

# Probing neutron-proton asymmetry dependence of source temperature with light charged particles

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## Abstract

The dependence of the nuclear temperature on the source neutron-proton ( $N/Z$ ) asymmetry was experimentally investigated with the light charged particles (LCPs) generated from thirteen reaction systems with different  $N/Z$  asymmetries,  $^{64}\text{Zn}$  on  $^{112}\text{Sn}$  and  $^{70}\text{Zn}$ ,  $^{64}\text{Ni}$  on  $^{112,124}\text{Sn}$ ,  $^{58,64}\text{Ni}$ ,  $^{197}\text{Au}$ ,  $^{232}\text{Th}$  at 40 MeV/u. A rather weak  $N/Z$  asymmetry dependence of the source temperature was qualitatively inferred from the extracted  $N/Z$  asymmetry dependence of the apparent temperature and that of the relative temperature change by the sequential decay effects with the help of the theoretical simulations. Comparing the present result with those from our previous work and other available experimental results, a weak  $N/Z$  asymmetry dependence of nuclear temperature is commonly observed in different independent experiments and with different thermometers, except for the result reported by McIntosh *et al.* [Phys. Lett. **B 719**, 337 (2013)]. The origin of the difference between the conclusion of the former group and that of McIntosh *et al.* was addressed, using statistical multifragmentation model (SMM) simulations.

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## 1 I. INTRODUCTION

2 The concept of nuclear temperature was introduced about seven decades ago in pioneer-  
3 ing works performed by Bethe [1] and Weisskopf [2], to describe the formation and decay of  
4 a compound nucleus formed in reactions induced by light projectiles, mostly neutrons [3].  
5 Nuclear temperature was later extended to nuclear reactions [4] and applied for studies  
6 about nuclear instabilities and the liquid-gas phase transition in nuclear matter [5, 6]. To  
7 experimentally extract temperature information, several nuclear “thermometers” have been  
8 proposed based on various experimental observables, i.e, energy spectra [7, 8], momentum  
9 fluctuations [9], double isotope yield ratios [10] and excited state populations [11] among  
10 others. These nuclear thermometers rely on critical thermodynamic conditions such as  
11 chemical and thermal equilibrium. In practical applications, however, non-equilibrium pro-  
12 cesses or mechanisms in nuclear reactions, such as multi-source emission, emission time  
13 difference, secondary decay processes among others [3], may significantly influence the ac-  
14 curacy of the nuclear thermometers and result in significant temperature difference deduced  
15 from the different thermometers [12]. In spite of these complications, the thermometers  
16 are still applicable and widely used in “dependence” studies of nuclear temperature with  
17 specific considerations by characterizing the source and examining sequential decay effects.  
18 Among these thermometers, the double isotope ratio thermometer has been used to study  
19 thermodynamic properties of fragmenting sources, i.e., temperature as a function of exci-  
20 tation energy [13–16], source (or system) neutron-proton asymmetry [17–20], and fragment  
21 emission time [21].

22 The dependence of nuclear temperature on the source neutron-proton ( $N/Z$ ) asymmetry,  
23 also called isotopic dependence of nuclear temperature, is of interest and has been studied  
24 for years, as one may expect crucial information on the  $N/Z$  asymmetry dependence of the  
25 nuclear forces, the properties of excited nuclei and the postulated nuclear liquid-gas phase  
26 transition from these studies [4, 22–24]. However, no certain conclusion has been drawn on  
27 the  $N/Z$  asymmetry dependence of nuclear temperature in both experiments and theories  
28 until now. For instance, Wuenschel *et al.* [9] found that the experimentally deduced temper-  
29 atures from the proton quadrupole momentum fluctuation thermometer show a rather weak  
30 source  $N/Z$  asymmetry dependence, whereas McIntosh *et al.* [25] found that the deduced  
31 temperatures from the same thermometer are notably higher for relatively proton-richer

32 systems than those for neutron-richer systems. Later, applying the double isotope ratio  
 33 thermometers to the same data set, McIntosh *et al.* [17] obtained a weak temperature de-  
 34 pendence on the source  $N/Z$  asymmetry as well. In theoretical work, some predicted that  
 35 limiting temperatures, defined as the plateau temperature of the caloric curve, are higher  
 36 for neutron-poor systems [26], whereas others made the opposite prediction [27–29]. To fully  
 37 understand the  $N/Z$  asymmetry dependence of the nuclear temperature, more experimental  
 38 and theoretical efforts are required.

39 In this article, measured light charged particle (LCP) yields from thirteen reaction sys-  
 40 tems with different  $N/Z$  asymmetry were used to investigate the  $N/Z$  asymmetry depen-  
 41 dence of nuclear temperature. For the present isotope yield measurements, the double  
 42 isotope ratio thermometer of Albergo *et al.* [10] was adopted to deduce temperature values.  
 43 As the measured isotope yields are perturbed by sequential decay, the temperature from  
 44 experimental yields, namely “apparent temperature”, may have been significantly altered  
 45 from that at the freezeout, referred as the “real (source) temperature” in this article. The  
 46 sequential decay effect on temperature determination was taken into account following the  
 47 analysis strategy in Ref. [19], which was also used in our previous work [30]. This article is  
 48 organized as follows. In Sec. II, we briefly describe the experiment and data analysis. In  
 49 Secs. III and IV, the  $N/Z$  asymmetry dependence of the nuclear temperature deduced from  
 50 measured LCP yields is given and discussed. In Sec. V, a summary is given.

## 51 II. EXPERIMENT AND DATA ANALYSIS

52 The experiment was performed at the K-500 superconducting cyclotron facility at Texas  
 53 A&M University.  $^{64,70}\text{Zn}$  and  $^{64}\text{Ni}$  beams were used to irradiate  $^{58,64}\text{Ni}$ ,  $^{112,124}\text{Sn}$ ,  $^{197}\text{Au}$  and  
 54  $^{232}\text{Th}$  targets at 40 MeV/u. Data from thirteen reaction systems,  $^{64}\text{Zn}$  on  $^{112}\text{Sn}$  and  $^{70}\text{Zn}$ ,  
 55  $^{64}\text{Ni}$  on  $^{112,124}\text{Sn}$ ,  $^{58,64}\text{Ni}$ ,  $^{197}\text{Au}$ ,  $^{232}\text{Th}$ , were analyzed in this work. In the experiment, IMFs  
 56 were detected by a detector telescope placed at  $20^\circ$  in a spherical scattering chamber. The  
 57 telescope consisted of four Si detectors with the same size of  $5\text{ cm} \times 5\text{ cm}$  and different  
 58 nominal thicknesses of 129, 300, 1000,  $1000\ \mu\text{m}$ , respectively. All four Si detectors were seg-  
 59 mented into four sections and each quadrant had a  $5^\circ$  opening in the polar angle. Telescope  
 60 signals were taken inclusively as the main trigger for all detected events. In coincidence  
 61 with IMFs, LCPs were measured using 16 single-crystal CsI(Tl) detectors of 3 cm length

