

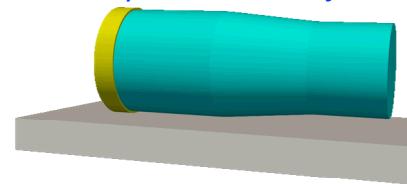
## Modular Hyperelastic Plasticity Modeling

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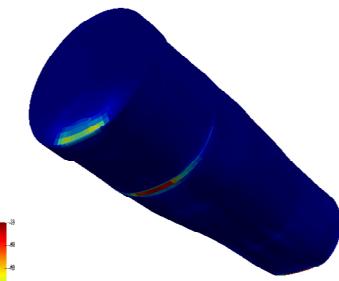
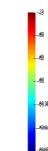
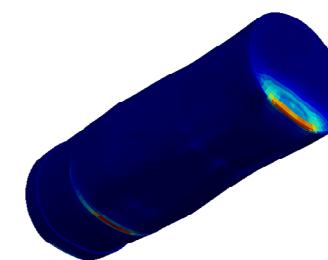
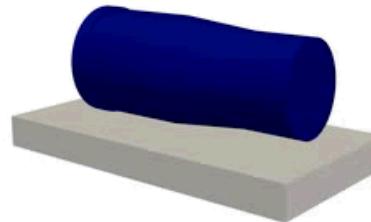
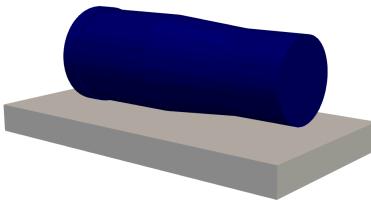
Sandia National Laboratories / California

WCCM13; Manhattan, NY; July 22-27, 2018

# Motivation



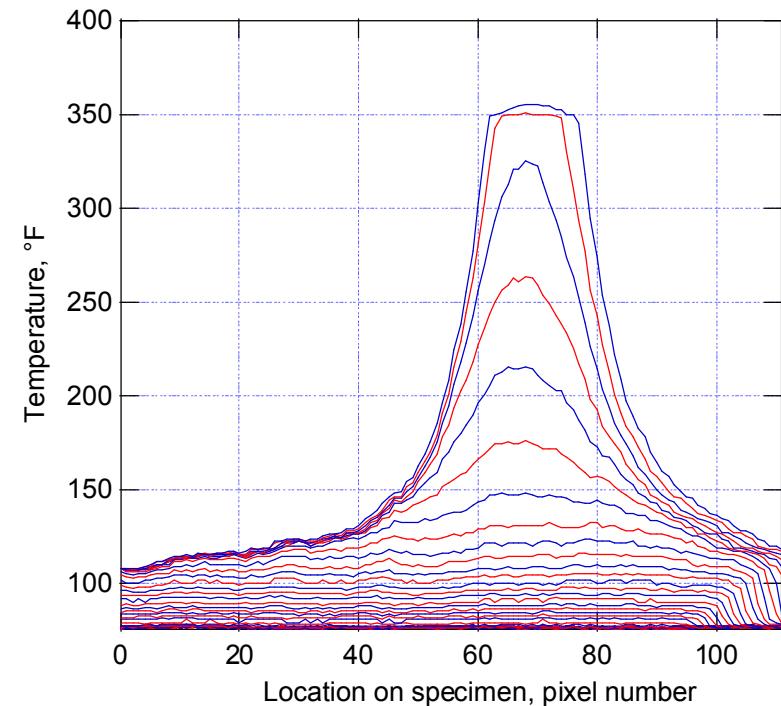
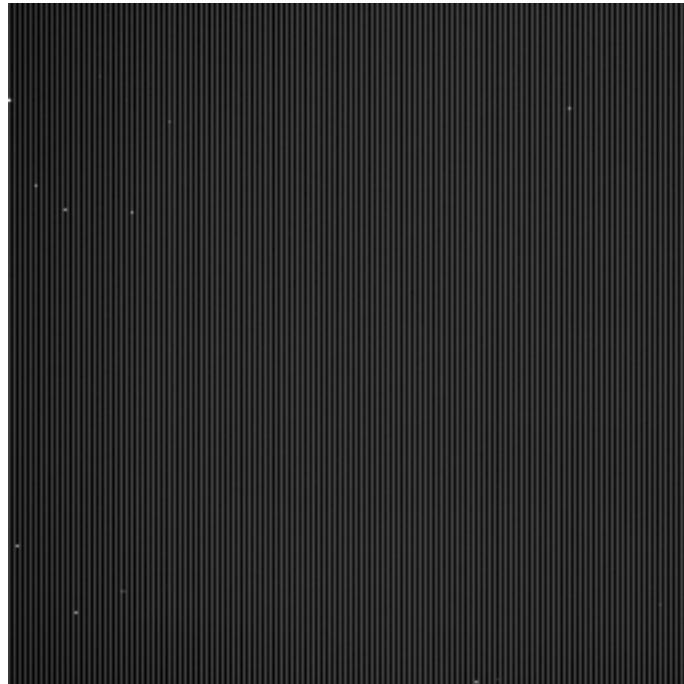
- Engineering environments demand constitutive models that can resolve large plastic deformations
- Structural alloys typically have strain rate dependence, temperature dependence, and anisotropy
- Often questions to be answered include “when does it break?”
- Objective: *develop a program to understand the application environment (loading, strain rate, temperature), test materials within and at the bounds of that environment, and develop constitutive models to capture the observed physical behavior of the data*



