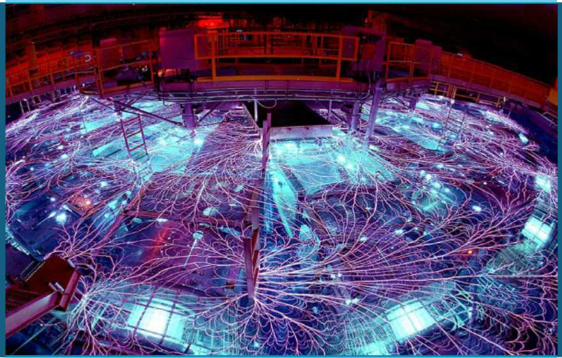
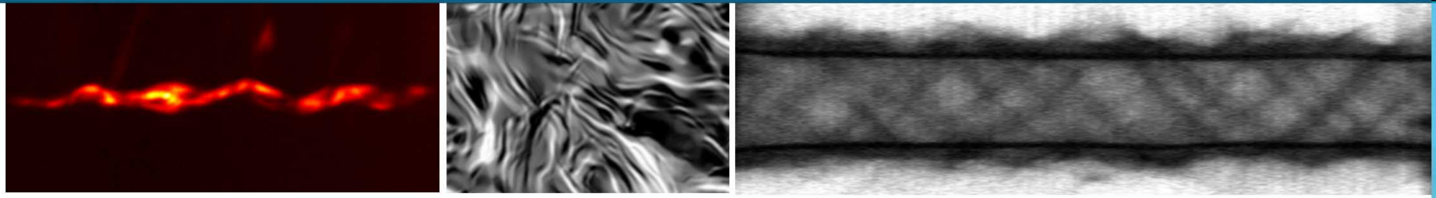
SAND2018-12838C

The Mallat Scattering Transform (MST) in high energy density plasmas: a new look at nonlinear, multiscale physics in HED





PRESENTED BY

Michael Glinsky

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What will be presented

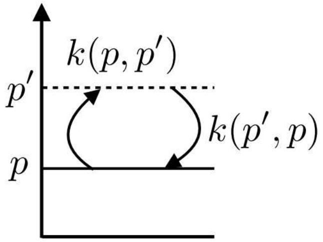
- Nonlinear physics vs. linear physics
  - linear instability vs. nonlinear stability
  - steady state structures (emergent behavior)
  - 2D inverse cascade vs 3D normal cascade
  - 2D Navier-Stokes with conserved vorticity and 3D MHD with topological helicity invariant
- Intuitive description of Mallat Scattering Transformation (MST)
- Connection of MST to nonlinear physics
  - Enhanced Wigner-Weyl transformation (manifold safe)
  - S-matrix (multiple scale, 1/momentum, scattering cross sections)
- Evidence for nonlinear stability, that is large scale emergent behavior in MagLIF implosions
  - mode merger
  - helical structure in liner with modes below linear mode with maximal growth rate
  - unexpected convergence to double helical structures with extreme CR>200
- Analysis of stagnation morphology with MST
  - regression to helical parameters (remarkably linear)
  - advanced background subtraction
  - quantitative metric of morphology (that is, steady state nonlinear structure or emergent behavior)

Difference between linear and nonlinear physics

$p \equiv 1/\lambda$,canonical momentum, or quantum numbers

$$k(p,p') \equiv \frac{\bar f_2(p,p',t)}{\bar f_1(p,t)}$$

Generalized Master Equation $\frac{\partial \bar f_1(p,t)}{\partial t} - \frac{\partial f_{\text{source}}(p)}{\partial t} \backsim \int dp'\ \bar f_1(p',t)\,k(p',p) - \bar f_1(p,t)\,k(p,p')$



linear instability analysis

$$\bar f_1(p,t) \approx f_0(p,t) + \delta f(p,t) \quad \text{, where } \delta f/f_0 \ll 1$$

$$\Rightarrow \text{ dispersion relation, } D(p,t) = 0 \Rightarrow p = p_0(t)$$

$$p_0 = k + i\gamma$$

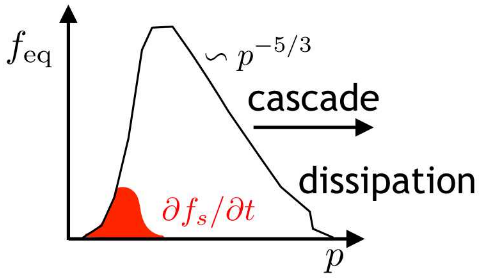
oscillation ↗ ↖ instability/stability

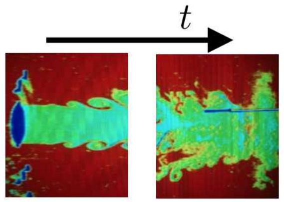
nonlinear steady state analysis

$$f_{\text{eq}}(p) \equiv \lim_{t\to\infty} \bar f_1(p,t)$$

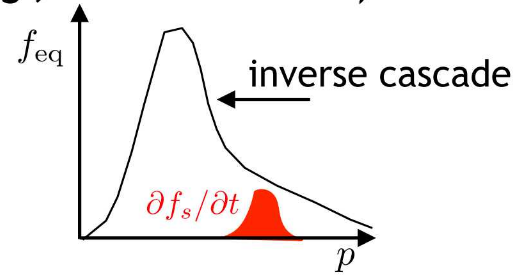
$$\int dp'\ f_{\text{eq}}(p)\,k(p,p') - f_{\text{eq}}(p')\,k(p',p) \backsim \frac{\partial f_{\text{source}}(p)}{\partial t}$$

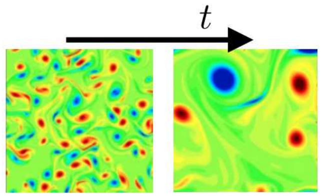
emergent behavior
(e.g., 3D Kolmogorov scaling)





large scale self organization
(e.g., 2D Navier-Stokes)

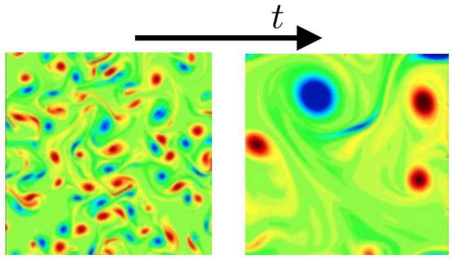




Why 3D MHD can exhibit a 2D Navier-Stokes inverse cascade with a resulting large scale, self organized, nonlinear, helical structure?

3D Navier-Stokes when constrained to 2D conserves total vorticity, relaxes energy while maintaining circulation

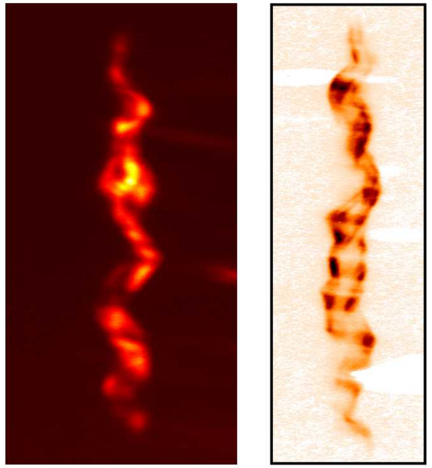
$$\text{total vorticity} = \int \nabla \times u \; d^2x$$



3D MHD when magnetized has total magnetic and cross helicity as a topological invariants, dissipates energy but must maintain helical twist

$$\text{total magnetic helicity} = \int A \cdot B \; d^3x$$

$$\text{total cross helicity} = \int v \cdot B \; d^3x$$

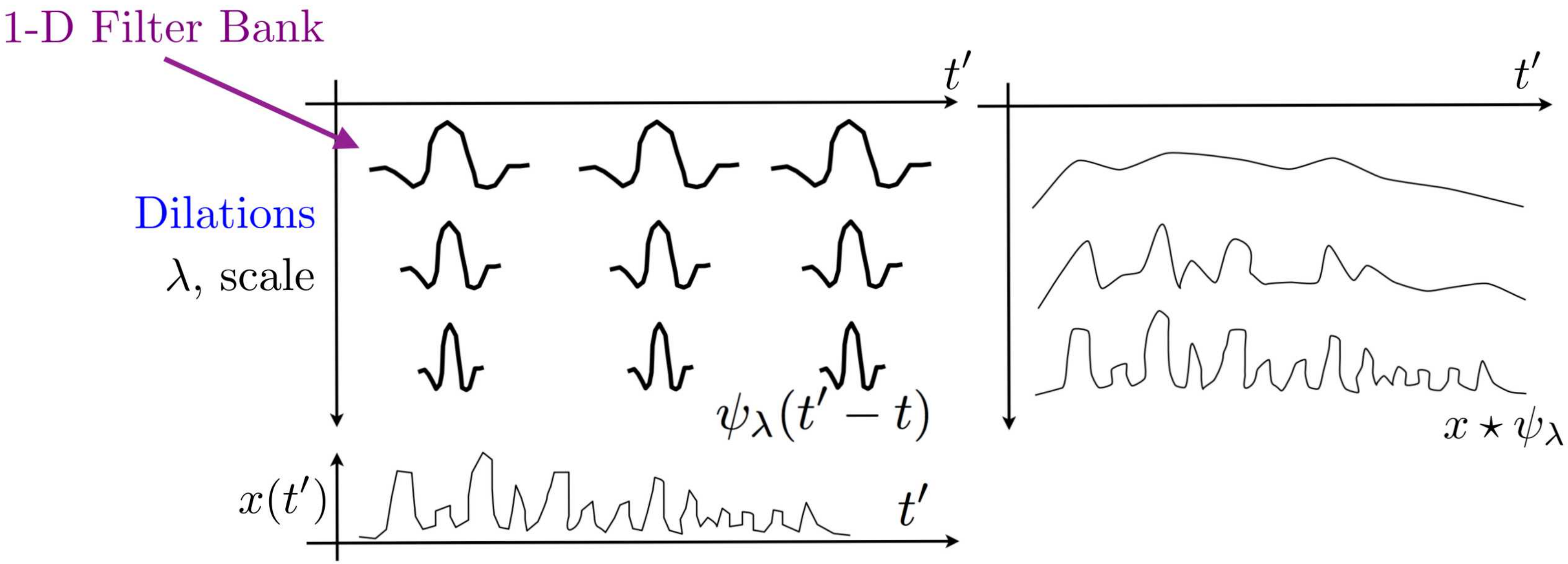


What is a Wavelet Transform?

- Wavelet Transform, $\mathcal{W}$
  - Convolutions of a signal with dilated Mother Wavelets, $\psi_\lambda(t)$ (i.e. a bank of band-pass filtered signals)
  - $\psi_\lambda(t'-t)$ consists of dilations and translations of the Mother Wavelet $\psi(t)$

