

1 Detecting an underground tunnel by applying joint traveltime and waveform inversion

2 Wenbin Jiang^{1,3}, Colin A. Zelt², and Jie Zhang³

³ ¹Nanjing University, School of Earth Sciences and Engineering, Nanjing, Jiangsu, China.

⁴²Rice University, Department of Earth, Environmental & Planetary Sciences, Houston, Texas, USA.

7 Corresponding author: Jie Zhang; Email: jzhang25@ustc.edu.cn

8 Email: wbjiang@nju.edu.cn

9 E-mail: czelt@rice.edu

10

11 **Abstract**

12 Underground tunnel and void detection is a challenging geophysical problem, and many methods have
13 been proposed. Seismic techniques are promising because of the large seismic velocity contrasts
14 between the air-filled void and the surrounding sediment and concrete. We apply a joint seismic
15 travelttime and waveform inversion method to image a buried tunnel with concrete walls and a void
16 space inside. The joint inversion images the top of the concrete tunnel as a high-velocity anomaly and
17 the void space as a low-velocity anomaly. The location of the velocity anomalies predicted by the
18 method agrees with the known location of the tunnel. As a comparison, the stand-alone full waveform
19 inversion is also applied to the data. The first-arrival travelttime tomography shows weak nonlinearity
20 but fails to image the hidden low-velocity layer. Full waveform inversion is able to image complex
21 near-surface structures, but fitting waveforms may not honor the travelttime fit, especially when the
22 data contain noise. Synthetic and real data tests show that the joint inversion method retains the
23 advantages of both travelttime inversion and full waveform inversion and overcomes their respective
24 drawbacks at the same time. The field example shows the joint inversion provides a better
25 reconstruction of the high-velocity feature representing the top of the concrete wall in terms of its
26 magnitude and location.

27

28 **Keywords:** Underground tunnel detection; Full waveform inversion; Joint inversion; Seismic data
29 processing

30

31

32 **1. Introduction**

33 Over the past several decades, many geophysical methods have been proposed to detect underground
34 tunnels and void spaces, such as electromagnetic (EM), gravity, ground-penetrating radar (GPR), and
35 seismic refraction, diffraction, and surface-wave methods. These techniques are based on the large
36 contrast in physical properties such as electrical resistivity, density, and seismic velocity between the
37 air-filled cavity, the surrounding concrete, if any, and the host geologic medium (Butler, 1984; Belfer et
38 al. 1998; Van Schoor, 2002; Mochales et al. 2008; Kaufmann et al. 2011; Sloan et al. 2015; Chen et al.
39 2017; Hauquin and Mourey, 2019; Lai et al. 2018; Alsharabi et al. 2019).

40

41 Butler (1984) detects a shallow air-filled cavity system and a deeper water-filled cavity system using a
42 microgravity survey. The use of the vertical gravity gradient along the profile line helps detect shallow
43 (less than 6 m) anomalous features. Mochales et al. (2008) present a combination of gravity, magnetic,
44 and GPR surveys to detect underground cavities. These three methods are based on different physical
45 properties. The sequential use of three methods allows the detection of cavities with different fill, such
46 as air, water, and debris, that are difficult to resolve with only one or two approaches. Kaufmann (2011)
47 applies gravimetry and electrical resistivity imaging to detect underground caves in the southern Harz
48 mountains of Germany. The density of the air- and water-filled voids is much lower than the
49 surrounding host rock. The water-filled cavity is more electrically conductive than the host rock, while
50 the air-filled cavity is less conductive. Hauquin and Mourey (2019) apply 2D electrical resistivity
51 tomography technique to detect masonry tunnels above subsurface cavities. The tunnels are imaged as
52 low resistivity anomalies because of a highly resistive layer (caused by moisture and seepage water
53 deposits) around it. Hong Kong highways department carry out a blind test to evaluate the feasibility of
54 GPR method in underground void detection (Lai et al. 2018). An acceptance criterion is designed

55 according to the international standard and local experience. Results provided by six local service
56 companies are not satisfactory, which indicates the difficulty of void detection in the urban area. Kilic
57 and Eren (2018) conduct a GPR survey to obtain an image of voids and karst conduits. Furthermore, a
58 three-layer neural network is applied in the detection process.

59

60 All the above nonseismic methods have advantages and limitations in detecting embedded anomalies.
61 Gravity and GPR methods are effective in identifying shallow cavities. However, gravity data are
62 insensitive to the void if it is buried deeper than the size of the void (Wightman et al., 2003). The GPR
63 method depends on the contrast in the dielectric properties between the target and host overburden. The
64 penetration depth of the GPR signal is limited in lossy sediments (Slob et al. 2010; Luo et al. 2019),
65 and the void cannot be detected if it is filled with water. The EM approach provides no useful
66 information on the material below high electrically conductive areas. Moreover, the EM approach
67 requires longer electrode arrays in the case of deeper voids, which makes the detection difficult
68 (Wightman et al., 2003). Therefore, the potential-field methods are usually applied in regions where the
69 target is large and shallow. Compared with the gravity, GPR and EM approaches, seismic methods are
70 more reliable for detecting smaller objects because of better resolution and penetration under various
71 conditions.

72

73 Belfer et al. (1998) apply refraction tomography to detect short-wavelength velocity variations.
74 Furthermore, a diffraction stack is used to delineate local scattering objects. Seismic diffractions result
75 from a subsurface discontinuity that acts as a secondary radiation source and produces hyperbolic
76 events in seismic records. Backscattered surface waves are caused by surface waves that scatter back

77 toward the source when reaching a lateral interface. Conventional seismic data processing workflows
78 seek to enhance reflected energy while suppressing diffractions and surface waves because both are
79 considered to be noise in reflection imaging. Xia et al. (2007) demonstrate that the Raleigh-wave
80 diffractions in the shot gather can be used to detect 2D void and fault. However, a sophisticated data
81 preprocessing is required to preserve weak diffracted surface waves. Sloan et al. (2015) apply P-wave
82 diffraction and surface-wave backscatter approaches to detect a subterranean tunnel. Compared with
83 reflections, the amplitude of the diffractions is weak. The backscattered surface waves are mixed with
84 the forward propagating surface-wave energy. A sophisticated preprocessing workflow needs to be
85 applied to the seismic data to extract diffractions and backscattered surface-wave energy. Tran et al.
86 (2013) apply 2D time-domain full waveform inversion to detect an air-filled sinkhole. Both body
87 waves and Rayleigh waves are used to invert P- and S-wave velocities. Chen et al. (2017) demonstrate
88 the capability of using frequency-dependent travelttime tomography (FDTT) and frequency-domain full
89 waveform inversion (FWI) to detect a known target tunnel. The embedded air-filled void space is
90 imaged as a low-velocity anomaly. The workflow of applying FDTT followed by FWI shows the
91 capability of detecting subwavelength-scale features. Chen et al. (2017) present frequency-domain FWI
92 of P and SH waves. In the present study, joint inversion is implemented in the time-domain with an
93 acoustic approximation, and only P-wave data are inverted.

94

95 Ray-based first-arrival travelttime tomography has been widely applied for estimating near-surface
96 velocity (Zhu et al. 1992; Zhang and Toksöz, 1998; Leung and Qian, 2006; Park and Pyun, 2018).
97 However, travelttime inversion usually produces a suboptimal estimate of subsurface velocity because it
98 assumes an infinite frequency approximation of the data, although Zelt and Chen (2016) present a

99 frequency-dependent traveltime inversion methodology. FWI is a robust tool for imaging near-surface
100 structures, which improves the velocity estimation with higher resolution (Tarantola, 1984; Pratt et al.
101 1998; Wang et al. 2019). A fundamental challenge of FWI is to avoid a local minimum in the model
102 space caused by the nonlinearity of the inverse problem. FWI requires a good starting model to avoid
103 cycle-skipping between the synthetic and observed data (Virieux and Operto, 2009). Researchers have
104 made significant efforts to develop new FWI algorithms and strategies that can avoid or mitigate the
105 cycle-skipping problem. Zhang and Chen (2014) propose a joint first-arrival travelttime and waveform
106 inversion method to estimate near-surface velocities. The method minimizes the misfit function for
107 both traveltimes and waveform simultaneously, which helps mitigate the nonlinearity of the inversion.
108 Jiang and Zhang (2015) apply the joint travelttime and waveform inversion method to image complex
109 near-surface structures in the Yumen oil field. Fan et al. (2015) demonstrate the effectiveness of the
110 joint inversion method using real data from the North Sea.

111

112 The goal of this study is to detect a buried tunnel surrounded by concrete walls and filled with air. The
113 burial depth of the anomalies is shallow and the geologic structure in the survey area is quite simple.
114 We apply seismic methods to detect the embedded anomalies, specifically, we apply joint travelttime
115 and waveform inversion to detect the underground tunnel and void space. We demonstrate the synthetic
116 test results with two starting models: 1D linear gradient velocity model and travelttime tomography
117 model. As a comparison, stand-alone FWI results are also presented using these two workflows. We
118 also present real data results from FWI and the joint inversion.

119

120 In the following sections, we shall introduce the basic theory of joint inversion. Then, we present a

121 synthetic test that is designed according to the prior knowledge of the target. Finally, we test the joint
 122 inversion method on real data acquired on the Rice University campus and previously analyzed using
 123 frequency-domain FWI by Chen et al. (2017).

124

125 **2. Method**

126 In this section, we review the basic theory of the joint inversion method. The joint inversion minimizes
 127 the traveltimes and waveform residuals simultaneously by solving a nonlinear inverse problem. We refer
 128 readers to Zhang and Chen (2014) for more details.

129

130 In the time domain, the 2D acoustic-wave equation can be expressed as

$$131 \quad \begin{aligned} -\frac{1}{\kappa} \frac{\partial \mathbf{P}}{\partial t} &= \nabla \cdot \mathbf{v} + \mathbf{S}, \\ \rho \frac{\partial \mathbf{v}}{\partial t} &= -\nabla \mathbf{P}. \end{aligned} \quad (1)$$

132 where \mathbf{P} is pressure, \mathbf{v} denotes particle velocity, \mathbf{K} and ρ are the bulk modulus and density,
 133 respectively, and \mathbf{S} represents the source time function. The synthetic seismograms are generated by a
 134 staggered-grid finite-difference method with fourth-order accuracy in space and second-order accuracy
 135 in time (Zhang and Zhang, 2011). The free-surface boundary condition is applied on the top of the
 136 model, and the perfectly matched layer (PML) boundary conditions are used at the other three
 137 boundaries (Zhang and Shen, 2010). The traveltimes are calculated by wavefront raytracing (Zhang and
 138 Toksöz, 1998).

139

140 The objective function of the joint inversion is:

$$141 \quad \Phi(\mathbf{m}) = (1 - \omega) \|\mathbf{P}_{obs} - \mathbf{P}_{syn}(\mathbf{m})\|^2 + \omega \|\mathbf{t}_{obs} - \mathbf{t}_{syn}(\mathbf{m})\|^2 + \tau \|\mathbf{L}(\mathbf{m} - \mathbf{m}_0)\|^2,$$

142 (2)

143 where \mathbf{P}_{obs} and \mathbf{P}_{syn} represent the observed data and synthetic waveform, respectively. \mathbf{t}_{obs} and \mathbf{t}_{syn} are
144 the picked first-arrival traveltimes and the synthetic traveltimes. \mathbf{m} is the velocity model, and \mathbf{m}_0 is a
145 prior model for the joint inversion. \mathbf{L} denotes a Laplacian operator for regularization, and ω is a
146 weighting factor between the waveform and traveltimes misfits. τ is the regularization parameter. The
147 joint inversion gradient is the summation of the weighted traveltimes and waveform gradients. An
148 optimal step length is calculated at each iteration to minimize the joint inversion misfit. The nonlinear
149 optimization problem is solved by the conjugate gradient method. Only P wave velocities are inverted
150 during the inversion. The density model is converted from P-wave velocity model according to
151 Gardner's law (Gardner et al., 1974) in each iteration. And the converted density model is used in the
152 forward modeling of synthetic waveforms.

153

154 Selection of weighting factor ω is an important issue in the joint inversion. We design an
155 adaptive weighting factor selection strategy in this study. A large traveltimes weighting factor is
156 used at the beginning of the inversion process. The weighting factor gradually decreases as the
157 inversion iterates. We run the joint inversion in the beginning to constrain the shallow velocities,
158 and end up with waveform inversion to reconstruct the details. To be specific, we smoothly
159 decrease the traveltimes weighting factor from 0.8 to 0.2 during the inversion. The decreasing rate
160 depends on the iteration number. The regularization parameter is fixed as 0.1 during the whole
161 inversion process. The starting model is used as prior model \mathbf{m}_0 in the synthetic and field
162 examples.

