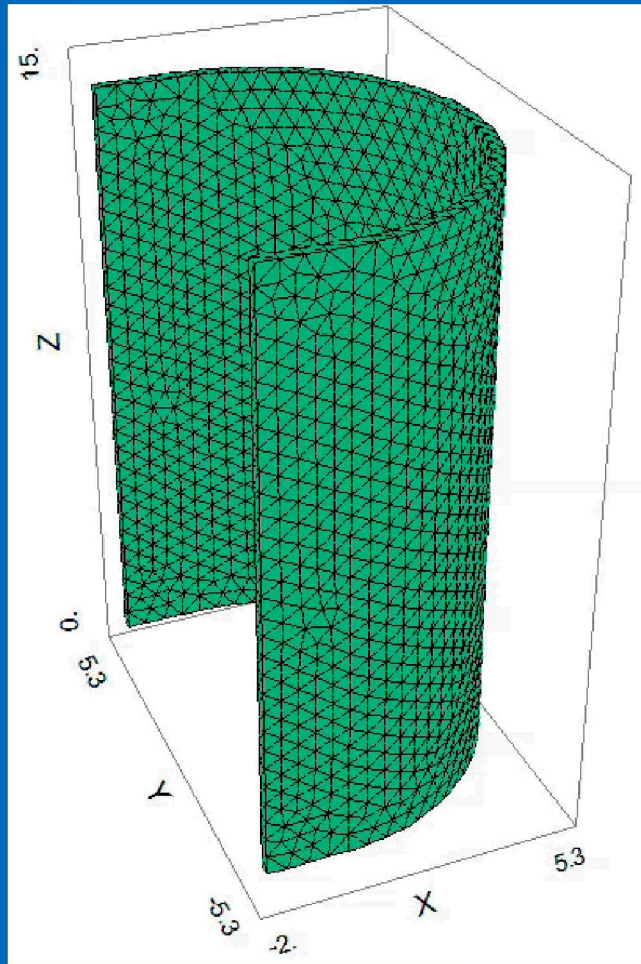
[Click on Video]



