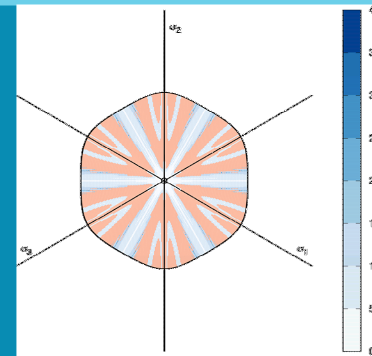
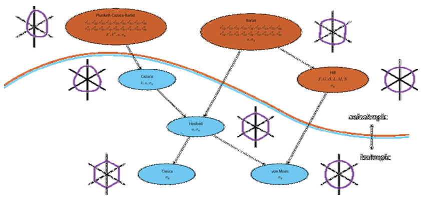
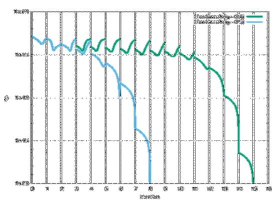
SAND2018-XXXXC

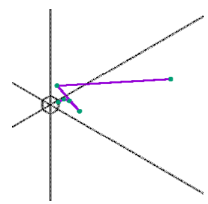
SAND2018-7980C

Robust Numerical Integration of Plasticity Models









William M. Scherzinger

Brian Lester

Jake Ostien

Sandia National Laboratories

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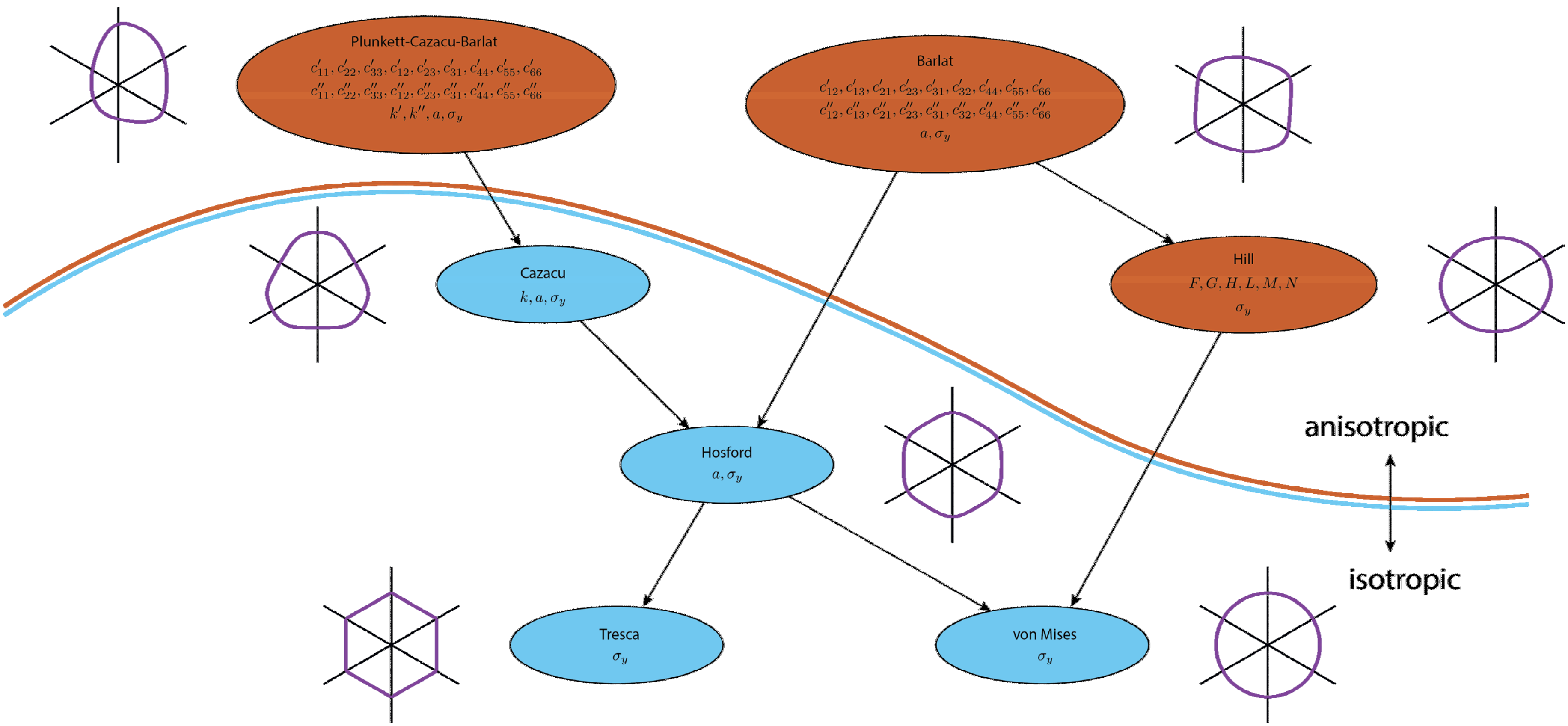
Motivation

Family of yield surfaces provides flexible modeling of plastic deformation…
… IF we can integrate these models



Closest Point Projection using Newton-Raphson Augmented with Line Search

Plasticity Model

- Yield function

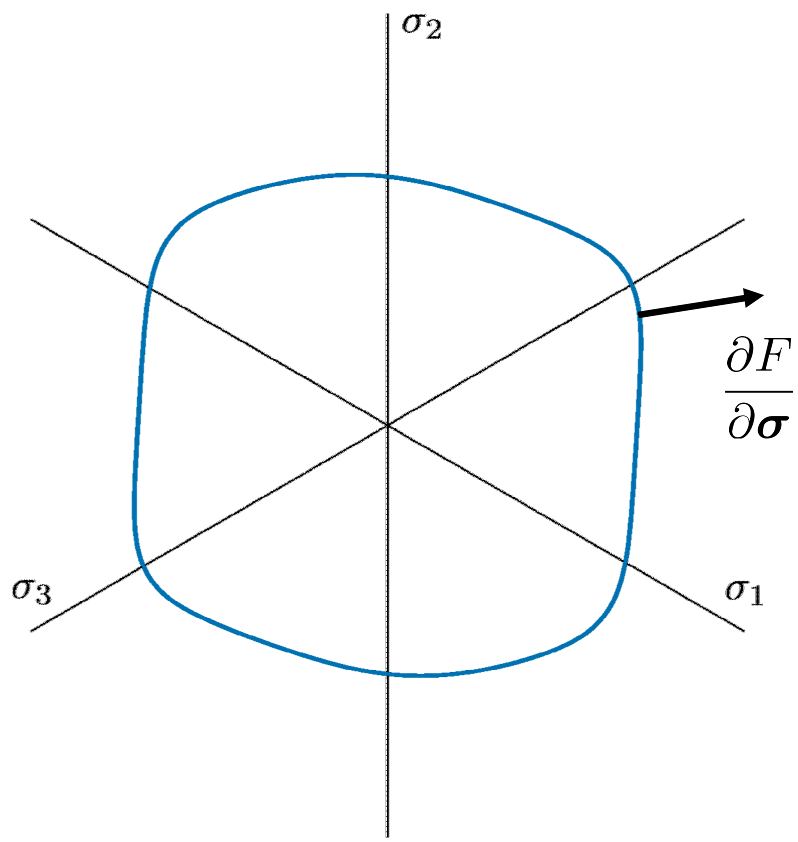
$$F\left(\boldsymbol{\sigma}, \bar{\varepsilon}^{p}\right) = \phi\left(\boldsymbol{\sigma}\right) - \bar{\sigma}\left(\bar{\varepsilon}^{p}\right) \leq 0$$

- Flow direction

$$\dot{\boldsymbol{\varepsilon}}^{p} = \dot{\gamma}\frac{\partial F}{\partial \boldsymbol{\sigma}}$$

- Stress rate

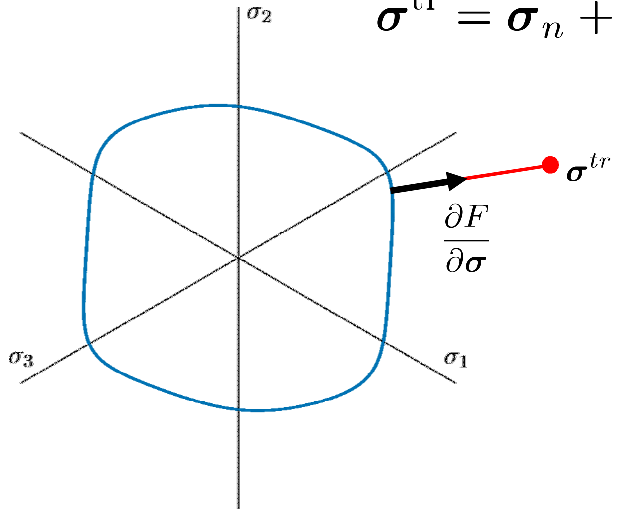
$$\dot{\boldsymbol{\sigma}} = \mathbb{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}\right)$$



Discretization

- Closest point projection
  - Trial stress state is outside of elastic region
  - Return stress to the yield surface

$$\boldsymbol{\sigma}^{\mathrm{tr}} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\boldsymbol{\varepsilon}$$



$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{\mathrm{tr}} - \mathbb{C} : \Delta\boldsymbol{\varepsilon}^p$$

backward Euler $\Delta\boldsymbol{\varepsilon}^p = \Delta\gamma \left(\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)_{n+1}$

$$F(\boldsymbol{\sigma}, \Delta\gamma) = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\Delta\gamma) = 0$$

$$\mathbf{R}(\boldsymbol{\sigma}, \Delta\gamma) = -\Delta\boldsymbol{\varepsilon}^p + \Delta\gamma \frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \mathbf{0}$$

Newton-Raphson

$$\Delta\gamma_{(k+1)} = \Delta\gamma_{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

solution variables

Newton-Raphson

$$\Delta\gamma_{(k+1)} = \Delta\gamma_{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

$$\boxed{\begin{aligned} \Delta\left(\Delta\gamma\right) &= \frac{F^{(k)} - \mathbf{R}^{(k)} : \mathscr{L}^{(k)} : \dfrac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}}}{\dfrac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}} : \mathscr{L}^{(k)} : \dfrac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}} + H'_{(k)}} \\ \Delta\boldsymbol{\sigma} &= -\mathscr{L}^{(k)} : \left(\mathbf{R}^{(k)} + \Delta\left(\Delta\gamma\right) \frac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}}\right) \end{aligned}}$$

increments in
solution variables

Newton-Raphson

$$\Delta\gamma_{(k+1)} = \Delta\gamma_{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

$$\Delta\left(\Delta\gamma\right) = \frac{F^{(k)} - \mathbf{R}^{(k)} : \mathscr{L}^{(k)} : \dfrac{\partial F^{(k)}}{\partial\boldsymbol{\sigma}}}{\dfrac{\partial F^{(k)}}{\partial\boldsymbol{\sigma}} : \mathscr{L}^{(k)} : \dfrac{\partial F^{(k)}}{\partial\boldsymbol{\sigma}} + H'_{(k)}}$$

$$\Delta\boldsymbol{\sigma} = -\mathscr{L}^{(k)} : \left(\mathbf{R}^{(k)} + \Delta\left(\Delta\gamma\right)\frac{\partial F^{(k)}}{\partial\boldsymbol{\sigma}}\right)$$

Slope of hardening curve

$$H' = \frac{d\bar{\sigma}}{d\Delta\bar{\varepsilon}^p} \quad ; \quad \left(\Delta\bar{\varepsilon}^p = \Delta\gamma\right)$$

Hessian

$$\mathscr{L}^{-1} = \mathbb{C}^{-1} + \Delta\gamma\frac{\partial^2 F}{\partial\boldsymbol{\sigma}\partial\boldsymbol{\sigma}}$$

Newton-Raphson

$$\Delta\gamma_{(k+1)} = \Delta\gamma_{(k)} + \Delta\left(\Delta\gamma\right)$$

$$\boldsymbol{\sigma}^{(k+1)} = \boldsymbol{\sigma}^{(k)} + \Delta\boldsymbol{\sigma}$$

$$\Delta\left(\Delta\gamma\right) = \frac{F^{(k)} - \mathbf{R}^{(k)} : \mathscr{L}^{(k)} : \dfrac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}}}{\dfrac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}} : \mathscr{L}^{(k)} : \dfrac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}} + H'_{(k)}}$$

$$\Delta\boldsymbol{\sigma} = -\mathscr{L}^{(k)} : \left(\mathbf{R}^{(k)} + \Delta\left(\Delta\gamma\right)\frac{\partial F^{(k)}}{\partial \boldsymbol{\sigma}}\right)$$

Slope of hardening curve

$$H' = \frac{d\bar{\sigma}}{d\Delta\bar{\varepsilon}^p} \quad ; \quad \left(\Delta\bar{\varepsilon}^p = \Delta\gamma\right)$$

Hessian

$$\mathscr{L}^{-1} = \mathbb{C}^{-1} + \Delta\gamma\frac{\partial^2 F}{\partial\boldsymbol{\sigma}\partial\boldsymbol{\sigma}}$$

von Mises

$$\Delta\left(\Delta\gamma\right) = \frac{F^{(k)}}{3\mu + H'_{(k)}} \qquad \Delta\boldsymbol{\sigma} = -\Delta\left(\Delta\gamma\right)\frac{3}{2\phi}\mathscr{L}^{(k)} : \mathbf{s}$$

Line Search*

$$\Delta\gamma_{(k+1)}(\alpha) = \Delta\gamma_{(k)} + \alpha\, \Delta\left(\Delta\gamma\right)$$

search direction
(found from Newton-Raphson)

$$\boldsymbol{\sigma}^{(k+1)}(\alpha) = \boldsymbol{\sigma}^{(k)} + \alpha\, \Delta\boldsymbol{\sigma}$$

search magnitude

$$\alpha \in (0, 1]$$

Residual $\mathbf{r}(\alpha) = \left(\frac{F}{2\mu} \,,\, \mathbf{R} \right)$

Merit function $\psi\left(\alpha\right) = \frac{1}{2}\mathbf{r}\left(\alpha\right) \cdot \mathbf{r}\left(\alpha\right)$

* A. Perez-Foguet and F. Armero, *Int. J. Num. Meth. Eng.*, **52** (2002) 331-374

Line Search*

Iteration algorithm for line search parameter

$$\alpha_{(0)} = 1$$

check

$$\psi\left(\alpha_{(j)}\right) < \psi(0)$$

Yes $\alpha = \alpha_{(j)}$

find minimum using quadratic approximation

No $\hat{\psi} = \psi_0 + \alpha_{(j)}\psi_1 + \alpha_{(j)}^2\psi_2$

$$\hat{\psi} = \frac{1}{\alpha_{(j)}^2}\left[\left(\alpha + \alpha_{(j)} - 2\alpha\alpha_{(j)}\right)\left(\alpha_{(j)} - \alpha\right)\psi(0) + \alpha^2\psi\left(\alpha_{(j)}\right)\right]$$

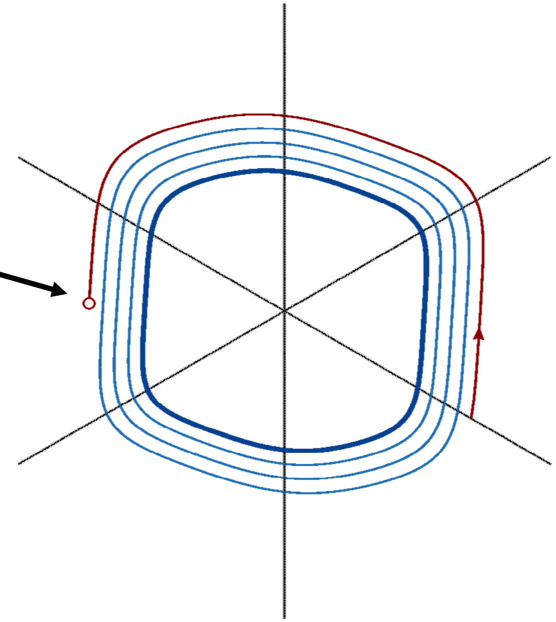
$$\alpha_{(j+1)} = \max\left\{\eta\alpha_{(j)}, \frac{\alpha_{(j)}^2\psi(0)}{\psi(\alpha_{(j)}) - \left(1 - 2\alpha_{(j)}\right)\psi(0)}\right\} \qquad \eta = 0.1$$

* A. Perez-Foguet and F. Armero, *Int. J. Num. Meth. Eng.*, **52** (2002) 331-374

