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Co-optimization to Integrate Power System Reliability Decisions with Resiliency Decisions

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Project Team

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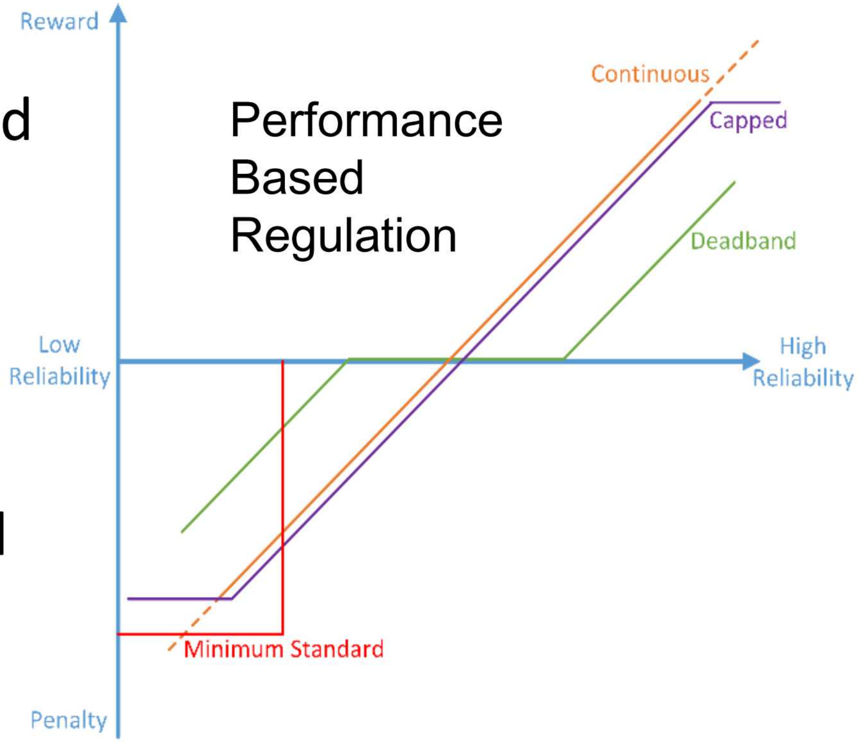
Resilience vs. Reliability

- Reliability – Low consequence high probability
  - Squirrels, birds, etc.
  - Traffic accidents
  - Trees/wind
  - Lightning
- Resilience - High consequence low probability events
  - Severe winter storms
  - Hurricanes
  - Earthquakes
  - EMPs and GMDs
  - Large wildfires

Utilities are incentivized to be reliable not resilient

- Utilities are often incentivized to be more reliable (improve their SAIDI and SAIFI metrics)
- Some utilities have performance based regulation (PBR)
- Large scale events (severe winter storms, hurricanes, etc.) are removed from the SAIDI and SAIFI metrics
- Less incentive to invest in resiliency

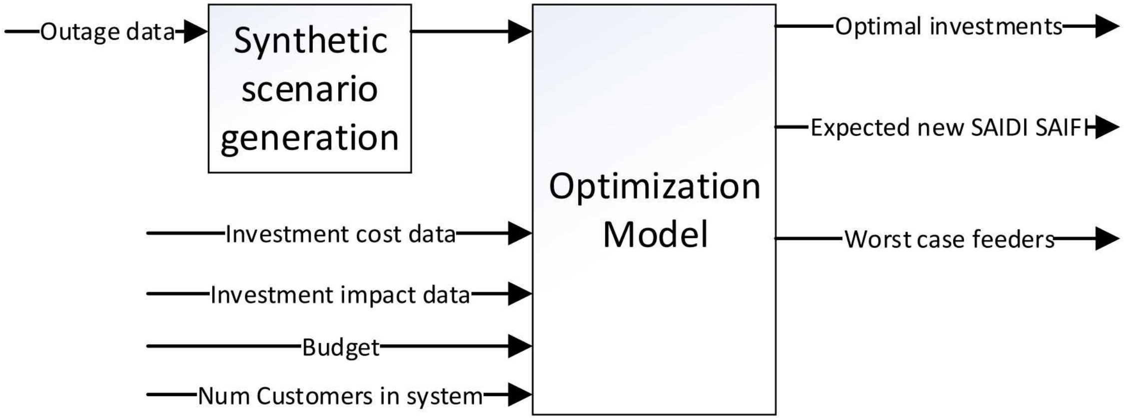


Primary project goals

- Develop optimization models which find the optimal investments to improve reliability, resiliency, and a weighted combination of the two.
- Help utilities see the trade-offs between investing more heavily in reliability or resiliency.
- Help utilities develop rate recovery cases to justify large scale investments, by quantifying how that investment will improve their reliability and resiliency.
- Inform utilities and their stakeholders, DOE, DHS, and policy makers of cost-effective infrastructure investment decisions that simultaneously improve both reliability and resilience.

Investment optimization for Reliability

- Objective is to minimize SAIDI and SAIFI
- Input is historical outage data
- Investments are mostly smaller scale compared to the investments for resilience



Stochastic mixed integer program for optimal reliability investments

Objective function

$$\text{minimize} \frac{\text{SAIDI}_{up}}{\text{SAIDI}_{syn}} + \frac{\text{SAIFI}_{up}}{\text{SAIFI}_{syn}} \quad (5)$$

subject to

$$\sum_{i,d,u \in U_{i,d}} c_u y_{i,d,u} \le B \quad (6)$$

$$SAIDI_{up} = \frac{1}{N} \sum_{o \in O} CO_o TO_o \quad (7)$$

$$SAIFI_{up} = \frac{1}{N} \sum_{o \in O} CO_o \quad (8)$$

$$CO_o = \min_{u \in U_o} \{ C_{o,u} y_{i_o,d_o,u} + C_o (1 - y_{i_o,d_o,u}) \} \quad \forall o \in O \quad (9)$$

$$TO_o = \min_{u \in U_o} \{ T_{o,u} y_{i_o,d_o,u} + T_o (1 - y_{i_o,d_o,u}) \} \quad \forall o \in O \quad (10)$$

The above model is non-linear so linearize with:

$$CO_o \le C_{o,u} y_{i_o,d_o,u} + C_o (1 - y_{i_o,d_o,u}) \quad \forall o \in O, u \in U_o \quad (11)$$

$$CO_o \ge C_{o,u} [y_{i_o,d_o,u} + C_o (1 - y_{i_o,d_o,u})] m_{o,u} \quad \forall o \in O, u \in U_o \quad (12)$$

$$\sum_{u \in U_o} m_{o,u} = 1 \quad \forall o \in O \quad (13)$$

$$TO_o \le T_{o,u} y_{i_o,d_o,u} + T_o (1 - y_{i_o,d_o,u}) \quad \forall o \in O, u \in V_o \quad (14)$$

$$TO_o \ge T_{o,u} [y_{i_o,d_o,u} + T_o (1 - y_{i_o,d_o,u})] n_{o,u} \quad \forall o \in O, u \in V_o \quad (15)$$

$$\sum_{u \in V_o} n_{o,u} = 1 \quad \forall o \in O \quad (16)$$

$$my_{o,u} \le m_{o,u} \quad \forall o \in O, u \in U_o \quad (17)$$

$$my_{o,u} \le y_{i_o,d_o,u} \quad \forall o \in O, u \in U_o \quad (18)$$

$$my_{o,u} \ge m_{o,u} + y_{i_o,d_o,u} + 1 \quad \forall o \in O, u \in U_o \quad (19)$$

$$ny_{o,u} \le n_{o,u} \quad \forall o \in O, u \in V_o \quad (20)$$

$$ny_{o,u} \le y_{i_o,d_o,u} \quad \forall o \in O, u \in V_o \quad (21)$$

$$ny_{o,u} \ge n_{o,u} + y_{i_o,d_o,u} + 1 \quad \forall o \in O, u \in V_o \quad (22)$$

$$COTO_o = \sum_{u \in U_o} \sum_{u' \in V_o} C_{o,u} T_{o,u'} m_{o,u} n_{o,u'} \quad \forall o \in O \quad (23)$$

$$mn_{o,u,u'} \le m_{o,u} \quad \forall o \in O, u \in U_o, u' \in V_o \quad (24)$$

$$mn_{o,u,u'} \le n_{o,u'} \quad \forall o \in O, u \in U_o, u' \in V_o \quad (25)$$

$$mn_{o,u,u'} \ge m_{o,u} + n_{o,u'} + 1 \quad \forall o \in O, u \in U_o, u' \in V_o \quad (26)$$