1-D Wavelet Trasform

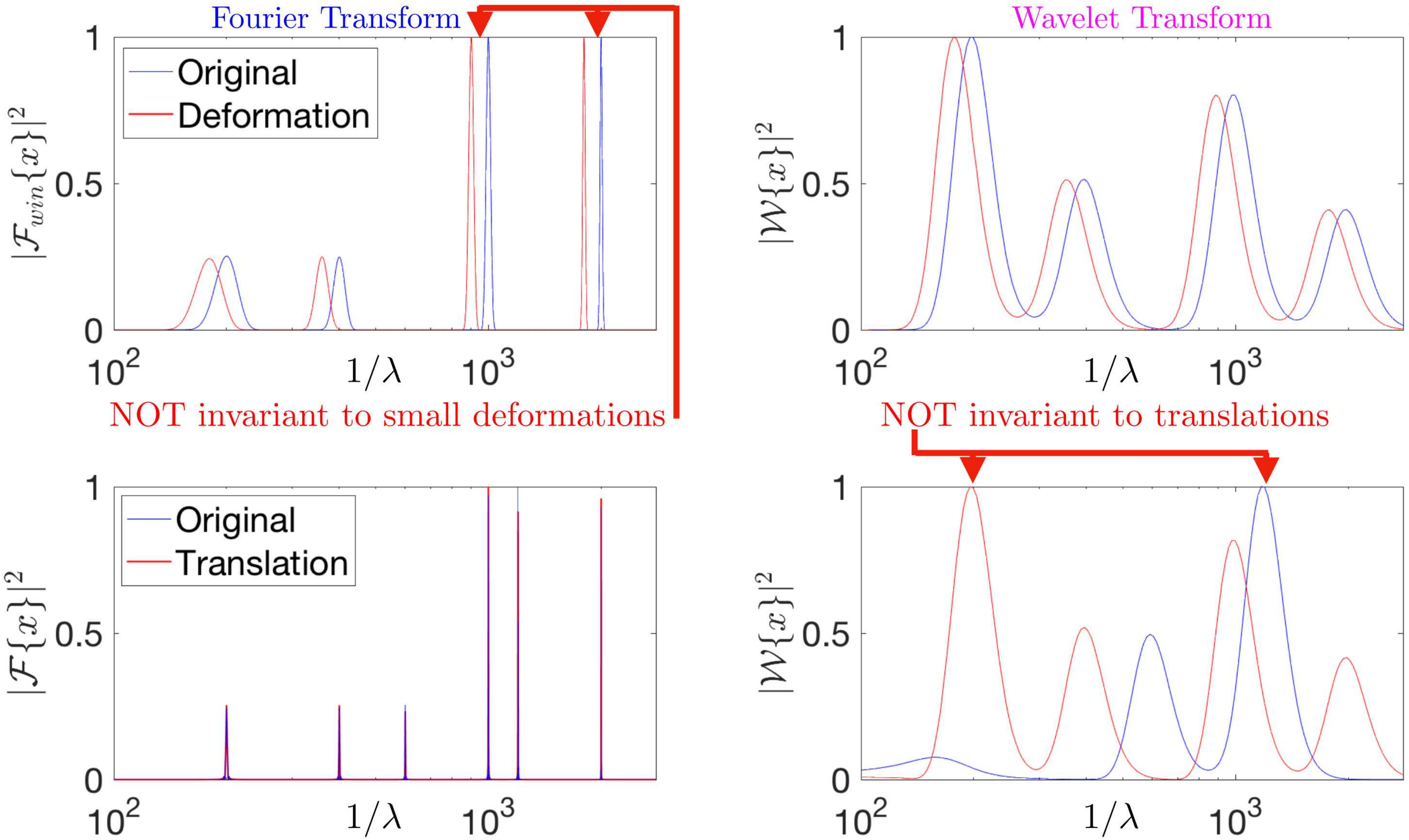
$$x[\lambda](t) = \mathcal{W}\{x(t)\} = x \star \psi_\lambda = \int x(t')\psi_\lambda(t'-t)dt'$$



The Good, the Bad and the Ugly

Lipschitz Continuous - invariant to small deformations
Stationary - invariant to translation

	$\mathcal{F}$	$\mathcal{W}$
Lipschitz Continuous		✓
Stationary	✓	



NOT invariant to small deformations

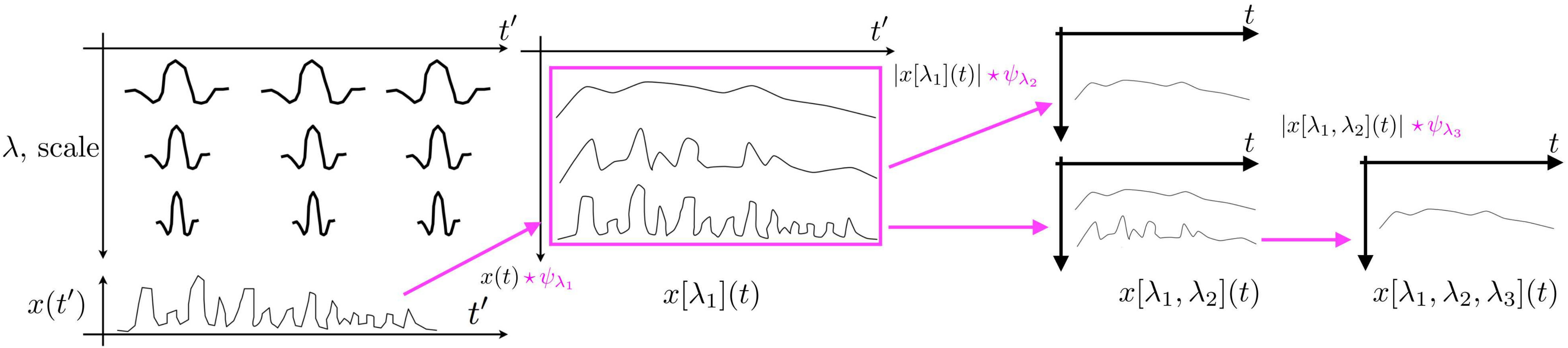
NOT invariant to translations

Wavelet Transformations can be iterated

- Wavelet Transform, $\mathcal{W}$
  - convolutions of a signal with dilated Mother Wavelets, $\psi_\lambda(t)$ (i.e. a bank of band-pass filtered signals)
  - $\psi_\lambda(t'-t)$ consists of dilations and translations of the Mother Wavelet $\psi(t)$
  - because $x[\lambda](t)$ is a function of time we can take its Wavelet Transform

Wavelet Transform of a Wavelet Transform

$$x[\lambda_1, \lambda_2](t) = |x \star \psi_{\lambda_1}| \star \psi_{\lambda_2}$$



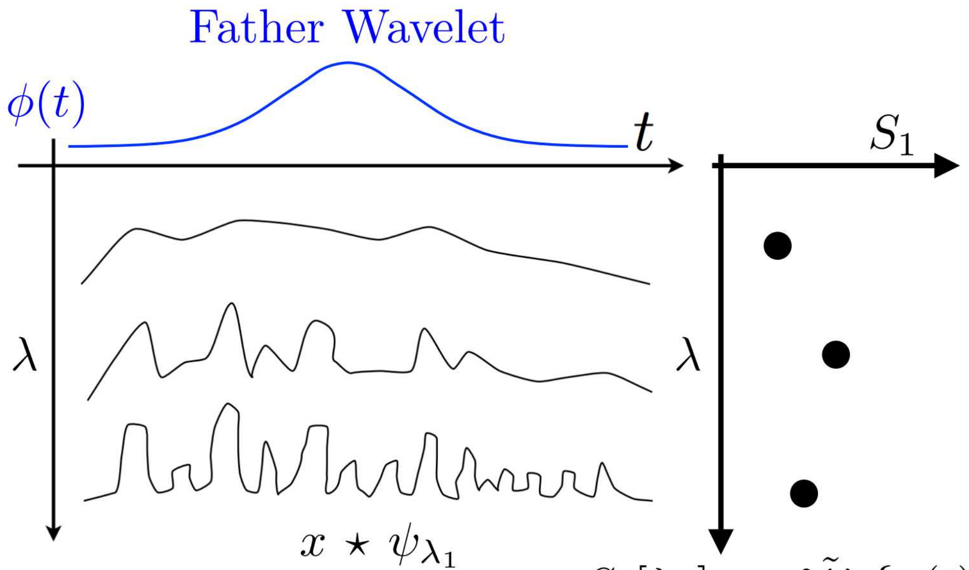
The Mallat Scattering Transform (MST)

[Mallat 2012; Bruna and Mallat 2013; Mallat 2016]

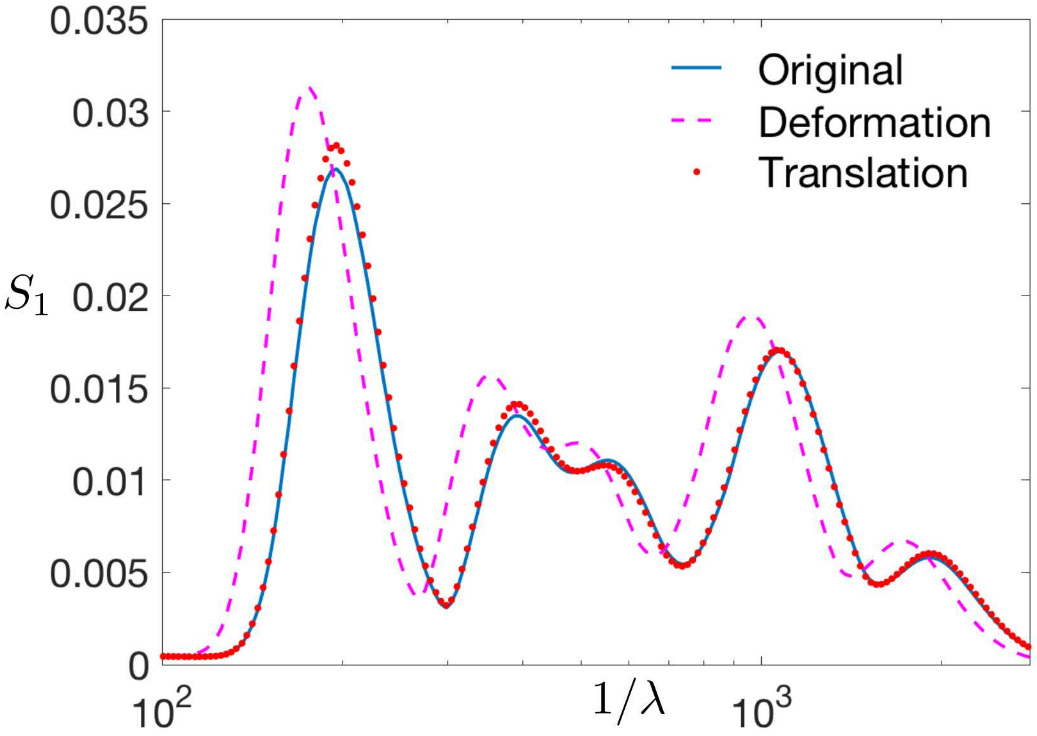
Mallat Scattering Transform, $\tilde{\mathcal{W}}$

$$S_m[\lambda \equiv \sum_{n=1}^{m} \lambda_n]x = \tilde{W}_m\{x(t)\} = |||x \star \psi_{\lambda_1}| \star \psi_{\lambda_2}| \cdots \star \psi_{\lambda_m}| \star \phi$$

	$\mathcal{F}$	$\mathcal{W}$	$\tilde{\mathcal{W}}$
Lipschitz Continuous		✓	✓
Stationary	✓		✓



$$S_1[\lambda_1]x = \tilde{\mathcal{W}}_1\{x(t)\} = |x \star \psi_{\lambda_1}| \star \phi$$



Our systems are diffeomorphisms

- Liouville equation
- BBGKY hierarchy
- Master equation
- Vlasov equation
- Boltzmann equation
- multi fluid equations
- Navier-Stokes equations
- MHD equations
- Heat diffusion
- Radiation transport
- Quantum field theory
- Quantum mechanics
- Maxwell's equations
- Newton's equations
- etc.

that is, advection by a vector field

$$\boxed{\frac{\partial \rho^{(N)}}{\partial t} + \mathscr{L}_{u^{(N)}} \rho^{(N)} = 0 \qquad \textbf{Generalized Liouville Equation}}$$

$$\rho^{(n)} \equiv f_n \tau^{(n)} = \text{n-particle distribution form, where } \tau^{(n)} \equiv \prod_{i=1}^{n} \wedge \omega_i$$

$$\mathrm{i}_{u^{(n)}} \omega^{(n)} = -\mathrm{d}H^{(n)}, \text{ where } \omega^{(n)} \equiv \sum_{i=1}^{n} \omega_i$$

$$\boxed{\frac{\partial \rho^{(n)}}{\partial t} + \mathscr{L}_{u^{(n)}} \rho^{(n)} = -n_0 \int_{T^*M} \mathscr{L}_{u^{(n)}_{\text{int}}} \rho^{(n+1)} \qquad \textbf{Generalized BBGKY Hierarchy}}$$

$$u^{(n)}_{\text{int}} \equiv \sum_{i=1}^{n} u_{i,n+1}$$

this is why Lipschitz continuity (invariance under diffeomorphism, deformation, or advection) is such a big deal

$$\boxed{\rho = \text{statistical distribution or QFT state}}$$

Relation of MST to Generalized Master Equation: ideas of Bogoliubov

f_1 relaxes at dynamic rate $= \Omega$

$\bar{f}_1$ evolves at collisional rate $= \dfrac{d\Omega/dt}{\Omega} \ll \Omega$

f_2 relaxes at collision rate

$\bar{f}_2$ evolves at correlation rate $= \dfrac{d^2\Omega/dt^2}{\Omega^2} \ll \dfrac{d\Omega/dt}{\Omega} \ll \Omega$

pullback of first two equations in BBGKY hierarchy,

$$\frac{\partial f_1}{\partial t} + \{f_1, H_1\} = -n_0 \int dp_2\, dq_2\, \{f_2, H_{12}\}$$

$$\frac{\partial f_2}{\partial t} + \{f_2, H_1 + H_2 + H_{12}\} = -n_0 \int dp_3\, dq_3\, \{f_3, H_{13} + H_{23}\}$$

can be reduced to, assuming the separation of rates,

$$\boxed{\frac{\partial \bar{f}_1(p)}{\partial t} \backsim \int dp'\ \bar{f}_2(p', p) - \bar{f}_2(p, p') \qquad \textbf{Generalized Master Equation}}$$

$$= \int dp'\ \bar{f}_1(p')\, k(p', p) - \bar{f}_1(p)\, k(p, p') \qquad k(p, p') \equiv \frac{\bar{f}_2(p, p')}{\bar{f}_1(p)}$$

Relation of MST to Generalized Master Equation: manifold safe Wigner-Weyl transformation

Wigner-Weyl transformation takes operators to/from classical phase space (1927).
The Key is a modified Wigner-Weyl transform that is manifold safe.
Need a local Fourier kernel (Mother Wavelet) with a partition of unity (Father Wavelet).

$$\text{modified Wigner map} = \tilde{W}[\hat{A}] \equiv \int ds\, \psi_p^*(-s) \left\langle q+s \middle| \hat{A} \middle| q-s \right\rangle \psi_p(s) = A(q,p)$$

$$\text{modified Wigner function} = \tilde{W}[\hat{\rho}] = \tilde{W}[\,|f\rangle\langle f|\,] = |f \star \psi_p|^2 = \tilde{W}_f(q,p)$$

Now we can identify and calculate,

$$\bar{f}_1(p) \equiv \mathrm{E}(\tilde{W}[\hat{f}]) = |f \star \psi_p| \star \phi = S_1[p]f$$

$$\bar{f}_2(p,p') \equiv \mathrm{E}(\tilde{W}[\hat{f}\hat{f}]) = ||f \star \psi_p| \star \psi_{p'}| \star \phi = S_2[p,p']f$$

This is the Mayer Cluster expansion on the manifold.