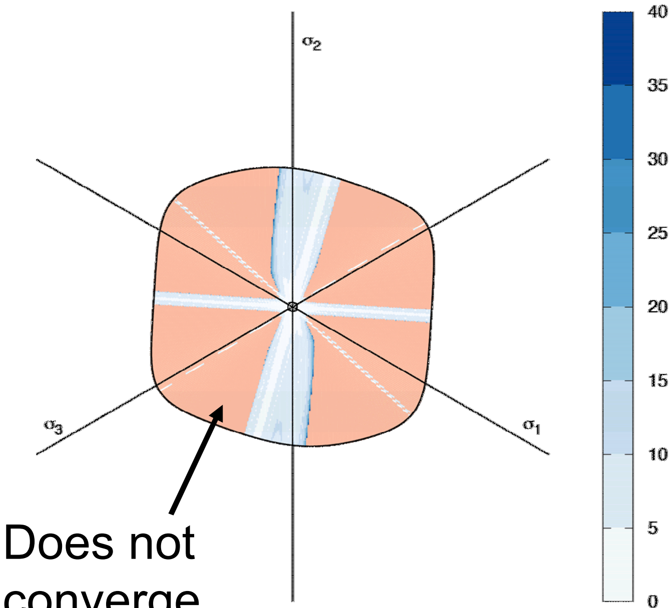
Yield Surfaces

Scan Trial Stress Space

For every trial stress state
- Record the number of iterations required for convergence



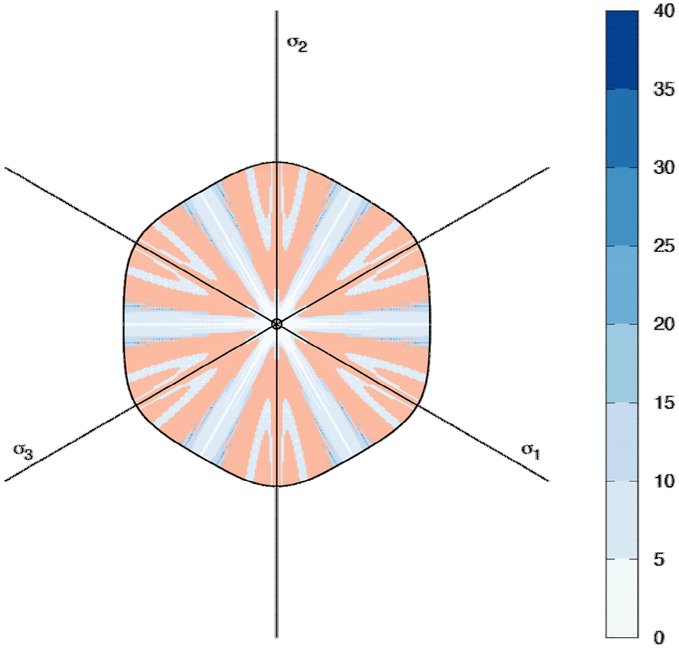
Analysis
- Trial stresses out to 30 times size of yield surface
- 95,718 trial stress states
- Perfect plasticity
- Results for both Newton-Raphson and line search



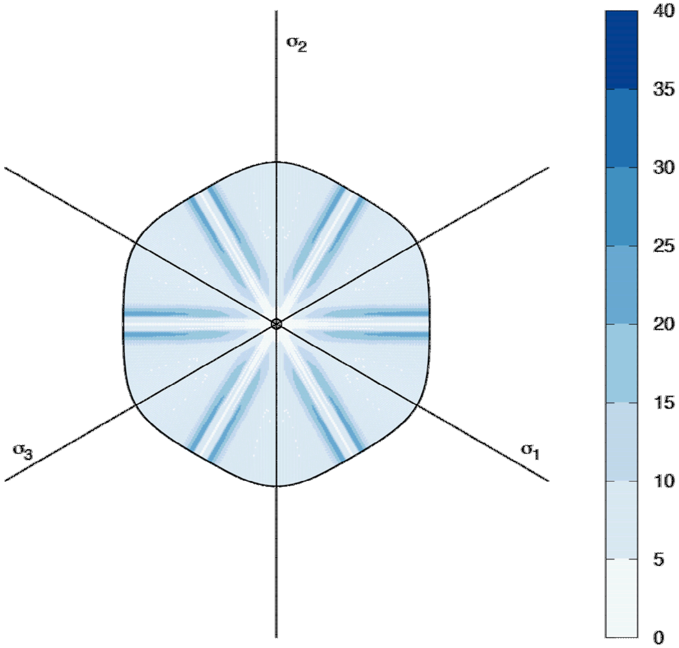
Hosford*

Isotropic, non-quadratic

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{2} \left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a \right] \right\}^{1/a} \qquad a = 8$$



Newton



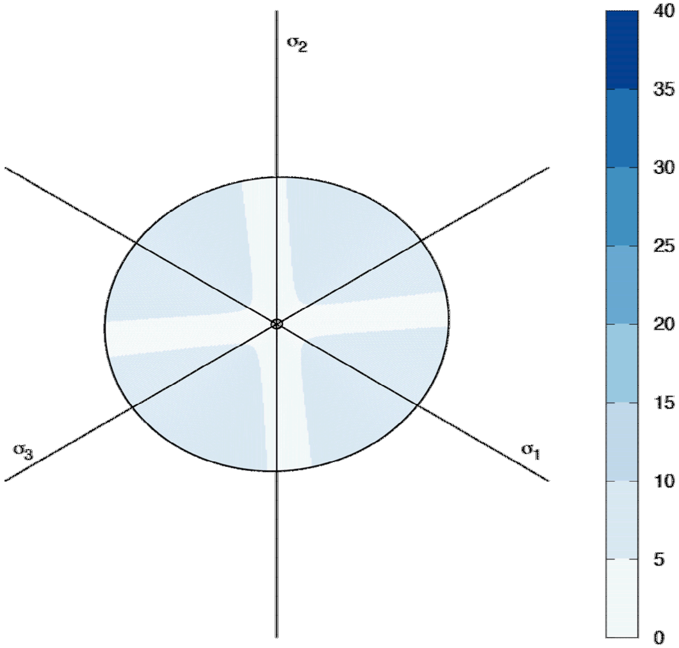
Line Search

* W. F. Hosford, *J. Appl. Mech.*, **39** (1972) 607-609

Hill

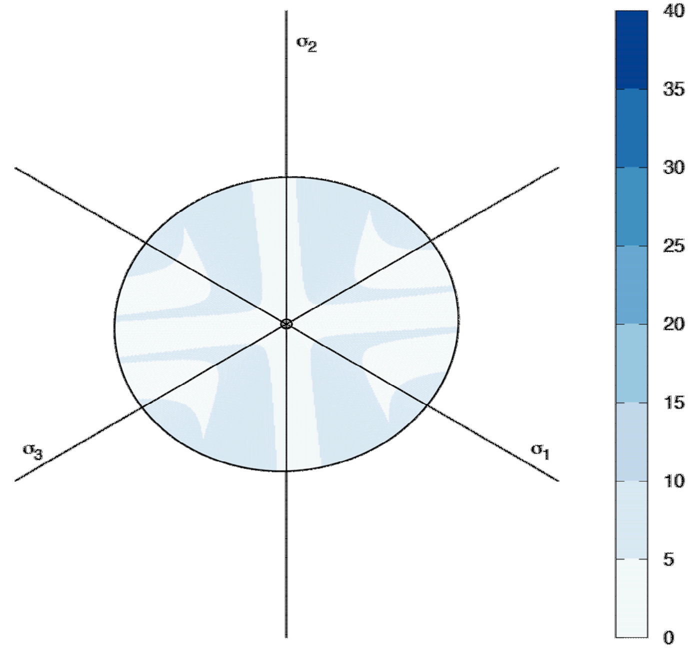
Anisotropic, quadratic

$$\phi^2\left(\boldsymbol{\sigma}\right) = F\left(\hat{\sigma}_{22} - \hat{\sigma}_{33}\right)^2 + G\left(\hat{\sigma}_{33} - \hat{\sigma}_{11}\right)^2 + H\left(\hat{\sigma}_{11} - \hat{\sigma}_{22}\right)^2 + 2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2$$



Newton

$$\begin{aligned} F &= 0.583 \\ G &= 0.364 \\ H &= 0.634 \\ L &= 1.815 \\ M &= 2.069 \\ N &= 2.349 \end{aligned}$$



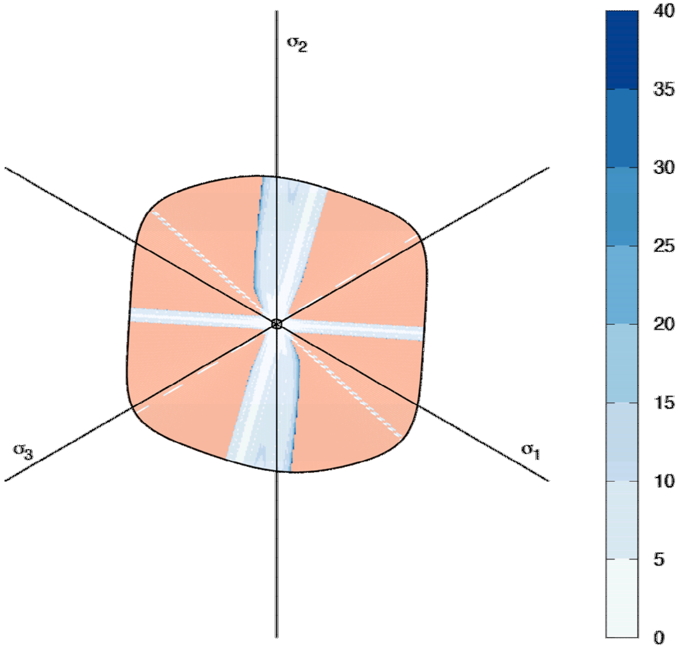
Line Search

Barlat* (Yld2004-18p)

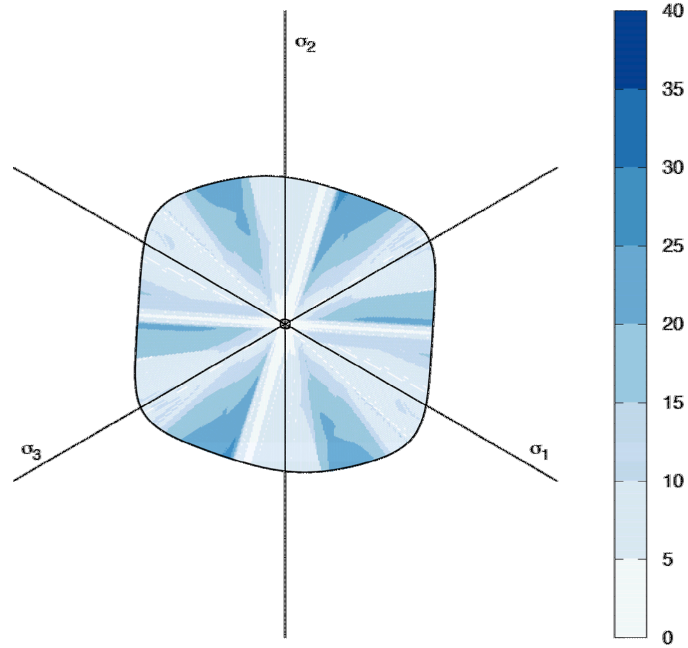
Anisotropic, non-quadratic

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{4} \Big[|s_1' - s_1''|^a + |s_1' - s_2''|^a + |s_1' - s_3''|^a + |s_2' - s_1''|^a + |s_2' - s_2''|^a + |s_2' - s_3''|^a + |s_3' - s_1''|^a + |s_3' - s_2''|^a + |s_3' - s_3''|^a \Big] \right\}^{1/a}$$

$$a = 8$$



Newton



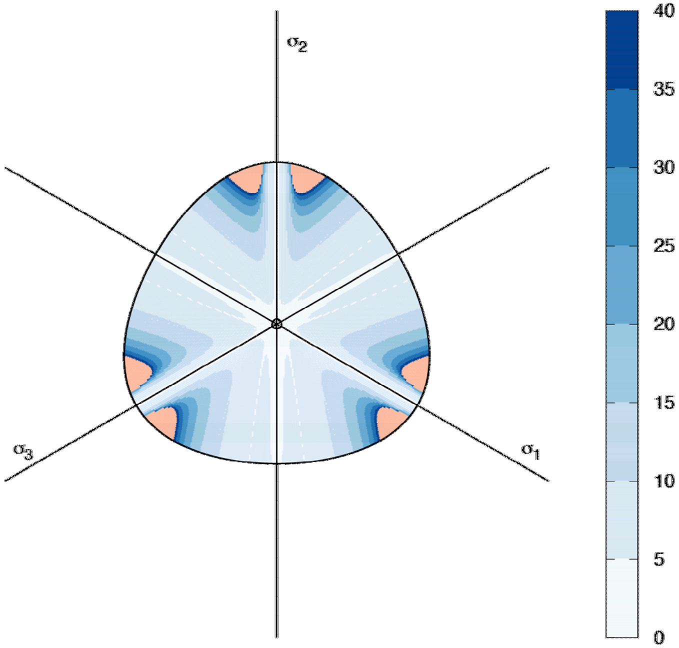
Line Search

* F. Barlat et. al., *Int. J. Plast.*, **19** (2005) 1009-1039

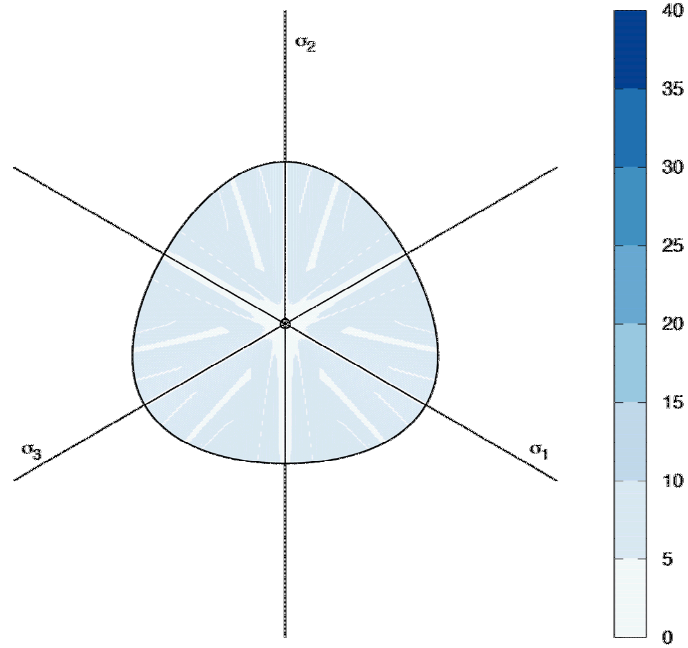
Cazacu*

Isotropic, non-quadratic, tension/compression asymmetry

$$\phi(\boldsymbol{\sigma}) = \left\{ g(a,k) \left[(|s_1| - ks_1)^a + (|s_2| - ks_2)^a + (|s_3| - ks_3)^a \right\}^{1/a} \qquad k = 0.2 \quad ; \quad a = 4$$



Newton



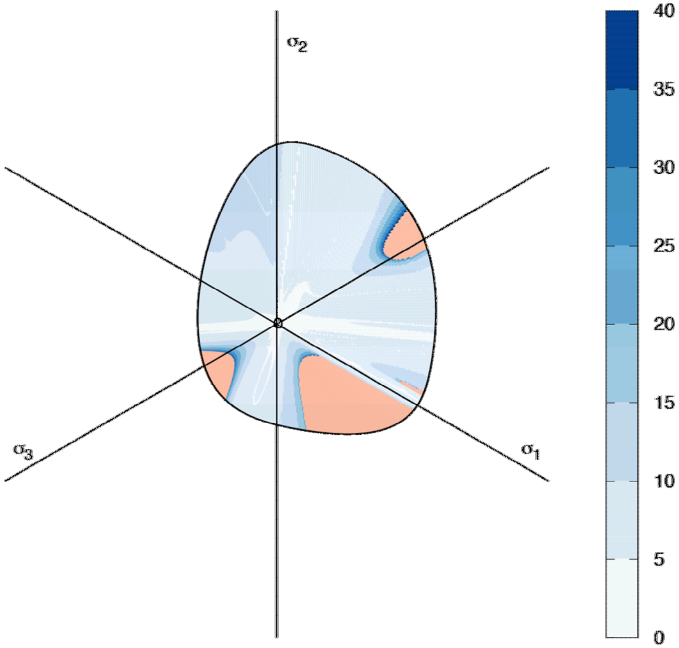
Line Search

* B. Plunkett et. al., *Int. J. Plast.*, **24** (2008) 847-866

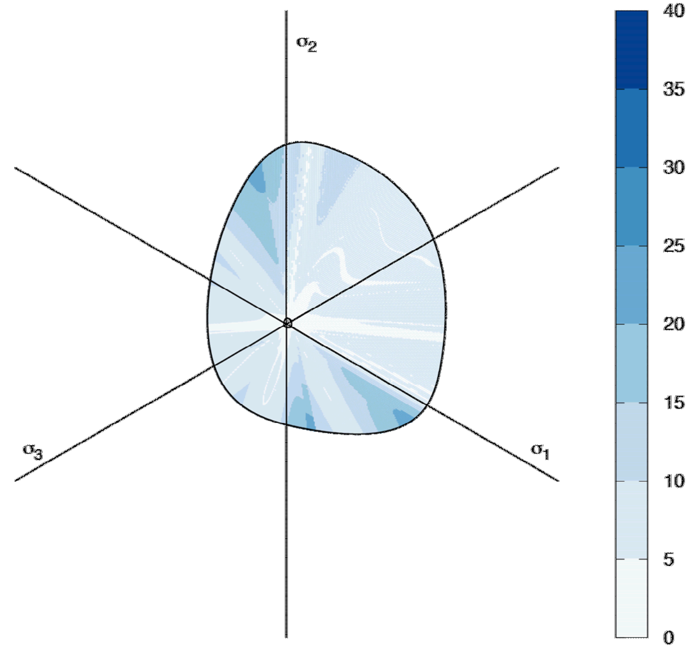
Plunkett-Cazacu-Barlat* (CPB06ex2)