Sets

D	Device types
I	Feeder IDs
U	Upgrade options
$U_{i,d}$	Upgrade options for device type d in feeder i
O	Outages
U_o	Upgrade options that improve the number of customers outaged in outage o if applied
V_o	Upgrade options that improve the duration of outage in outage o if applied
S	Outage causes

Parameters

C_o	Number of customers outage o affects
T_o	Duration of outage o
d_o	Device type of outage o
i_o	Device ID of outage o (also gives feeder ID/location)
s_o	Cause of outage o
c_u	Cost to purchase upgrade u
$C_{o,u}$	Number of customers outage o affects after upgrade u
$T_{o,u}$	Duration of outage o after upgrade u
$SAIDI_{syn}$	Baseline SAIDI value
$SAIFI_{syn}$	Baseline SAIFI value
B	Budget
N	Number of customers in total system

Variables

$y_{i,d,u}$	Binary indicating whether or not to apply upgrade $u \in U_{i,d}$ to device type d in feeder i
$SAIDI_{up}$	SAIDI value after upgrades
$SAIFI_{up}$	SAIFI value after upgrades
$m_{o,u}$	Binary indicating that upgrade u gives minimal customer outage during outage o. Necessary for when multiple upgrades are selected that affect one outage.
$n_{o,u}$	Binary indicating that upgrade u gives minimal outage duration during outage o. Necessary for when multiple upgrades are selected that affect one outage.
$mn_{o,u,u'}$	The product $m_{o,u}\, n_{o,u'}$. Can also be interpreted as a binary indicating upgrade u gives minimal customer outage and upgrade u_0 gives minimal outage duration during outage o
$my_{o,u}$	The product $m_{o,u}\, y_{i_o,d_o,u}$. Can also be interpreted as a binary indicating that upgrade u is applied and results in minimal number of customers affected during outage o
$ny_{o,u}$	The product $n_{o,u}\, y_{i_o,d_o,u}$. Can also be interpreted as a binary indicating that upgrade u is applied and results in minimal outage duration during outage o
CO_o	Number of customers which outage o affects after upgrade
TO_o	Duration of outage o after upgrade
$COTO_o$	The product $CO_o\, TO_o$

Model details

Goal: Determine the optimal investments to improve power distribution system reliability.

Inputs to model: Historical outage data, investment impact data, investment cost data

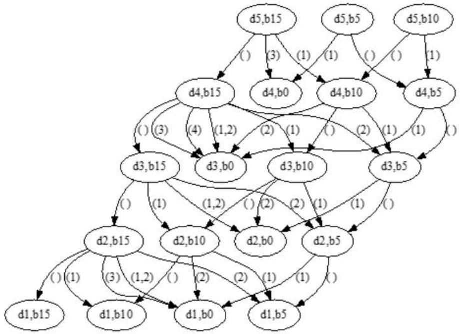
Model type: Nonlinear mixed integer program Linearized through new and old techniques

Model efficiency (scalability): Great efficiency, especially for larger systems, but worse for large budgets and large outage sets

Generalized dynamic programming method for optimal reliability investments

GRDP Algorithm

```
# Precondition: Package_bundles is a list of upgrade
#             bundles which can be used to upgrade
#             the bundles' respective outages.
# Postcondition: Returns the package bundle whose
#              contribution to the objective function
#              is optimal.
function max_obj(package_bundles)
  max = -1
  for each bundle in package_bundles do
    objective_contribution = 0
    for each package in bundle do
      increment objective_contribution by the package's
      contribution to the objective function
    if objective_contribution > max:
      max = objective_contribution
      optimal_bundle = bundle

    return optimal_bundle

global cache = []

function GRDP(feeder_device_pairs, budget)
  if (feeder_device_pairs, budget) is in cache do
    return cache[feeder_device_pairs, budget]

  if budget < 0 do
    return empty list

  for each package in applicable upgrade packages for
      feeder and device given in first pair from
      feeder_device_pairs do
    if the cost of package > budget do
      return empty list
    upgrade_package_bundles = a list with package followed
            by GRDP(feeder_device_pairs with first element
                    removed, budget – cost of package)
    cache[feeder_device_pairs, budget] =
          max_obj(upgrade_package_bundles)
  return cache[feeder_device_pairs, budget]
```



Model details

Goal: Determine the optimal investments to improve power distribution system reliability.

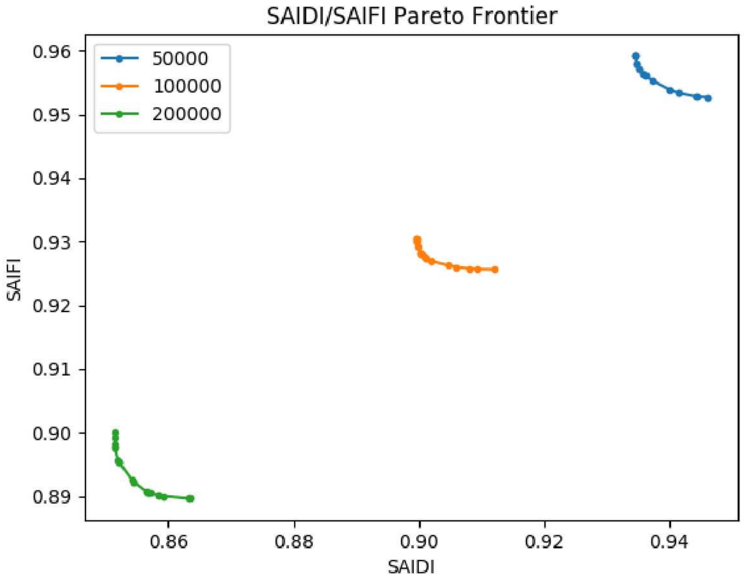
Inputs to model: Historical outage data, investment impact data, investment cost data

Model type: Generalized dynamic programming – decision tree – based on classic Knapsack algorithm

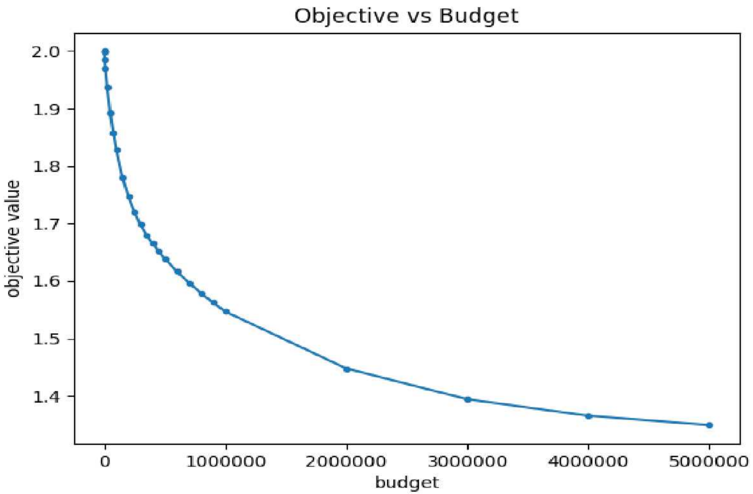
Model efficiency (scalability): Good efficiency, especially for large budgets and large outage sets, worse on large systems than previous model