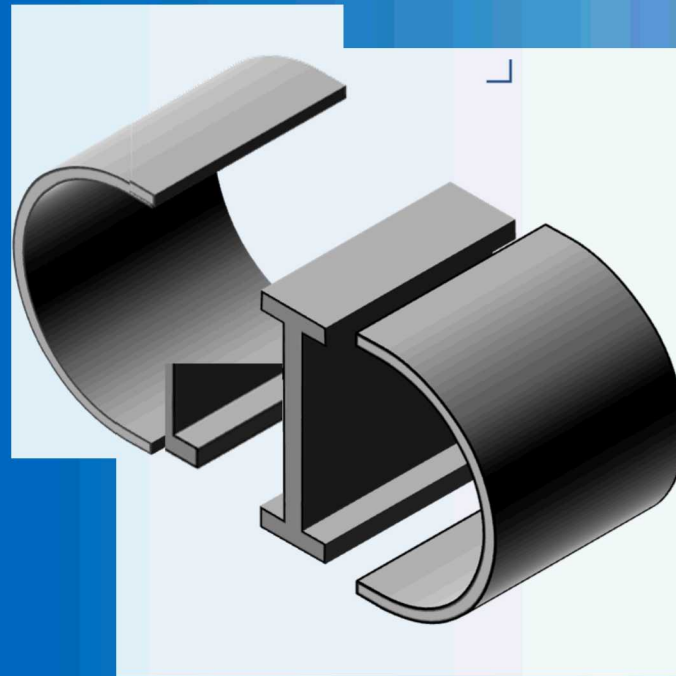
Displacements



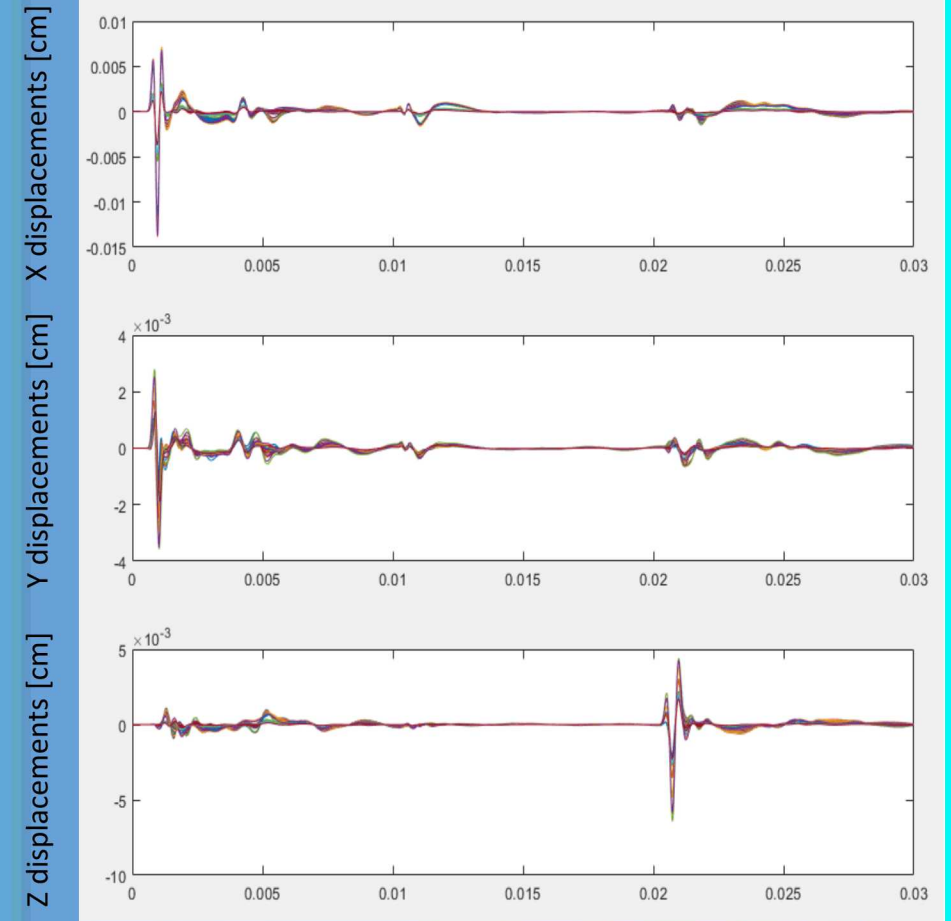