Anisotropic, non-quadratic, tension/compression asymmetry

$$\phi(\boldsymbol{\sigma}) = \Big\{ g(a, c'_{ij}, c''_{ij}, k', k'') \Big[(|s'_1| - k's'_1)^a + (|s'_2| - k's'_2)^a + (|s'_3| - k's'_3)^a + (|s''_1| - k''s''_1)^a + (|s''_2| - k''s''_2)^a + (|s''_3| - k''s''_3)^a \Big] \Big\}^{1/a}$$



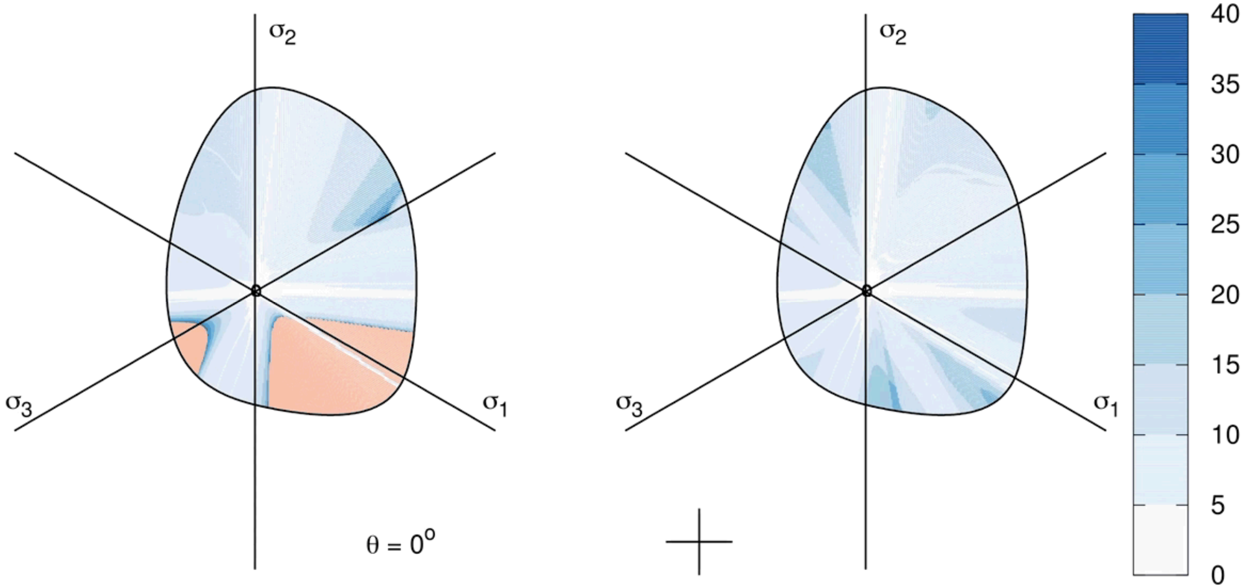
Newton



Line Search

* B. Plunkett et. al., *Int. J. Plast.*, **24** (2008) 847-866

Plunkett-Cazacu-Barlat (CPB06ex2)

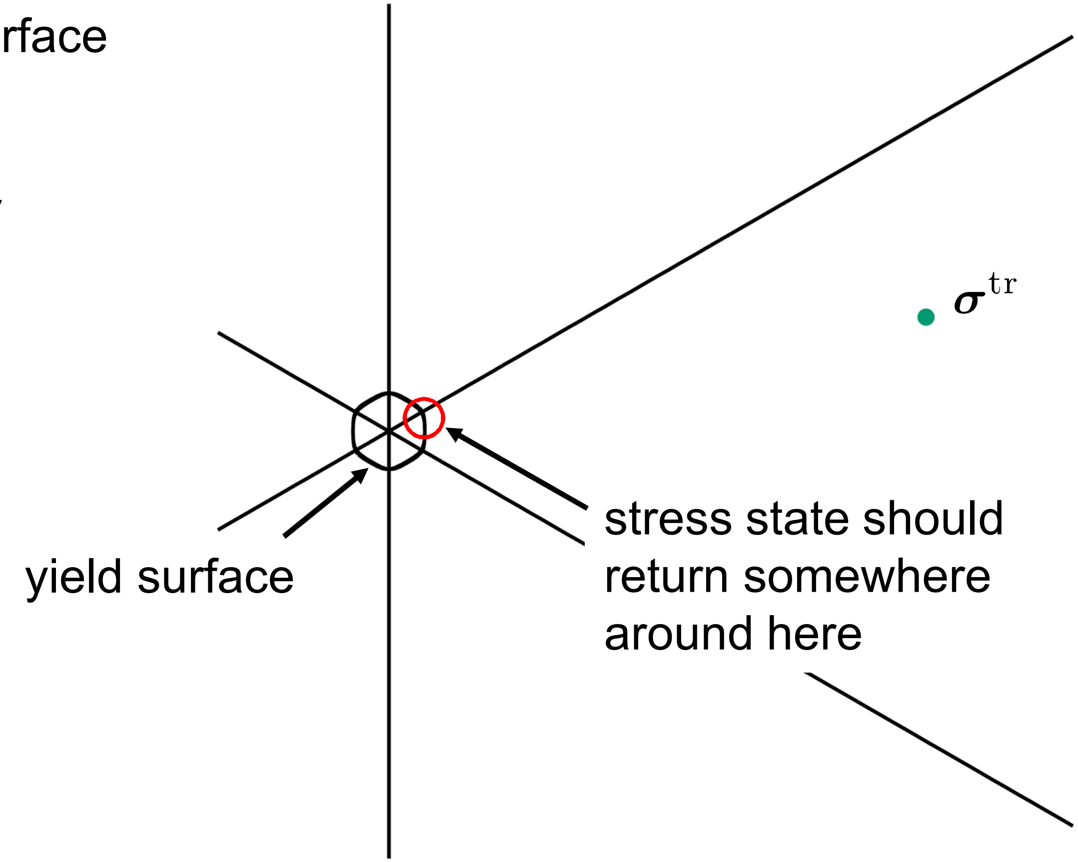


Return Mapping

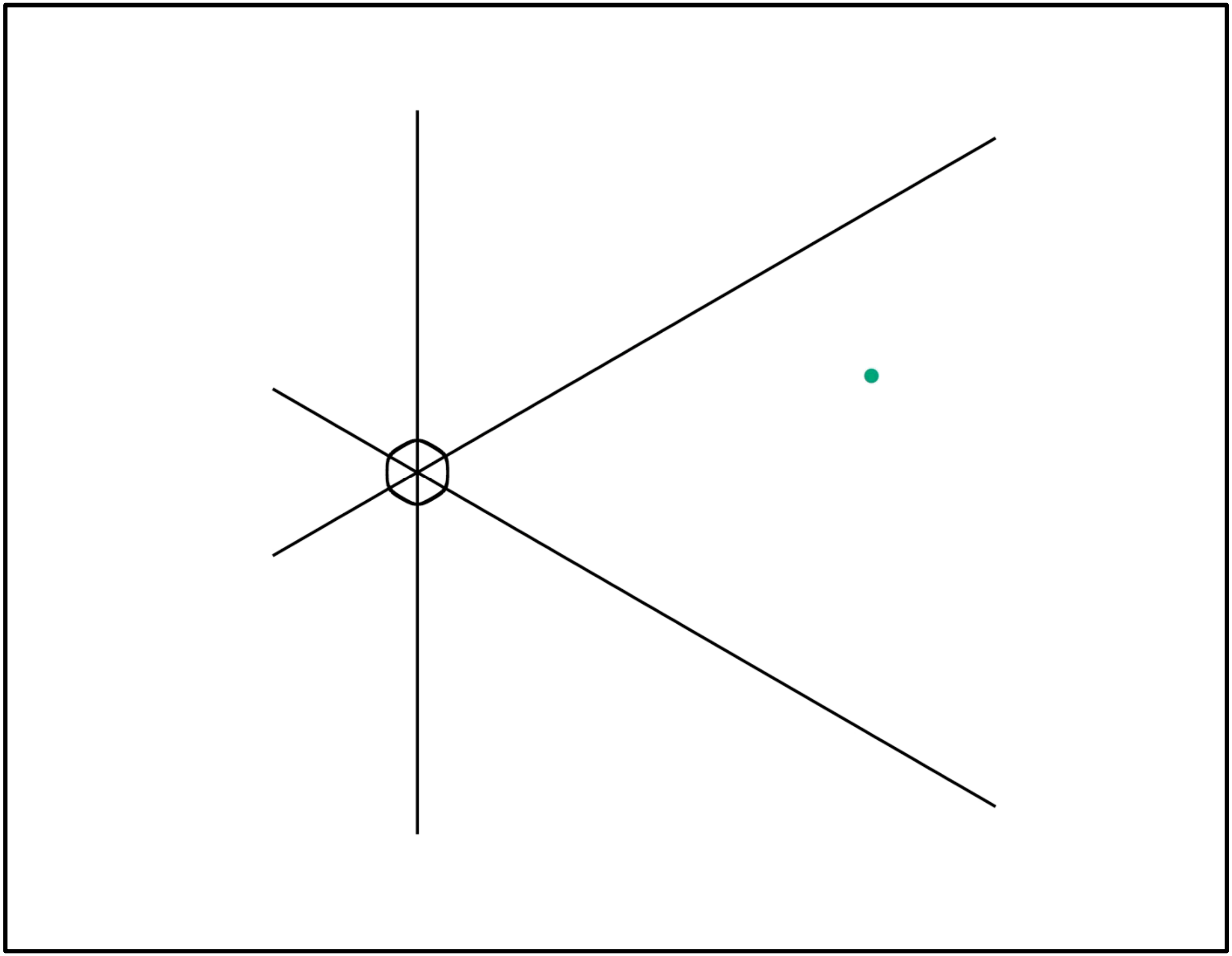
Hosford yield surface

$$a = 8$$

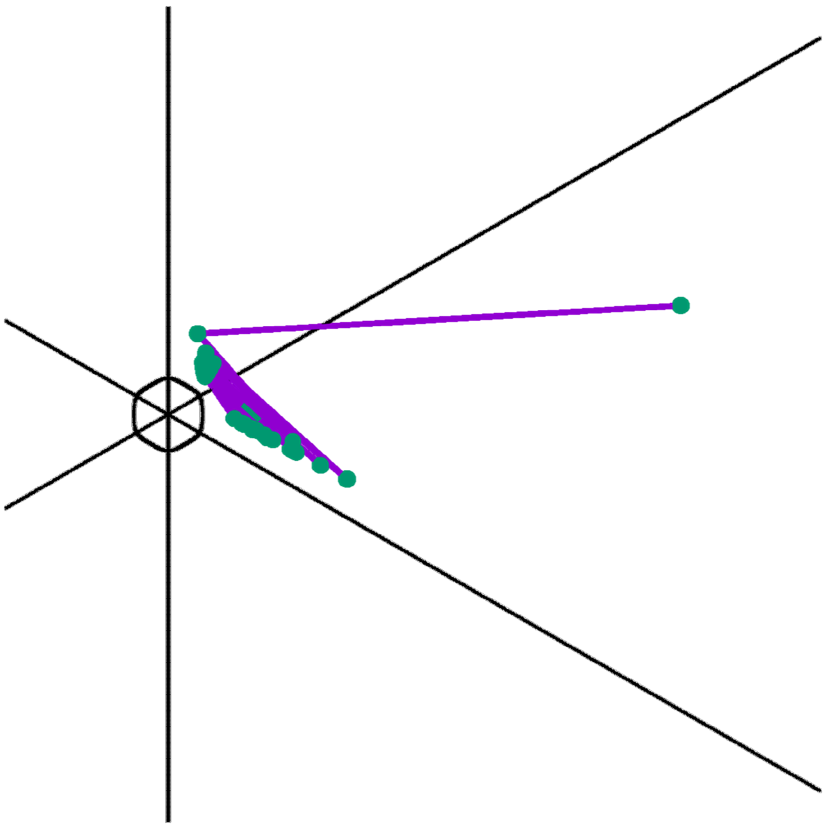
perfect plasticity



Return Mapping Trajectory – Newton-Raphson



Return Mapping Trajectory – Newton-Raphson



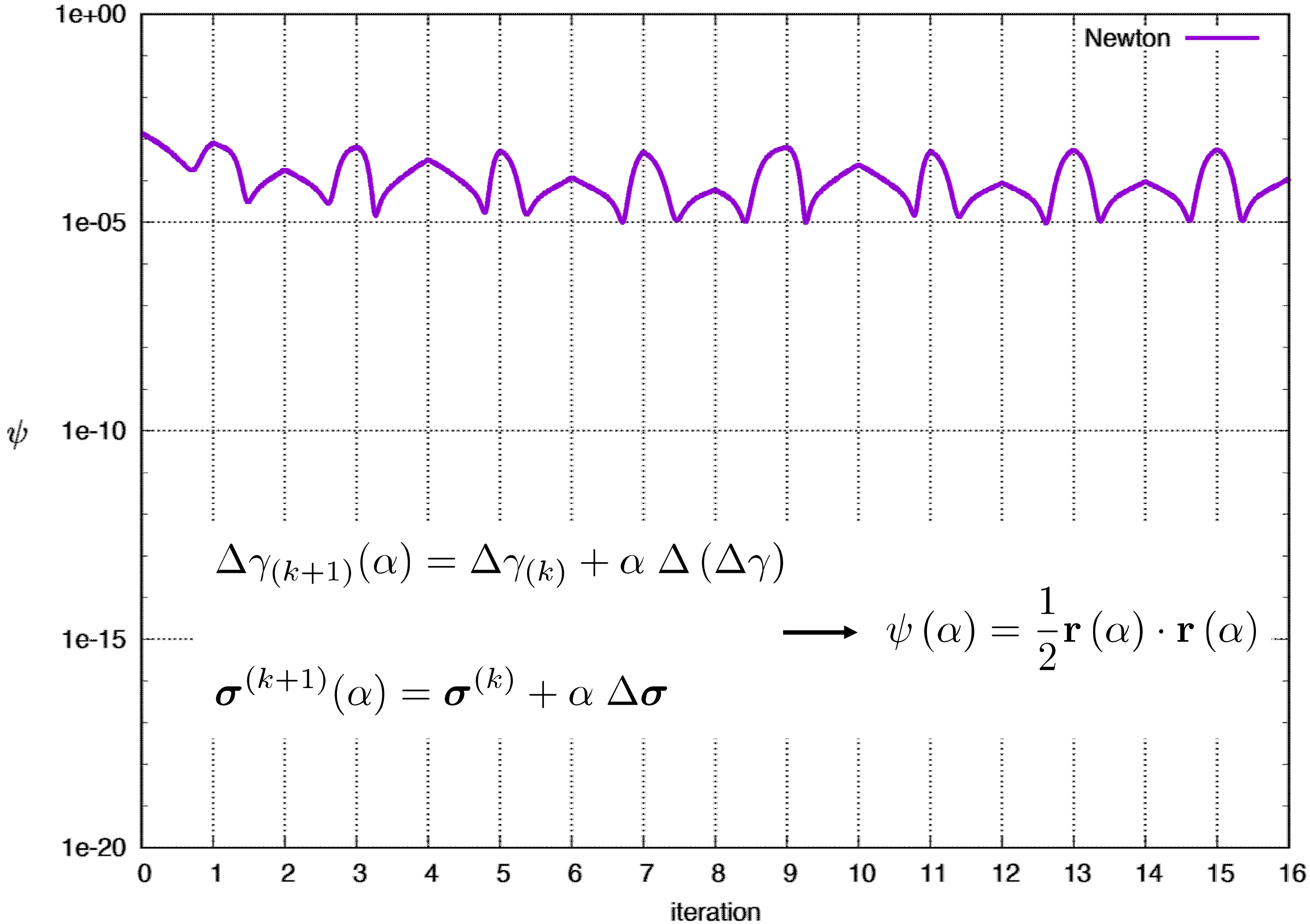
- Newton-Raphson has difficulty near the corner
- With any yield surface other than von Mises this will occur

$$F = \phi(\boldsymbol{\sigma}) - \bar{\sigma}(\Delta\gamma) = 0$$

$$\boxed{\mathbf{R} = -\Delta\boldsymbol{\varepsilon}^p + \Delta\gamma \frac{\partial \phi}{\partial \boldsymbol{\sigma}} = \mathbf{0}}$$