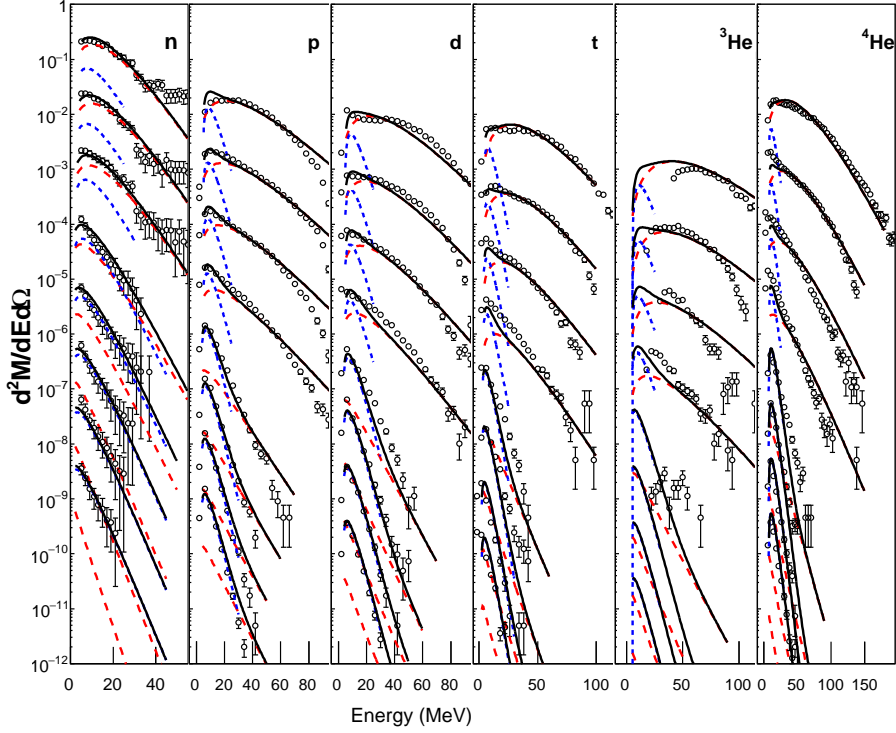


FIG. 1. (Color online) Neutron and light charged particle energy spectra at different  $\theta_{lab}$  in coincidence with IMFs with  $Z \geq 3$  for the  $^{64}\text{Ni} + ^{112}\text{Sn}$  reaction. The differential multiplicity is shown on an absolute scale, but multiplied by a factor of  $10^n$  ( $n = 0 - 7$ ) from the top to the bottom spectra. For neutrons,  $\theta_{lab} = 25^\circ, 31^\circ, 40^\circ, 67^\circ, 85^\circ, 104^\circ, 120^\circ, 140^\circ$  and for LCPs,  $\theta_{lab} = 36^\circ, 47^\circ, 57^\circ, 70^\circ, 115^\circ, 135^\circ, 145^\circ, 155^\circ$  from top to bottom. Red dashed lines represent the IV source component, blue dotted lines for the TLF source component and the black solid lines for the summation of them.

62 set around the target at angles between  $\theta_{Lab} = 27^\circ$  and  $\theta_{Lab} = 155^\circ$ . The light output from  
 63 each detector was read by a photomultiplier tube. The pulse shape discrimination method  
 64 was used to identify  $p$ ,  $d$ ,  $t$ ,  $^3\text{He}$  and  $\alpha$  particles. The energy calibrations of these particles  
 65 were performed using Si detectors ( $50 \sim 300 \mu\text{m}$ ) in front of the CsI detectors in separate  
 66 runs. Sixteen detectors of the Belgian-French neutron detector array DEMON (Detecteur  
 67 Modulaire de Neutrons) outside the chamber were used to measure neutrons, covering polar  
 68 angles of  $15^\circ \leq \theta_{IMF-n} \leq 160^\circ$  between the telescope and the neutron detectors.

69 In off-line analyses, simulations of the antisymmetrized molecular dynamics (AMD) [31]

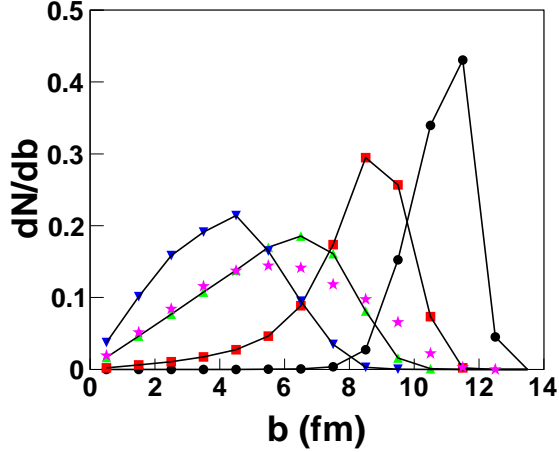


FIG. 2. (Color online) Simulated impact parameter distributions for violent (downward triangles), semi-violent (upward triangles), semi-peripheral (squares) and peripheral (dots) collisions of  $^{64}\text{Zn}+^{112}\text{Sn}$  at 40 MeV/nucleon. Stars indicate the events in which at least one IMF ( $Z \geq 3$ ) is emitted at  $15^\circ$ - $25^\circ$ . The summed distribution for a given event class is normalized to 1. The figure is taken from Ref. [34].

70 incorporating with the statistical decay code GEMINI as an afterburner [32] were performed  
 71 to characterize the measured events. Fig. 2 presents the calculated impact parameter distri-  
 72 butions for the system of  $^{64}\text{Zn}+^{112}\text{Sn}$  at 40 MeV/nucleon. In this figure, the violence of the  
 73 reaction for each event was determined in the same way as that in Ref. [33], in which the  
 74 multiplicity of light particles, including neutrons, and the transverse energy of light charged  
 75 particles were used. The resultant impact parameter distributions for each class of events  
 76 are shown together with that of the events in which at least one IMF is emitted at the polar  
 77 angles within  $15^\circ$ - $25^\circ$ . The comparison between the experiment and AMD-GEMINI sim-  
 78 ulations shown in the figure suggests that the events selected by the inclusive IMF triggers  
 79 at the polar angles of  $15^\circ$ - $25^\circ$  are corresponding to semi-violent collisions.

81 To further characterize the fragmenting source to isolate the reaction mechanisms involved  
 82 in the reaction products, a moving source fit technique [35] was employed. In the moving  
 83 source fit, the sources were classified as projectile-like (PLF), intermediate-velocity (IV)  
 84 (also called as nucleon-nucleon-like (NN) [36]), and target-like (TLF) sources according to  
 85 the source velocity. For neutrons and LCPs, since the measured angles were greater than  
 86  $\theta_{lab} > 20^\circ$  where the PLF source component has negligible contributions to the spectra,  
 87 two sources, the IV source and TLF source, were used in the present moving-source fit. In

88 the fitting procedures, the IV source [was](#) described with a volume type Maxwellian with a  
 89 velocity around a half beam velocity, whereas the TLF source [was](#) described with a surface  
 90 type Maxwellian with a small source velocity [35]. The Minuit in the Cern library [was](#) used  
 91 to optimize the four parameters for each source, isotope yield, slope parameter, Coulomb  
 92 energy, and source velocity. Typical fitting results for neutrons, protons, deuterons, tritons,  
 93  $^3\text{He}$  and  $^4\text{He}$  are shown in Fig. 1 from the left to the right panels, respectively. In the  
 94 figure, the IV (red dashed lines) and TLF (blue dotted lines) source components dominate  
 95 in two distinct angular ranges where the IV source component dominates in the top 3-4  
 96 spectra whereas the TLF source component dominates in the bottom 3-4 spectra for all cases  
 97 presented. The parameter errors from the moving source fits [were](#) evaluated by performing  
 98 different optimizations with different initial values within a wide range, rather than by  
 99 using the errors given by the Minuit which are much smaller in general, because there are  
 100 many local minima in the present multiple parameter fits. Therefore, large parameter errors  
 101 (including that of isotope yield) [were](#) assigned the multiplicity of the IV source for LCPs.  
 102 As indicated in Refs. [37, 38], the experimental extraction of nuclear matter properties,  
 103 such as temperature and density, is strongly influenced by the complicated multi-source  
 104 emission mechanism. Here, the yields of LCPs from the IV source are used in the following  
 105 investigation of the  $N/Z$  asymmetry dependence of nuclear temperature.

