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Nonlinear Interface Reduction for Time Domain Analysis of Coupled Hurty/Craig-Bampton Superelements

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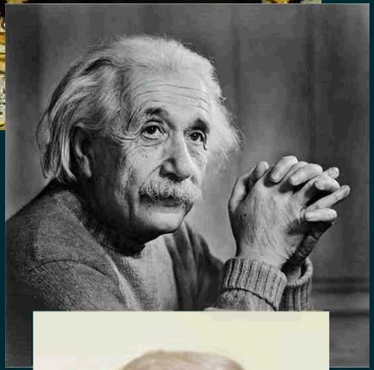
Introduction

- Accurate and efficient modeling of contact is growing in importance and prevalence
- Any explicit consideration of a structure's inter-component stiffness and damping requires a robust contact model
- Associated Problems
  - Demand for sophisticated models is outpacing the availability of adequate computing resources
  - Engineers often need to repeat dynamic analyses for design and evaluation purposes

"Entities should not be multiplied without necessity."
- William of Ockham (1300s)

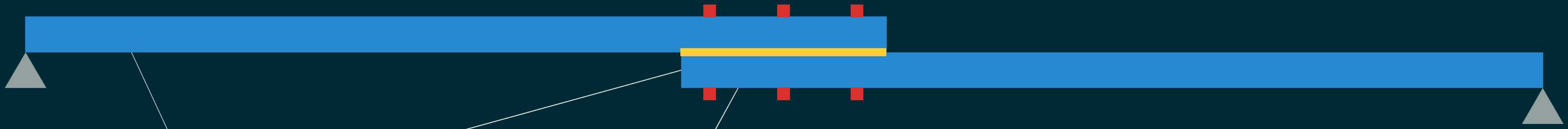


"Everything should be made as simple as possible, but not simpler."
- Albert Einstein (1900s)

"[models should be] as simple as possible and as accurate as necessary."
- Karl Popp (2000s)

Projection-Based Model Order Reduction

- Algebraic projection of higher-order kinematics onto a lower-order subspace that:
  - captures the essential physics within an acceptable margin of error (**accuracy**)
  - is much smaller in dimension than the original space (**computational savings**)
- Example: truncated modal analysis of a linear elastic continuum

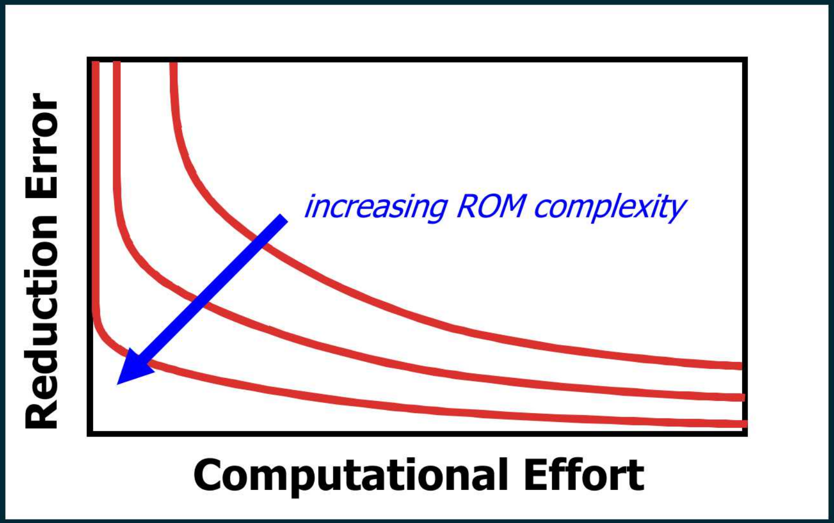


Interface DOF

- Contact nonlinearity
- Reduce the number of DOF using ***nonlinear mode shapes***
  - Characteristic constraint modes
  - Joint interface modes
  - Preload modes
  - Modal derivatives

Interior DOF

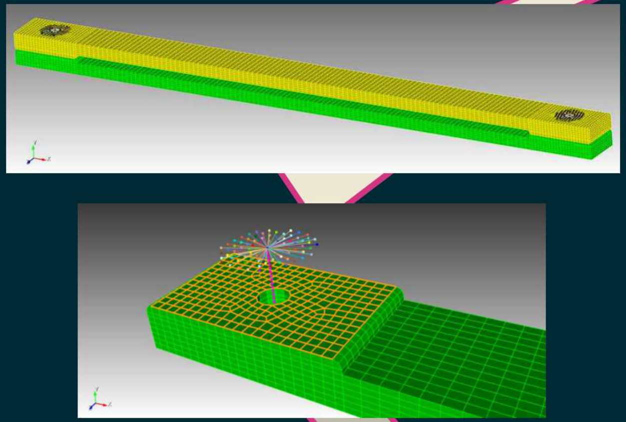
- Assume linear elastic behavior
- Reduce the number of DOF using ***linear mode shapes***
  - Fixed-interface modes
  - Free-interface modes



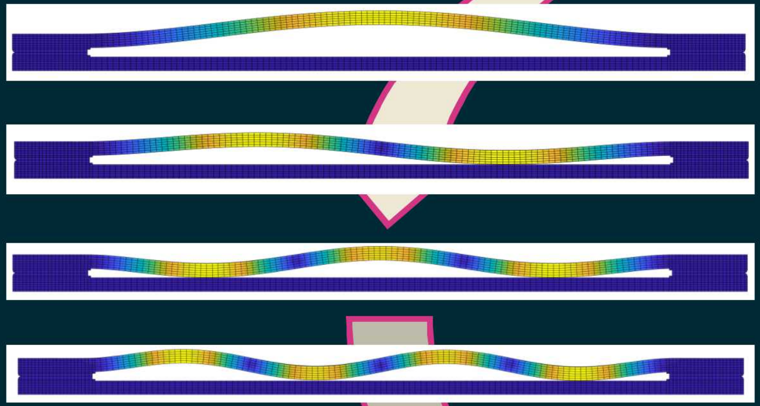
Model Order Reduction Strategies

modal displacements *modal displacements*

Full-Order FE Model → **Hurty/Craig-Bampton (HCB) Model** → **Interface-Reduced (IR) Model**

