

# ***Predictive Guidance and Control for Robotic Obstacle Avoidance and Hypersonic Trajectory Generation***

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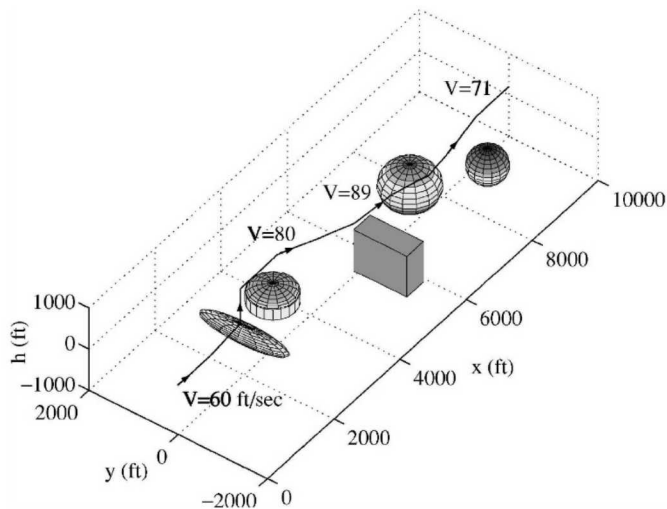
## **Thesis Committee**

Assistant Professor Hyeongjun Park (Chair)  
Assistant Professor Liang Sun  
Professor James McAteer (Dean's Representative)

# Overview



<https://www.popularmechanics.com/technology/robots/a27542297/meet-the-new-flying-robots-that-nasa-is-sending-into-space/>



[Yang et al. 2004]



<https://www.nbcnews.com/mach/science/hypersonic-airliner-would-take-you-los-angeles-tokyo-under-two-ncna1045986>

- Introduction
  - Motivation and Background
  - NMPC Framework
  - Research Contributions
  
- NMPC based robotic obstacle avoidance
  - Astrobbee free-flyer system architecture and purpose
  - Obstacle avoidance case studies and results
  
- NMPC based hypersonic trajectory generation
  - HFV system architectures
  - 2D and 3D model formulations
  - Trajectory generation case studies and results
  
- Conclusions and future work

# Introduction

- Model Predictive Control (MPC)
  - Effective feedback control for constrained control problems with multi-variable systems
  - Various applications [Camacho 2007; Qin & Badgwell 2003]
    - Petrochemical, Power Management, Industrial Process Control
  
- Linear MPC
  - Widely applied & mature technology [Mayne et al. 2000; Allgöwer et al. 2004]
  
- Nonlinear Model Predictive Control (NMPC)
  - Used for full fidelity nonlinear system dynamics and constraints integration into MPC framework
  - **Main challenge: computational complexity** for real-time implementation

- Methodologies to simplify real-time computations
  - Model order reduction/ simplification techniques [Choroszuca 2015; Hovland et al. 2007; Dufour et al. 2003]
    - Focusing on dominant dynamics / linear approximation
    - Degraded closed-loop performance with significant nonlinearities
  - Explicit MPC [Bemporad et al. 2002; Johansen 2004; Tøndel 2003]
    - Explicit solutions are obtained offline and stored locally
    - Large local memory may be required
    - If problem conditions change, the solution can not adapt
  - **Numerical methods and algorithms** [Diehl et al. 2009; Cannon 2004]
    - Real-time optimization solvers
    - Advantageous for
      - Nonlinear systems
      - Systems with changing parameters

- Interior Point (IP) Optimization
  - Effective method for solving nonlinear constrained optimization problems [Wang & Boyd 2008; Ding et al. 2016; Klintberg & Gros 2017 ]
    - Directly integrates equality and inequality constraints into objective function optimization
  - Widely available in solvers in commercial and open source spaces
    - MATLAB
    - COIN-OR IPOPT (Interior Point OPTimizer) [Wächter & Bigeler 2005]
  - IP optimization will be used in all NMPC implementations discussed in this work

- Constrained optimal control problem over a receding horizon

$$\begin{aligned}
 & \min_{u(\cdot)} J(x(\cdot), u(\cdot)), \\
 & \text{where } J(x(\cdot), u(\cdot)) = \Phi(x(t+N)) + \sum_{k=t}^{t+N-1} L(x(k), u(k)), \\
 & \text{subject to} \\
 & x(k+1) = f(x(k), u(k)), \quad f: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^n, \\
 & x(t) = x_t, \quad x_t \in \mathbb{R}^n, \\
 & C(x(k), u(k)) \leq 0, \quad C: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^l, \quad k = t, \dots, t+N-1, \\
 & \bar{C}(x(k)) \leq 0, \quad \bar{C}: \mathbb{R}^n \rightarrow \mathbb{R}^q, \quad k = t, \dots, t+N.
 \end{aligned} \tag{1}$$

- Minimize the cost function over a prediction horizon
- Obtain an optimal input sequence given an initial state
- Implement the first element of the input sequence

## NMPC controller based on IP optimization for real-time robotics and flight vehicle control

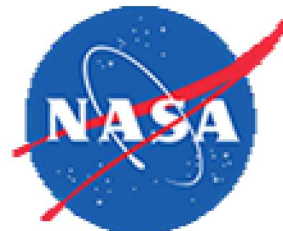
### Robotic obstacle avoidance

- ❑ **Astrobbee Free-Flyer**
  - Flight in 3D constrained by hardware and environment
- ❑ **Case Studies**
  - Static obstacle avoidance cases w/ nonlinear keep-out zones

### Hypersonic trajectory generation

- ❑ **Hypersonic Flight Vehicles**
  - 2D and 3D models constrained by hardware and mission requirements
  - Highly nonlinear system dynamics w/ multi-variable dependencies
- ❑ **Case Studies**
  - Trajectory generation considering No-Fly-Zone avoidance in tandem with waypoint following

# Robotic Obstacle Avoidance



This research was funded by the New Mexico Space Grant Consortium.

New Mexico Space Grant Consortium (NMSGC) is a statewide collaboration with industry, government and academic partners. New Mexico State University (NMSU) is the lead Land Grant and Space Grant University in the state of New Mexico. NMSGC is a member of the congressionally funded National Space Grant College and Fellowship Program that has been administered by the National Aeronautics and Space Administration (NASA) since 1989.

## Free Flyers

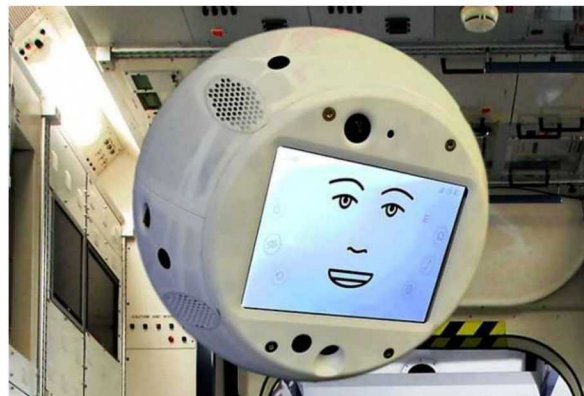
- Robots flying with 6-DOF on ISS
- Manual or Semi-Autonomous Control

## Uses

- Repetitive/simple environmental monitoring
- Remote agent to assist astronauts
- Guest science



<https://www.electronicweekly.com/news/research-news/inertia-steers-int-ball-drone-international-space-station-2017-07/>



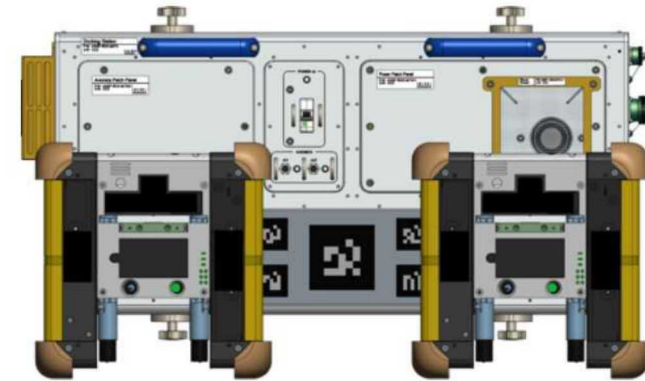
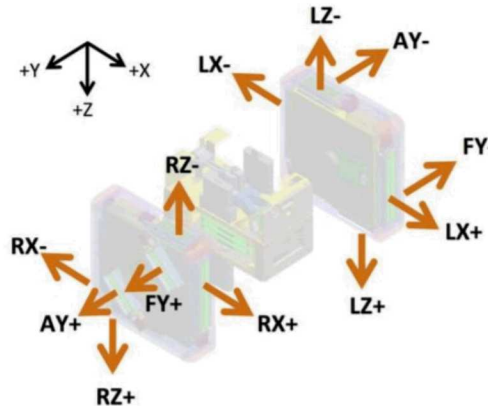
<https://www.bbc.com/news/technology-44655675>



<https://www.nasa.gov/astrobee/>

## Physical Hardware

- Electric impeller propulsion
- Modular structure

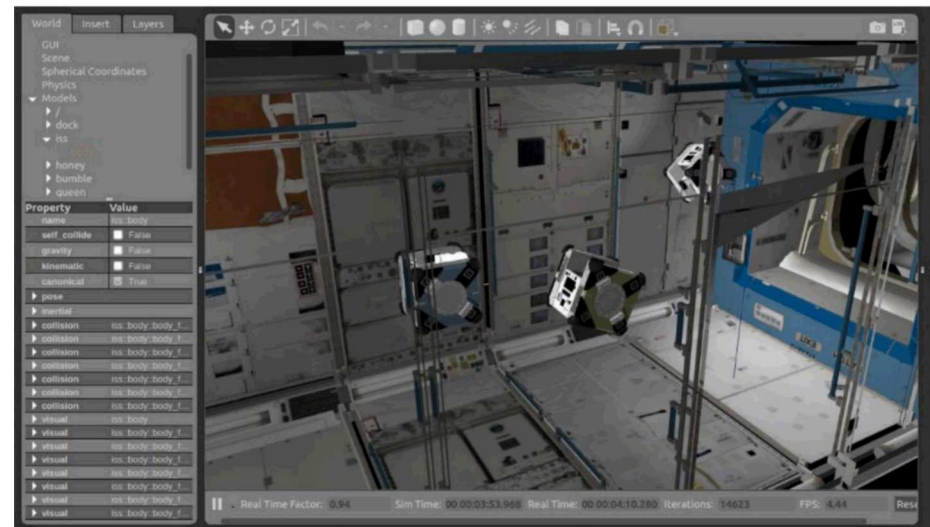


## Electronic Hardware

- ARM processor and Android OS
- Multiple cameras/sensors

## Software

- Programming with ROS
- Hardware simulation with Simulink
- Operation visualization with Gazebo



## Motivation

1. Astronaut and ISS structural safety requires collision avoidance during Free-Flyer maneuvering to guarantee
  2. Minimize maneuvering time while maximizing battery life
- **Goal: Autonomous real-time obstacle avoidance**

## Approach

1. Create G&C algorithm based on NMPC framework
2. Simulate realistic scenarios with the G&C algorithm (avoidance of static obstacles wrt. ISS spatial limitations)
3. Verify and validate the G&C algorithm with the Gazebo/ROS simulator

## NMPC problem formulation

Minimize cost function J

where

$$J(x(\cdot), u(\cdot)) = (x(t + N) - x_f)^T P (x(t + N) - x_f) + \sum_{k=t}^{t+N-1} (x(k) - x_f)^T Q (x(k) - x_f) + u(k)^T R u(k),$$

Terminal Cost Weight  
(Final Positioning & Velocity)

Incremental Cost Weight  
for Position & Velocity

Incremental Cost Weight for  
Control Input (Battery Use)

## Constraints

1. Astrobee's equations of motion
2. Propulsion system constraints (maximum thrust)
3. Maximum safe velocity
4. ISS corridor area
5. Collision avoidance from obstacle keep out zones

# Constraints (1/2)

1. Astrobee's equations of motion (translational motion only)

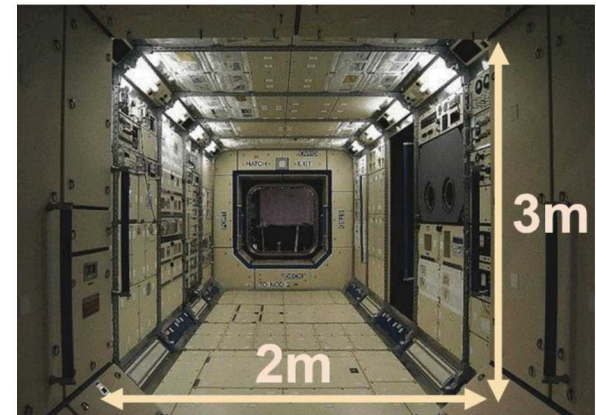
$$\ddot{x} = \frac{F_x}{m}, \quad \ddot{y} = \frac{F_y}{m}, \quad \ddot{z} = \frac{F_z}{m}$$

2. Propulsion system constraints (maximum thrust)

$$-0.6 \leq F_i \leq 0.6 \text{ [N]}$$

3. Maximum velocity  $-0.5 \leq V_i \leq 0.5 \text{ [m/s]}$

4. 2 m x 3 m x 2 m area  $0 \leq y \leq 2 \text{ [m]}$   
 $0 \leq z \leq 3 \text{ [m]}$