IEEE

Results from reliability investment models on utility data



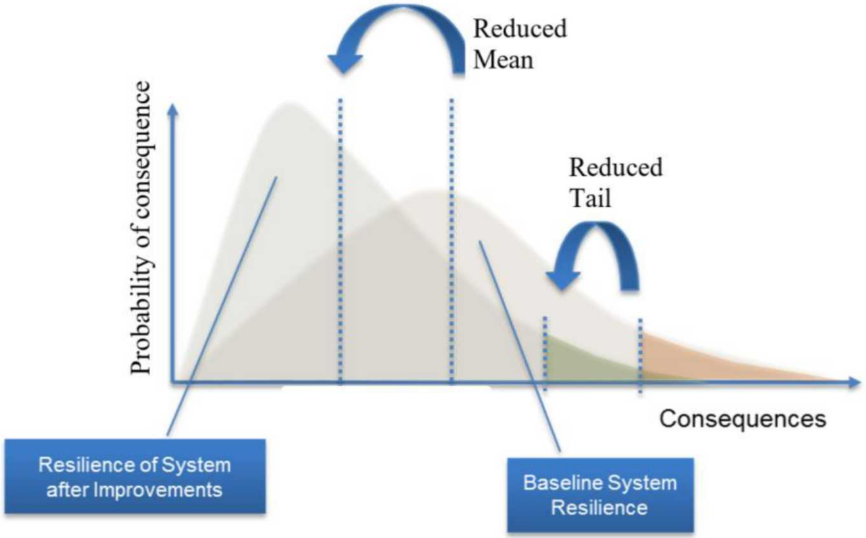
Pareto frontiers of weighting SAIDI or SAIFI more. Whether you weight SAIDI (duration) more or SAIFI (frequency of events) more, the results are similar.



The improvement in reliability at budget increases. The optimal investments are chosen for each budget

Resiliency investment optimization

- The goals are to push the mean consequence and the tail of the consequence to the left.
- Reducing the tail, reduces the consequence from the large worst-case scenarios
- Resilience metrics used in this project are Loss of Load and Duration.



Stochastic mixed integer program for optimal resilience investments

Minimize

$$Resiliency\ metric = \frac{1}{B_{LSWD}} LSWD + \frac{1}{B_{LS}} LS \quad (1)$$

subject to

$$\text{LSWD} = \sum_{t \in T} t \sum_{b \in B} A_b \sum_{w \in \Omega} P_w p_{b,t}^{w} \quad (2)$$

$$LS = \sum_{b \in B} A_b \sum_{b \in B} P_w p_{b,0}^{w} \quad (3)$$

$$\sum_{b \in B} C_b i_b + \sum_{l \in L} C_l i_l + \sum_{g \in G} C_g i_g \le K \quad (2)$$

$$\sum_{g \in G_b} p_{g,t}^{w} + \sum_{l \in L_b^{to}} p_{l,t}^{w} - \sum_{l \in L_b^{from}} p_{l,t}^{w} = D_b - p_{b,t}^{w} \quad \forall b \in B, \forall t \in T, \forall w \in \Omega \quad (5)$$

$$p_{l,t}^{w} = y_{l,t}^{w} S_l \left(\theta_{B_l^{to},t}^{w} - \theta_{B_l^{from},t}^{w}\right) \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (6)$$

$$p_{g,t}^{w} \le p_{g,t-1}^{w} + RU_g y_{g,t}^{w} + SU_g\left(y_{g,t}^{w} - y_{g,t}^{w}\right) + \overline{P}_g\left(1 - y_{g,t}^{w}\right) \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (7)$$

$$p_{g,t-1}^{w} \le \overline{P}_g y_{g,t}^{w} + SD_g\left(y_{g,t-1}^{w} - y_{g,t}^{w}\right) \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (8)$$

$$p_{g,t-1}^{w} - p_{g,t}^{w} \le RD_g y_{g,t}^{w} + SD_g\left(y_{g,t-1}^{w} - y_{g,t}^{w}\right) + \overline{P}_g\left(1 - y_{g,t}^{w}\right) \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (9)$$

$$-\frac{\pi}{3} \le \theta_{B_l^{to},t}^{w} - \theta_{B_l^{from},t}^{w} \le \frac{\pi}{3} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (10)$$

$$-\overline{P}_l y_l^{w} \le p_{l,t}^{w} \le \overline{P}_l y_l^{w} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (11)$$

$$\underline{P}_g y_g^{w} \le p_{g,t}^{w} \le \overline{P}_g y_g^{w} \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (12)$$

$$0 \le p_{b,t}^{w} \le D_b \quad \forall b \in B, \forall t \in T, \forall w \in \Omega \quad (13)$$

$$y_{l,t}^{w} \le i_l + \frac{t}{X_l^{w}} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (14)$$

$$y_{l,t}^{w} \le i_{B_l^{from}} + \frac{t}{X_{B_l^{from}}^{w}} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (15)$$

$$y_{l,t}^{w} \le i_{B_l^{to}} + \frac{t}{X_{B_l^{to}}^{w}} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (16)$$

$$y_{g,t}^{w} \le i_g + \frac{t}{X_g^{w}} \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (17)$$

$$y_{g,t}^{w} \le i_{B_g} + \frac{t}{X_{B_g}^{w}} \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (18)$$

$$y_{b,t}^{w} \le i_b + \frac{t}{X_b^{w}} \quad \forall b \in B, \forall t \in T, \forall w \in \Omega \quad (19)$$

$$y_{l,t}^{w} \le y_{B_l^{from},t}^{w} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (20)$$

$$y_{l,t}^{w} \le y_{B_l^{to},t}^{w} \quad \forall l \in L, \forall t \in T, \forall w \in \Omega \quad (21)$$

$$y_{g,t}^{w} \le y_{B_g,t}^{w} \quad \forall g \in G, \forall t \in T, \forall w \in \Omega \quad (22)$$

Resilience Sets

L	Transmission lines
G	Generators
B	Buses
Ω	Outage scenarios
Ω_l	Set of scenarios under which transmission line l goes offline
Ω_g	Set of scenarios under which generator g goes offline
Ω_b	Set of scearios under which bus b goes offline
T	Discrete set of times: duration each component is out of service
G_b	Set of generators connected to bus b
L_b^{from}	Set of transmission lines leaving bus b
L_b^{to}	Set of tranmssion lines entering bus b
I	Set of investments for buses, generators, and transmission lines

Resilience Parameters

B_l^{from}	Bus from which transmission line l leaves
B_l^{to}	Bus transmission line l enters
S_l	Susceptance of tranmission line l
$\overline{P}_l$	Thermal limit of transmission line l
B_g	Bus containing generator g
RU_g	Ramp-up limit of generator g dispatch level
RD_g	Ramp-down limit of generator g dispatch level
SU_g	Start-up limit of generator g distpatch level
SD_g	Shut-down limit of generator g distpatch level
$\overline{P}_g$	Upper limit of generator g dispatch level
$\underline{P}_g$	Lower limit of generator g dispatch level
D_b	Demand at bus b
A_b	Load weighting factor at bus b
C_l	Cost of hardening transmission line l
C_g	Cost of hardening generator g
C_b	Cost of hardening bus b
P_w	Probability of scenario w occuring
X_l^w	Number of time periods line l is affected by event in scenario w with no hardening
X_g^w	Number of time periods generator g is affected by event in scenario w with no hardening
X_b^w	Number of time periods bus b is affected by event in scenario w with no hardening
P_w	Probability of scenario w occuring
B_{LSWD}	First term in objective during baseline model run with 0 budget
B_{LS}	Second term in objective during baseline model run with 0 budget

Resilience Variables

$LSWD$	Load Shed With Duration in MW – similar to the SAIDI reliability metric
LS	Load Shed – similar to the SAIFI reliability metric
$p_{l,t}^w$	Power flow through transmission line l at time t in scenario w
$p_{g,t}^w$	Generator dispatch level for generator g at time t in scenario w
$p_{b,t}^w$	Load shed at bus b at time t in scenario w
$\theta_{b,t}^w$	Phase angle for bus b at time t in scenation w
$y_{l,t}^w$	On/off status of line l at time t during scenario w
$y_{g,t}^w$	On/off status of generator g at time t during scenario w
$y_{b,t}^w$	On/off status of bus b at time t during scenario w
i_l	Binary indicating whether or not transmission line l is hardened
i_g	Binary indicating whether or not generator g is hardened
i_b	Binary indicating whether or not bus b is hardened

Model details

Goal: Determine the optimal investments to improve power system resilience (loss of weighted load and duration).

