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(Invited) Towards computational imaging for intelligence in highly scattering aerosols

PRESENTED BY

Brian Z. Bentz

Brian J. Redman, Andres L. Sanchez, John D. Van Der Laan, Karl Westlake, and Jeremy B. Wright

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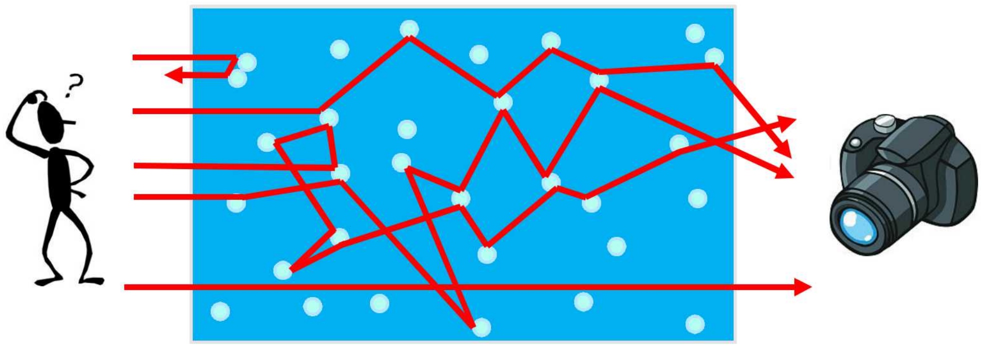
Light Scattering Limits Visibility

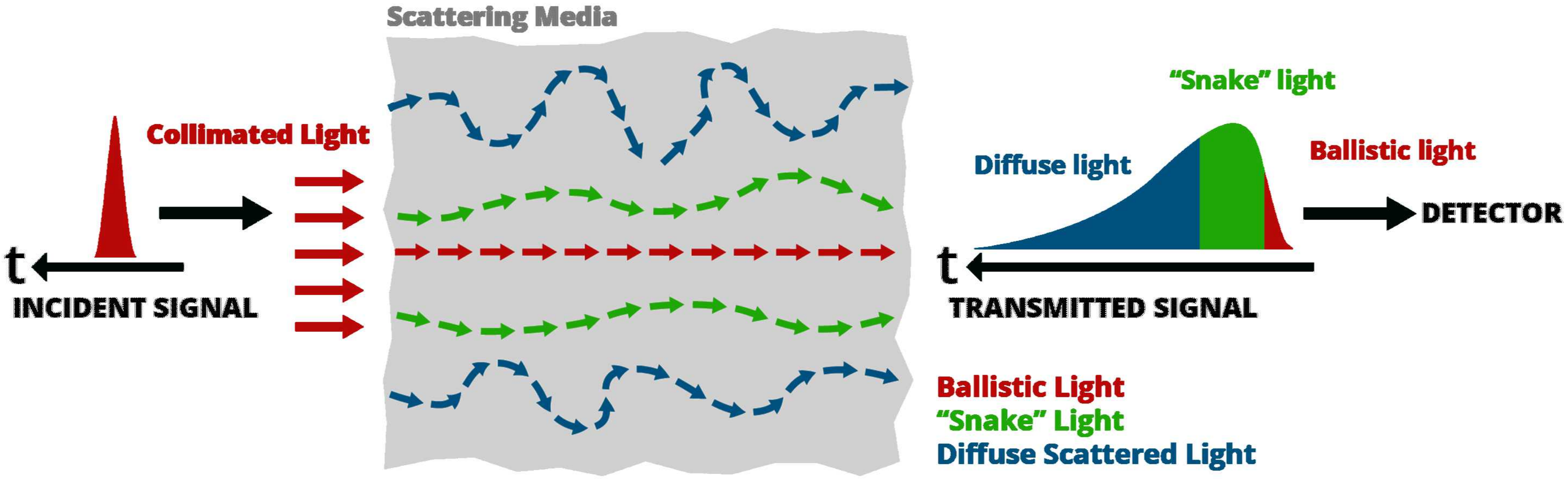
- Aerosols like fog reduce visibility and cause down-time that for critical systems or operations are unacceptable
- Information is lost due to the random scatter of photons from tiny particles
- Impacts physical security, site surveillance, navigation, and tactical scenarios



Simulated degraded visual environment at the Sandia Fog Chamber Facility

[1] B. J. Redman, et al., *Optical Engineering* **58**(5), 051806, (2019).