163

164 The first-arrival traveltimes and the early arrival waveforms represent different attributes of the seismic
165 data. The infinite frequency traveltime tomography is based on ray theory, that can reconstruct
166 macro-velocity structure. The waveform inversion takes the frequency content of the data into account.
167 Therefore, it is more sensitive to small-scale heterogeneities. Furthermore, FWI resolves low-velocity
168 anomalies better than traveltime tomography because first-arrival ray coverage is poor in low-velocity
169 zone. The joint inversion method minimizes the misfit function for both traveltimes and waveforms in
170 the inversion process. In this way, the joint inversion method can fit both data by combining different
171 physical imaging theories (Zhang and Chen, 2014).

172

173 **3. Numerical example**

174 We apply the joint inversion method to data from a synthetic model. The true model is shown in Fig. 1a,
175 which is designed according to the known information of the tunnel structure on the campus of Rice
176 University (Chen et al. 2017). The velocity model is discretized with 240×100 cells, with a square cell
177 size of 0.1 m. The tunnel is surrounded with concrete walls that are 0.6 m thick on the top and 0.3 m
178 thick on the bottom and two sides. The void space within the tunnel is filled with air. The velocity of
179 the concrete is 4000 m/s, and the air velocity is 340 m/s. In this study, we are dealing with a void
180 surrounded by concrete. Therefore, there are two large velocity contrasts: between sediment and
181 concrete, and between concrete and air. The background velocity increase from 200 m/s at the surface
182 to 1000 m/s at the bottom. A 1D linear gradient initial model (Fig. 1b) is based on the true model. No
183 prior information of the tunnel walls and the voids are included in the starting model.

184

185 To test the spatial resolving power and reliability of the survey geometry, we use the same shot and

186 receiver intervals in the synthetic test as in the real data survey. The seismic geometry is perpendicular
187 to the buried tunnel. The seismic survey line is 24 m long and includes 25 shots with an interval of 1 m
188 and 72 receivers for each shot with an interval of 0.333 m. A 50 Hz peak frequency Ricker wavelet is
189 used as the source wavelet in the finite-difference forward modeling. The first-arrival traveltimes are
190 generated with wavefront raytracing (Zhang and Toksöz, 1998). Uncorrelated Gaussian-distributed
191 random noise with a standard deviation of 5% is added to the traveltimes. The noise is proportional to
192 the absolute traveltimes, the standard deviation is approximately 4 ms for the longest offset (24 m).

193

194 We perform forward modeling tests to explore how the tunnel structure affects the waveform. Figure 2
195 shows the synthetic traveltimes and waveform generated with the 1D linear-gradient background model
196 and the true model. We observe an advance of the traveltimes and early arrival waveform because of the
197 high-velocity concrete tunnel walls (indicated by the black arrows).

198

199 The 1D linear-gradient model is used as the starting model for conventional ray-theory
200 infinite-frequency traveltimes tomography (Zhang and Toksöz, 1998). The traveltimes misfit decreases to
201 the same level as the reciprocal errors. The traveltimes tomography model images the top of the
202 high-velocity concrete wall at the correct position (Fig. 1c). However, traveltimes tomography fails to
203 image the low-velocity void space below the high-velocity top of the tunnel. FWI utilizes full
204 wavefield information to improve the resolution of the geophysical properties estimation. The 1D
205 background velocity model is used as a starting model for FWI and the joint inversion. The starting
206 density model is converted from initial P wave velocity model according to Gardner's law, and it does
207 not include the void and concrete. Both the FWI and joint inversion image the low-velocity anomaly

208 associated with the void space. The joint inversion result appears to image the high-velocity anomaly
209 associated with the concrete top wall more accurately than FWI (Figs. 3(a)–(b)). This is because the
210 model gradient from traveltime helps reconstruct the velocity at the top of the concrete wall in the joint
211 inversion. However, the high-velocity anomaly at the top of the concrete in the joint inversion result is
212 much weaker than the one in the traveltime tomography result (Fig. 1c). The comparison between the
213 observed and synthetic waveform data is presented in Fig. 4 for shot 9. The waveform match between
214 observed and synthetic waveforms is excellent, but the traveltimes calculated with FWI and joint
215 inversion model do not fit the observed traveltimes. The results suggest that a 1D laterally
216 homogeneous velocity model is insufficient for this test using FWI and joint inversion. This inspires us
217 to use the traveltime tomography result as an initial model for FWI and the joint inversion.

218

219 Figures 3(c)–(d) show the FWI and joint inversion results using the traveltime tomography result as the
220 starting model. Compared with the traveltime tomography model, the position of the high-velocity
221 tunnel is more accurate. The top concrete wall is centered at the correct lateral and vertical position.
222 The joint inversion method reconstructs the tunnel features better than the FWI model with a stronger
223 magnitude. Figure 5 shows the waveform overlay for shot 9. For the FWI, the waveform match is
224 excellent but we still observe large traveltime differences between traveltimes calculated with the FWI
225 result and observed traveltimes (Fig. 5b). Compared with FWI, the joint inversion improves matches of
226 waveform and traveltime (Fig. 5c). The differences between the observed traveltimes and the synthetic
227 traveltimes in figure 5b are partially caused by the random noise in the observed traveltimes. But we
228 can also observe that the synthetic traveltimes are earlier than the observed traveltimes in the
229 near-offset area (< 6 m), and larger than the observed traveltimes in the far-offset area (> 6 m). It is not

230 randomly distributed, suggests that the traveltime differences are mainly caused by velocity error
231 instead of noise. The excellent traveltime and waveform match suggests that the traveltime tomography
232 result as a starting model is sufficient for joint inversion. The synthetic experiments verify the
233 possibility of the workflow to image the tunnel in the real data application because the source-receiver
234 geometry and the data frequency band are the same.

235

236 Figure 6 shows 1D velocity profiles through the middle of the tunnel to illustrate the performance of
237 the inversion methods. The joint inversion models are superior to the comparable FWI models because
238 they better reconstruct the high-velocity top concrete wall. In this case, the traveltime tomography
239 model is comparable with the FWI model and joint inversion model in recovering the high-velocity top
240 concrete wall. However, the FWI and joint inversion models better reconstruct the low-velocity void
241 space. In addition, the velocity profiles show that the traveltime tomography model is a better starting
242 model than the 1D linear gradient model.

243

244 Figure 7a shows the normalized waveform misfit curves of FWI and the joint inversion using the 1D
245 starting model. The waveform misfit curves depict that the joint inversion misfit is slightly larger than
246 the FWI misfit. Figure 7b is the traveltime misfit curve of the joint inversion. The traveltime misfit
247 decreases from 3.6 ms to 0.9 ms. This clearly demonstrates that the joint inversion fits the traveltimes
248 and waveforms simultaneously in the inversion process. Figure 8 shows the normalized waveform
249 misfit and traveltime misfit using the traveltime tomography starting model. The waveform data misfits
250 of the two methods decrease to about the same level. The traveltime misfit of the joint inversion
251 increases at first because the traveltime tomography result served as the starting model. The traveltime

252 misfit starts to decrease after four iterations. The final traveltime misfit is slightly larger than the initial
253 value, but still acceptable given the data uncertainties.

254

255 Traveltime tomography followed by the joint inversion successfully detects the high-velocity top
256 concrete wall and low-velocity air-filled void. However, FWI and joint inversion fail to image the two
257 sides and the bottom concrete walls. There are several reasons for this. First, the central frequency of
258 the data is about 50 Hz, the wavelength varies from 5-10 m in this case, which is too large compared
259 with the 0.3 m thick concrete walls. Second, the first-arrival wave paths are focused along the top
260 high-velocity concrete wall and are not sensitive to the sides and bottom of the tunnel underneath the
261 top concrete wall. Therefore, the two sides and bottom concrete walls are below the resolution of FWI
262 and the joint inversion.

263

264 **4. Real data example**

265 A seismic experiment to image a known concrete tunnel with air-filled void space was carried out on
266 Rice University campus, Houston, Texas, USA, in 2011 (Chen et al. 2017). Data were collected on a
267 grass field. The total length of the survey line is 24 m. The orientation of the tunnel is perpendicular to
268 the survey line. The geometry includes 25 shots and 72 receivers, for a total of 1800 traces. The seismic
269 data were collected by stacking ten hammer blows on a trailer hitch ball vertically mounted on the
270 ground. A two-component version of a Galperin geophone was used for each receiver such that
271 stacking the two components retains the vertical ground velocity and cancels out the horizontal motion
272 (Chen et al. 2017). Figure 9 shows a raw shot gather collected at X=24 m and the corresponding

273 average amplitude spectrum. Similar to the synthetic data, we can observe a traveltime and waveform
274 advance due to the high-velocity concrete walls of the tunnel (indicated by the black boxes).

275

276 In this study, we use the first-arrival traveltimes picked by Chen et al. (2017). They used a
277 semi-automated picking scheme and manually corrected a few picks. The average reciprocal error for
278 all the shots is about 1 ms (Fig. 10), it is acceptable given the sampling rate and frequency band of
279 0.2 ms and 10-60 Hz, respectively. We obtained a best-fit 1D linear gradient velocity model using the
280 Zelt and Smith (1992) algorithm. The 1D model serves as the starting model for 2D traveltime
281 tomography (Jiang and Zhang, 2017). Figure 11a shows the traveltime tomography result. The
282 high-velocity concrete top wall is imaged. The low-velocity anomaly below the top concrete wall is not
283 nicely reconstructed due to the low illumination in this area.

284

285 The preprocessing of the waveform data is a crucial part of the workflow for FWI and the joint
286 inversion of the field data. The primary objectives of the data processing are to improve the
287 signal-to-noise ratio and to transform the field data such that they reflect the approximations made in
288 the acoustic forward modeling. Therefore, it is necessary to preprocess the observed data to remove the
289 seismic noise and elastic effects. Otherwise, the seismic noise and non-acoustic wavefield will be
290 projected into the reconstructed P-wave velocity models. We apply a preprocessing workflow to the
291 field data to preserve early-arrival waveforms. The early-arrival waveform data mainly includes
292 refractions and diving waves, which are useful to build an accurate shallow velocity model. In this
293 study, the field data are bandpass filtered with a 5-10-60-120 Hz bandwidth. The filtering aims to
294 remove the low frequencies with a poor signal-to-noise ratio and to limit the maximum frequency

295 content of the data to mitigate the cycle skipping problem. The dominant frequency of the early arrival
296 waveform is about 50 Hz. We mute the seismic data before the picked first-arrival times to remove
297 noise. A time window with a 10 ms cosine taper is applied to the data to exclude the surface waves and
298 converted S-waves. During the inversion process, the near-offset data (< 4 m) are muted due to the
299 strong surface wave present in the near-offset traces. Figure 9 shows the shot gather at X=24 m after
300 preprocessing, including the windowing of the data and the associated amplitude spectrum.

301

302 Since FWI and joint inversion are performed in 2D, the source is modeled as a line source. While a
303 hammer blow is a point source, the 3D effects may limit the capability of detecting underground tunnel
304 and voids. As shown in Forbriger et al. (2014), a hybrid transformation to the point-source data is
305 equivalent to line-source response. The simplest way to correct the phase is to convolve the waveforms
306 with $\sqrt{t^{-1}}$, which corresponds to a phase shift of $\pi/4$. In this study, we have applied the amplitude
307 correction to the observed data. We did not correct the phase of the observed data because applying a
308 time shift to synthetic waveform is equivalent to a phase correction on narrow-band observed data. For
309 a single frequency data, a phase shift of $\pi/4$ corresponds to a time shift of $T/8$ (T is the period of the
310 data). The field data are bandpass filtered, and the dominant frequency of the early arrival waveform is
311 about 50 Hz. Applying a phase shift of $\pi/4$ on narrow-band data is approximately equal to a time shift
312 on synthetic waveform. However, we should mention that the method is not applicable to the
313 broadband seismic data.

314

315 The wavelet extraction is tricky in the real data inversion. The source wavelet can be extracted through
316 deconvolution when the minimum phase or zero phase of the source wavelet is true (Yilmaz, 2001).