Generic engineered system constructed from structural alloys impacting a hard target. Loading produces multi-axial states of stress throughout the components with variable strain rates, inducing temperature change due to plastic work.

# Motivation

PH13-8 sheet, strain rate  $\sim 1 \text{ s}^{-1}$



*Structural alloys can generate a significant amount of heat during plastic deformation, even at moderate strain rates.*

*Courtesy of Bonnie Antoun, Mechanics of Materials, Sandia/CA.*

# Theoretical and practical considerations



- Material behavior in application environments suggests a plasticity formulation that includes a number of physical phenomena: specifically temperature dependence, rate dependence, anisotropy, and ultimately fracture/failure
- Strive to develop such a formulation as rigorously as possible
- Start with a Helmholtz free energy, and incorporate ideas from continuum thermodynamics
- The plasticity solve incorporates ideas from the optimization literature to perform an implicit return mapping procedure
- To achieve impact the target is Sandia's production finite element code for solid mechanics, SIERRA-SM, imposing a few constraints in terms of thermo-mechanical coupling, etc...
- To achieve impact we are mindful of the trade off between accuracy and efficiency

# Formulation - plasticity

- Multiplicative decomposition of the deformation gradient

$$\mathbf{F} = \mathbf{F}_m \mathbf{F}_\theta, \quad \mathbf{F}_m = \mathbf{F}_e \mathbf{F}_p, \quad \det \mathbf{F}_p = 1$$

- Isotropic thermal expansion via thermal stretch ratio

$$\mathbf{F}_\theta = \beta(\theta) \mathbf{I} \rightarrow \mathbf{F}_m = \mathbf{F} \mathbf{F}_\theta^{-1} = \frac{1}{\beta(\theta)} \mathbf{F}$$

- Choice of strain – log strain

$$\mathbf{E}_e = \frac{1}{2} \log \mathbf{C}_e$$

- Helmholtz free energy

$$\psi = \mu(\theta) |\text{dev}(\mathbf{E}_e)|^2 + \frac{1}{2} \kappa(\theta) \text{tr}(\mathbf{E}_e)^2 + \frac{1}{2} h(\theta) \alpha^2$$

- Stresses, Mandel and Cauchy

$$\mathbf{M} = \frac{\partial \psi}{\partial \mathbf{E}_e} = 2\mu(\theta) \text{dev } \mathbf{E}_e + \kappa(\theta) (\text{tr } \mathbf{E}_e) \mathbf{I}, \quad \boldsymbol{\sigma} = \frac{1}{J} \mathbf{F}_e^{-T} \mathbf{M} \mathbf{F}_e^T$$