## 5. Collision avoidance constraints with keep-out zone

Astrobee: **Sphere approximation**

$$(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 \geq r^2$$

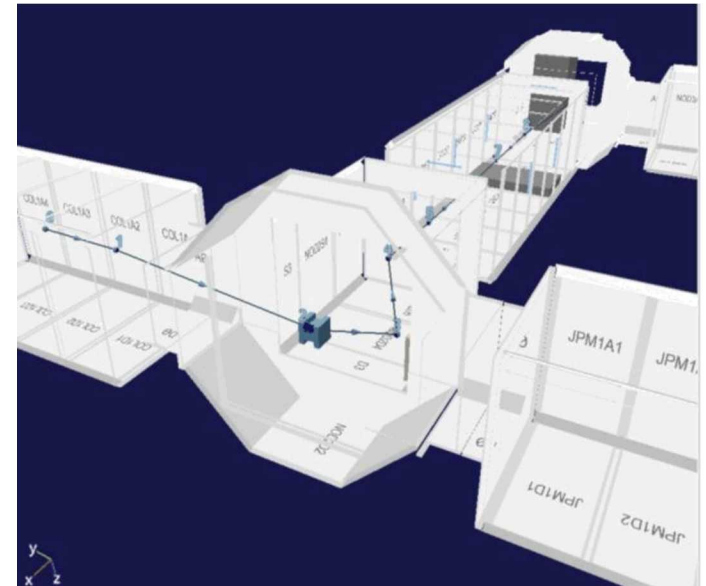
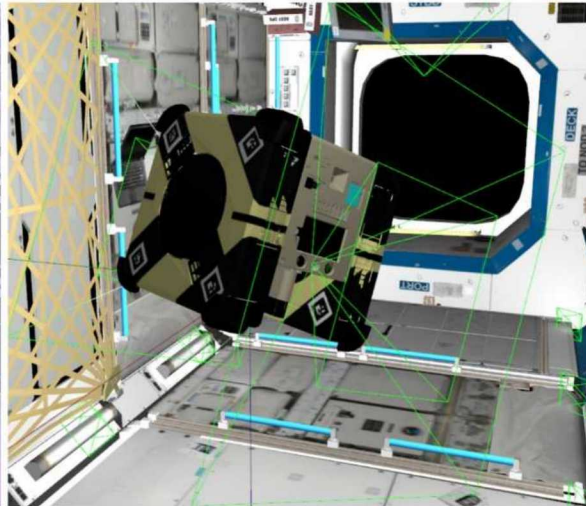
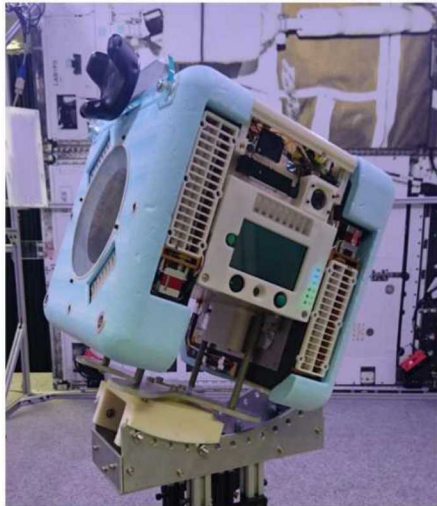


Astronaut: **Cylinder approximation**

$$(x - x_o)^2 + (y - y_o)^2 \geq r^2$$



- Cases for obstacle avoidance demonstration
  - Case 1: Static Astrobeer avoidance
  - Case 2: Static astronaut avoidance

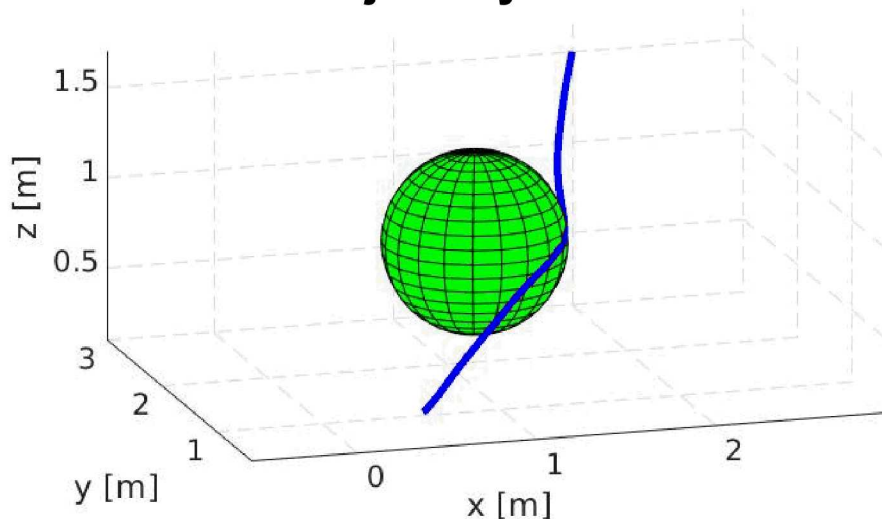


# Case 1 – Static Astrobees (1/2)

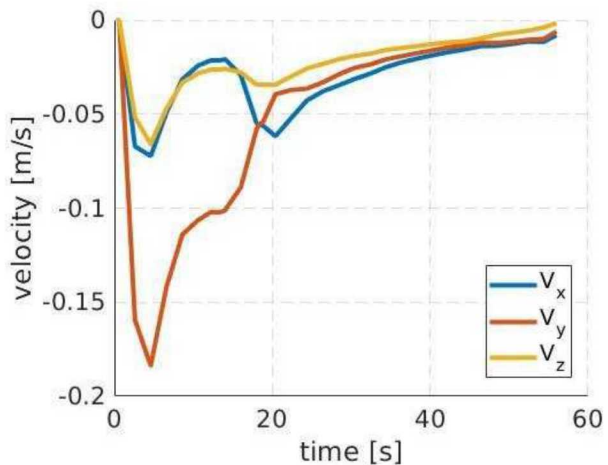
## Case 1 problem set-up

Parameter	Value
Initial Position	(2, 3, 1.5) m
Desired Position	(0.3, 0.3, 0.3) m
Other Astrobees Position	(1, 1.5, 0.8) m
Collision Avoidance Keep-out Radius	0.5 m
Sample Time	2 s
Prediction Horizon	40

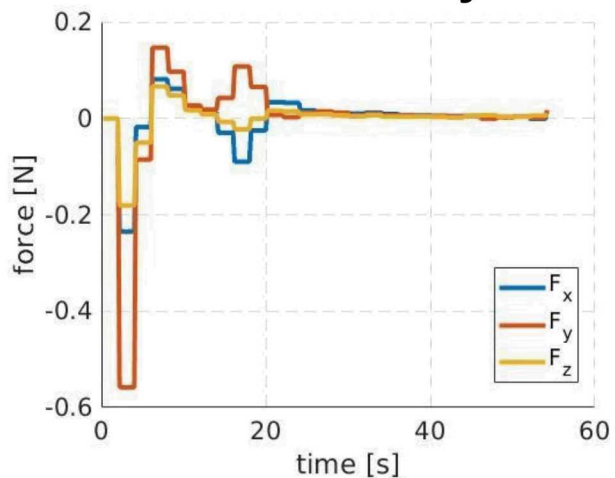
## Trajectory results



## Velocity history

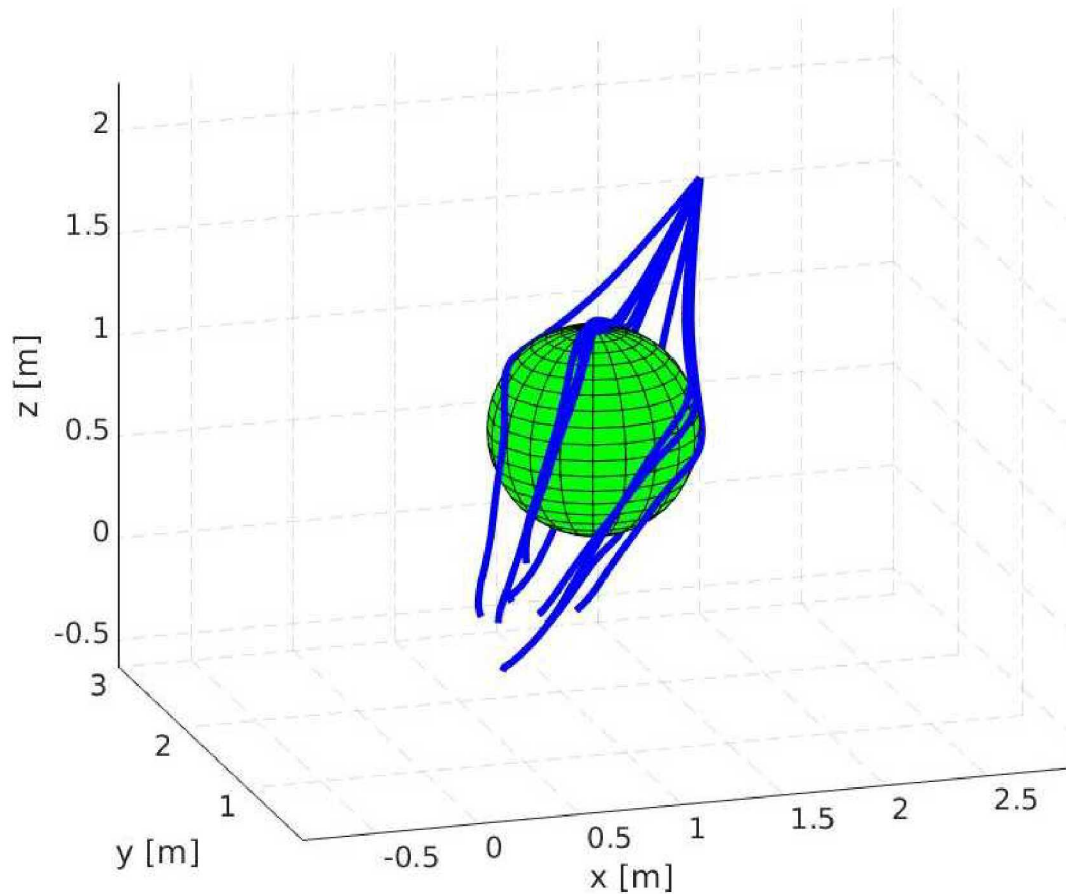


## Force history



# Case 1 - Static Astrobees (2/2)

**Multiple trajectories for different position goals**

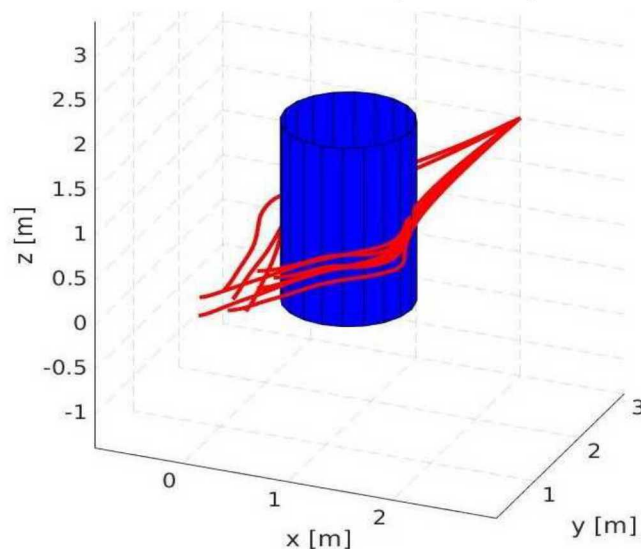


# Case 2 – Static Astronaut

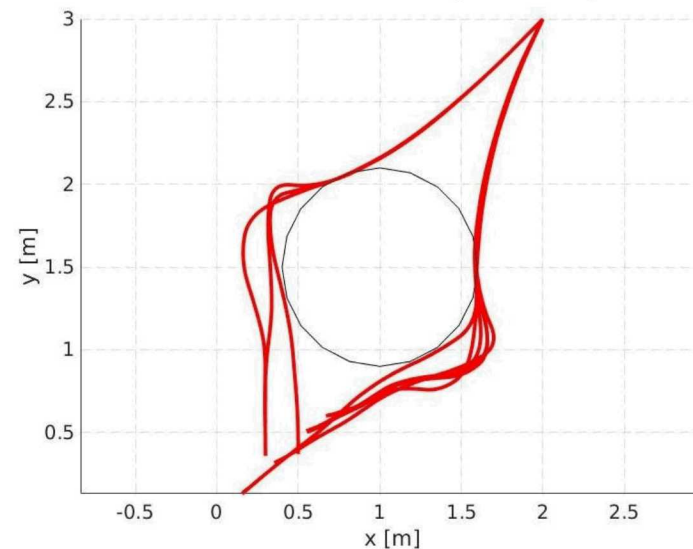
## Case 2 problem set-up

Parameter	Value
Initial Position	(2, 3, 1.5) m
Desired Position	(0.3, 0.3, 0.3) m
Other Astrobees's Position	(1, 1.5, 0.8) m
Collision Avoidance Keep-out Radius	0.5 m
Sample Time	2 s
Prediction Horizon	40