- Algorithm can't find the flow direction

Merit Function – Newton-Raphson

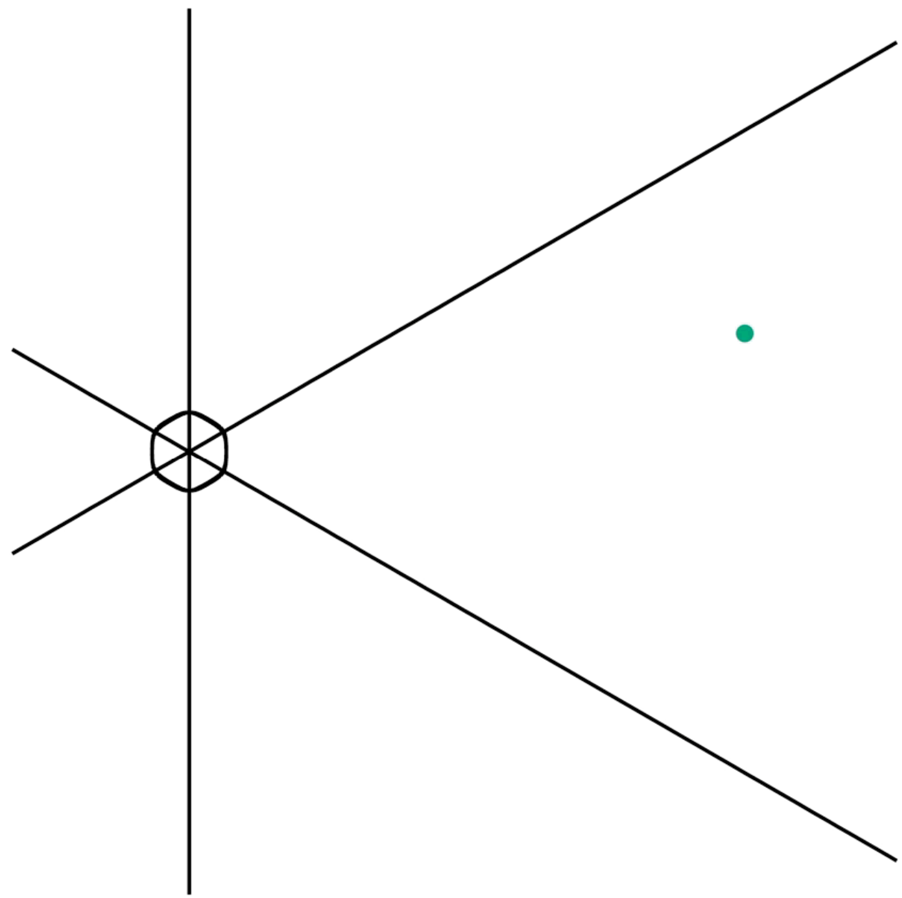


$$\Delta\gamma_{(k+1)}(\alpha) = \Delta\gamma_{(k)} + \alpha\,\Delta\left(\Delta\gamma\right)$$

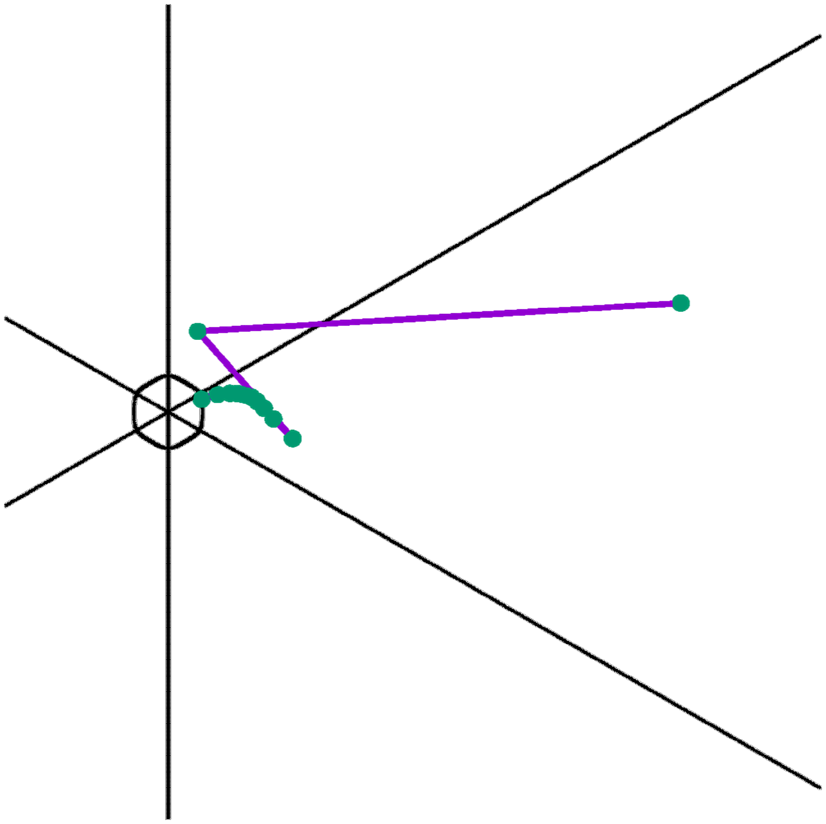
$$\boldsymbol{\sigma}^{(k+1)}(\alpha) = \boldsymbol{\sigma}^{(k)} + \alpha\,\Delta\boldsymbol{\sigma}$$

$$\longrightarrow \quad \psi\left(\alpha\right) = \frac{1}{2}\mathbf{r}\left(\alpha\right)\cdot\mathbf{r}\left(\alpha\right)$$

Return Mapping Trajectory – Line Search

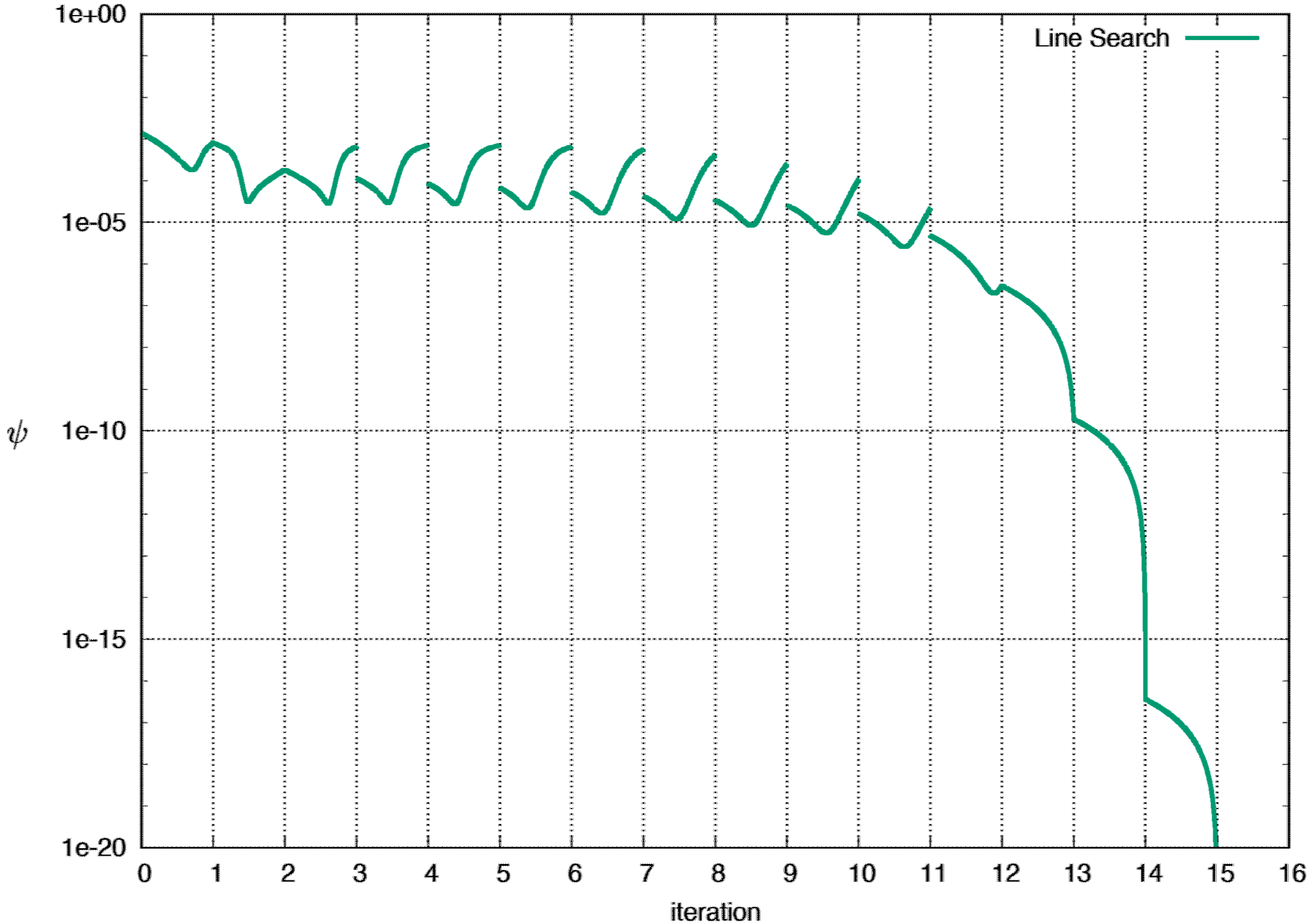


Return Mapping Trajectory – Line Search

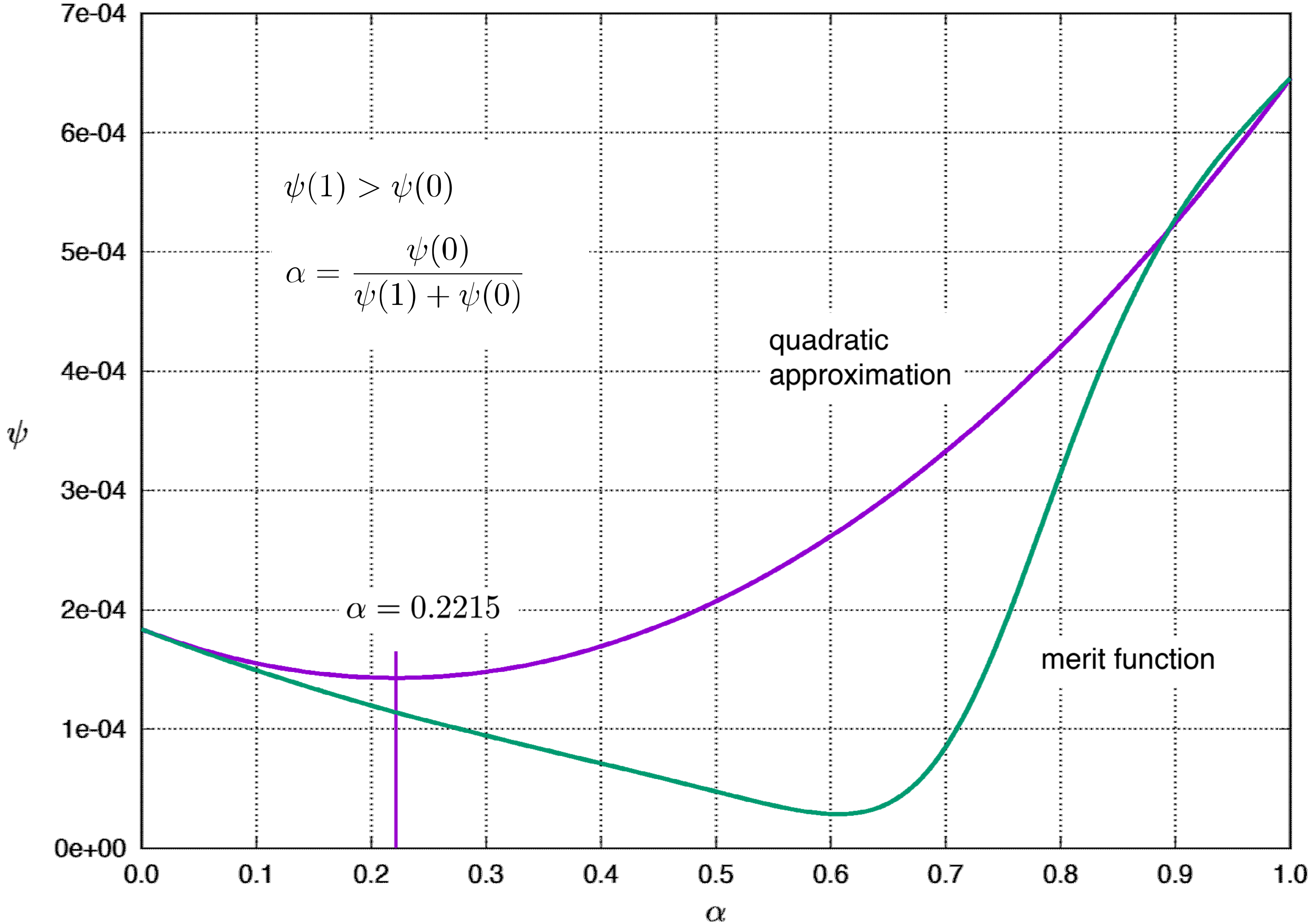


- Line search converges near the corner
- Algorithm finds the flow direction
- Convergence is slow
- Many small steps

Merit Function – Line Search



Merit Function – Quadratic Approximation



Effect of η

Quadratic approximation

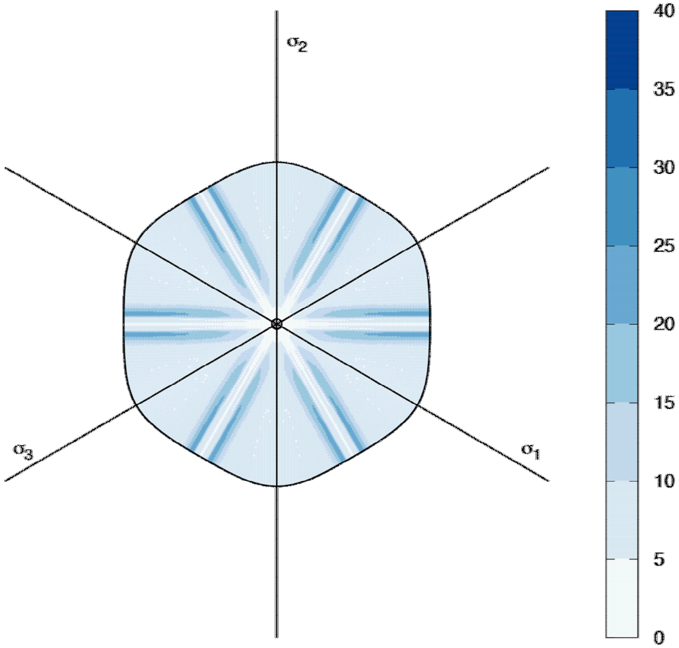
$$\alpha_{(j+1)} = \max \left\{ \eta \alpha_{(j)}, \frac{\alpha_{(j)}^2 \psi(0)}{\psi(\alpha_{(j)}) - \left(1 - 2\alpha_{(j)}\right) \psi(0)} \right\}$$

parameter for the line search algorithm

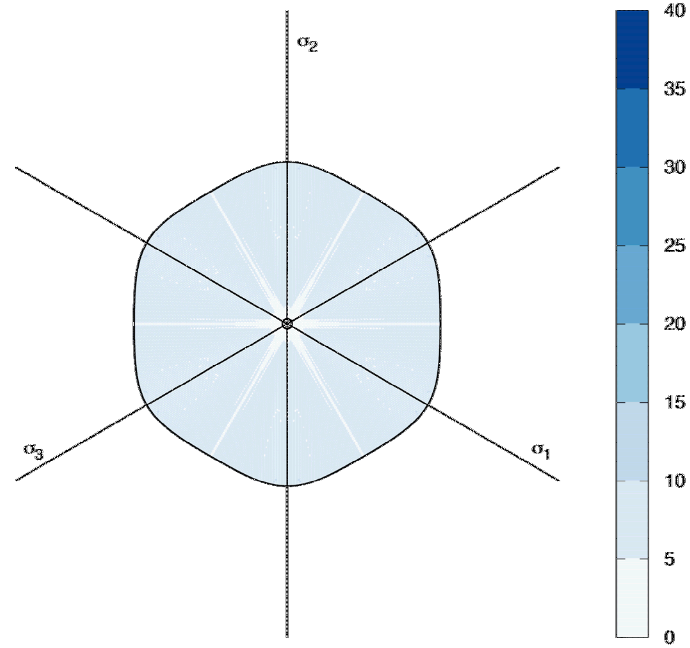
$\eta = 0.5$

Hosford – η

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{2} \left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a \right] \right\}^{1/a} \qquad a = 8$$



