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Computing the Maximum Expected Environment of a Small Data Set

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1 Abstract

There is often very little data available from which to compute the maximum expected environment (MEE) of a vibration or shock event. With very little data, the true distribution of the data is unknown and there is no data at the tails of the distribution where we wish to make statistical inferences. The MEE depends on assumptions that are made about the true distribution. This paper describes a statistical simulation that computes the MEE using the Normal Tolerance Limit, Karhunen-Loeve Expansion, and the Space and Missile Systems Center (SMC) Standard. The MEE is computed from a small data set randomly drawn from a much larger data set. The MEE is compared with the true MEE, and findings are discussed given the size of the small data set and the MEE computation method.

2 Introduction

The Maximum Expected Environment (MEE) is the one-sided, upper tail extreme of the mechanical environment. The MEE vibration or shock specification is used to test systems and components to be sure they can survive or function through the environment of interest. Typically, the MEE is defined as:

1. P95/50: The limit that will exceed the response spectral values for at least 95% of the data points with 50% confidence. This is often used for acceptance testing.
2. P99/90: The limit that will exceed the response spectral values for at least 99% of the data points with 90% confidence. This is often used for qualification testing.

Figure 1 shows the probability density of a random variable, X, that is normally distributed with mean of 0 and standard deviation of 1. The probability that X is less than 1.65 is 0.95. In other words, P95 is 1.65. Similarly, P99 is 2.33. There is no need to state a confidence value of the P95 and P99 of the standard normal distribution because the confidence is 100%.

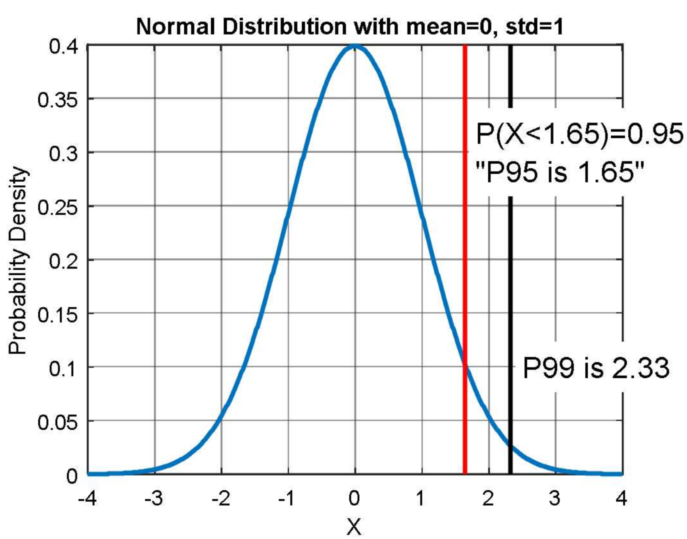

Figure 1: Standard Normal Distribution

The MEE is at the tail of the distribution of the shock or vibration event. With a small data set, it may not be possible to determine how the data is distributed, and it is even more problematic to make inferences about the tails

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of the distribution at P95 or P99. Figure 2 shows kernel density estimates of random draws from the standard normal distribution. The kernel density estimate is a way to estimate the probability density, in this case, of random draws from the standard normal distribution. In the top subplot of Figure 2, the probability density is estimated from 5 random draws. This is repeated 30 times and each probability density estimate is plotted to get an idea of the variation in the 30 estimates. In the second subplot, the probability density is estimated from 10 random draws. In the third subplot, the probability density is estimated from 15 random draws. In the bottom subplot, the probability density is estimated from 100 random draws. The standard normal distribution P95 and P99 are also on each subplot. With very few samples from which to estimate the probability density, the estimate often greatly differs from the standard normal distribution.

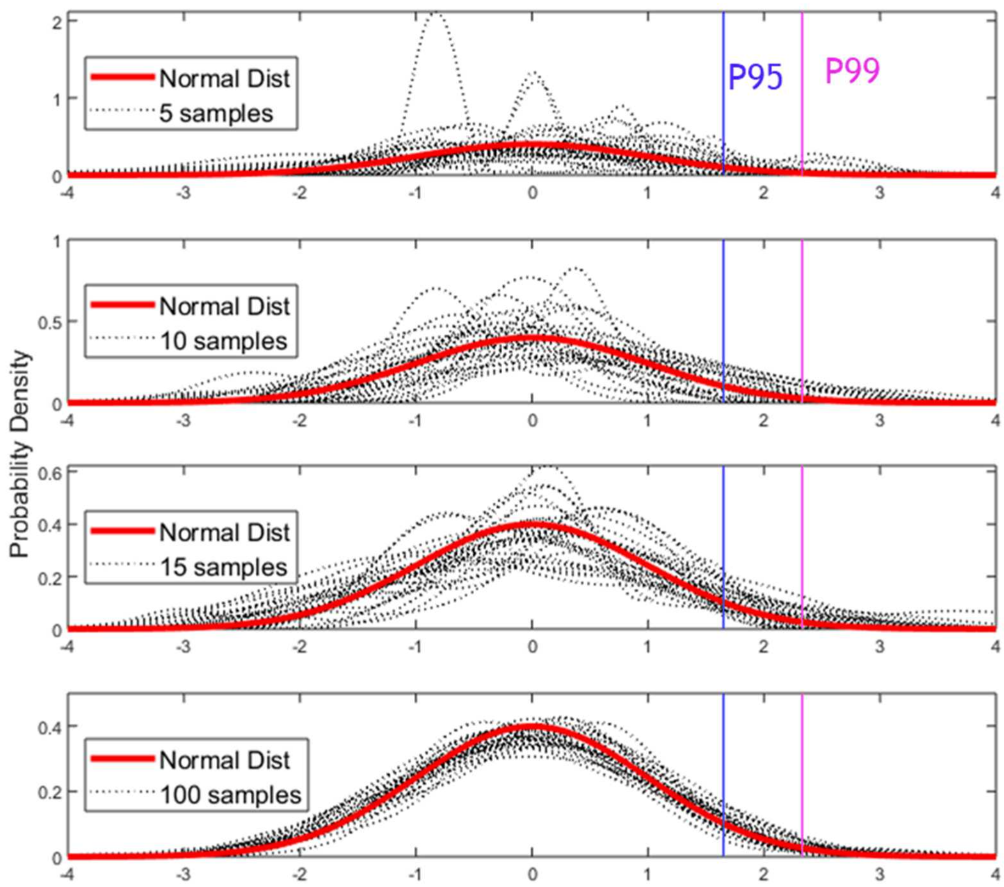


Figure 2: 30 Kernel Density Estimates of Random Draws from Standard Normal Distribution

In practice, we sometimes must compute the MEE with only a few data points. In this study, we will compute the MEE with a small number of randomly selected data points from a large data set and compare that to the MEE computed from the full data set. We'll do that many times to get an understanding of the variation of the MEE.