### Isometric trajectory



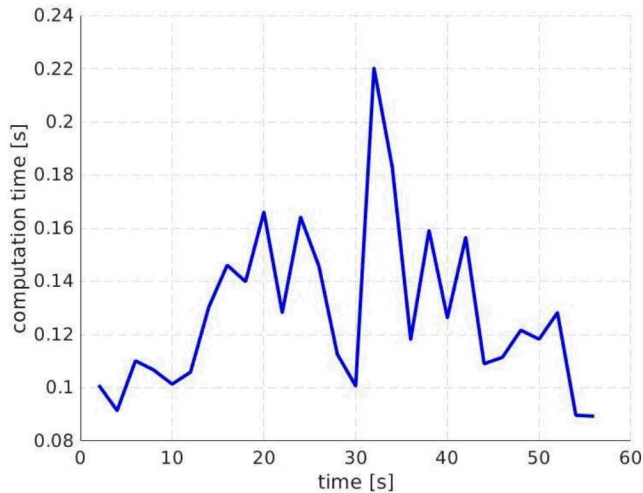
### Top-down trajectory



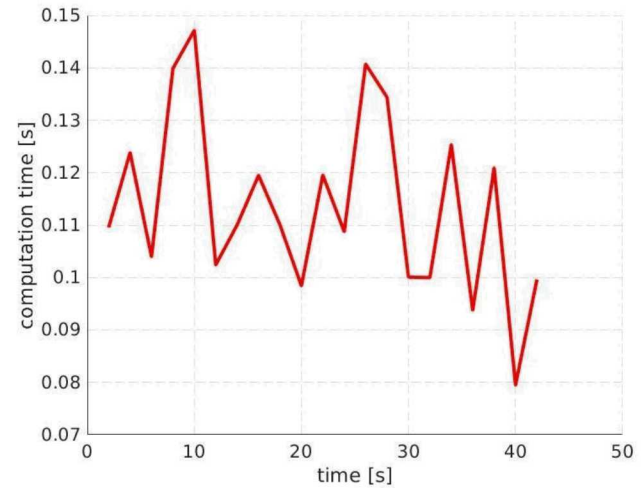
## IPOPT Computation Time Comparison

	Case 1	Case 2	Sample Time
Avg / Max Comp. Time [s]	0.13 / 0.22	0.11 / 0.15	2

### Case 1 Computation Time History

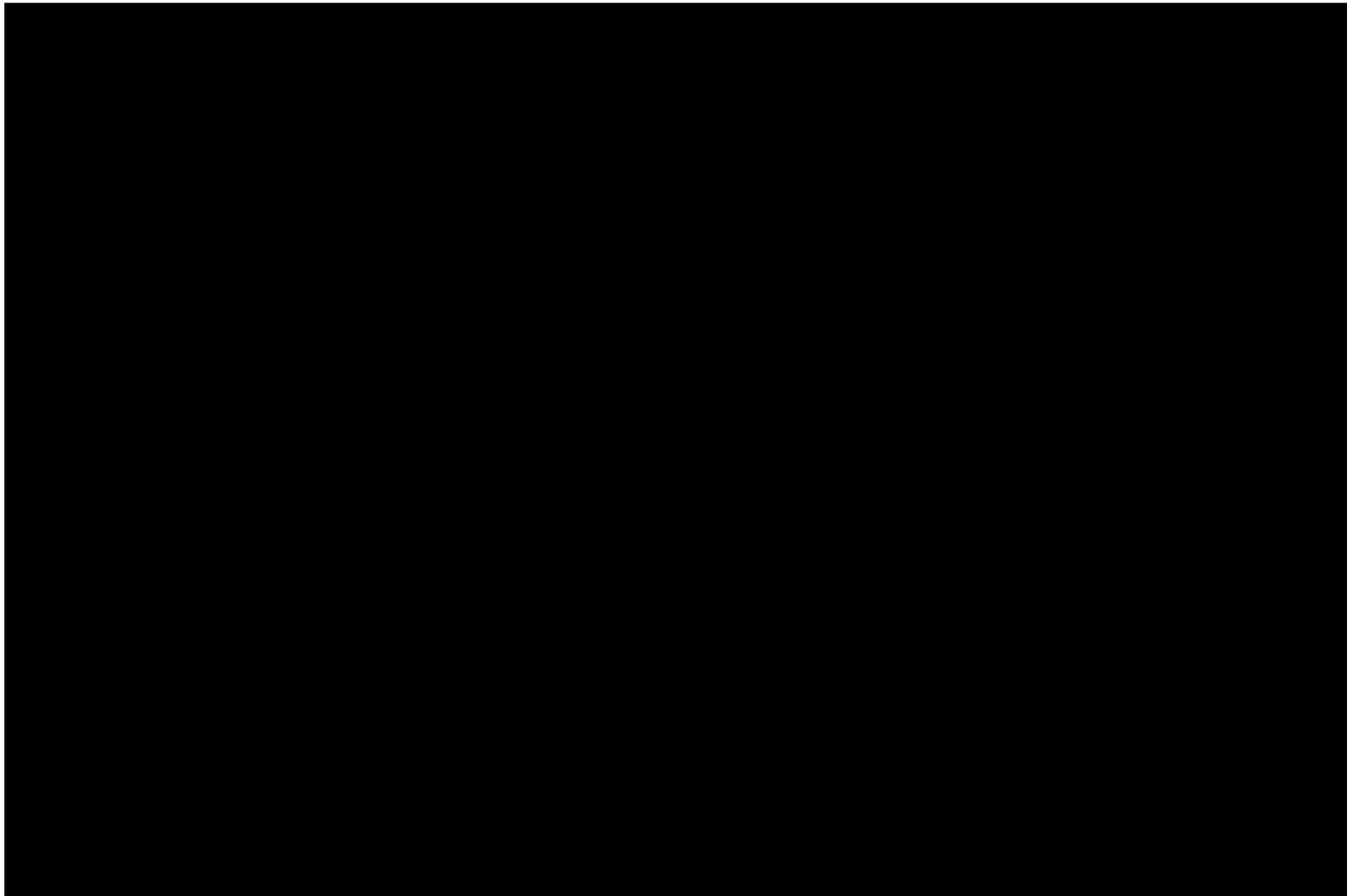


### Case 2 Computation Time History

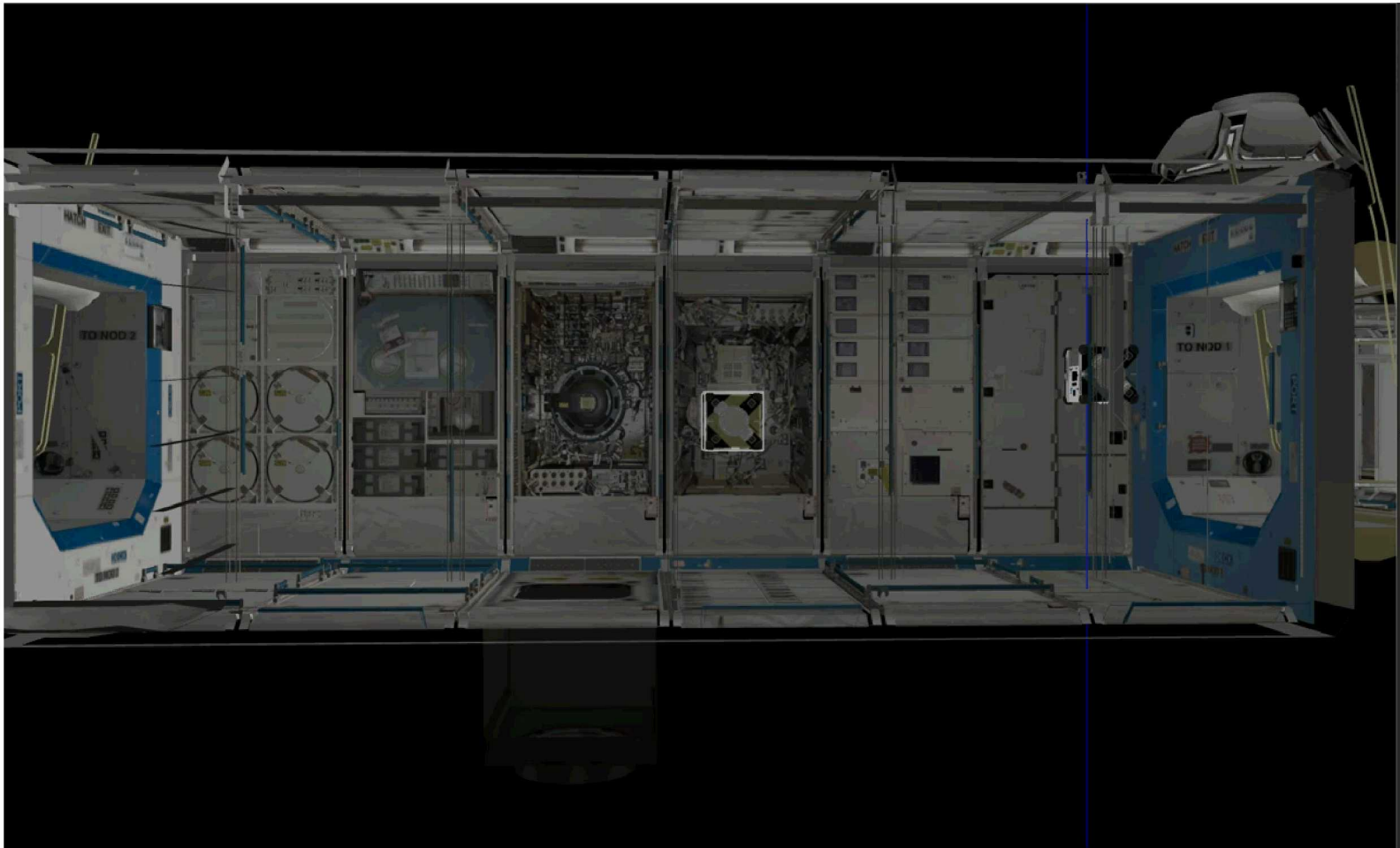


– Simulations on a laptop with Intel(R) CPU @ 1.60 GHz

## Kinematic Model Visualization



## Dynamic Model Visualization



## Completed Work

- Static obstacle avoidance with NMPC controller
- Integration obstacle avoidance results into ROS/Gazebo visualization framework

## Future work

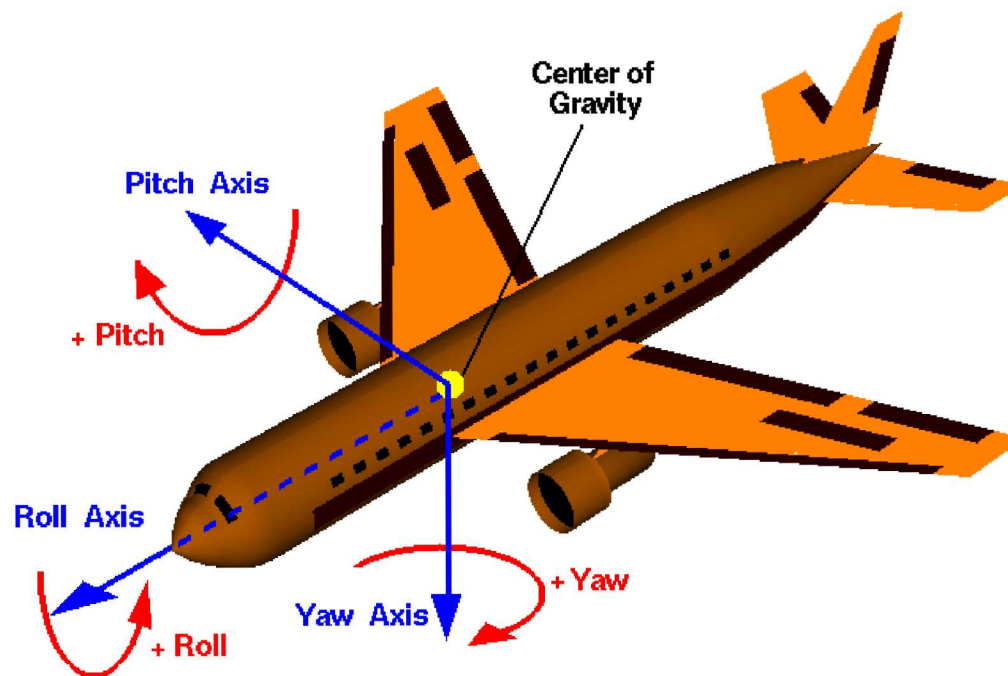
- Moving obstacle / Multiple Obstacle cases both offline and implemented into ROS/Gazebo
- Validating controller design using NASA Ames test facilities

# Hypersonic Trajectory Generation



This research was funded by the Sandia National Laboratories Laboratory-Directed Research and Development Program. Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525

# Aircraft Principal Axes



NASA Glenn Research Center  
<https://www.grc.nasa.gov/WWW/K-12/airplane/rotations.html>

- **Hypersonic Glide Vehicles (HGVs)** are reentry vehicles with dynamic flight capabilities but no on-board propulsion
- **Hypersonic Cruise Vehicles (HCVs)** hypersonic flight platforms with on-board propulsion
  - Fully atmosphere based booster platforms or suborbital reentry vehicles

## Aircraft vehicle dynamics models

Model	States	Controls	EOM	Constraints
2 DOF navigation + 1 DOF point mass	$x, y, \epsilon$	$\dot{\epsilon}$	$\dot{x} = V_o \sin(\epsilon)$ $\dot{y} = V_o \cos(\epsilon)$ $\epsilon = \int \dot{\epsilon} dt$	$\dot{\epsilon} \leq \frac{V_o}{R_{\min}}$
3 DOF navigation + 1 DOF point mass	$x, y, z, \epsilon$	$\dot{\epsilon}, \gamma$	$\dot{x} = V_o \sin(\epsilon) \cos(\gamma)$ $\dot{y} = V_o \cos(\epsilon) \cos(\gamma)$ $\dot{z} = -V_o \sin(\gamma)$	$\gamma \leq \gamma_{\max}$
3 DOF navigation + 2 DOF point mass	$x, y, z, \gamma, \epsilon$	$V, C_L, \phi$	$\dot{\epsilon} = \frac{1}{2} \rho \frac{m}{S}^{-1} V C_L \sin(\phi)$ $\dot{\gamma} = \frac{1}{2} \rho \frac{m}{S}^{-1} V C_L \cos(\phi) - \frac{g \cos(\gamma)}{V}$	$V \leq V_{\max}$ $C_L < C_{L_{\max}}$ $\phi \leq \phi_{\max}$
3 DOF navigation + 3 DOF point mass	$x, y, z, \gamma, \epsilon, V$	$T, C_L, \phi$	$\dot{V} = \frac{T}{m} - \frac{g}{m} \sin(\gamma) - \frac{1}{2} \rho \frac{m}{S}^{-1} V^2 C_D(C_L)$	$T \leq T_{\max}$
Many more models!	....	....	....	....

+dynamics  
+variables  
+complexity  
+details

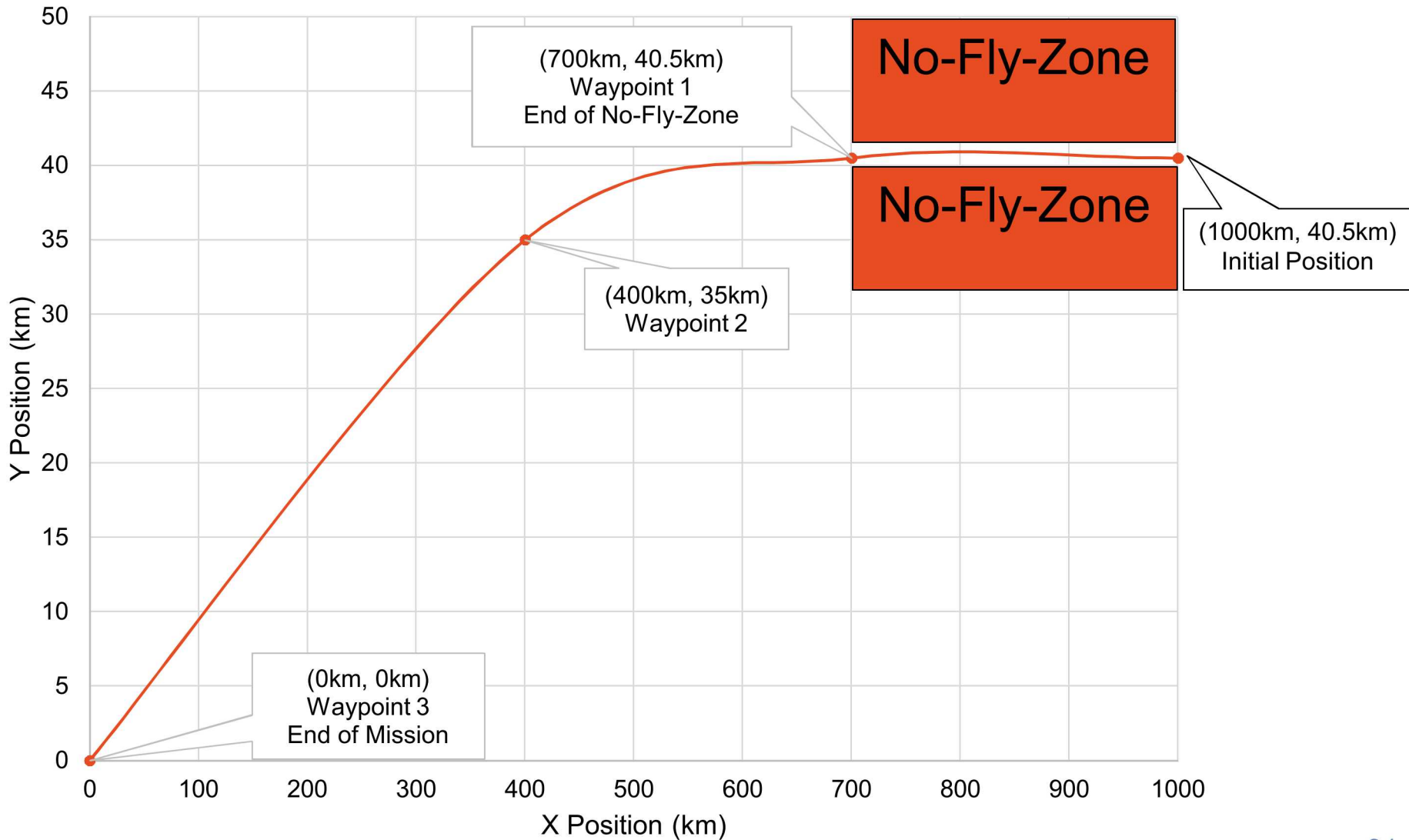
# ***HCV 2D Model Formulation***

## Mission Structure

1. Hypersonic release with constant thrust to maintain velocity & altitude
2. Hypersonic cruise through **No-Fly-Zone** corridor (Corridor is 300km in length and 1km in width)
3. Waypoint 1 at the end of hypersonic corridor
4. Turn to waypoint 2
5. Mission end at waypoint 3