317 While the source wavelet is mixed phase in the real case. The source wavelet inversion is linear under
318 the assumption that the current velocity model is correct, which can be inverted during the first
319 iteration in the frequency domain FWI (Pratt, 1999). Another approach is to invert the source wavelet
320 and velocity model simultaneously in FWI. For the data with simple near-surface structures, the source
321 wavelet can be extracted by stacking the near-offset first arrivals along the first breaks. However, the
322 near-offset data are contaminated with high amplitude surface waves, extract source wavelet from
323 near-offset first arrivals is difficult. Since the effective early arrival waveform are bandpass filtered
324 to a specific bandwidth, a Ricker wavelet with a central frequency of 50 Hz is acceptable in this
325 study.

326

327 The traveltime tomography result (Fig. 11a) is used as the starting model for FWI and the joint
328 inversion to avoid cycle skipping and converging to a solution that represents a local minimum. The
329 FWI and joint inversion models contain more small-scale structure than in the travelttime tomography
330 model, and the position of the high-velocity top concrete wall is more accurate (Figs. 11(b)-(c)). To
331 compare the performance of the methods in terms of how well they reconstruct the high-velocity top of
332 the concrete tunnel and the low-velocity air below, we plot the 1D velocity-depth profiles (Fig. 12).
333 The FWI and joint inversion models clearly image the low-velocity inside the tunnel better than the
334 travelttime model, and they both image the top of the tunnel better in terms of location.

335

336 The input waveform data and synthetic data from the travelttime model are shown in Fig. 13a for a shot
337 at X=24 m. The waveform advances (Fig. 9b) that are interpreted due to the high-velocity top of the
338 concrete tunnel, are matched well by the travelttime tomography model predicted data. The far-offset

339 waveforms (-24 to -18 m) show some mismatches, suggesting that the initial model needs to be updated
340 to fit the observed data. Figures 13(b)-(c) show the final waveform overlay from FWI and the joint
341 inversion. The far-offset data are significantly better matched by the FWI and joint inversion predicted
342 data. A low-velocity anomaly is presented in the FWI and joint inversion to compensate the phase
343 advances of the far-offset data. The matches of the first positive peak become worse. This is because
344 the amplitude of the latter waveform is stronger, which dominates the waveform match in the inversion.
345 We observe large traveltimes difference between traveltimes calculated with the FWI result and picked
346 traveltimes in far offset (Fig. 13b). The joint inversion matched traveltimes better than the waveform
347 inversion (Fig. 13c). It suggests that FWI improves the waveform match but does not take traveltime
348 into account. While the joint inversion fits both traveltime and waveform. In this study, we focus on the
349 traveltime and early waveform advances (indicated by the black boxes in Fig. 9) due to the tunnel walls.
350 Although we apply a mute function to exclude near-offset traces and later arrivals. Note that there are
351 still some seismic events (surface waves, converted S-waves) that cannot be modeled by acoustic
352 modeling, and were regarded as P-waves in the inversion. Since the arrival times of P-waves are earlier
353 than surface waves and S-waves, these events might distort the deeper part of the inversion result,
354 which is beneath the target area. An elastic full waveform inversion of recorded data is required in the
355 future to quantitative analyze the influence of surface waves on the inversion result. Figure 14 shows
356 the normalized waveform misfit and traveltimes misfit of the FWI and joint inversion. The waveform
357 data misfit of the joint inversion is slightly larger than FWI. Similar to the joint inversion traveltimes
358 misfit curve in the synthetic test, the traveltimes misfit increases at first because the traveltimes
359 tomography result served as the starting model. The traveltimes misfit starts to decrease after few
360 iterations. The final traveltimes misfit is also slightly larger than the initial value.

361

362 Comparisons of the FWI and joint inversion models show that the joint inversion model has a higher
363 magnitude for the high-velocity feature representing the top of the concrete wall. The traveltime
364 advances (Fig. 13c) which are interpreted to be due to the high-velocity top of the concrete tunnel, are
365 significantly better matched by the joint inversion model's predicted traveltime data. FWI images the
366 low-velocity inside the tunnel slightly better than the joint inversion. There are two reasons for this.
367 First, the joint inversion fits the traveltimes at the far-offsets (21-24 m) better than FWI (Figs. 13b and
368 13c), and it thereby favors higher velocities in the void space. Second, the joint inversion traveltime
369 gradient contains the high-velocity concrete wall feature, which would affect the low-velocity anomaly
370 below due to the smoothing regularization.

371

372 As a comparison, the frequency domain FWI result that uses the FDTT model as the starting model
373 (FDTT-FWI) in Chen et al. (2017) paper is also presented (Fig. 11d). The 1D velocity-depth profiles
374 show that FDTT-FWI model presents the low-velocity feature representing the void space with a
375 smaller value than in our FWI and joint inversion model. *e.g.*, the minimum velocity is approximately
376 230 m/s in the FDTT-FWI model, while it is approximately 380 m/s in our FWI model. On the other
377 hand, Our FWI and joint inversion models better reconstruct the tunnel features than the FDTT-FWI
378 model with a stronger magnitude, *e.g.*, a maximum of approximately 950 m/s for the top of the concrete
379 tunnel in the FDTT-FWI model compared with a maximum of approximately 1250 m/s in the joint
380 inversion model (Fig. 12). There are several possible reasons for the differences. First, the FDTT model
381 contains a low-velocity anomaly below the top of the concrete corresponding to the void space, while
382 our infinite frequency traveltime tomography model does not contain the low-velocity feature. The

383 starting model would affect the final inversion result. Second, the smoothing strategies are different.
384 No regularization was used in FDTT-FWI, but a low-pass wavenumber filter was applied to smooth the
385 gradient. However, we apply Tikhonov regularization to stabilize the inversion. Finally, the difference
386 between the frequency-domain and time-domain FWI code could also lead to the model difference.

387

388 **5. Discussion**

389 The air-filled void space is interpreted as a low-velocity zone in the acoustic FWI and joint inversion
390 tests. A certain amount of waves will be trapped into the low-velocity zone, while 100% of the incident
391 waves are scattered back at the air-solid interface in the real case. Since there are some differences
392 between the air-filled void space and the low-velocity zone, we conduct several forward modeling tests
393 to figure out whether the void space can be replaced by a low-velocity zone. The best replacement
394 velocity is obtained by evaluating the waveform difference. The benchmark model contains an
395 air-filled hole (Fig. 15a), which is the same size as the void space in the field data experiment. We then
396 replace the air-filled hole with velocity anomalies ranging from 200 m/s to 480 m/s. The velocity
397 interval is 10 m/s. Compared with the benchmark model, the differences are the surface topography and
398 velocities in the anomaly area. We evaluate the differences of the waveform fit for the forward
399 modeling results from these models and benchmark model. Figure 16 shows the normalized waveform
400 difference with different replacement velocity. Forward modeling results show that a replacement
401 velocity of 350 m/s yields the smallest waveform difference. Figure 15b compares the waveform
402 between the benchmark model and model with 350 m/s replacement velocity. The overall waveform
403 matches well except the strong near offset reflections from the left side of the air-filled hole and late
404 arrivals in the far offset. The near offset seismic traces and late arrivals are usually muted in the early

405 arrival waveform inversion of real data. Therefore, the inverted velocity models image the air-filled
406 void space of the tunnel as a low-velocity anomaly.

407

408 **6. Conclusions**

409 In this study, we demonstrate the application of FWI and joint inversion to detect a known target. The
410 size of the target anomalies is less than the dominant seismic wavelength. First-arrival traveltime
411 tomography provides a better starting model for FWI and the joint inversion than a 1D linear velocity
412 gradient model. FWI and the joint inversion use waveform information to improve the resolution to
413 detect subwavelength scales. Both FWI and the joint inversion reconstruct the high-velocity top
414 concrete wall and low-velocity void space inside the tunnel. The joint inversion images the top of the
415 tunnel with better magnitude and spatial extent than the FWI, while FWI better reconstructs the
416 low-velocity anomaly corresponding to the void space in the tunnel. The fit of the traveltime data in the
417 joint inversion improves detection of the high-velocity concrete wall but degrades the image of the void
418 space below, for reasons described previously. The workflow of applying traveltime tomography
419 followed by the joint inversion shows the ability to detect near-surface subwavelength features.
420 Compared with the FDTT-FWI result of Chen et al. (2017), our FWI and joint inversion models better
421 recover the high-velocity tunnel features but produce a poorer low-velocity feature representing the
422 void space in terms of magnitude, probably because of a better starting model for FWI provided by
423 FDTT.

424

425 The extremely strong velocity variation and subwavelength dimensions of the target, make the goal of
426 the study to be detection instead of imaging. Therefore, the magnitude of the anomalies is inaccurate,
427 but it represents a success from the standpoint of detection.

428

429

430 **Acknowledgments**

431 We thank the editor and reviewers for their constructive comments and suggestions, which
432 helped to improve the paper. We thank the financial support from National Natural Science Foundation
433 of China (Grant No: 41674120) and the China Postdoctoral Science Foundation (Grant No:
434 2019M661791). The IRIS PASSCAL Instrument Center provided the seismic acquisition instruments.

435 The acquisition of the real data and previous analysis by Chen et al. (2017) was funded by National
436 Science Foundation grant EAR-1056073 and Department of Energy grant DE-FG07-97ER14827. We
437 thank the volunteers in the Rice Earth, Environmental & Planetary Sciences Department for their help
438 in acquiring the data. Wenbin Jiang thanks Rice University for making it possible for him to serve as a
439 visiting student for one year. We thank Jianxiong Chen for providing his picks and model (Fig. 11d).

440 We appreciate the support from GeoTomo, allowing us to use the TomoPlus software package to
441 conduct the research.

442

443

444 **References**

445 Alsharabi, G., Faize, A., Louazni, M., Mostapha, A.M.M., Bayjja, M., Driouach, A., 2019. Detection
446 of cavities and fragile areas by numerical methods and GPR application. *Journal of Applied
447 Geophysics*. 164, 225–236.

448 Belfer, I., Bruner, I., Keydar, S., Kravtsov, A., Landa, E., 1998. Detection of shallow objects using
449 refracted and diffracted seismic waves. *Journal of Applied Geophysics*. 38, 155–168.
450 [https://doi.org/10.1016/s0926-9851\(97\)00025-6](https://doi.org/10.1016/s0926-9851(97)00025-6).

451 Butler, D.K., 1984. Microgravimetric and gravity gradient techniques for detection of subsurface
452 cavities. *Geophysics*. 49, 1084–1096. <https://doi.org/10.1190/1.1441723>.

453 Chen, J.X., Zelt, C.A., Jaiswal, P., 2017. Detecting a known near-surface target through application of
454 frequency-dependent traveltimes tomography and full-waveform inversion to P-and SH-wave
455 seismic refraction data. *Geophysics*. 82, R1-R17. <https://doi.org/10.1190/geo2016-0085.1>.

456 Fan, H., Bell, L., Deng, Y.L., Zhang, J., 2015. Imaging shallow complex structures in North Sea with
457 joint traveltimes and waveform inversion. *in* 2015 Workshop: Depth Model Building:
458 Full-waveform Inversion, Beijing, China, Society of Exploration Geophysicists. 40-43.
459 <https://doi.org/10.1190/fwi2015-010>.

460 Forbriger, T., Groos, L., Schäfer, M., 2014. Line-source simulation for shallow-seismic data. Part 1:
461 Theoretical background: *Geophysical Journal International*, 198(3), 1387-1404.

462 Gardner, G.H.F., Gardner, L.W., Gregory A.R., 1974. Formation velocity and density - the
463 diagnostic basics for stratigraphic traps: *Geophysics*, 39(8), 770–780.

464 Hauquin, T., Mourey, Q., 2019. The detection of old masonry tunnels as low electrical resistivity
465 anomalies-application to one covered stream tunnel of the Cevennes Mountain region (France).

466 Journal of Applied Geophysics. 168, 12-23. <https://doi.org/10.1016/j.jappgeo.2019.05.010>.

467 Jiang, W., Zhang, J., 2015. Imaging complex near-surface land area with joint traveltime and waveform
468 inversion. *in Expanded Abstracts: 85th Annual International Meeting, Society of Exploration
469 Geophysicists.* 1441-1445. <https://doi.org/10.1190/segam2015-5863831.1>.

470 Jiang, W., Zhang, J., 2017. First-arrival traveltime tomography with modified total-variation
471 regularization. *Geophysical prospecting.* 65(5), 1138-1154.

