



Optimal Power Flow Problems and PTDFs

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Introduction: Optimal Power Flow

- The full AC optimal power flow (AC OPF) problem is a mixed-integer, non-linear optimization problem, and hence very difficult to solve.
- The current standard practice is to use a linearization known as DC optimal power flow (DC OPF), which is a fast and reasonably accurate approximation.
- There is much interest in and potential economic applications to the problem of scheduling generating units in order to meet power demands and observe the power flow equations. This is called the *unit commitment* problem.

Problem: Incorporate PTDFs into a unit commitment model

- We computationally investigate replacing the DC optimal power flow with equations based on the Power Transfer Distribution Factor (PTDF) matrix.
- PTDFs measure linear sensitivity of power flow through transmission lines under the influence of power injections.
- If B_d is the diagonal matrix where the diagonal entries are the line susceptances, and A is the incidence matrix of the power grid, then we can write:
- $\text{PTDF} = (B_d * A) * (A^T * B_d * A)^{-1}$
- Using the Pyomo modeling environment, we program a set of DC OPF equations and PTDF equations in order to compare the efficiencies of the two formulations.
- It is hoped that PTDFs provide a computational speedup while reproducing the DC OPF solutions.

PTDF Example:

The column of zeros indicates that the third bus is the reference bus for the PTDF computations.

.27	-.45	0	-.18	-.09
.73	.45	0	.18	.09
.27	.55	0	-.18	-.09
-.18	-.36	0	-.55	-.27
-.09	-.18	0	-.27	-.64
.09	.18	0	.27	-.36

Observations and Results

- The first step is to use PTDFs in line flow equations, so that an optimization problem equivalent to DC OPF is obtained.
- Preliminary tests on the IEEE-1996 bus test case and smaller test cases indicate a computational speedup using PTDFs.
- Work will continue on larger test cases, and under adversarial conditions like the removal of some generating units.

Discussion and Future Work

- Much of the work is still in progress and will be incorporated into larger models.
- PTDF matrices are in general not sparse, but one advantage is that they only need to be computed once.
- PTDFs are also dependent on the DC OPF assumptions, which give only a linearization of the full AC OPF formulation.

References

- Hinojosa, V. and Gutierrez-Alcaraz, G. *A computational comparison of 2 mathematical formulations to handle transmission network constraints in the unit commitment problem*, Int. Trans. Electr. Energ. Syst., 2017.
- Van den Bergh, K. et al., *DC power flow in unit commitment models*, TME Working Paper - Energy and Environment, May 2014.