- The HCV (**Hypersonic Cruise Vehicle**) governed by 2D eq. of motion
- This model used as baseline for future higher fidelity models

# 2D Mission Visualization



# 2D Model Assumptions & Control Input

- Assumptions
  - Altitude is maintained as constant
  - Deceleration is **zero** (decouples velocity change from bank angle control input)
- Control input
  - Control  $u$  is a function of bank angle **normalized** by tangent of the maximum bank angle
  - Thrust is constant and **maintains acceleration at 0**

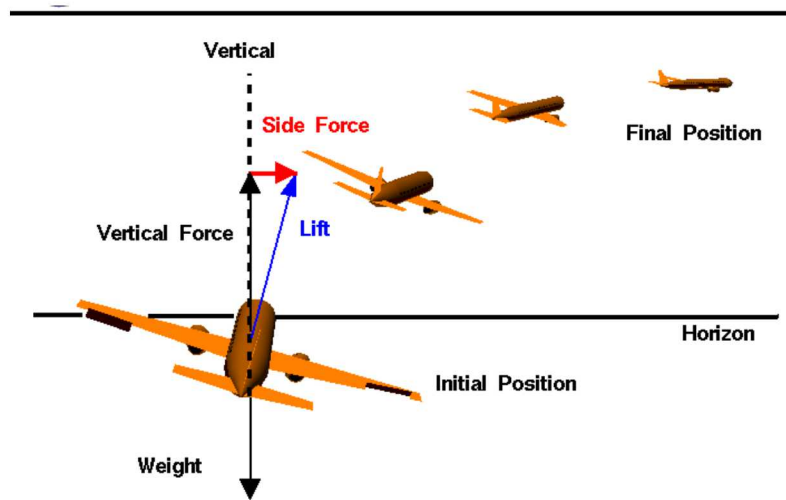
$$u = \frac{\tan(\sigma)}{\tan(\sigma_{\max})}$$

$$\sigma_{\max} = \pm 20^\circ$$

$$-1 \leq u(k) \leq 1$$

[Jorris 2007]

## Banking Turn



<https://www.grc.nasa.gov/www/k-12/airplane/turns.html>

## Equations of Motion

$$\dot{x} = V \cos(\theta)$$

$$\dot{y} = V \sin(\theta)$$

$$\dot{\theta} = \frac{\tan(\sigma_{max})}{V} u$$

$$\dot{V} = 0$$

## State Space Model

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \\ V \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad \dot{\mathbf{x}} = \begin{bmatrix} x_4 \cos(x_3) \\ x_4 \sin(x_3) \\ \frac{0.363u}{x_4} \\ 0 \end{bmatrix}$$

$$u = \frac{\tan(\sigma)}{\tan(\sigma_{max})}$$

$$\tan(\sigma_{max}) = \tan(20^\circ) = 0.363$$

$$\dot{\mathbf{x}} = [\dot{x} \ \dot{y} \ \dot{\theta} \ \dot{V}]^T = f(\mathbf{x}, \mathbf{u})$$

Symbol	Parameter
$x$	x position
$y$	y position
$\theta$	Heading angle
$V$	Velocity
$a$	Acceleration
$\sigma$	Bank angle
$\sigma_{max}$	Max. bank angle

- NMPC problem formulation

$$\begin{aligned} \text{Minimize } J(x(\cdot), u(\cdot)) &= (x(t+N) - x_f)^T P (x(t+N) - x_f) \\ &+ \sum_{k=t}^{t+N-1} (x(k) - x_f)^T Q (x(k) - x_f) + u(k)^T R u(k), \end{aligned}$$

subject to

$$x(k+1) = f(x(k), u(k)),$$

$$x(t) = x_i,$$

$$-1 \leq u(k) \leq 1,$$

$$40 \leq x_2(k) \leq 41, \quad \text{if } 700 \leq x_1(k) \leq 1000$$

$$x(t+N) = x_f \quad x_f \text{ is updated from waypoints}$$

- Interior-point optimization algorithms require
  - Hessian of Hamiltonian
  - Jacobian of state constraints

$$H(k) = L(\mathbf{x}(k), \mathbf{u}(k)) + \sum_{l=1}^n \lambda_l g_l(\mathbf{x}(k), \mathbf{u}(k))$$

$$\frac{\partial^2 H}{\partial z_i \partial z_j} = \frac{\partial^2 L(\mathbf{x}(k), \mathbf{u}(k))}{\partial z_i \partial z_j} + \sum_{l=1}^n \lambda_l \frac{\partial^2 g_l(\mathbf{x}(k), \mathbf{u}(k))}{\partial z_i \partial z_j}$$

where

$$\begin{aligned} \mathbf{z} &= [x_1, x_2, x_3, x_4, u]^T \\ &= [z_1, z_2, z_3, z_4, z_5]^T \end{aligned}$$

Symbol	Parameter
$H$	Hamiltonian
$L$	Incremental Cost Function
$g$	Constraint state formulation
$\lambda$	Lagrangian multiplier
$\mathbf{x}$	State vector
$\mathbf{u}$	Control vector
$\mathbf{z}$	Total states & controls vector
$n$	Total number of state constraints

## State Constraint Formulation

$-1 \leq u(k) \leq 1 \Rightarrow$  Handled by explicit control constraint implementation – no state formulation needed

$x(k + 1) = f(x(k), u(k)) \Rightarrow$  one constraint formulation for every state (total of 4)  
 $g_1, g_2, g_3, g_4$

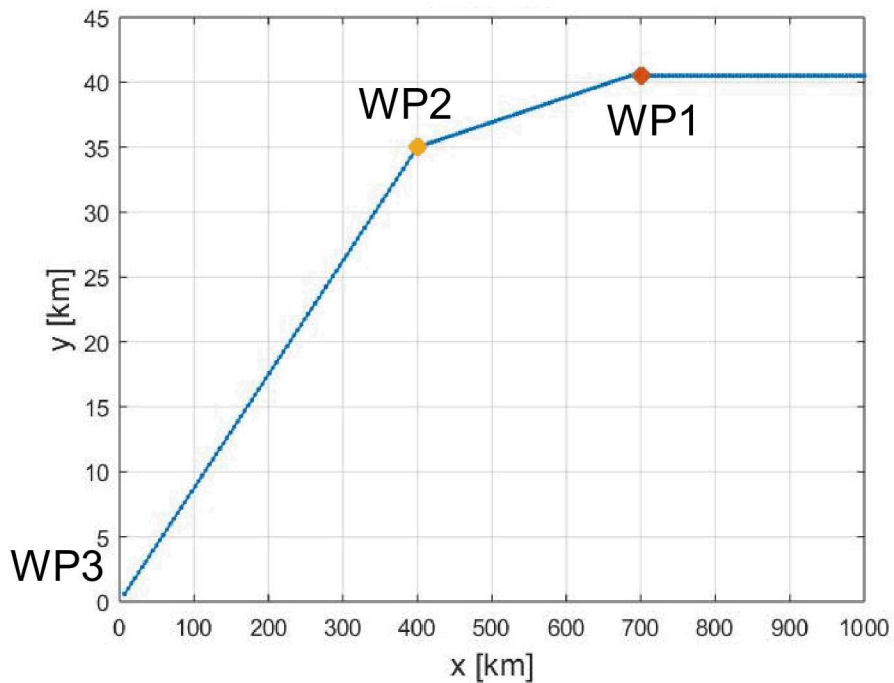
$40 \leq x_2(k) \leq 41 \Rightarrow$  only considers 1 state (y-pos =  $x_2$ )  
 $g_5$

$$g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{bmatrix} = \begin{bmatrix} x_1(k + 1) - (x_1(k) + Ts \cdot f(x(k), u(k))) \\ x_2(k + 1) - (x_2(k) + Ts \cdot f(x(k), u(k))) \\ x_3(k + 1) - (x_3(k) + Ts \cdot f(x(k), u(k))) \\ x_4(k + 1) - (x_4(k) + Ts \cdot f(x(k), u(k))) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} x_6 - x_1 + Ts \cdot x_4 \cos(x_3) \\ x_7 - x_2 + Ts \cdot x_4 \sin(x_3) \\ x_8 - x_3 + Ts \cdot 0.363x_5/x_4 \\ x_9 - x_4 \\ x_2 \end{bmatrix}$$

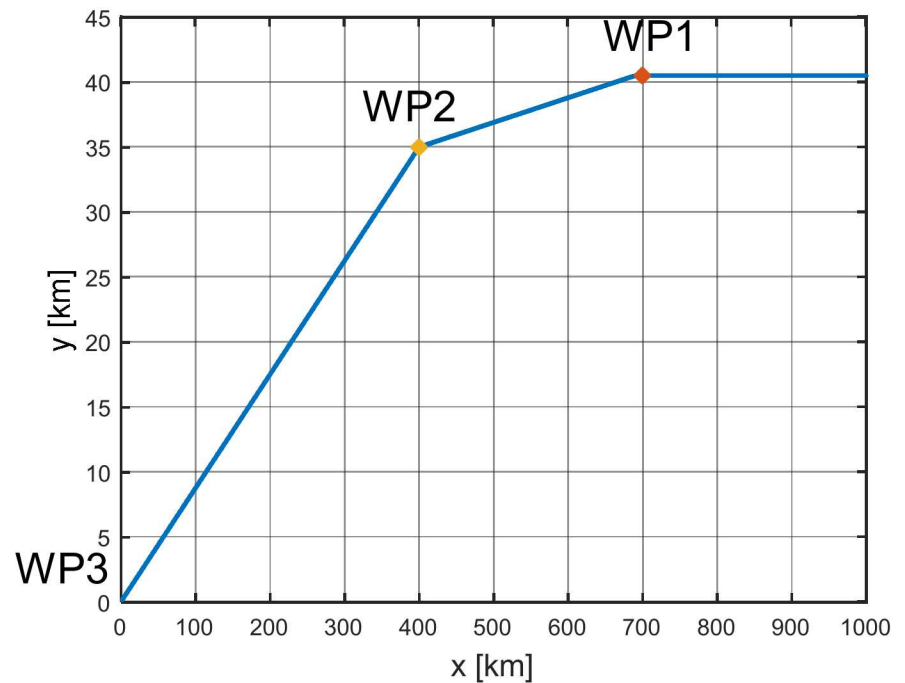
- **MATLAB** implementation using 'fmincon' function with interior-point optimization algorithm was used for baseline
- COIN-OR **IPOPT** implemented into a Simulink model in an Ubuntu Linux environment
- Ad-hoc **conditional logic** handles transition between NFZ corridor and sequential waypoint state objectives
  - Initial waypoint: WP1
  - If distance from WP1 is within 10 km
    - Check x-position range, and if it is out of NFZ corridor, then switch to WP2
  - If distance from WP2 is within 10 km
    - Check x-position range and switch to WP3
  - If distance from WP3 is within 10 km
    - **Terminate simulation**