Light Scattering Fundamentals

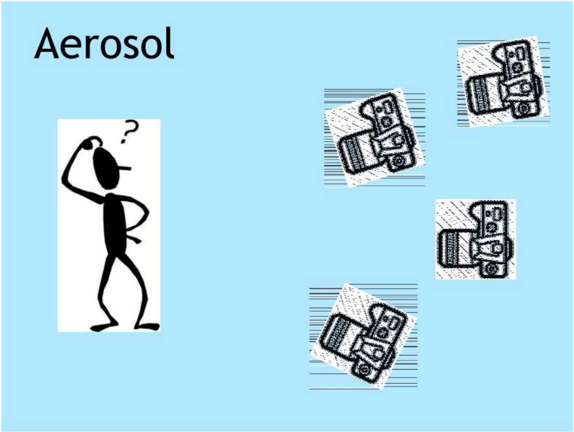


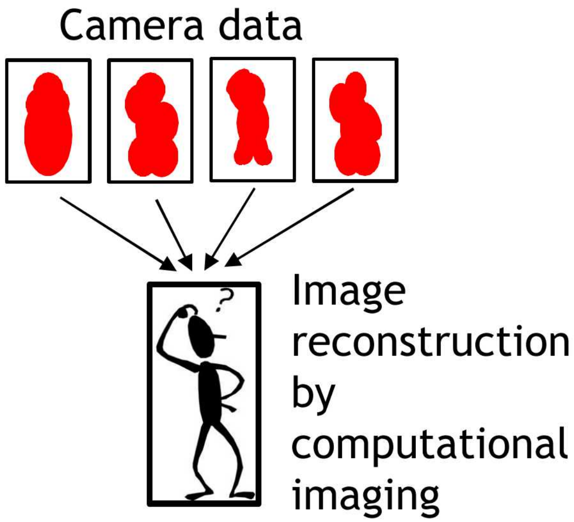
- Ballistic light is exponentially attenuated with distance: $I = I_o \exp(-L/\mathrm{MFP})$ [Beer-Lambert law]
- Time and coherence gating reject scattered light limiting imaging to $L{\sim}10$ MFP
- Diffuse optical imaging using all photons allows imaging to $L{\sim}100$ MFP or 10 times deeper

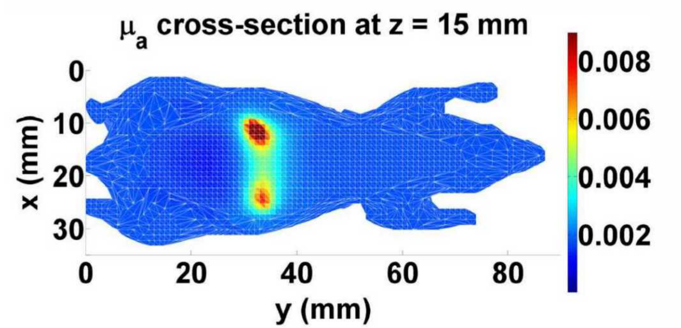
[1] C. Dunsby and P. M. W. French, *Journal of Physics D: Applied Physics* **36**, R207-R227 (2003).
[2] A. Mosk, Y. Silberberg, K. J. Webb, and C. Yang, *Defense Technical Information Center* ADA627354 (2015).

Computational Diffuse Optical Imaging

- Data from many detectors is combined to provide new information
  - (detection, localization, imaging, spectroscopic information)
  - Potentially provided in real time for rapid decision making using existing infrastructure
- **Key question**
  - **What can be done with diffuse optical imaging (DOI) methods in aerosols like fog?**







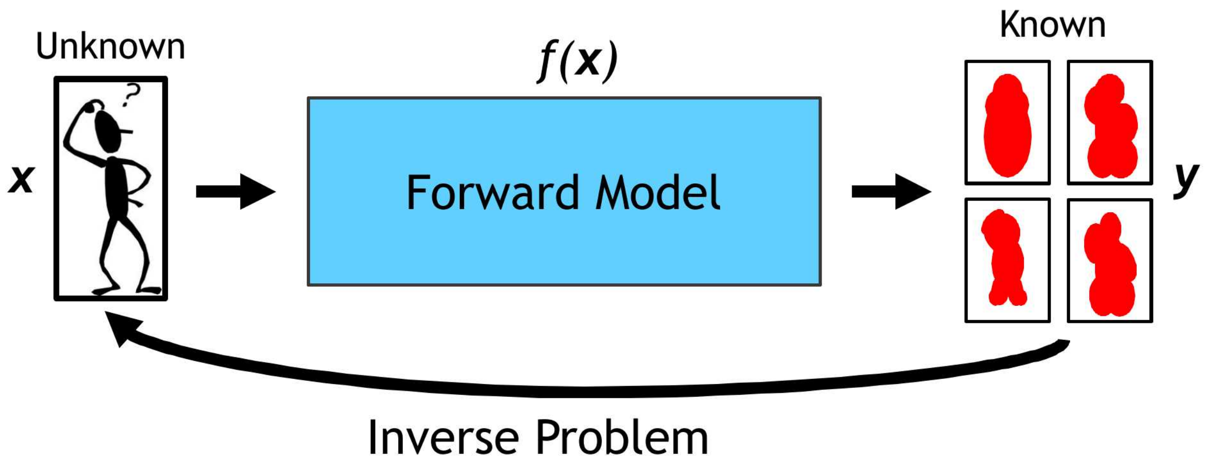
Example from biomedical imaging: Diffuse Optical Tomography (DOT) [1]

[1] B. Z. Bentz, A. V. Chavan, D. Lin, E. H-R Tsai, and K. J. Webb, *Applied Optics* **55**(2), 280-287, (2016).