η = 0.1

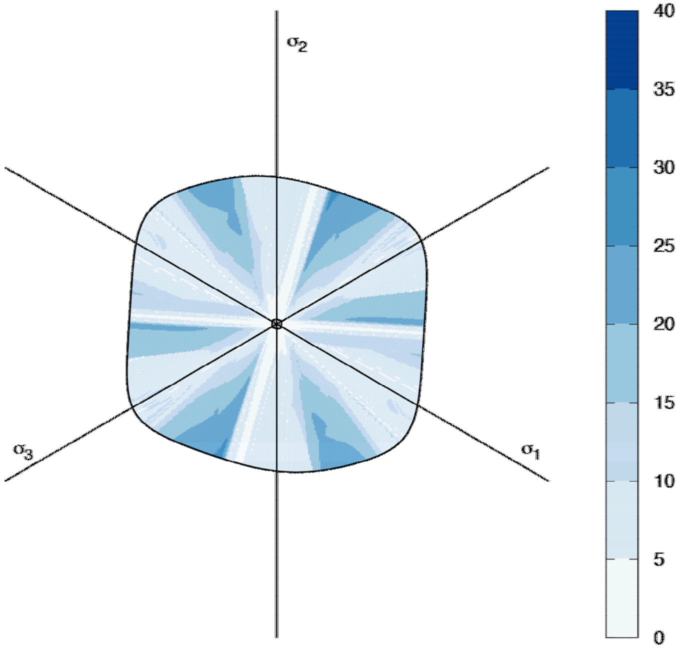


η = 0.5

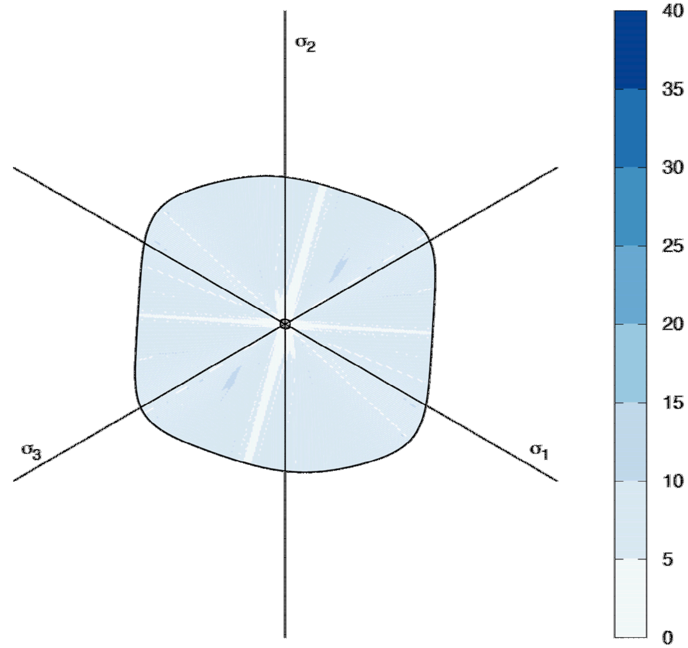
Barlat – η

$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{4} \left[|s'_1 - s''_1|^a + |s'_1 - s''_2|^a + |s'_1 - s''_3|^a + |s'_2 - s''_1|^a + |s'_2 - s''_2|^a \right.\right.$$
$$\left.\left. + |s'_2 - s''_3|^a + |s'_3 - s''_1|^a + |s'_3 - s''_2|^a + |s'_3 - s''_3|^a \right] \right\}^{1/a}$$

$a = 8$



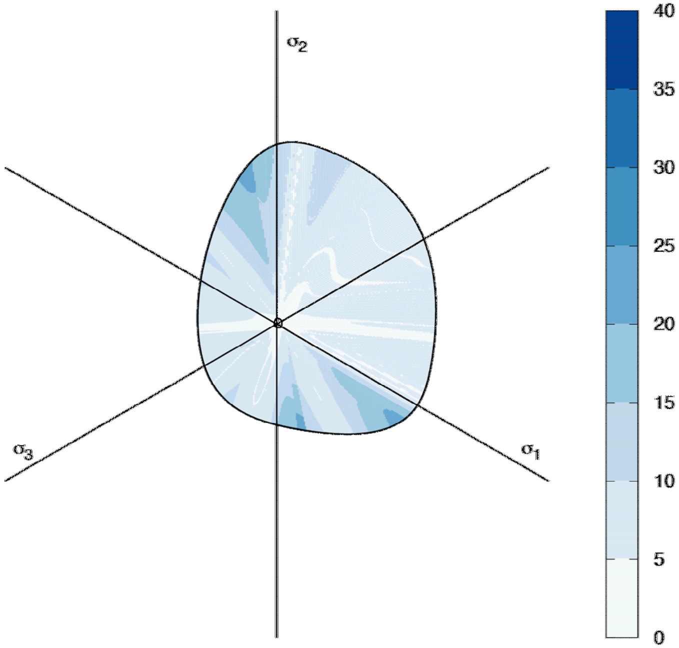
η = 0.1



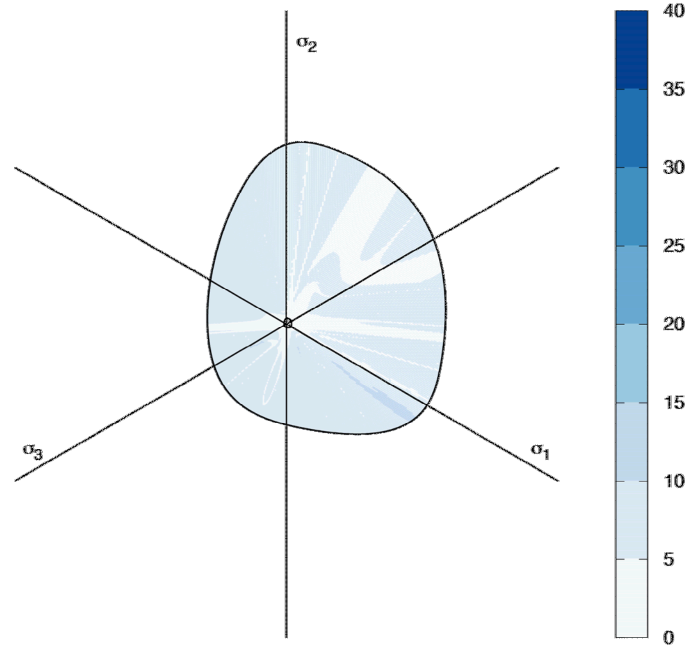
η = 0.5

Plunkett-Cazacu-Barlat – η

$$\phi(\boldsymbol{\sigma}) = \Big\{ g(a, c'_{ij}, c''_{ij}, k', k'') \Big[(|s'_1| - k's'_1)^a + (|s'_2| - k's'_2)^a + (|s'_3| - k's'_3)^a + (|s''_1| - k''s''_1)^a + (|s''_2| - k''s''_2)^a + (|s''_3| - k''s''_3)^a \Big] \Big\}^{1/a}$$

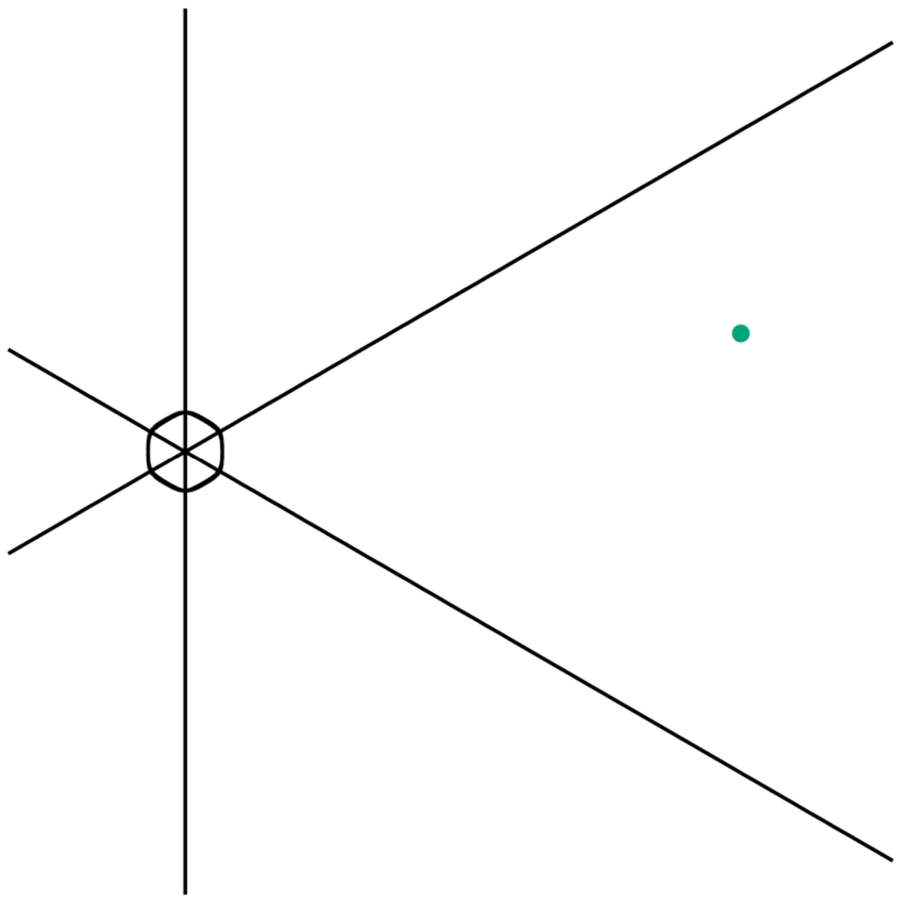


η = 0.1

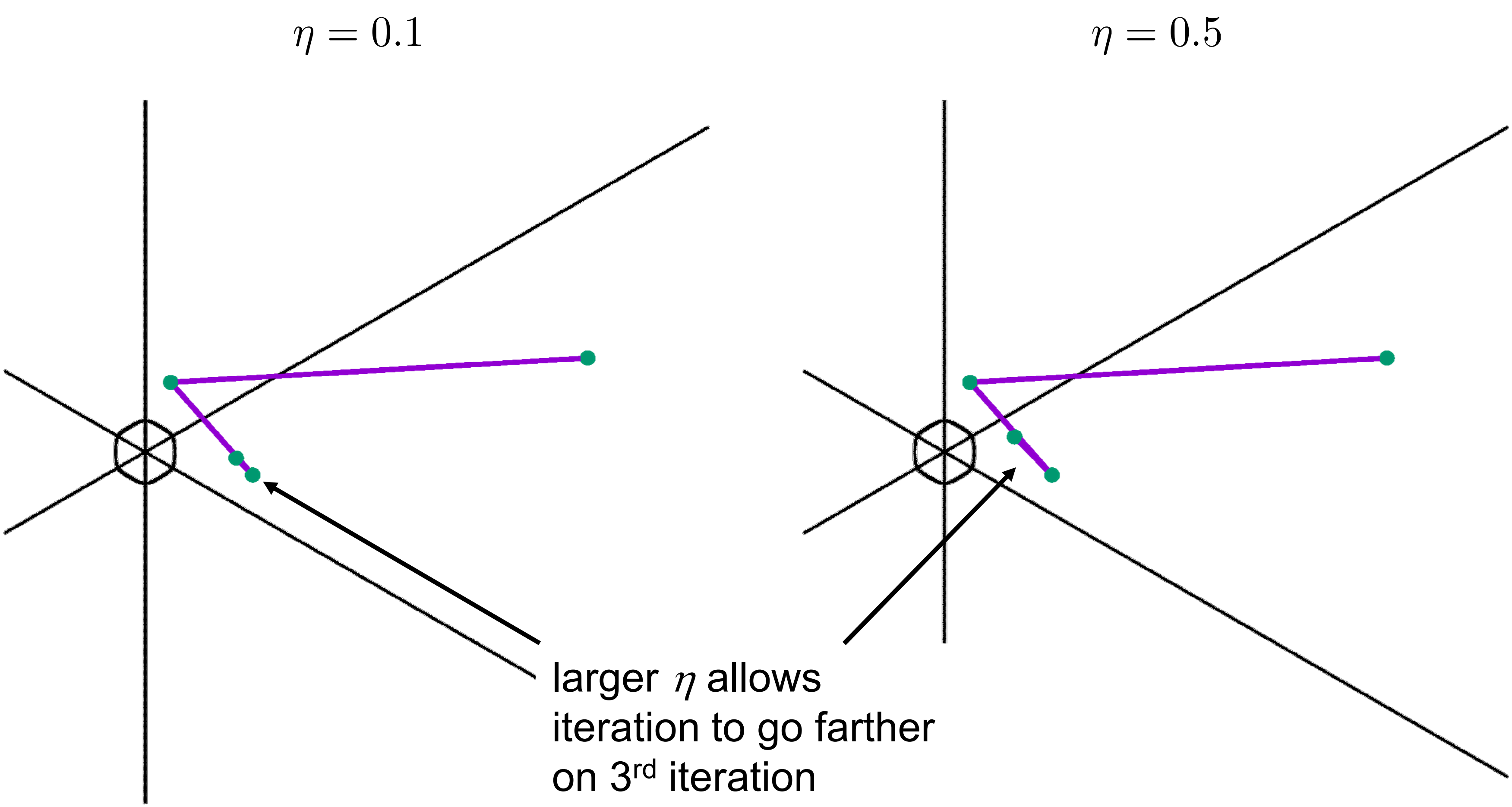


η = 0.5

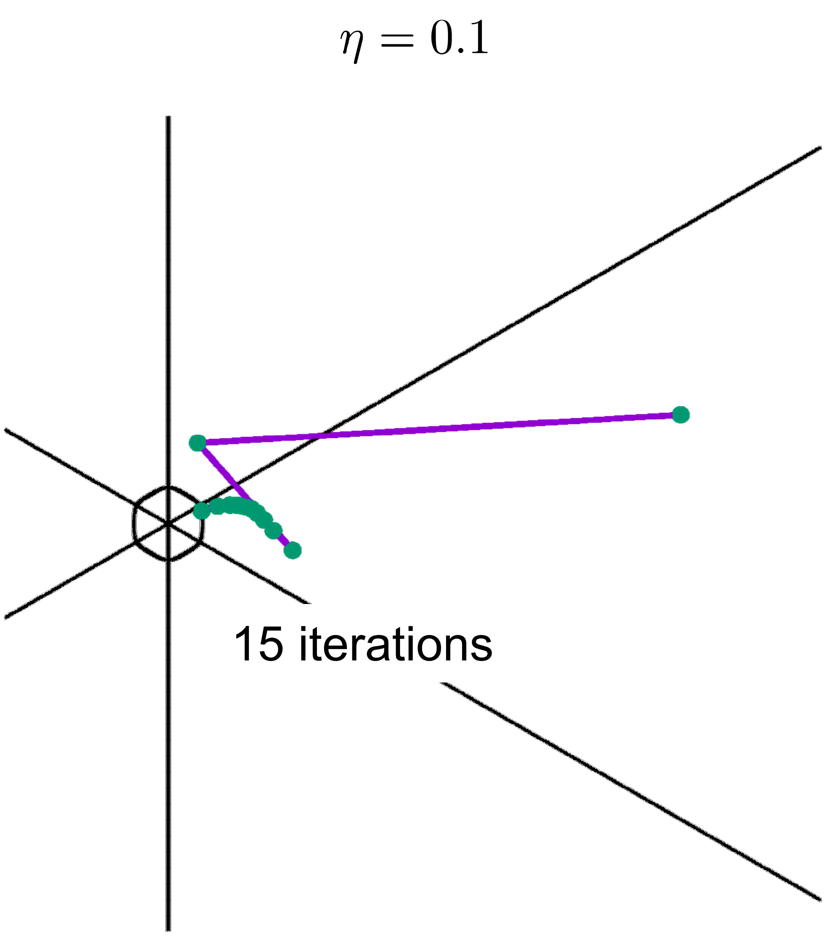
Return Mapping Trajectory – Line Search ($\eta = 0.5$)