Inputs to model: Scenario data from threats listing component outages and recovery time. Investment cost data.

Model type: Linear mixed integer program (MIP).

Model efficiency (scalability): Poor efficiency, especially for larger systems, and a large number of scenarios. Can solve the IEEE RTS96 system with 50 scenarios.

$$\min \sum_{\omega\in\Omega} q_\omega \left(\frac{1}{B_1}\sum_{t\in T} t \sum_{b\in B} A_b p^{\omega}_{b,t} + \frac{1}{B_2}\sum_{b\in B} A_b p^{\omega}_{b,1}\right) \quad (22)$$

$$\text{s.t.} \sum_{b\in B} k_b + \sum_{l\in \mathcal{L}} k_l + \sum_{g\in\mathcal{G}} k_g \le K \quad (23)$$

$$\sum_{g\in G_b} p^{\omega}_{g,t} + \sum_{l\in \mathcal{L}^{to}_b} p^{\omega}_{l,t} - \sum_{l\in \mathcal{L}^{from}_b} p^{\omega}_{l,t} = D_b - p^{\omega}_{b,t} \quad \forall b\in\mathcal{B}, \forall t\in T, \forall\omega\in\Omega \quad (24)$$

$$\begin{bmatrix} y^{on}_{g,t,\omega} \\ \underline{P}_g \le p^{\omega}_{g,t} \\ \underline{P}_g \le p^{\omega}_{g,t-1} \\ -RD_g \le p^{\omega}_{g,t} - p^{\omega}_{g,t-1} \\ p^{\omega}_{g,t} - p^{\omega}_{g,t-1} \le RU_g \end{bmatrix} \vee \begin{bmatrix} y^{off}_{g,t,\omega} \\ p^{\omega}_{g,t} = 0 \\ p^{\omega}_{g,t-1} \le SD_g \end{bmatrix} \vee \begin{bmatrix} y^{startup}_{g,t,\omega} \\ \underline{P}_g \le p^{\omega}_{g,t} \le SU_g \\ p^{\omega}_{g,t-1} = 0 \end{bmatrix} \quad \forall g\in\mathcal{G}, \forall t\in T, \forall\omega\in\Omega \quad (25)$$

$$y^{on}_{g,t,\omega} \veebar y^{off}_{g,t,\omega} \veebar y^{startup}_{g,t,\omega} = True \quad \forall g\in\mathcal{G}, \forall t\in T, \forall\omega\in\Omega \quad (26)$$

$$\begin{bmatrix} y^{\omega}_{l,t} \\ p^{\omega}_{l,t} = S_l(\theta^{\omega}_{B^{to}_l,t} - \theta^{\omega}_{B^{from}_l,t}) \end{bmatrix} \vee \begin{bmatrix} \neg y^{\omega}_{l,t} \\ p^{\omega}_{l,t} = 0 \end{bmatrix} \quad \forall l\in\mathcal{L}, \forall t\in T, \forall\omega\in\Omega \quad (27)$$

$$\begin{bmatrix} z_l \\ k_l = C_l \end{bmatrix} \vee \begin{bmatrix} \neg z_l \\ k_l = 0 \\ p^{\omega}_{l,t} = 0 \; \forall t \le X_l \end{bmatrix} \quad \forall l\in\mathcal{L} \quad (28)$$

$$\begin{bmatrix} z_g \\ k_g = C_g \end{bmatrix} \vee \begin{bmatrix} \neg z_g \\ k_g = 0 \\ p^{\omega}_{g,t} = 0 \; \forall t \le X_g \end{bmatrix} \quad \forall g\in\mathcal{G} \quad (29)$$

$$\begin{bmatrix} z_b \\ k_b = C_b \end{bmatrix} \vee \begin{bmatrix} \neg z_b \\ k_b = 0 \\ p^{\omega}_{l,t} = 0 \quad \forall t \le X_b, \forall l\in\mathcal{L}_b \\ p^{\omega}_{g,t} = 0 \quad \forall t \le X_b, \forall g\in G_b \end{bmatrix} \quad \forall b\in\mathcal{B} \quad (30)$$

$$0 \le p^{\omega}_{b,t} \le D_b \quad \forall b\in\mathcal{B}, \forall t\in T, \forall\omega\in\Omega \quad (31)$$

$$0 \le p^{\omega}_{g,t} \le \overline{P}_g \quad \forall g\in\mathcal{G}, \forall t\in T, \forall\omega\in\Omega \quad (32)$$

$$-\overline{P}_l \le p^{\omega}_{l,t} \le \overline{P}_l \quad \forall l\in\mathcal{L}, \forall t\in T, \forall\omega\in\Omega \quad (33)$$

$$-\frac{\pi}{3} \le \theta^{\omega}_{B^{to}_l,t} - \theta^{\omega}_{B^{from}_l,t} \le \frac{\pi}{3} \quad \forall l\in\mathcal{L}, \forall t\in T, \forall\omega\in\Omega \quad (34)$$

Algorithm 1: Modified Benders Decomposition

Input: Master problem $\mathcal{M}$, subproblems $\mathcal{P}_\omega(z)$ for all $\omega\in\Omega\setminus\{\omega_1\}$, absolute tolerance, ϵ

Output: Lower and upper bounds for partially relaxed problem, LB and UB, and optimal solution z^* within tolerance ϵ.

```
LB ← −∞
UB ← +∞
while UB - LB < ε do
    solve M.
    let the (ẑ, y_ω1, θ) be the optimal solution and v be the
      optimal value
    let v_ω1 = v − Σ_{ω∈Ω\{ω1}} θ_ω
    LB ← v
    foreach ω ∈ Ω \ {ω1} do
        solve rP_ω(ẑ)
        let v_ω be the optimal value, and z_ω and λ_ω be the
          optimal primal and dual solutions respectively
    end
    u ← Σ_{ω∈Ω} v_ω
    if UB > u then
        UB ← u
        z* ← ẑ
    end
    foreach ω ∈ Ω \ {ω1} do
        add θ_ω ≥ v_ω − λ_ω^T (z_ω − z) to M
    end
end
```

Sets

- $\mathcal{L}$ Transmission lines
- $\mathcal{G}$ Generators
- $\mathcal{B}$ Buses
- T Discrete set of times after a scenario occurs, starting with time 1
- G_b Set of generators contained in bus b
- $\mathcal{L}^{from}_b$ Set of transmission lines leaving bus b
- $\mathcal{L}^{to}_b$ Set of transmission lines entering bus b
- $\mathcal{L}_b$ Set of transmission lines either leaving or entering bus b