Vibration Example – Shell



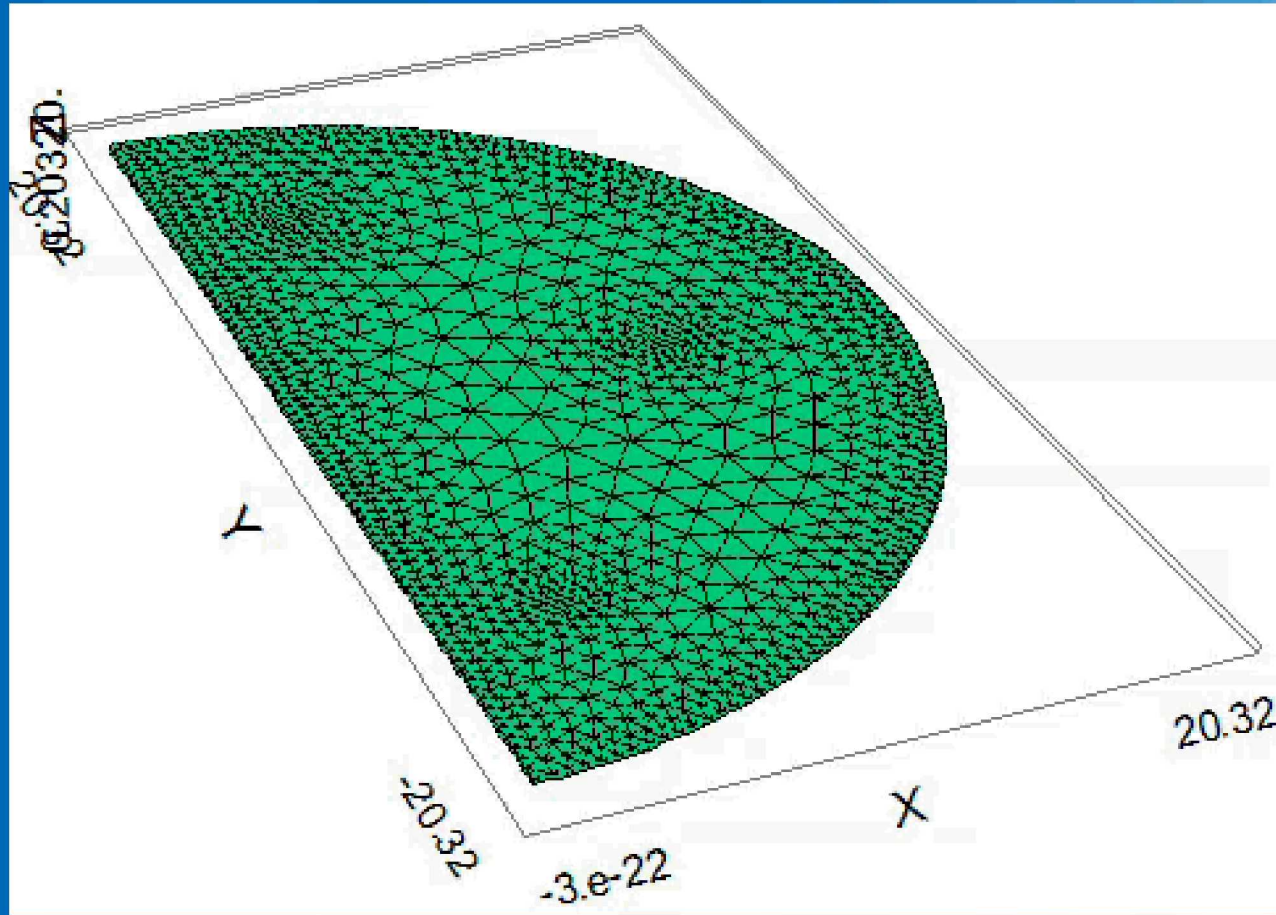
[Click on Video]