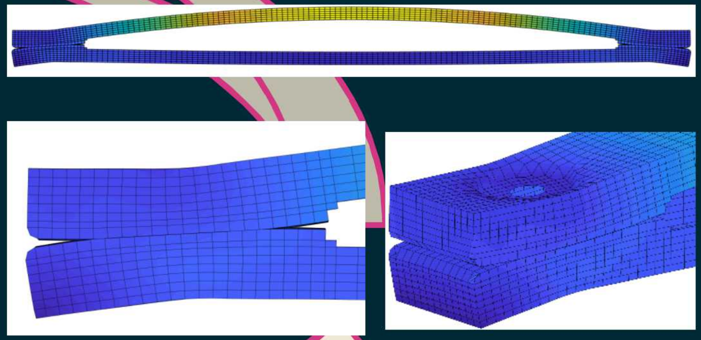


- interior
  - 10^5 - 10^6 physical DOF
- interface
  - 10^3 - 10^4 physical DOF
- TOTAL: **10^5 - 10^6 DOF**



- interior
  - 10^1 modal DOF
- interface
  - 10^3 - 10^4 physical DOF
- TOTAL: **10^3 - 10^4 DOF**

modal



- interior
  - 10^1 modal DOF
- interface
  - 10^2 - 10^3 modal interface DOF
- TOTAL: **10^2 - 10^3 DOF**

used on multi-party projects to abstract model details & protect proprietary information

Research Objective

Previous studies of nonlinear interface reduction techniques, in general:

1. require the full-order finite element model
2. are restricted to single-mode analysis in the frequency domain
3. ignore the accuracy of the interface kinematics

Research Objective

Adapt existing interface reduction schemes such that the new method:

1. only requires the HCB model
2. is applicable to nonlinear multi-modal analysis in the time domain
3. accurately captures the interface kinematics

Outline of the Presentation

- Theoretical Background
  - Hurty/Craig-Bampton (HCB) Transformation
  - Interface reduction basis
- Application Example
  - Prototype substructure assembly: C-Beams
  - Time history results for interface-reduced (IR) model & HCB baseline
  - Performance assessment: error-effort curves
- Conclusions & Future Work
  - Overall accuracy & computational savings
  - Place within the tapestry of reduced-order models
  - Ongoing & future developments

Theoretical Background

Hurty/Craig-Bampton Transformation

Full-order Mass Matrix

- from original finite element model

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix}$$

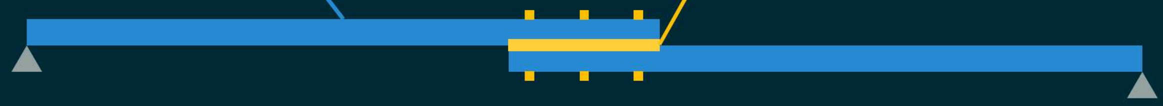
HCB Transformation Matrix

- requires $\mathbf{M}$, $\mathbf{K}$

$$\mathbf{T} = \begin{bmatrix} \boldsymbol{\Phi}^{\mathrm{FI}} & \boldsymbol{\Psi} \end{bmatrix}$$

$$\boxed{\left[\mathbf{K}_{ii} - \left(\omega_j^{\mathrm{FI}}\right)^2 \mathbf{M}_{ii}\right] \boldsymbol{\phi}_j^{\mathrm{FI}} = \mathbf{0}_i}$$

$$\boxed{\boldsymbol{\Psi} = -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib}}$$



HCB Mass Matrix

- $\bar{\mathbf{M}} = \mathbf{T}^{\mathrm{T}}\mathbf{M}\mathbf{T}$

$$\bar{\mathbf{M}} = \begin{bmatrix} \mathbf{I}_{ii} & \bar{\mathbf{M}}_{ib} \\ \bar{\mathbf{M}}_{bi} & \bar{\mathbf{M}}_{bb} \end{bmatrix}$$

$$\begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} = \underbrace{\begin{bmatrix} \bar{\boldsymbol{\Phi}}^{\mathrm{FI}} & \boldsymbol{\Psi} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}}_{\mathbf{T}} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_b \end{Bmatrix}$$

Gram-Schmidt Interface (GSI) Modes

HCB Mass Matrix

- from HCB transformation

$$\bar{\mathbf{M}} = \begin{bmatrix} \mathbf{I}_{ii} & \bar{\mathbf{M}}_{ib} \\ \bar{\mathbf{M}}_{bi} & \bar{\mathbf{M}}_{bb} \end{bmatrix}$$

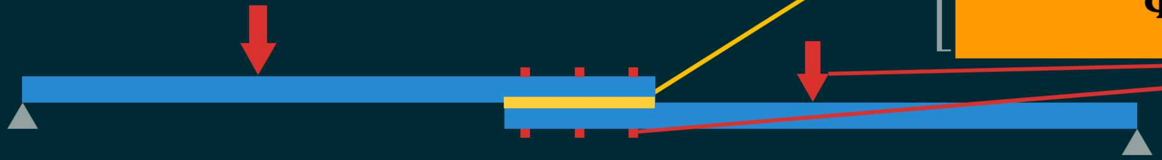
Boundary Eigenmodes

- from b-b partition of HCB system matrices:

$$\left[\bar{\mathbf{K}}^{c}_{bb} - \left(\bar{\omega}_{bb,j}\right)^{2}\bar{\mathbf{M}}_{bb}\right]\bar{\boldsymbol{\Phi}}_{bb,j} = \mathbf{0}_{b}$$

- split into reduced (r) and active (a) partitions

$$\bar{\boldsymbol{\Phi}}_{bb} = \begin{bmatrix} \bar{\boldsymbol{\Phi}}_{rr} & \bar{\boldsymbol{\Phi}}_{ra} \\ \bar{\boldsymbol{\Phi}}_{ar} & \bar{\boldsymbol{\Phi}}_{aa} \end{bmatrix}$$