The two data sets that will be used to evaluate variations in small sample MEE are the standard normal distribution and 255 truck transportation shocks identified from field testing. It is convenient to use the standard normal distribution because of the ease of drawing random samples from the distribution from which the small sample P95/50 and P99/90 can be computed and compared to the true P95 and P99. Although one of the MEE methods (SMC Standard) is written specifically for flight data, it is problematic to obtain a large enough number of flights to fully define the distribution of the data. It is much easier to obtain a large set of data from over-the-road truck transportation data.

Small data sets can yield under or over conservative MEE. An overly conservative specification may cause failures during lab testing that lead to higher program cost, delayed schedule, or compromises in the mission. An under conservative specification may result in products passing lab qualification testing that fail during the mission.

3 MEE Methods

There are many methods which can be used to compute the MEE including a simple envelope, distribution free tolerance limit, empirical tolerance limit, and normal prediction limit. Three popular methods were selected to estimate the small sample P95/50 and P99/90:

1. Normal Tolerance Limit (NTL)
2. SMC-S-016 Standard (SMC)
3. Karhunen-Loeve (KL)

The NTL and SMC methods are industry-standard methods while the KL method is not an industry standard method. The NTL and SMC methods both assume a lognormal distribution. This has shown to be generally true for vibration and shock spectra. The KL method makes no assumptions on how the data is distributed.

3.1 Normal Tolerance Limit (NTL)

The NTL method is in *NASA Handbook 7005* [1] and MIL-STD 810H [2]. The data, x, is assumed to be lognormally distributed. The log of the data is then normally distributed.

$$y = log_{10}(x) \tag{1}$$

The normal tolerance limit for y, defined in equation 2, is the value of y that will exceed β portion of all values of y, with γ confidence. For example, β =0.95, γ =0.5 is P95/50.

$$NTL_y(n, \beta, \gamma) = \bar{y} + k_{n,\beta,\gamma} s_y \tag{2}$$

The sample mean, $\bar{y}$, and sample standard deviation, s_y, are defined in equations 3 and 4.

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{3}$$

$$s_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})} \tag{4}$$

The normal tolerance factor, $k_{n,\beta,\gamma}$ is a tabulated value that can be found in *NASA Handbook 7005* [1], MIL-STD 810H [2], and other sources.

3.2 SMC-S-016 Standard (SMC)

The SMC Standard is the Air Force space and missile systems center standard formerly called MIL-STD 1540 [3]. It assumes the data is lognormally distributed. The test level, $L_{P/C}$, in dB above the mean spectrum is given by equation 5.

$$L_{P/C} = \sigma\left[z_P + \frac{z_C}{\sqrt{N}}\right] dB \tag{5}$$

The number of flights is N and the standard deviation of spectra from the mean is σ and is baselined to be 3 dB. z_p and z_c are cumulative distribution values from the standard normal distribution. The assumption of log-normal distribution with 3 dB standard deviation is based on repeated measurements on 24 static firings and over 40 flights of a launch vehicle [5]. The P95/50 and P99/90 test level are given in equations 6 and 7.

$$L_{95/50} = 3\left[1.64 + \frac{0}{\sqrt{N}}\right] dB = 4.9 \text{ dB for any N} \tag{6}$$

$$L_{99/90} = 3\left[2.33 + \frac{1.28}{\sqrt{N}}\right] dB \tag{7}$$

The P95/50 is 4.9 dB above the mean spectrum independent of N. The margin between P99/90 and P95/50 not allowed to be less than 3 dB. This occurs with more than 17 flights of data. When there is enough data, the distribution and standard deviation may be based on the data.

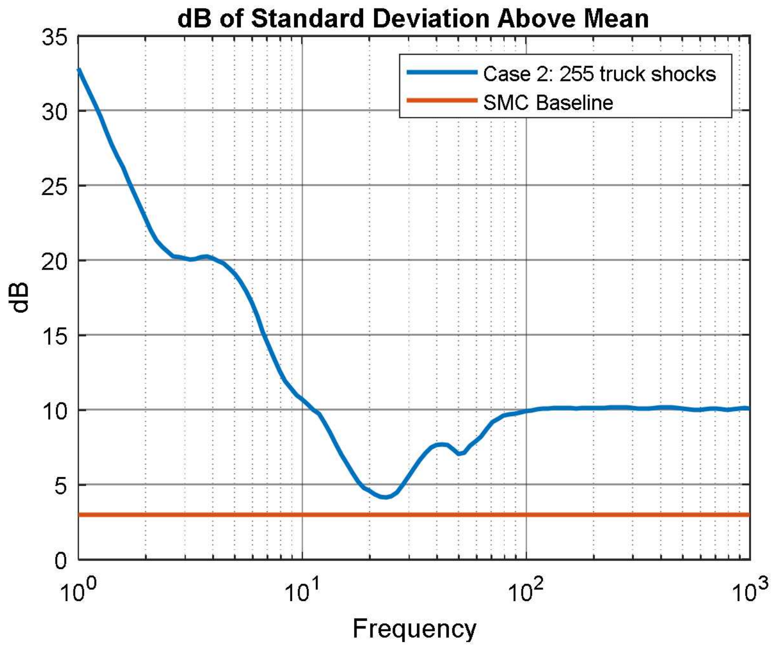


Figure 3: dB of Standard Deviation Above the Mean

Figure 3 shows the dB of the standard deviation above the mean computed from 255 truck shocks identified from field testing. The SMC standard small sample P95/50 and P99/90 estimate will use the baseline standard deviation of spectra above the mean of 3. Because the true standard deviation is larger than the baseline, it is expected that this method will underestimate the P95 and P99.