Optimization-Based Imaging

- Long term goal is to solve the optimization problem (inversion)
  - Properties of interest, $\boldsymbol{x}$
  - Numerical forward solution, $f(\boldsymbol{x})$
  - Measurement, $\boldsymbol{y}$
- Bayesian framework – maximum *a posteriori* (MAP) estimation

$$\hat{\boldsymbol{x}}_{MAP} = \arg\max_{x \geq 0} \{\log p_{y|x}(\boldsymbol{y}|\boldsymbol{x}) + \log p_x(\boldsymbol{x})\}$$



[1] J. C. Ye, K. J. Webb, C. A. Bouman, and R. P. Millane, *JOSA A* **16**(10), 2400-2412, (1999).

Model Development: Radiative Transfer Equation (RTE)

$$\frac{1}{c}\frac{\partial I(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}})}{\partial t} + \widehat{\boldsymbol{\Omega}} \cdot \nabla I(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}}) + (\mu_a + \mu_s) I(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}}) = \mu_s \int_{4\pi} d\widehat{\boldsymbol{\Omega}}' f(\widehat{\boldsymbol{\Omega}}' \to \widehat{\boldsymbol{\Omega}}) I(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}}') + Q(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}})$$

Where

- $I(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}})$ is the radiance ($W/m^2/s/sr$) at position $\boldsymbol{r}$ in direction $\widehat{\boldsymbol{\Omega}}$
- $\mu_a = \sigma_a n$ is the absorption coefficient (m^{-1})
- $\mu_s = \sigma_s n$ is the scattering coefficient (m^{-1})
- σ is cross section and n is density
- $f(\widehat{\boldsymbol{\Omega}}' \to \widehat{\boldsymbol{\Omega}})$ is the in-line scattering phase function for incident direction $\widehat{\boldsymbol{\Omega}}'$ and scattering direction $\widehat{\boldsymbol{\Omega}}$
- $Q(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}})$ is the radiance source function ($W/m^3/s/sr$)

[1] S. Chandrasekhar, *Radiation Transfer*, Oxford, (1950).

Mie Theory: Determining the Scattering Parameters

- For spherical particles of constant area A, Mie Theory allows calculation of

$$\mu_s = Q_{sca} A n$$
$$\mu_a = Q_{abs} A n$$

- Where Q_{sca} and Q_{abs} are the scattering and absorption efficiencies and n is the density of particles

$$\mu_s = \frac{3}{2} f_v \sum_i \frac{Q_{sca_i} v_i}{d_i}$$

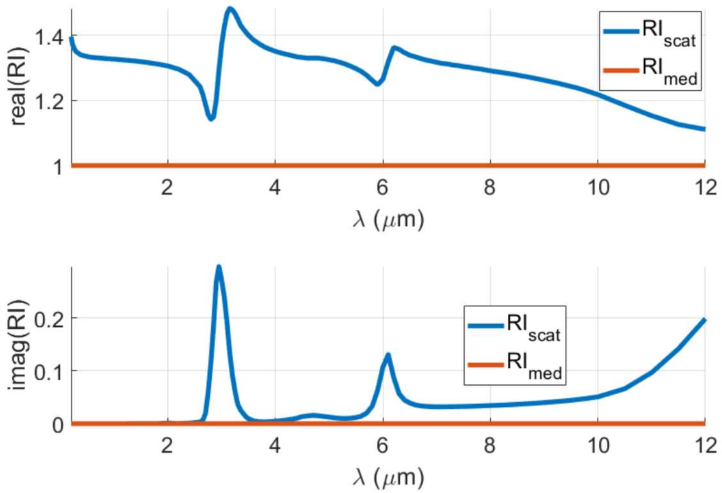
$$\mu_a = \frac{3}{2} f_v \sum_i \frac{Q_{abs_i} v_i}{d_i}$$

$$\mu_s' = \frac{3}{2} f_v \sum_i \left(\frac{Q_{sca_i} v_i}{d_i} \right) (1 - g_i)$$

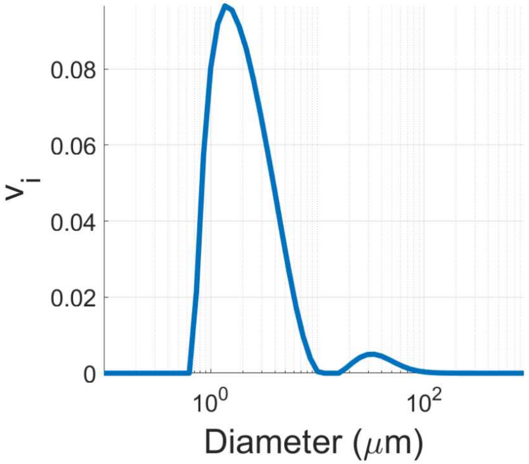
- Where
  - f_v is the particle volume fraction
  - v_i is the percent of total volume by particles with diameter d_i