### 106 III. RESULTS

#### 107 1. $N/Z$ asymmetry dependence of apparent temperature

108 To deduce the nuclear temperature, the double isotope ratio thermometer of Albergo *et*  
 109 *al.* [10] [was](#) adopted according to the present isotope yield measurements. Under the assump-  
 110 tion that chemical equilibrium is established between free nucleons and composite fragments  
 111 contained within a certain freezeout (fragmentation) volume, the nuclear temperature can  
 112 be deduced as ([see details in Appendix I](#))

$$T = \frac{B_{diff}}{\ln(aR)}. \quad (1)$$

113 In this work, two commonly used double isotope ratios,  $^1,2\text{H}/^3,4\text{He}$  and  $^2,3\text{H}/^3,4\text{He}$ , [were](#)  
 114 applied. The corresponding thermometers are, respectively,

$$T_{1,2H/3,4He} = \frac{18.4}{\ln(5.6R)}, \quad (2)$$

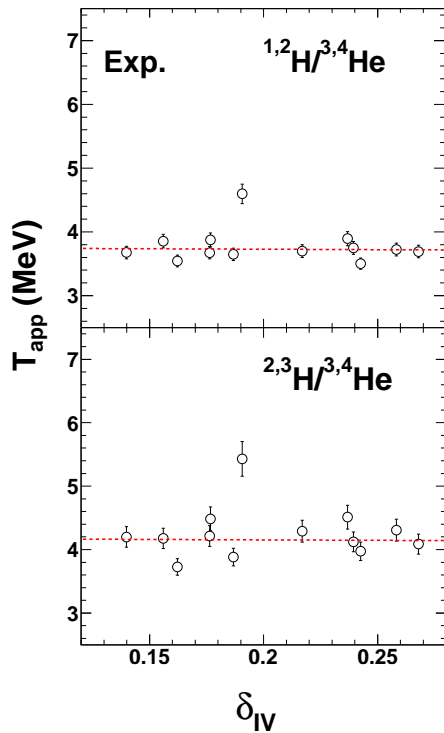


FIG. 3. (Color online) Apparent temperatures from  $^{1,2}\text{H}/^{3,4}\text{He}$  and  $^{2,3}\text{H}/^{3,4}\text{He}$  thermometers as a function of source  $N/Z$  asymmetry  $\delta_{IV}$ . Red dashed lines show the global fits with linear functions with one common slope  $k_{app}$  and different intercepts.

115 and

$$T_{2,3\text{H}/^{3,4}\text{He}} = \frac{14.3}{\ln(1.6R)}. \quad (3)$$

116 Note that, as the experimental LCP yields which are perturbed by sequential decay are  
 117 used in Eqs. 2 and 3, the deduced temperature is the “apparent temperature” rather than  
 118 the “real temperature”. In this work, similar to that in our previous work [30], the analysis  
 119 strategy of Sfienti *et al.* [19] was adopted. That is, instead of using the double isotope  
 120 thermometer as an absolute thermometer, we used it as a relative thermometer and divide  
 121 the  $N/Z$  asymmetry dependence of the real source temperature into two “parts”: one is the  
 122  $N/Z$  asymmetry dependence of the apparent temperature,  $T_{app}$ , and the other is that of the  
 123 relative temperature change,  $\Delta T$ , between the apparent and real temperatures due to the  
 124 sequential decays. The former can be directly determined from the experimental yields, and  
 125 the latter can be deduced with the aid of theoretical calculations.

126 The deduced apparent temperature values from  ${}^1,2\text{H}/{}^{3,4}\text{He}$  and  ${}^2,3\text{H}/{}^{3,4}\text{He}$  thermometers  
 127 are plotted in Fig. 3 as a function of the IV source  $N/Z$  asymmetry,  $\delta_{IV} = (N_{IV} - Z_{IV})/A_{IV}$ ,  
 128 where  $N_{IV}$ ,  $Z_{IV}$  and  $A_{IV}$  are the neutron, proton and mass of the fragmenting source  
 129 calculated from summing over the experimentally measured IV component yields of neutrons,  
 130 LCPs and IMFs with  $Z \leq 18$ . Errors shown in the figure were calculated from the isotope  
 131 multiplicity errors. As seen in the figure, the apparent temperatures from both thermometers  
 132 fluctuate around certain mean values, except for one point from the  ${}^{70}\text{Zn}+{}^{64}\text{Ni}$  system,  
 133 and exhibit almost no dependence on  $\delta_{IV}$ . The origin of the significant overestimation  
 134 of the temperature for the  ${}^{70}\text{Zn}+{}^{64}\text{Ni}$  system is unknown, but this data point does not  
 135 have a significant influence on the overall trend of  $T_{app}$  versus  $\delta_{IV}$ . A global fit to the  
 136 two  $T_{app}$  versus  $\delta_{IV}$  plots with linear functions with one common slope  $k_{app}$  and individual  
 137 intercepts was performed. The common slope,  $k_{app}$ , in the fit reflects the average increasing  
 138 or decreasing trend of  $T_{app}$  as a function of  $\delta_{IV}$ , whereas the individual intercepts are sensitive  
 139 to the extracted values of the apparent temperature. From the fit,  $k_{app} = -0.1 \pm 0.5$  MeV  
 140 is obtained, where the error is the fitting error. The obtained  $k_{app}$  value is rather small  
 141 indicating that the apparent temperature decreases slightly as  $\delta_{IV}$  increases in the presently  
 142 measured  $\delta_{IV}$  region,  $0.14 \lesssim \delta_{IV} \lesssim 0.27$ .

## 143 **2. $N/Z$ asymmetry dependence of temperature change due to sequential de-** 144 **cays**

145 To determine the  $N/Z$  asymmetry dependence of nuclear temperature change due to se-  
 146 quential decays, the statistical multifragmentation model (SMM) of Bondorf *et al.* [39] was  
 147 used. SMM assumes that the fragmentation takes place in equilibrated nuclear matter and  
 148 the break-up configuration determined by statistical weights. Within the thermodynamic  
 149 limit, this process is consistent with a possible nuclear liquid-gas phase transition. Here the  
 150 relative temperature change,  $\Delta T$ , was defined as the difference between the temperatures  
 151 from the secondary and primary isotope yields, where the primary fragments were identi-  
 152 fied as those directly from the fragmentation processes, and the secondary fragments were  
 153 generated using the default encapsulated sequential decay code as an afterburner. In our  
 154 previous work [40–42], the symmetry entropy effect was added into the SMM of Bondorf *et*  
 155 *al.* and the reconstructed hot fragment yield distributions from the reaction  ${}^{64}\text{Zn} + {}^{112}\text{Sn}$   
 156 at 40 MeV/u were well reproduced by the new SMM. Following analyses are based on the  
 157 new SMM (referred to “SMM” hereafter).