3.3 Karhunen-Loeve (KL)

This method makes no assumption about how the data is distributed. Karhunen-Loeve (KL) expansion can be used to describe the data as the data mean and a set of eigenvectors. This is analogous to a time history represented by a Fourier series. This method uses a special case of the KL expansion to generate realizations of the underlying random process. The method generates a large set of eigenvalues using the data mean and standard deviation of the as measured eigenvectors and a Gaussian model. Eigenvalue analysis is performed on bootstrap data from the original data. The realizations are used to make P95/50 and P99/90 estimates. More information on this method can be found in Ref. 4.

4 Computing P95/50 and P99/90 from Random Draws from Standard Normal Distribution

Eight test cases were performed in which the P99/90 and P95/50 were computed with samples drawn from a standard normal distribution. The eight test cases had sample sizes of N = 3, 4, 5, 6, 8, 10, 14, and 20. Each test case was repeated 500 times to examine the general trend in accuracy of the MEE with respect to the sample size. KL and NTL methods were used to compute the P95/50 and P99/90. The SMC method was not included in Case 1 because it assumes a standard deviation of 3 dB above the mean.

It is expected that 50% of the P95/50 estimates computed should be less than the actual P95, and 50% should be greater than the actual P95 because the confidence of the P95/50 is 50%. It is expected that 90% of the P99/90 estimates computed should be greater than the actual P99, and 10% should be less than the actual P99 because the confidence of the P99/90 is 90%. For the standard normal distribution, the P95 is 1.65 and the P99 is 2.33.

4.1 P95/50 Simulation Results

The results are displayed in box plots in Figure 4 and Figure 5. Each box plot represents the distribution of P95/50s of 500 simulations of each test case. The data median is represented as a circle in the middle of the box plot. The rectangular box in the box plot contains 50% of the data. Additional information about the data representation in box plots can be found in the Appendix.

The results show that both methods have large dispersion when few samples are used to compute the P95/50. The NTL P95/50 distribution median is close to the actual P95; however, the KL median is less than the true P95. The NTL and KL methods have similar results for larger sample sizes, but the KL estimate is biased lower than the NTL estimate. Figure 4 shows the true probability of the P95/50 estimate. This figure shows that more samples can greatly reduce the dispersion of the data.

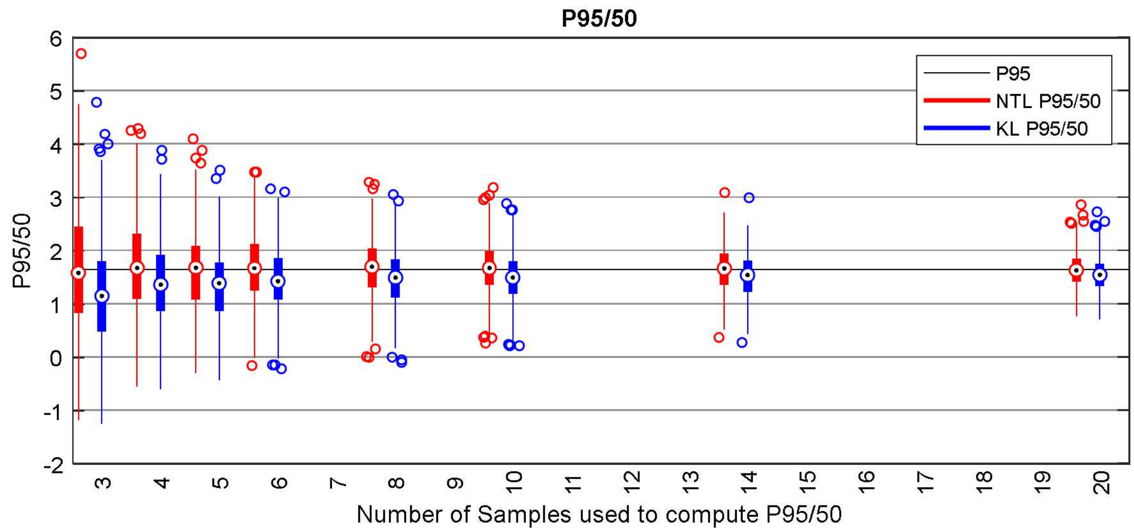


Figure 4: P95/50 Estimate, True P95 = 1.64

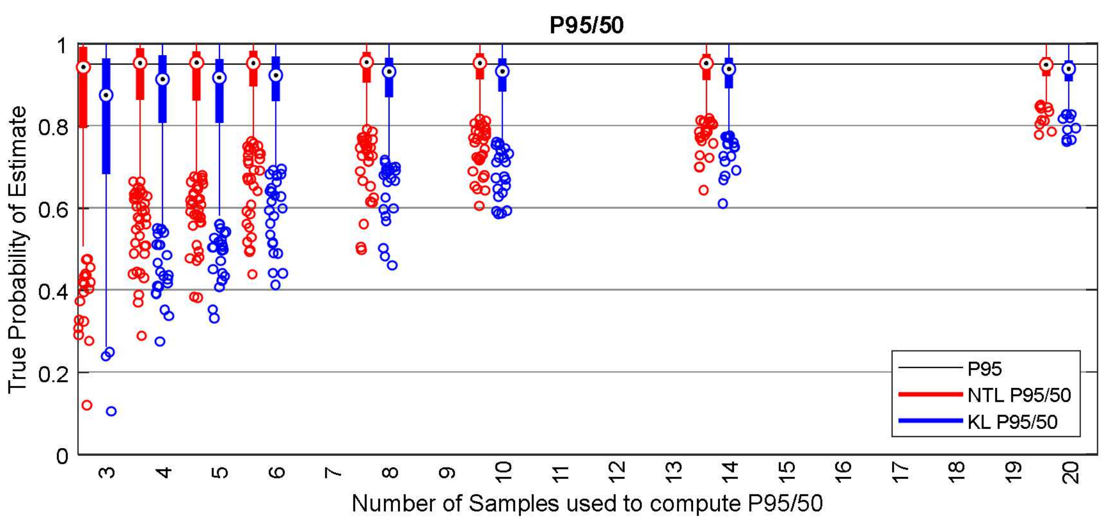


Figure 5: True Probability of P99/90 Estimate

4.2 P99/90 Simulation Results

We would expect that 90% of the data to be greater than the true P99 due to the 90% confidence value. This is approximately true for the NTL method, but the KL median is approximately equal to the true P99. The NTL method has significantly larger dispersion than the KL method for small samples. NTL and KL have similar results for larger sample sizes.