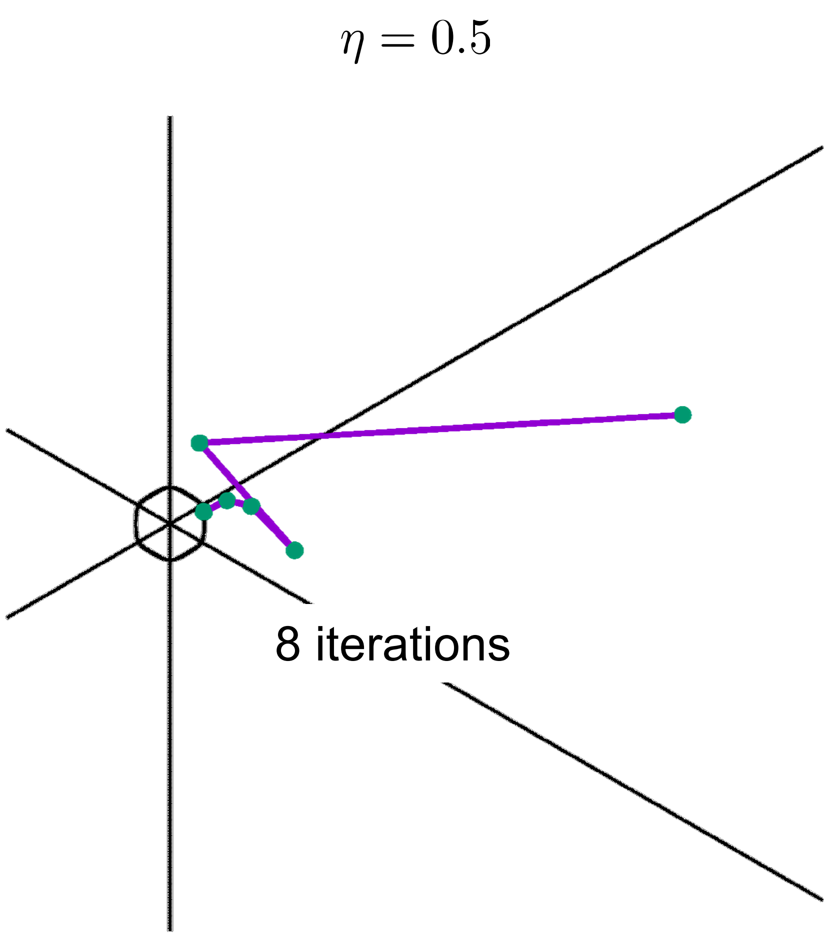


Return Mapping Trajectories

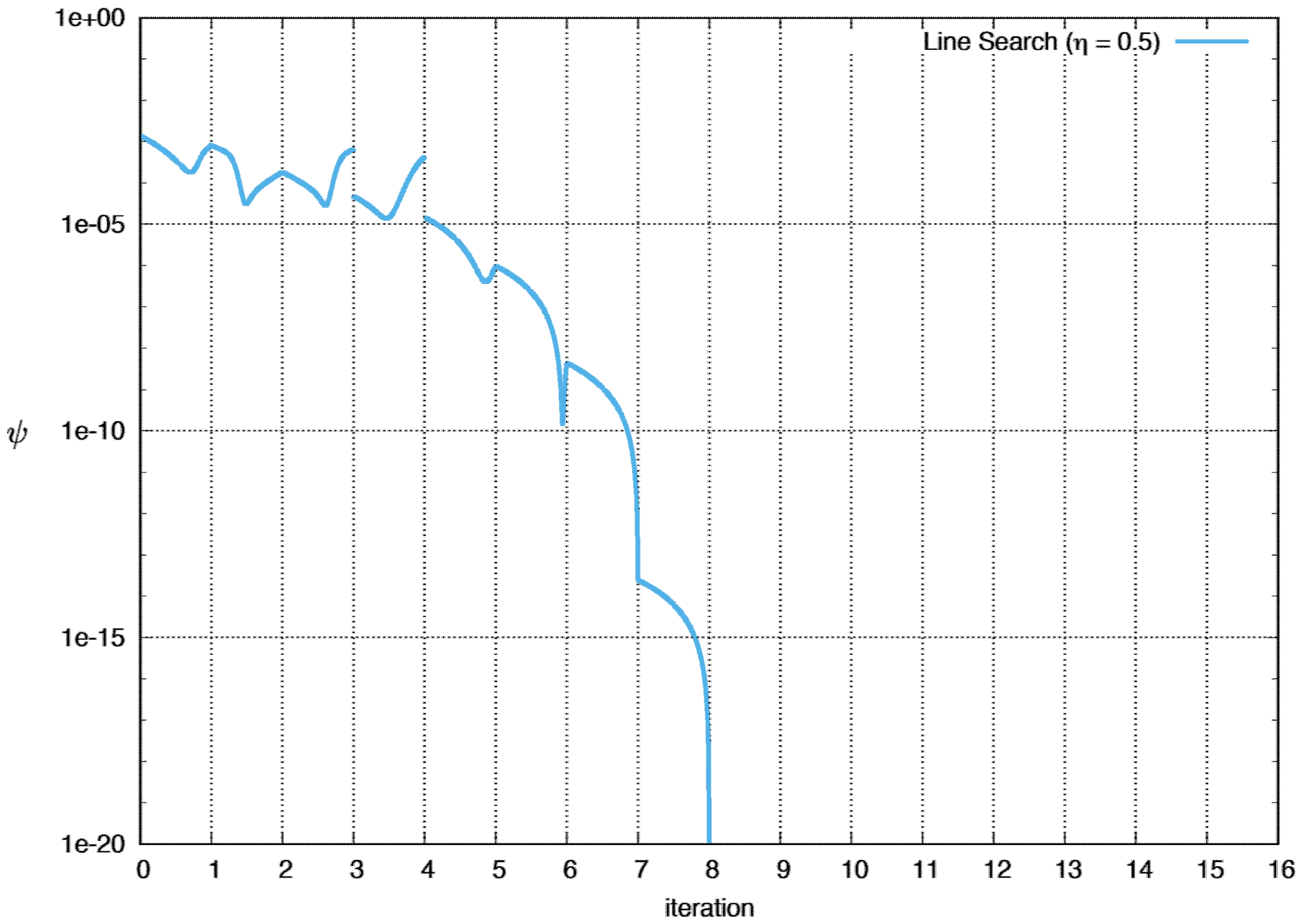


Return Mapping Trajectories

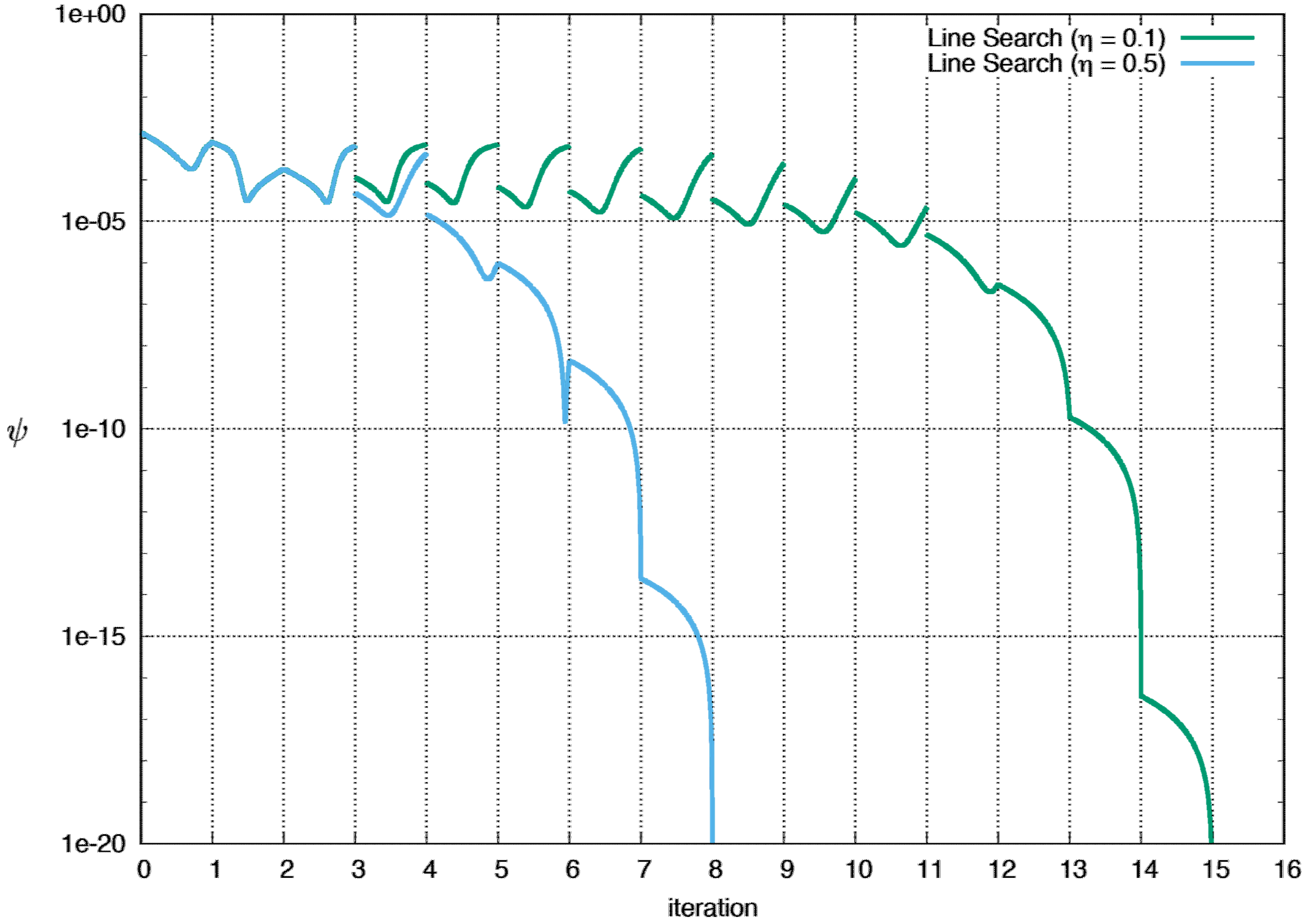




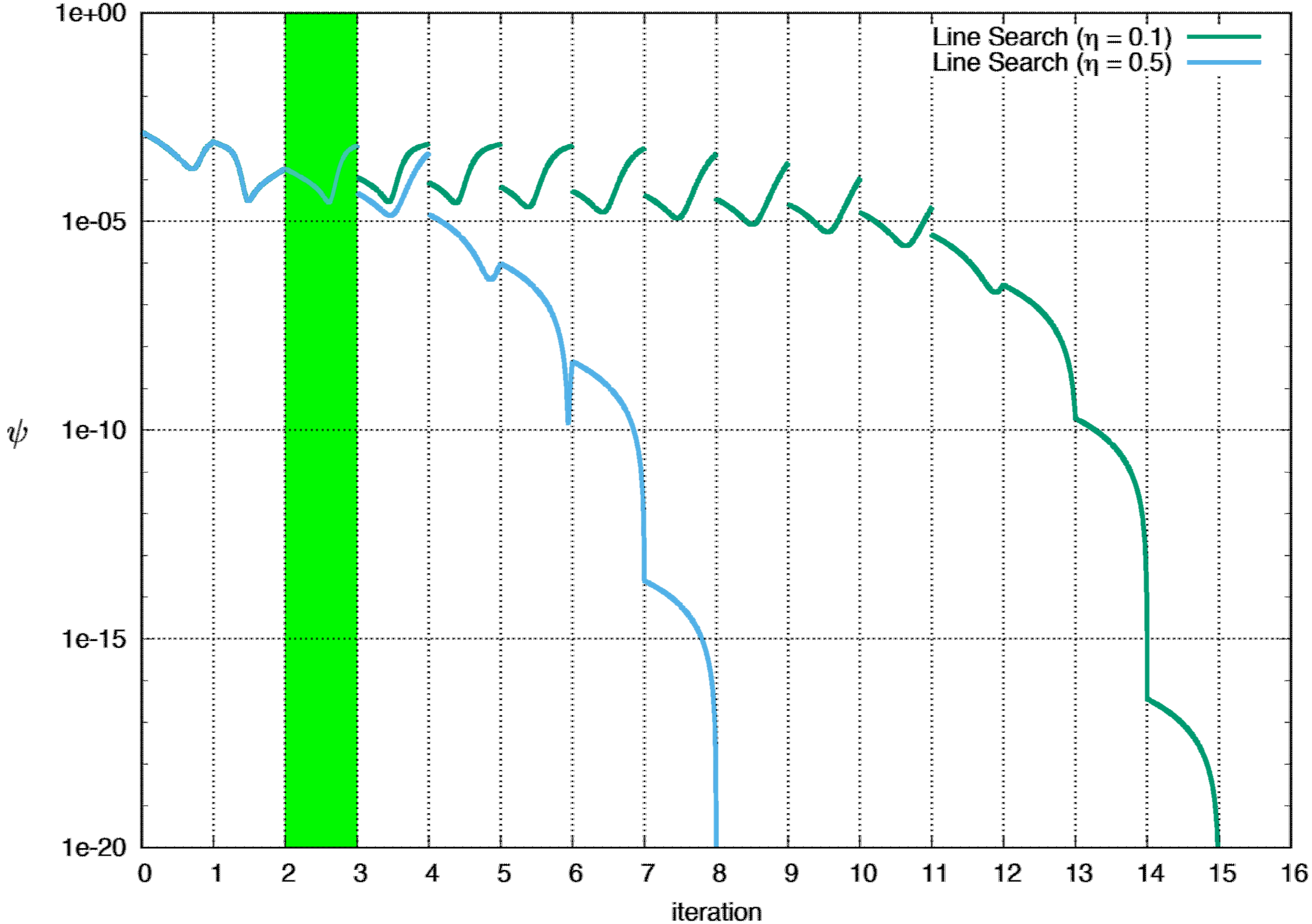
Merit Function – η = 0.5



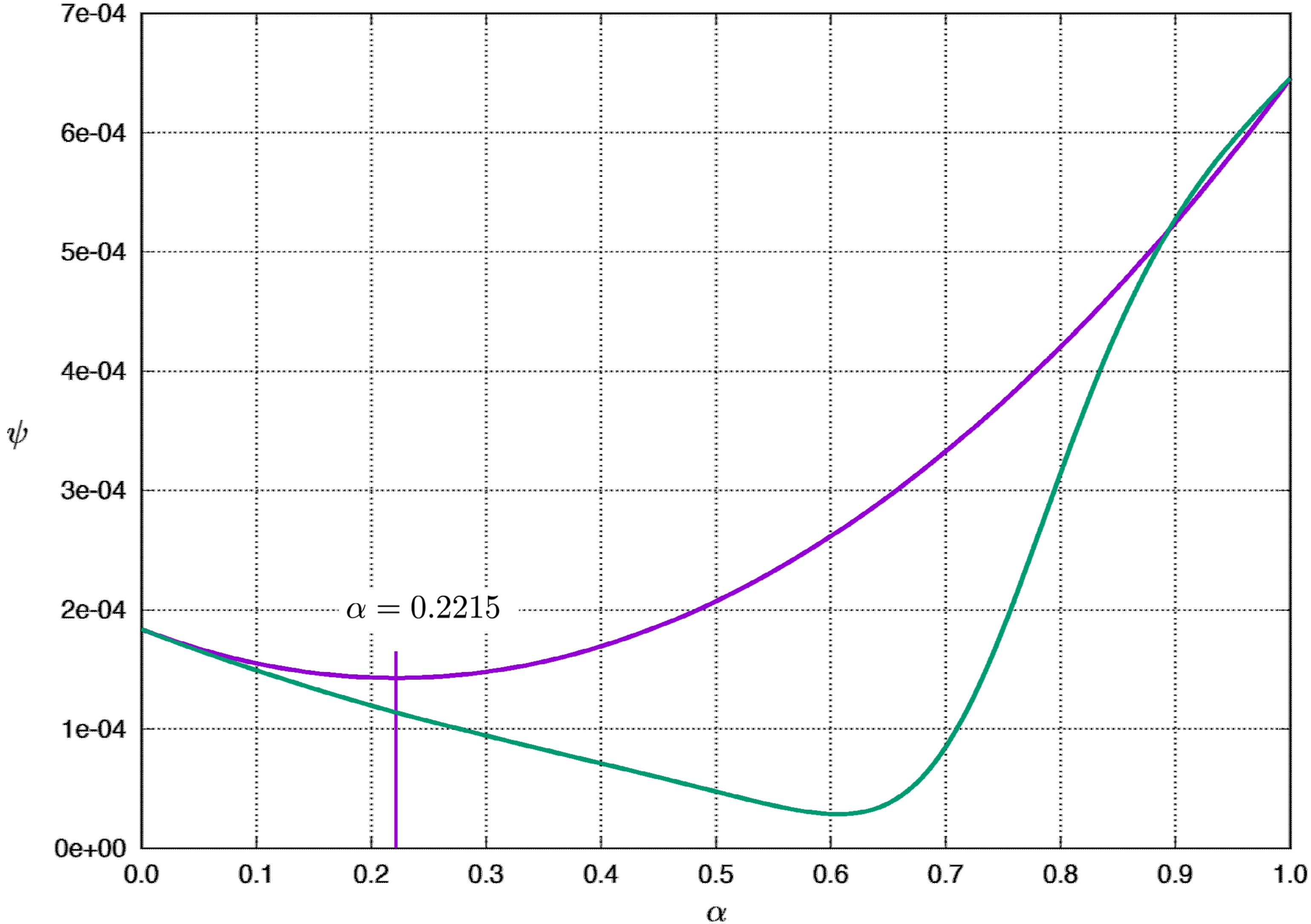
Merit Function – Comparison



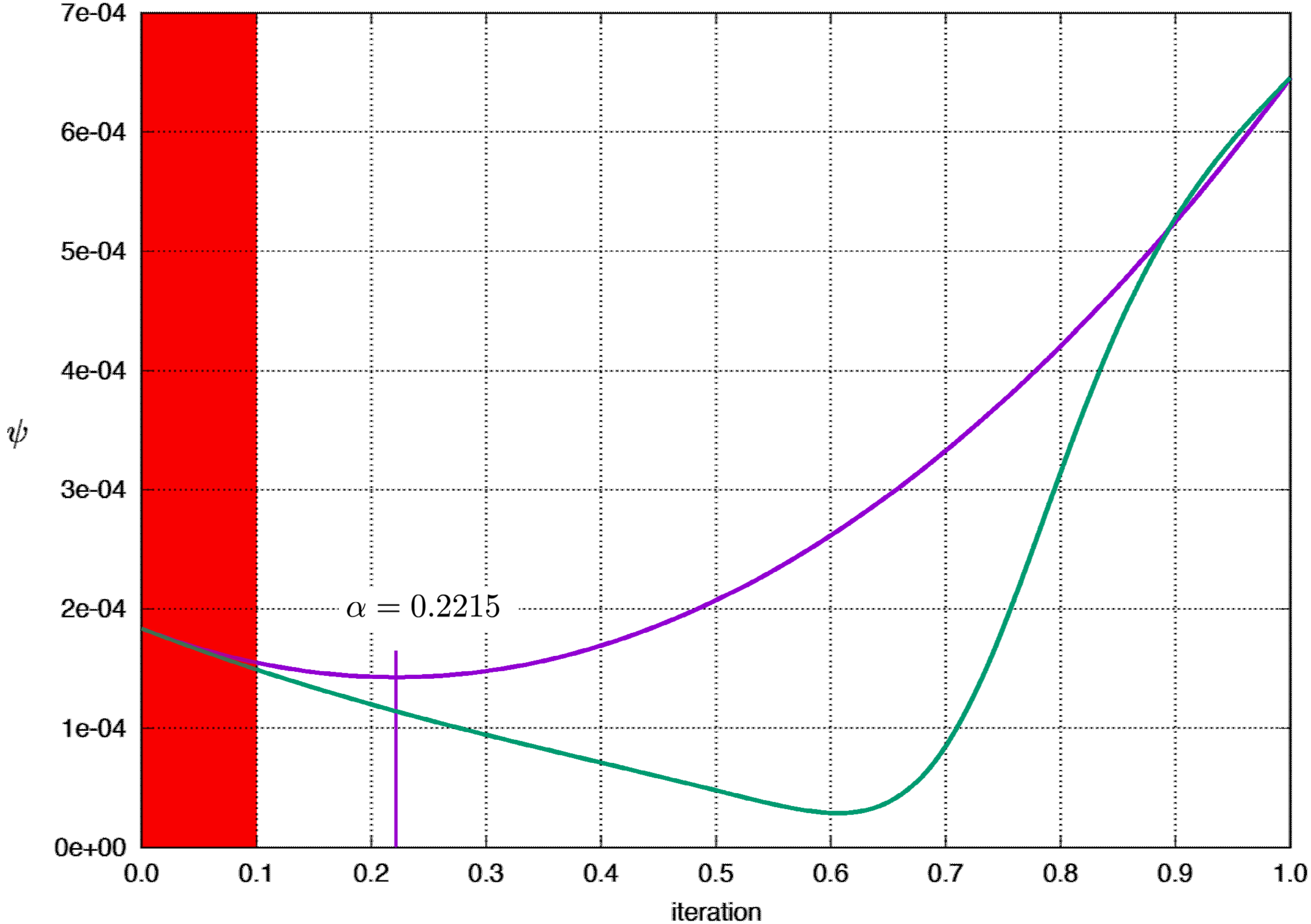
Merit Function – Comparison to η=0.1



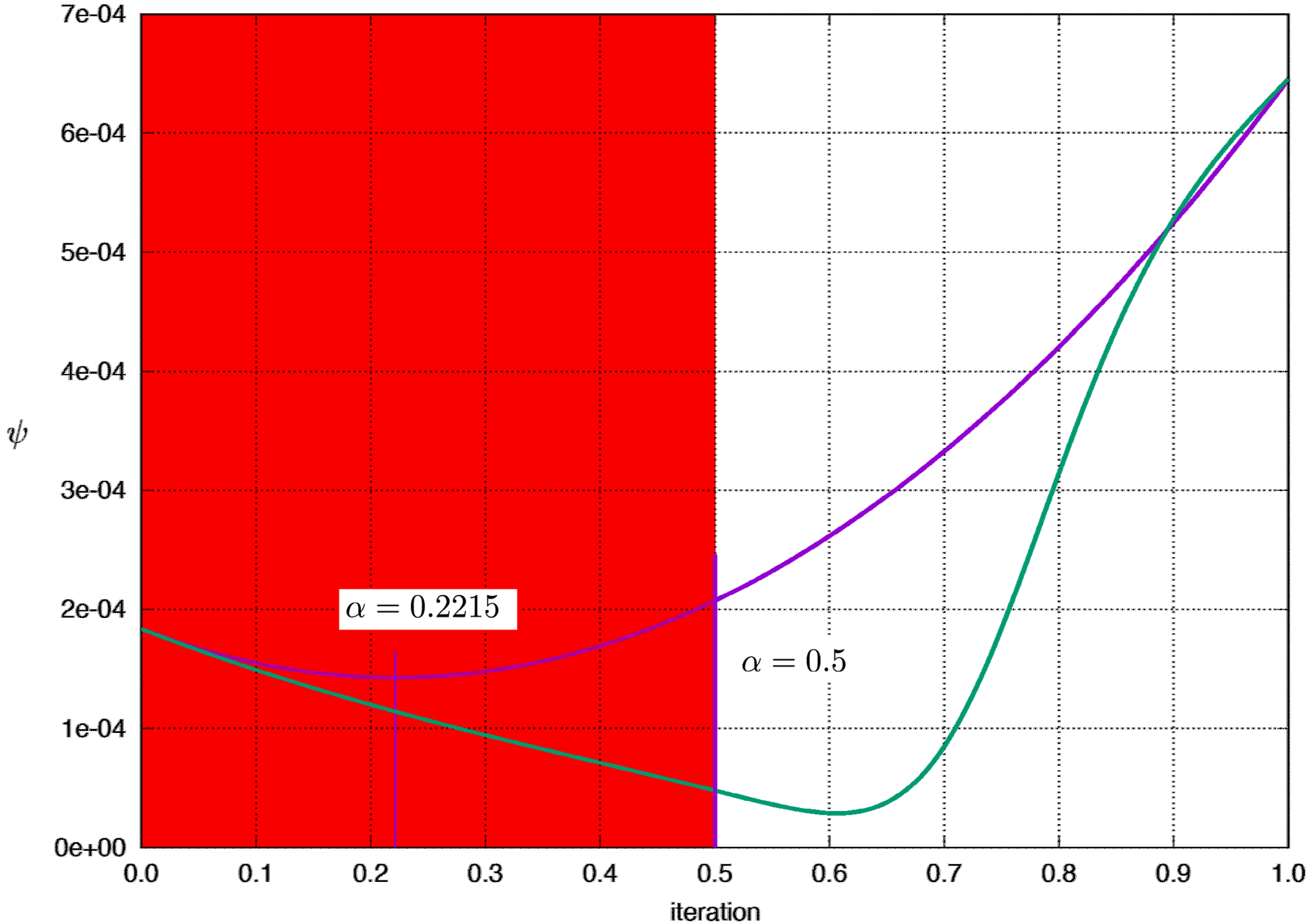
Merit Function – Comparison