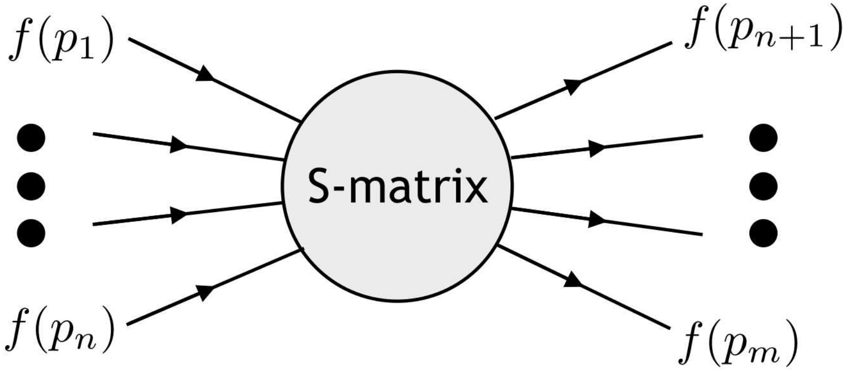
MST as the S-matrix: an alternative dynamical interpretation (I)

From the Lagrangian perspective define the generating function:

$$Z[J] = N \int [df(p)]\, \mathrm{e}^{(i/\hbar)\, S_0[f(p)] + (i/\hbar) \int dp\, J(p)\, f(p)}$$

the connection to the canonical formulation is:

$$S_m(|f\rangle) = \mathrm{E}(T_p(\hat{f}(p_1) \ldots \hat{f}(p_m))\, F(f)) = ||f \star \psi_{p_1}| \cdots \star \psi_{p_m}| \star \phi = \frac{1}{Z[J]} \frac{\delta}{\delta J(p_1)} \cdots \frac{\delta}{\delta J(p_m)} Z[J] \bigg|_{J=0}$$



scattering cross section
generalized Green's function

MST as the S-matrix: an alternative dynamical interpretation (II)

define the effective action through Legendre transform:

$$S[\varphi(p)] = -\ln Z[J] + \int dp\, J(p)\, \varphi(p)$$

expanding in S and φ it can be shown that:

$$S_1(|f\rangle) = |f \star \psi_p| \star \phi = \mathrm{E}(\hat{f}(p)\, F(f)) = \frac{1}{Z[J]} \frac{\delta Z[J]}{\delta J(p)}\bigg|_{J=0} = \varphi_0(p) =$$

classical action averaged over fluctuations as a function of inverse renormalization scale

$$= \bar{f}_1(p)$$

$$S_2(|f\rangle) = ||f \star \psi_p| \star \psi_{p'}| \star \phi = \mathrm{E}(\hat{f}(p)\, \hat{f}(p')\, F(f)) = \frac{1}{Z[J]} \frac{\delta^2 Z[J]}{\delta J(p) \delta J(p')}\bigg|_{J=0} = \frac{1}{m(p,p')} =$$

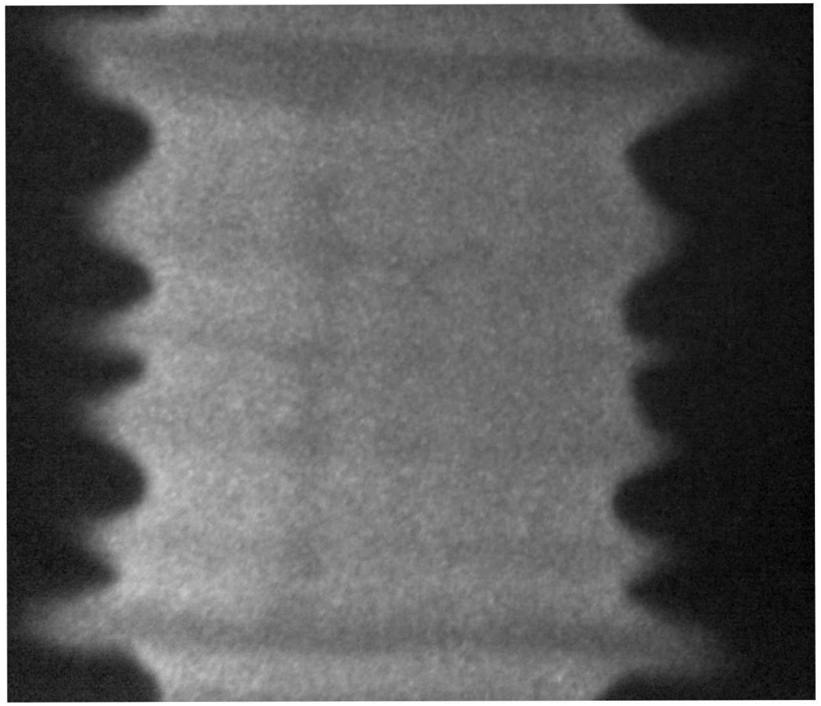
two state scattering cross section (scale dependent renormalization mass) as a function of initial and final inverse renormalization scale

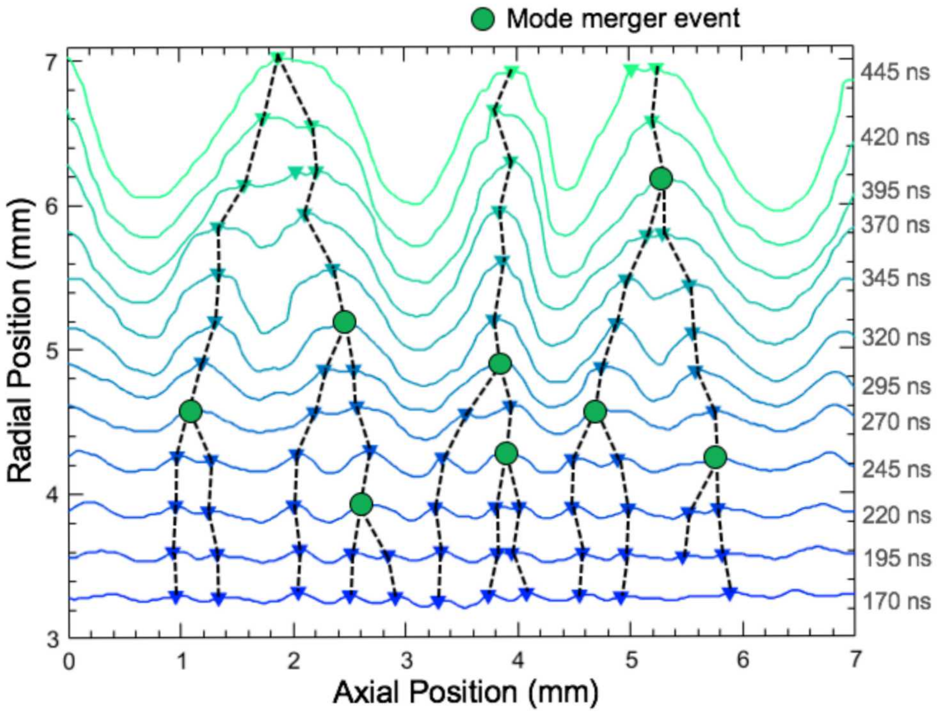
$$= \bar{f}_2(p,p')$$

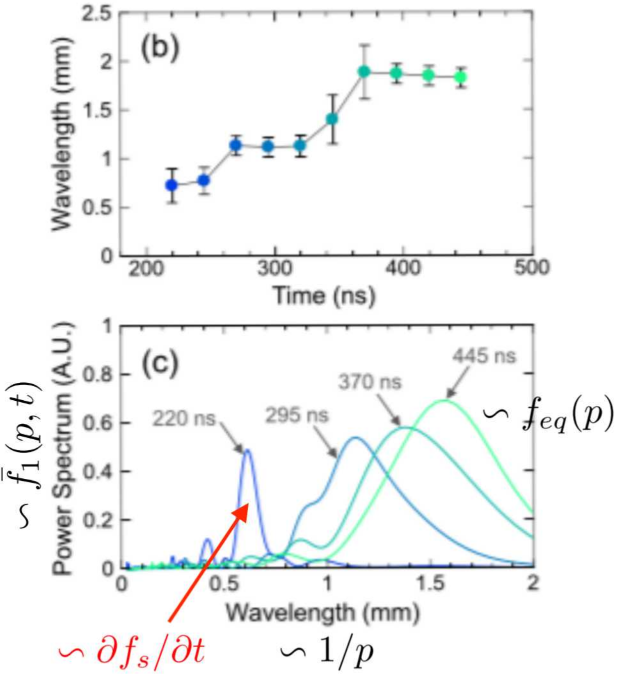
MST and kinetics (that is PDE solution)

- MST is advected with the diffeomorphism of a vector field on a manifold
  - leads to a set of “extended energies” or topological invariants, that are advected by the flow
- group symmetries can be built into the transformation
  - leads to additional constants advected by the flow
- MST is the “pull back” of the set of N particle distribution forms (i.e., density operators) using a modified version of the Wigner-Weyl transformation (mother wavelet with compact support replaces Fourier kernel, father wavelets are partition of unity)
  - Nth order MST is the Nth order Wigner function, that is N-particle correlation function
  - BBGKY hierarchy on manifolds gives evolution of the Nth order distribution function as an advection modified by a “collision operator” resulting from interaction with the N+1 particle (advective functional of the N+1 order distribution function)
- therefore, MST is the natural coordinate system to analyze statistical mechanics and kinetics
  - MST are constants for a steady state system allowing construction of the canonical ensemble following ideas of Jaynes
  - examples of generalized advective-collisional systems are:
    - Liouville equation
    - Boltzmann equation
    - Vlasov equation
    - MHD
    - Navier-Stokes
    - quantum field theory
    - quantum mechanics

University of Michigan experiments track mode merger of liner structure



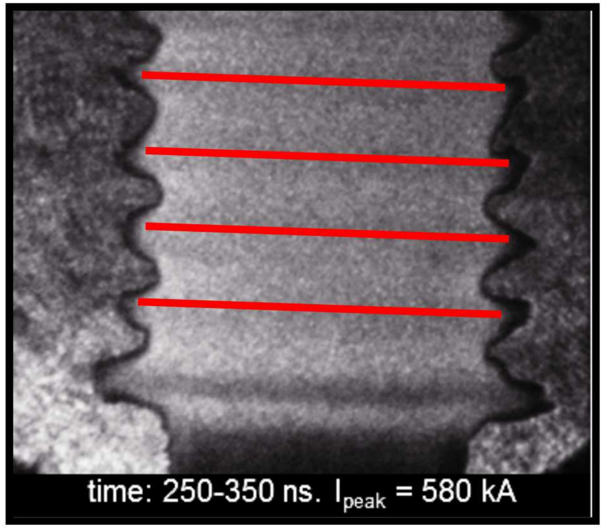




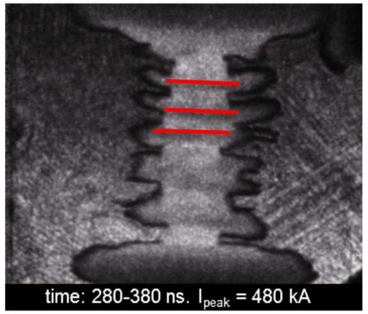
both with 1 T axial magnetic field and without