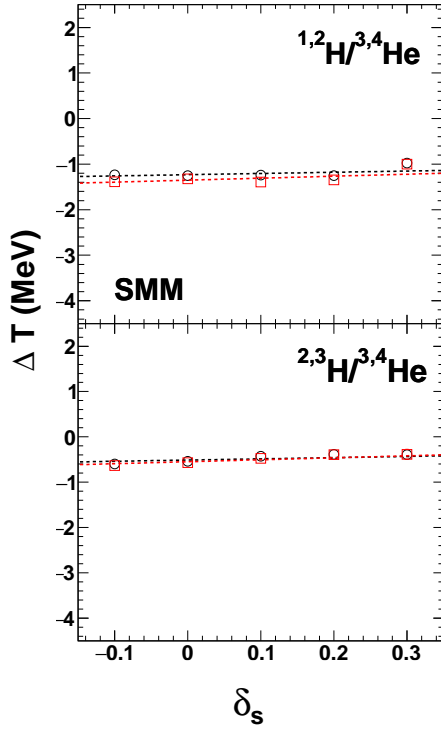


FIG. 4. (Color online) Difference between the temperatures from the secondary and primary isotope yields from the SMM simulations determined using the  ${}^1,2\text{H}/{}^3,4\text{He}$  and  ${}^2,3\text{H}/{}^3,4\text{He}$  thermometers as a function of source neutron-proton asymmetry  $\delta_s$ . Initial fragmentation conditions are  $E_x/A = 5$  MeV and  $V/V_0=5$  (squares),  $E_x/A = 5$  MeV and  $V/V_0=10$  (circles). Dashed lines represent the corresponding global fits with linear functions with one common slope  $k_{\Delta T}^{SMM}$  and different intercepts.

158 In the SMM simulations, fragmenting sources with the same mass number  $A_s = 100$   
159 but different charge numbers, i.e.,  $Z_s = 35, 40, 45, 50$  and  $55$ , were used. The source  $N/Z$   
160 asymmetry,  $\delta_s = 1 - 2Z_s/A_s$ , ranges from  $-0.1$  to  $0.3$ , and fully covers the measured  $\delta_{IV}$  region.  
161 The fragmentation conditions were specified with excitation energies  $E_x/A = 5$  MeV and  
162 fragmentation volumes  $V/V_0=5$  and  $10$ , where  $V_0$  is the volume with the normal saturation  
163 density. The selection of  $E_x/A = 5$  MeV corresponds to a fragmentation temperature of  $\sim 5$   
164 MeV which has been previously extracted from the IMF yields of the reaction  ${}^{64}\text{Zn} + {}^{112}\text{Sn}$   
165 at  $40$  MeV/u using a self-consistent method [43]. In Fig. 4, the resultant  $\Delta T$  versus  $\delta_s$   
166 relations are plotted for  $E_x/A = 5$  MeV and  $V/V_0=5$  (squares), and  $E_x/A = 5$  MeV and  
167  $V/V_0=10$  (circles). Similar weak dependences of  $\Delta T$  on the  $N/Z$  asymmetry are observed for  
168 both thermometers, although the absolute  $\Delta T$  values are slightly different. This indicates

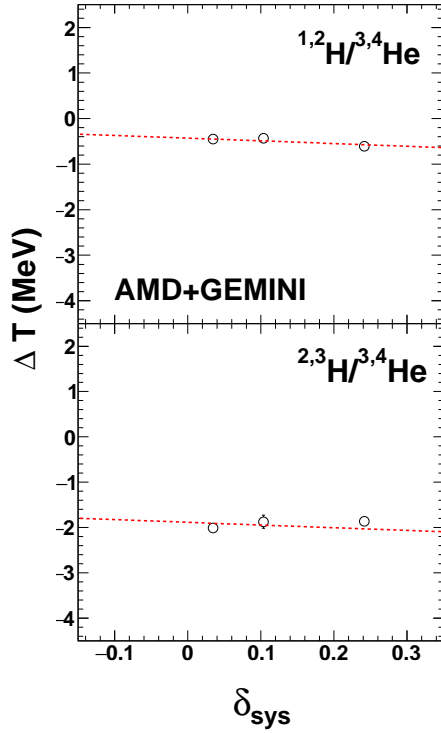


FIG. 5. (Color online) Difference between the temperatures from the secondary and primary isotope yields from the AMD+GEMINI simulations determined using the  $^{1,2}\text{H}/^{3,4}\text{He}$  and  $^{2,3}\text{H}/^{3,4}\text{He}$  thermometers as a function of system neutron-proton asymmetry  $\delta_{sys}$ . Red dashed lines represent the global fits with linear functions with one common slope  $k_{\Delta T}^{AMD}$  and different intercepts.

169 that the nuclear structure characteristics in the secondary decay process is nearly the same  
 170 for a given double isotope ratio selection among the reaction systems with different  $N/Z$   
 171 asymmetries, once the initial condition is fixed even if any incomplete inclusion of nuclear  
 172 structure characteristics is involved. For the results under a given fragmentation condition,  
 173 the same global fit as that of Fig. 3 was applied, and slopes  $k_{\Delta T}^{SMM} = 0.4 \pm 0.2$  MeV for  
 174  $E_x/A = 5$  MeV and  $V/V_0=5$  (squares) and  $0.3 \pm 0.2$  MeV for  $E_x/A = 5$  MeV and  $V/V_0=10$   
 175 (circles) are obtained, respectively, where the errors are from the fits. Both  $k_{\Delta T}^{SMM}$  values  
 176 show close agreement and have a consistent magnitude with the deduced  $|k_{\Delta T}| \lesssim 2.5$  MeV  
 177 from the observation reported by Sfienti *et al.* [19], in which the deviation of the secondary  
 178 decay corrections is less than 300 keV as the  $N/Z$  asymmetry changes from 0.07 to 0.19  
 179 among the projectile-like fragmenting sources,  $^{107}\text{Sn}$ ,  $^{124}\text{La}$  and  $^{124}\text{Sn}$ .

180 To further clarify the dynamical effect on the  $N/Z$  asymmetry dependence of  $\Delta T$ , simu-

181 lations of the  $^{58}\text{Ti} + ^{58}\text{Ti}$ ,  $^{58}\text{Fe} + ^{58}\text{Fe}$  and  $^{58}\text{Ni} + ^{58}\text{Ni}$  reaction systems at 40 MeV/u, were  
 182 also performed using the AMD code of Ono *et al.* [31]. For AMD, the dynamical calculation  
 183 was performed up to 300 fm/c, and the primary fragments from the dynamical process were  
 184 then de-excited to the ground state using the GEMINI code of Charity *et al.* [32]. Inclusive  
 185 primary and secondary LCPs from an impact parameter range of 0-8 fm were taken to calcu-  
 186 late the real and apparent temperatures, respectively. The resultant  $\Delta T$  values as a function  
 187 of system neutron-proton asymmetry  $\delta_{sys}$  are shown in Fig. 5. Applying the global fit to  
 188 the  $\Delta T$  versus  $\delta_{sys}$  plot from the AMD+GEMINI simulations leads to  $k_{\Delta T}^{AMD} = -0.6 \pm 0.4$   
 189 MeV. Even though the signs are opposite, the absolute values of  $k_{\Delta T}^{SMM}$  and  $k_{\Delta T}^{AMD}$  are small,  
 190 indicating a weak  $N/Z$  asymmetry dependence of  $\Delta T$ . AMD and SMM follow completely  
 191 different scenarios for the fragment production, that is, AMD is dynamical, whereas SMM  
 192 is statistical. This consistency suggests an insensitivity of the  $N/Z$  asymmetry dependence  
 193 of  $\Delta T$  to the fragmentation mechanism and further confirms the weak dependence of the  
 194 relative temperature change due to sequential decays on the  $N/Z$  asymmetry.

