

Accurate Calculations of a Solid State Test Set with Quantum Monte Carlo Methods

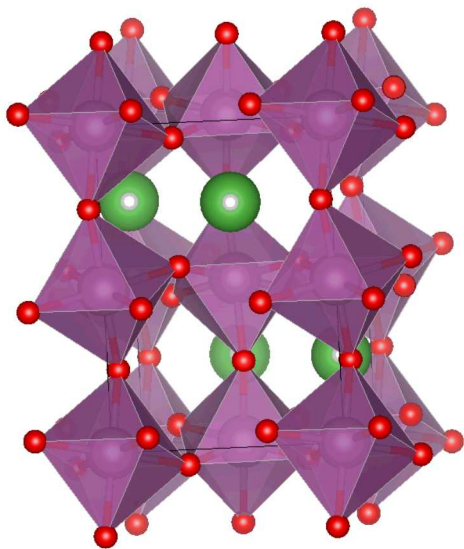


Cody A. Melton

Jaron Krogel, Fionn Malone, Miguel A. Morales, Luke Shulenburger

CPSFM | Center for Predictive Simulation
of Functional Materials

Motivation



Accurate predictions for the properties of functional materials requires accurate *ab-initio* solutions to the many-body Schrödinger equation

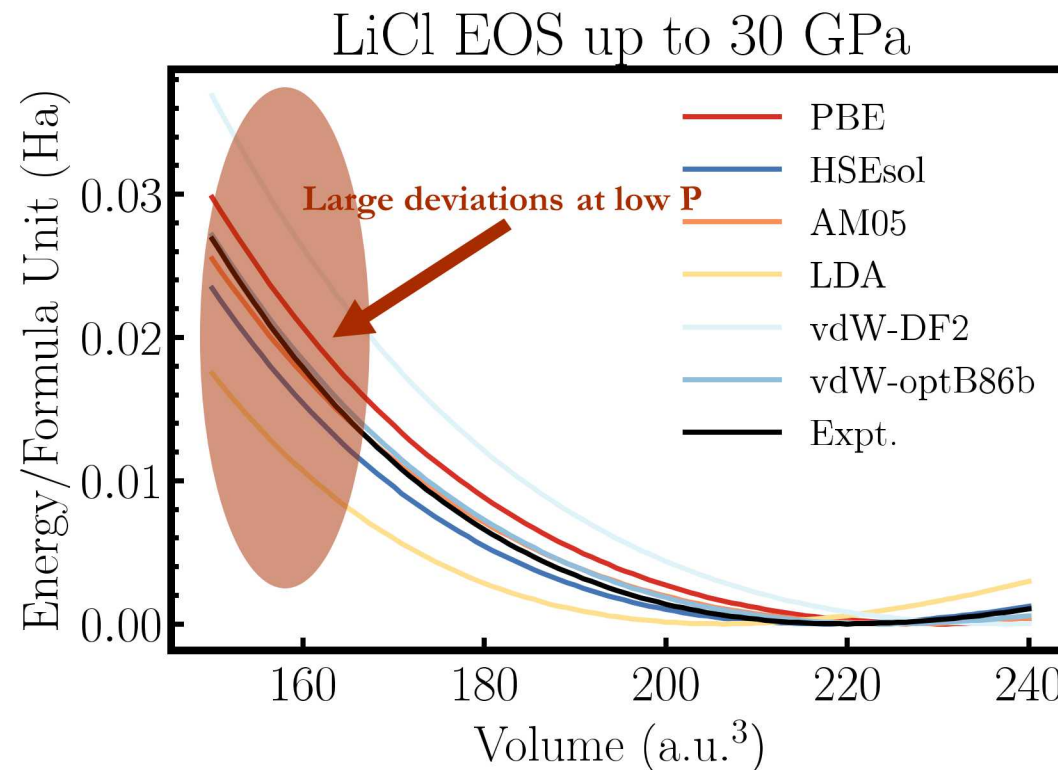
$$\left(\hat{T} + \hat{V}_{\text{Ion-Ion}} + \hat{V}_{\text{Elec-Ion}} + \hat{V}_{\text{Elec-Elec}} \right) |\Psi\rangle = E|\Psi\rangle$$

Properties: Cohesion, Optical Properties (gaps), Magnetic Phases, Structural Phase Transitions, etc.