Simulation of Fog Optical Properties

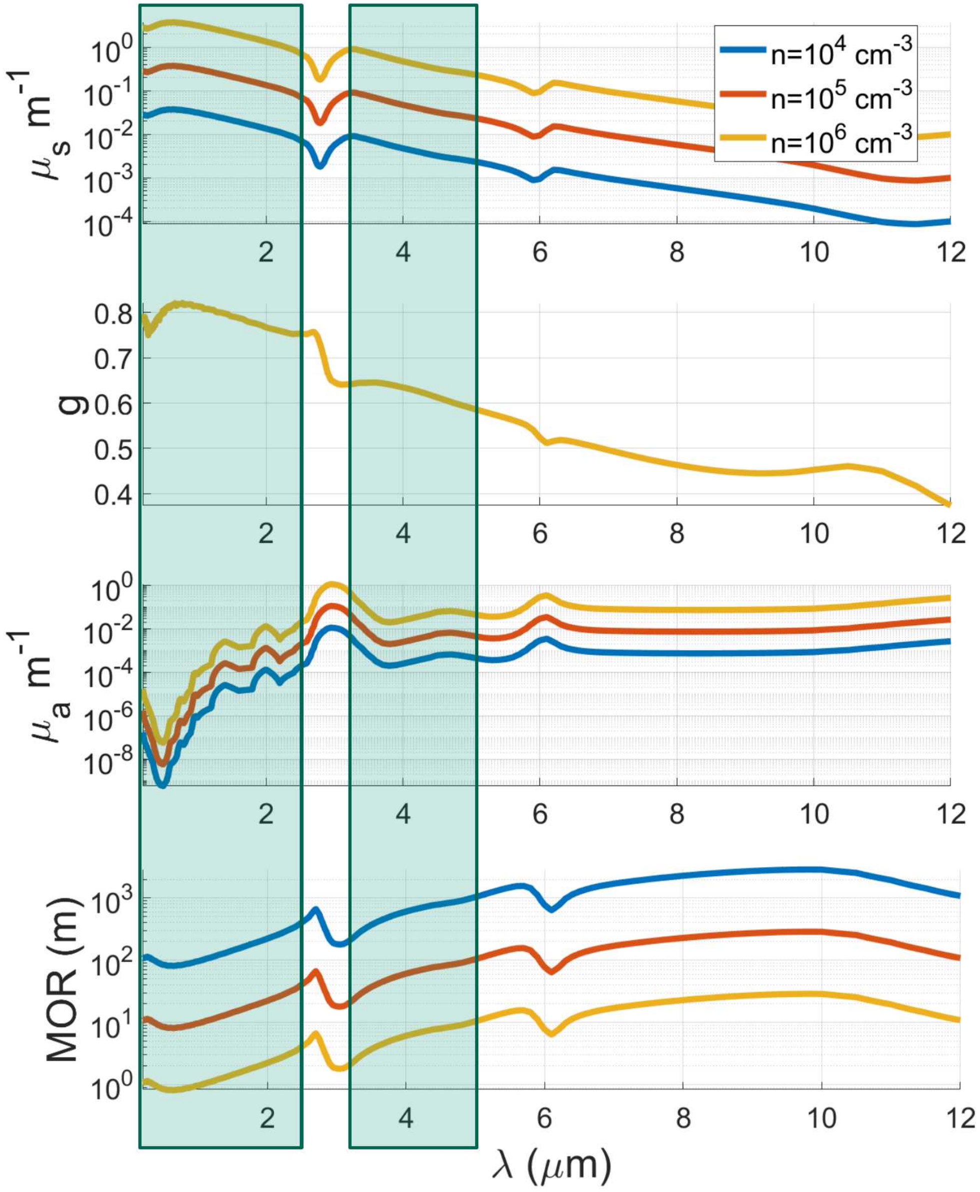
Water Refractive Index



Sandia Fog v_i



Diffusion approximation may be valid ($\mu_s > \mu_a$)



Simplifying the RTE: Continuity and Diffusion Equations

$$\frac{1}{c}\frac{\partial \Phi(\boldsymbol{r},t)}{\partial t} + \nabla \cdot \boldsymbol{J}(\boldsymbol{r},t) + \mu_a(\boldsymbol{r})\Phi(\boldsymbol{r},t) = S(\boldsymbol{r},t)$$

Where

- $\phi(\boldsymbol{r},t) = \int_{4\pi} d\Omega\, I\left(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}}\right)$ is the fluence rate (W/m²/s)
- $\boldsymbol{J}(\boldsymbol{r},t) = \int_{4\pi} d\Omega \widehat{\boldsymbol{\Omega}} I\left(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}}\right)$ is the flux density (W/m²/s)
- $S(\boldsymbol{r},t) = \int_{4\pi} d\Omega\, Q\left(\boldsymbol{r},t,\widehat{\boldsymbol{\Omega}}\right)$ is the source (W/m³/s)

Let: $\boldsymbol{J}(\boldsymbol{r},t) = -D(\mathbf{r})\nabla\phi(\boldsymbol{r},t)$ (Fick's first law of diffusion)

- $D = 1/3(\mu_s' + \mu_a)$ is the diffusion coefficient (m)
- $\mu_s' = \mu_s(1-g)$ is the reduced scattering coefficient for anisotropy g

$$\frac{1}{c}\frac{\partial \Phi(\boldsymbol{r},t)}{\partial t} - \nabla \cdot D(\boldsymbol{r})\nabla\phi(\boldsymbol{r},t) + \mu_a(\boldsymbol{r})\phi(\boldsymbol{r},t) = S(\boldsymbol{r},t)$$