Axial magnetic field nonlinearly stabilizes liner perturbations into helical structure

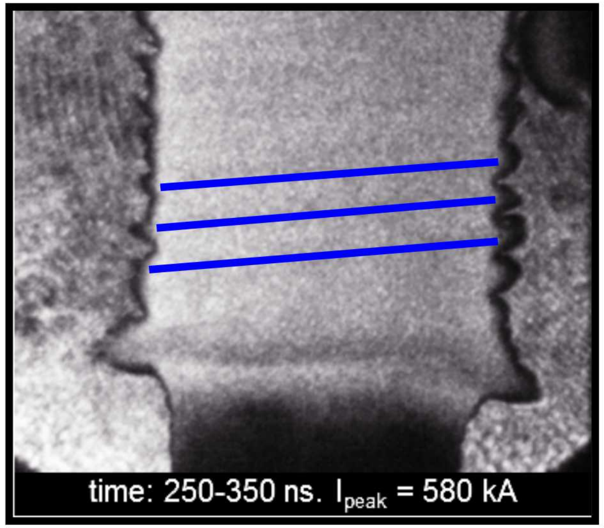
Bz = 0



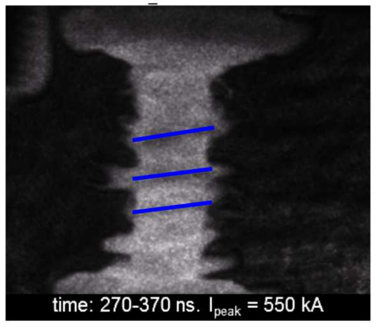
Unmagnetized
- Horizontal striations
- *m* = 0 sausage mode

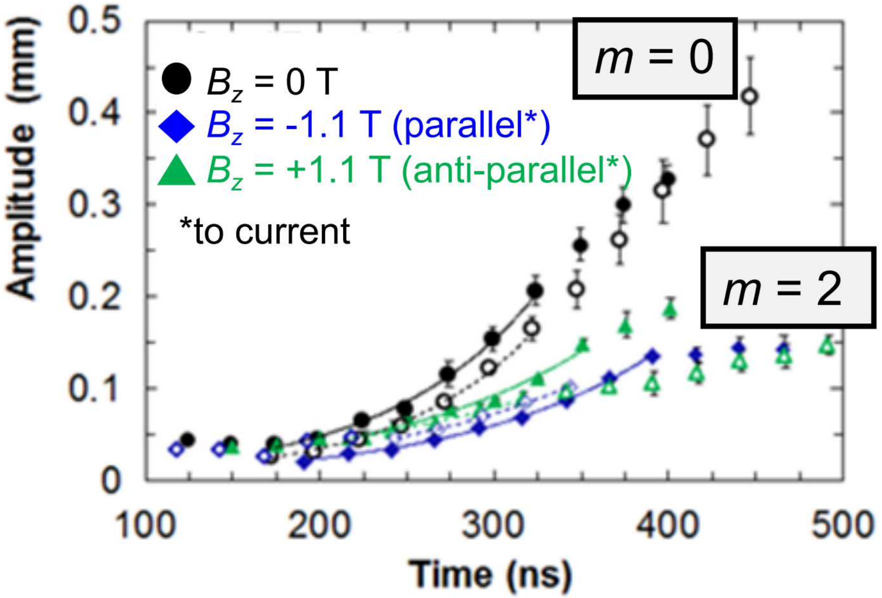


Bz = 1.1 T

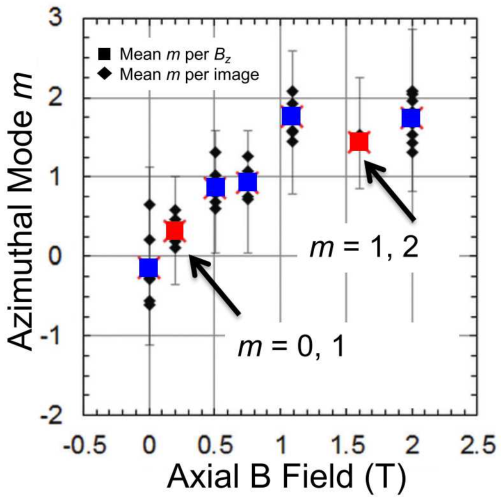


Magnetized
- *m* = 2 helical mode
- Reduced amplitude
- Reverse B_z, striations also reverse



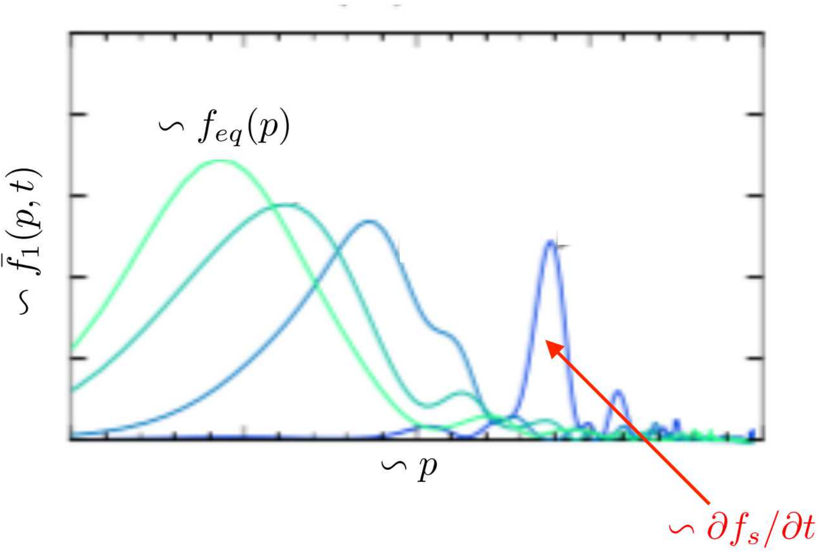


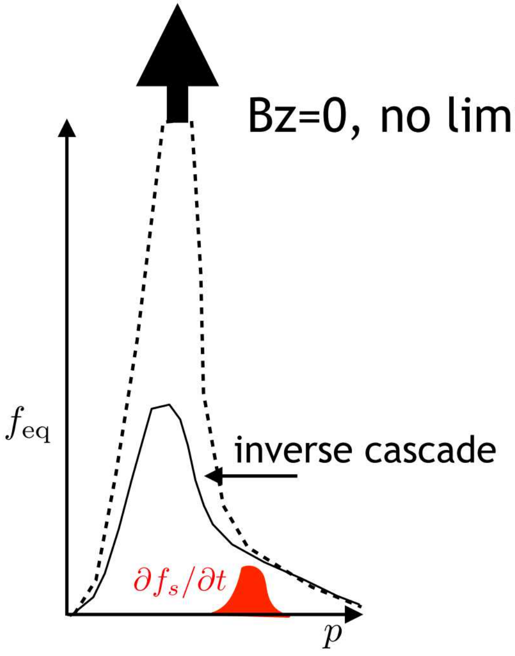
mode dependent on B, smaller than mode with largest linear growth rate



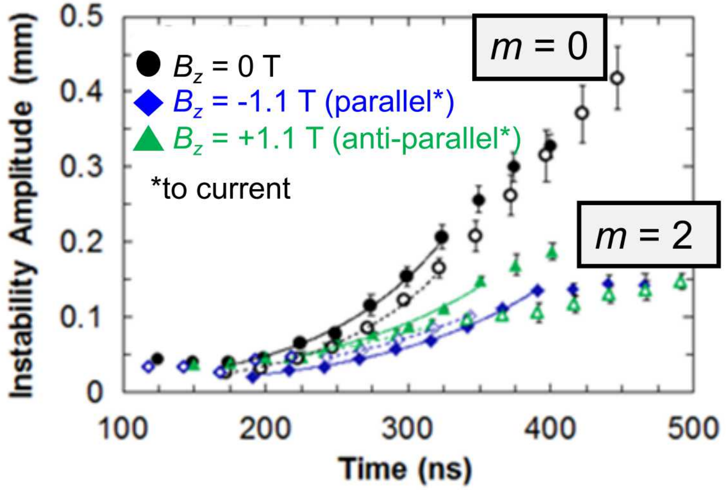
structure persists throughout implosion and bounce of liner

Look at what is happening to distribution function

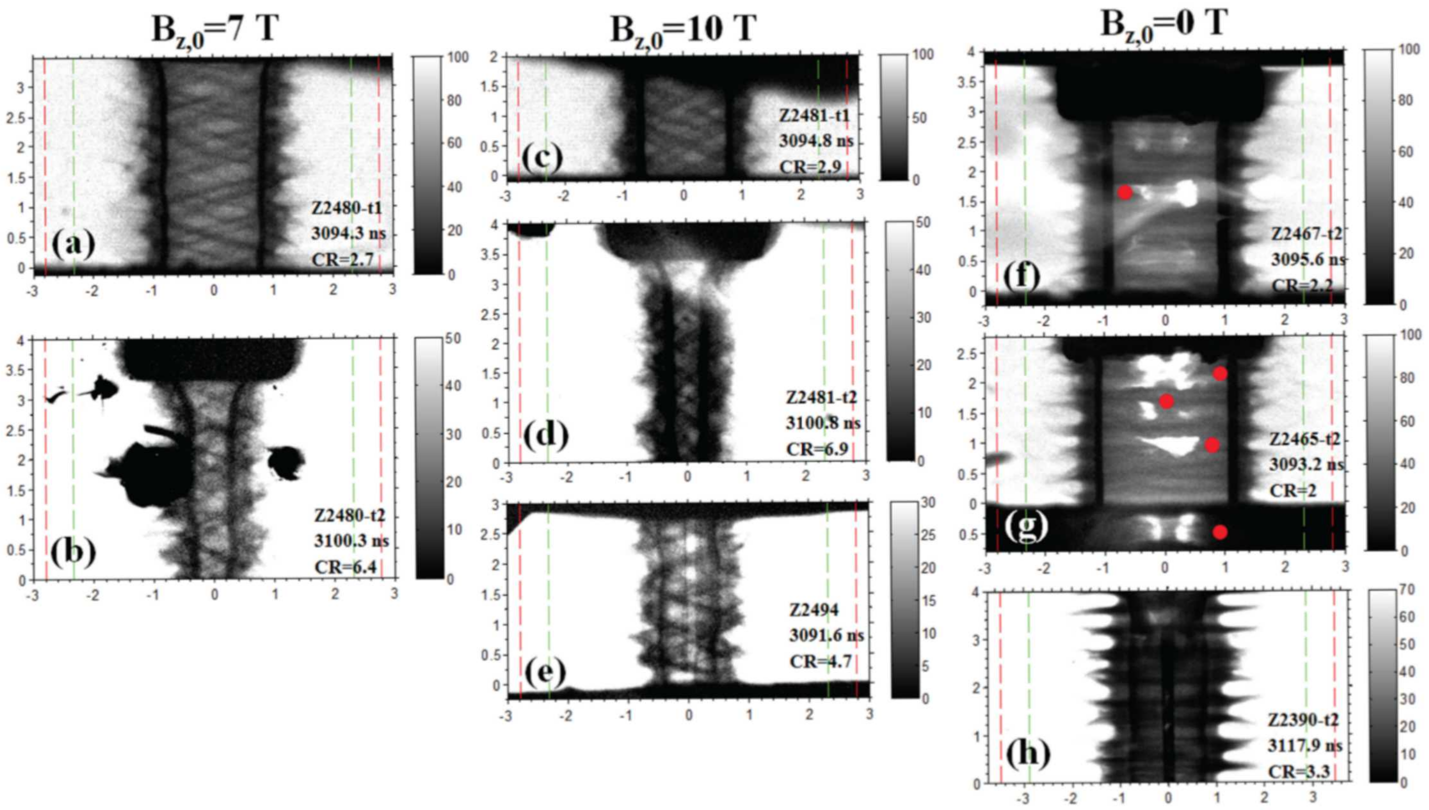


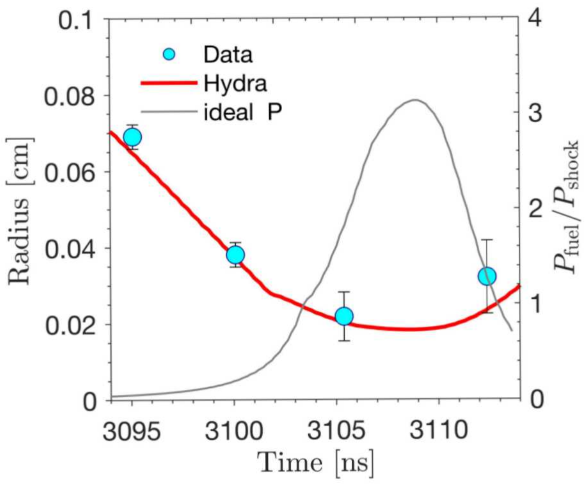


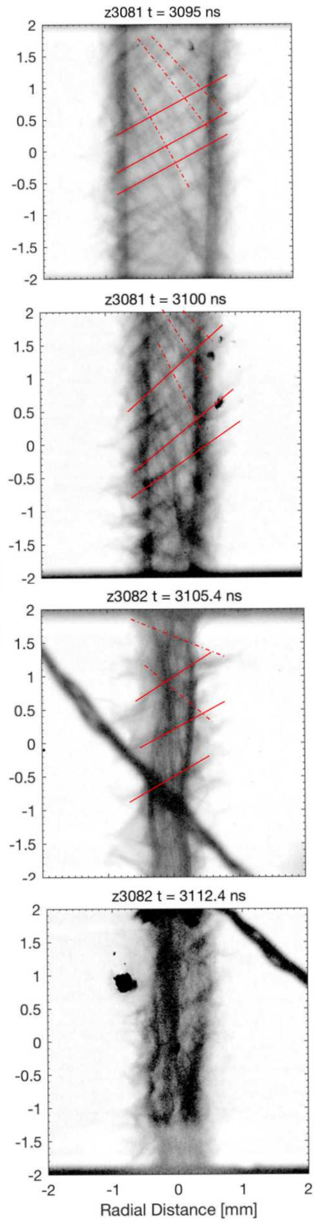
Bz=0, no limit on size of probability container