### 195 3. $N/Z$ asymmetry dependence of real temperature

196 From the weak  $N/Z$  asymmetry dependence of the apparent temperature deduced from  
 197 the experimental LCP yields with  $k_{app} = -0.1 \pm 0.5$  MeV and the weak  $N/Z$  asymmetry  
 198 dependence of the relative temperature change deduced from the theoretical predictions with  
 199  $k_{\Delta T}^{SMM} = 0.3 \sim 0.4 \pm 0.2$  MeV and  $k_{\Delta T}^{AMD} = -0.6 \pm 0.4$  MeV, one can conclude that a change  
 200 of 1 unit in source  $N/Z$  asymmetry corresponds to a maximum absolute change in real  
 201 temperature by about 0.5 MeV. Therefore, a weak  $N/Z$  asymmetry dependence of the real  
 202 source temperature is inferred. We note that the source mass has a negligible contribution to  
 203 this result, since no significant size dependence was experimentally observed for the reactions  
 204 with system sizes and incident energies similar to those of this work [21]. The present weak  
 205  $N/Z$  asymmetry dependence of temperature deduced from LCP thermometers is rather  
 206 consistent with that from IMF thermometers in our previous work [30]. This consistency  
 207 is an indication for early chemical equilibrium prior to the source fragmentation, as LCPs  
 208 and IMFs involve different emission time scales in the collisions [44, 45]. To fully address  
 209 this issue, on the other hand, nuclear density information during fragment emission is still  
 210 required.

211 **IV. DISCUSSION**

212 Detailed comparisons about the available experimental results and ours from IMF yields  
 213 are given in the previous work [30]. Since the present result is similar to that in the previous  
 214 work, the comparisons presented in Ref. [30] are valid. From those comparisons, one may  
 215 find that a weak  $N/Z$  asymmetry dependence of nuclear temperature is commonly observed  
 216 in different reactions and with different thermometers [9, 17, 19, 20], except for the result  
 217 reported by McIntosh *et al.* [25]. Since Wuenschel *et al.* and McIntosh *et al.* used the same  
 218 proton quadrupole momentum fluctuation thermometer as a probe, we focus in this article  
 219 on pursuing the origin of the different result of McIntosh *et al.* [25] by comparing with that  
 220 of Wuenschel *et al.* [9]. To address the difference, we first briefly describe their analyses and  
 221 conclusions.

222 1. Wuenschel *et al.* measured the temperature (caloric curve) of reconstructed quasi-  
 223 projectiles (QPs) from the reactions of  $^{86,78}\text{Kr} + ^{64,58}\text{Ni}$  at 35 MeV/nucleon collected with  
 224 the NIMROD-ISiS array housed inside the TAMU Neutron Ball [46]. Charged particle  
 225 yields were obtained using the Si-CsI telescopes in NIMROD-ISiS. Free neutron yields were  
 226 provided by the Neutron Ball in conjunction with the event reconstruction with isotopically  
 227 resolved charged particles. The reconstructed QP source was constrained to be in the QP  
 228 charge range of  $Z_{QP}=30-34$ . The fragments in an accepted event were then cut on the  
 229 longitudinal velocity relative to that of the largest fragment. The average neutron-corrected  
 230  $N/Z$  asymmetries of the QPs obtained from the  $^{86}\text{Kr} + ^{64}\text{Ni}$  and  $^{78}\text{Kr} + ^{58}\text{Ni}$  systems were  
 231 0.14 and 0.06, respectively [47]. The excitation energies per nucleon of the QPs were from  
 232 1.5 MeV to 8.5 MeV. A weak  $N/Z$  asymmetry dependence in good agreement with our  
 233 present result was concluded.

234 2. McIntosh *et al.* performed [their](#) experiment using the same detector array, NIMROD-  
 235 ISiS and Neutron Ball. The temperature of the reconstructed QPs from the reactions of  
 236  $^{70}\text{Zn} + ^{70}\text{Zn}$ ,  $^{64}\text{Zn} + ^{64}\text{Zn}$  and  $^{58}\text{Ni} + ^{58}\text{Ni}$  at 35 MeV/nucleon was deduced. The mass  
 237 of the reconstructed QPs was constrained to be  $48 \leq A_{QP} \leq 52$ . To select QPs that  
 238 were equilibrated, it was also [required that the emission of the QPs should be spherical](#) on  
 239 average. The  $N/Z$  asymmetries of the reconstructed QPs ranged from 0.04 to 0.24. The  
 240 excitation energies per nucleon of the QPs were from 2.5 MeV to 8.5 MeV. A significant  
 241  $N/Z$  asymmetry dependence on the source temperature was concluded, that an increase in

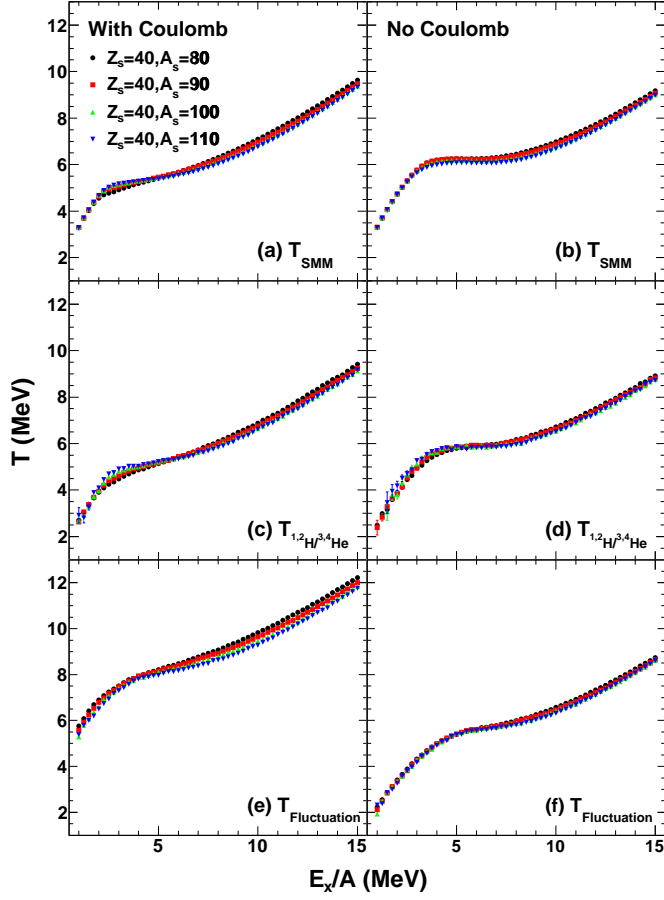


FIG. 6. (Color online) Temperature values as a function of excitation energy per nucleon from SMM calculations with fragmenting sources with the same charge number  $Z_s=40$  and different mass numbers  $A_s=80, 90, 100,$  and  $110$ . Left panels are those from SMM calculations that include the Coulomb effects, whereas right panels are those SMM calculations without the Coulomb effect. Upper, middle and lower panels correspond to different temperature determinations. Temperatures in the upper panels are those directly from the SMM,  $T_{SMM}$ , and those in the middle and lower panels are deduced using the  $^{1,2}\text{H}/^{3,4}\text{He}$  thermometer,  $T_{1,2\text{H}/3,4\text{He}}$ , and the proton momentum fluctuation thermometer,  $T_{fluctuation}$ .