Solving the RTE with Weak Angular Dependence

- Radiance at a detector at position $\boldsymbol{r}$ in homogeneous fog

$$I(\boldsymbol{r}, \widehat{\boldsymbol{\Omega}}) = \mu_s \int_{\boldsymbol{0}}^{\infty} dR \exp[-(\mu_s + \mu_a)R] \int_{4\pi} d\widehat{\boldsymbol{\Omega}}' \, f(\widehat{\boldsymbol{\Omega}}' \to \widehat{\boldsymbol{\Omega}}) I(\boldsymbol{r} - R\widehat{\boldsymbol{\Omega}}, \widehat{\boldsymbol{\Omega}}')$$

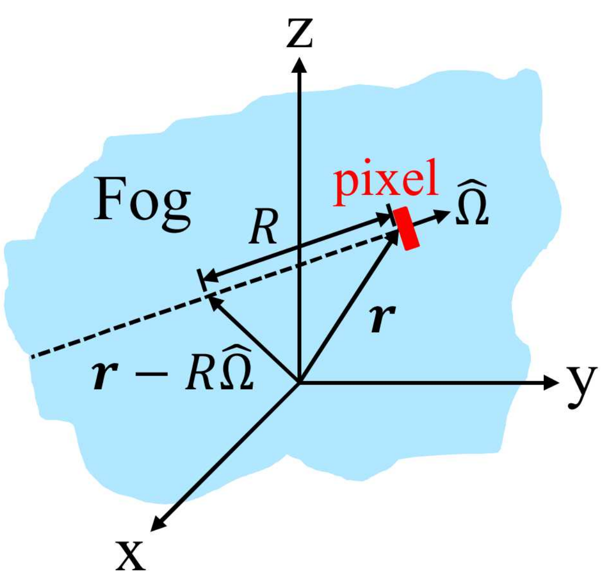
- Assuming isotropic scatter

$$I(\boldsymbol{r}, \widehat{\boldsymbol{\Omega}}) = \frac{\mu_s}{4\pi} \int_{\boldsymbol{0}}^{\infty} dR \exp[-(\mu_s + \mu_a)R] \, \phi(\boldsymbol{r} - R\widehat{\boldsymbol{\Omega}})$$

- Assuming weak angular dependence

$$I(\boldsymbol{r}, \widehat{\boldsymbol{\Omega}}) = \frac{\mu_s}{4\pi} \int_{\boldsymbol{0}}^{\infty} dR \exp[-(\mu_s + \mu_a)R] \left[\phi(\boldsymbol{r} - R\widehat{\boldsymbol{\Omega}}) + 3g\boldsymbol{J}(\boldsymbol{r} - R\widehat{\boldsymbol{\Omega}}) \cdot \widehat{\boldsymbol{\Omega}}\right]$$

Additional anisotropic term



[1] G. I. Bell and S. Glasstone, *Nuclear Reactor Theory*, U.S. Atomic Energy Commission, Washington DC, (1970).
[2] J. J. Duderstadt and L. J. Hamilton, *Nuclear Reactor Analysis*, John Wiley & Sons, Inc., New York, (1976).

Simulating Fluence Rate (Φ) and Flux Density ($\boldsymbol{J}$)

- Analytic Green's function solutions to the diffusion equation

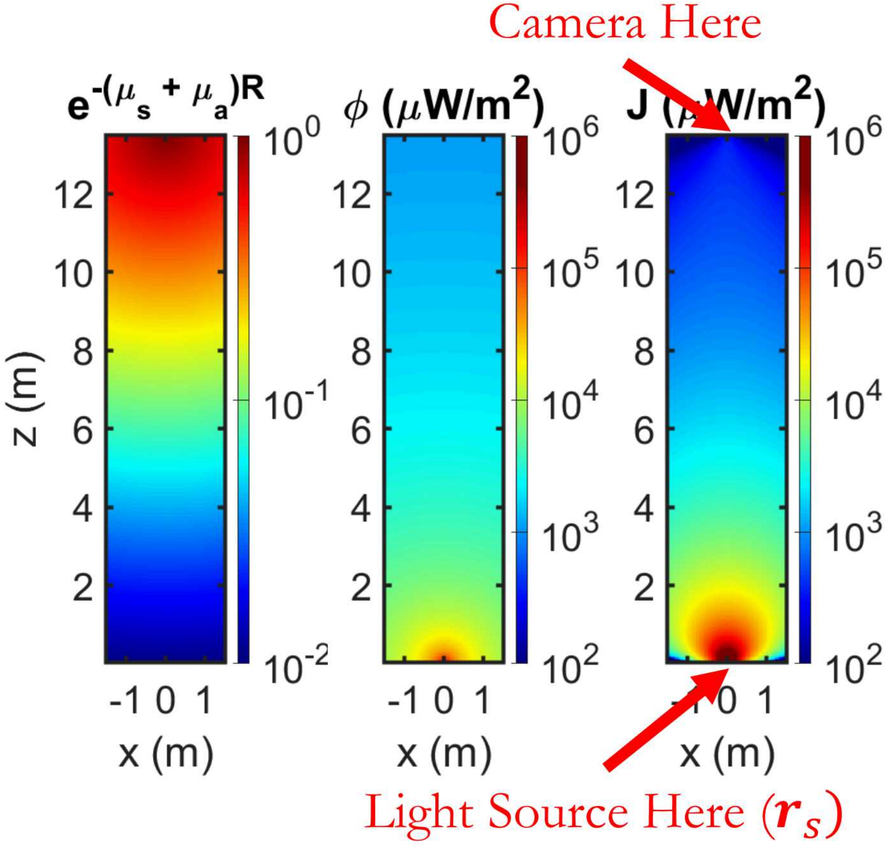
$$\phi(\boldsymbol{r}) = \left(\frac{S_o}{4\pi D}\right)\frac{\exp\left(\sqrt{\frac{\mu_a}{D}}|\boldsymbol{r}-\boldsymbol{r}_s|\right)}{|\boldsymbol{r}-\boldsymbol{r}_s|}$$