Further evidence for nonlinear helical structure of liner, stabilized by axial magnetic field





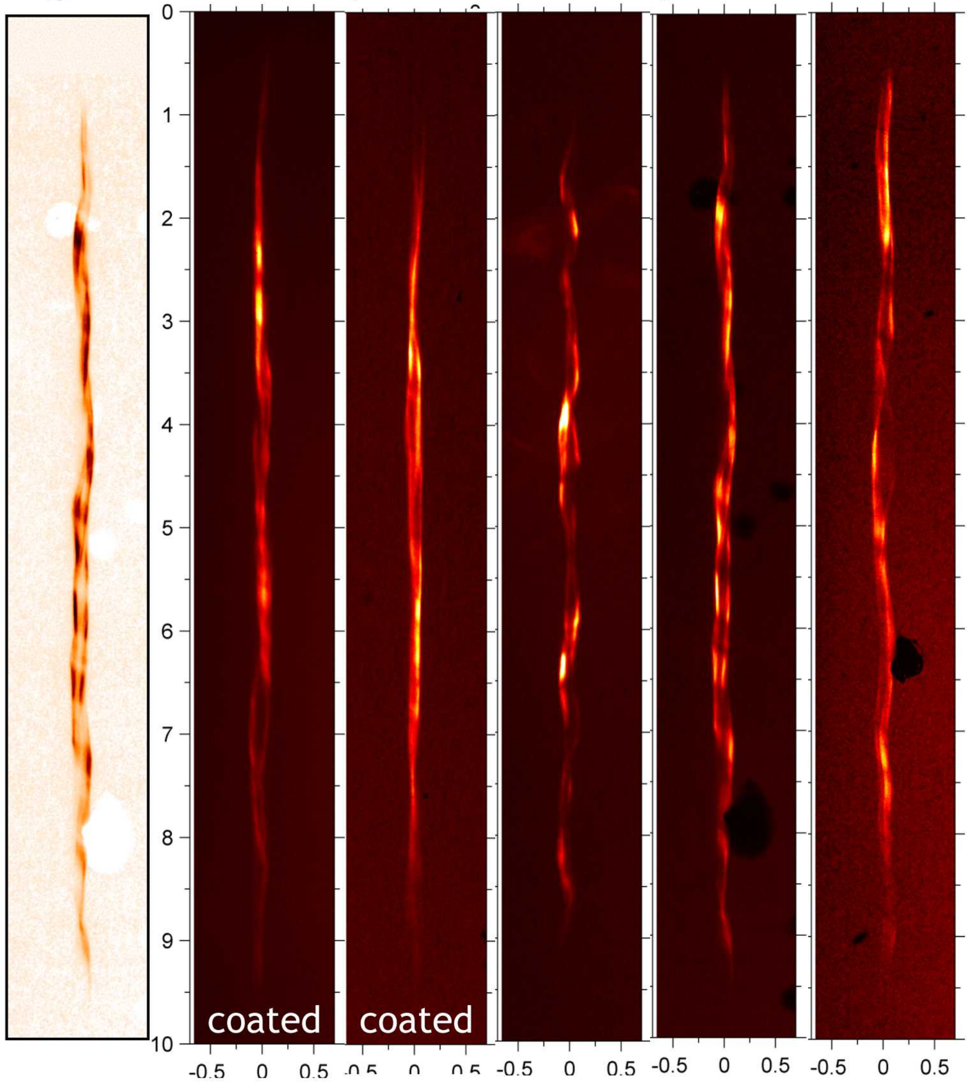


Self emission images of stagnation shows axial magnetic field nonlinearly stabilizes plasma into helical structures

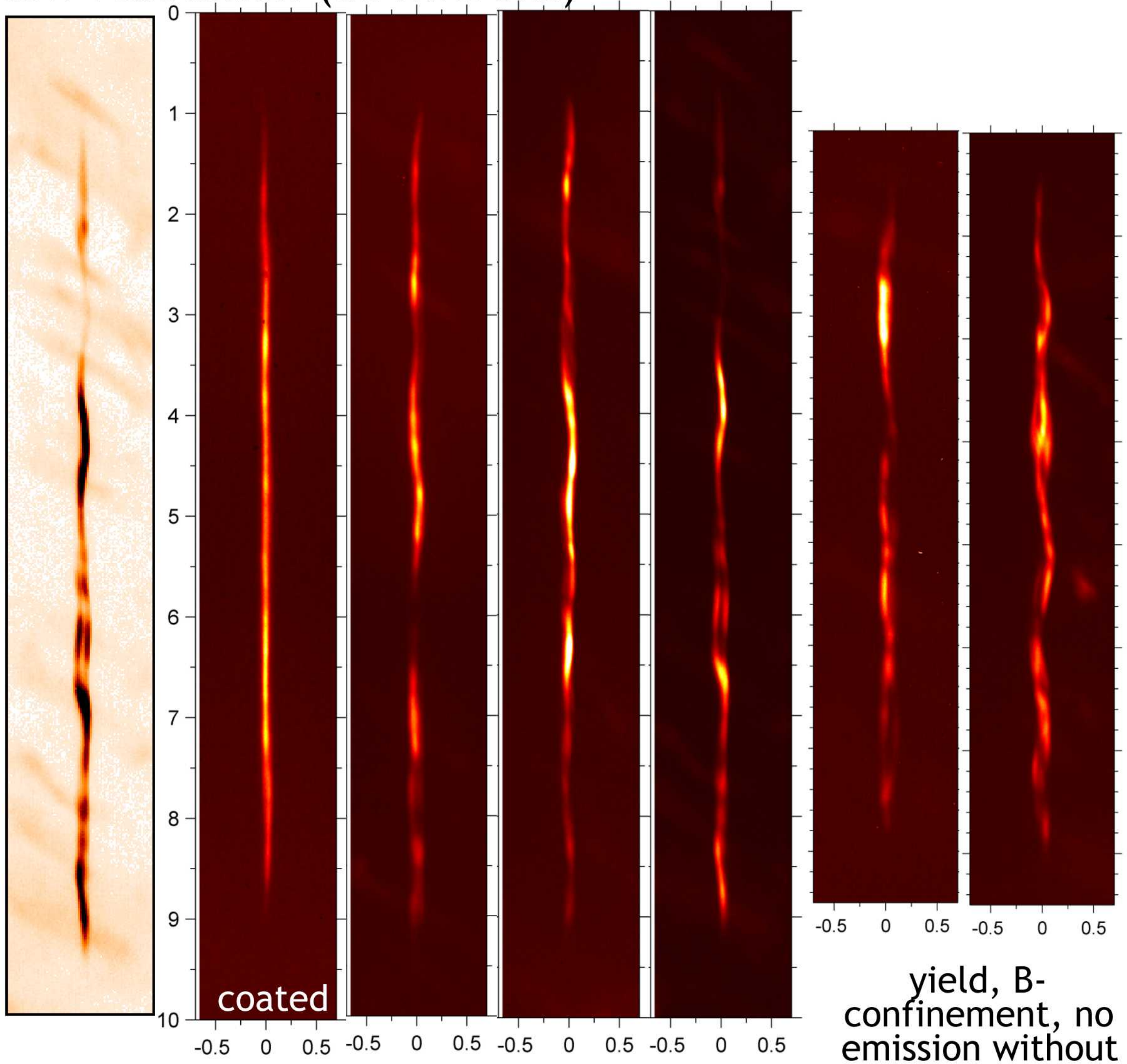
no Bz

no yield or B-confinement or bulk convergence

high resolution (15-20 microns)



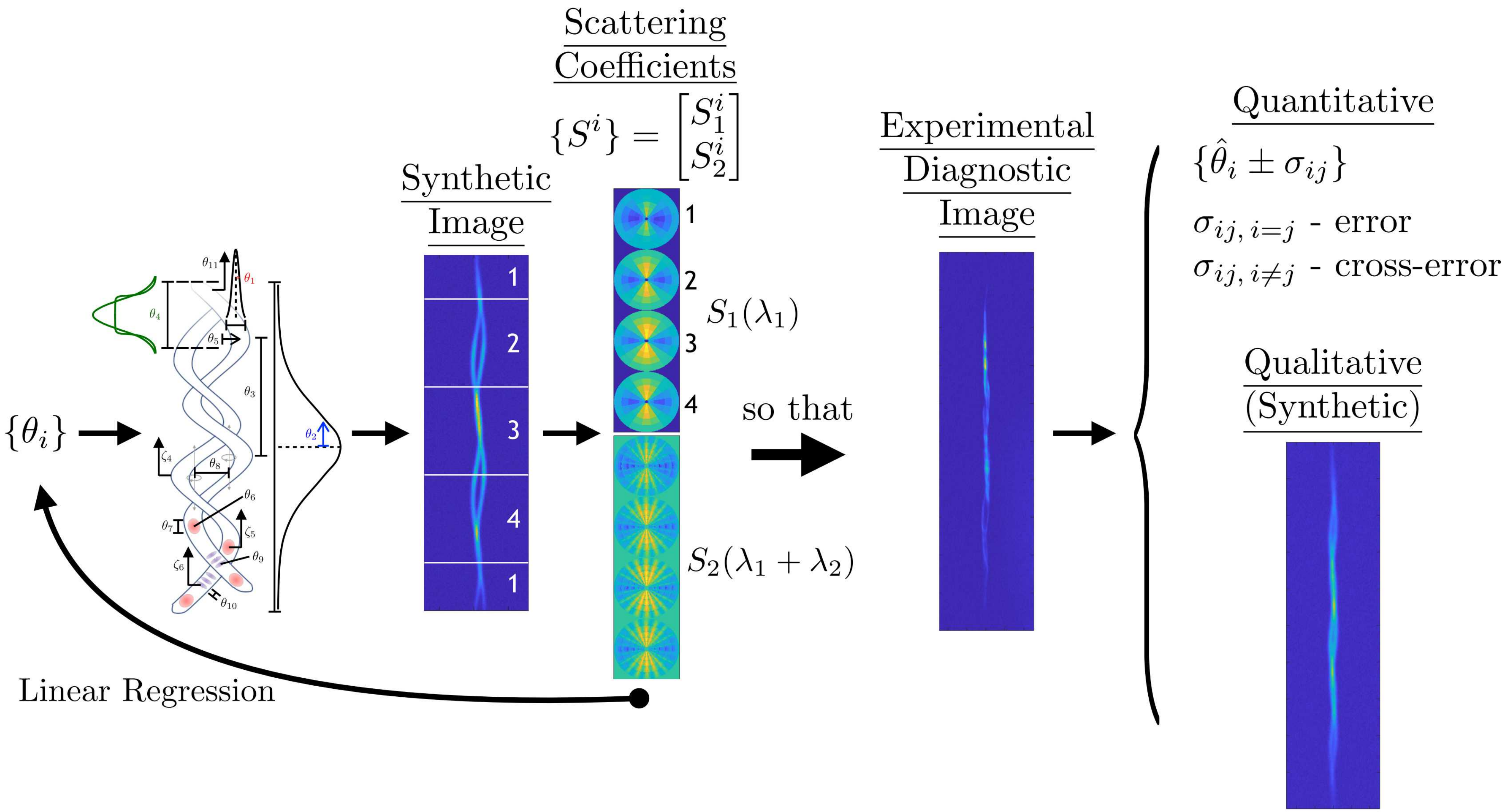
low resolution (60 microns)