GSI Modes

- orthogonalize r-r partition of boundary eigenmodes:

$$\bar{\boldsymbol{\Phi}}_{rr} \xrightarrow{Gram-Schmidt} \bar{\boldsymbol{\Phi}}^{\mathrm{GSI}}$$

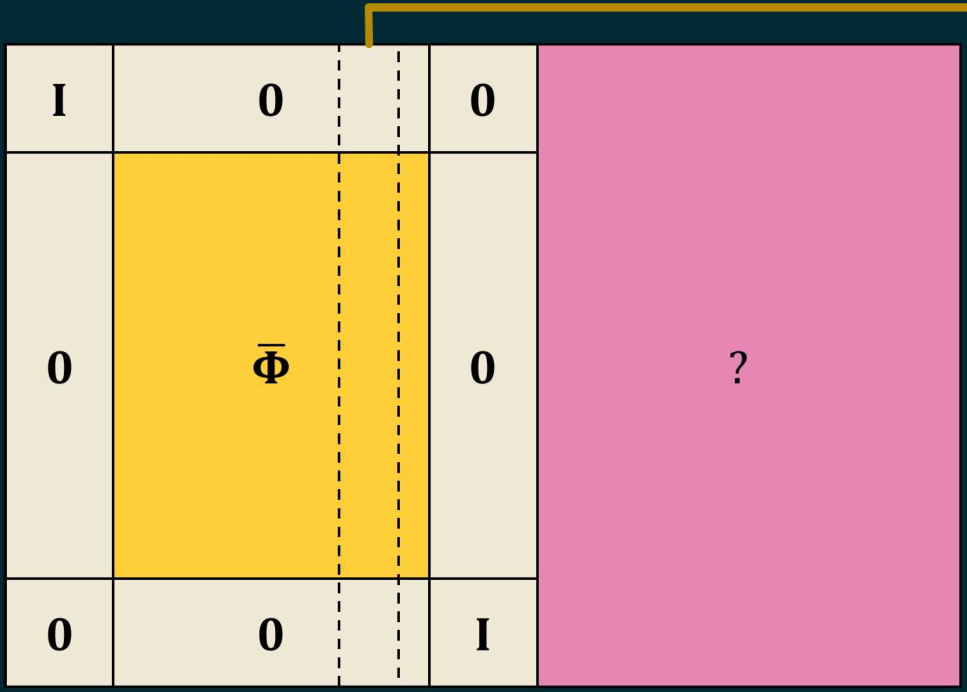
- truncate first n GSI modes

$$\bar{\boldsymbol{\Phi}}^{\mathrm{GSI}} = \begin{bmatrix} \bar{\boldsymbol{\phi}}^{\mathrm{GSI}}_{1} & \cdots & \bar{\boldsymbol{\phi}}^{\mathrm{GSI}}_{n} \end{bmatrix}$$

$$\begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_r \\ \mathbf{u}_a \end{Bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{\boldsymbol{\Phi}}^{\mathrm{GSI}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{\bar{\mathbf{T}}^{\mathrm{GSI}}} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{q}_r \\ \mathbf{u}_a \end{Bmatrix}$$

Modal Derivatives

$\overline{\mathbf{T}}^{\mathrm{GSI}} =$



$$\left(\overline{\mathbf{K}}^{\mathrm{c}} - \bar{\omega}_j^{\,2}\overline{\mathbf{M}}\right)\overline{\boldsymbol{\phi}}_j = \mathbf{0}$$

$$\frac{\partial}{\partial \mathrm{q}_k}\left(\overline{\mathbf{K}}^{\mathrm{c}} - \bar{\omega}_j^{\,2}\overline{\mathbf{M}}\right)\overline{\boldsymbol{\phi}}_j = \frac{\partial}{\partial \mathrm{q}_k}\mathbf{0}$$

$$\vdots$$

$$\bar{\boldsymbol{\theta}}_{jk} = \frac{\partial \overline{\boldsymbol{\phi}}_j}{\partial \mathrm{q}_k} = \left(\overline{\mathbf{K}}^{\mathrm{c}}\right)^{-1}\frac{\partial \overline{\mathbf{K}}^{\mathrm{c}}}{\partial \mathrm{q}_k}\overline{\boldsymbol{\phi}}_j$$

n^2 modal derivatives!

$$\vdots$$

$$\mathrm{E}(d) = \frac{\sum_{i=1}^{d}\tilde{\lambda}_{\mathrm{i}}}{\sum_{i=1}^{n^2}\tilde{\lambda}_{\mathrm{i}}} \quad \rightarrow \quad \mathrm{E}(d) \approx 0.99$$

retain first d modal derivatives ($d < n^2$) from proper orthogonal decomposition (POD)

$$\vdots$$

$$\widetilde{\boldsymbol{\Theta}} = \left[\tilde{\boldsymbol{\theta}}_1 \quad \tilde{\boldsymbol{\theta}}_2 \quad \cdots \quad \tilde{\boldsymbol{\theta}}_d\right]$$

$\overline{\boldsymbol{\Phi}}$ captures the essential linear characteristics

Augment $\overline{\mathbf{T}}^{\mathrm{GSI}}$ with modal derivatives (MDs) to capture the nonlinear behavior

$$\begin{Bmatrix}\mathbf{q}_i\\ \mathbf{u}_r\\ \mathbf{u}_a\end{Bmatrix} = \underbrace{\begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \overline{\boldsymbol{\Phi}}^{\mathrm{GSI}} & \mathbf{0} & \widetilde{\boldsymbol{\Theta}}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0}\end{bmatrix}}_{\overline{\mathbf{T}}^{\mathrm{GSI}}}\begin{Bmatrix}\mathbf{q}_i\\ \mathbf{q}_r\\ \mathbf{u}_a\end{Bmatrix}$$

Reduction Basis Summary

$$\bar{\mathbf{T}} = \left[\begin{array}{c|c|c|c} \mathbf{I} & \mathbf{0} & \mathbf{0} & \\ \hline \mathbf{0} & \bar{\boldsymbol{\Phi}} & \mathbf{0} & \tilde{\boldsymbol{\Theta}} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{I} & \end{array}\right]$$

Gram-Schmidt Interface (GSI) Modes

Modal Derivatives (MDs)