Merit Function – η = 0.1

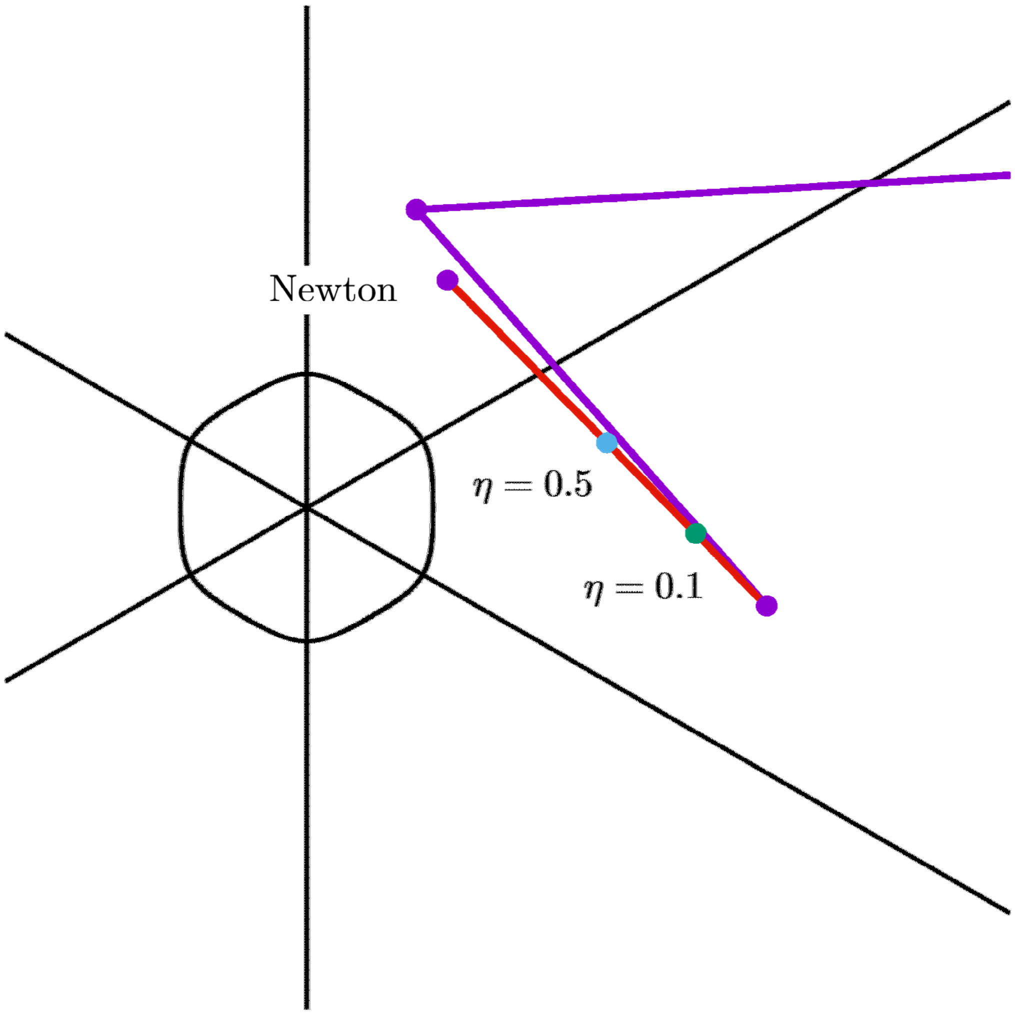


Merit Function – η = 0.5



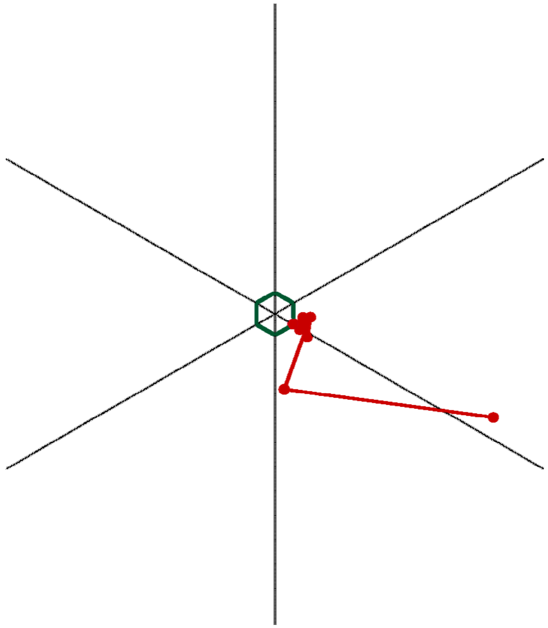
Return Map

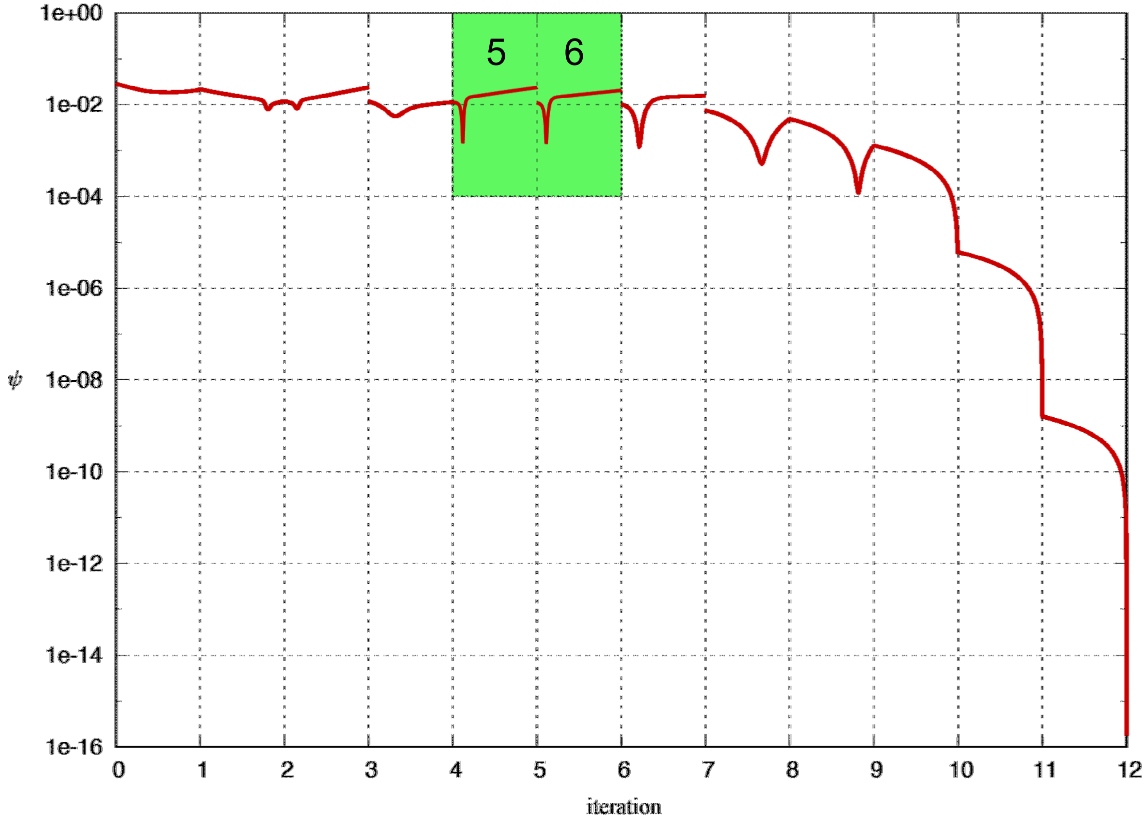
3rd iteration



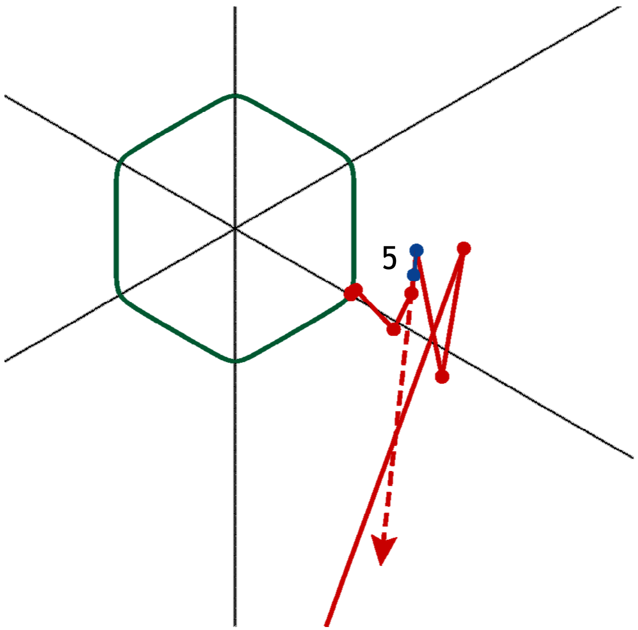
Merit Function for "Tresca"

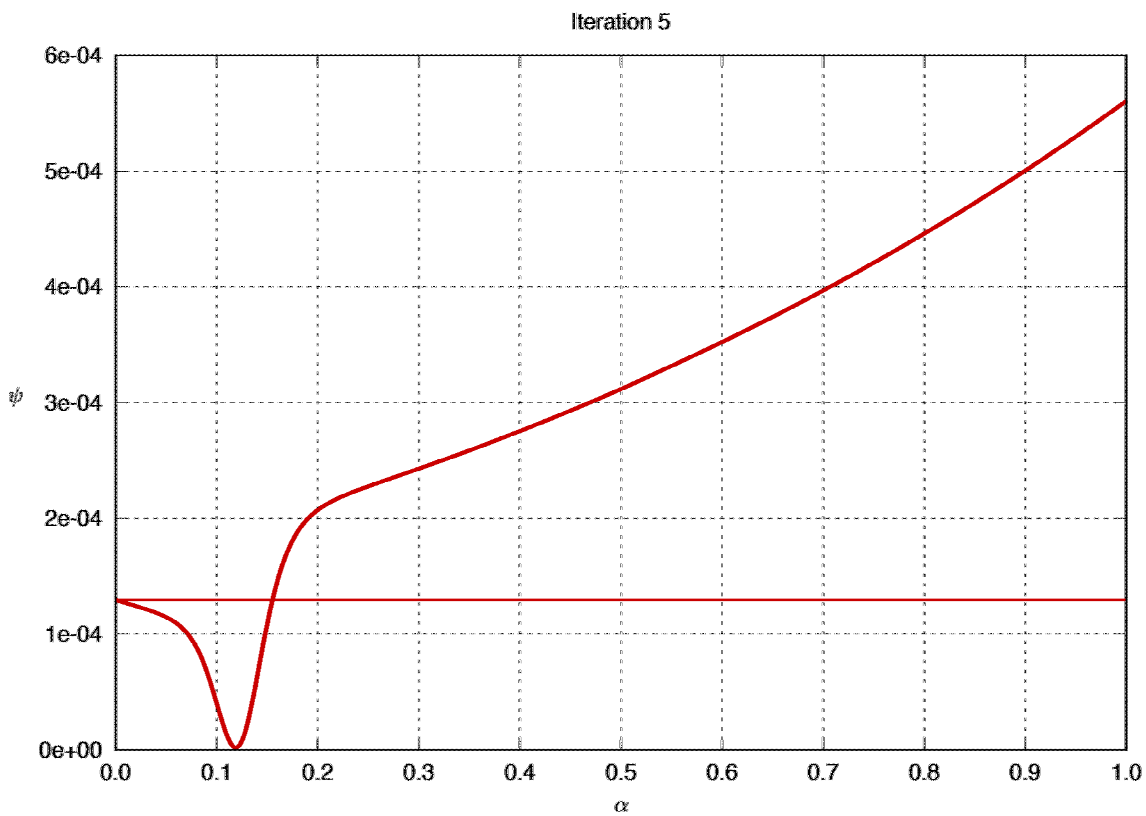
$$\phi(\boldsymbol{\sigma}) = \left\{ \frac{1}{2} \left[|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a \right] \right\}^{1/a} \qquad a = 20$$