Displacements

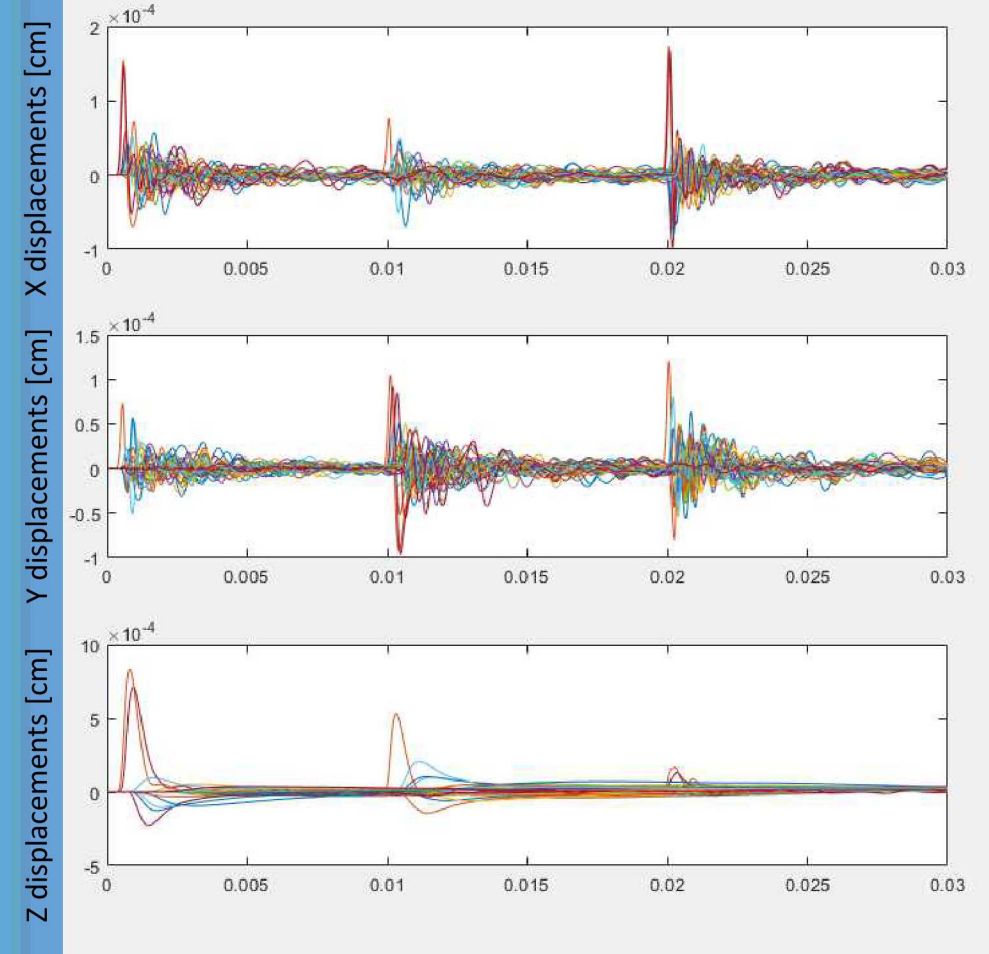


Vibration Example – Circuit Board



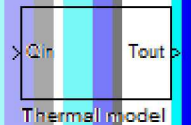
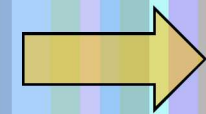
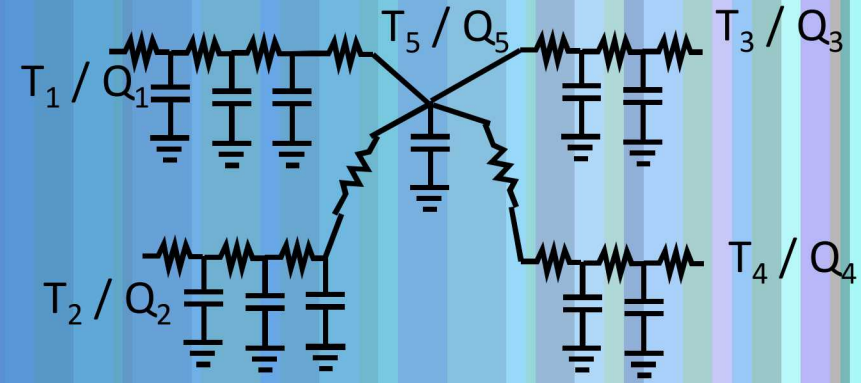
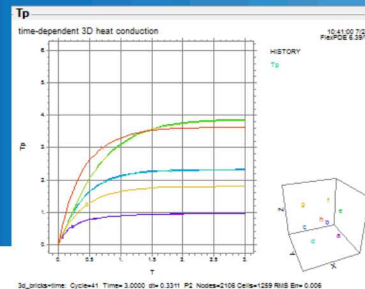
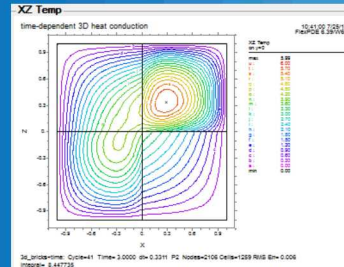
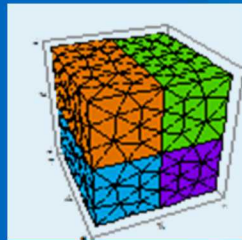
[Click on Video]

Displacements



Thermal Modeling

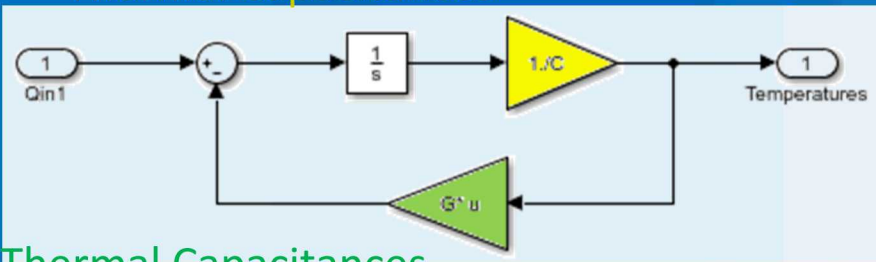
- Conduction
 - Thermal Resistance
 - Thermal Capacitance
- Convection
 - Free
 - Forced
- Radiation