472 Kaufmann, G., Romanov, D., Nielbock, R., 2011. Cave detection using multiple geophysical methods:
473 Unicorn cave, Harz Mountains, Germany. *Geophysics.* 76(3), B71–B77.
474 <https://doi.org/10.1190/1.3560245>.

475 Kılıç, G., Eren, L., 2018. Neural network based inspection of voids and karst conduits in hydro–electric
476 power station tunnels using GPR. *Journal of Applied Geophysics.* 151, 194-204.
477 <https://doi.org/10.1016/j.jappgeo.2018.02.026>

478 Lai, W.W., Chang, R.K., Sham, J.F., 2018. A blind test of nondestructive underground void detection by
479 ground penetrating radar (GPR). *Journal of Applied Geophysics.* 149, 10-17.
480 <https://doi.org/10.1016/j.jappgeo.2017.12.010>

481 Leung, S., Qian, J., 2006. An adjoint state method for three-dimensional transmission traveltime
482 tomography using first-arrivals. *Communications in Mathematical Sciences.* 4, 249-266.
483 <https://doi.org/10.4310/cms.2006.v4.n1.a10>.

484 Luo, T.X.H., Lai, W.W.L., Chang, R.K.W., Goodman, D., 2019. GPR imaging criteria. *Journal of
485 Applied Geophysics.* 165, 37–48. <http://dx.doi.org/10.1016/j.jappgeo.2019.04.008>.

486 Mochales, T., Casas, A.M., Pueyo, E.L., Pueyo, O., Roman, M.T., Pocovi, A., Soriano, M.A., Anson, D.,
487 2008. Detection of underground cavities by combining gravity, magnetic and ground penetrating

488 radar surveys: A case study from the Zaragoza area, NE Spain. *Environmental Geology*. 53, 1067–
489 1077. <https://doi.org/10.1007/s00254-007-0733-7>.

490 Park, Y., Pyun, S., 2018. Refraction traveltimes tomography based on damped wave equation for
491 irregular topographic model. *Journal of Applied Geophysics*. 150, 160–171.
492 <https://doi.org/10.1016/j.jappgeo.2018.01.025>.

493 Pratt, R. G., 1999. Seismic waveform inversion in the frequency domain. Part 1: Theory and
494 verification in a physical scale model. *Geophysics*. 64, 888–901.

495 Pratt, R.G., Shin, C., Hicks, G.J., 1998. Gauss-Newton and Full Newton Methods in Frequency-space
496 Seismic Waveform Inversion. *Geophysical Journal International*. 133, 341–362.
497 <https://doi.org/10.1046/j.1365-246x.1998.00498.x>.

498 Sloan, S.D., Peterie, S.L., Miller, R.D., Ivanov, J., Schwenk, J.T., McKenna, J.R., 2015. Detecting
499 clandestine tunnels using near-surface seismic techniques. *Geophysics*. 80(5), EN127–EN135.
500 <https://doi.org/10.1190/geo2014-0529.1>.

501 Slob, E., Sato, M., Olhoeft, G., 2010. Surface and borehole ground-penetrating-radar developments.
502 *Geophysics*. 75, 75A103–75A120. <https://doi.org/10.1190/1.3480619>.

503 Tarantola, A., 1984. Inversion of Seismic-reflection Data in the Acoustic Approximation. *Geophysics*.
504 49, 1259–1266. <https://doi.org/10.1190/1.1441754>.

505 Tran, K.T., McVay, M., Faraone, M., Horhota, D., 2013. Sinkhole detection using 2D full seismic
506 waveform tomography. *Geophysics*. 78(5), R175–R183. <https://doi.org/10.1190/geo2013-0063.1>.

507 Van Schoor, M., 2002. Detection of sinkholes using 2D electrical resistivity imaging. *Journal of
508 Applied Geophysics*. 50, 393–399. [https://doi.org/10.1016/s0926-9851\(02\)00166-0](https://doi.org/10.1016/s0926-9851(02)00166-0).

509 Virieux, J., Operto, S., 2009. An Overview of Full-waveform Inversion in Exploration Geophysics.

510 Geophysics. 74, WCC1–WCC26. <https://doi.org/10.1190/1.3238367>.

511 Wang, Z.Y., Huang, J.P., Liu, D.J., Li, Z.C., Yong, P., Yang, Z.J., 2019. 3D variable-grid

512 full-waveform inversion on GPU. Petroleum Science. 16(5), 1001-1014.

513 Wightman, W., Jalinoos, F., Sirles, P., Hanna, K., 2003. Applications of geophysical methods to related

514 highway problems: FHWA Technical manual.

515 Xia, J., Nyquist, J.E., Xu, Y., Roth, M.J., Miller, R.D., 2007. Feasibility of detecting near-surface

516 feature with Rayleigh-wave diffraction. Journal of Applied Geophysics. 62(3), 244-253.

517 <https://doi.org/10.1016/j.jappgeo.2006.12.002>.

518 Yilmaz, O., 2001, Seismic data analysis: Processing, inversion, and interpretation of seismic data:

519 SEG.

520 Zelt, C.A., Chen, J., 2016. Frequency-dependent travelttime tomography for near-surface seismic

521 refraction data. Geophysical Journal International. 207, 72–88. <https://doi.org/10.1093/gji/ggw269>

522 Zelt, C.A., Smith, R.B., 1992. Seismic travelttime inversion for 2-D crustal velocity structure.

523 Geophysical journal international. 108, 16-34. <https://doi.org/10.1111/j.1365-246x.1992.tb00836.x>

524 Zhang, J., Chen, J., 2014. Joint Seismic Travelttime and Waveform Inversion for Near Surface Imaging.

525 *in* Expanded Abstracts: 84th Annual International Meeting, Society of Exploration Geophysicists.

526 934-937. <https://doi.org/10.1190/segam2014-1501.1>.

527 Zhang, J., Toksöz, M.N., 1998. Nonlinear refraction travelttime tomography. Geophysics. 63, 1726–

528 1737. <https://doi.org/10.1190/1.1826562>.

529 Zhang, W., Zhang, J., 2011. Full-waveform tomography with consideration for large topography
530 variations. *in* Expanded Abstracts: 81st Annual International Meeting, Society of Exploration
531 Geophysicists. 2539–2542. <https://doi.org/10.1190/1.3627719>.

532 Zhang, W., Shen, Y., 2010. Unsplit complex frequency-shifted PML implementation using auxiliary
533 differential equations for seismic wave modeling. *Geophysics*. 75(4), T141-T154.
534 <https://doi.org/10.1190/1.3463431>.

535 Zhu, X., Sixta, D.P., Angstman, B.G., 1992. Tomostatics: turning-ray tomography+static corrections.
536 *The Leading Edge*. 11(12), 15–23. <https://doi.org/10.1190/1.1821921>.

537

538 **Figure captions**

539 Fig. 1. (a) True model. (b) 1D linear gradient starting model. (c) Traveltime tomography models from
 540 synthetic data. The known concrete walls of the tunnel are indicated by black line.

541

542 Fig. 2. Synthetic traveltimes and waveform data. The blue dots and red waveforms represent traveltimes
 543 and waveform calculated with true model. The green dots and black waveforms denote traveltimes and
 544 waveform calculated with 1D linear gradient model. Near-offset waveform data are muted. The
 545 traveltimes and waveform advances due to the tunnel walls are indicated by the black arrows. (a) Shot
 546 gather for $X=0$ m, (b) $X=9$ m, and (c) $X=13$ m.

547

548 Fig. 3. FWI and the joint inversion models from synthetic data. (a) FWI model that uses the 1D model
 549 as the starting model. (b) The joint inversion model that uses the 1D model as the starting model. (c)
 550 FWI model that uses the traveltime tomography model as the starting model. (d) The joint inversion
 551 model that uses the traveltime tomography model as the starting model.

552

553 Fig. 4. Waveform overlay for shot gather at $X=9$ m using the 1D model as the starting model. (a) Initial
 554 waveform overlay. (b) FWI final waveform overlay. (c) Joint inversion final waveform overlay. Black
 555 waveforms are observed data and red waveforms represent synthetic data. Blue dots denote observed
 556 traveltimes, green dots represent synthetic traveltimes.

557

558 Fig. 5. Waveform overlay for shot gather at $X=9$ m using the traveltime tomography model as the
 559 starting model. (a) Initial waveform overlay. (b) FWI final waveform overlay. (c) Joint inversion final

560 waveform overlay. Black waveforms are observed data and red waveforms represent synthetic data.

561 Blue dots denote observed traveltimes, green dots represent synthetic traveltimes.

562

563 Fig. 6. 1D velocity profiles as labeled at X=8 m in the middle of the tunnel.

564

565 Fig. 7. Data misfit of the synthetic tests using a 1D starting model. (a) Normalized waveform misfit of

566 FWI and the joint inversion. (b) Traveltime misfit of the joint inversion.

567

568 Fig. 8. Data misfit of the synthetic tests using the travelttime tomography model as the starting model.

569 (a) Normalized waveform misfit of FWI and the joint inversion. (b) Traveltime misfit of the joint

570 inversion.

571

572 Fig. 9. (a) The raw shot gather of the real data at X=24 m. (b) The shot gather after preprocessing. (c)

573 The average amplitude spectrum of the raw shot gather. (d) The average amplitude spectrum of the shot

574 gather after preprocessing. Blue dots represent picked traveltimes. The waveform advances due to the

575 tunnel walls are indicated by the black boxes.

576

577 Fig. 10. The reciprocal error of the shots. The average reciprocal error is about 1 ms.

578

579 Fig. 11. Final models from real data. (a) Traveltime tomography model that uses the 1D model as the

580 starting model. (b) FWI model that uses the travelttime tomography model as the starting model. (c)

581 The joint inversion model that uses the traveltime tomography model as the starting model. (d) The
582 frequency domain FWI result from Chen et al. (2017).

583

584 Fig. 12. 1D velocity profiles as labeled at X=8 m in the middle of the tunnel.

585

586 Fig. 13. Waveform overlays of a shot gather at X=24 m. (a) Waveform overlay between observed data
587 (black) and synthetics (red) associated with travelttime tomography result. (b) Waveform overlay
588 between observed data (black) and synthetics (red) associated with FWI result. (c) Waveform overlay
589 between observed data (black) and synthetics (red) associated with the joint inversion result. Blue dots
590 denote picked traveltimes, green dots represent synthetic traveltimes.

591

592 Fig. 14. Data misfit of the real data test (a) Normalized waveform misfit of FWI and the joint inversion.
593 (b) Traveltime misfit of the joint inversion.