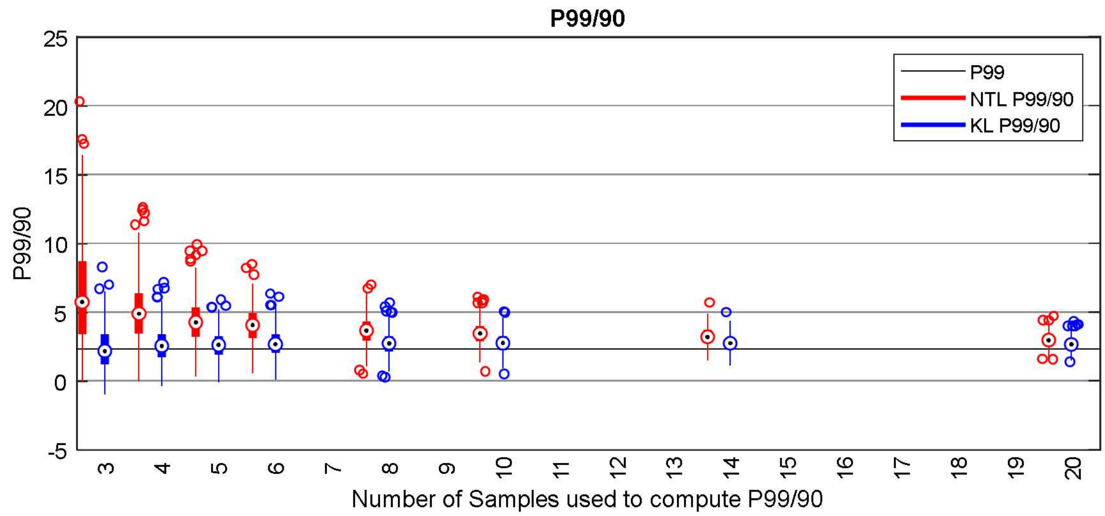


Figure 6:P99/90 Estimate, True P99=2.33

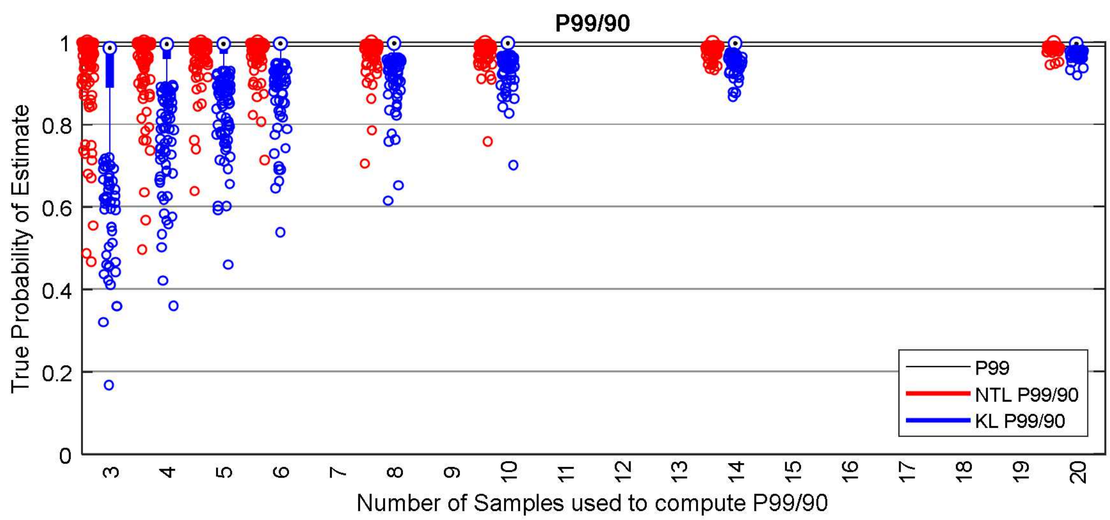


Figure 7: True Probability of P99/90 Estimate

4.3 Correlation Between NTL and KL Methods

This section examines the correlation between the NTL and KL methods. The same randomly drawn data from the standard normal distribution was used to compute the MEE using the KL and NTL methods. This allows us to determine if both methods produce similarly under- or over-conservative MEE with the same input data. The correlation coefficient was computed for each simulation of each test case. There were 500 simulations run for each of the eight test cases with sample size N = 3, 4, 5, 6, 8, 10, 14, and 20.

Over all eight P95/50 test cases, the mean correlation between NTL and KL is 0.99, and the minimum correlation between NTL and KL is 0.97. Over all eight P99/90 test cases, the mean correlation between NTL and KL is 0.97 and the minimum correlation between NTL and KL 0.97. This indicates strong correlation between the NLT and KL methods. If the NLT estimate is overconservative, then the KL estimate is will very likely be overconservative as well. Both methods may be wrong or right together, and agreement between the methods is not a reason to have confidence in the P95/50 or P99/90 estimate.

Figure 8 shows a comparison of the NTL and KL estimates for 50 simulation of one test case, N = 6. The figure shows strong correlation between the NTL and KL methods. It also shows that the KL estimate is lower than the NTL estimate which is what was also seen in Sections 4.1 and 4.2.