Transformations

kinematics:

$$\underbrace{\bar{\mathbf{u}}}_{HCB} = \bar{\mathbf{T}} \cdot \underbrace{\bar{\bar{\mathbf{u}}}}_{IR}$$

force vectors:

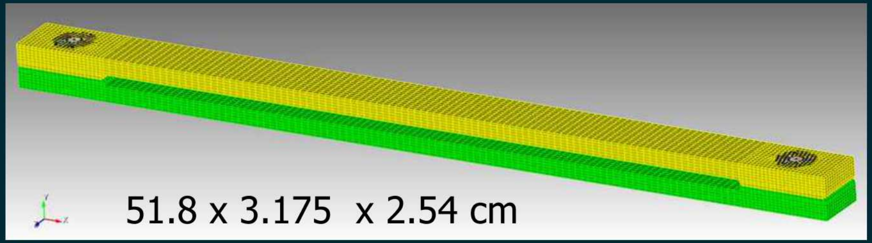
$$\underbrace{\bar{\bar{\mathbf{f}}}}_{IR} = \bar{\mathbf{T}}^{\mathrm{T}} \cdot \underbrace{\bar{\mathbf{f}}}_{HCB}$$

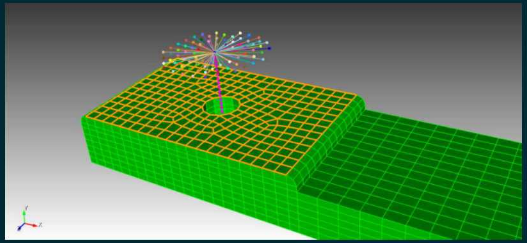
system matrices:

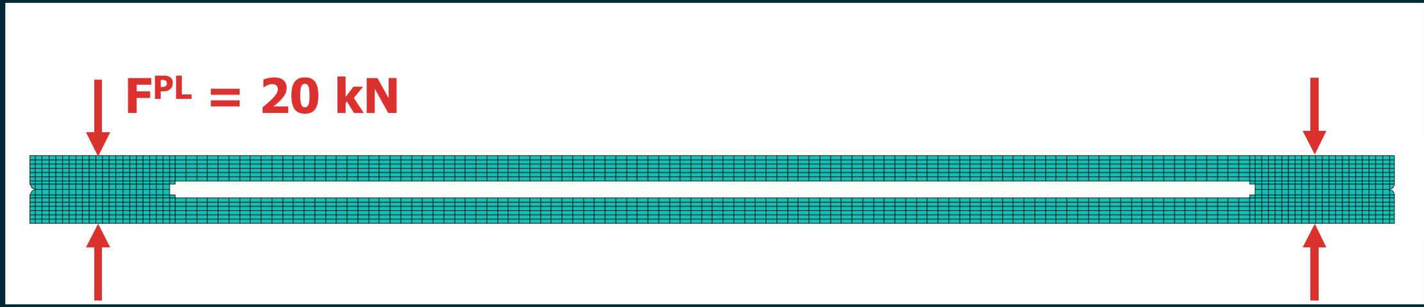
$$\bar{\bar{\mathbf{M}}} = \bar{\mathbf{T}}^{\mathrm{T}} \bar{\mathbf{M}} \bar{\mathbf{T}}$$
$$\bar{\bar{\mathbf{C}}} = \bar{\mathbf{T}}^{\mathrm{T}} \bar{\mathbf{C}} \bar{\mathbf{T}}$$
$$\bar{\bar{\mathbf{K}}} = \bar{\mathbf{T}}^{\mathrm{T}} \bar{\mathbf{K}} \bar{\mathbf{T}}$$