242  $N/Z$  asymmetry of 0.15 unit corresponds to a decrease in fluctuation temperature on the  
 243 order of 1 MeV.

244 Wuenschel *et al.* reconstructed the QPs with a constraint on the charge number  $Z_{QP}$ ,  
 245 whereas McIntosh *et al.* reconstructed the QPs with a constraint on the mass number  $A_{QP}$ .  
 246 As a consequence, the reconstructed QPs of Wuenschel *et al.* were with similar  $Z_{QP}$  but

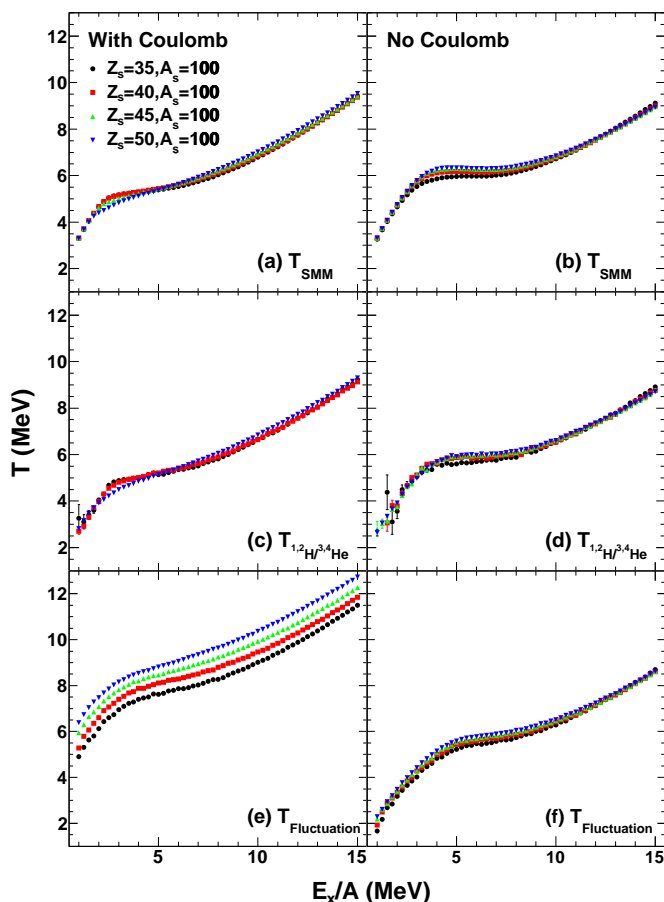


FIG. 7. (Color online) Similar plots to those shown in Fig. 6 but from the SMM calculations with fragmenting sources with the same mass number  $A_s=100$  and different mass numbers  $Z_s=35, 40, 45,$  and  $50$ .

247 different  $A_{QP}$ , whereas those of McIntosh *et al.* were with similar  $A_{QP}$  but different  $Z_{QP}$ .  
 248 These different constraints on the QP mass and charge may lead to the difference in the  
 249 temperature dependence on the  $N/Z$  asymmetry deduced by Wuenschel *et al.* and McIntosh  
 250 *et al.*. It should be mentioned that in some studies, especially in the symmetry energy  
 251 studies [48, 49], reconstructed sources or systems with the constant mass are commonly  
 252 used. This is because for the sources with the same mass, the effects from the volume and  
 253 surface of the sources are eliminated, so that one may isolate the symmetry energy term  
 254 with the aid of dynamical or statistical calculations [without a Coulomb term](#).

255 To clarify the effect of the different constraints, additional SMM simulations [were](#) per-  
 256 formed besides those in Sec.III.2 where the fragmenting sources have the same charge number

257  $Z_s=40$  but different mass numbers  $A_s=80, 90, 100,$  and  $110$ . In the additional group of the  
 258 SMM simulations, the fragmenting sources have the same mass number  $A_s=100$  but different  
 259 charge numbers  $Z_s=35, 40, 45,$  and  $50$ . Two group calculations are essentially corresponding  
 260 to the two QP reconstruction constraints of Wuenschel *et al.* [47] and McIntosh *et al.* [25],  
 261 respectively. The source  $N/Z$  asymmetries for both groups fully cover the measured QP  
 262  $N/Z$  asymmetry regions. Fragmentation volume was set as  $6V_0$ . The excitation energies  
 263 were in the same range of  $E_x/A=1-15$  MeV with a step of  $0.25$  MeV, which also fully cover  
 264 the measurement region. More than 1 million events were generated for each given  $E_x/A$ .  
 265 SMM calculations without the Coulomb force were separately performed under the same  
 266 initial conditions for comparison. Sequential decay effect in the SMM was ignored, since in  
 267 the  $N/Z$  asymmetry study the thermometers can be used as a relative temperature probe  
 268 so that the sequential decay effects are not important. Indeed in Ref. [19], the sequential  
 269 decay effects were examined and no significant effect on the  $N/Z$  asymmetry dependence of  
 270 the source temperature was concluded.

271 In Fig. 6, temperature values as a function of excitation energy per nucleon for the frag-  
 272 menting sources with the same  $Z_s$  (corresponding to the QP mass constraint of Wuenschel  
 273 *et al.*) are plotted, where left and right panels represent those from calculations with and  
 274 without the Coulomb force, respectively. Upper, middle and lower panels correspond to  
 275 different temperature determinations, where temperatures in the upper panels are the frag-  
 276 mentation (or equilibrium) temperatures calculated from the energy balance in the SMM  
 277 ( $T_{SMM}$ ) as references. Those in the middle and lower panels are deduced using the  ${}^{1,2}\text{H}/{}^{3,4}\text{He}$   
 278 thermometer,  $T_{1,2\text{H}/3,4\text{He}}$ , and the proton quadrupole momentum fluctuation thermometer,  
 279  $T_{fluctuation}$ , respectively. Different symbols represent the results for the sources with different  
 280  $A_s$  as indicated in the top-left panel. Error bars are smaller than the data points. From  
 281 Figs. 6(a)-(f), weak  $N/Z$  asymmetry dependences of nuclear temperature are observed for all  
 282 cases with different temperature determination methods, and with or without the Coulomb  
 283 effect. These results are consistent with those from our works and those from Kunde *et*  
 284 *al.* [20], Sfienti *et al.* [19] and Wuenschel *et al.* [9]. Similar results to those in Fig. 6 but from  
 285 the SMM calculations with fragmenting sources with the same  $A_s$  are presented in Fig. 7,  
 286 which correspond to the QP mass constraint of McIntosh *et al.*. From Figs. 7(a)-(d), weak  
 287  $N/Z$  asymmetry dependences of both  $T_{SMM}$  and  $T_{1,2\text{H}/3,4\text{He}}$  are also observed both with and  
 288 without the Coulomb effect. In contrast, the proton momentum fluctuation thermometer

289 gives a significant  $N/Z$  asymmetry dependence of temperature as shown in Fig. 7(e). The  
 290 increase of 1 unit in  $\delta_s$  corresponds to a decrease in  $T_{fluctuation}$  on an average order of 5-6  
 291 MeV in the overall excitation energy region, in good agreement with the result of McIntosh  
 292 *et al.* [25]. After turning off the Coulomb force in the SMM, however, the significant  $N/Z$   
 293 asymmetry dependence of  $T_{fluctuation}$  disappears as shown in Fig. 7(f). This fact strongly  
 294 indicates a close correlation between the significant  $N/Z$  asymmetry dependence of temper-  
 295 ature obtained by McIntosh *et al.* and the Coulomb effect.