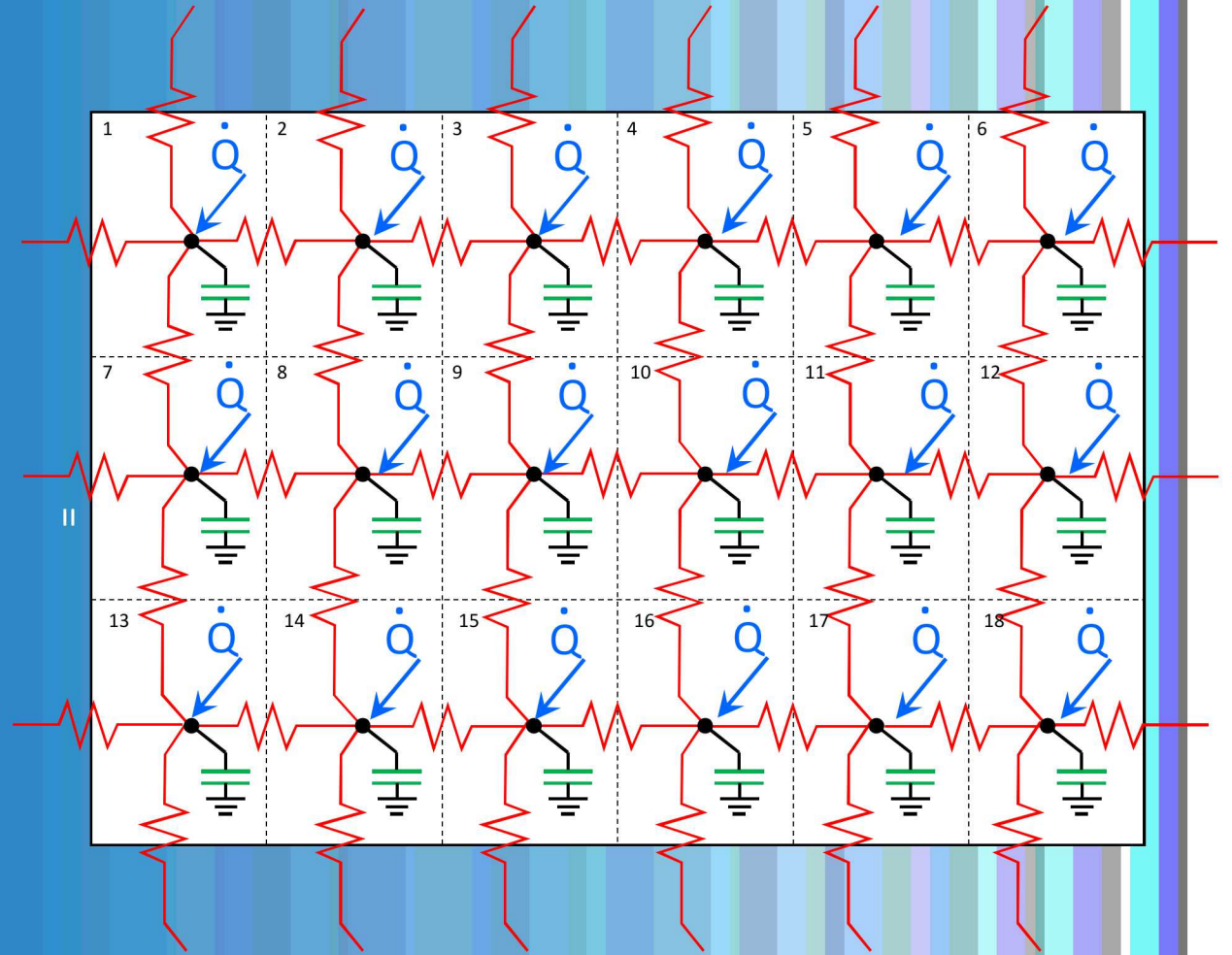
Thermal Modeling - Conduction

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ \vdots \\ \vdots \\ \vdots \\ Q_{18} \end{bmatrix} - \begin{bmatrix} G \\ G \\ G \\ G \\ G \\ G \\ G \\ G \\ \vdots \\ \vdots \\ \vdots \\ G \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ \vdots \\ \vdots \\ \vdots \\ T_{18} \end{bmatrix} = S \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ \vdots \\ \vdots \\ \vdots \\ C_{18} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ \vdots \\ \vdots \\ \vdots \\ T_{18} \end{bmatrix}$$

Thermal Capacitances



Thermal Capacitances



Thermal Modeling – Conduction Matrix



$$[G] = \begin{bmatrix} -\Sigma_{row} & -1/R_{12} & 0 & 0 & 0 & 0 & -1/R_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/R_{12} & -\Sigma_{row} & -1/R_{23} & 0 & 0 & 0 & 0 & -1/R_{28} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/R_{23} & -\Sigma_{row} & -1/R_{34} & 0 & 0 & 0 & 0 & -1/R_{39} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/R_{34} & -\Sigma_{row} & -1/R_{45} & 0 & 0 & 0 & 0 & -1/R_{410} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/R_{45} & -\Sigma_{row} & -1/R_{56} & 0 & 0 & 0 & 0 & -1/R_{511} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/R_{56} & -\Sigma_{row} & 0 & 0 & 0 & 0 & 0 & -1/R_{612} & 0 & 0 & 0 & 0 & 0 \\ -1/R_{17} & 0 & 0 & 0 & 0 & -1/R_{67} & -\Sigma_{row} & -1/R_{67} & 0 & 0 & 0 & 0 & -1/R_{713} & 0 & 0 & 0 & 0 \\ 0 & -1/R_{28} & 0 & 0 & 0 & 0 & -1/R_{78} & -\Sigma_{row} & -1/R_{89} & 0 & 0 & 0 & 0 & -1/R_{814} & 0 & 0 & 0 \\ 0 & 0 & -1/R_{39} & 0 & 0 & 0 & 0 & -1/R_{89} & -\Sigma_{row} & -1/R_{910} & 0 & 0 & 0 & 0 & -1/R_{915} & 0 & 0 \\ 0 & 0 & 0 & -1/R_{410} & 0 & 0 & 0 & 0 & -1/R_{910} & -\Sigma_{row} & -1/R_{1011} & 0 & 0 & 0 & 0 & -1/R_{1016} & 0 \\ 0 & 0 & 0 & 0 & -1/R_{511} & 0 & 0 & 0 & 0 & -1/R_{910} & -\Sigma_{row} & -1/R_{1011} & 0 & 0 & 0 & 0 & -1/R_{1117} \\ 0 & 0 & 0 & 0 & 0 & -1/R_{612} & 0 & 0 & 0 & 0 & -1/R_{1011} & -\Sigma_{row} & -1/R_{1112} & 0 & 0 & 0 & -1/R_{1218} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/R_{713} & 0 & 0 & 0 & 0 & -1/R_{1112} & -\Sigma_{row} & -1/R_{1213} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/R_{814} & 0 & 0 & 0 & 0 & -1/R_{1213} & -\Sigma_{row} & -1/R_{1314} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/R_{915} & 0 & 0 & 0 & 0 & -1/R_{1314} & -\Sigma_{row} & -1/R_{1415} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/R_{1016} & 0 & 0 & 0 & 0 & -1/R_{1415} & -\Sigma_{row} & -1/R_{1516} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/R_{1117} & 0 & 0 & 0 & 0 & -1/R_{1516} & -\Sigma_{row} & -1/R_{1617} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/R_{1218} & 0 & 0 & 0 & 0 & -1/R_{1617} & -\Sigma_{row} \end{bmatrix}$$