Parameters

- B^{from}_l Bus from which transmission line l leaves
- B^{to}_l Bus transmission line l enters
- S_l Susceptance of transmission line l
- $\overline{P}_l$ Thermal limit for transmission line l
- B_g Bus containing generator g
- RU_g Ramp-up limit of generator g dispatch level
- RD_g Ramp-down limit of generator g dispatch level
- SU_g Start-up limit of generator g dispatch level
- SD_g Shut-down limit of generator g dispatch level
- $\underline{P}_g$ Lower limit of generator g dispatch level
- $\overline{P}_g$ Upper limit of generator g dispatch level
- D_b Demand at bus b
- A_b Load conversion factor at bus b
- C_l Cost of hardening transmission line l
- C_g Cost of hardening generator g
- C_b Cost of hardening bus b
- K Budget
- X_l Number of time periods line l is affected by event with no hardening
- X_g Number of time periods generator g is affected by event with no hardening
- X_b Number of time periods bus b is affected by event with no hardening
- B_1 Baseline load shed, calculated by taking first term in objective during model run with 0 budget
- B_2 Baseline load shed at time 1, calculated by taking second term in objective during model run with 0 budget
- A_b Priority level of bus b for restoration

Variables

Common to both models

- $p_{l,t}$ Power flow through transmission line l at time t
- $p_{g,t}$ Generator dispatch level for generator g at time t
- $p_{b,t}$ Load shed at bus b during time t
- $\theta_{b,t}$ Phase angle for bus b at time t
- $y_{l,t}$ On/off status of line l at time t
- z_l Binary indicating whether or not transmission line l is hardened
- z_g Binary indicating whether or not generator g is hardened
- z_b Binary indicating whether or not bus b is hardened
- k_l Cost incurred by line l
- k_g Cost incurred by generator g
- k_b Cost incurred by bus b
- $y^{on}_{g,t}$ Indicator if generator g is on but not in startup at time t
- $y^{off}_{g,t}$ Indicator if generator g is off or in shutdown at time t
- $y^{startup}_{g,t}$ Indicator if generator g is starting up at time t

A two-stage stochastic generalized disjunctive programming formulation for optimal resilience investments

Model details

Goal: Determine the optimal investments to improve power system resilience (loss of weighted load and duration).

Inputs to model: Scenario data from threats listing component outages and recovery time. Investment cost data.

Model type: A two-stage stochastic generalized disjunctive program.

Model efficiency (scalability): Faster deterministic model solve time than MIP version resulting in speedups for stochastic model.

A co-optimization stochastic mixed integer model to improve reliability and resiliency

$$minimize[Resiliency\ metric + Reliability\ metric]$$

subject to:

$$\sum_{b\in B} C_b i_b + \sum_{l\in L} C_l i_l + \sum_{g\in G} C_g i_g + \sum_{i,d,u\in U_{i,d}} c_u y_{i,d,u} \le K$$

$$Resiliency\ metric = \frac{1}{B_{LSWD}} LSWD + \frac{1}{B_{LS}} LS \quad (1)$$

$$\text{LSWD} = \sum_{t\in T} t \sum_{b\in B} A_b \sum_{w\in \Omega} P_w p_{b,t}^w \quad (2)$$

$$LS = \sum_{b\in B} A_b \sum_{b\in B} P_w p_{b,0}^w \quad (3)$$

$$\sum_{g\in G_b} p_{g,t}^w + \sum_{l\in L_b^{to}} p_{l,t}^w - \sum_{l\in L_b^{from}} p_{l,t}^w = D_b - p_{b,t}^w \quad \forall b\in B, \forall t\in T, \forall w\in\Omega \quad (5)$$

$$p_{l,t}^w = y_{l,t}^w S_l \left(\theta_{B_l^{to},t}^w - \theta_{B_l^{from},t}^w\right) \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (6)$$

$$p_{g,t}^w \le p_{g,t-1}^w + RU_g y_{g,t}^w + SU_g\left(y_{g,t}^w - y_{g,t}^w\right) + \overline{P}_g\left(1 - y_{g,t}^w\right) \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (7)$$

$$p_{g,t-1}^w \le \overline{P}_g y_{g,t}^w + SD_g\left(y_{g,t-1}^w - y_{g,t}^w\right) \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (8)$$

$$p_{g,t-1}^w - p_{g,t}^w \le RD_g y_{g,t}^w + SD_g\left(y_{g,t-1}^w - y_{g,t}^w\right) + \overline{P}_g\left(1 - y_{g,t}^w\right) \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (9)$$

$$-\frac{\pi}{3} \le \theta_{B_l^{to},t}^w - \theta_{B_l^{from},t}^w \le \frac{\pi}{3} \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (10)$$

$$-\overline{P}_l y_l^w \le p_{l,t}^w \le \overline{P}_l y_l^w \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (11)$$

$$\underline{P}_g y_g^w \le p_{g,t}^w \le \overline{P}_g y_g^w \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (12)$$

$$0 \le p_{b,t}^w \le D_b \quad \forall b\in B, \forall t\in T, \forall w\in\Omega \quad (13)$$

$$y_{l,t}^w \le i_l + \frac{t}{X_l^w} \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (14)$$

$$y_{l,t}^w \le i_{B_l^{from}} + \frac{t}{X_{B_l^{from}}^w} \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (15)$$

$$y_{l,t}^w \le i_{B_l^{to}} + \frac{t}{X_{B_l^{to}}^w} \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (16)$$

$$y_{g,t}^w \le i_g + \frac{t}{X_g^w} \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (17)$$

$$y_{g,t}^w \le i_{B_g} + \frac{t}{X_{B_g}^w} \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (18)$$

$$y_{b,t}^w \le i_b + \frac{t}{X_b^w} \quad \forall b\in B, \forall t\in T, \forall w\in\Omega \quad (19)$$

$$y_{l,t}^w \le y_{B_l^{from},t}^w \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (20)$$

$$y_{l,t}^w \le y_{B_l^{to},t}^w \quad \forall l\in L, \forall t\in T, \forall w\in\Omega \quad (21)$$

$$y_{g,t}^w \le y_{B_g,t}^w \quad \forall g\in G, \forall t\in T, \forall w\in\Omega \quad (22)$$

$$Relibaility\ metric = \frac{1}{B_{SAIDI}} \text{SAIDI}_{up} + \frac{1}{B_{SAIFI}} \text{SAIFI}_{up} \quad (23)$$

$$SAIDI_{up} = \frac{1}{N} \sum_{o\in O} CO_o TO_o \quad (24)$$

$$SAIFI_{up} = \frac{1}{N} \sum_{o\in O} CO_o \quad (25)$$

$$CO_o = \min_{u\in U_o}\left\{C_{o,u} y_{i_o,d_o,u} + C_o\left(1 - y_{i_o,d_o,u}\right)\right\} \quad \forall o\in O \quad (27)$$

$$TO_o = \min_{u\in U_o}\left\{T_{o,u} y_{i_o,d_o,u} + T_o\left(1 - y_{i_o,d_o,u}\right)\right\} \quad \forall o\in O \quad (28)$$

Sets

L	Transmission lines
G	Generators
B	Buses
Ω	Outage scenarios
Ω_l	Set of scenarios under which transmission line l goes offline
Ω_g	Set of scenarios under which generator g goes offline
Ω_b	Set of scearios under which bus b goes offline
T	Discrete set of times: duration each component is out of service
G_b	Set of generators connected to bus b
L_b^{from}	Set of transmission lines leaving bus b
L_b^{to}	Set of tranmssion lines entering bus b
I	Set of investments for buses, generators, and transmission lines
D	Device types
J	Feeder IDs
U	Upgrade options
$U_{i,d}$	Upgrade options for device type d in feeder i
O	Outages
U_o	Upgrade options that improve the number of customers outaged in outage o if applied
V_o	Upgrade options that improve the duration of outage in outage o if applied
S	Outage causes