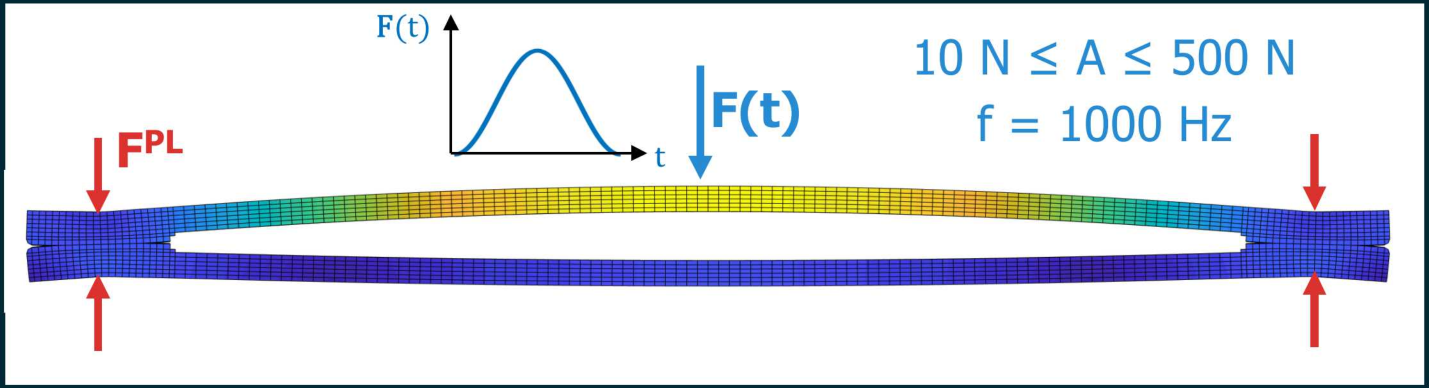
Application Example

Example: C-Beams



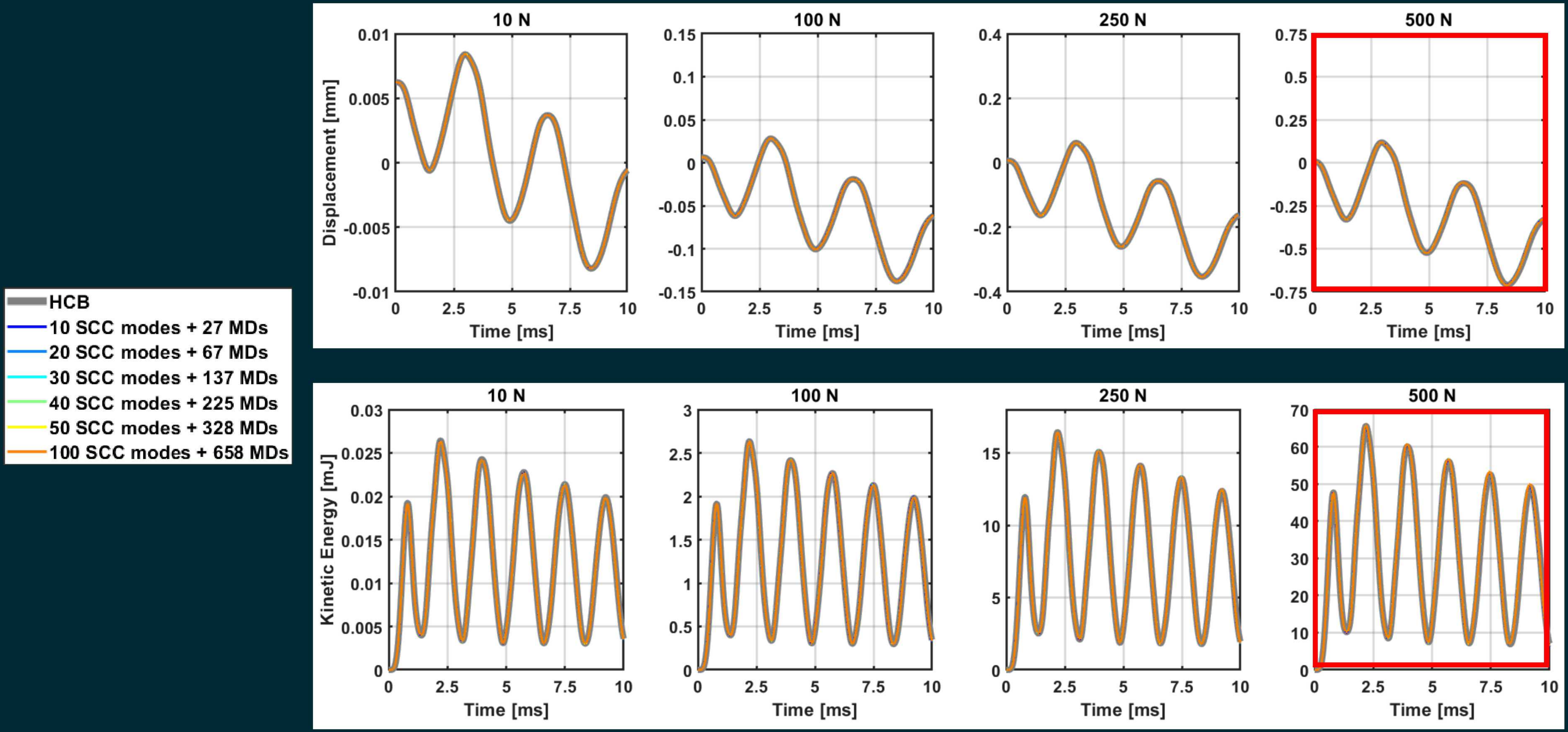




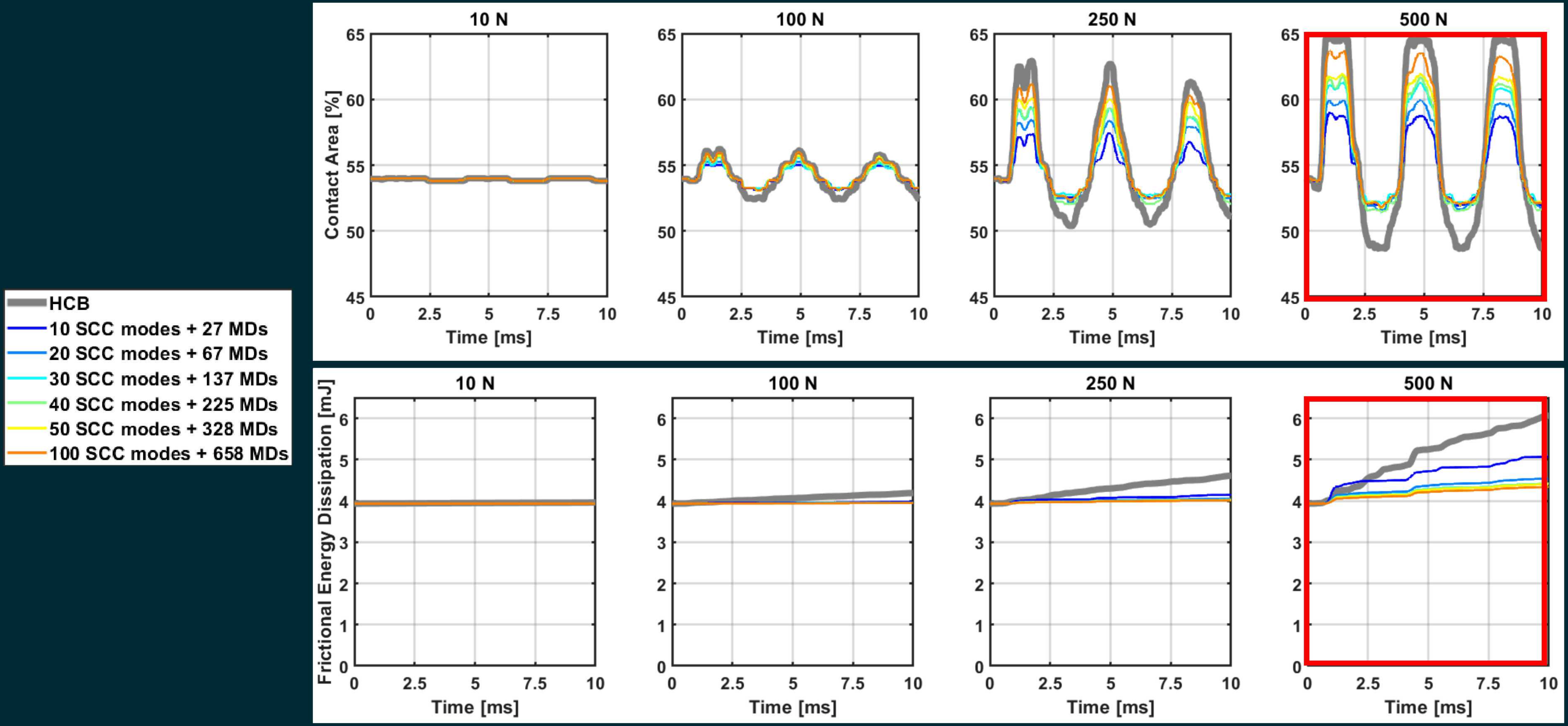


- Node-to-node contact
  - **Normal:** penalty springs
  - **Tangential:** Jenkins friction elements
- Explicit time integration
  - $\Delta t = \Delta t_{crit}/2$ for all models
- Full-order FE model
  - Interior (i): ~300,000 physical DOF
  - Interface (b): 12,444 physical DOF
  - TOTAL: **94,244 DOF**
- Hurty/Craig-Bampton (HCB) model
  - Interior (i): 16 modal DOF
  - Interface (r): 12,444 physical DOF
  - Active (a): 5 physical DOF
  - TOTAL: **3,700 DOF**
- Interface-reduced (IR) model
  - Interior (i): 16 modal DOF
  - Interface (r): ~300 physical DOF