# 2D Model Simulation Results (1/3)

### MATLAB fmincon Trajectory



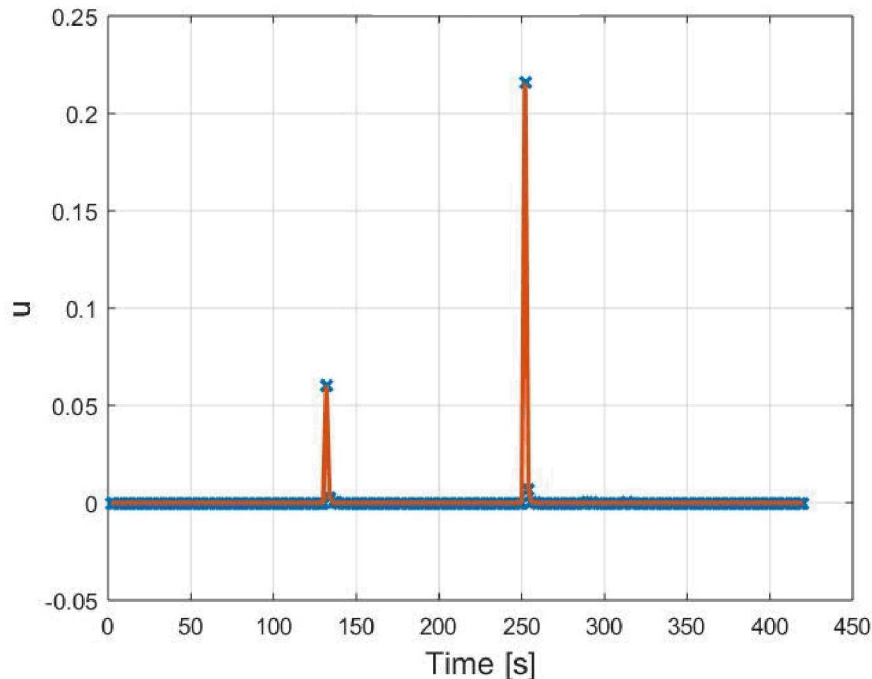
### IPOPT/Simulink Trajectory



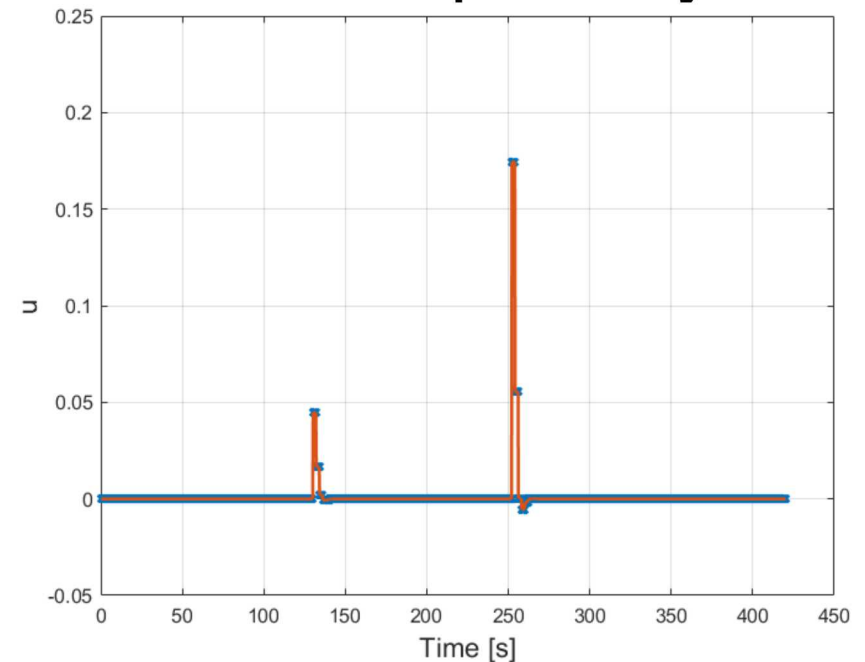
- Identical trajectory performance

# 2D Model Simulation Results (2/3)

**MATLAB fmincon  
Control Input History**



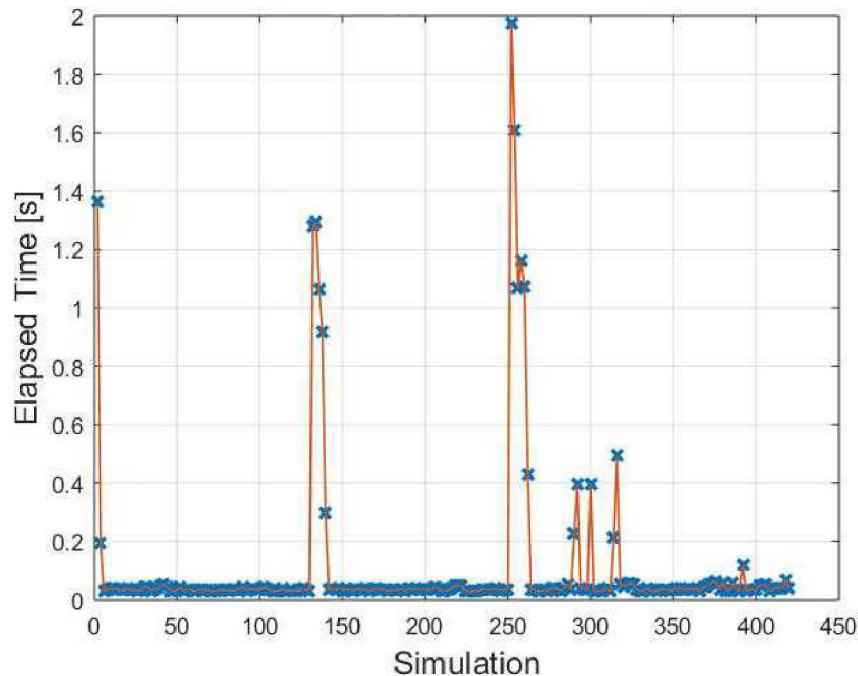
**IPOPT/Simulink  
Control Input History**



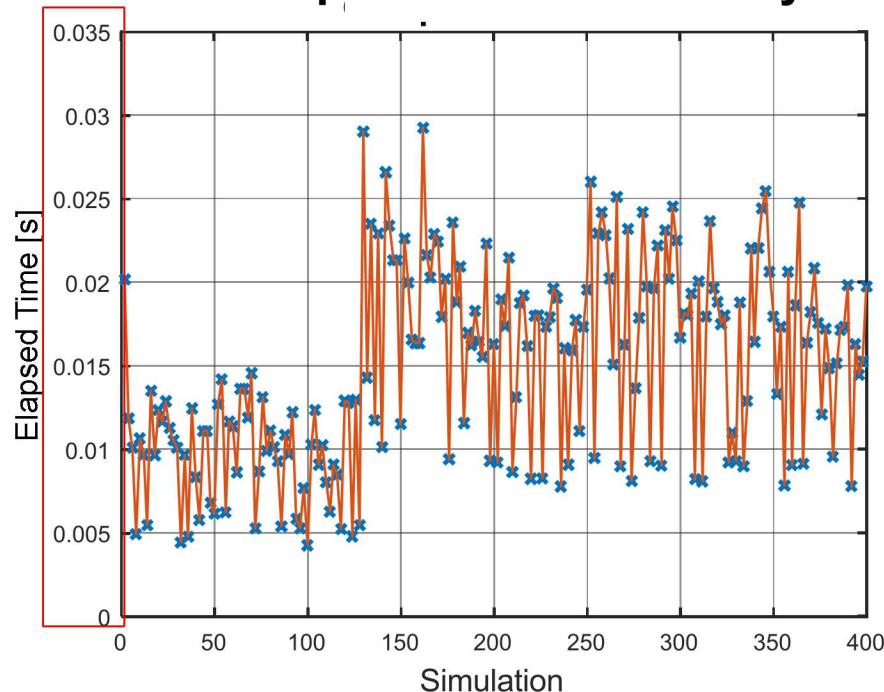
- Inputs are slightly different
- Simulink/IPOPT has smaller control inputs, but distributed in 2-3 sampling instants
- Overall similar results with slightly different algorithm setup

# 2D Model Simulation Results (3/3)

## MATLAB fmincon Computation Time History



## IPOPT/Simulink Computation Time History



	MATLAB fmincon	Simulink/IPOPT
Avg. Comp. Time	0.1179 sec	0.0148 sec (~15 ms)
Max. Comp. Time (worst case)	1.9733 sec	0.0293 sec (~29 ms)

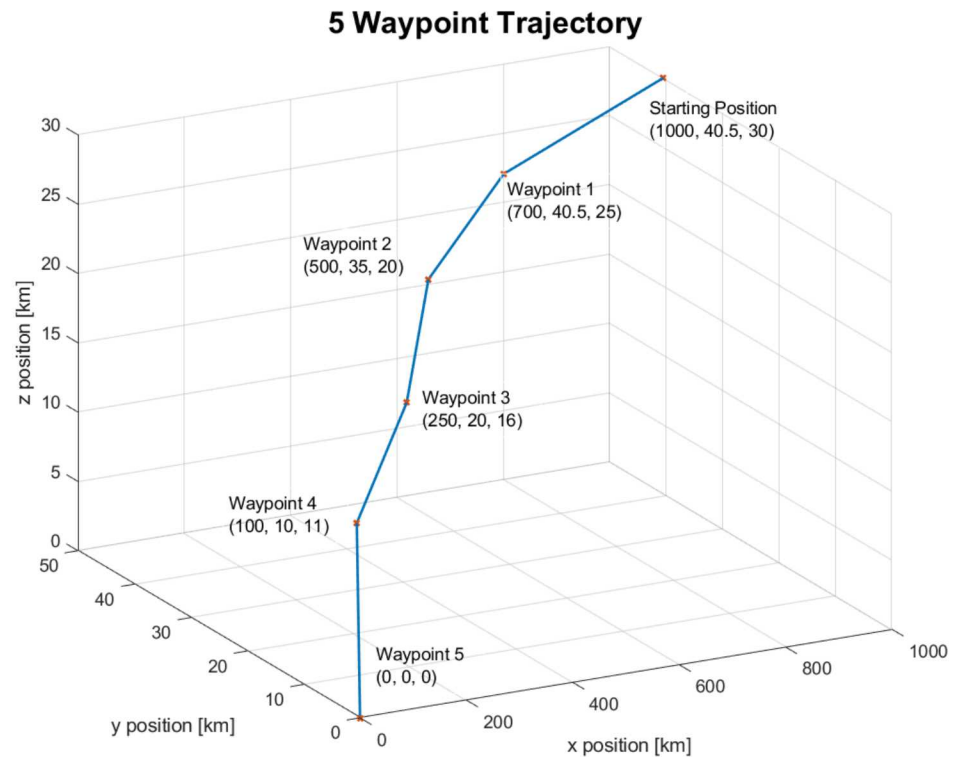


# ***HGV 3D Model Formulations and Results***

- HGV (Hypersonic Glide Vehicle) governed by 3D Eq. of motion
- No propulsion power (**thrust will not be a control variable**)

## Mission Structure

1. Hypersonic release (**no engine thrust**)
2. Hypersonic glide through No-Fly-Zone corridor (Corridor is 300km in length and 1km in width)
3. Waypoint 1 at the end of NFZ corridor (altitude decreases)
4. Waypoint following sequence from 1 to 5 decreasing in altitude



# 3D Eq. of Motion and State Space Model

## Hood Reformulation

$$\dot{x} = V \cos(\gamma) \cos(\theta)$$

$$\dot{y} = V \cos(\gamma) \sin(\theta)$$

$$\dot{h} = V \sin(\gamma)$$

$$\dot{V} = -\frac{D}{m} - g \sin(\gamma)$$

$$\dot{\gamma} = \frac{L \cos(\sigma) - S \sin(\sigma)}{mV} - \frac{g \cos(\gamma)}{V}$$

$$\dot{\theta} = \frac{L \sin(\sigma) + S \cos(\sigma)}{mV \cos(\gamma)}$$

## Modified Formulation

$$\dot{x} = V \cos(\gamma) \cos(\theta)$$

$$\dot{y} = V \cos(\gamma) \sin(\theta)$$

$$\dot{h} = V \sin(\gamma)$$

$$\dot{V} = -\frac{\rho V^2 S C_L^* (1 + c_l^2)}{4E^* m} - g \sin(\gamma)$$

$$\dot{\gamma} = \frac{\rho c_l C_L^* S V \cos(\sigma)}{2m} - \frac{S \sin(\sigma)}{mV} - \frac{g \cos(\gamma)}{V}$$

$$\dot{\theta} = \frac{\rho c_l C_L^* S V \sin(\sigma)}{2m \cos(\gamma)} + \frac{S \cos(\sigma)}{mV \cos(\gamma)}$$

where

$$L = \frac{1}{2} \rho V^2 c_l C_L^* S$$

$$D = \frac{1}{2} \rho V^2 \frac{C_L^* (1 + c_l^2)}{2E^*} S$$

\*Note: Multiple formulations of 3D model were evaluated.  
Flight path angle **small angle** &  
**Three waypoints**

\*Note: Air density is recalculated at every sampling time  
( $\rho = \rho_{sl} e^{-\beta h}$ )  
 $\rho_{sl}$  = density at sea level ( $1.225 \frac{kg}{m^3}$ )  
 $\beta$  = air density decay factor ( $-0.14 km^{-1}$ )  
 $h$  = altitude (km)

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ h \\ V \\ \gamma \\ \theta \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} c_l \\ \sigma \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} x_4 \cos(x_5) \cos(x_6) \\ x_4 \cos(x_5) \sin(x_6) \\ x_4 \sin(x_5) \\ -\frac{\rho C_L^* S x_4^2 (1 + u_1^2)}{4mE^*} - g \sin(x_5) \\ \frac{\rho C_L^* S x_4 u_1 \cos(u_2)}{2m} - \frac{S \sin(u_2)}{m x_4} - \frac{g \cos(x_5)}{x_4} \\ \frac{\rho C_L^* S x_4 u_1 \sin(u_2)}{2m \cos(x_5)} + \frac{S \cos(u_2)}{m x_4 \cos(x_5)} \end{bmatrix}$$

[Jorris 2007]

[Hood 2019]

# Aerodynamic Heating

- Formulation for nose heating was added to the model to guide future considerations for **thermal or structural constraints**

$$\dot{q} = \frac{k}{\sqrt{r_{nose}}} \left( \frac{\rho}{\rho_{sl}} \right) \left( \frac{V}{\sqrt{g_0 r_0}} \right)^3$$

where  $k = 17000 \frac{BTU \cdot ft^{-3}}{s}$

- Note:  $\dot{q}$  is calculated in BTU/s and converted to Watts afterwards

Symbol	Parameter
$k$	Heating constant
$r_{nose}$	Radius of aircraft nose
$\rho$	Local air density
$\rho_{sl}$	Sea level air density
$V$	Velocity
$g_0$	Earth Gravity
$r_0$	Radius from earth center

[Jorris 2007]

## State constraint formulation

$x(k + 1) = f(x(k), u(k)) \Rightarrow$  one constraint formulation for every state (total of 6)

$g_1, g_2, g_3, g_4, g_5, g_6$

$40 \leq x_2(k) \leq 41 \Rightarrow$  only considers 1 state (y-pos =  $x_2$ )

$g_7$

$$g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \\ g_7 \end{bmatrix} = \begin{bmatrix} x_9 - x_1 - Ts \cdot x_4 \cos(x_5) \cos(x_5) \\ x_{10} - x_2 - Ts \cdot x_4 \cos(x_5) \sin(x_5) \\ x_{11} - x_3 - Ts \cdot x_4 \sin(x_5) \\ x_{12} - x_4 - Ts \cdot \left( -\frac{\rho C_L^* S x_4^2 (1+u_1^2)}{4mE^*} - g \sin(x_5) \right) \\ x_{13} - x_5 - Ts \cdot \left( \frac{\rho C_L^* S x_4 u_1 \cos(u_2)}{2m} - \frac{S \sin(u_2)}{m x_4} - \frac{g \cos(x_5)}{x_4} \right) \\ x_{14} - x_6 - Ts \cdot \left( \frac{\rho C_L^* S x_4 u_1 \sin(u_2)}{2m \cos(x_5)} + \frac{S \cos(u_2)}{m x_4 \cos(x_5)} \right) \\ x_2 \end{bmatrix}$$

## MATLAB fmincon Implementation

### Optimization Parameters

$n_s = 6$  - Number of State Variables  
 $n_u = 2$  - Number of Control Variables  
 $n_a = 8$  - State and Control Variables  
 $n_c = 7$  - Number of Constraints  
 $T_s = 2$  - Sampling Time [sec]  
 $N = 20$  - Prediction Horizon