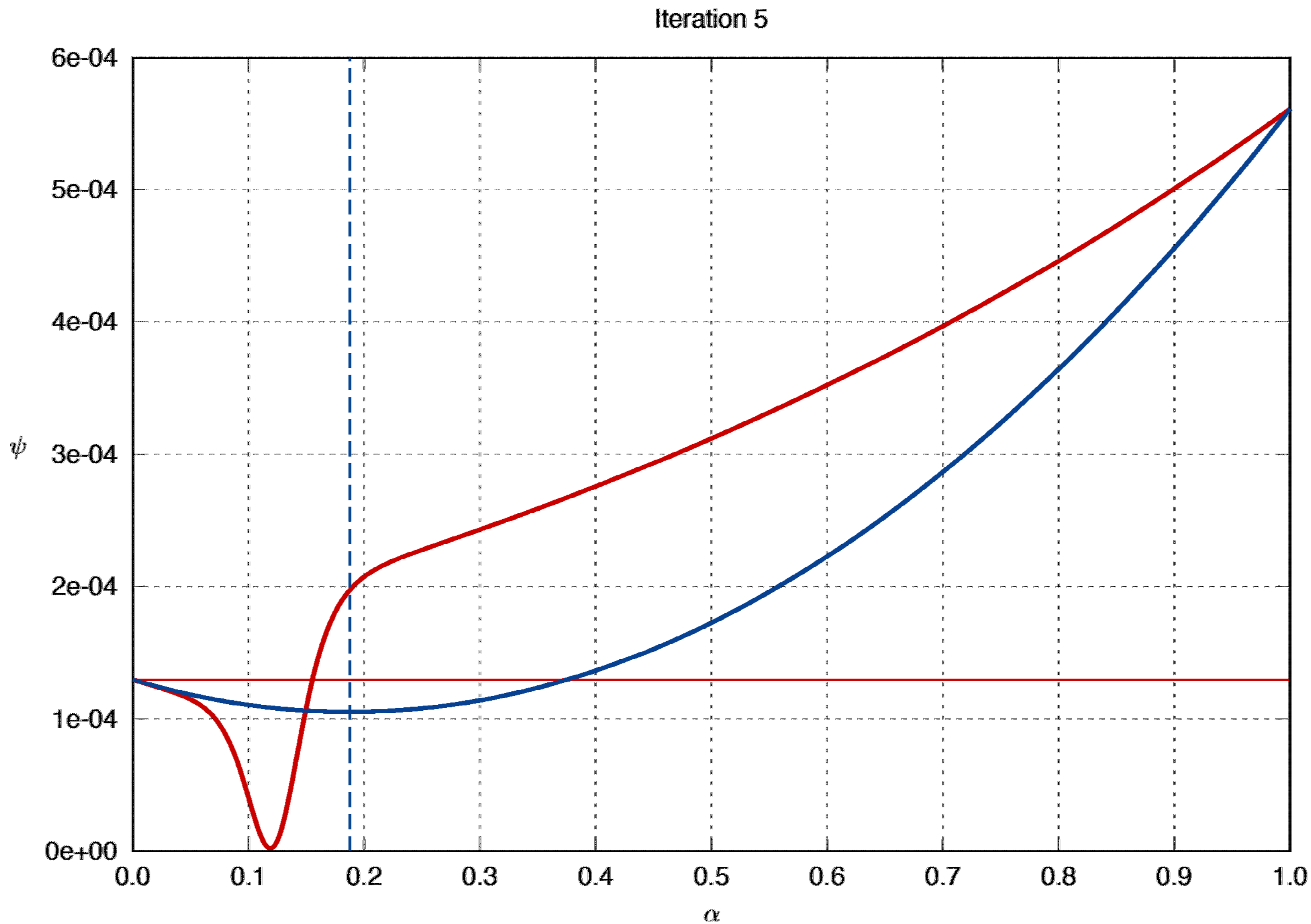


Merit Function



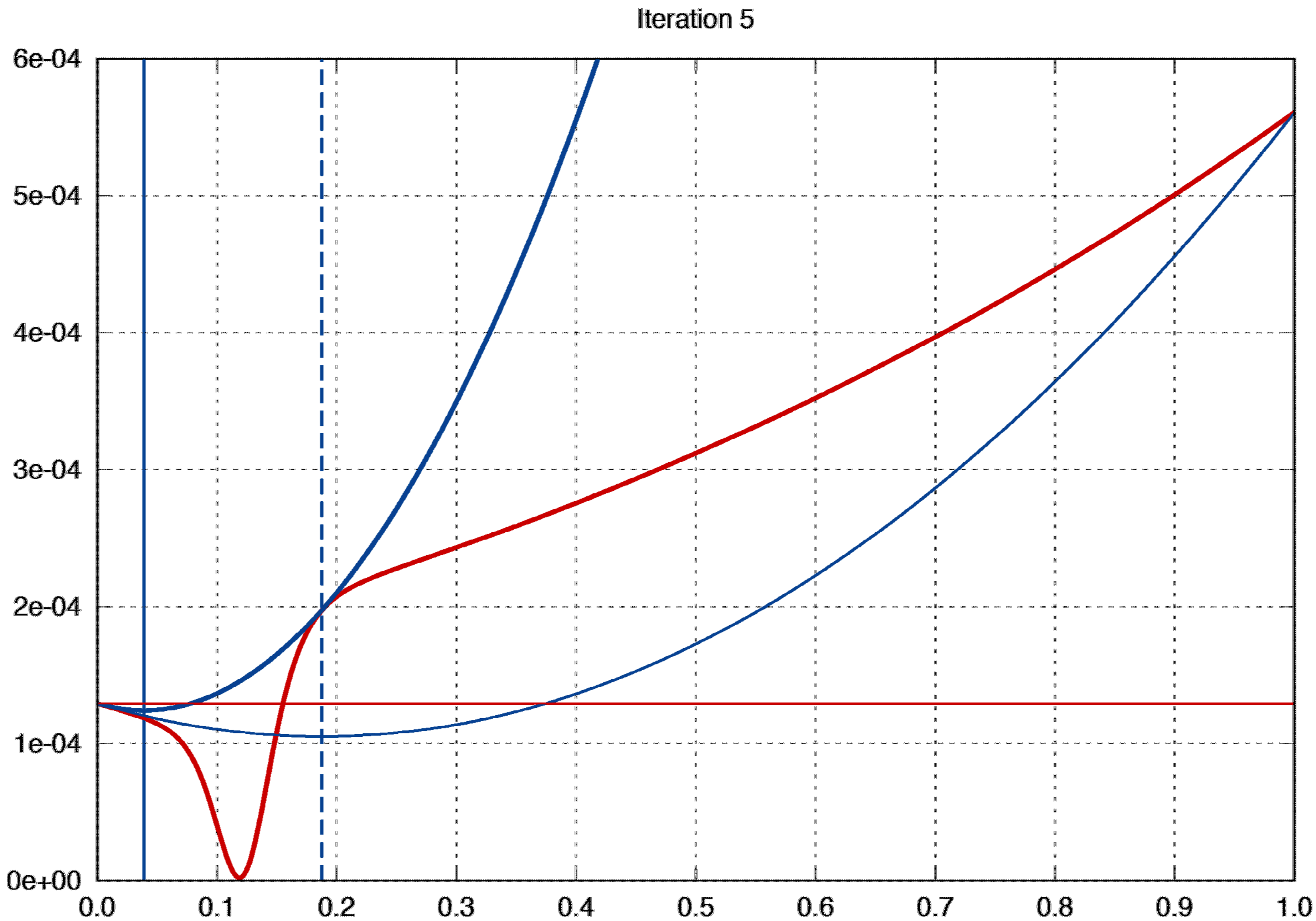


Iteration for α



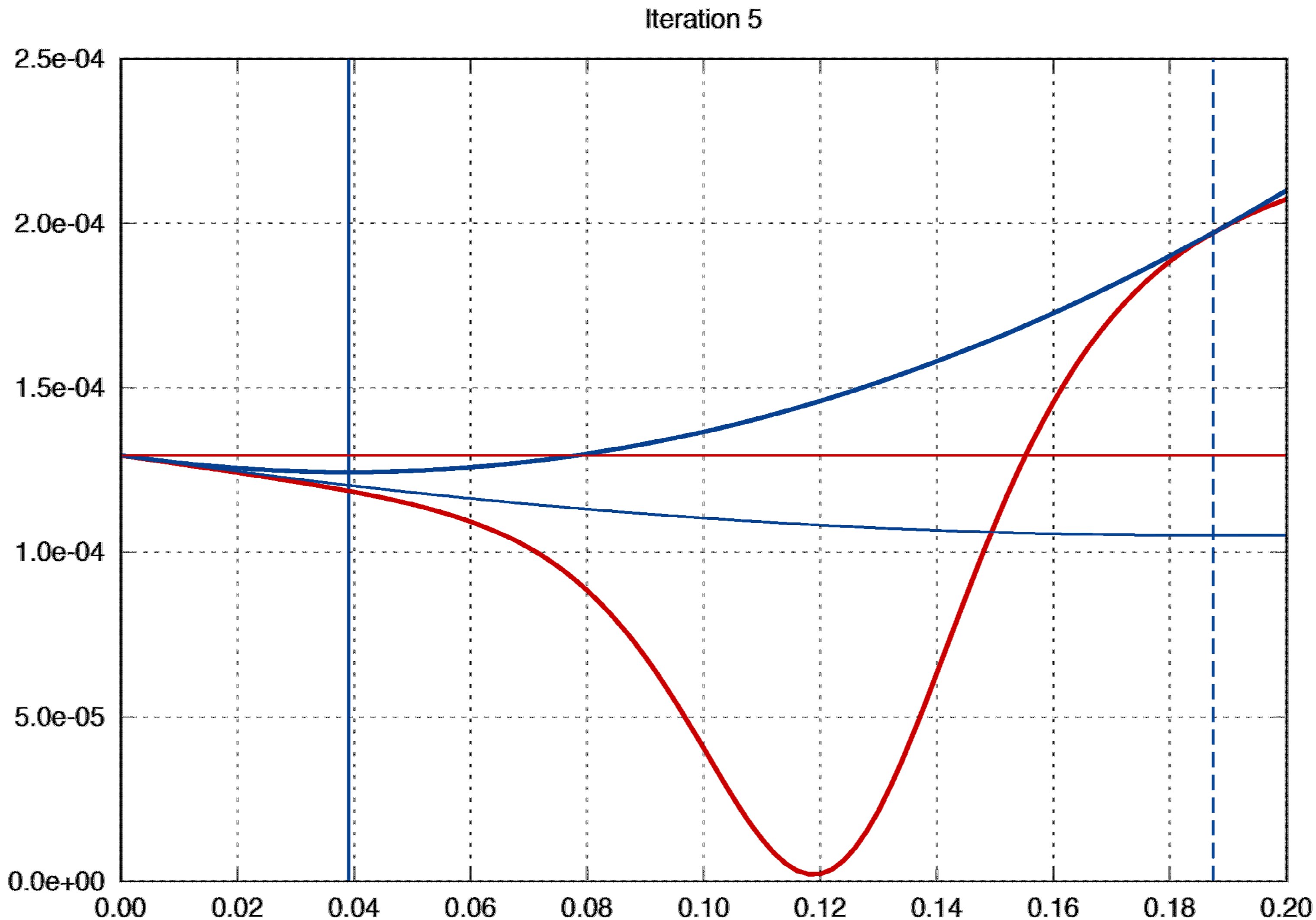
α = 0.1874

Iteration for α



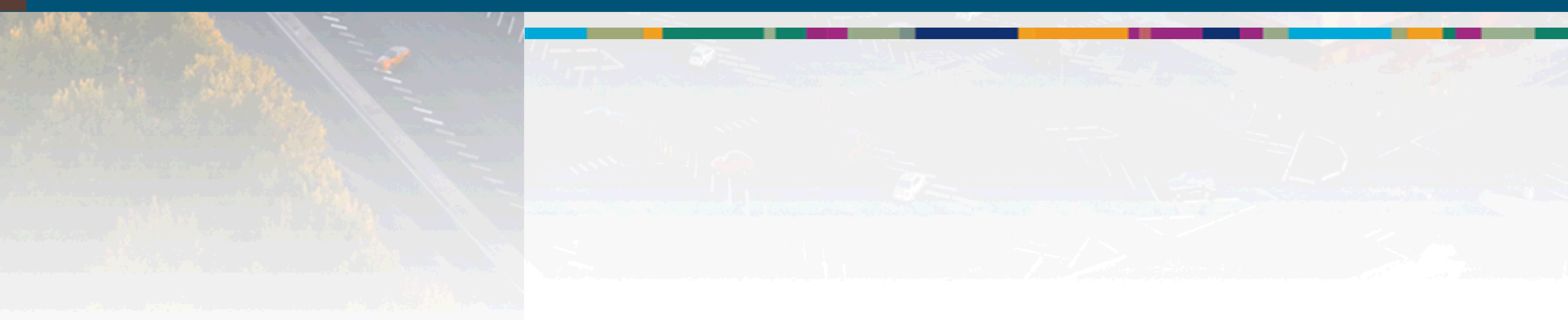
α = 0.0391

Iteration for α



α = 0.0391

Future Work

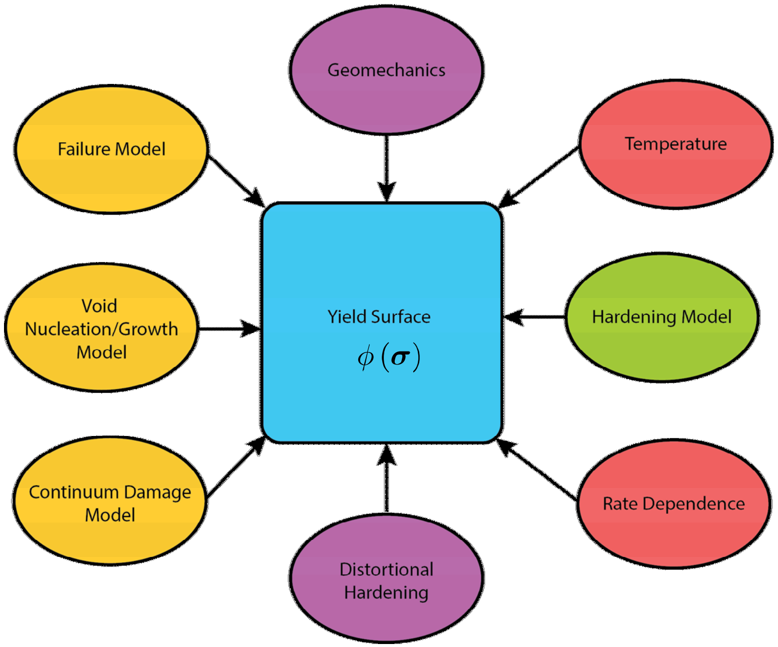


Capability Expansion

Robust integration for any yield surface provides a foundation for further developments

- Hardening models
- Temperature and rate dependence
- Models for material failure
- Pressure dependent models
- Yield surface distortion

$$F\left(\boldsymbol{\sigma}, \bar{\varepsilon}^{p}\right)=\phi\left(\boldsymbol{\sigma}\right)-\bar{\sigma}\left(\bar{\varepsilon}^{p}\right)\leq 0$$



Conclusions

- A robust integration algorithm has been developed, augmenting a return mapping algorithm with a line search method
  - Flexibility
  - Understanding
  - Extensibility

1. W. M. Scherzinger, “A return mapping algorithm for isotropic and anisotropic plasticity models using a line search method” *Comp. Meth. App. Mech. Eng.,* **317** (2017) 526-553

2. B. T. Lester and W. M. Scherzinger, “Trust-region based return mapping algorithm for implicit integration of elastic-plastic constitutive models” *Int. J. Num. Meth. Eng.,* **112** (2017) 257-282

3. B. T. Lester and W. M. Scherzinger, “An evolving effective stress approach to anisotropic distortional hardening” *Int. J. Solid Struct.,* **143** (2018) 194-208

Extra Slides