Density Functional Theory (DFT) has been the method of choice due to its cheap computational cost and reasonable accuracy across many systems and scales.

EOS ($T=0\text{K}$) is one of the simplest metrics for measuring the accuracy of an electronic structure method.

Even for simple systems, various DFT can produce vastly large errors in the EOS, especially at high pressure!

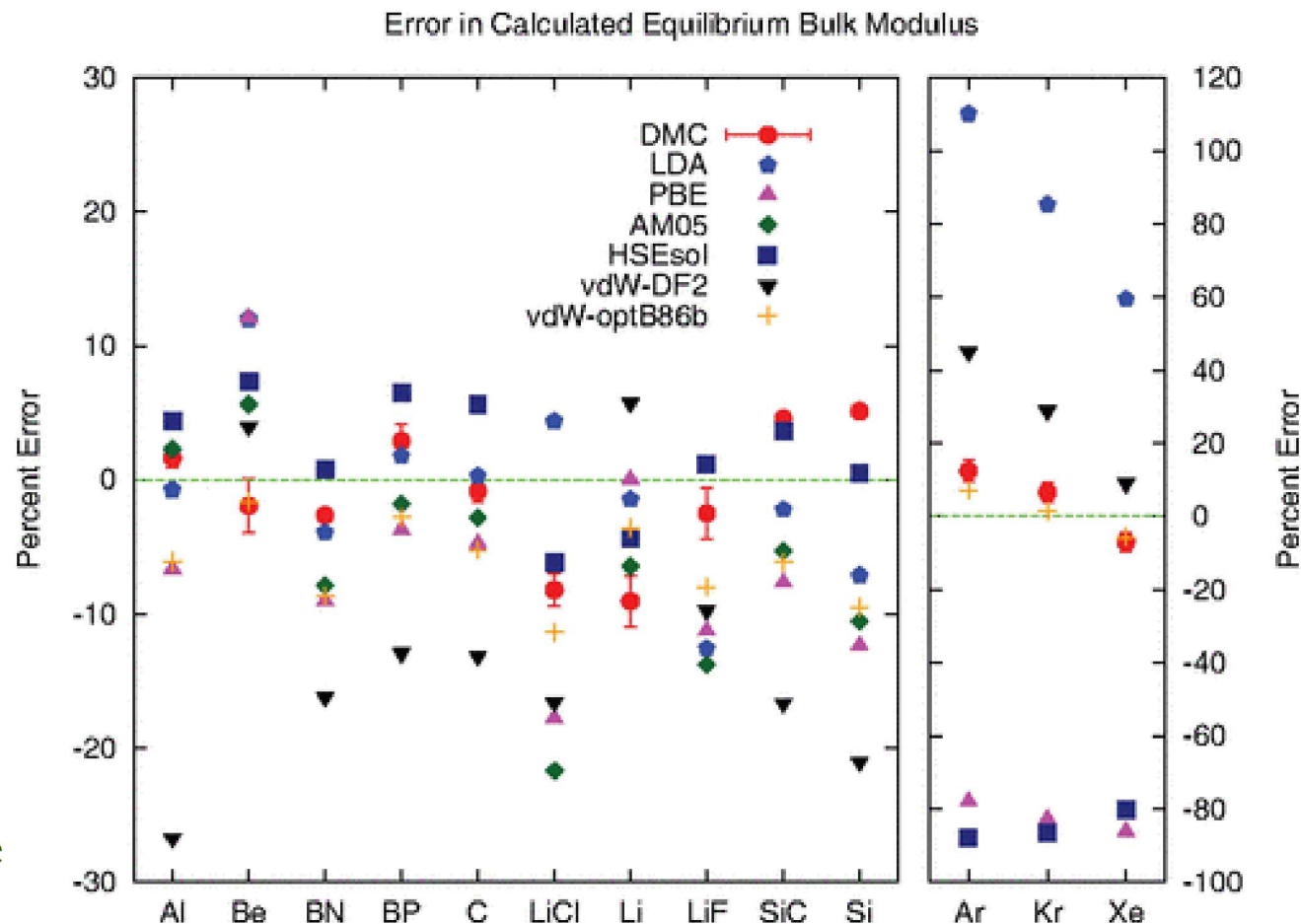


Solid State Test Set and Previous Results

- Test set for EOS probes a variety of simple solids with various types of bonding...ionic, covalent, metallic, and vdW
- Various DFT have MARE(%) ranging from 6%-26% error from expt.
- Previous state-of-the-art DMC makes overall improvement 5% MARE (%). Still room for improvement

$$E(V) = E_0 + \frac{2B_0V_0}{(B'_0 - 1)^2} \left[2 - \left(5 + 3 \left(\frac{V}{V_0} \right)^{1/3} (B'_0 - 1) - 3B'_0 \right) \right] \\ \times \exp \left(-\frac{3}{2}(B'_0 - 1) \left[\left(\frac{V}{V_0} \right)^{1/3} - 1 \right] \right)$$

