Multilevel CMFD Method Development in MPACT

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INTRODUCTION

MPACT [1] is the deterministic neutron transport code in VERA. It calculates the neutron power distribution and the characteristic eigenvalue of the system. The transport problem is solved using the 2D/1D method, with the method of characteristics (MOC) solving 2D radial planes and 1D NEM-P_3 for the axial calculation. To improve the efficiency of the calculation, the coarse-mesh finite difference (CMFD) solver is introduced to accelerate the higher order transport solution and to couple the 2D and 1D transport problems [1]. The CMFD solver is very effective in accelerating the transport solution, but improvements in other parts of the code, including MOC [2], subgroup, [3] and cross section processing, have led to CMFD taking up a significant portion of the runtime. An artificially diffusive CMFD solver has been implemented in MPACT which is unconditionally stable, unlike standard CMFD which is unstable for cells with large optical thickness [4]. This method was shown to converge as fast or faster than standard CMFD for all optical thicknesses. To further reduce runtime and cost of the CMFD solver and to improve the code's efficiency, multigrid methodologies can be leveraged. Multigrid methods reduce the total work performed in the problem by moving some of the work to a set of equations with reduced dimensionality. In CMFD, the two main variables in which this coarsening can be accomplished are energy and space. Several CMFD solvers have been developed which use two energy levels in energy to reduce the work in the CMFD problem [5–6]. A multilevel in space and energy diffusion (MSED) solver was developed for MPACT which uses two levels in energy and a multigrid-in-space linear solver [7]. MSED has also been implemented as a solver for CMFD. Multilevel Using CMFD solvers with multiple grids in energy was shown to have advantages over using only two levels [8].

This paper discusses several multilevel CMFD methods that were developed by reducing the dimensionality in energy and space and implemented in MPACT. These methods are (1) a multilevel-in-energy (MLE) CMFD solver with multiple energy grids [9], (2) a multilevel-in-space (MLS) CMFD solver with three spatial levels [10–12], and (3) a multilevel-in-space-and-energy (MLSE) CMFD solver that combines the MLE and MLS CMFD solvers [13]. These methods are described herein, and their performances are compared for reactor problems of interest.

MULTILEVEL CMFD IN MPACT

Conventional CMFD

CMFD was originally developed for reducing the storage for nodal diffusion problems [14] but much later was found to be an excellent acceleration scheme for the fine mesh transport method that was used to generate the nodal diffusion parameters [15]. The CMFD equations consist of the balance equation and a generalized Fick's Law equation:

$$\sum_s J_{j,g,s}A_{j,s} + \Sigma_{t,j,g}\phi_{i,g}V_j = \sum_{g'=1}^{G}\left(\Sigma_{s0,j,g'\to g} + \frac{\chi_g}{k_{eff}}\nu\Sigma_{f,j,g'}\right)\phi_{j,g'}V_j \ , \quad (1)$$

$$\mathrm{J}_{j,g,s} = -\widetilde{D}_{j,g,s}\left(\phi_{j,g} - \phi_{j,g,s}\right) + \widehat{D}_{j,g,s}\left(\phi_{j,g} + \phi_{j,g,s}\right) \quad (2)$$

where j is the CMFD cell index, g is the group, s is a surface of cell j, and

$$\widetilde{D}_{j,g,s} = \frac{2D_{j,g}D_{j,g,s}}{h_{j,s}\left(D_{j,g} + D_{j,g,s}\right)} \ .$$

A correction term is introduced to the diffusion coefficient and defined such that it yields an exact closure preserving the net current between the transport and modified diffusion equations:

$$\widehat{D}_{j,g,s} = \frac{J^{trans}_{j,g,s} + \widetilde{D}_{j,g,s}\left(\phi^{trans}_{j,g} - \phi^{trans}_{j,g,s}\right)}{\left(\phi^{trans}_{j,g} + \phi^{trans}_{j,g,s}\right)}. \quad (3)$$

Upon convergence, the fine mesh transport solution and CMFD solutions will be the same. The CMFD equations are solved on a coarser mesh than the transport solution. The restriction from the transport to the CMFD problem is defined by a flux-volume homogenization. The coarse CMFD spatial cell is usually defined as a homogenized pin cell.