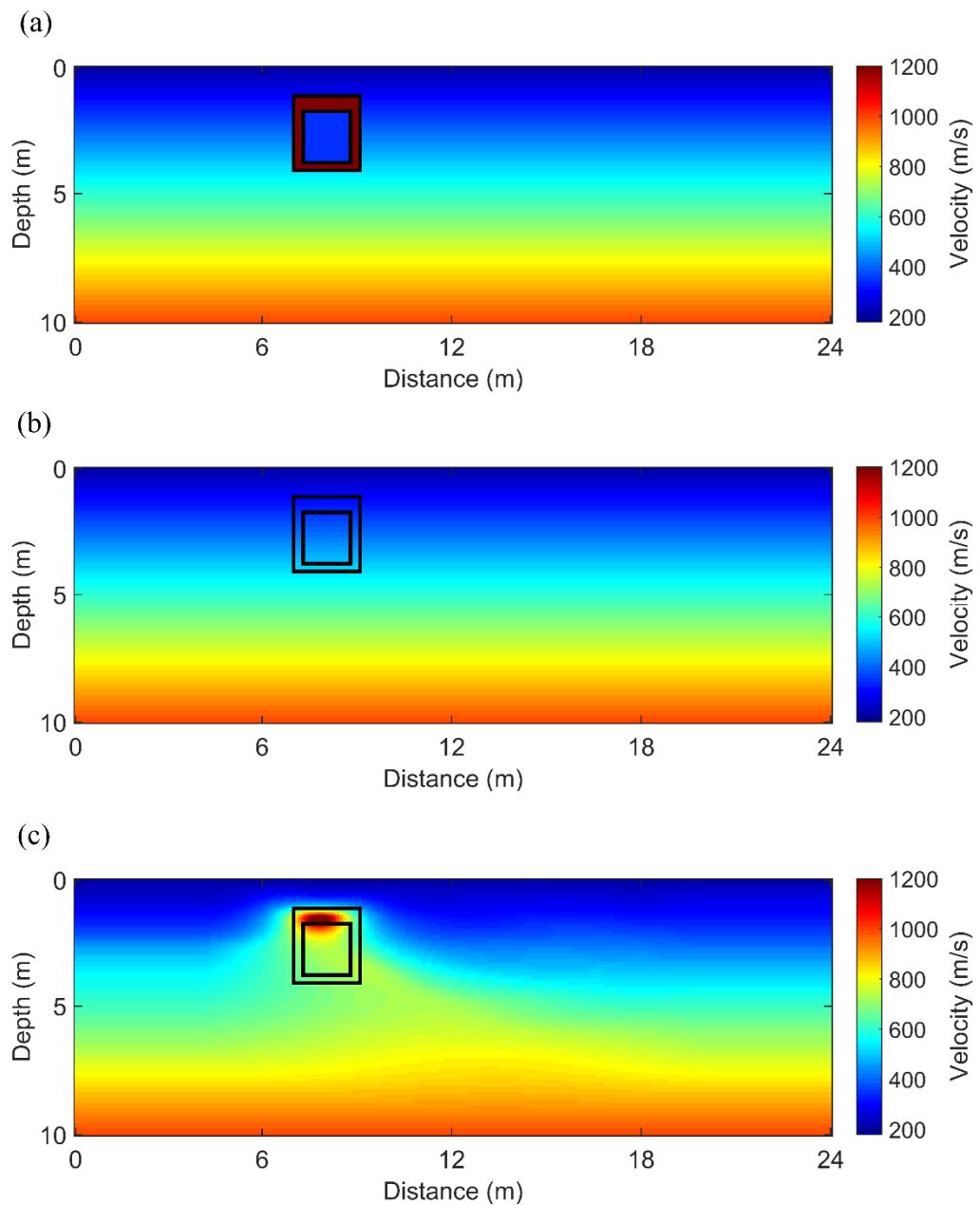
594

595 Fig. 15. (a) Benchmark model with an air-filled hole. The red star denotes shot, yellow triangles
596 represent receivers. The red line is the surface topography. (b) Waveform overlay of the shot gather
597 from Benchmark model (black) and model with 350 m/s replacement velocity (red).

598

599 Fig. 16. Normalized waveform difference between velocity models filled with replacement velocity and
600 benchmark model.

601

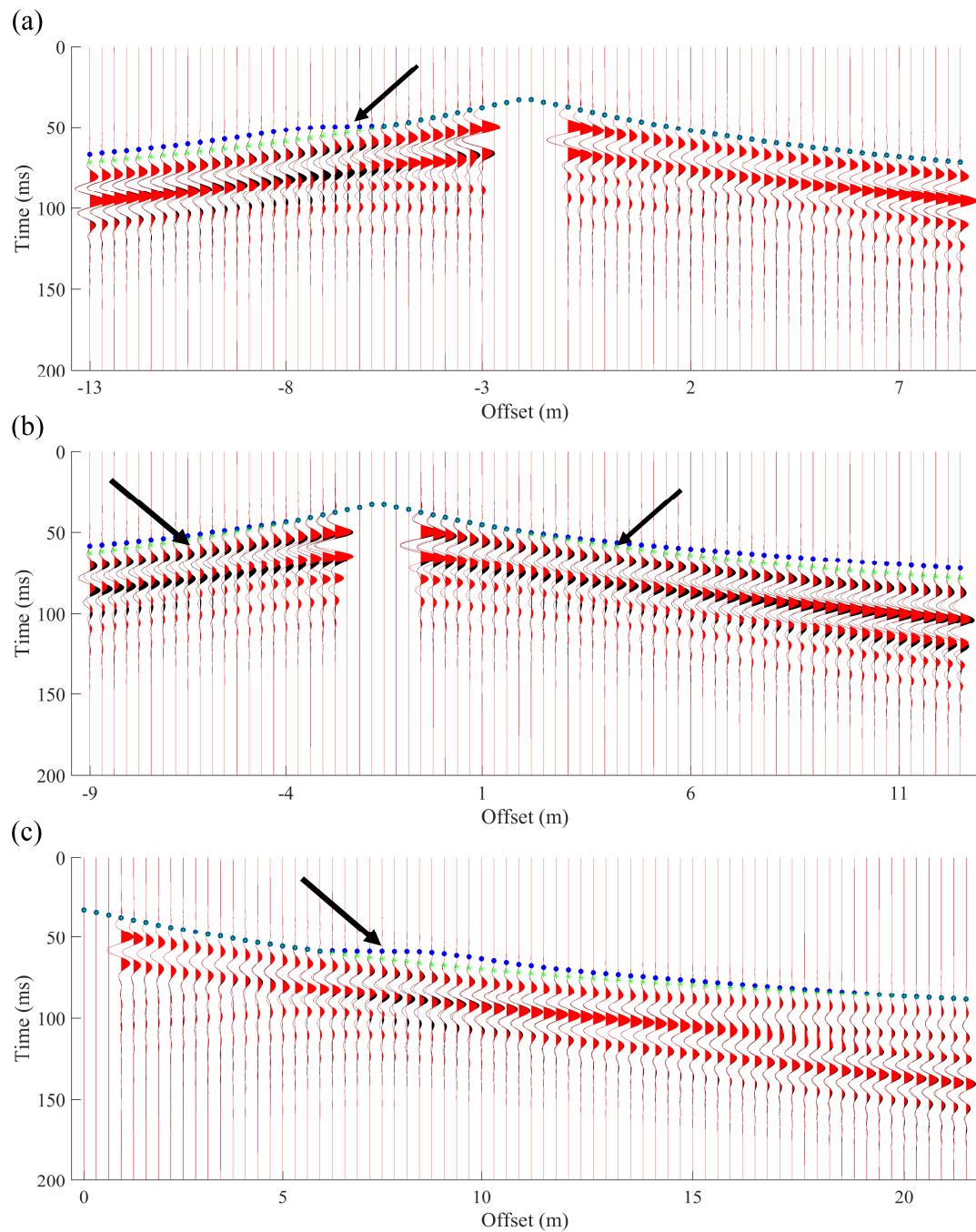


602

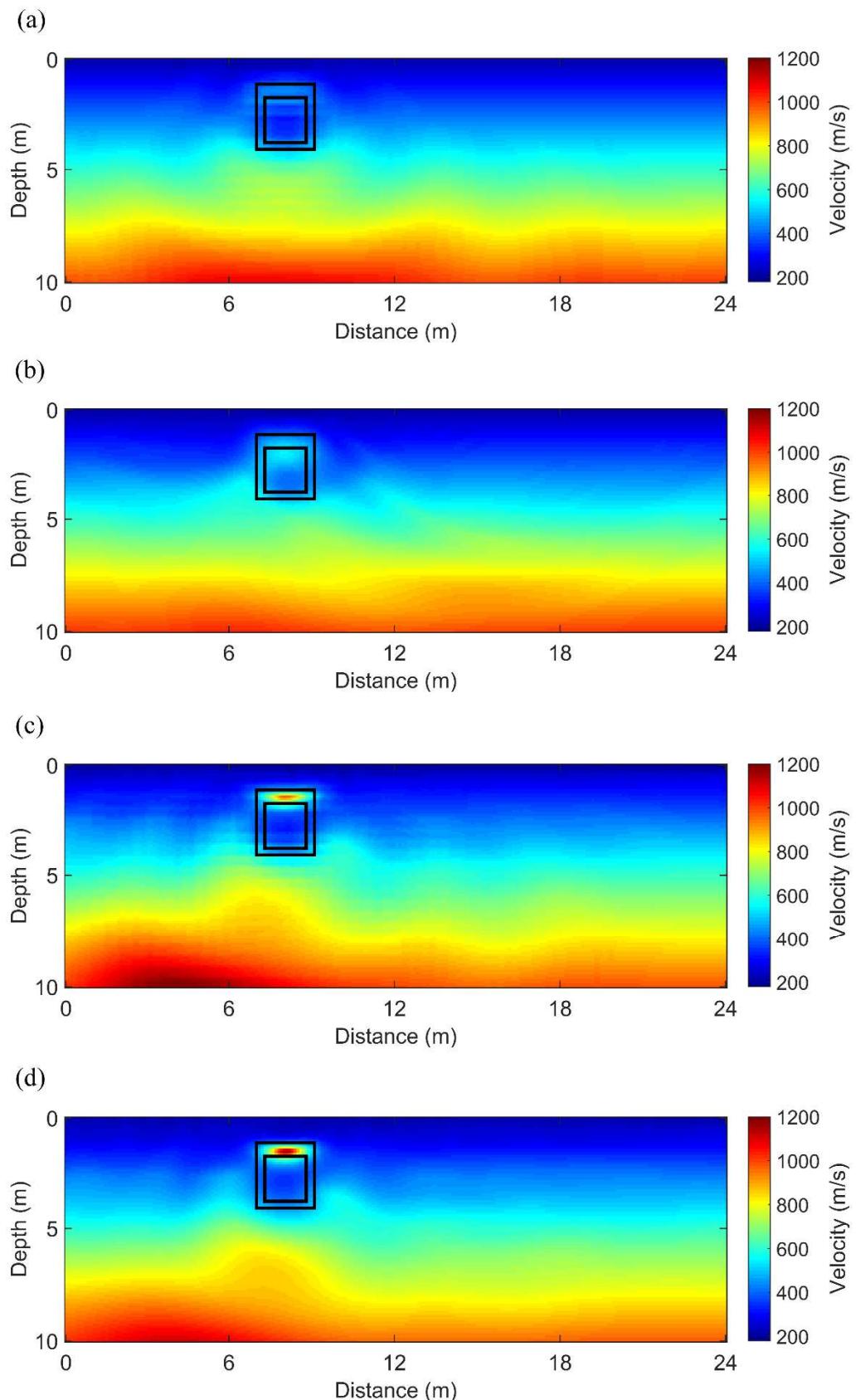
603 Fig. 1. (a) True model. (b) 1D linear gradient starting model. (c) Traveltime tomography models from

604 synthetic data. The known concrete walls of the tunnel are indicated by black line.

605



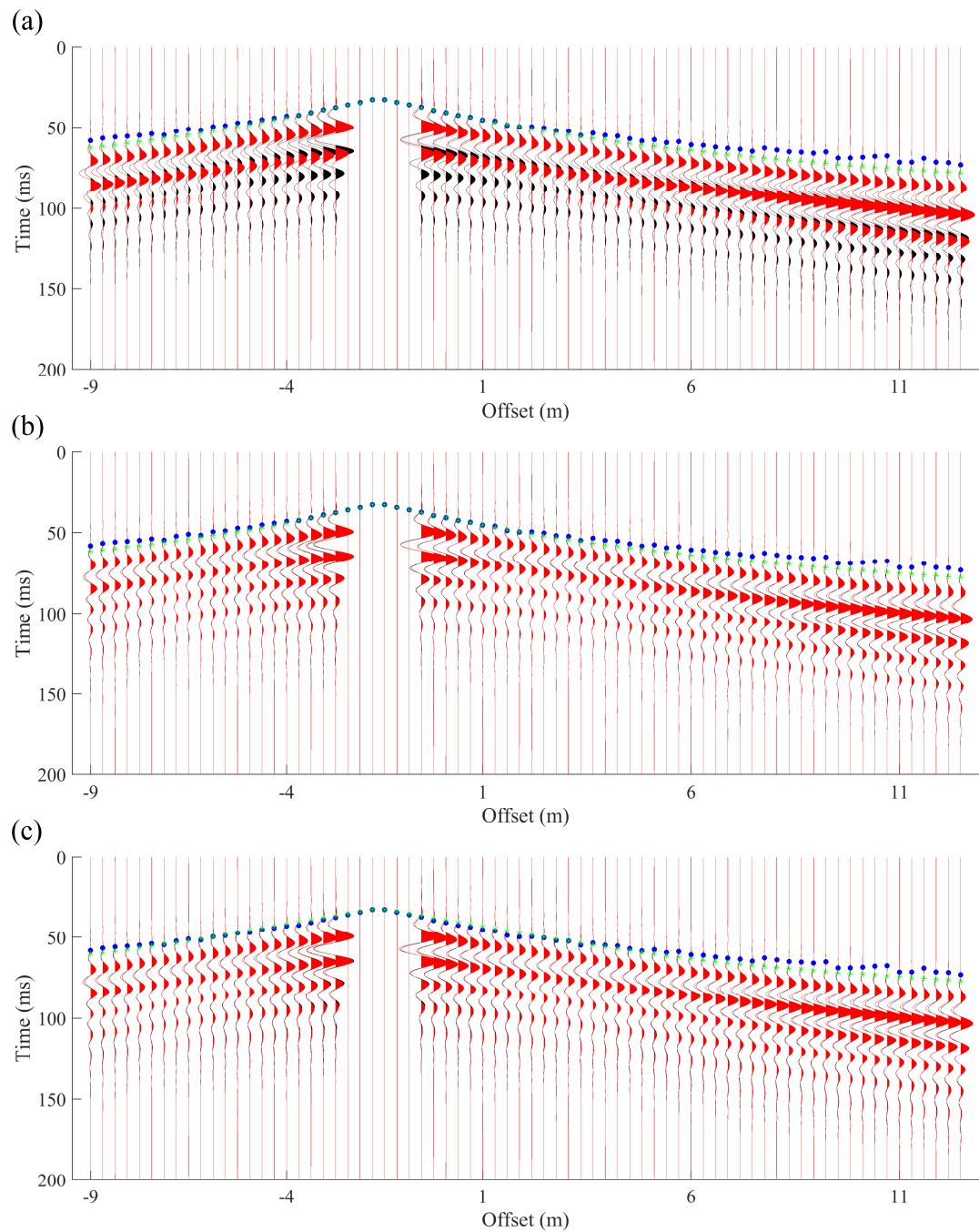
606 Fig. 2. Synthetic traveltime and waveform data. The blue dots and red waveforms represent traveltime
 607 and waveform calculated with true model. The green dots and black waveforms denote traveltime and
 608 waveform calculated with 1D linear gradient model. Near-offset waveform data are muted. The
 609 traveltime and waveform advances due to the tunnel walls are indicated by the black arrows. (a) Shot
 610 gather for $X=0$ m, (b) $X=9$ m, and (c) $X=13$ m.
 611
 612