# Formulation - damage

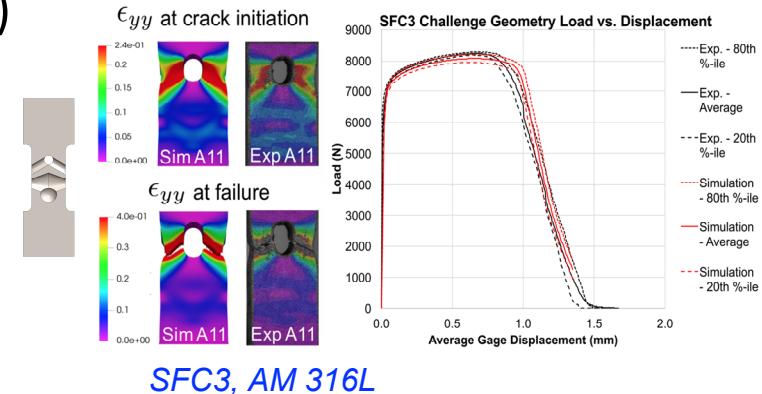
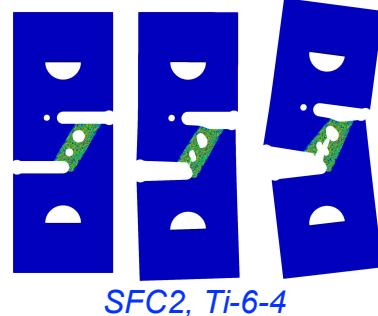
- Historically the computational mechanics group at Sandia/CA has explored usage of the void growth model due to Cocks and Ashby

$$\dot{\phi} = \sqrt{\frac{2}{3}} \dot{\epsilon}_p \frac{1 - (1 - \phi)^{m+1}}{(1 - \phi)^m} \sinh \left[ \frac{2m - 1}{2m + 1} \frac{p}{\bar{\sigma}_e} \right]$$

- Additionally we have employed an additive set of terms intended to treat shear behavior through void nucleation

$$\dot{\eta} = \eta \dot{\epsilon}_p \left( N_1 \left[ \frac{4}{27} - \frac{J_3^2}{J_2^3} \right] \right) + N_3 \left[ \frac{\langle p \rangle}{\bar{\sigma}_e} \right]$$

- The combination of damage models has successfully been applied to the Sandia Fracture Challenges (SFC2, SFC3)



# Implementation/Design

- This work fits in the context of the previous talk, i.e. robust integration algorithms for a modular set of plasticity models (Scherzinger, Lester)
- High level algorithm

$$\mathbf{F}_{e,n+1}^{tr} = \mathbf{F}_{m,n+1} \mathbf{F}_{p,n}$$

$$\mathbf{E}_{e,n+1}^{tr} = \frac{1}{2} \log ([\mathbf{F}_{e,n+1}^{tr}]^T \mathbf{F}_{e,n+1}^{tr})$$

$$\mathbf{M}^{tr} = 2\mu(\theta) \text{dev } \mathbf{E}_e^{tr} + \kappa(\theta) (\text{tr } \mathbf{E}_e^{tr}) \mathbf{I}$$

if  $\phi(\mathbf{M}^{tr}) > \bar{M}(\bar{\varepsilon}^p) \rightarrow \text{RMA}$

$$\mathbf{F}_{p,n+1} = \exp \left( \Delta \gamma \frac{\partial \phi}{\partial \mathbf{M}} \right) \mathbf{F}_{p,n}$$

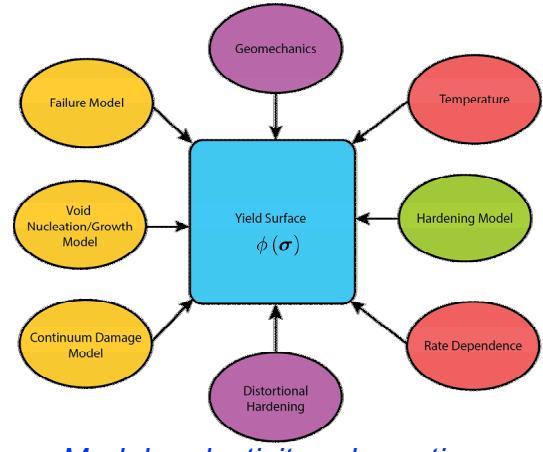
$$\text{dev}(\mathbf{M})_{n+1} = \text{dev}(\mathbf{M}^{tr}) - 2\mu \Delta \gamma \frac{\partial \phi}{\partial \mathbf{M}}$$

$$\mathbf{M}_{n+1} = \text{dev}(\mathbf{M})_{n+1} + \kappa \text{tr}(\mathbf{E}_e^{tr}) \mathbf{I}$$