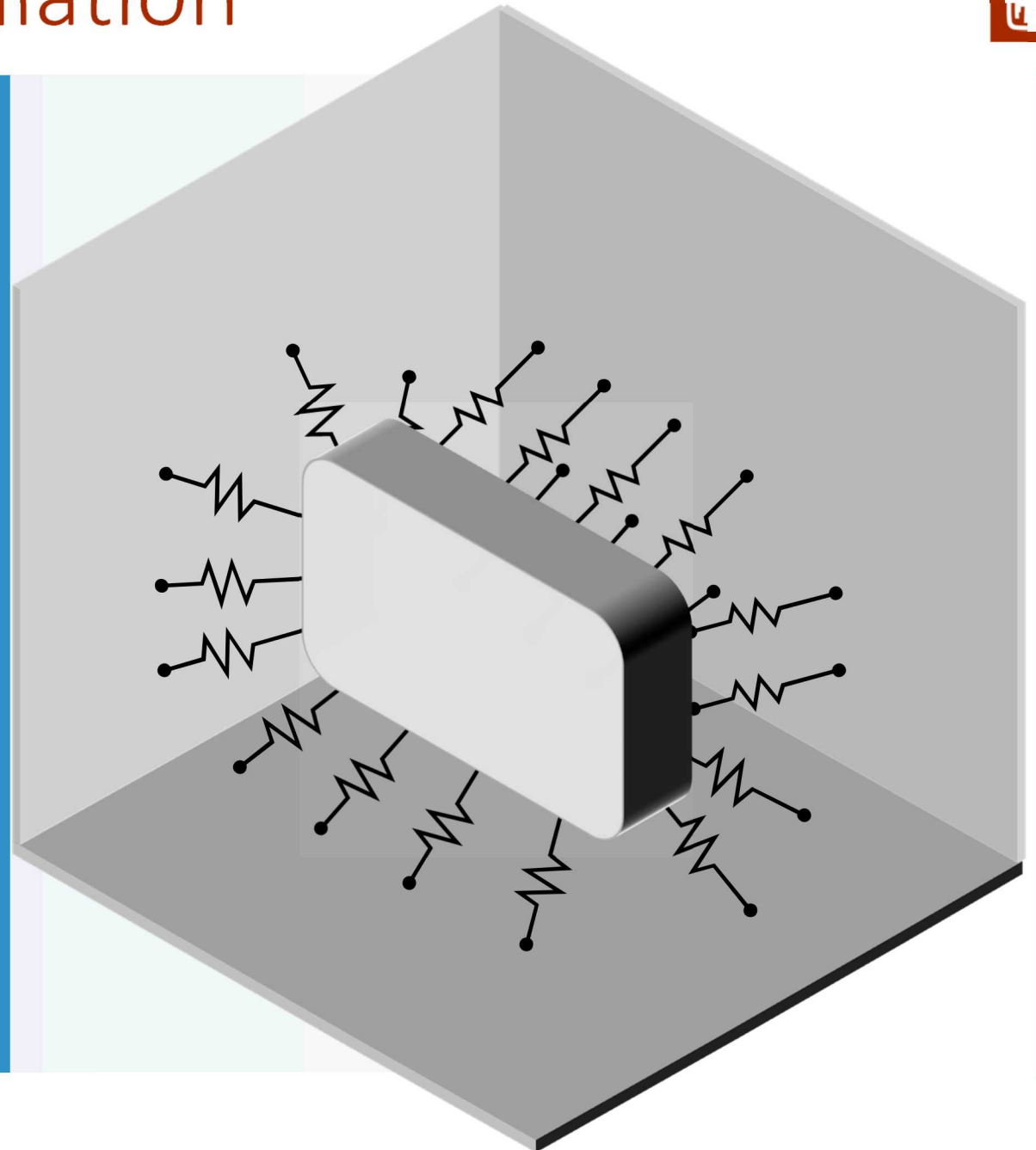
Thermal Modeling - Radiation



$$\frac{q}{A} = \epsilon k_b (T_1^4 - T_2^4) = \frac{\Delta T}{R}$$



$$R = \frac{(T_1 - T_2)}{\epsilon k_b (T_1^4 - T_2^4)}$$



- Motors / Actuators
- Electrical Connectors ← Thermal ← shock/vibe
- Pyro ← Thermal ← Pressure Waves

Gasses / Pneumatics
Pressure Waves / Pyrotechnics

G

Electro-Magnetic Fields

Radiated EMI (Power Converters)

Radar

Other..



Extraction of Parameters

Four Element Example in 1D

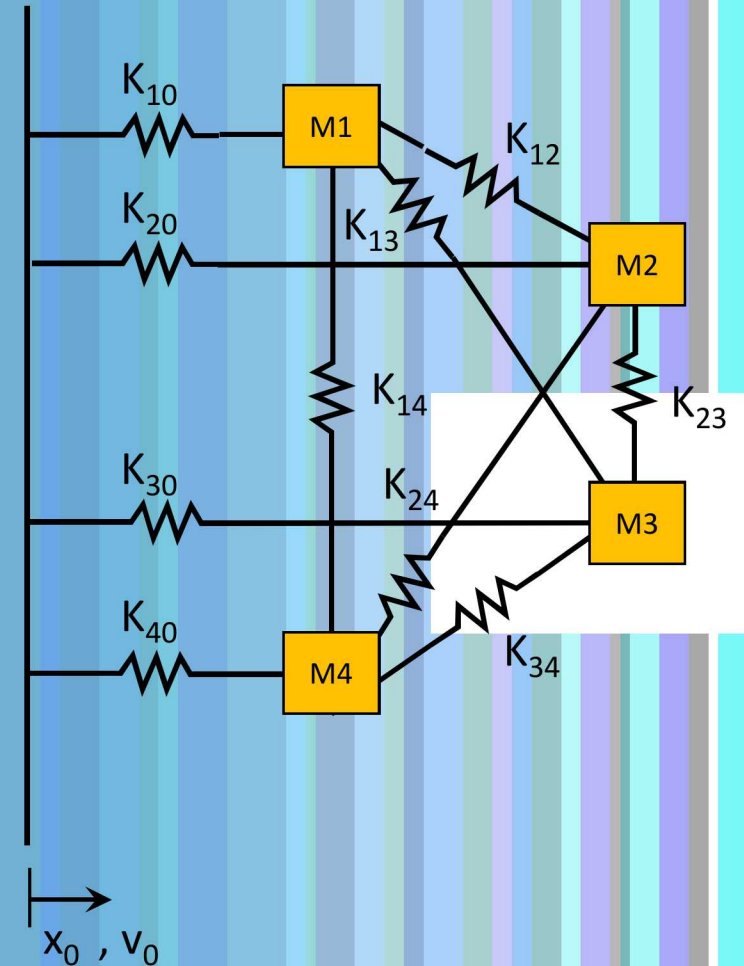
$$\begin{aligned} \underline{M_1} \\ (k_{01}+sb_{01})(x_0-x_1) + (k_{21}+sb_{21})(x_2-x_1) + (k_{31}+sb_{31})(x_3-x_1) + (k_{41}+sb_{41})(x_4-x_1) &= M_1 s^2 x_1 \\ \underline{M_2} \\ (k_{02}+sb_{02})(x_0-x_2) + (k_{21}+sb_{21})(x_1-x_2) + (k_{32}+sb_{32})(x_3-x_2) + (k_{42}+sb_{42})(x_4-x_2) &= M_2 s^2 x_2 \\ \underline{M_3} \\ (k_{03}+sb_{03})(x_0-x_3) + (k_{13}+sb_{13})(x_1-x_3) + (k_{23}+sb_{23})(x_2-x_3) + (k_{43}+sb_{43})(x_4-x_3) &= M_3 s^2 x_3 \\ \underline{M_4} \\ (k_{04}+sb_{04})(x_0-x_4) + (k_{14}+sb_{14})(x_1-x_4) + (k_{24}+sb_{24})(x_2-x_4) + (k_{34}+sb_{34})(x_3-x_4) &= M_4 s^2 x_4 \end{aligned}$$