613

614 Fig. 3. FWI and the joint inversion models from synthetic data. (a) FWI model that uses the 1D model
 615 as the starting model. (b) The joint inversion model that uses the 1D model as the starting model. (c)

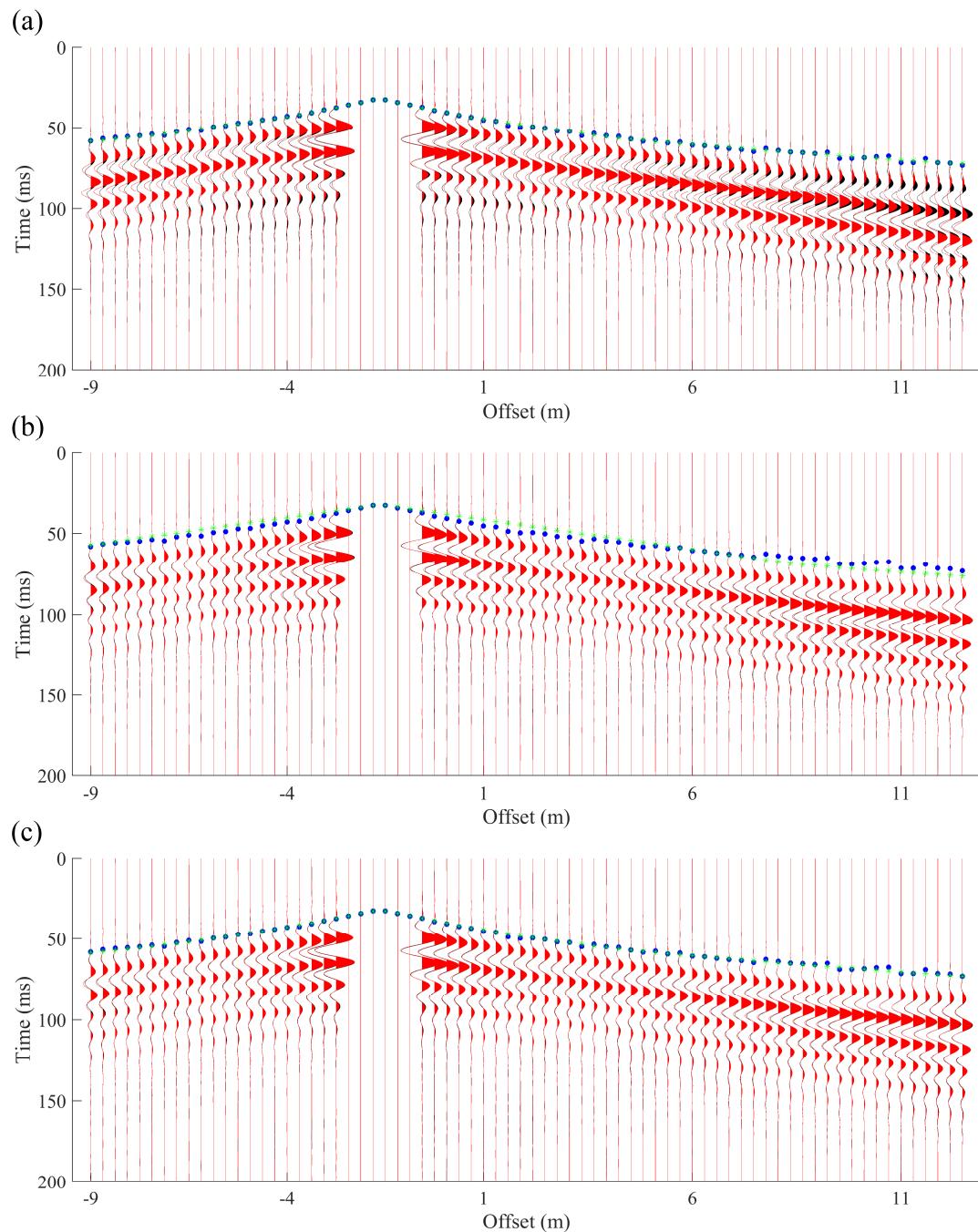
616 FWI model that uses the traveltime tomography model as the starting model. (d) The joint inversion
617 model that uses the traveltime tomography model as the starting model.
618



619

620 Fig. 4. Waveform overlay for shot gather at $X=9$ m using the 1D model as the starting model. (a) Initial
 621 waveform overlay. (b) FWI final waveform overlay. (c) Joint inversion final waveform overlay. Black
 622 waveforms are observed data and red waveforms represent synthetic data. Blue dots denote observed
 623 traveltimes, green dots represent synthetic traveltimes.

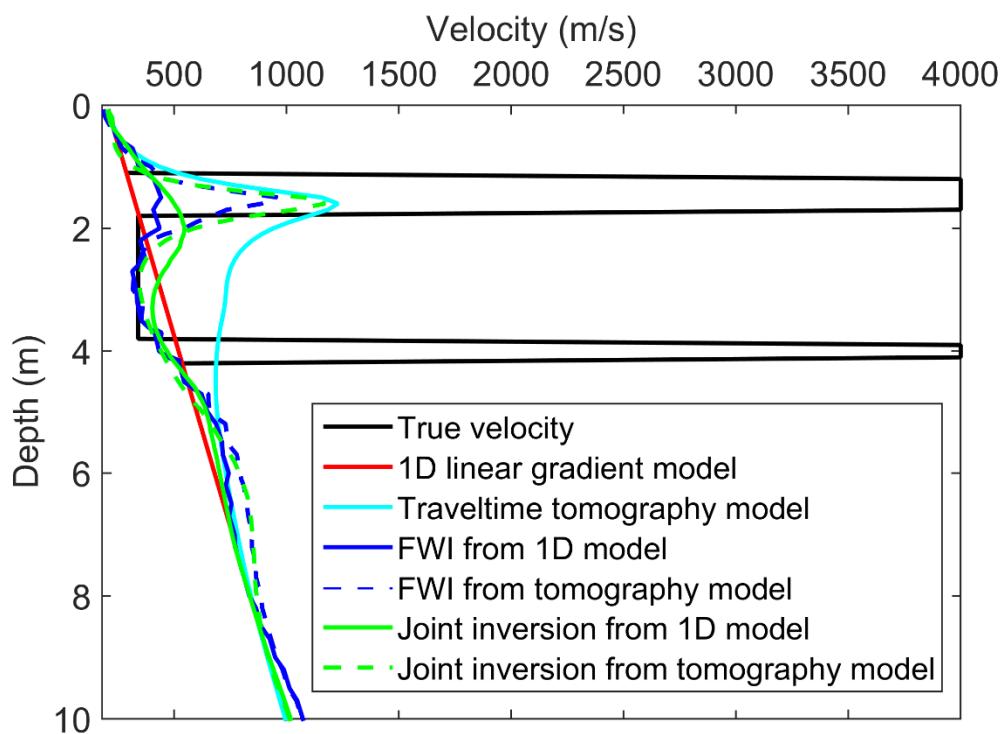
624



625

626 Fig. 5. Waveform overlay for shot gather at $X=9$ m using the travelttime tomography model as the
 627 starting model. (a) Initial waveform overlay. (b) FWI final waveform overlay. (c) Joint inversion final
 628 waveform overlay. Balck waveforms are observed data and red waveforms represent synthetic data.
 629 Blue dots denote observed traveltimes, green dots represent synthetic traveltimes.