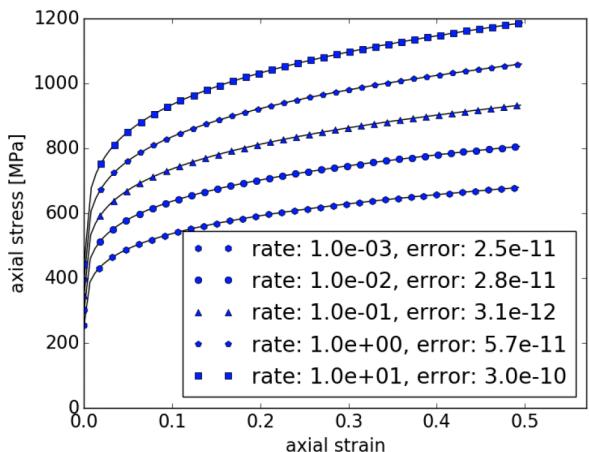
$$\mathbf{F}_{e,n+1} = \mathbf{F}_{m,n+1} \mathbf{F}_{p,n+1}$$

$$\boldsymbol{\sigma}_{n+1} = \frac{1}{J} \mathbf{F}_{e,n+1}^{-T} \mathbf{M}_{n+1} \mathbf{F}_{e,n+1}^T$$

$$F(\boldsymbol{\sigma}, \bar{\varepsilon}^p) = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\bar{\varepsilon}^p) \leq 0$$



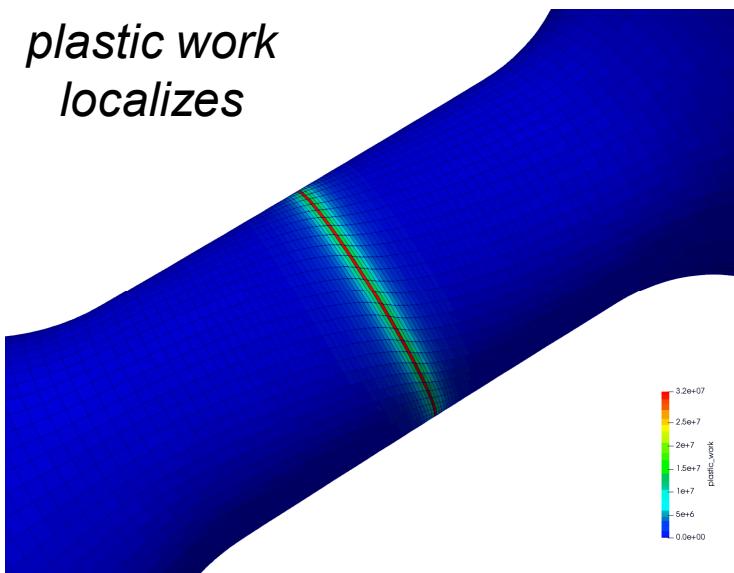
Rate Dependent Power Law Hardening: 33 direction



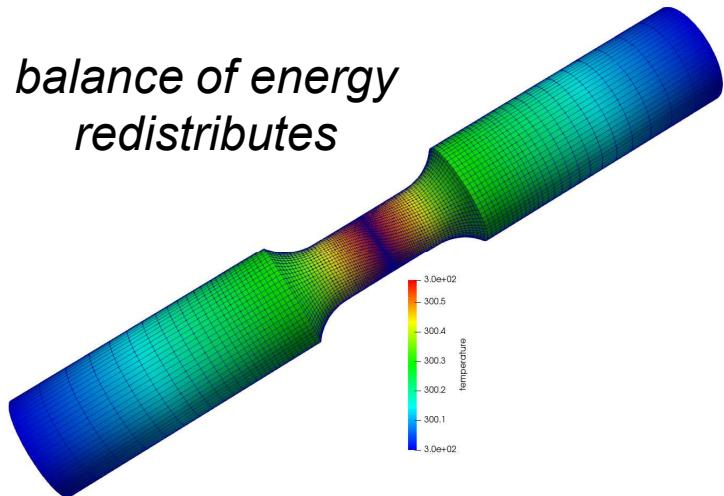
Verification of rate dependent implementation