### Weighting matrices

$Q = \text{diag}[0.005, 0.03, 0.03, 0, 0, 0]$

$R = [1, 1]$

$S = 2 \cdot Q$

*For waypoint 4 to 5  $S = 3 \cdot Q$*

### Initial Conditions

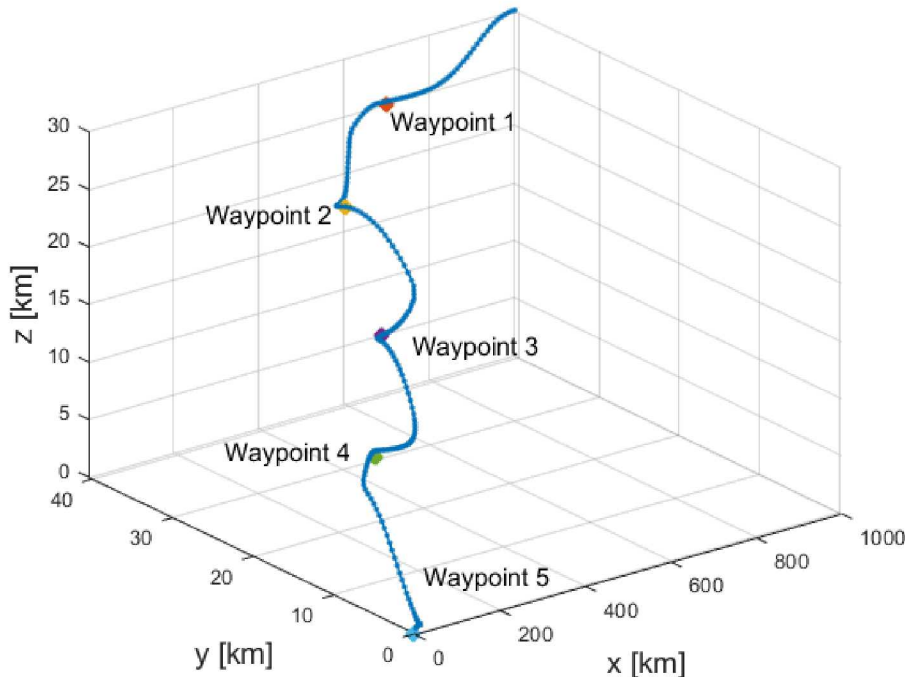
$x_1$  (x-pos) initial = 1000 km  
 $x_2$  (y-pos) initial = 40.5 km  
 $x_3$  (z-pos) initial = 30 km  
 $x_4$  ( $V$ ) initial = 4.08 km/s (Mach 12)  
 $x_5$  ( $\gamma$ ) initial = 0 rad (0 deg)  
 $x_6$  ( $\theta$ ) initial =  $\pi$  rad (180 deg)

### Waypoints (Target)

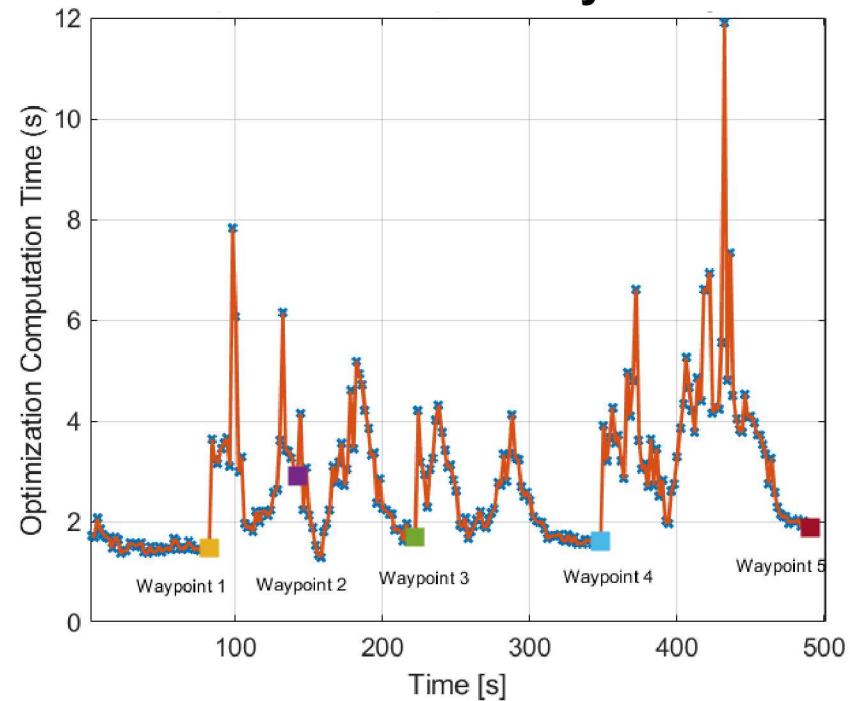
WP1: (700, 40.5, 23) [km]  
 WP2: (500, 35, 18) [km]  
 WP3: (300, 20, 10) [km]  
 WP4: (100, 10, 6) [km]  
 WP5: (0, 0, 0) [km]

# 3D Model Simulation Results (1/2)

## 3D Model 5 Waypoint Trajectory



## 3D Model Computation Time History



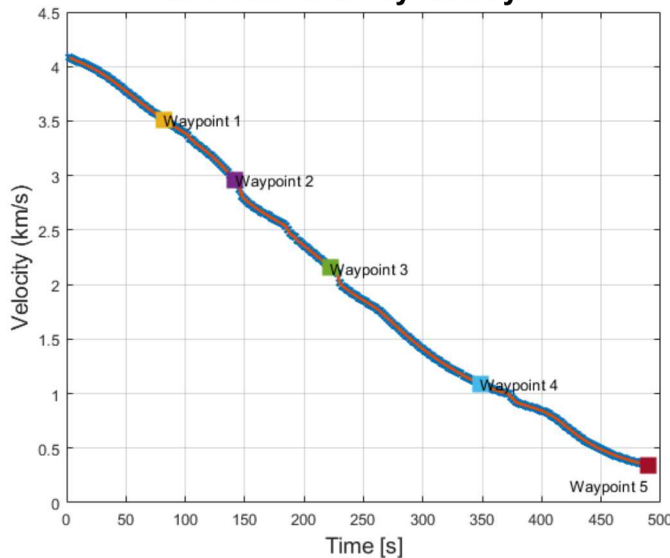
Avg. Time = 2.78 s

- Trajectory smoothness better but still room for improvement
- Aircraft takes discrete turns in y and z before completing x state goal

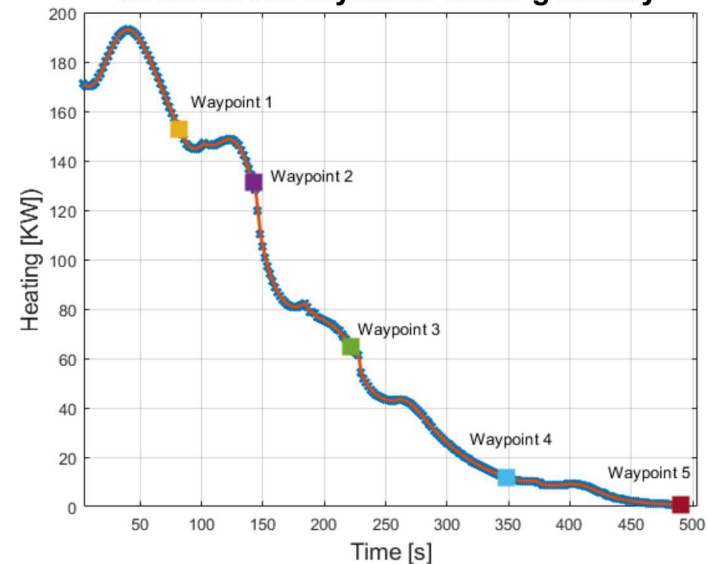
# 3D Model Simulation Results (2/2)

- Velocity is well maintained by the 3D **no-small-angle** model.
- Aerodynamic heating reaches a **maximum of approx. 195 kW**
- Control input in large bursts as the model maneuvers before and after waypoint checks

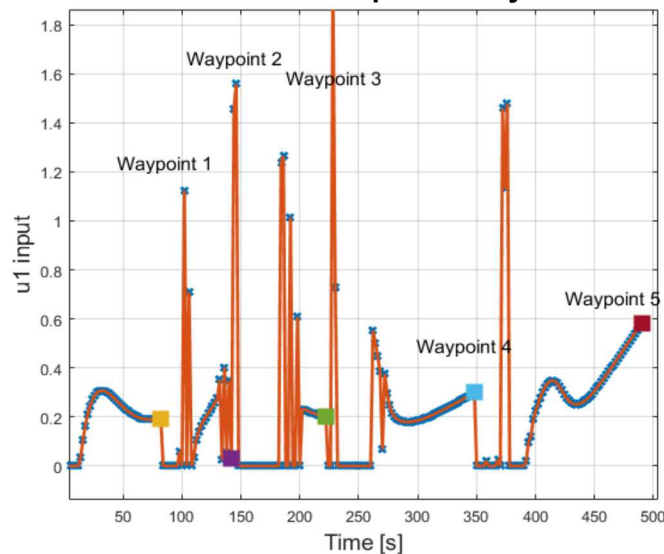
### 3D Model Velocity History



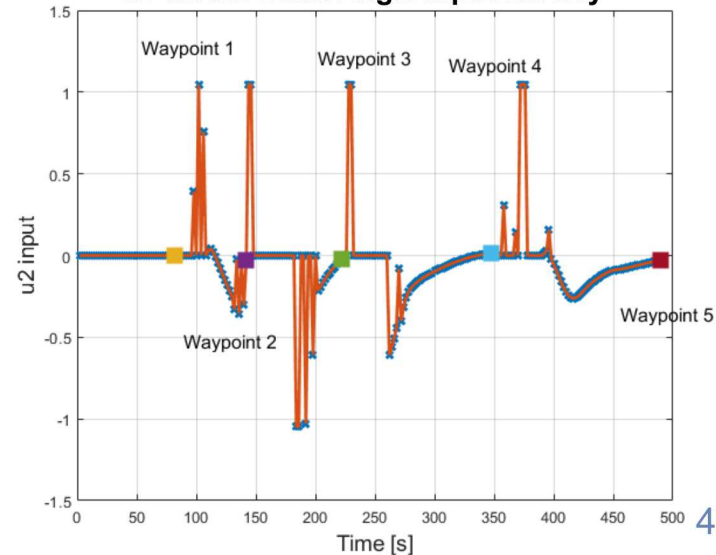
### 3D Model Aerodynamic Heating History



### 3D Model $c_l$ Input History



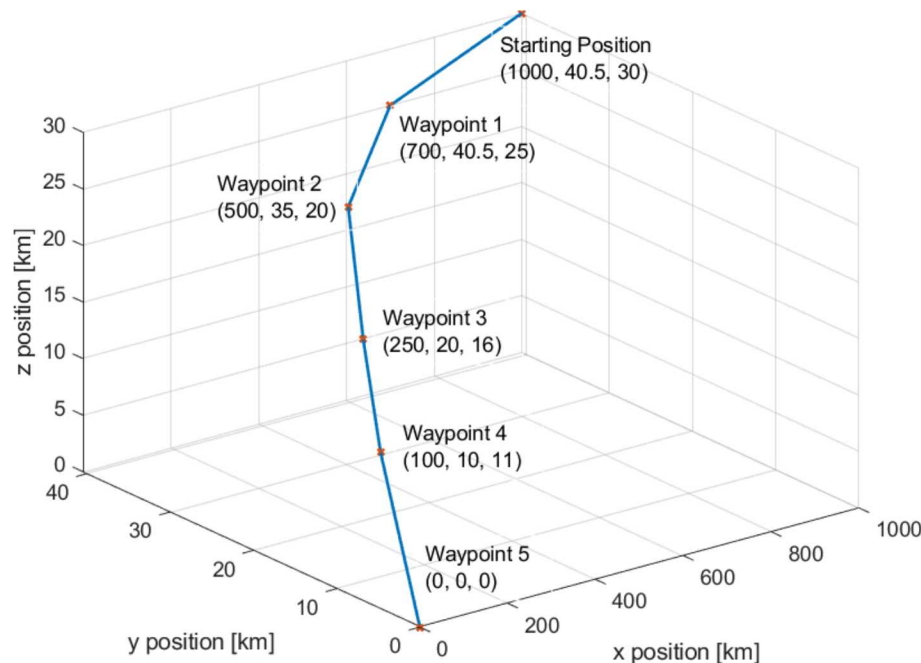
### 3D Model Bank Angle Input History



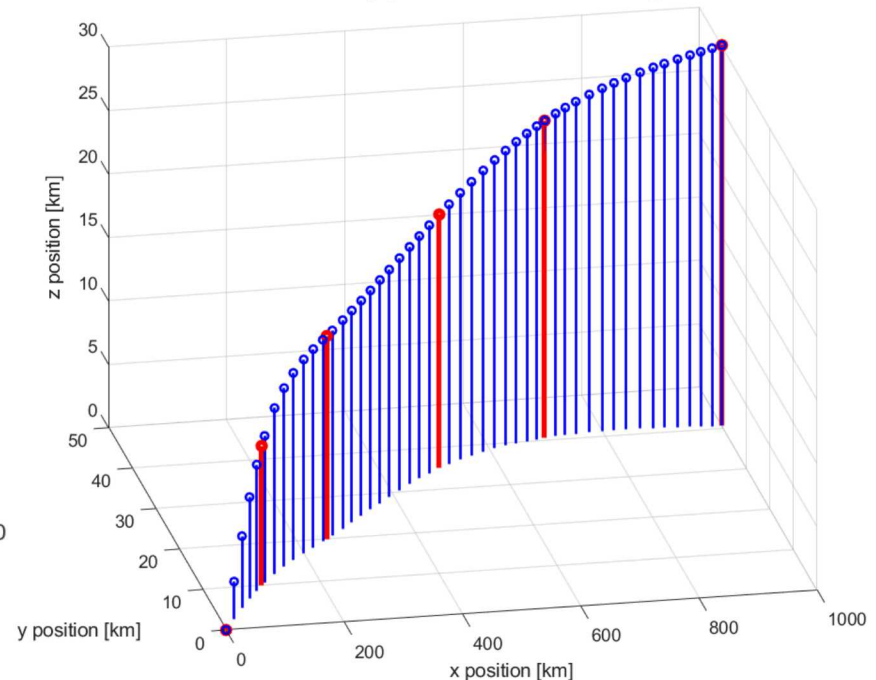
# Waypoint Reformulation

- Waypoint refined to generate a smoother trajectory
- 5 waypoints should be used to generate a smooth **spline** of several dozen waypoints to **enforce strict waypoint following**

**Reformulated 5 Waypoints**



**50 Sub-Waypoint Point Spline**



## MATLAB fmincon Implementation

### Optimization Parameters

$n_s = 6$  - Number of State Variables  
 $n_u = 2$  - Number of Control Variables  
 $n_a = 8$  - State and Control Variables  
 $n_c = 7$  - Number of Constraints  
 $T_s = 2$  - Sampling Time [sec]  
 $N = 20$  - Prediction Horizon