  - Active (p): 5 physical DOF
  - TOTAL: **~300 DOF**

Interface Reduction Results: Global Response



Interface Reduction Results: Local Response

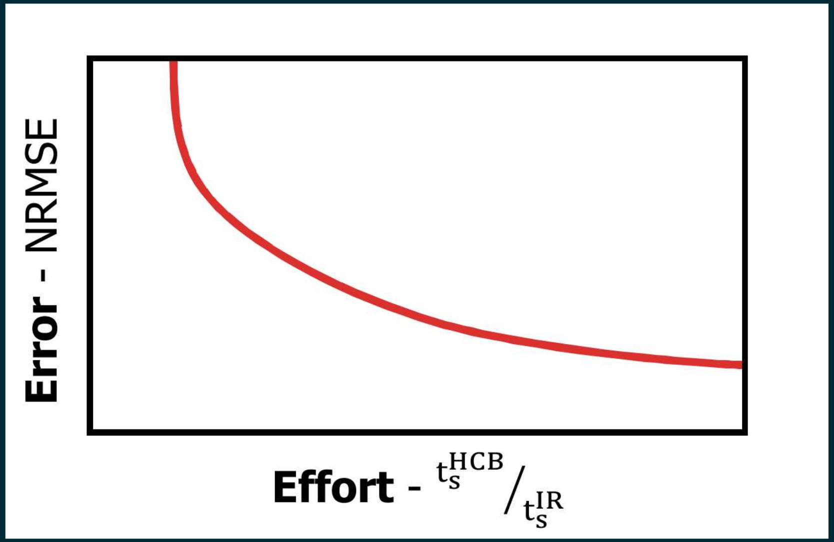


How good is the method, really?

- **Accuracy:** normalized root mean square error (NRMSE)

$$\mathrm{NRMSE} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\mathrm{x}_i^{\mathrm{HCB}} - \mathrm{x}_i^{\mathrm{IR}}\right)^2}}{\max(\mathrm{x}^{\mathrm{HCB}}) - \min(\mathrm{x}^{\mathrm{HCB}})} \times 100\%$$

"Error-Effort" Curves



- **Computational savings:** solve time reduction

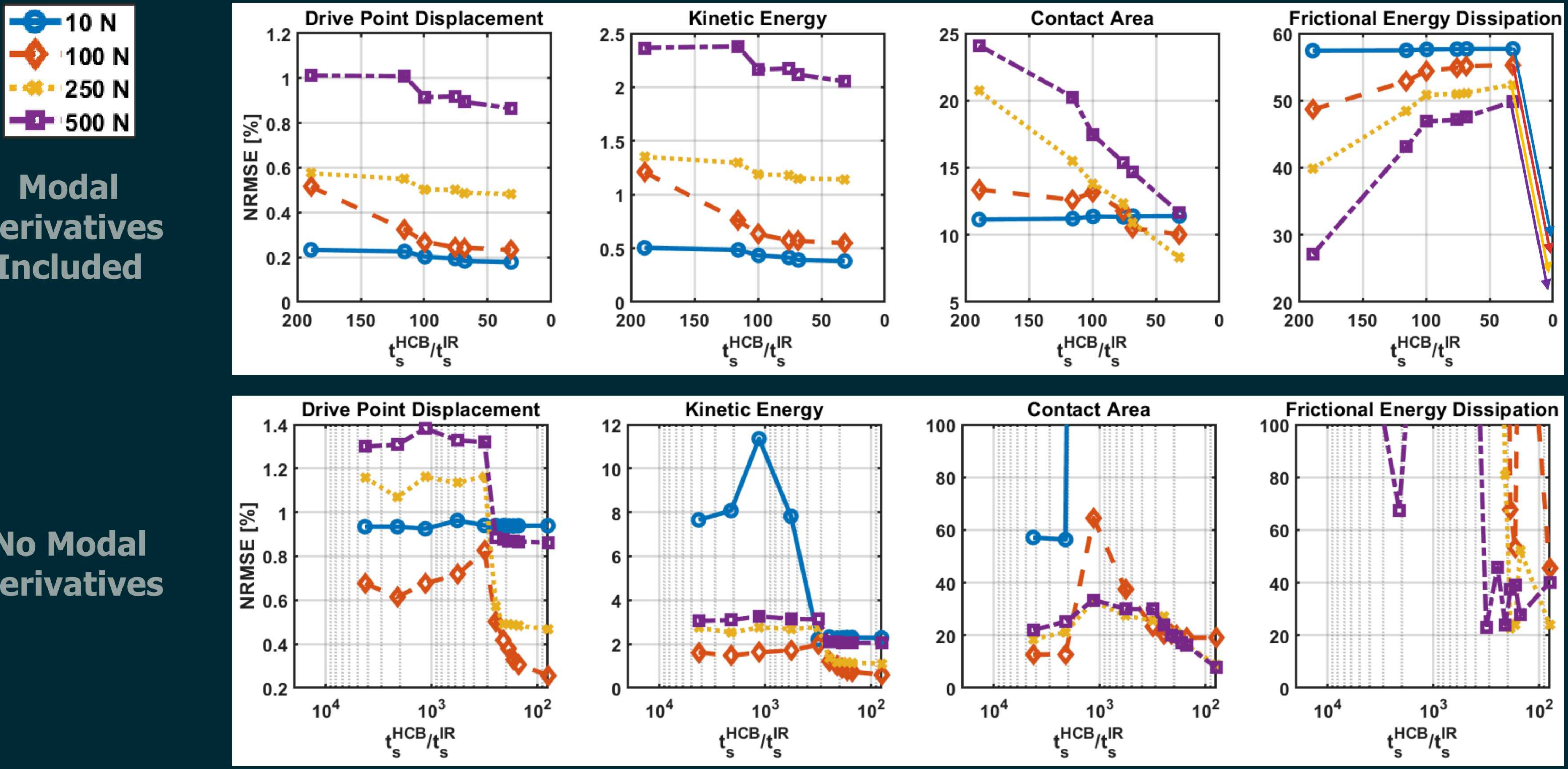
$$\frac{\mathrm{t_s^{HCB}}}{\mathrm{t_s^{IR}}} = \frac{\{\text{HCB solve time}\}}{\{\text{IR solve time}\}}$$