Parameters

K	Budget
B_l^{from}	Bus from which transmission line l leaves
B_l^{to}	Bus transmission line l enters
S_l	Susceptance of tranmission line l
$\overline{P}_l$	Thermal limit of transmission line l
B_g	Bus containing generator g
RU_g	Ramp-up limit of generator g dispatch level
RD_g	Ramp-down limit of generator g dispatch level
SU_g	Start-up limit of generator g distpatch level
SD_g	Shut-down limit of generator g distpatch level
$\overline{P}_g$	Upper limit of generator g dispatch level
$\underline{P}_g$	Lower limit of generator g dispatch level
D_b	Demand at bus b
A_b	Load weighting factor at bus b
C_l	Cost of hardening transmission line l
C_g	Cost of hardening generator g
C_b	Cost of hardening bus b
P_w	Probability of scenario w occuring
X_l^w	Number of time periods line l is affected by event in scenario w with no hardening
X_g^w	Number of time periods generator g is affected by event in scenario w with no hardening
X_b^w	Number of time periods bus b is affected by event in scenario w with no hardening
P_w	Probability of scenario w occuring
B_{LSWD}	First term in objective during baseline model run with 0 budget
B_{LS}	Second term in objective during baseline model run with 0 budget
C_o	Number of customers outage o affects
T_o	Duration of outage o
d_o	Device type of outage o
i_o	Device ID of outage o (also gives feeder ID/location)
s_o	Cause of outage o
c_u	Cost to purchase upgrade u
$C_{o,u}$	Number of customers outage o affects after upgrade u
$T_{o,u}$	Duration of outage o after upgrade u
B_{SAIDI}	Baseline SAIDI value
B_{SAIFI}	Baseline SAIFI value
N	Number of customers in total system

Variables

$LSWD$	Load Shed With Duration in MW – similar to the SAIDI reliability metric
LS	Load Shed – similar to the SAIFI reliability metric
$p_{l,t}^w$	Power flow through transmission line l at time t in scenario w
$p_{g,t}^w$	Generator dispatch level for generator g at time t in scenario w
$p_{b,t}^w$	Load shed at bus b at time t in scenario w
$\theta_{b,t}^w$	Phase angle for bus b at time t in scenation w
$y_{l,t}^w$	On/off status of line l at time t during scenario w
$y_{g,t}^w$	On/off status of generator g at time t during scenario w
$y_{b,t}^w$	On/off status of bus b at time t during scenario w
i_l	Binary indicating whether or not transmission line l is hardened
i_g	Binary indicating whether or not generator g is hardened
i_b	Binary indicating whether or not bus b is hardened
$y_{i,d,u}$	Binary indicating whether or not to apply upgrade $u \in U_{i,d}$ to device type d in feeder i
$SAIDI_{up}$	SAIDI value after upgrades
$SAIFI_{up}$	SAIFI value after upgrades
CO_o	Number of customers which outage o affects after upgrade
TO_o	Duration of outage o after upgrade

Model details

Goal: Determine the optimal investments to improve power system reliability and resilience. See the trade offs between the two.

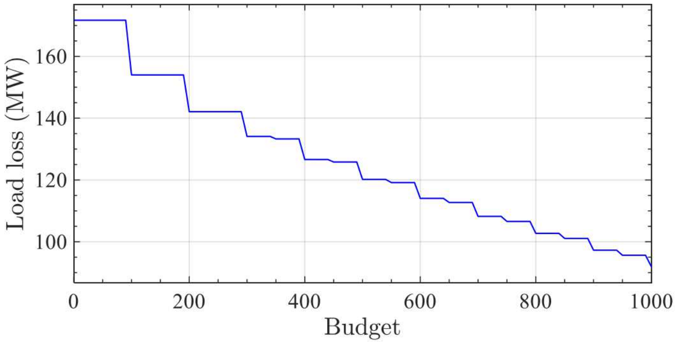
Inputs to model: Scenario data based on historical large scale events that include outaged components and time off and time recovered. In addition, utility historical outage data, investment impact data, and investment cost data.

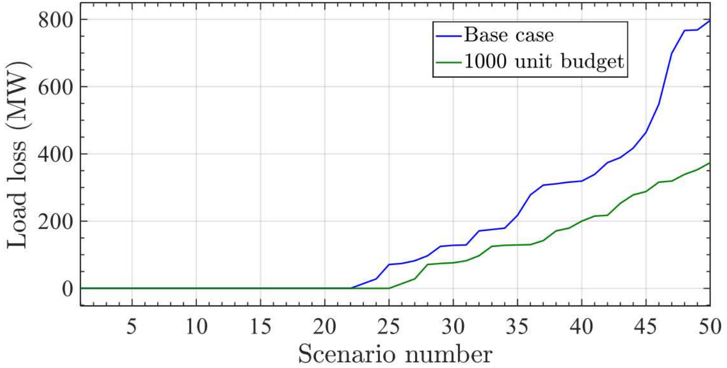
Model type: Nonlinear mixed integer program, linearized through new and old techniques

Model efficiency (scalability): Poor efficiency, especially for larger systems, and a large number of scenarios

IEEE

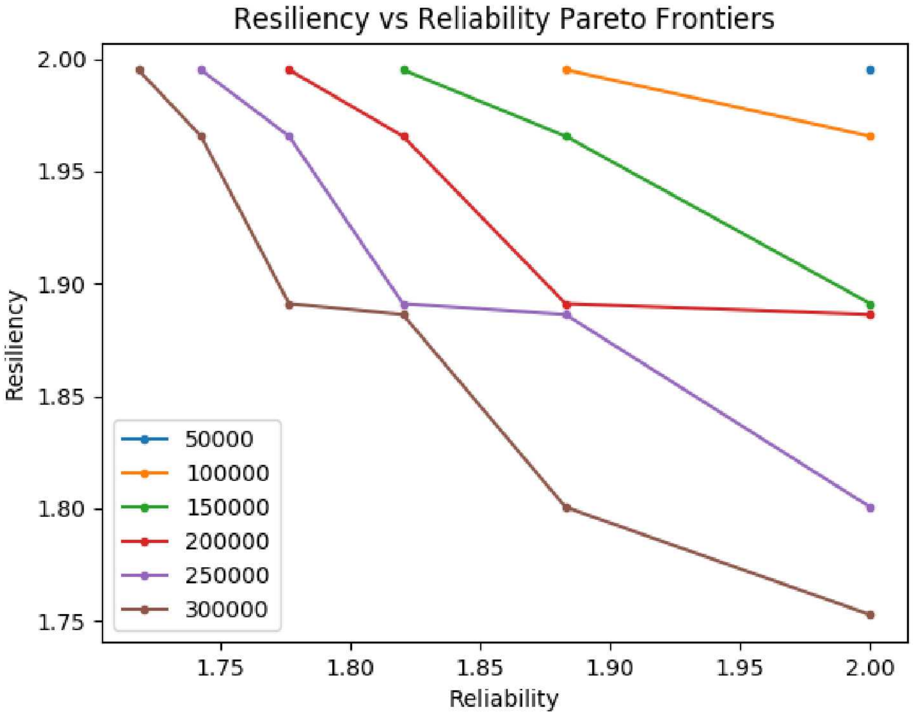
Resilience results on IEEE RTS-96 system





The expected loss of load from 50 winter storm scenarios vs. the investment budget

Co-op results on IEEE RTS-96 system



Formulation with SOCP Network Constraints

Sets

- $\mathcal{L}$ Set of transmission lines l
- $\mathcal{L}_b^{to}$ Set of lines to bus b
- $\mathcal{L}_b^{fr}$ Set of lines from bus b
- $\mathcal{L}^{t,s}$ Set of lines out in the scenario at time t
- $\mathcal{B}$ Set of buses b
- $\mathcal{B}^{t,s}$ Set of buses b out in the scenario at time t
- $\mathcal{G}$ Set of generators g
- $\mathcal{G}_b$ Set of generators g at bus b
- $\mathcal{G}^{t,s}$ Set of generators out in the scenario at time t
- $\mathcal{R}$ Set of renewable energy sources (RES) r
- $\mathcal{R}_b$ Set of RES r at bus b
- $\mathcal{R}^{t,s}$ Set of RES out in the scenario at time t