# Examples – Torsion

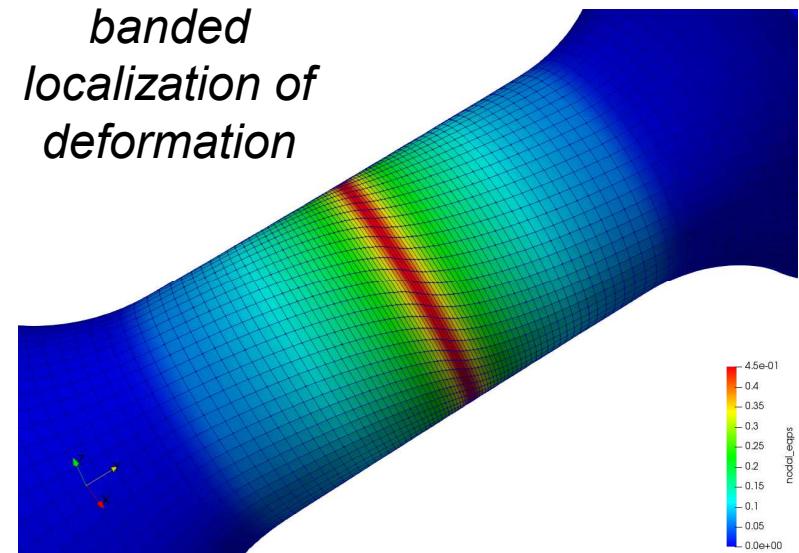
*plastic work  
localizes*



*balance of energy  
redistributes*



*banded  
localization of  
deformation*



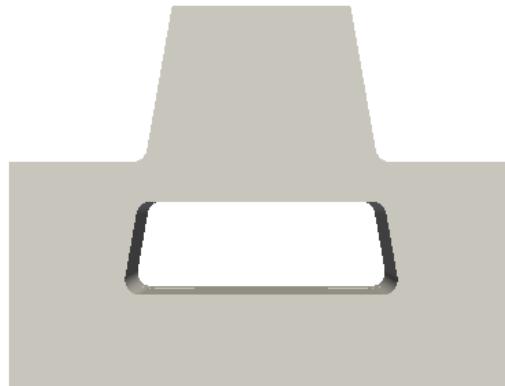
## Progress in modeling localization

- Mesh contains central band with axial bias to ease (symmetrize) process
- Hardening behavior is important
- Small thermal gradients may be important
- Solving thermomechanical problem to simulate the localization of deformation
- Interaction between solver + material model

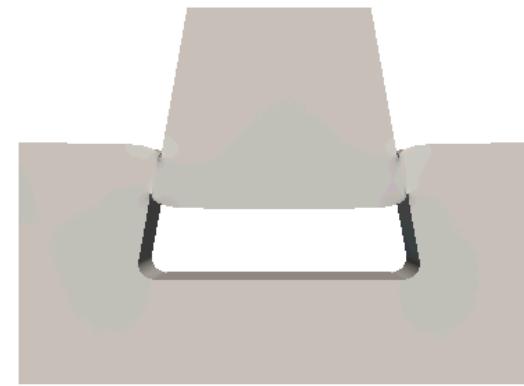
*Courtesy of and in collaboration with Jay Foulk, Sandia/CA*

# Examples – Shear compression

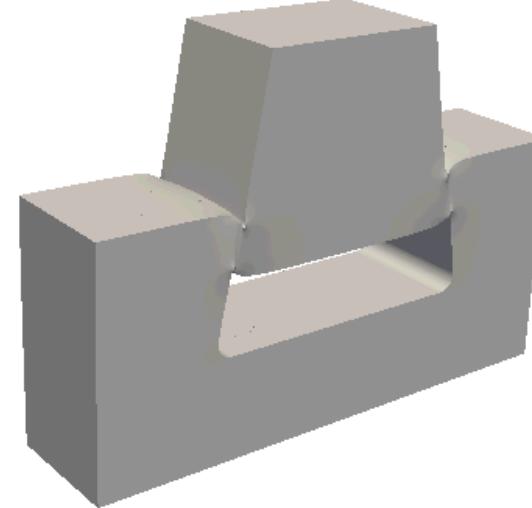
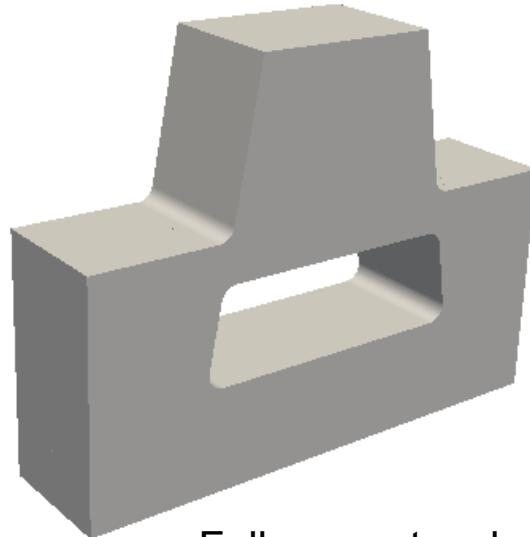
Boundary conditions: fixed (Y) on top, applied velocity (Y) on bottom of 2.54 m/s (100 in/s)