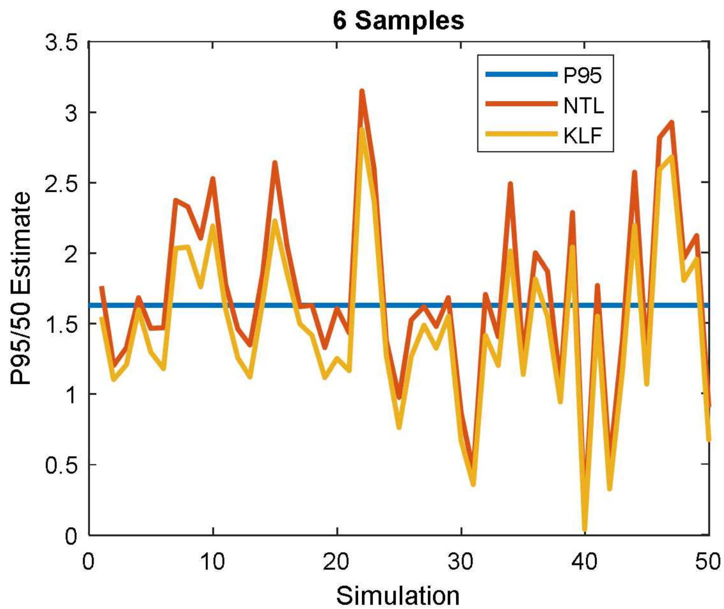


Figure 8:Subset of Simulation Results Demonstrating Strong Correlation

5 Computing P95/50 and P99/90 from Truck Shock Data

Eight test cases were performed in which the P99/90 and P95/50 were computed from a set a Shock Response Spectra (SRS) randomly drawn from 255 truck transportation shocks identified from continuously measured accelerometer data. These are the same test cases that were performed in Section 4. The eight test cases had sample sizes of N = 3, 4, 5, 6, 8, 10, 14, and 20. Each test case was repeated 500 times to examine the general trend in accuracy of the MEE with respect to the sample size. The KL, NTL, and SMC methods were used to compute the P95/50 and P99/90.

The full set of truck shocks is shown in Figure 9 as the gray lines. The full data set NTL P95/50 and P99/90 is taken to be the true P99 and P95 that will be compared the small sample P95/50 and P99/90. The full data set NTL and KL P99/90 and P95/50 agree very closely. Figure 9 shows a small number of randomly selected SRS (green lines in the plot) are used to compute the small sample P95/50 (blue line in the plot). This is repeated 500 times per sample size and comparisons are made with the true P95. A P95/50 and P99/90 is computed at each frequency line of the SRS.

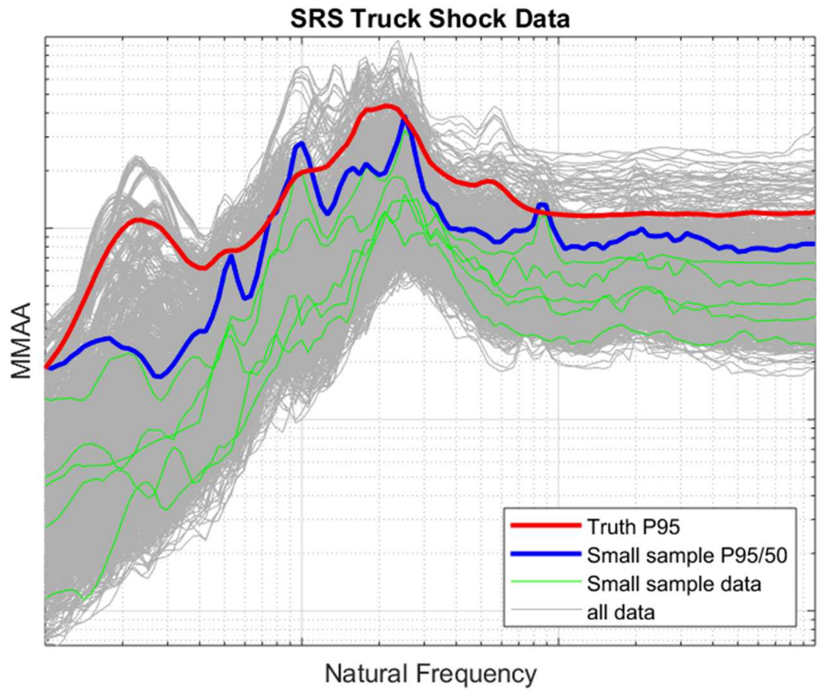


Figure 9: Example of Computing the P95/50 from 6 SRS Curves Randomly Selected from the Full Data Set

5.1 Simulation Results with Five Randomly Selected Samples per Simulation

Figure 10 shows the small sample P95/50 and P99/90 (green lines on the plot) computed with five randomly selected SRS. There are 500 simulations per subplot (500 green lines per subplot). Each subplot also has the true P95 or P99 and the expected value (P50). The figure shows that the SMC method has the least dispersion of the MEE while the NTL has the most dispersion.