Typically, there is one MOC plane for each axial CMFD node. The 2D radial MOC sweeps have a high dimensionality which makes them quite expensive, so it is advantageous to reduce the number of axial planes on which that problem is solved. The subplane method allows one MOC plane to be used to characterize multiple axial levels [16–17]. The CMFD problem is solved with the original axial resolution to

preserve the accuracy of the solution. The subplane method is effective in reducing the number of MOC planes that need to be solved, but it also moves more of the work into the CMFD problem, making an efficient solver even more important.

Multilevel-in-Energy (MLE) CMFD

The MLE CMFD problem is defined by formulating the CMFD equations on a sequence of successively coarsening in energy grids. The first energy grid has the same energy group structure as the transport problem. The coarsest energy grid is defined to be two groups. The solver iterates through each CMFD level using a multigrid V-cycle structure, illustrated in Fig. 1. The cycle begins with restriction steps moving recursively from the finest energy level to coarser energy grids. Restriction from a parent grid to a coarser child grid is performed by averaging the cross sections and CMFD coefficients with a flux-weighted homogenization. Pre-sweep smoothing is performed on each level besides the coarsest one, by performing a fixed source sweep using the flux from the last sweep on that grid and the most recent eigenvalue. On the coarsest grid, the eigenvalue problem is solved on two groups using shifted power iterations. During the prolongation steps fixed source sweeps are solved as post-sweep smoothing on every grid from the second coarsest to the finest. The fission source on each grid is calculated using the updated eigenvalue and prolongated flux from previous coarser grid.

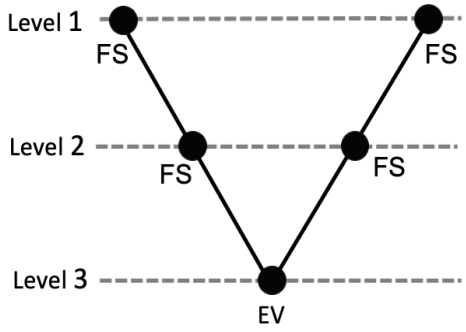

Fig. 1. Multigrid V-Cycle for three levels, FS – fixed source sweep, EV – eigenvalue iterations.

Different numbers of levels and different groups on each level were investigated. It was found that three levels with a uniform constriction factor (i.e., $(1/2\,G)^{1/\Gamma-1}$, where G is the original number of groups, and where Γ is the number of energy levels) between each level has given the best and most consistent results for a variety of problems [13].

Multilevel-in-Space (MLS) CMFD

An MLS CMFD solver has also been implemented with coarsening in space. This solver formulates the CMFD problem on three levels of spatial discretization. The first level is the original CMFD mesh with pin cell homogenization and full axial resolution. The second level collapses radially to quarter assemblies with the same axial mesh as the first level. The third and coarsest level is collapsed axially to MOC subplanes and radially to quarter assemblies. If subplanes are not employed, then this effectively becomes a two-level method with radial collapse only. Restriction in space is performed using a flux-volume homogenization CMFD-like operator. This hierarchy of levels is solved using the same V-cycle type algorithm illustrated in Fig. 1 above.

Multilevel-in-Space-and-Energy (MLSE) CMFD

Both the MLS and MLE CMFD solvers were found to be effective in reducing the work in the CMFD problem [13]. It is possible to combine these two strategies to create the multilevel-in-space-and-energy (MLSE) CMFD solver, which is even more efficient. This solver has three levels, with coarsening in space and energy between each level. The first level has the original pin and axial spatial mesh and the full energy group structure. The third grid is quarter assembly and subplane collapsed with two energy groups.