We revisit the test set using **Diffusion Monte Carlo (DMC)** and **Auxiliary Field QMC (AFQMC)**, utilizing recent methodological advances to show improved accuracy of EOS



Shulenburger & Mattsson, PRB **88**, 245117 (2013)

DMC and AFQMC both utilize imaginary time propagation from an initial Ψ_T to obtain *accurate* and *explicitly correlated* energies

$$|\Psi_0\rangle \propto \lim_{\tau \rightarrow \infty} \exp[-\tau \mathcal{H}] |\Psi_T\rangle$$

$$E_0 = \lim_{\tau \rightarrow \infty} \frac{\langle \Psi(\tau) | \mathcal{H} | \Psi_T \rangle}{\langle \Psi(\tau) | \Psi_T \rangle}$$

Diffusion Monte Carlo

- 1st Quantization: Samples Configuration Space

$$\langle \mathbf{R}' | \Psi^{(n+1)} \rangle = \int d\mathbf{R} G(\mathbf{R} \rightarrow \mathbf{R}'; \tau) \langle \mathbf{R} | \Psi^{(n)} \rangle$$
- Fixed-Node Approximation to deal with FSP
- FN is typically largest bias

Auxiliary Field QMC

- 2nd Quantization: Samples Determinant Space

$$|\Psi^{(n+1)}\rangle = \int d\mathbf{x} p(\mathbf{x}) \hat{B}(\mathbf{x}) |\Psi^{(n)}\rangle$$
- Phaseless Approximation to deal with FSP
- Bias is smaller, but has large basis set errors