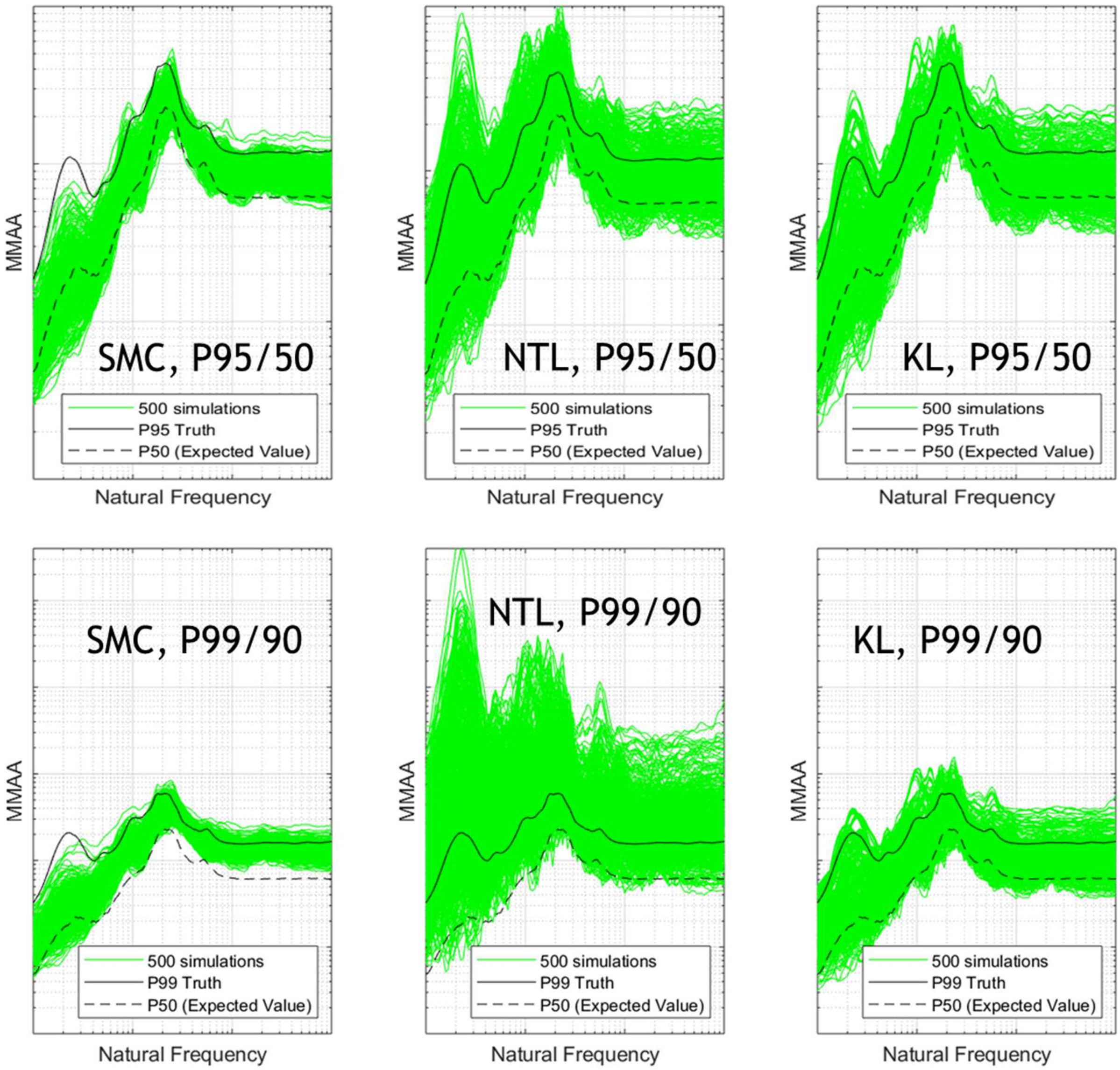


Figure 10: MEE Results for Five Randomly Selected Shocks per Simulation

5.2 Simulation Results with N Randomly Selected Samples per Simulation

Each small sample MEE that is computed will be compared with the true MEE using two error metrics shown in equation 8 and 9. Equation 8 is the average dB error of the small sample MEE. The MEE is computed at each frequency line of the SRS, so this metric quantifies the average error over all the frequency lines. The average dB error indicates the under or overconservativeness of the small sample MEE. Equation 9 is the RMS dB error and is an indicator of the total error of the small sample MEE. A small sample MEE may have 0 average dB error but large RMS dB error. The errors are displayed in box plots in the following two sections. Each box plot represents 500 simulations for that small sample size.

$$Average\ dB\ error = \frac{1}{N}\sum_{i=1}^{N} 20log_{10}\left(\frac{P95_{small\ sample\ at\ freq\ i}}{P95_{full\ data\ set\ at\ freq\ i}}\right) \tag{8}$$

$$RMS\ dB\ error = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(20log_{10}\left(\frac{P95_{small\ sample\ at\ freq\ i}}{P95_{full\ data\ set\ at\ freq\ i}}\right)\right)^2} \tag{9}$$

5.2.1 P95/50 Simulation Results

Figure 11 and Figure 12 show that the dispersion of the error of all the methods decreases with increasing number of samples. With larger numbers of samples, the NTL and KL methods closely agree, and the SMC method is more under conservative than the other two methods.