yield, B-confinement, no emission without preheat

diameter ranges from 120 to <23 microns, CR from 38 to >200

Regression for parameters of double helical structure from MST



$\{\theta_i\}$ →

Synthetic Image →

Scattering Coefficients

$$\{S^i\} = \begin{bmatrix} S^i_1 \\ S^i_2 \end{bmatrix}$$

$S_1(\lambda_1)$

$S_2(\lambda_1 + \lambda_2)$

so that →

Experimental Diagnostic Image →

Quantitative

$\{\hat{\theta}_i \pm \sigma_{ij}\}$

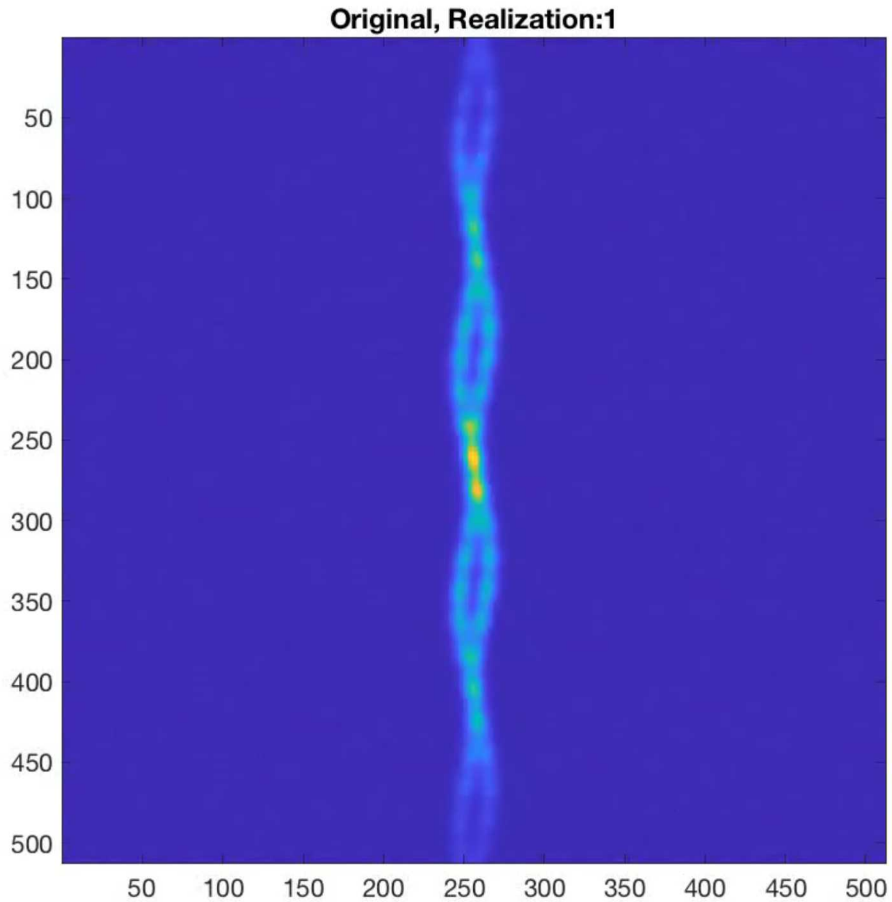
$\sigma_{ij,\, i=j}$ - error

$\sigma_{ij,\, i\neq j}$ - cross-error

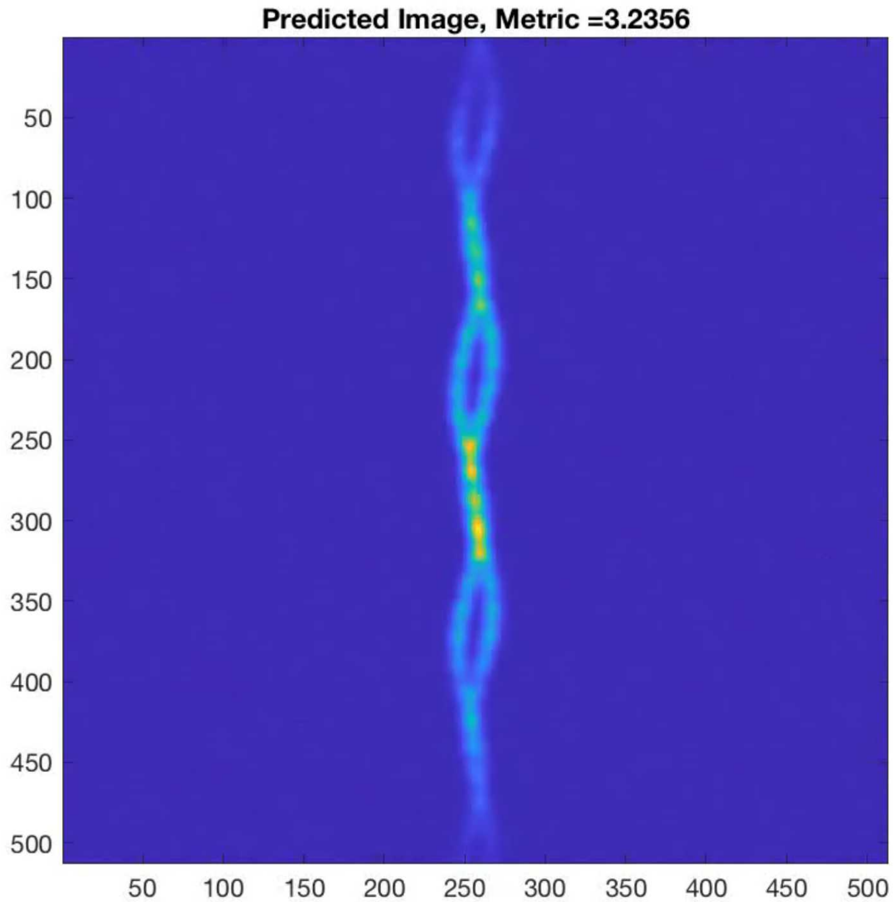
Qualitative (Synthetic)