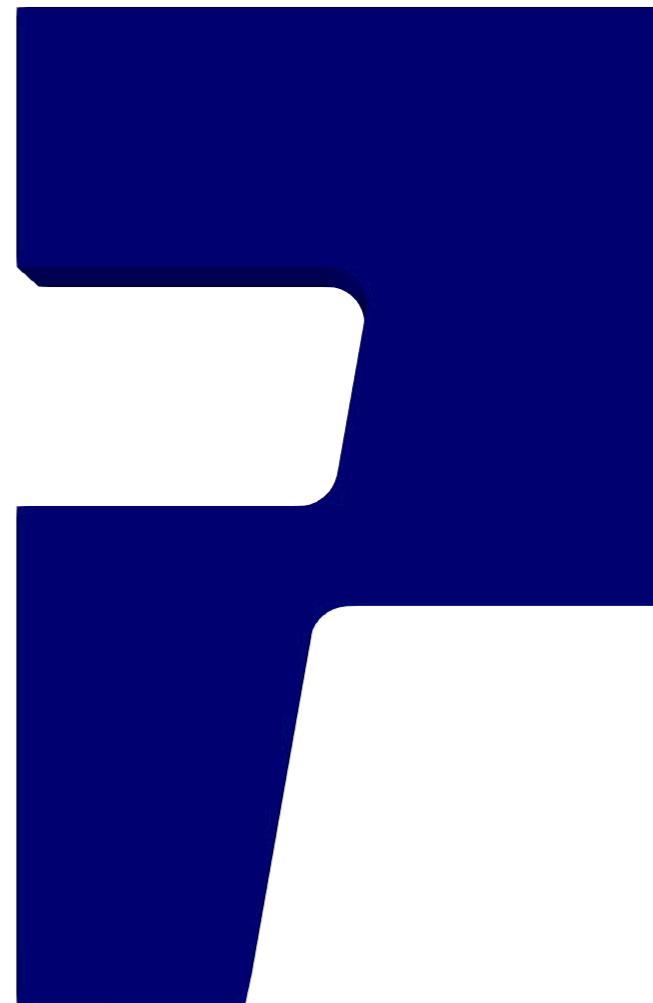
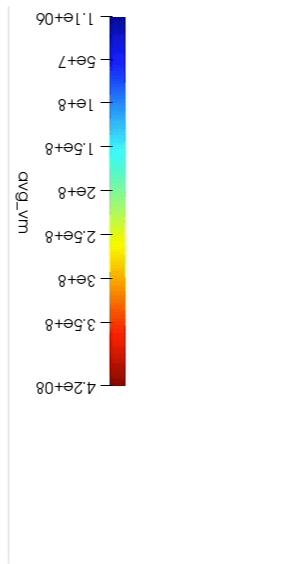
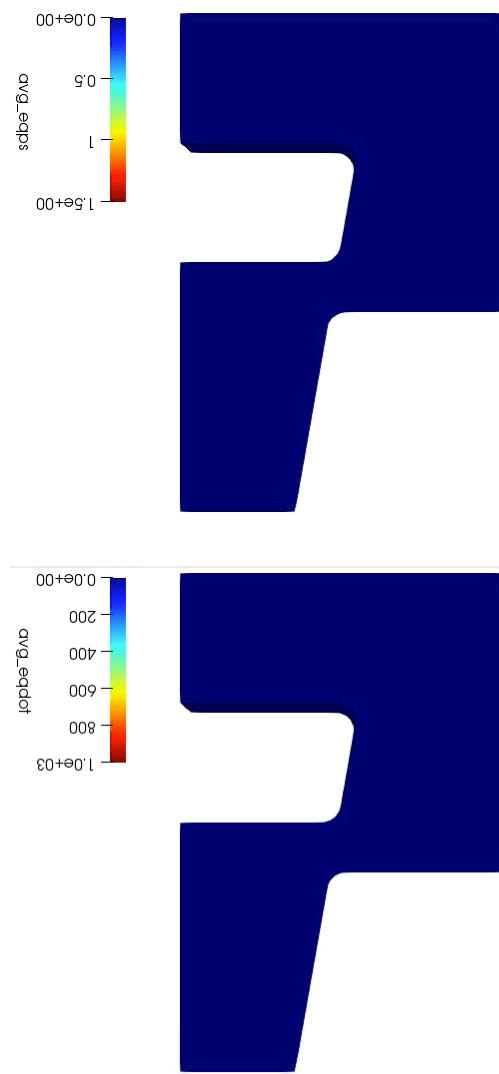
*undeformed*