influenced by reduced model size & increased critical timestep length

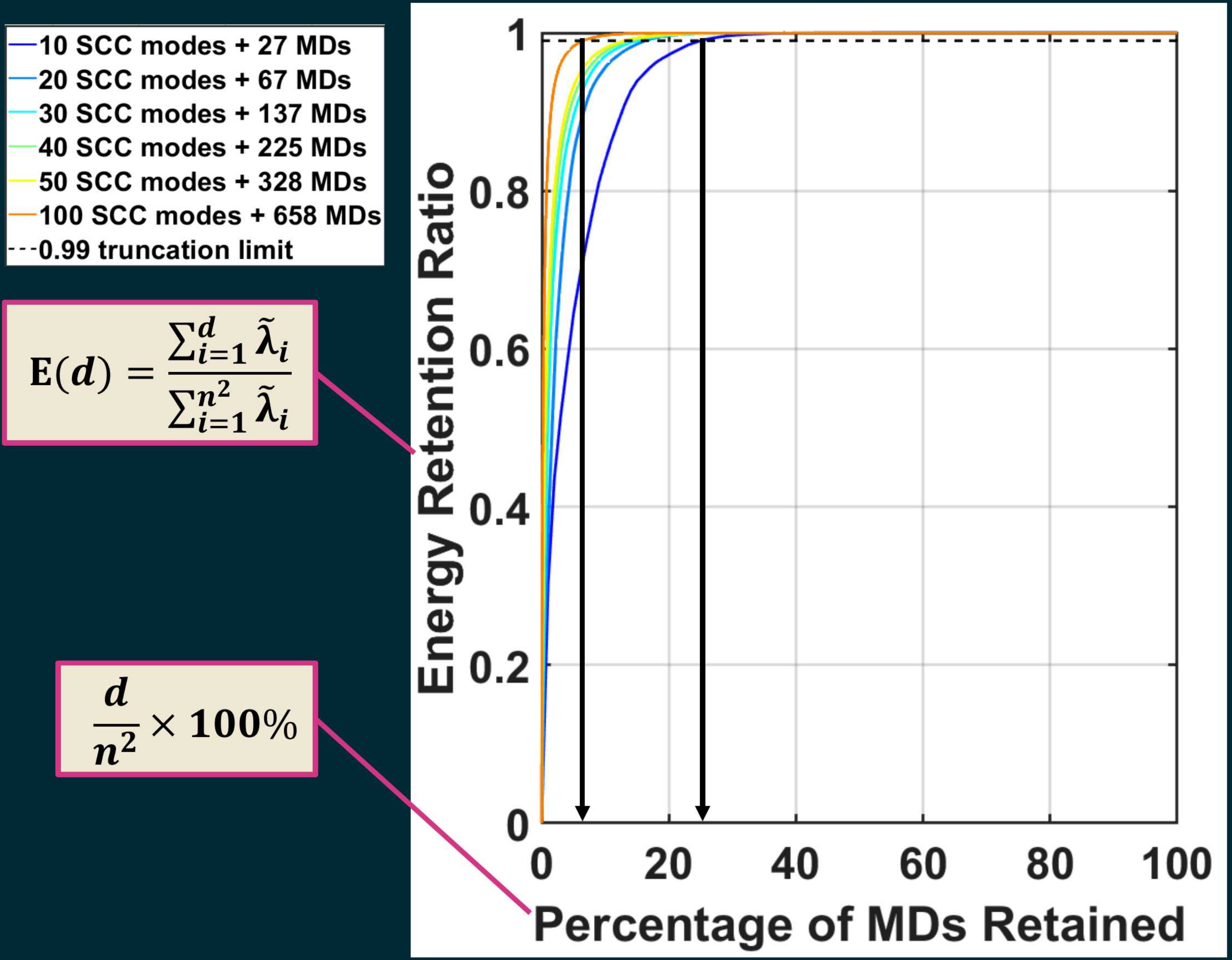
Error-Effort Curves

Modal Derivatives Included

No Modal Derivatives



Modal Derivatives (MDs)



$$\mathbf{E}(\boldsymbol{d}) = \frac{\sum_{i=1}^{d} \tilde{\boldsymbol{\lambda}}_i}{\sum_{i=1}^{n^2} \tilde{\boldsymbol{\lambda}}_i}$$

$$\frac{\boldsymbol{d}}{\boldsymbol{n}^2} \times \mathbf{100\%}$$

Conclusions & Future Work

Conclusions

- Global, linear response metrics (displacement, kinetic energy) converge to the HCB solution with very few modes
  - Slight frequency error observable at simulation end; may be significant for longer simulations
- Local, nonlinear response metrics (contact area, FED) converge slower
  - Good for 10 N - 100 N loads, OK for 250 - 500 N loads
- Method is tunable:
  - High accuracy in the linear region, very low accuracy in nonlinear region → 4,000X reduction
  - High accuracy in the linear region, moderate accuracy in nonlinear region → 25X reduction

Future Work

- Evaluate the scalability of the method for different mesh sizes
- Determine effectiveness for implicit time marching schemes
- Investigate non-monotonic convergence (esp. for FED)

Acknowledgments

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Thank You

- Questions?