296 In the SMM, fragmentations are constrained by the conservation of mass, charge, mo-  
 297 mentum, and energy. The collective radial motion of the fragments from the Coulomb force  
 298 is superimposed upon the internal random thermal motion. As a consequence of the su-  
 299 perposition of the radial collective motion (expansion) on the thermal motion, the deduced  
 300 quadrupole momentum fluctuation temperature becomes larger in sources with larger  $Z_s$ ,  
 301 in which the Coulomb effect is more significant. This results in the significant dependence  
 302 of  $T_{fluctuation}$  on the source  $N/Z$  asymmetry in Fig. 7(e). In the case of Fig. 6(e), since  
 303 the Coulomb contributions among the sources with different  $N/Z$  asymmetries are almost  
 304 equal due to the same given  $Z_s$ , the significant dependence of  $T_{fluctuation}$  on the source  $N/Z$   
 305 asymmetry disappears. To further investigate the Coulomb effect, we applied the classical  
 306 Coulomb correction of Ref. [50] to the  $T_{fluctuation}$  values in panels (e) of Figs. 6 and 7. The  
 307 corrected temperature ( $T_{Corr.}$ ) values as a function of  $E_x/A$  are shown in Fig. 8. Upper and  
 308 lower panels correspond to results from sources with the same  $Z$  number and the same  $A$   
 309 number, respectively. It is found that the Coulomb correction reduces the absolute tempera-  
 310 ture values significantly, that is, the  $T_{Corr.}$  values in both panels become significantly smaller  
 311 and similar to those of  $T_{SMM}$  in Figs. 6(a) and 7(a), though their values are still around  
 312 1-2 MeV higher in the overall  $E_x/A$  region. One also may notice that for the case with  
 313 the same  $A$  number (lower panel), the offset of the temperature  $N/Z$  dependence remains,  
 314 but significantly reduces comparing to that in Fig. 7(e). In the SMM, the Coulomb energy  
 315 creates a radial expansion flow under a simultaneous multifragmentation scenario, whereas  
 316 the Coulomb correction of Zheng *et al.* [50] is made for the acceleration of the probe particle  
 317 in the source Coulomb field alone, and thus the Coulomb correction in the latter is only par-  
 318 tial. Therefore the above observed differences originate from the incomplete inclusion of the  
 319 collective radial flow before the Coulomb acceleration in the Coulomb correction. From the  
 320 above comparisons, we conclude that the significant  $N/Z$  dependence of the source tempera-

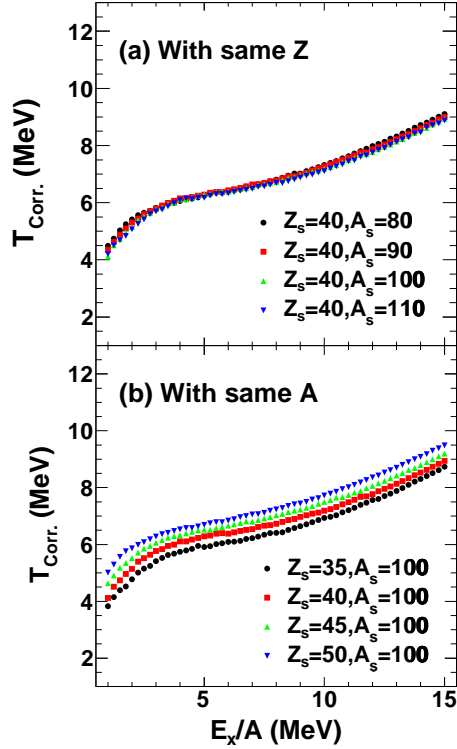


FIG. 8. (Color online) Similar plots to those in panels (e) of Figs. 6 and 7 but with the Coulomb correction of Ref. [50]. Upper and lower panels are corresponding to the results from sources with the same  $Z$  number and the same mass number, respectively.

321 ture observed by McIntosh *et al.* originates from the different Coulomb contributions in the  
 322 reconstructed QPs with different charges under the QP mass constraint. On the other hand,  
 323 since the double isotope ratio temperature is dominated by the chemical equilibrium during  
 324 fragmentations (commonly assumed in SMM simulations) rather than the Coulomb force,  
 325 the double isotope ratio temperature shows a weak source  $N/Z$  asymmetry dependence for  
 326 both cases with and without the Coulomb force.

327 Apart from the absolute values, the shapes of  $T_{1,2H/3,4}$  versus  $E_x/A$  and  $T_{fluctuation}$  versus  
 328  $E_x/A$  are similar to that of  $T_{SMM}$  versus  $E_x/A$  in Fig. 9(a), in which the results of  $Z_s = 45$  in  
 329 Fig. 7(a),(c) and (e) are depicted. This fact indicates an applicability in deducing the critical  
 330 behavior of hot nuclear matter using both  ${}^1,2\text{H}/{}^{3,4}\text{He}$  thermometer and proton momentum  
 331 fluctuation thermometer. After sequential decays, however, the temperature values and  
 332 their trends as a function of  $E_x/A$  from both the  ${}^1,2\text{H}/{}^{3,4}\text{He}$  thermometer and the proton  
 333 momentum fluctuation thermometer are significantly altered and become quite different from

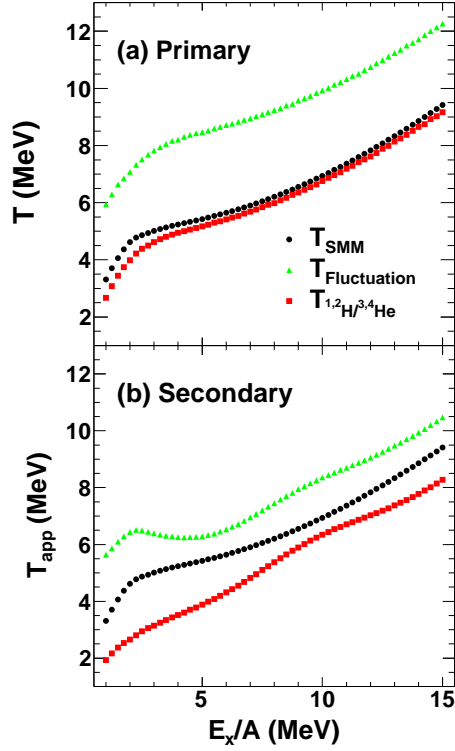


FIG. 9. (Color online) Temperature values as a function of excitation energy per nucleon for the  $Z_s = 45$  source. Upper and lower panels correspond to the results from primary hot fragments and secondary cool fragments. Different symbols represent different temperatures,  $T_{SMM}$  (dots),  $T_{1,2H/3,4He}$  (squares), and  $T_{fluctuation}$  (triangles). The same  $T_{SMM}$  is plotted in both upper and lower panels as a reference.

334 those of the primary yields, as shown in Fig. 9(b). This indicates that careful [treatment of](#)  
 335 [the sequential decay effects are important to](#) study the behavior of the hot nuclear matters  
 336 using the double isotope ratio thermometer and the quadrupole momentum fluctuation  
 337 thermometer.