### Weighting matrices

$Q = \text{diag}[ 0.0002, 0.25, 0.25, 0, 0, 0 ]$   
 $R = [1, 1]$   
 $S = 2 \cdot Q$

### Initial Conditions

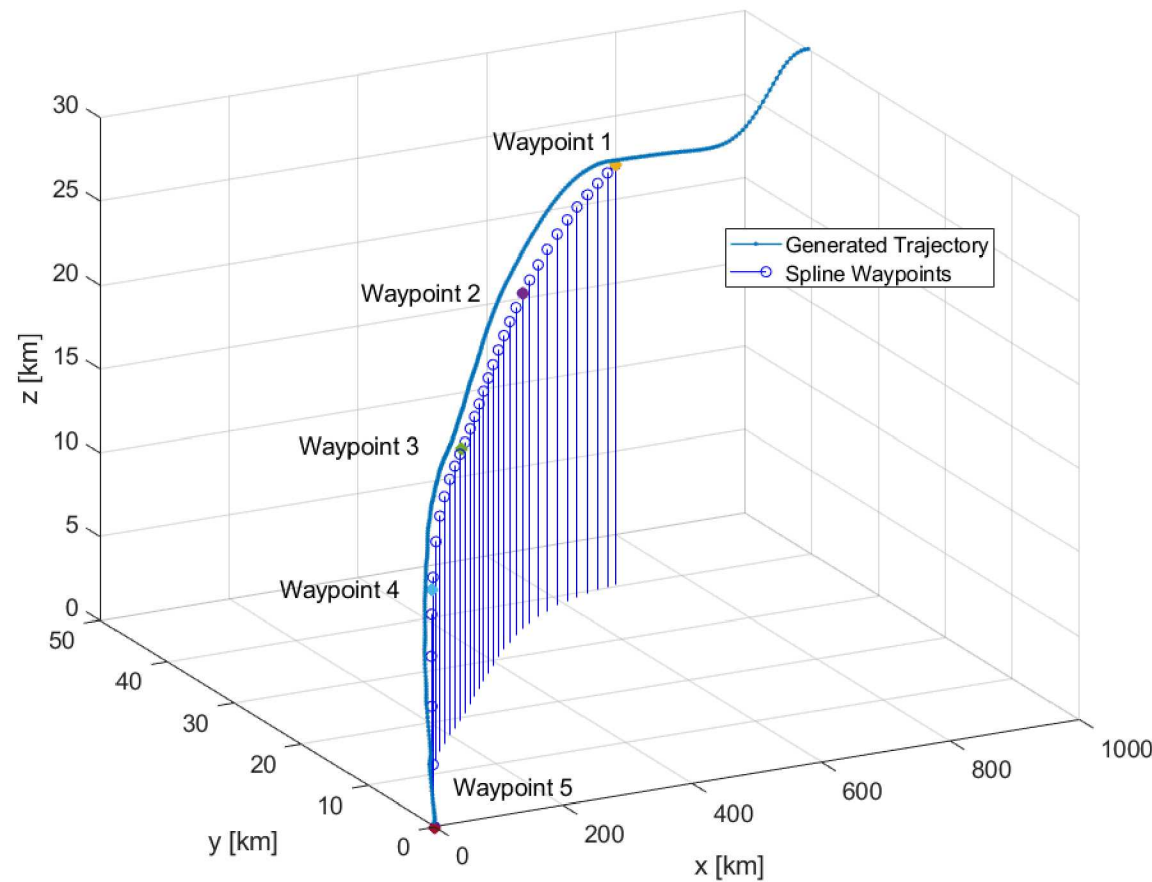
$x_1$  (x-pos) initial = 1000 km  
 $x_2$  (y-pos) initial = 40.5 km  
 $x_3$  (z-pos) initial = 30 km  
 $x_4$  ( $V$ ) initial = **2.72 km/s (Mach 8)**  
 $x_5$  ( $\gamma$ ) initial = 0 rad (0 deg)  
 $x_6$  ( $\theta$ ) initial =  $\pi$  rad (180 deg)

### Waypoints (Target)

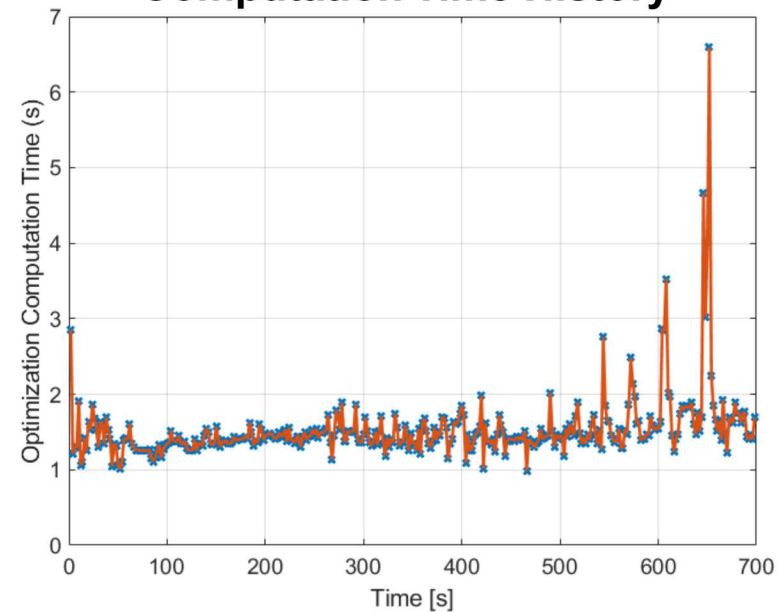
**33 Sub-waypoints** from end of No-Fly-Zone (700, 40.5, 25) to old Waypoint 5 (0,0,0)

# 3D Model Spline Results (1/2)

## 3D Model Spline Trajectory



## 3D Model Spline Computation Time History



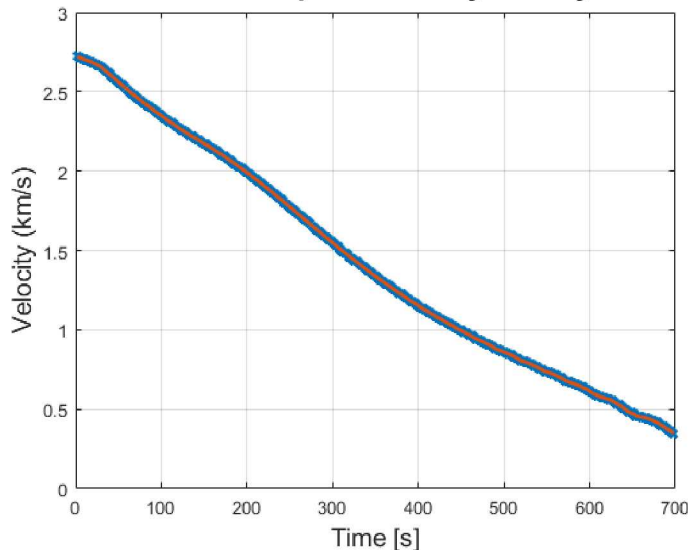
Avg. Time = 1.53 s

- Trajectory smoothness greatly improved
- Favorable computation time performance
- Computation time depends on **complex interactions between simulation params**

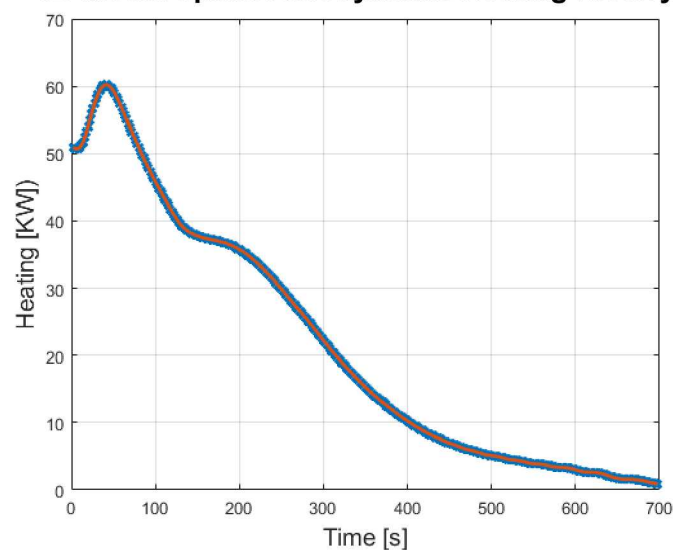
# 3D Model Spline Results (2/2)

- Velocity is well maintained despite lower initial velocity and **constant  $c_l$  control input**.
- Aerodynamic heating reaches a **maximum of approx. 60 kW**
- Control input in constant fluctuations with **no maximum constrained inputs**

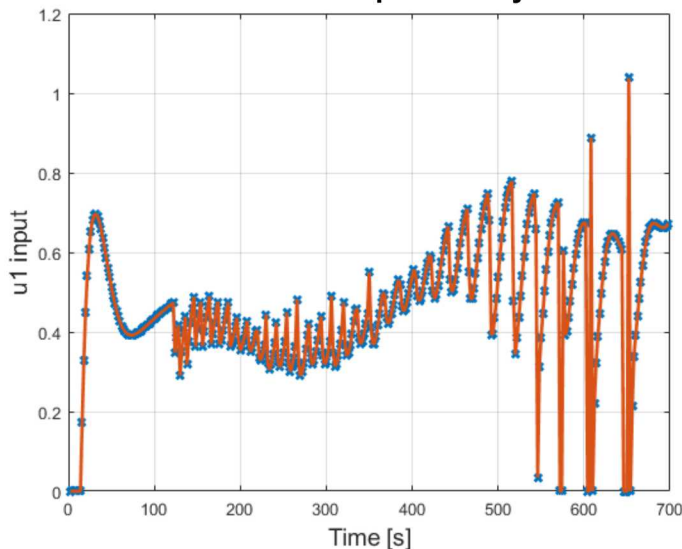
### 3D Model Spline Velocity History



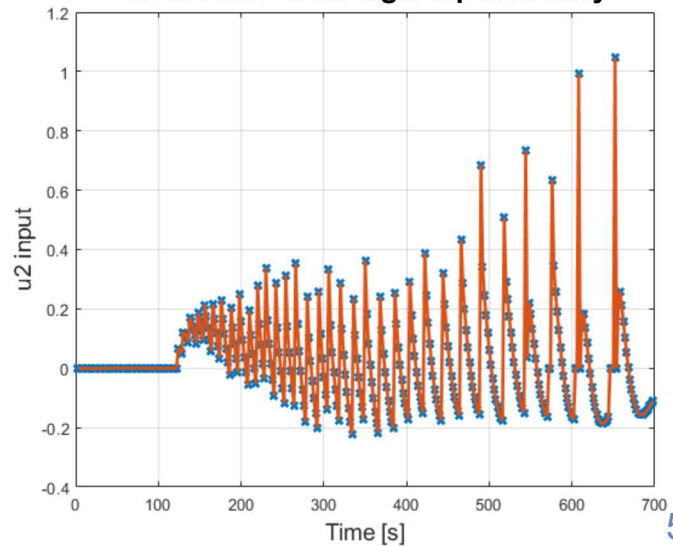
### 3D Model Spline Aerodynamic Heating History