Parameters

- $p_{D,b}^t$ real power load at bus b at time t
- $q_{D,b}^t$ reactive power load at bus b at time t
- fr(l) orgin bus of line l
- to(l) destination bus of line l
- B_l Susceptance of line l
- G_l Conductance of line l
- $\bar{S}_l$ Short term thermal limit of line l
- RU_g Ramp up limit of generator g
- RD_g Ramp down limit of generator g
- $\underline{P}_g$ Minimum real power output of generator g
- $\overline{P}_g$ Maximum real power output of generator g
- $\underline{Q}_g$ Minimum reactive power output of generator g
- $\overline{Q}_g$ Maximum reactive power output of generator g
- $\underline{V}_b$ Minimum voltage magnitude at bus b (p.u.)
- $\overline{V}_b$ Maximum voltage magnitude at bus b (p.u.)
- C_l Cost of hardening line l
- C_b Cost of hardening bus b
- C_g Cost of hardening generator g
- K Resiliency budget
- β_b Weight for importance of load at bus b

Variables

Common to both models

- $p_{S,b}^t$ real power load shed at bus b at time t
- p_g^t real power output generator g at time t
- x_l^t binary indicating status of line l at time t
- z_l binary indicating hardening decision for line l
- z_b binary indicating hardening decision for bus b
- z_g binary indicating hardening decision for generator g
- u_g^t binary indicating status of generator g at time t
- v_g^t binary indicating startup status of generator g at time t
- w_g^t binary indicating shutdown status of generator g at time t

SOCP Relaxation

- $q_{S,b}^t$ reactive power load shed at bus b at time t
- $p_{l,fr}^t$ real power flow though line l at time t from bus fr(l)
- $q_{l,fr}^t$ reactive power flow though line l at time t from bus fr(l)
- $p_{l,to}^t$ real power flow though line l at time t to bus to(l)
- $q_{l,to}^t$ reactive power flow though line l at time t to bus to(l)
- q_g^t reactive power output at generator g at time t

DC OPF

- θ_b^t voltage angle at bus b at time t
- p_l^t power flow through line l at time t

$$(\mathcal{P}_{ac}) \qquad \min \sum_{t\in\mathcal{T}}\sum_{b\in\mathcal{B}} \beta_b p_{S,b}^t \tag{1}$$

Minimize weighted load

subject to:

$$\sum_{b\in\mathcal{B}} C_b z_b + \sum_{g\in\mathcal{G}} C_g z_g + \sum_{l\in\mathcal{L}} C_l z_l \le K \tag{2}$$

Investment Budget

$$p_g^t - p_g^{t-1} \le (\underline{P}_g + RU_g)u_g^t - \underline{P}_g u_g^{t-1} \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T} \tag{3}$$

$$p_g^{t-1} - p_g^t \le (\underline{P}_g + RD_g)u_g^{t-1} - \underline{P}_g u_g^t \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T} \tag{4}$$

$$\underline{P}_g u_g^t \le p_g^t \le \overline{P}_g u_g^t \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T} \tag{5}$$

$$\underline{Q}_g u_g^t \le q_g^t \le \overline{Q}_g u_g^t \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T} \tag{6}$$

$$\underline{V}^2 \le c_b^t \le \overline{V}^2 \quad \forall b\in\mathcal{B}, \forall t\in\mathcal{T} \tag{7}$$

Generator real and reactive power ramping and operation limits

$$p_{l,fr}^t = x_l^t[c_{fr(l)}^t \tfrac{G_l}{\tau^2} + (\tfrac{-G_l}{\tau}c_{ft}^t - \tfrac{B_l}{\tau}s_{ft}^t)] \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{8}$$

$$q_{l,fr}^t = x_l^t[-\tfrac{1}{\tau^2}c_{fr(l)}^t\left(B_l + \tfrac{B_l^{ch}}{2}\right) + (\tfrac{B_l}{\tau}c_{ft}^t - \tfrac{G_l}{\tau}s_{ft}^t)] \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{9}$$

$$p_{l,to}^t = x_l^t[c_{to(l)}^t G_l + (-\tfrac{G_l}{\tau}c_{tf}^t - \tfrac{B_l}{\tau}s_{tf}^t)] \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{10}$$

$$q_{l,to}^t = x_l^t[-c_{to(l)}^t\left(B_l + \tfrac{B_l^{ch}}{2}\right) + (\tfrac{B_l}{\tau}c_{tf}^t - \tfrac{G_l}{\tau}s_{tf}^t)] \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{11}$$

$$(p_{l,to}^t)^2 + (q_{l,to}^t)^2 \le \bar{S}_l^2 \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{12}$$

$$(p_{l,fr}^t)^2 + (q_{l,fr}^t)^2 \le \bar{S}_l^2 \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{13}$$

Line power flows with optimal transmission switching

$$(c_{ft}^t)^2 + (s_{ft}^t)^2 \le x_l^t[c_{fr(l)}^t c_{to(l)}^t] \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T} \tag{14}$$

SOC Constraint

$$\underline{R}_r \le p_{c,r}^t \le \overline{R}_r \quad \forall r\in\mathcal{R}, \forall t\in\mathcal{T} \tag{15}$$

Renewable Curtailment

$$\sum_{g\in\mathcal{G}_b} p_g^t - G_b^{fs}c_b^t - \sum_{l\in\mathcal{L}_b^{to}} p_{l,to}^t - \sum_{l\in\mathcal{L}_b^{fr}} p_{l,fr}^t + \sum_{r\in\mathcal{R}_b}(p_r^t - p_{c,r}^t) = p_{D,b}^t - p_{S,b}^t, \quad \forall b\in\mathcal{B}, \forall t\in\mathcal{T} \tag{16}$$

$$\sum_{g\in\mathcal{G}_b} q_g^t + B_b^{fs}c_b^t - \sum_{l\in\mathcal{L}_b^{to}} q_{l,to}^t - \sum_{l\in\mathcal{L}_b^{fr}} q_{l,fr}^t = q_{D,b}^t - q_{S,b}^t, \quad \forall b\in\mathcal{B}, \forall t\in\mathcal{T} \tag{17}$$

Power Balance

$$0 \le p_{S,b}^t \le p_{D,b}^t \quad \forall b\in\mathcal{B}, \forall t\in\mathcal{T} \tag{18}$$

$$0 \le q_{S,b}^t \le q_{D,b}^t \quad \forall b\in\mathcal{B}, \forall t\in\mathcal{T} \tag{19}$$

$$q_{S,b}^t = \frac{q_{D,b}^t}{p_{D,b}^t}p_{S,b}^t \quad \forall b\in\mathcal{B}, \forall t\in\mathcal{T} \tag{20}$$

Real and reactive load shed

$$x_l^t \le z_l \quad \forall l\in\mathcal{L}^{t,s}, \forall t\in\mathcal{T} \tag{21}$$

$$u_g^t \le z_g \quad \forall g\in\mathcal{G}^{t,s}, \forall t\in\mathcal{T} \tag{22}$$

$$u_g^t \le z_b \quad \forall b\in\mathcal{B}^{t,s}, \forall g\in\mathcal{G}_b, \forall t\in\mathcal{T} \tag{23}$$

$$x_l^t \le z_b \quad \forall b\in\mathcal{B}^{t,s}, \forall l\in\mathcal{L}_b, \forall t\in\mathcal{T} \tag{24}$$