RESULTS

Numerical results are presented to demonstrate the efficiency of the various versions of CMFD solvers. The first problem is an AP1000-type reactor with Sanmen Unit 1 Cycle 1 core layout. This problem was run using 1,136 processors. Table I shows the number of transport outer iterations, the total time for each solver method, and the speedup compared to the standard CMFD solver for this problem. Typically, with the standard CMFD solver, the solution only takes 15–20 outer iterations to converge, but this challenging problem takes 55 iterations. Table I shows that the multilevel methods can reduce the number of outer iterations by more than half. This means that the multilevel CMFD solvers do a better job accelerating the transport problem than just CMFD. The MLE and MLS CMFD solvers more than double and triple the speed compared to standard CMFD. The combined MLSE CMFD has almost a 4× speedup.

TABLE I. AP1000 problem runtime.

Solver	Outer iterations	Total time [min]	Total speedup
CMFD	55	59.07	-
MLE CMFD	33	26.00	2.27
MLS CMFD	21	16.88	3.50
MLSE CMFD	19	15.37	3.84

The second problem is Consortium for Advanced Simulation of Light Water Reactors (CASL) Progression Problem 9 [18]. The problem consists of a Watts Bar Nuclear

Unit 1 full core with Westinghouse 17×17-type fuel assemblies and Cycle 1 depletion. Problem 9 includes thermal hydraulic feedback and has 32 depletion states. Table II summarizes the outer iterations and runtime for Problem 9. Both MLE and MLS CMFD speed up the solver for this problem, and MLSE CMFD shows the fastest time. For this problem with feedback and depletion, the transport and CMFD solver do not make up as large a component of the runtime, so the change in total runtime is not as drastic. Figure 2 plots the total runtime on each depletion state, demonstrating that the ML CMFD solvers perform consistently well, even for states in which the standard CMFD solver takes considerably longer.

TABLE II. Problem 9 runtime.

Solver	Outer iterations	Total time [hr]	Total speedup
CMFD	876	59.07	-
MLE CMFD	605	37.67	1.57
MLS CMFD	525	35.62	1.66
MLSE CMFD	514	34.68	1.70

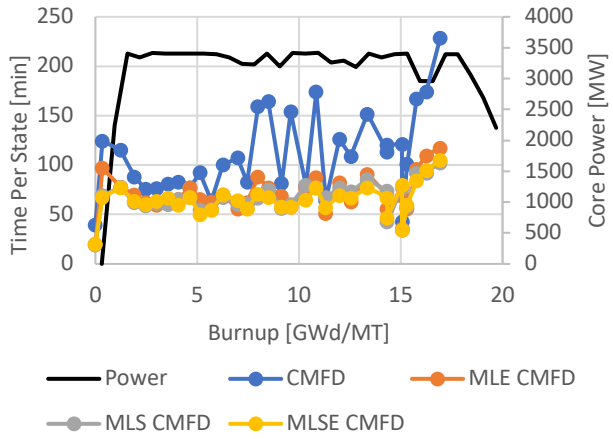

Fig. 2. Problem 9 runtime per state and core power over burnup.

CONCLUSIONS

Multilevel CMFD solvers were developed to improve the efficiency of the transport eigenvalue solver in MPACT by leveraging multilevel strategies to improve the efficiency of the CMFD solver and further accelerate the transport problem. The MLE CMFD solver used coarsening over groups to create two or more energy levels. MLS CMFD used a CMFD spatial collapse in the radial and axial dimensions on three levels. Each of these methods significantly speedup the CMFD solver. The MLSE CMFD solver combines these strategies to further improve their efficiently. All of these multilevel methods show increased speed compared to standard CMFD, but the MLSE CMFD solver shows the best performance for a variety of reactor physics problems. Work is being performed to make MLSE CMFD the default solver to provide MPACT and VERA users with the best performance possible.

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