*deformed*



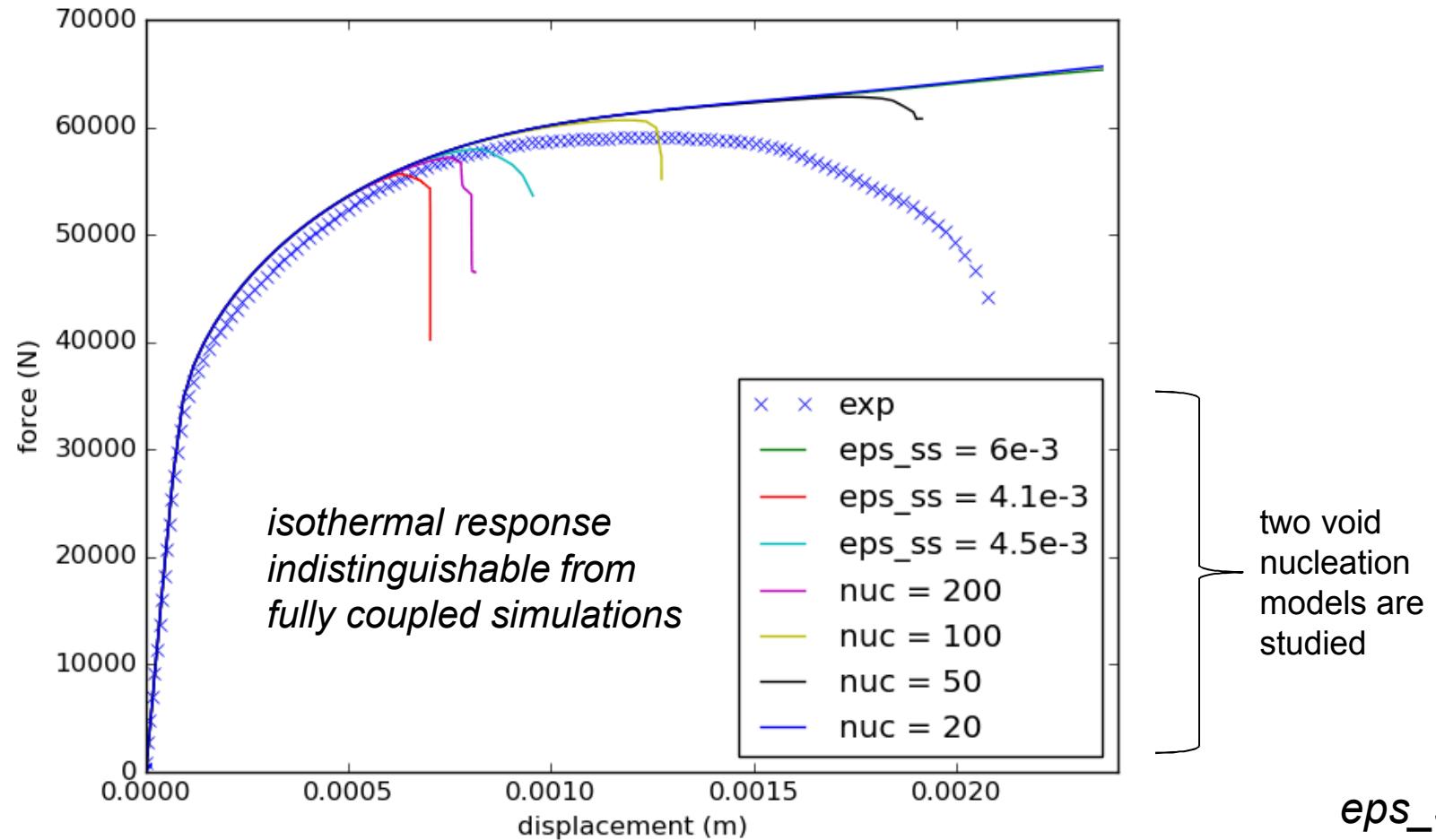
Full geometry shown, model exploits symmetry along 2 planes