Multivariable Regression

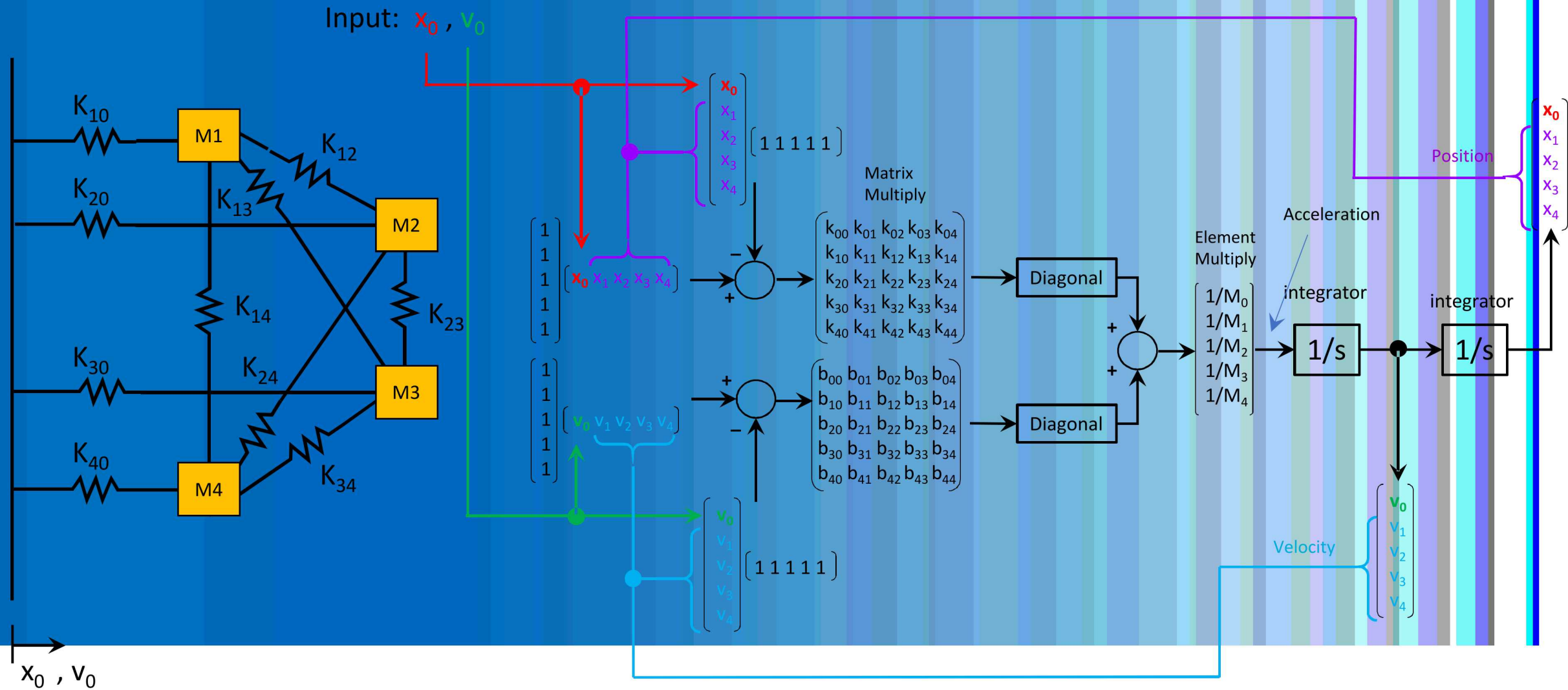
$$\underline{M_1} \quad \frac{k_{01}}{M_1}(x_0-x_1) + \frac{b_{01}}{M_1}(v_0-v_1) + \frac{k_{21}}{M_1}(x_2-x_1) + \frac{b_{21}}{M_1}(v_2-v_1) + \frac{k_{31}}{M_1}(x_3-x_1) + \frac{b_{31}}{M_1}(v_3-v_1) + \frac{k_{41}}{M_1}(x_4-x_1) + \frac{b_{41}}{M_1}(v_4-v_1) = a_1$$

Known (green arrow) and Unknown (red arrow) parameters are indicated.

Solve for Unknowns → Extract Stiffness and Damping tensor components



4 Element System in 1D



Full System Model Simulation Indicates when