- Solving for $\boldsymbol{J}(\boldsymbol{r}) = -D\nabla\boldsymbol{\phi}(\boldsymbol{r})$

$$\boldsymbol{J}(\boldsymbol{r}) = \left[\frac{S_o(\boldsymbol{r}-\boldsymbol{r}_s)}{4\pi}\right]\left(\frac{-\sqrt{\mu_a/D}}{|\boldsymbol{r}-\boldsymbol{r}_s|^2}+\frac{1}{|\boldsymbol{r}-\boldsymbol{r}_s|^3}\right)\exp\left(\sqrt{\frac{\mu_a}{D}}|\boldsymbol{r}-\boldsymbol{r}_s|\right)$$

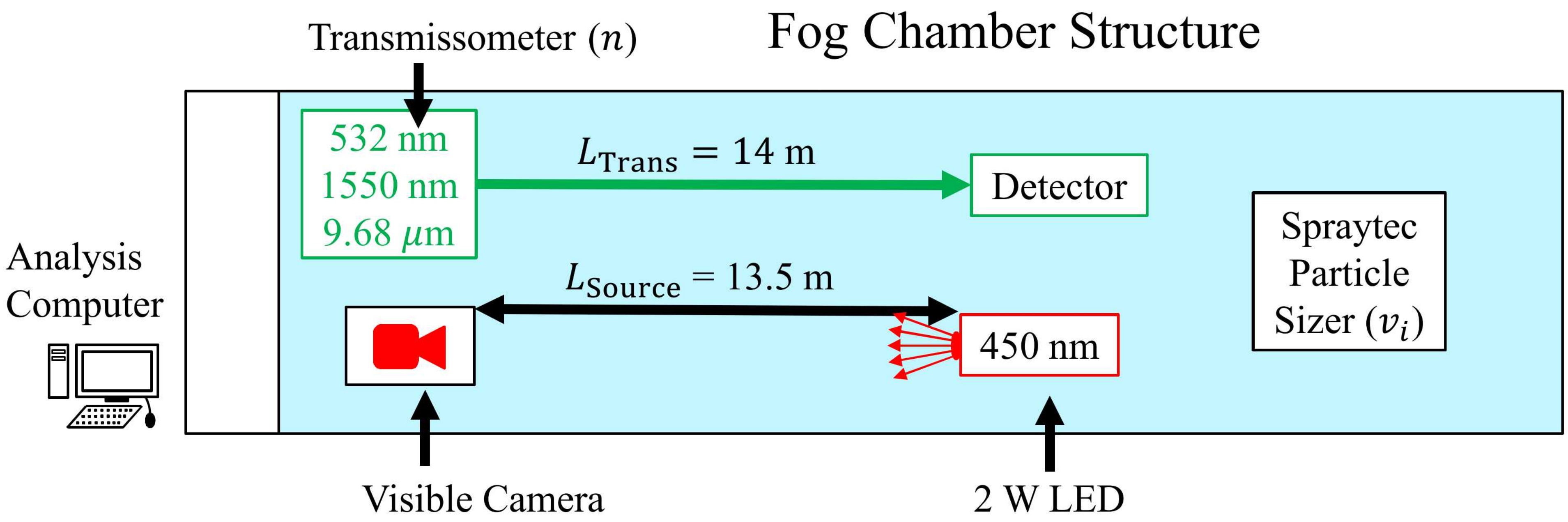
- Simulations use fog parameters at 450 nm and $n = 10^5$ cm^{-3}

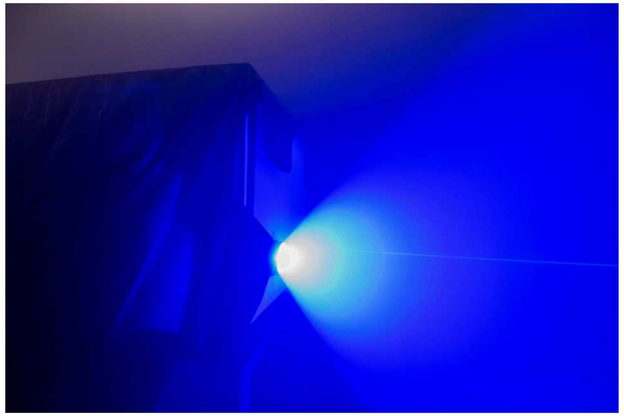
<u>Simulations</u>



[1] M. S. Patterson, B. Chance, and B.C. Wilson, *Applied Optics* **28**(12), 2331-2336, (1989).