Linear Regression

Regression ensemble: predicted vs actual



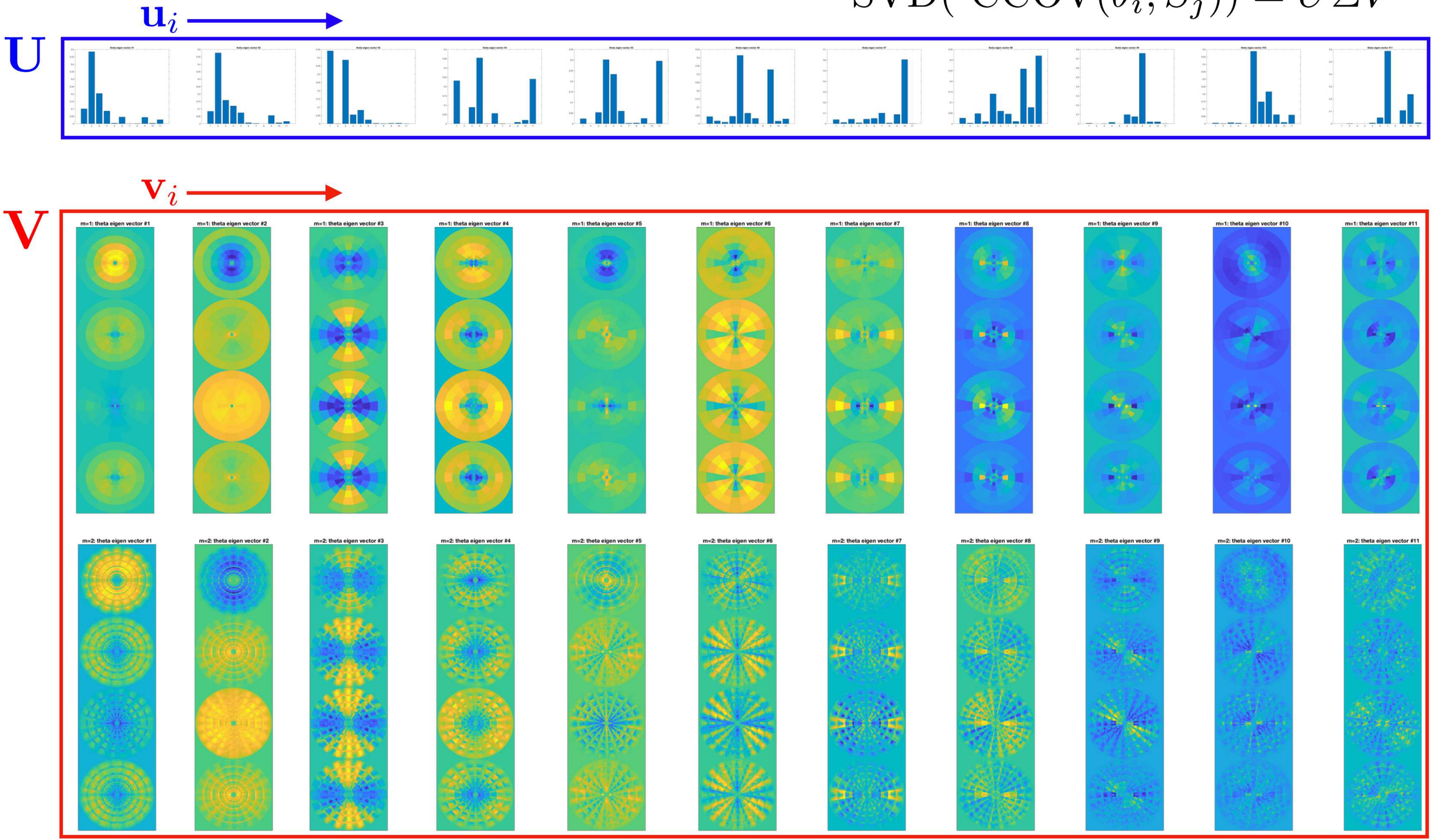
Actual



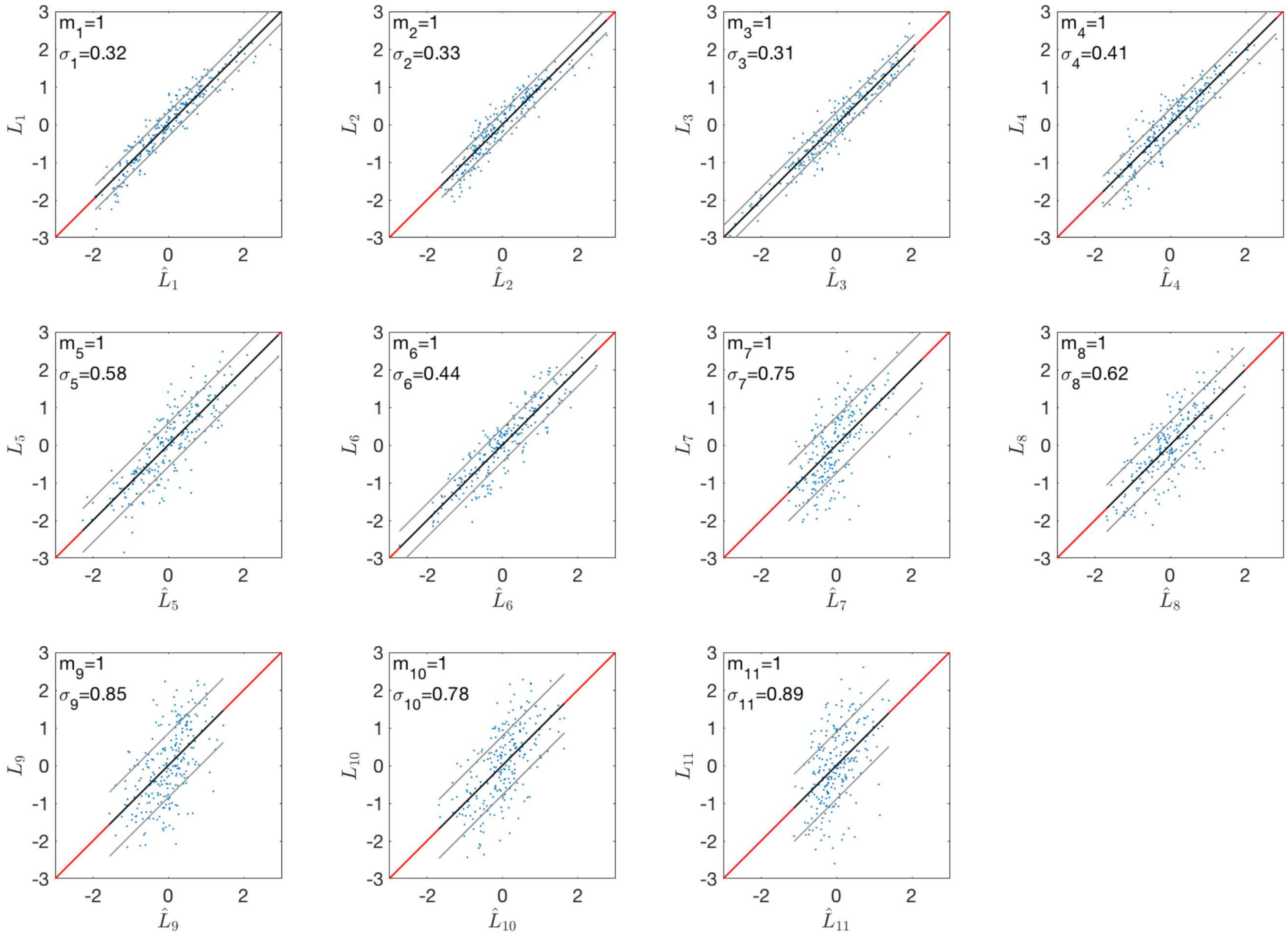
Predicted

Principal components

$$\mathrm{SVD}(\ \mathrm{CCOV}(\theta_i, S_j)) = U\Sigma V^T$$

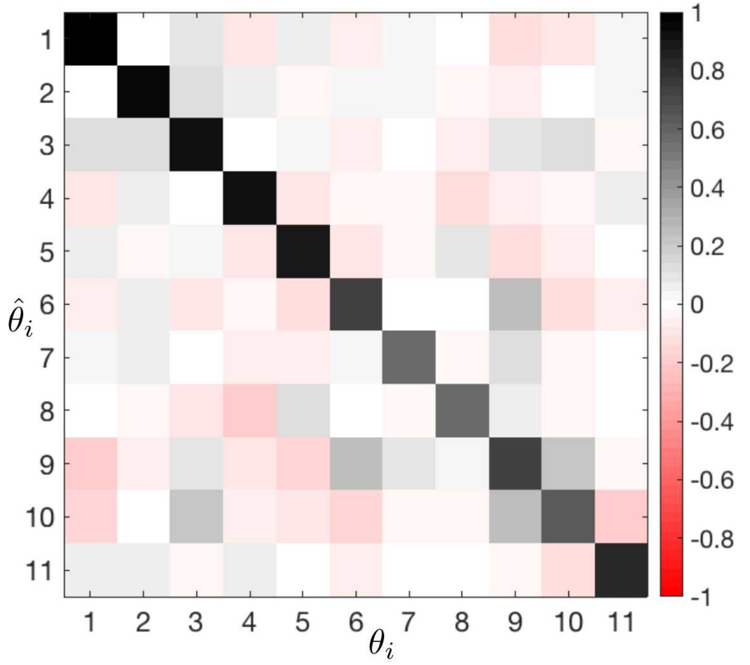


Linear regression performance is remarkable

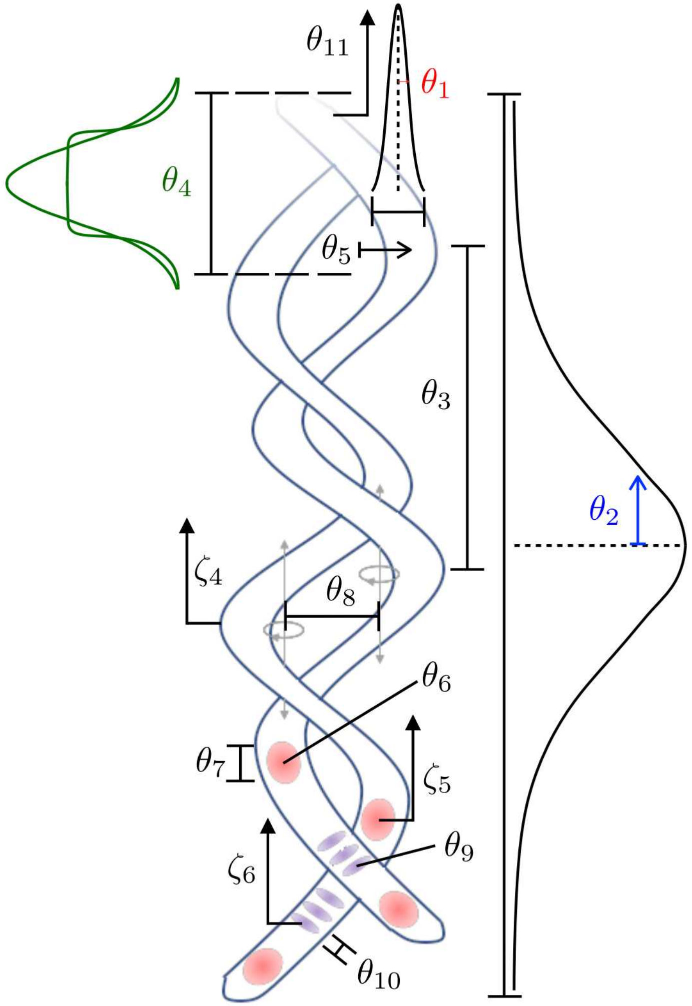


$$L \equiv \theta U$$
$$X \equiv SV$$
$$L_i = m_{ij} X_j + b_i$$

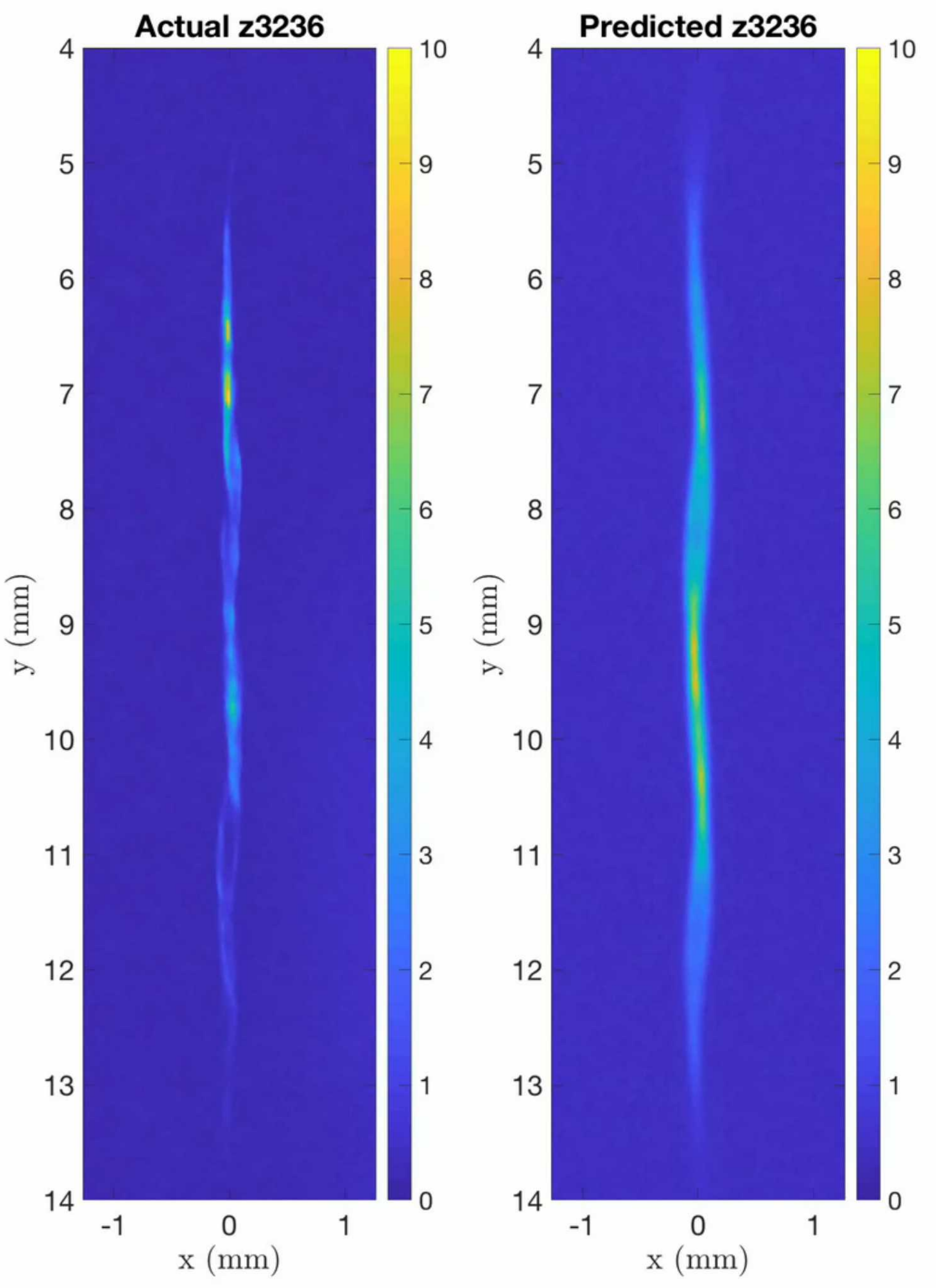
θ correlation



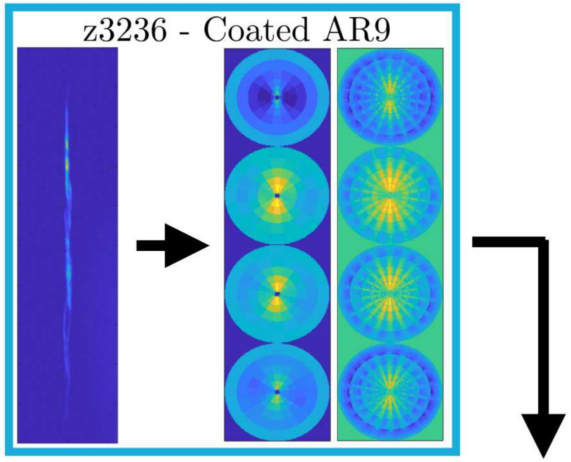
Fit to experimental image (coated AR9)

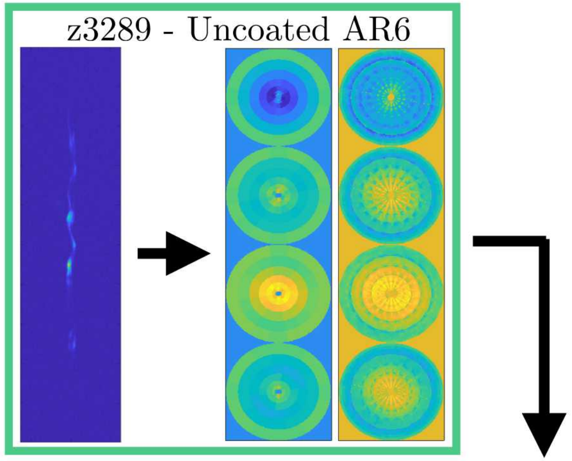


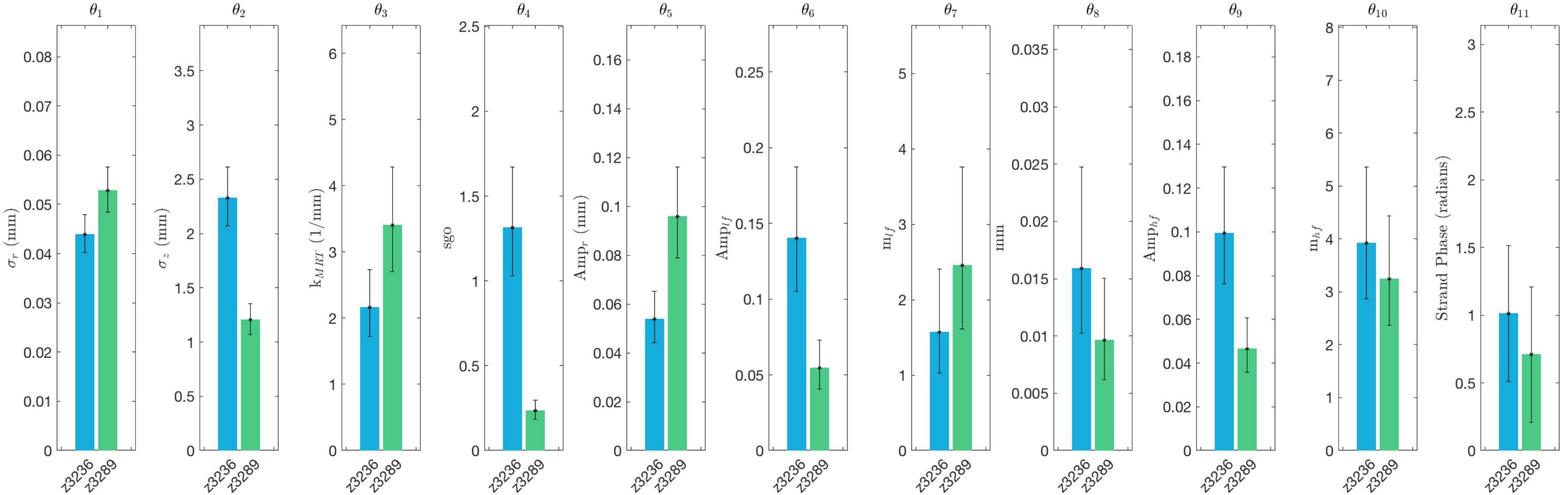
$$\begin{bmatrix}
\theta_1 = 0.0439 + (-0.0036\,, +0.0040) \\
\theta_2 = 2.3300 + (-0.2550\,, +0.2870) \\
\theta_3 = 2.1700 + (-0.4460\,, +0.5610) \\
\theta_4 = 1.3100 + (-0.2820\,, +0.3590) \\
\theta_5 = 0.0538 + (-0.0095\,, +0.0115) \\
\theta_6 = 0.1400 + (-0.0352\,, +0.0470) \\
\theta_7 = 1.5700 + (-0.5440\,, +0.8330) \\
\theta_8 = 0.0159 + (-0.0057\,, +0.0089) \\
\theta_9 = 0.0994 + (-0.0232\,, +0.0303) \\
\theta_{10} = 3.9200 + (-1.0500\,, +1.4400) \\
\theta_{11} = 1.0100 + (-0.5009\,, +0.5009)
\end{bmatrix}$$