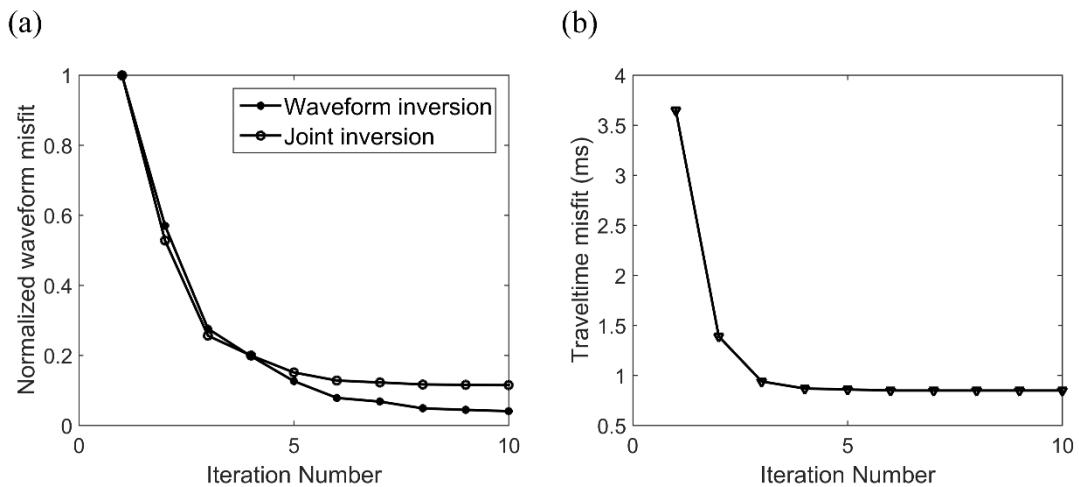
630



631

632 Fig. 6. 1D velocity profiles as labeled at X=8 m in the middle of the tunnel.

633

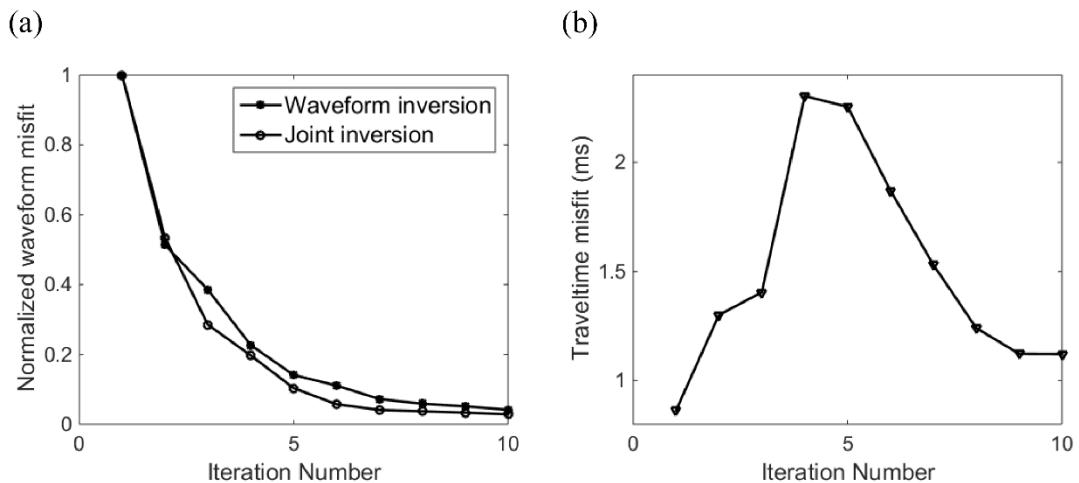


634

635 Fig. 7. Data misfit of the synthetic tests using a 1D starting model. (a) Normalized waveform misfit of

636 FWI and the joint inversion. (b) Travelttime misfit of the joint inversion.

637



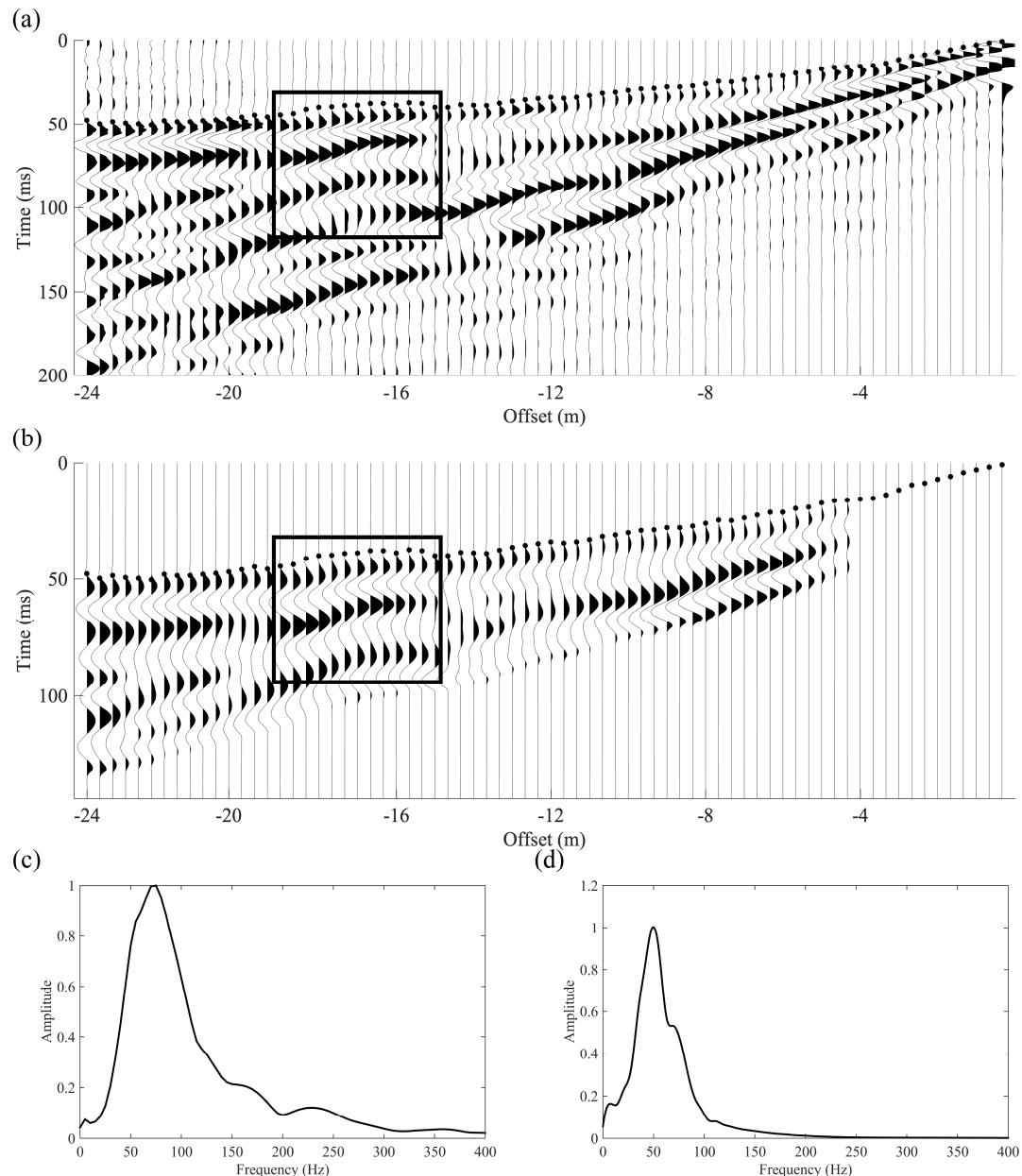
638

639 Fig. 8. Data misfit of the synthetic tests using the traveltome tomography model as the starting model.

640 (a) Normalized waveform misfit of FWI and the joint inversion. (b) Traveltome misfit of the joint

641 inversion.

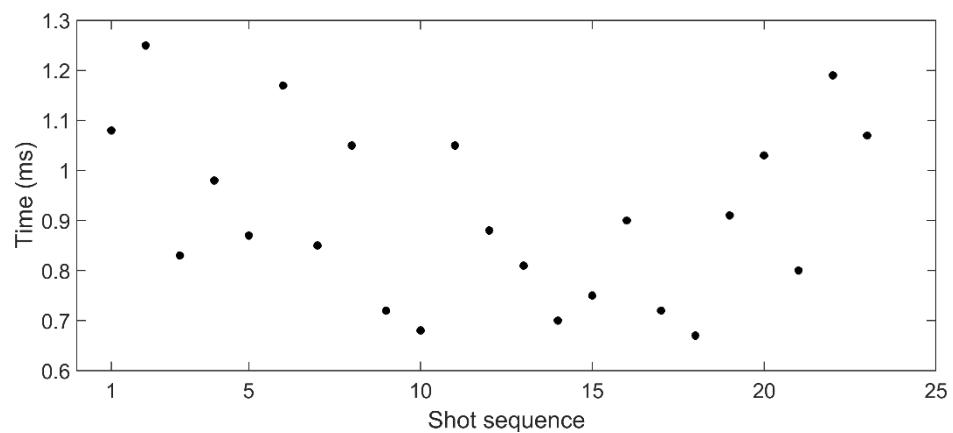
642



643

644 Fig. 9. (a) The raw shot gather of the real data at $X=24$ m. (b) The shot gather after preprocessing. (c)
 645 The average amplitude spectrum of the raw shot gather. (d) The average amplitude spectrum of the shot
 646 gather after preprocessing. Blue dots represent picked traveltimes. The waveform advances due to the
 647 tunnel walls are indicated by the black boxes.

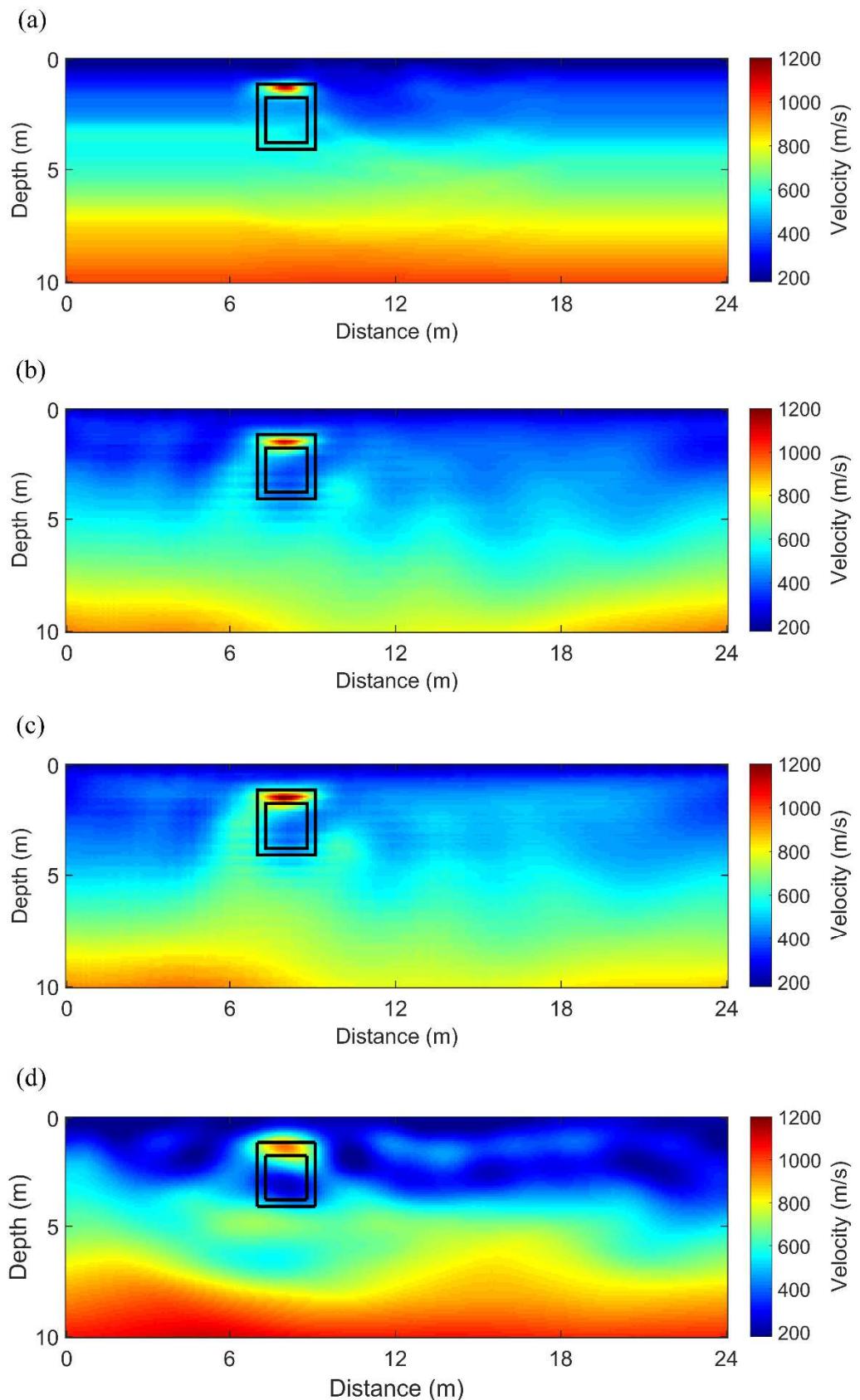
648



649

650 Fig. 10. The reciprocal error of the shots. The average reciprocal error is about 1 ms.

651

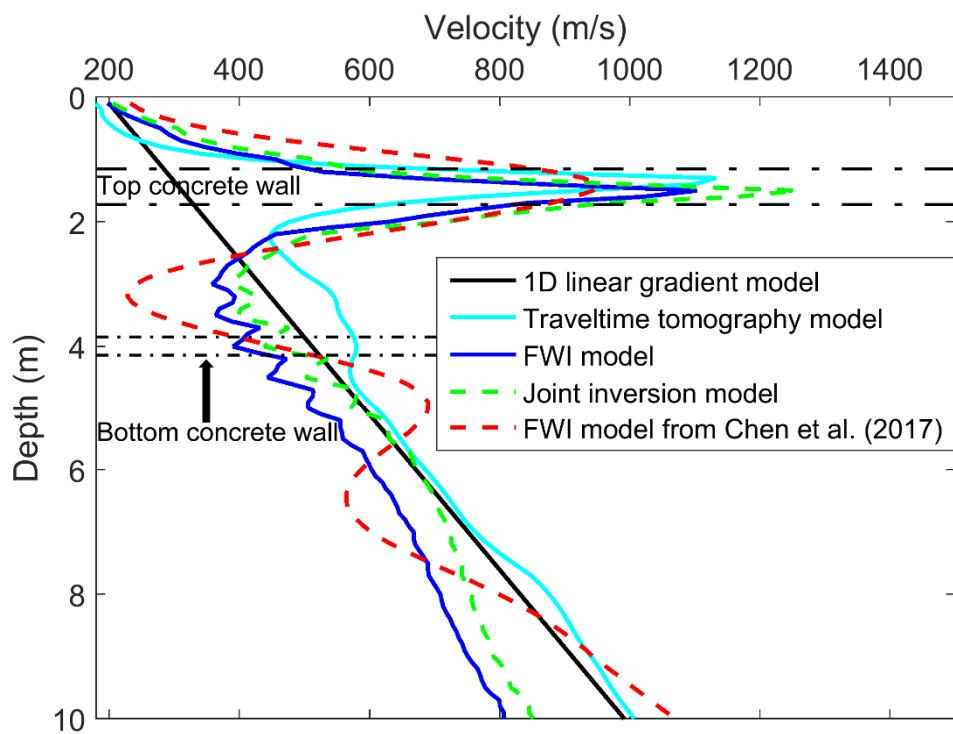


652

653 Fig. 11. Final models from real data. (a) Traveltime tomography model that uses the 1D model as the
 654 starting model. (b) FWI model that uses the travelttime tomography model as the starting model. (c)