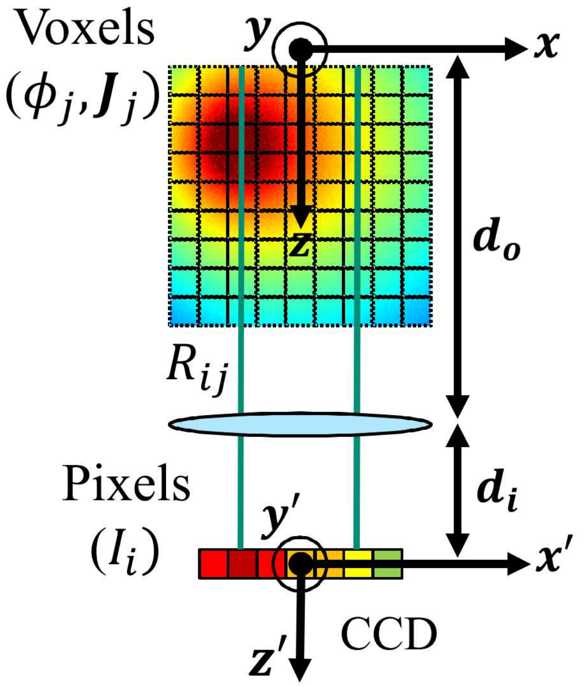
Experimental Setup





Line of Sight for Each Pixel Required for Model

- Perfectly linear imaging
  - Simple for single camera case, seemed to work
  - Requires that camera sensor dimension and d_i and d_o are known
  - $x' = \frac{-d_i}{d_o - z} x = -Mx$
  - $y' = \frac{-d_i}{d_o - z} y = -My$
- In general (aberration), $(x', y') = \boldsymbol{F}(x, y, z)$, where $\boldsymbol{F}$ is nonlinear
  - $\boldsymbol{F}$ can be approximated as a quadratic polynomial [1]
  - Coefficients can be estimated experimentally from calibration image plates
  - Allows co-registration of multiple cameras

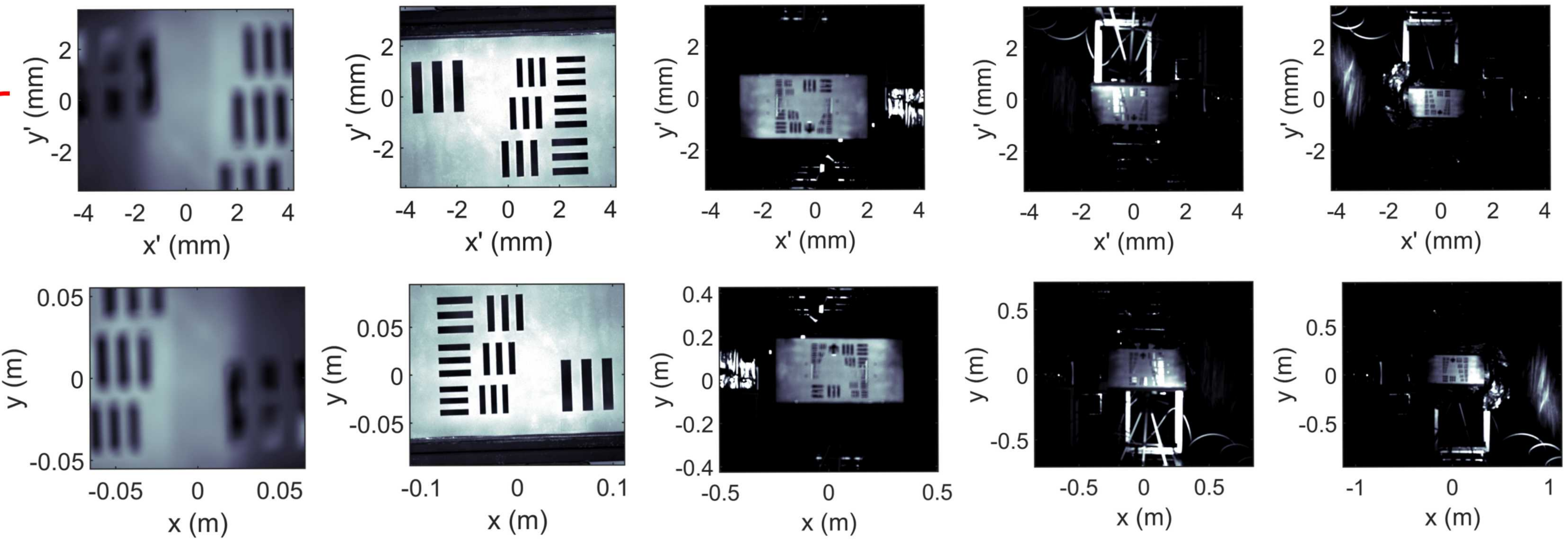


[1] S. M. Soloff, R. J. Adrian, and Z-C. Liu, *Measurement Science and Technology* **8**, 1441-1454 (1997).