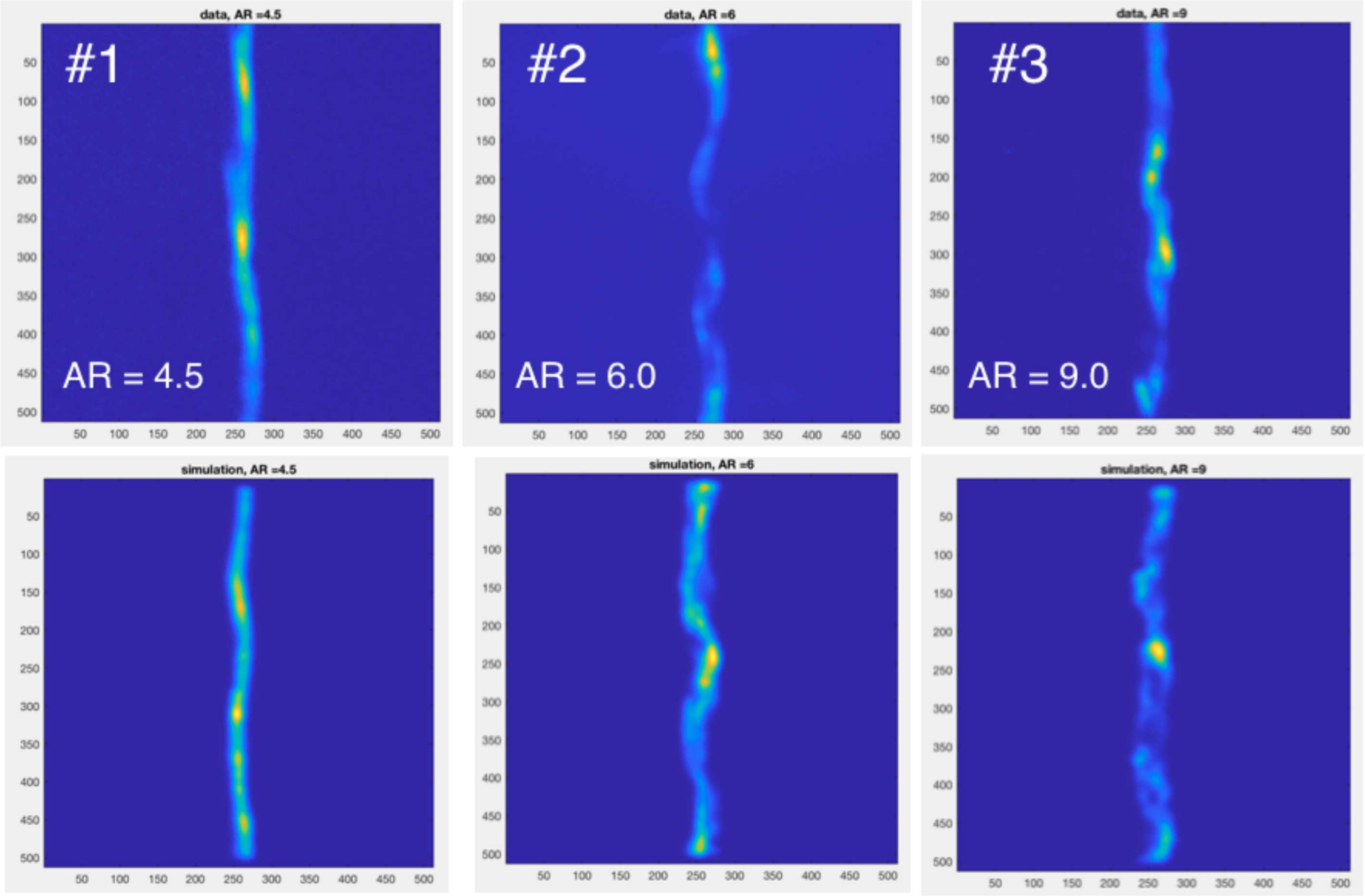
Comparative Results - coated AR9 vs uncoated AR6







Comparison of Gorgon computer simulation to experimental data



dat

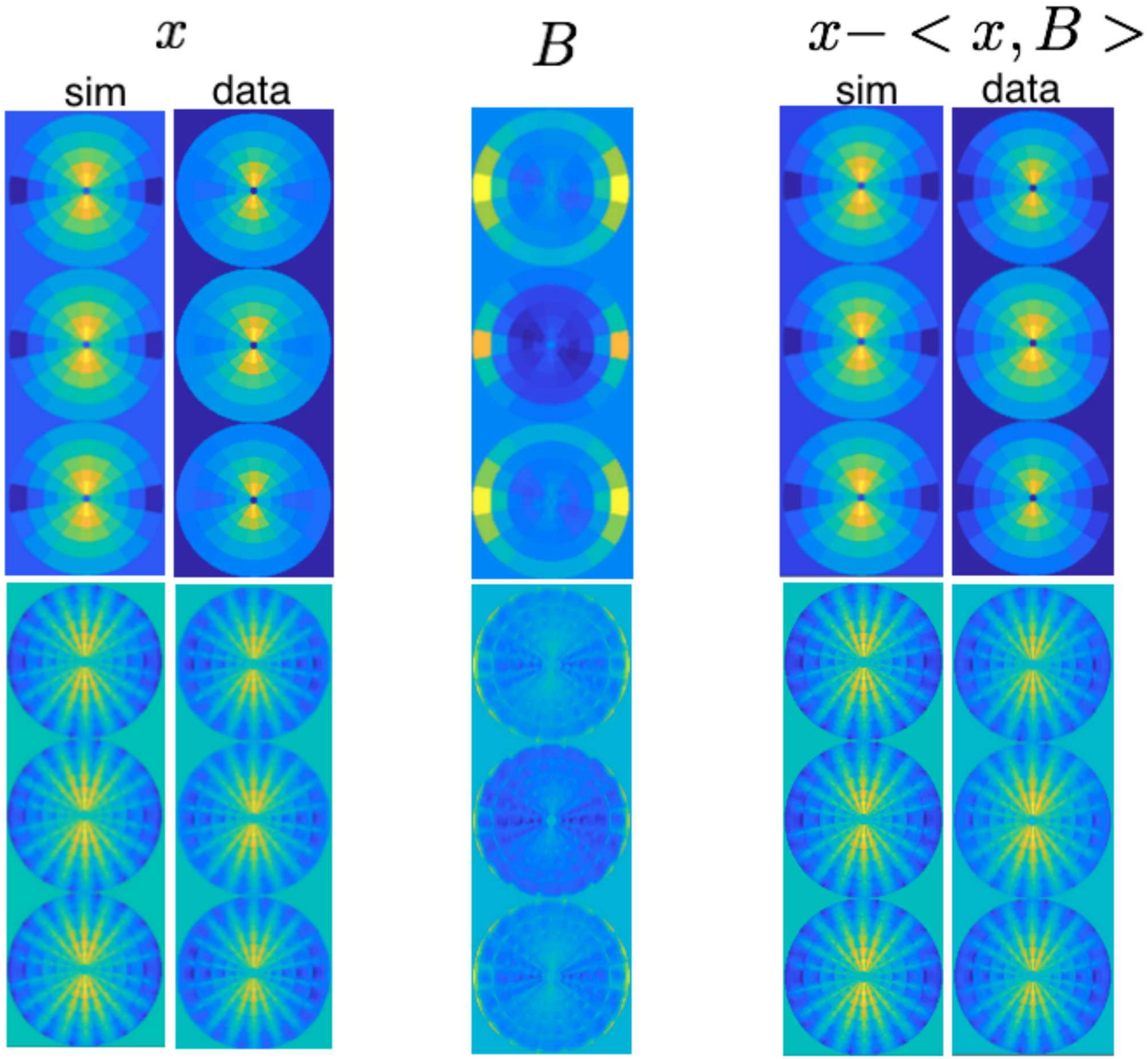
sim

Significant distortion between simulation and experiment

solution: first principal component of simulation to data covariance projected out (effective background subtraction)

AR = 4.5

note: average interclass distance is about 10-20, while the average intraclass distance is about 2-5 (in the synthetic dataset)



Quantification of stagnation image morphology

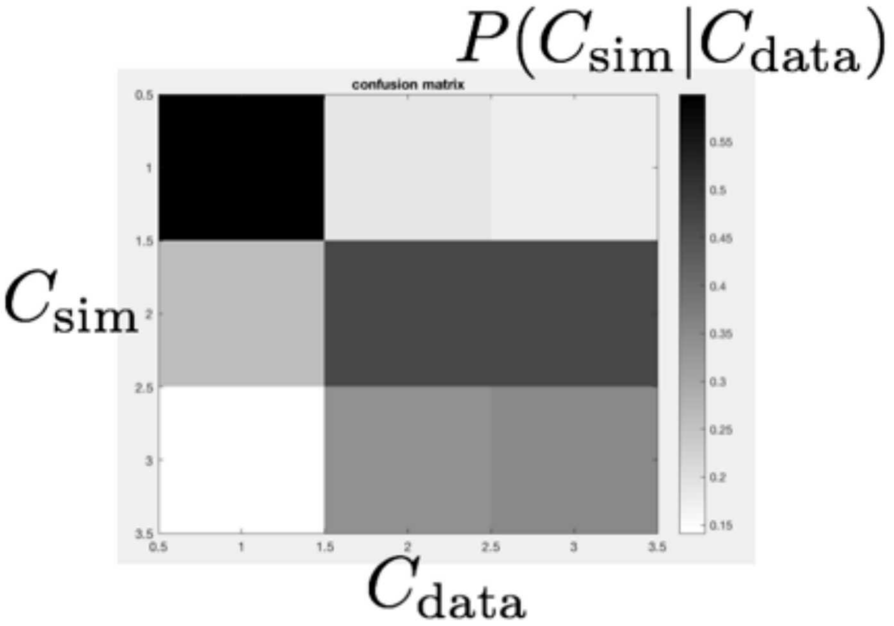
- metric quantifies similarities between simulation and experiment
  - enables use of images in UQV
  - allows quantified statements to be made about morphology
- for example here we can state:
  - little difference between AR6 and AR9 data
  - AR6 simulation matches both AR6 and AR9 data
  - AR4 data significantly different and matches simulation well

```
separation_matrix =

    1.0000    3.7279    2.4980
    4.1202    1.0000    0.5997
    7.6733    1.7911    1.0000

confusion_matrix =

    0.5998    0.1914    0.1753
    0.2588    0.4696    0.4687
    0.1414    0.3389    0.3560
```



Progress to date and the future

- progress to date
  - understanding of physical significance of MST
    - why it works so well
    - how it should be used
  - regression to helical parameters (remarkably linear)
  - advanced background subtraction
  - quantitative metric of morphology (that is, steady state nonlinear structure or emergent behavior)
- future
  - apply to radiographs as done to stagnation images
  - derive radiograph dynamics from MST of radiographs
  - establish connection between MagLIF implosion parameters and MST of stagnation image
    - **predict the scaling of MagLIF implosion morphology with uncertainty, that is establish "credible scaling" of morphology**