655 The joint inversion model that uses the traveltime tomography model as the starting model. (d) The
656 frequency domain FWI result from Chen et al. (2017).
657

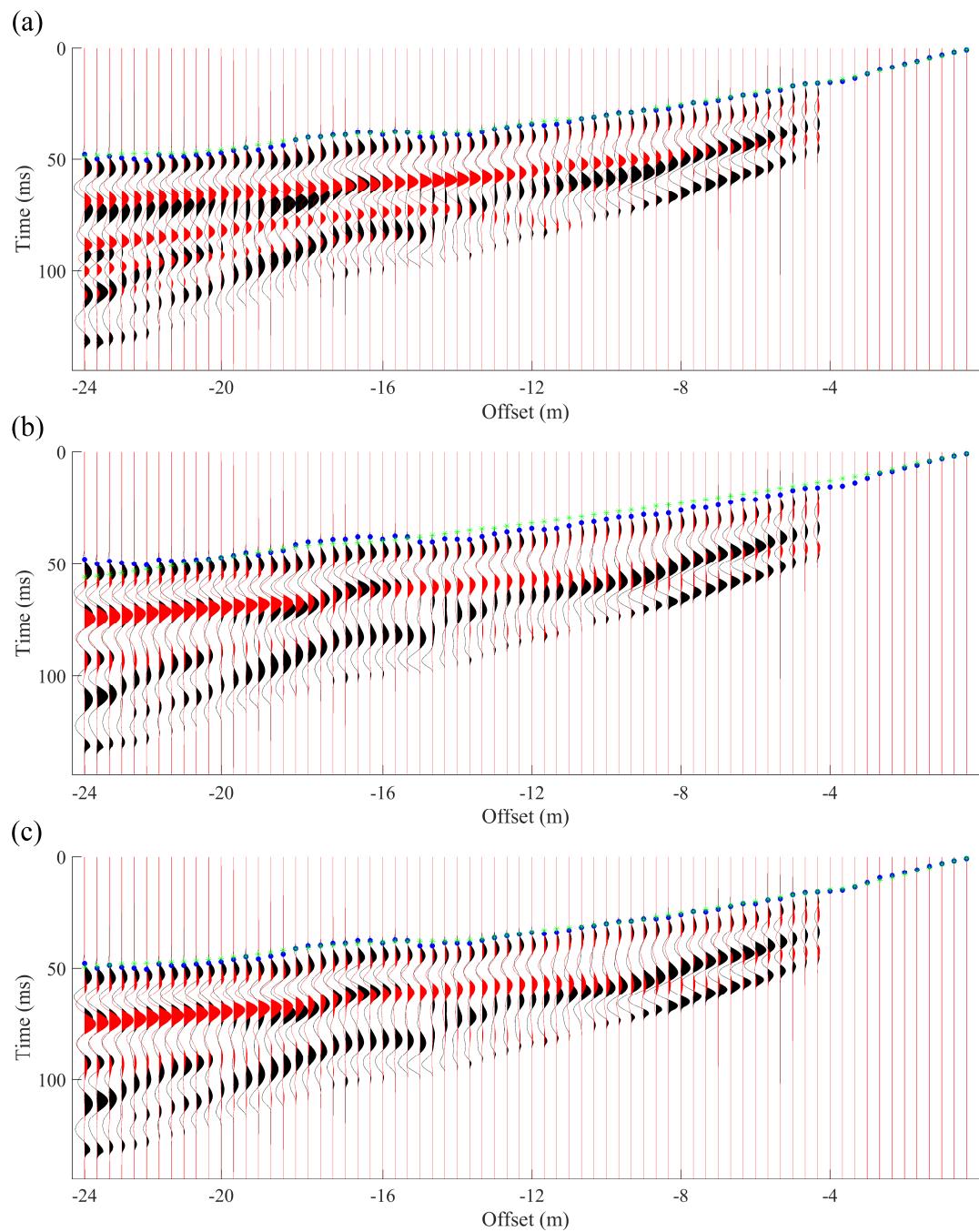
658



659

660 Fig. 12. 1D velocity profiles as labeled at $X=8$ m in the middle of the tunnel.

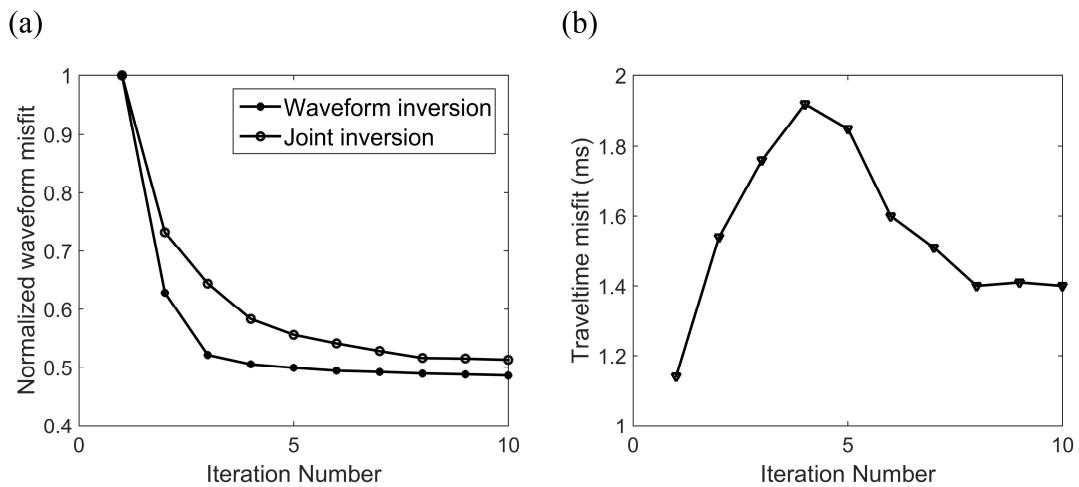
661



662

663 Fig. 13. Waveform overlays of a shot gather at $X=24$ m. (a) Waveform overlay between observed data
 664 (black) and synthetics (red) associated with traveltome tomography result. (b) Waveform overlay
 665 between observed data (black) and synthetics (red) associated with FWI result. (c) Waveform overlay
 666 between observed data (black) and synthetics (red) associated with the joint inversion result. Blue dots
 667 denote picked traveltimes, green dots represent synthetic traveltimes.

668

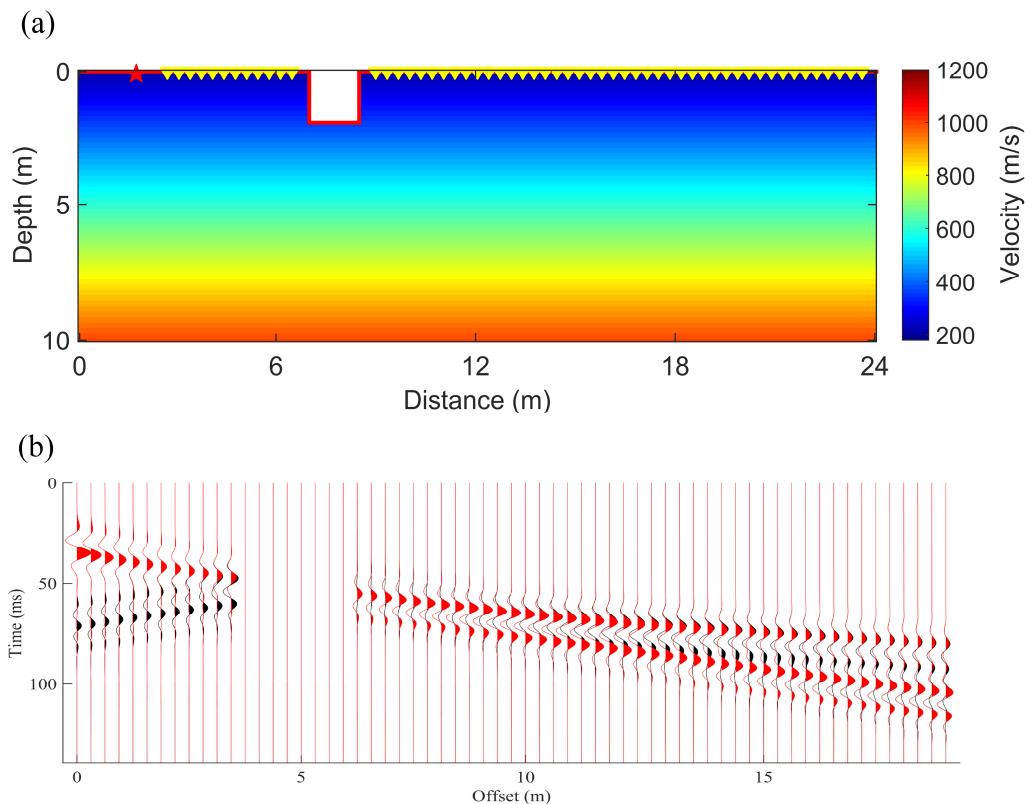


669

670 Fig. 14. Data misfit of the real data test (a) Normalized waveform misfit of FWI and the joint inversion.

671 (b) Traveltime misfit of the joint inversion.

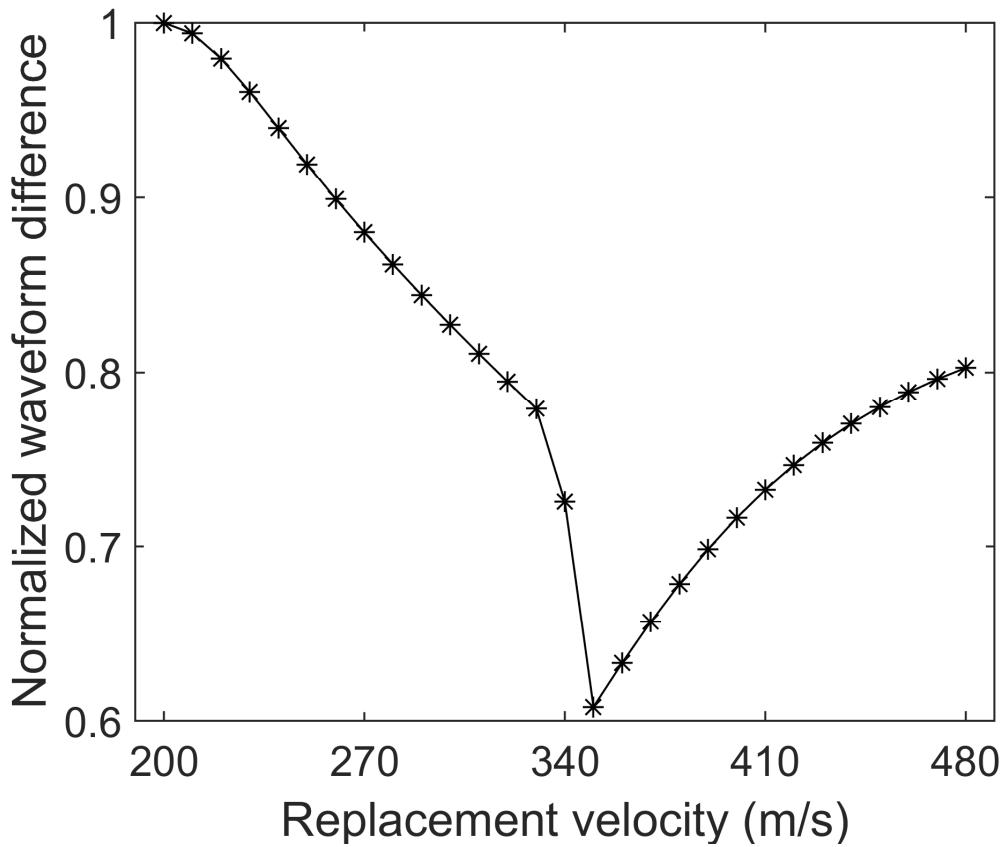
672



673

674 Fig. 15. (a) Benchmark model with an air-filled hole. The red star denotes shot, yellow triangles
 675 represent receivers. The red line is the surface topography. (b) Waveform overlay of the shot gather
 676 from Benchmark model (black) and model with 350 m/s replacement velocity (red).

677



678

679 Fig. 16. Normalized waveform difference between velocity models filled with replacement velocity and
680 benchmark model.