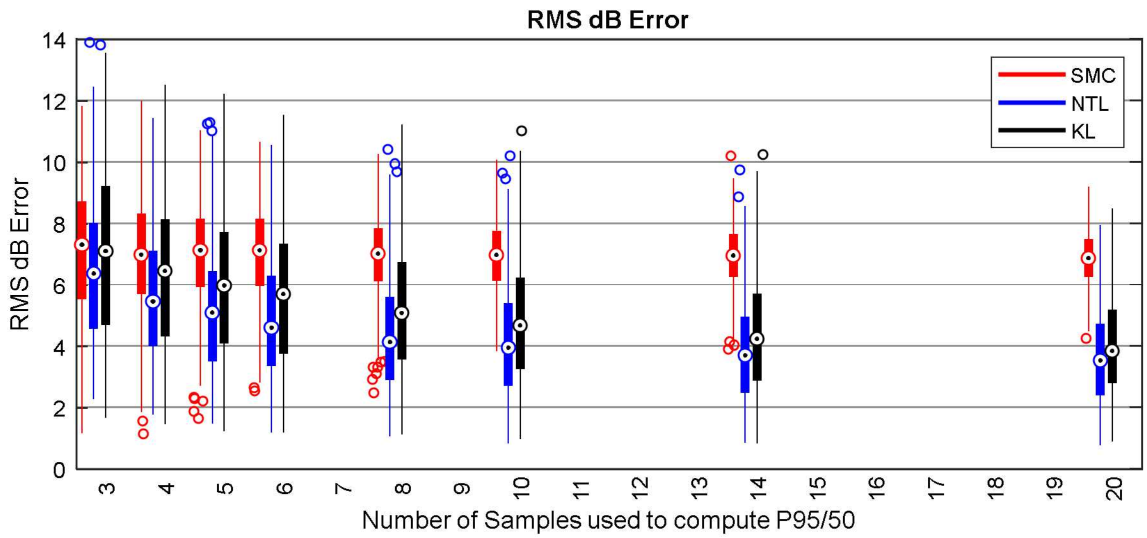


Figure 11: P95/50 RMS dB Error

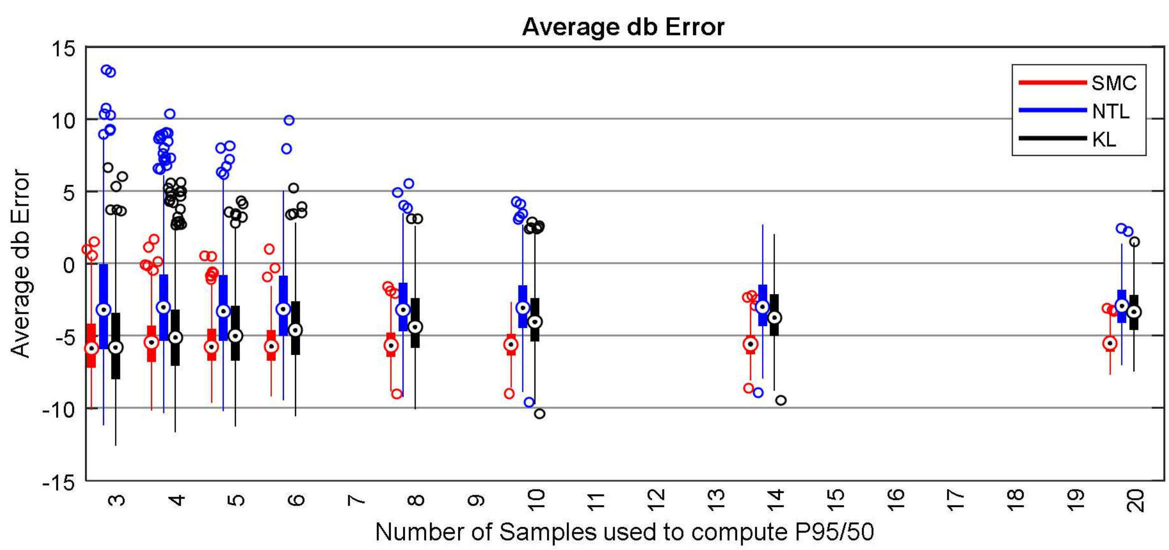


Figure 12: P95/50 Average dB Error

5.2.2 P99/90 Simulation Results

Figure 13 and Figure 14 show that the NTL method error has much greater dispersion in the P99/90 error than the P95/50 error especially for small sample sizes. The SMC and KL P99/90s are biased low, but the NTL method is biased high reflecting the 90% confidence value. The NTL and KL methods seem to be converging toward 0 error, but the SMC method is not converging to 0 error and this is because the SMC method assumes the standard deviation above the mean is 3 dB.

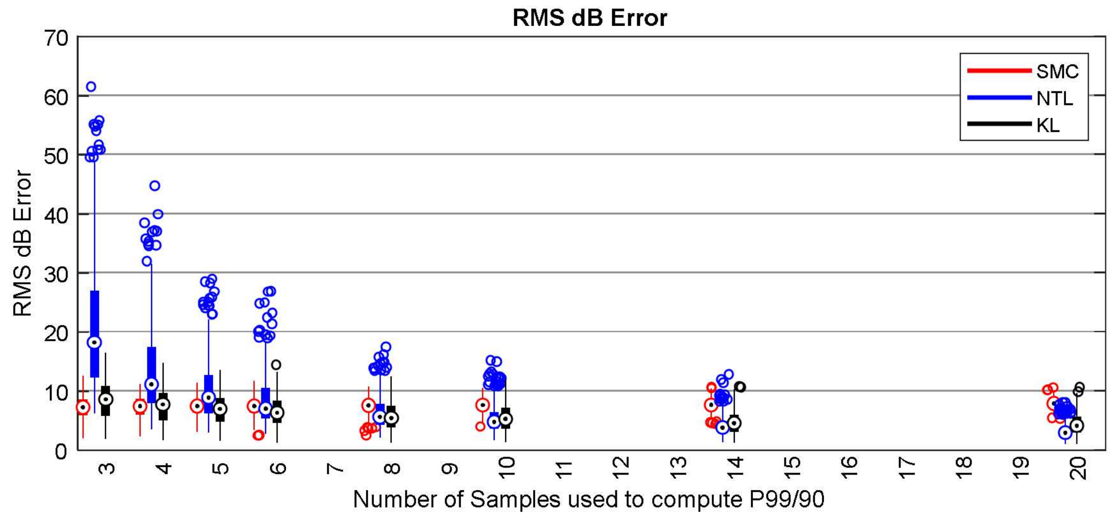


Figure 13: P99/90 RMS dB Error

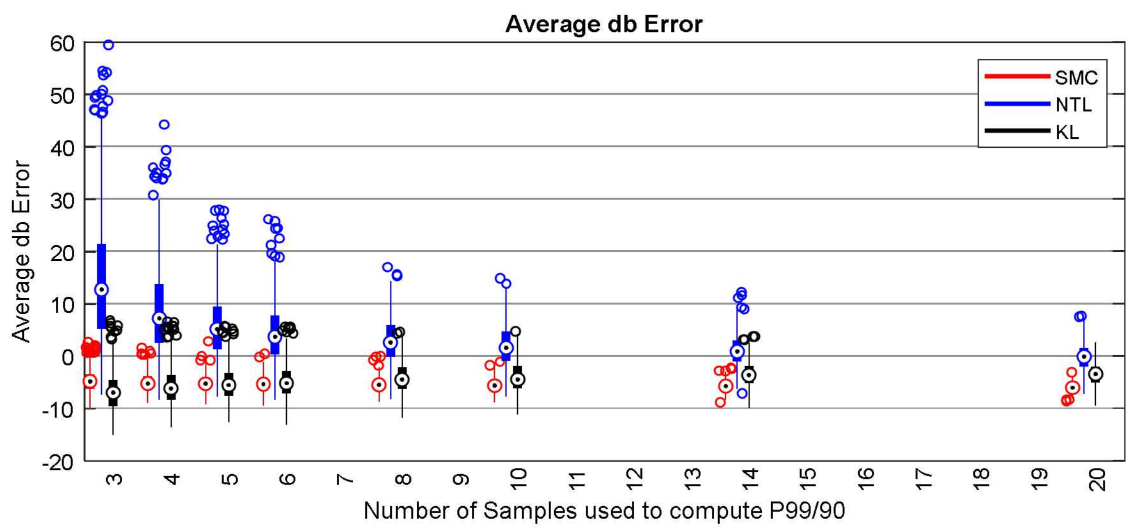


Figure 14: P99/90 Average dB Error

6 Conclusions

Early in the life of a system, with very little field data, it is very difficult to obtain a good estimate of the shock and vibration MEE. Components and systems are typically tested to the MEE to demonstrate they can survive the environments. An overly conservative specification may lead to higher cost, delayed schedule, or compromises in the mission. An under conservative specification may lead to mission failure. With very little data, the MEE estimate may be very misleading, and this must be kept in mind if the MEE is used to evaluate system performance.