Check Experimental Line of Sight Using Focus Images

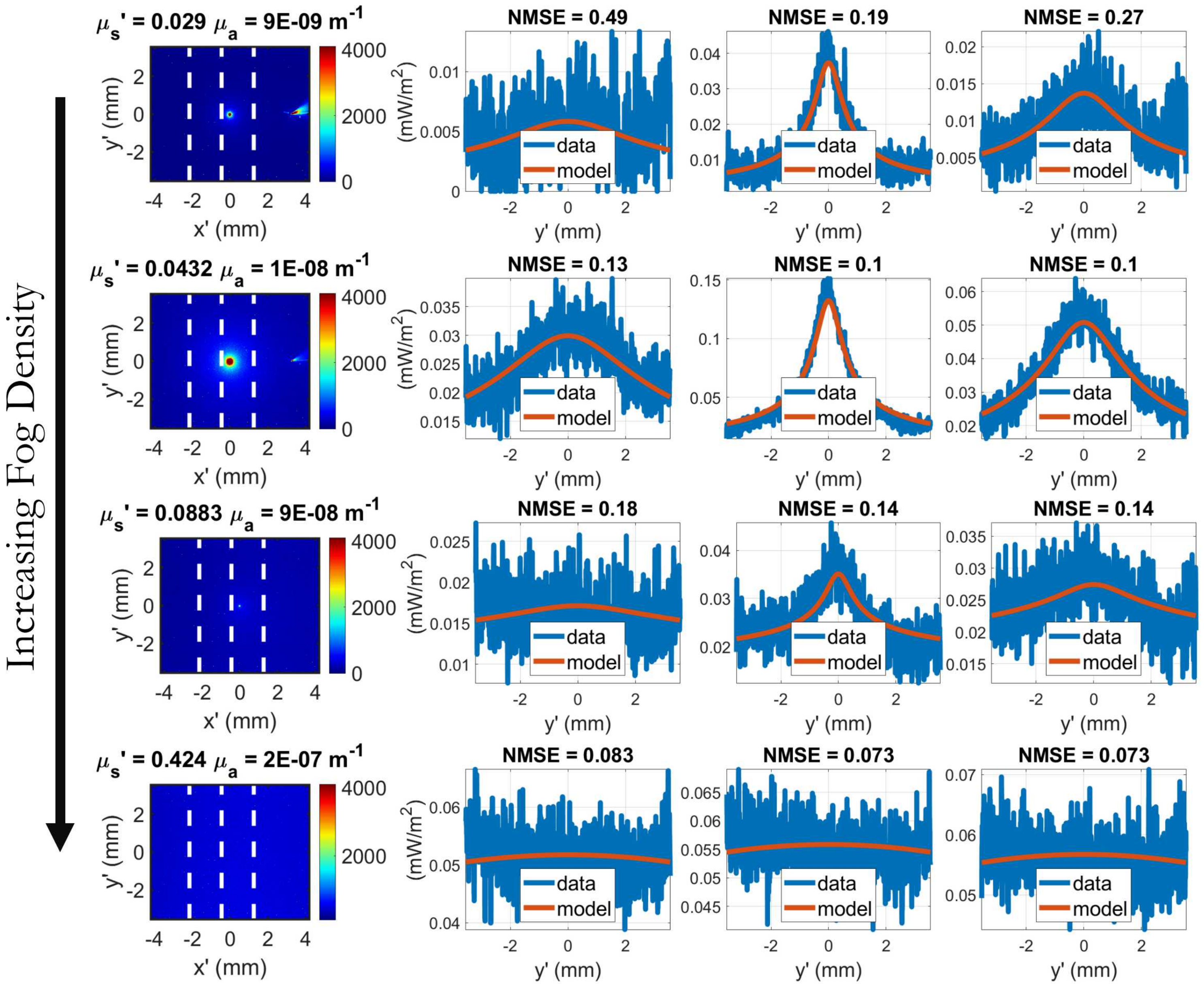
Target: 1 m Away | 1.5 m Away | 6 m Away | 10 m Away | 13.5 m Away

$$x = -x'/M$$
$$y = -y'/M$$



Line of Sight Error: 8.3 % | 2.8 % | 1.1 % | 11 % | 13 %

Comparing Model Predictions to Experimental Measurement



Conclusions

- The diffusion approximation to the RTE can be sufficient for modeling photon transport in fog
- Utilizing scattered photons has the potential to enhance system range by 10 times
- Success using computational diffuse imaging would improve situational awareness for
  - Navy, DOE (harbor and remote security, navigation)
  - DoD (tactical scenarios)
  - Aviation (take off and landing)
- **Future work:** leverage models for computational detection, localization, and imaging of objects

Thank you!

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