## Examples – Shear compression

# Examples – Shear compression

Isothermal plasticity simulations employing damage evolution



*nuc*

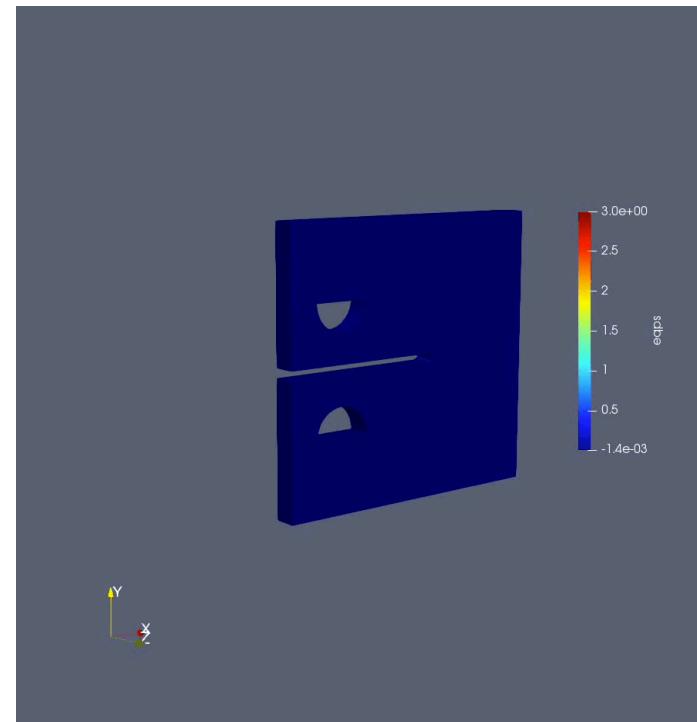
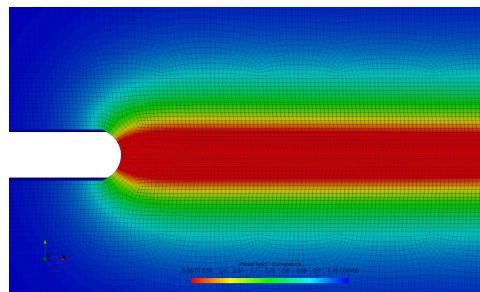
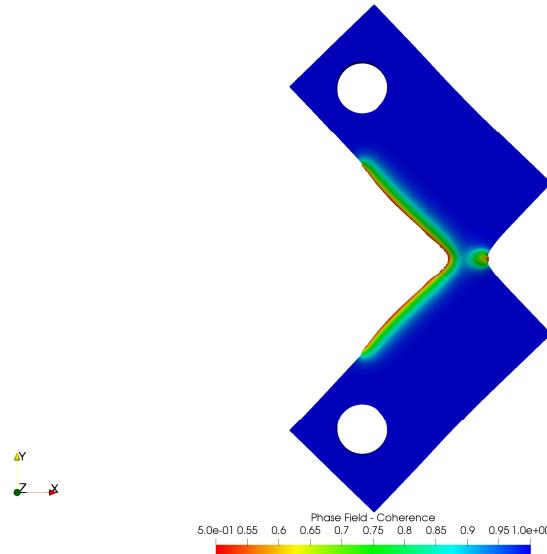
$$\dot{\eta} = \eta \dot{\epsilon}_p N_1 \left[ \frac{4}{27} - \frac{J_3^2}{J_2^3} \right]$$

*eps\_ss*

$$\phi_n(\epsilon_{ss}) = \frac{1}{2} \phi_{n,\infty} \left[ 1 + \operatorname{erf} \left( \frac{\epsilon_{ss} - \mu_{\epsilon_{ss}}}{\sqrt{2} \sigma_{\epsilon_{ss}}} \right) \right]$$

# Examples – Phase Field Fracture

- Hyperplastic model used as a basis for phase field fracture (Stershic, Talamini)



*Undeformed View (No visual threshold, zoomed):*

# Summary and Future Work

- Formulation of an isotropic hardening, rate and temperature dependent plasticity model based on a Helmholtz free energy and hyperelastic stress relation
- Implemented into SIERRA-SM, production finite element analysis code
- Certain features tested and verified in the context of a broader modular plasticity framework
- Next Steps
  - Modularization of damage and failure modeling, with and without gradients (phase field)
  - Further generalization of temperature dependence
  - Addressing Hyperelasticity in the context of anisotropic yield and hardening