7 References

[1] NASA Document. *NASA-HDBK-7005 Dynamic Environment Criteria*. March 2001.

[2] MIL-STD 810G. *Environmental Engineering Considerations and Laboratory Tests*. January 31, 2019.

[3] SMC-TR-06-11 (MIL-STD-1540E). *Test Requirements for Launch, Upper-Stage, and Space Vehicles*. September 6, 2006.

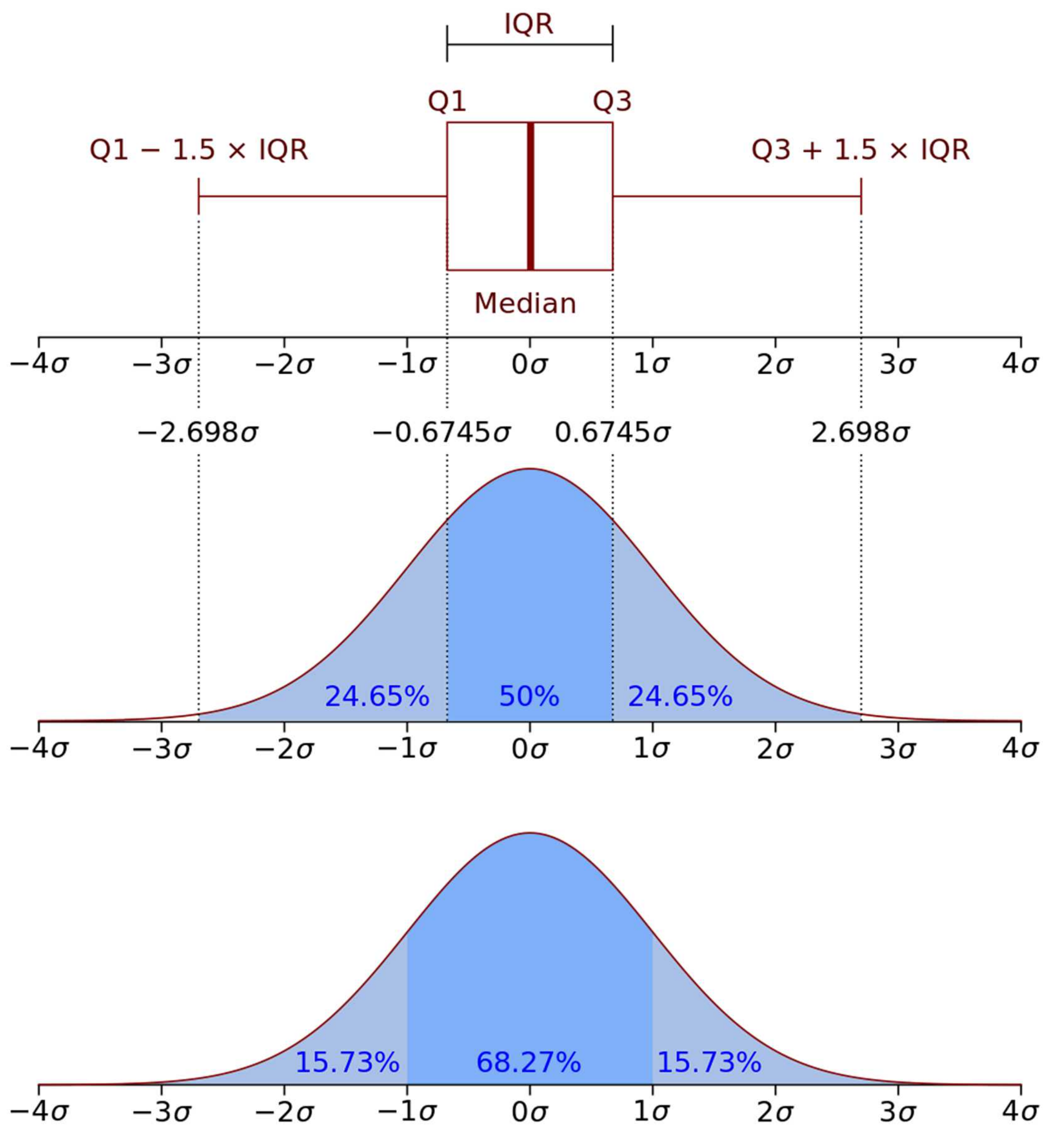
[4] J. S. Cap and T. L. Paez. "A Procedure for the Generation of Statistically Significant Transient Signals." *The Shock and Vibe Bulletin*.

[5] D. B. Owen. "Factors for One-Sided Tolerance Limits and for Variables Sampling Plans." Sandia Monograph, SC-R-607, Sandia Corp., Albuquerque, NM. March 1963.

[6] L. R. Pendleton and R. L. Henrikson. "Flight-to-Flight Variability in Shock and Vibration Levels Based on Trident I Flight Data." Proceedings of the 53rd Shock and Vibration Symposium, Danvers, MA, October 1983.

8 Appendix: Interpreting the Box Plots Used in This Paper

A box plot represents a distribution of data through quartiles. Quartiles divide the data into four equal quarters. The second quartile is the median of the data and is represented as a line or circle near the center of the box plot. The upper and lower box limits represent the 1st and 3rd quartile (Q1, Q3) where 50% of the data lies. The box plot whisker extends from Q1 to Q1-1.5*IQR and from Q3 to Q3+1.5*IQR where IQR is the interquartile region and is equal to Q3-Q1. In the plots in this paper each box plot represents 500 simulations for the given number of samples and the data beyond the whisker is represented as open circle outliers. The data that is represented with box plots in this paper is not all normally distributed and so does not all look like Figure 15.



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