Appendix

MD Computation

$$\left(\overline{\mathbf{K}}^{c} - \overline{\omega}_{j}{}^{2}\overline{\mathbf{M}}\right)\overline{\boldsymbol{\phi}}_{j} = \mathbf{0}$$

$$\frac{\partial}{\partial \mathrm{q}_k}\left(\overline{\mathbf{K}}^{c} - \overline{\omega}_{j}{}^{2}\overline{\mathbf{M}}\right)\overline{\boldsymbol{\phi}}_{j} = \frac{\partial}{\partial \mathrm{q}_k}\mathbf{0}$$

$$\left(\frac{\partial \overline{\mathbf{K}}^{c}}{\partial \mathrm{q}_k} - \frac{\partial \overline{\omega}_{j}{}^{2}}{\partial \mathrm{q}_k}\overline{\mathbf{M}}\right)\overline{\boldsymbol{\phi}}_{j} + \left(\overline{\mathbf{K}}^{c} - \overline{\omega}_{j}{}^{2}\overline{\mathbf{M}}\right)\frac{\partial \overline{\boldsymbol{\phi}}_{j}}{\partial \mathrm{q}_k} = \mathbf{0}$$

$$\frac{\partial \overline{\boldsymbol{\phi}}_{j}}{\partial \mathrm{q}_k} = -\left(\overline{\mathbf{K}}^{c} - \overline{\omega}_{j}{}^{2}\overline{\mathbf{M}}\right)^{-1}\left(\frac{\partial \overline{\mathbf{K}}^{c}}{\partial \mathrm{q}_k} - \frac{\partial \overline{\omega}_{j}{}^{2}}{\partial \mathrm{q}_k}\overline{\mathbf{M}}\right)\overline{\boldsymbol{\phi}}_{j}$$

$$\frac{\partial \overline{\boldsymbol{\phi}}_{j}}{\partial \mathrm{q}_k} = \overline{\boldsymbol{\theta}}_{jk} = -\left(\overline{\mathbf{K}}^{c}\right)^{-1}\frac{\partial \overline{\mathbf{K}}^{c}}{\partial \mathrm{q}_k}\overline{\boldsymbol{\phi}}_{j}$$

$$\overline{\mathbf{K}}^{c} = \overline{\mathbf{K}}(\overline{\mathbf{u}}_{\mathrm{PL}}) = \left.\frac{\partial \mathbf{f}^{c}}{\partial \overline{\mathbf{u}}}\right|_{\overline{\mathbf{u}}=\overline{\mathbf{u}}_{\mathrm{PL}}}$$

$$\frac{\partial \overline{\mathbf{K}}^{c}}{\partial \mathrm{q}_k} \approx \frac{\overline{\mathbf{K}}(\mathrm{u}_{\mathrm{PL}} + \mathrm{h}\cdot \mathrm{q}_k) - \overline{\mathbf{K}}(\overline{\mathbf{u}}_{\mathrm{PL}})}{\mathrm{h}}$$

choose h such that contact area (CA) changes by ~5%:

$$|\mathrm{CA}(\mathrm{u}_{\mathrm{PL}} + \mathrm{h}\cdot \mathrm{q}_k) - \mathrm{CA}(\overline{\mathbf{u}}_{\mathrm{PL}})| \approx 5\%$$

MD Computation

$$\bar{\boldsymbol{\theta}}_{jk} = -(\bar{\mathbf{K}}^{c})^{-1}\frac{\partial \bar{\mathbf{K}}^{c}}{\partial \mathrm{q}_k}\bar{\boldsymbol{\phi}}_j$$

$$\bar{\boldsymbol{\Theta}} = \left[\begin{array}{cccc|cccc|c|cccc}\bar{\boldsymbol{\theta}}_{11} & \bar{\boldsymbol{\theta}}_{12} & \cdots & \bar{\boldsymbol{\theta}}_{1n} & \bar{\boldsymbol{\theta}}_{21} & \bar{\boldsymbol{\theta}}_{22} & \cdots & \bar{\boldsymbol{\theta}}_{2n} & \cdots & \bar{\boldsymbol{\theta}}_{n1} & \bar{\boldsymbol{\theta}}_{n2} & \cdots & \bar{\boldsymbol{\theta}}_{nn}\end{array}\right]$$

$$\left(\bar{\boldsymbol{\Theta}}^{\mathrm{T}}\bar{\mathbf{K}}^{c}\bar{\boldsymbol{\Theta}} - \tilde{\lambda}_i\mathbf{I}\right)\tilde{\mathbf{s}}_i = \mathbf{0} \quad ; \quad i = 1, 2, \dots, n^2$$

$$\tilde{\boldsymbol{\theta}}_i = \frac{1}{\tilde{\lambda}_i}\bar{\boldsymbol{\Theta}}\tilde{\mathbf{s}}_i \quad ; \quad i = 1, 2, \dots, n^2$$

$$\mathrm{E}(d) = \frac{\sum_{i=1}^{d}\tilde{\lambda}_\mathrm{i}}{\sum_{i=1}^{n^2}\tilde{\lambda}_\mathrm{i}} \quad \rightarrow \quad \mathrm{E}(d) \approx 0.99$$

$$\tilde{\boldsymbol{\Theta}} = [\tilde{\boldsymbol{\theta}}_1 \quad \tilde{\boldsymbol{\theta}}_2 \quad \cdots \quad \tilde{\boldsymbol{\theta}}_d]$$