Investment Model

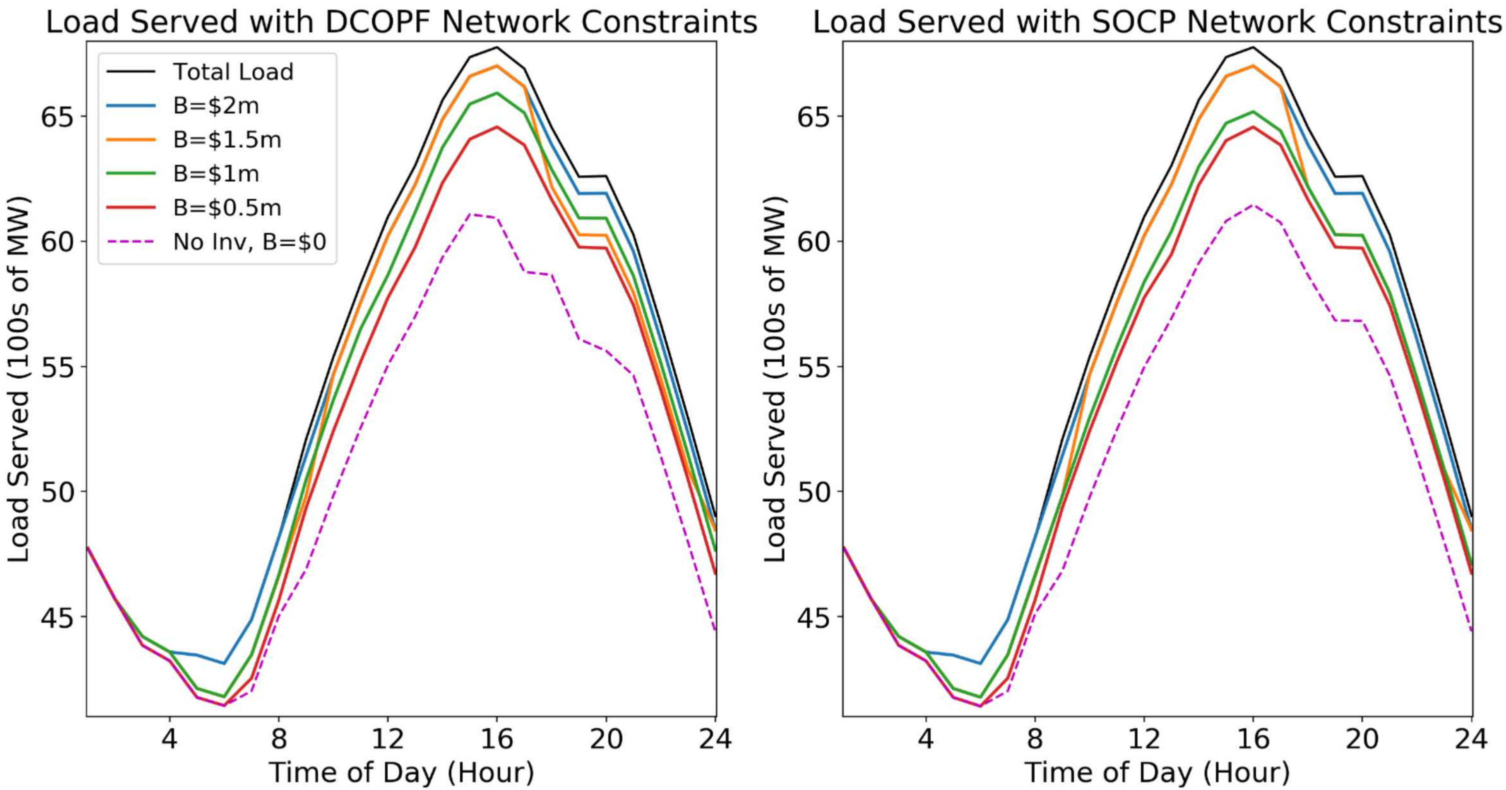
Model Details:

Goal: Determine the optimal investments (generators, lines, or buses) to improve power system resilience (minimize weighted load shed) *for various budget amounts*

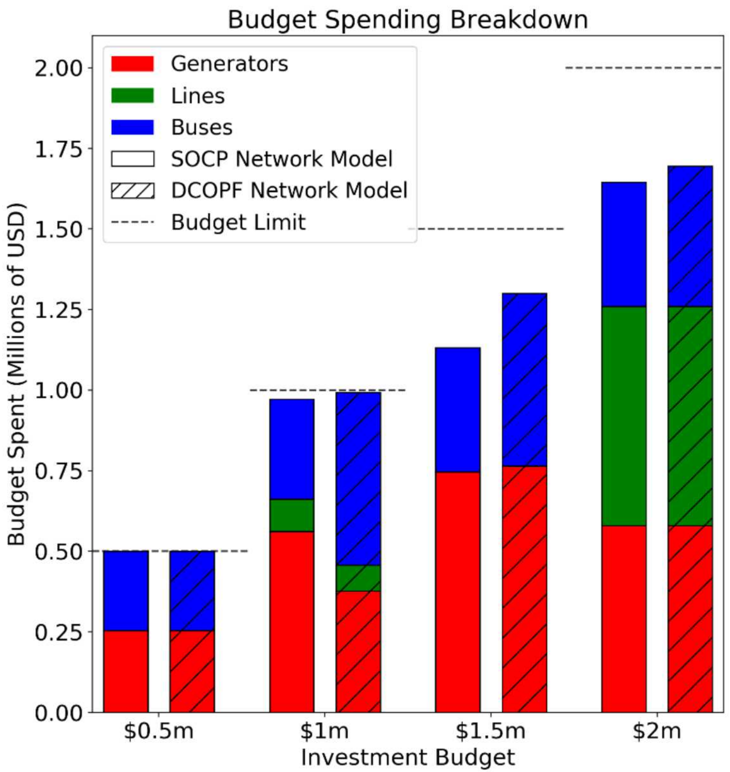
Inputs to model: Hurricane scenario, investment costs, network information

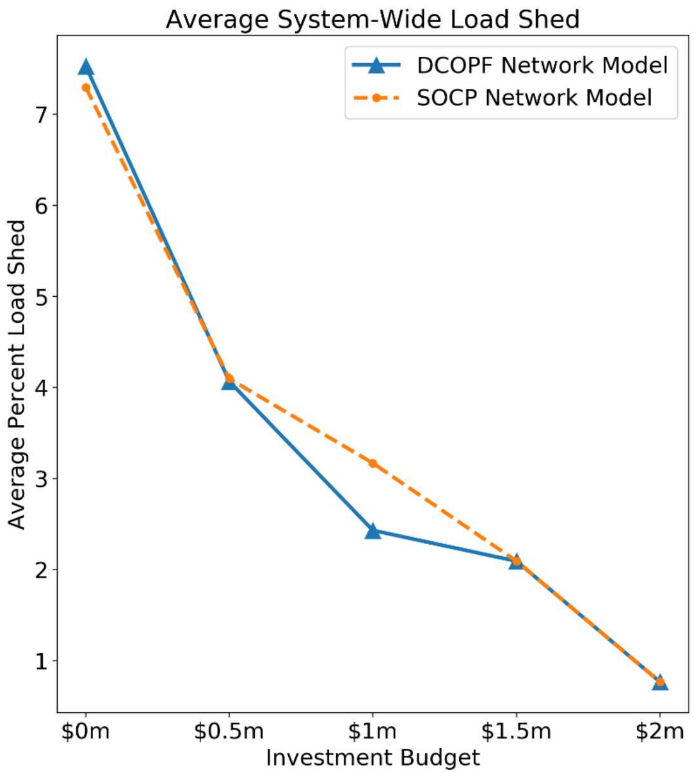
Model type: MISOCP

Results: Load Served Comparison with DC OPF Network Constraints



Results: Comparison of Network Models
Spending Breakdown & Percent of Total System Load Shed





Questions?

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