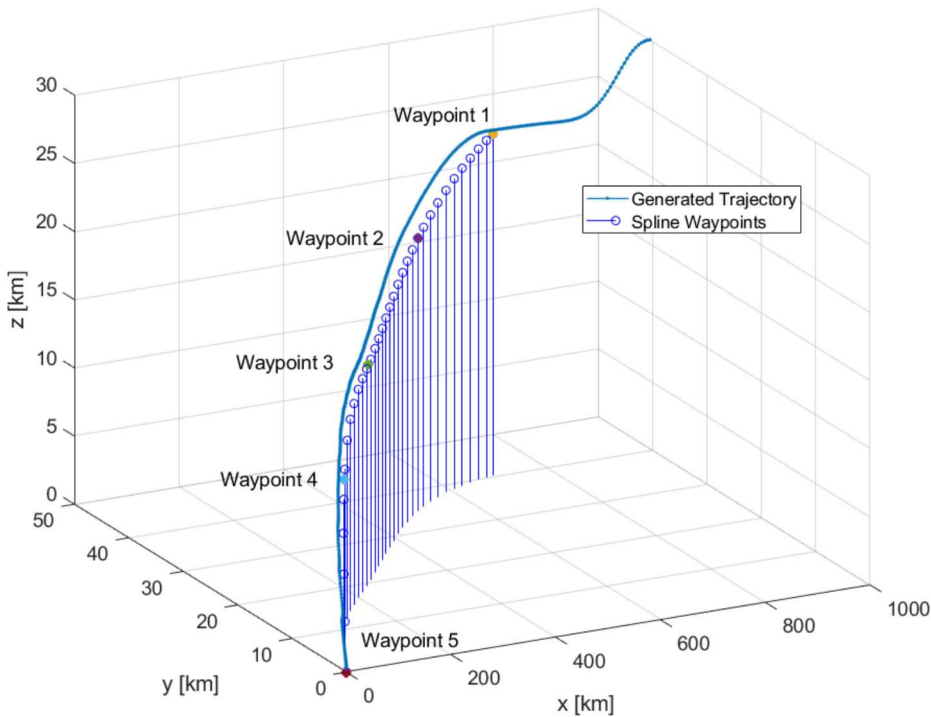
### 3D Model $c_l$ Input History



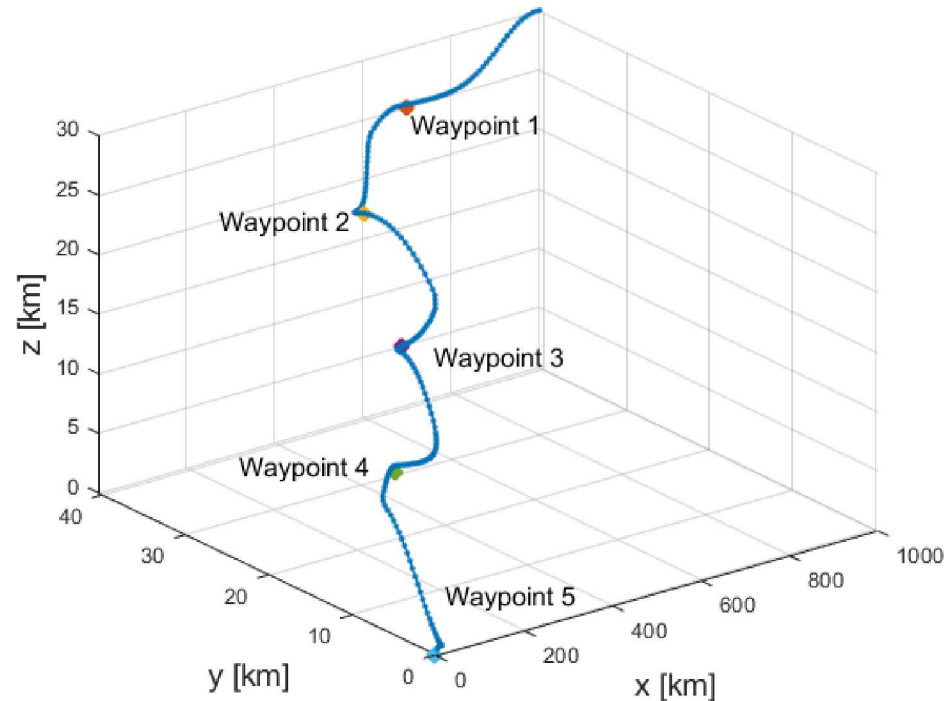
### 3D Model Bank Angle Input History



## 3D Model Spline Trajectory

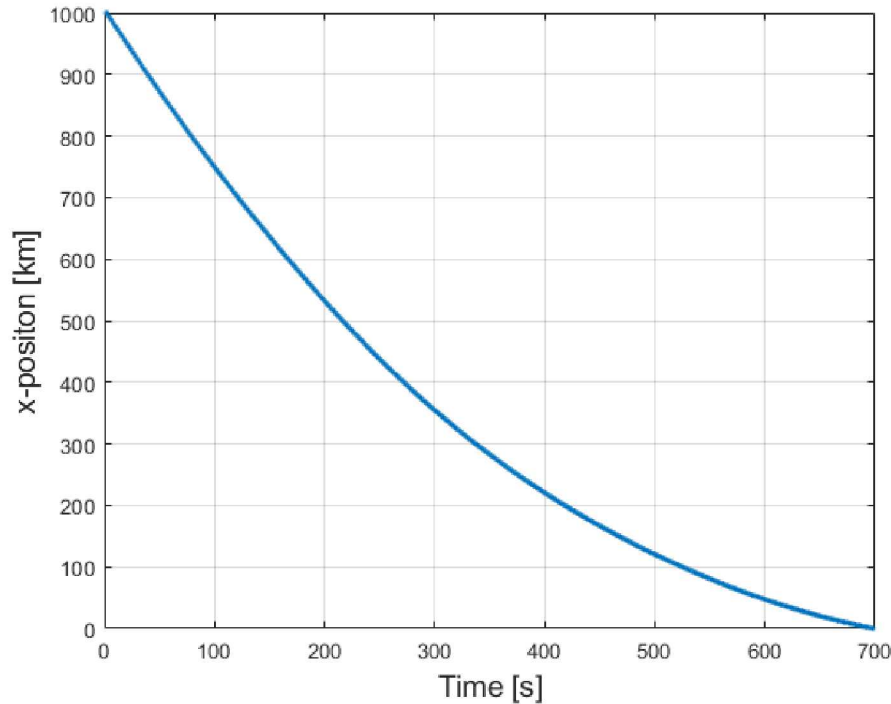


## 3D Model 5 Waypoint Trajectory

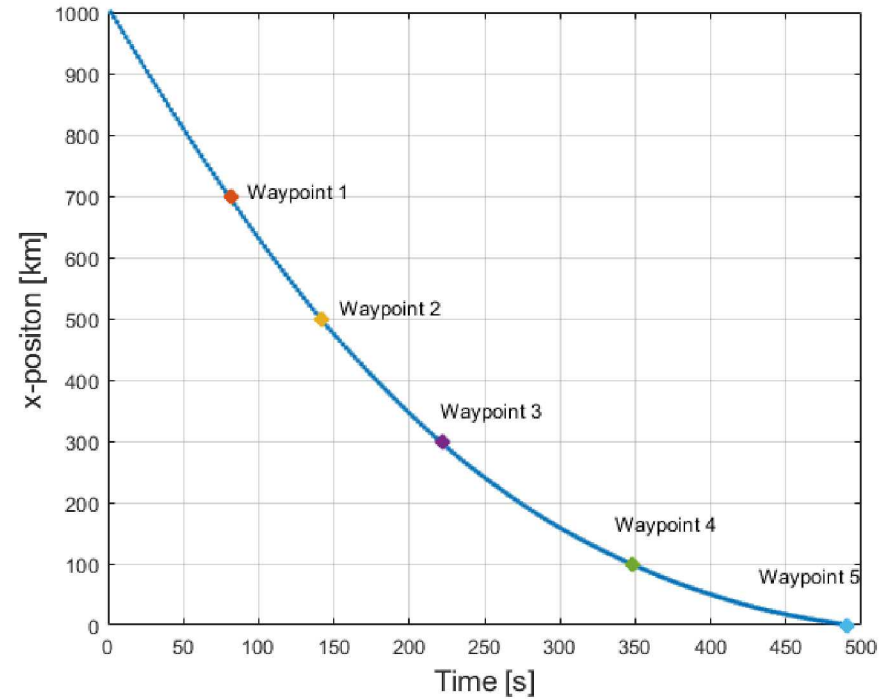


- NMPC framework for hypersonic flight works best with many **short term state objectives** rather than few long term objectives
- Spline model has **high accuracy for final waypoint** (Within 50m)

### Spline Model x-pos History



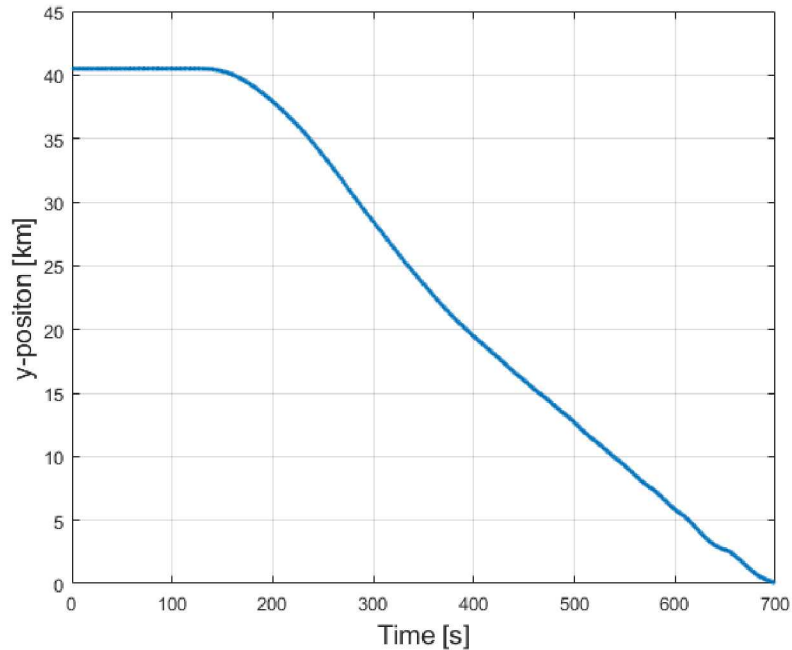
### 5 WP Model x-pos History



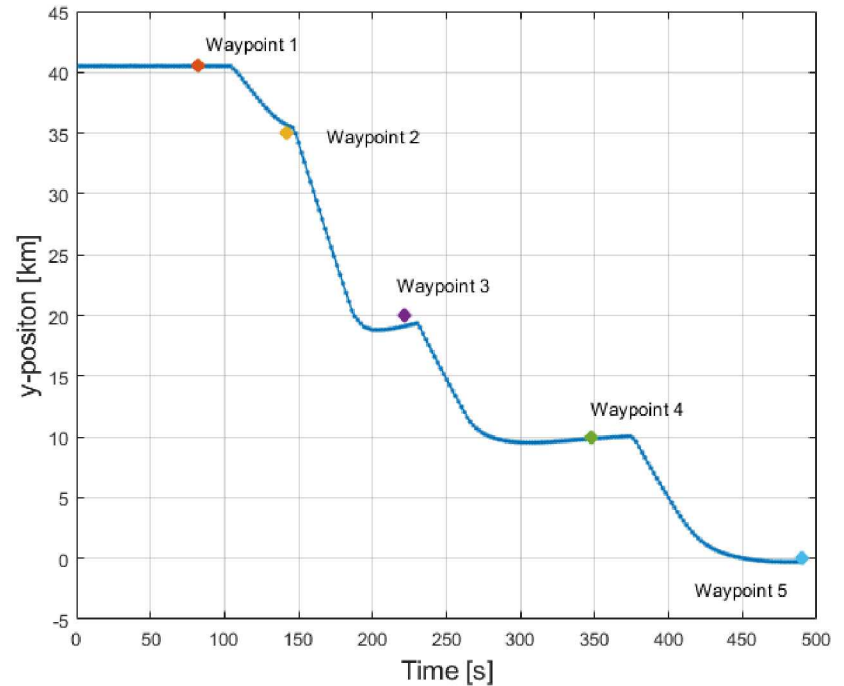
- Negligible difference in x-pos trajectory performance

# Discrete y-position Comparison

## Spline Model y-pos History

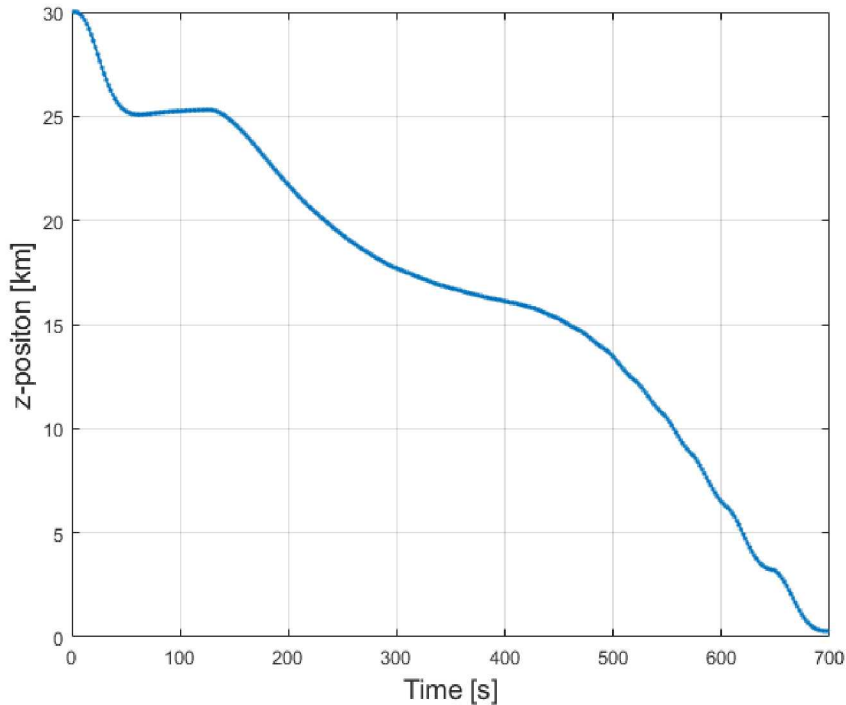


## 5 WP Model y-pos History

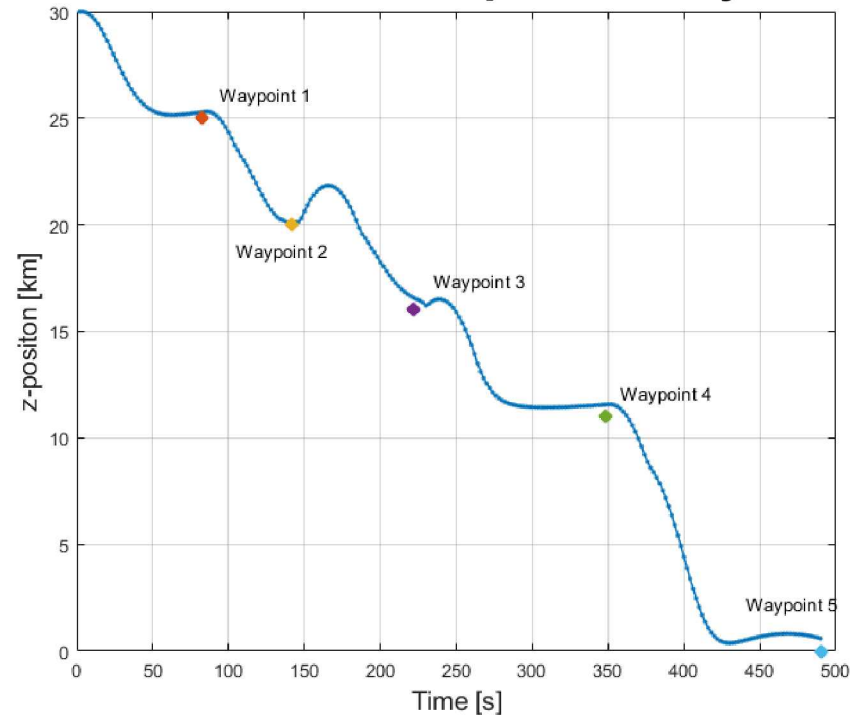


- Spline model is much improved. Less slope variation (i.e. **smoother trajectory**)
- Improvements in efficiency, reduction in structural loading

## Spline Model z-pos History



## 5 WP Model z-pos History



- Spline model is much improved. Less slope variation (i.e. **smoother trajectory**)
- Spline model has negligible in z-pos
- Improvements in efficiency, reduction in structural loading

## Robotic Obstacle Avoidance

- Developed NMPC framework for static obstacle avoidance
- Integrated obstacle avoidance case study results into ROS/Gazebo Simulation
- **Future Work:**
  - Multi-obstacle cases & moving obstacle cases
  - Integration of algorithms with real world hardware

## Hypersonic Trajectory Generation

- Developed NMPC framework for hypersonic flight vehicle trajectory generation
- Integrated multiple iterations of vehicle dynamics models to achieve best trajectory results
- **Future Work:**
  - 3D model IPOPT integration for real-time computational capability validation
  - 6DOF model with added thermo/structural constraints

## Advisor

Dr. Hyeongjun Park

## Committee

Dr. Hyeongjun Park (Chair)

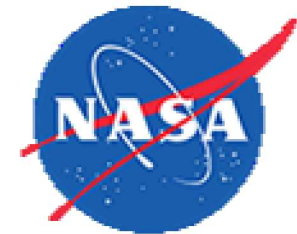
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Dr. James McAteer (Dean's Representative)



## New Mexico Space Grant Consortium

Dr. Paulo Oemig



## NASA Ames Research Center

Dr. Brian Coltin

Jose Benavides



## Sandia National Labs

Dr. Bethany Nicholson

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# Q & A



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