## 338 V. SUMMARY

339 The  $N/Z$  asymmetry dependence of the nuclear temperature [was](#) experimentally inves-  
 340 tigated with the yields of the LCP isotopes produced from thirteen reaction systems with  
 341 different  $N/Z$  asymmetries,  $^{64}\text{Zn}$  on  $^{112}\text{Sn}$  and  $^{70}\text{Zn}$ ,  $^{64}\text{Ni}$  on  $^{112,124}\text{Sn}$ ,  $^{58,64}\text{Ni}$ ,  $^{197}\text{Au}$ ,  $^{232}\text{Th}$  at  
 342 40 MeV/nucleon. The apparent temperatures for these systems [were](#) determined from the  
 343 measured LCP yields from the IV sources using the  $^{1,2}\text{H}/^{3,4}\text{He}$  and  $^{2,3}\text{H}/^{3,4}\text{He}$  thermome-

344 ters. A rather weak  $N/Z$  asymmetry dependence of the extracted apparent temperature is  
345 observed in the measured source  $N/Z$  asymmetry range in the present study. To take into  
346 account the alteration of the measured isotope yields by the sequential decay processes, the  
347  $N/Z$  asymmetry dependence of the relative temperature change, which was defined as the  
348 difference between the temperatures from the secondary and primary isotope yields, was  
349 investigated using the SMM and AMD+GEMINI simulations. The real source tempera-  
350 ture was then qualitatively inferred to have a rather weak dependence on the source  $N/Z$   
351 asymmetry from the deduced  $N/Z$  asymmetry dependence of the apparent temperature and  
352 the relative temperature change. The present result was compared with those from our  
353 previous work and other independent experiments. A weak  $N/Z$  asymmetry dependence  
354 of nuclear temperature is commonly observed from different independent experiments and  
355 with different thermometers, except for the result reported by McIntosh *et al.* [25]. With  
356 close examinations of the experimental details of Wuenschel *et al.* and McIntosh *et al.* and  
357 combining with SMM simulations, we conclude that the significant  $N/Z$  dependence of the  
358 source temperature observed by McIntosh *et al.* originates from the Coulomb contribution  
359 difference in the reconstructed QPs with different charges under the QP mass constraint.

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## 420 APPENDIX I

421 Under the assumption that equilibrium may be established between free nucleons and  
422 composite fragments contained within a certain freezeout volume  $V$  and a temperature  $T$ ,  
423 the density of an isotope with  $A$  nucleons and  $Z$  protons ( $A, Z$ ) may be expressed as

$$\rho(A, Z) = \frac{N(A, Z)}{V} = \frac{A^{3/2} \cdot \omega(A, Z)}{\lambda_T^3} \cdot \exp\left[\frac{\mu(A, Z)}{T}\right], \quad (4)$$

424 where  $N(A, Z)$  is the number of isotope ( $A, Z$ ) within the volume  $V$ ;  $\lambda_T = h/(2\pi m_0 T)^{1/2}$   
425 is the thermal nucleon wave-length, where  $m_0$  is the nucleon mass;  $\omega(A, Z)$  is the internal  
426 partition function of the isotope ( $A, Z$ ) and related to the ground- and excited-state spins  
427 (practically,  $\omega(A, Z)$  is limited to that at the ground state [10]);  $\mu(A, Z)$  is the chemical  
428 potential of the isotope ( $A, Z$ ). In chemical equilibrium,  $\mu(A, Z)$  is expressed as

$$\mu(A, Z) = Z\mu_p + (A - Z)\mu_n + B(A, Z), \quad (5)$$

429 where  $B(A, Z)$  is the binding energy of the isotope  $(A, Z)$ .  $\mu_p$  and  $\mu_n$  are the chemical  
 430 potentials of free protons and free neutrons, respectively. Calculating the densities of free  
 431 protons and neutrons,  $\rho_p$  and  $\rho_n$ , in the same volume using Eqs. 4 and 5, performing  
 432 transforms to obtain  $\mu_p$  and  $\mu_n$ , and then inserting  $\mu_p$  and  $\mu_n$  back into Eq. 4, one obtains,

$$\rho(A, Z) = \frac{N(A, Z)}{V} = \frac{A^{3/2} \cdot \omega(A, Z) \cdot \lambda_T^{3(A-1)}}{(2s_p + 1)^Z \cdot (2s_n + 1)^{A-Z}} \cdot \rho_p^Z \cdot \rho_n^{A-Z} \exp \left[ \frac{B(A, Z)}{T} \right], \quad (6)$$

433 where  $s_p$  and  $s_n$  are the spins of the free proton and neutron, respectively. The ratio between  
 434 the measured yields of two different isotopes is then

$$\frac{Y(A, Z)}{Y(A', Z')} = \frac{\rho(A, Z)}{\rho(A', Z')} = \left( \frac{A}{A'} \right)^{3/2} \left( \frac{\lambda_T^3}{2} \right)^{A-A'} \frac{\omega(A, Z)}{\omega(A', Z')} \rho_p^{(Z-Z')} \rho_n^{(A-Z)-(A'-Z')} \cdot \exp \left[ \frac{B(A, Z) - B(A', Z')}{T} \right], \quad (7)$$

435 The free proton density can be calculated from the yield ratio of two fragments with only  
 436 one proton difference, such as  $(A, Z)$  and  $(A + 1, Z + 1)$ ,

$$\rho_p = C \cdot \left( \frac{A}{A+1} \cdot T \right)^{3/2} \frac{\omega(A, Z)}{\omega(A+1, Z+1)} \cdot \exp \left[ \frac{B(A, Z) - B(A+1, Z+1)}{T} \right] \cdot \frac{Y(A+1, Z+1)}{Y(A, Z)}, \quad (8)$$

437 where  $C$  is the constant related to the unit conversion. Analogously, the free neutron density  
 438 is calculated from the yield ratio of two fragments with only one neutron difference, such as  
 439  $(A, Z)$  and  $(A + 1, Z)$ ,

$$\rho_n = C \cdot \left( \frac{A}{A+1} \cdot T \right)^{3/2} \frac{\omega(A, Z)}{\omega(A+1, Z)} \cdot \exp \left[ \frac{B(A, Z) - B(A+1, Z)}{T} \right] \cdot \frac{Y(A+1, Z)}{Y(A, Z)}. \quad (9)$$

440 For a given temperature  $T$ , the same free proton (or neutron) density must be evaluated  
 441 from Eq. 8 ( or 9). Choosing two ratios with one proton (or neutron) excess, one can deduce  
 442 the relation between  $T$  and the experimental yield ratios as

$$T = \frac{B_{diff}}{\ln(aR)}, \quad (10)$$

443 and the error of  $T$ ,  $\delta T$ , is deduced as

$$\delta T = \frac{B_{diff}}{\ln^2(aR)} \cdot \frac{\delta R}{R}, \quad (11)$$

444 where  $R = (Y_1/Y_2)/(Y_3/Y_4)$  is the double isotope yield ratio of the ground states for isotope  
 445 pairs (1, 2) and (3, 4), and  $\delta R$  is the error of  $R$ . One can find from Eq. 11 that  $\delta T$  depends

446 on both  $B_{diff}/\ln^2(aR)$  and  $\delta R/R$ . For the (1, 2) and (3, 4) ratios with same one-neutron  
 447 excess used this work,  $B_{diff}$  is the binding energy difference,  $B_{diff} = (B_1 - B_2) - (B_3 - B_4)$ ,  
 448 and  $a$  is the statistical weighting factor and is defined as

$$a = \frac{\omega_3/\omega_4}{\omega_1/\omega_2} \left[ \frac{A_3/A_4}{A_1/A_2} \right]^{1.5}, \quad (12)$$

449 where  $\omega_i = 2S_i + 1$  and  $S_i$  is the ground state spin of the  $i$ th isotope and  $A_i$  is the mass  
 450 number of the  $i$ th isotope.