reference workflow

- Large source of uncertainty in previous DMC came from inaccurate ECPs.**

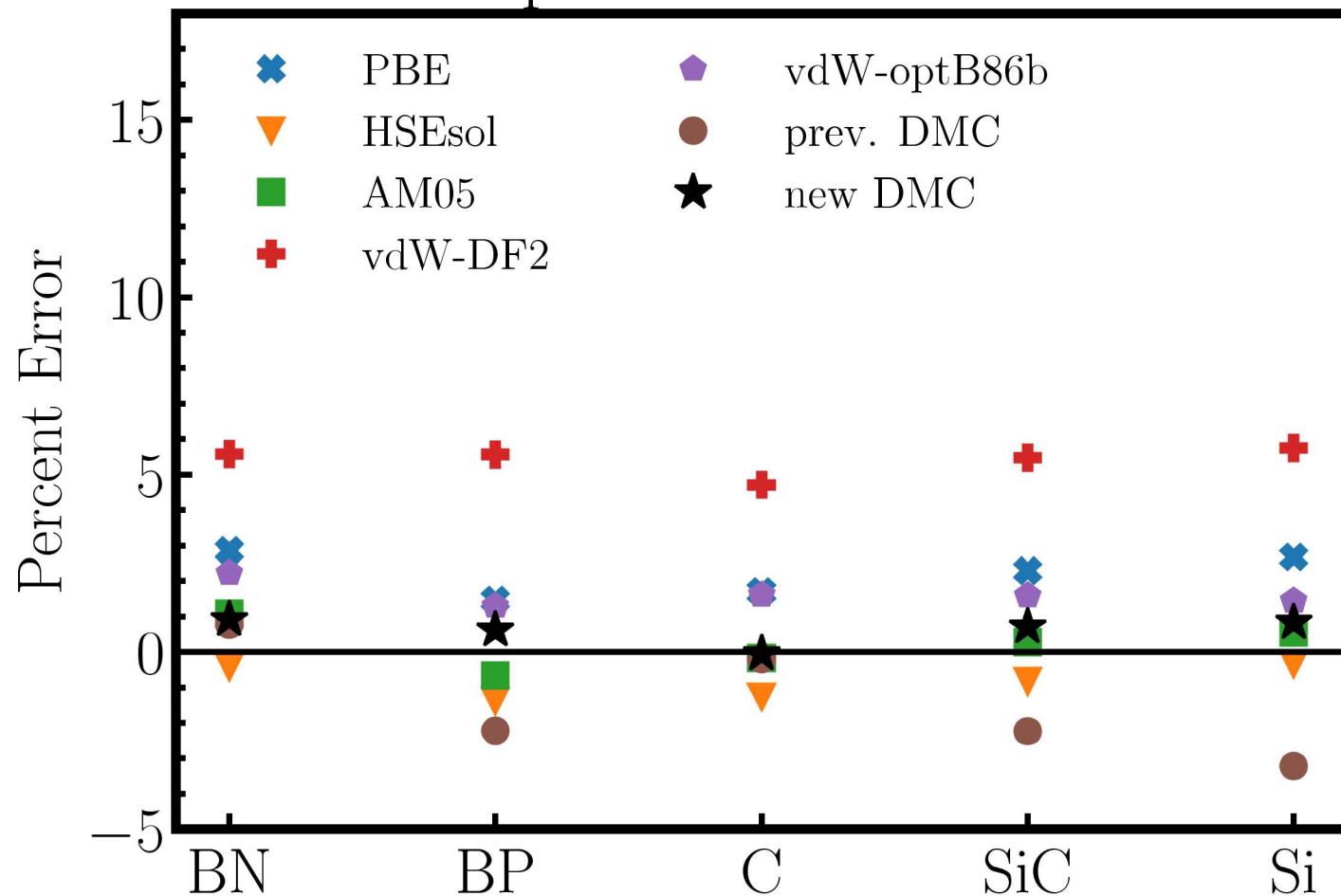
utilizing recent *explicitly correlated ECPs*

A community website for pseudopotentials/effective core potentials developed for high accuracy correlated many-body methods such as quantum Monte Carlo and quantum chemistry.

<http://pseudopotentiallibrary.org/>

Preliminary DMC Results

Equilibrium Volume



$$E(V) = E_0 + \frac{2B_0V_0}{(B'_0 - 1)^2} \left[2 - \left(5 + 3 \left(\frac{V}{V_0} \right)^{1/3} (B'_0 - 1) - 3B'_0 \right) \right] \times \exp \left(-\frac{3}{2} (B'_0 - 1) \left[\left(\frac{V}{V_0} \right)^{1/3} - 1 \right] \right)$$

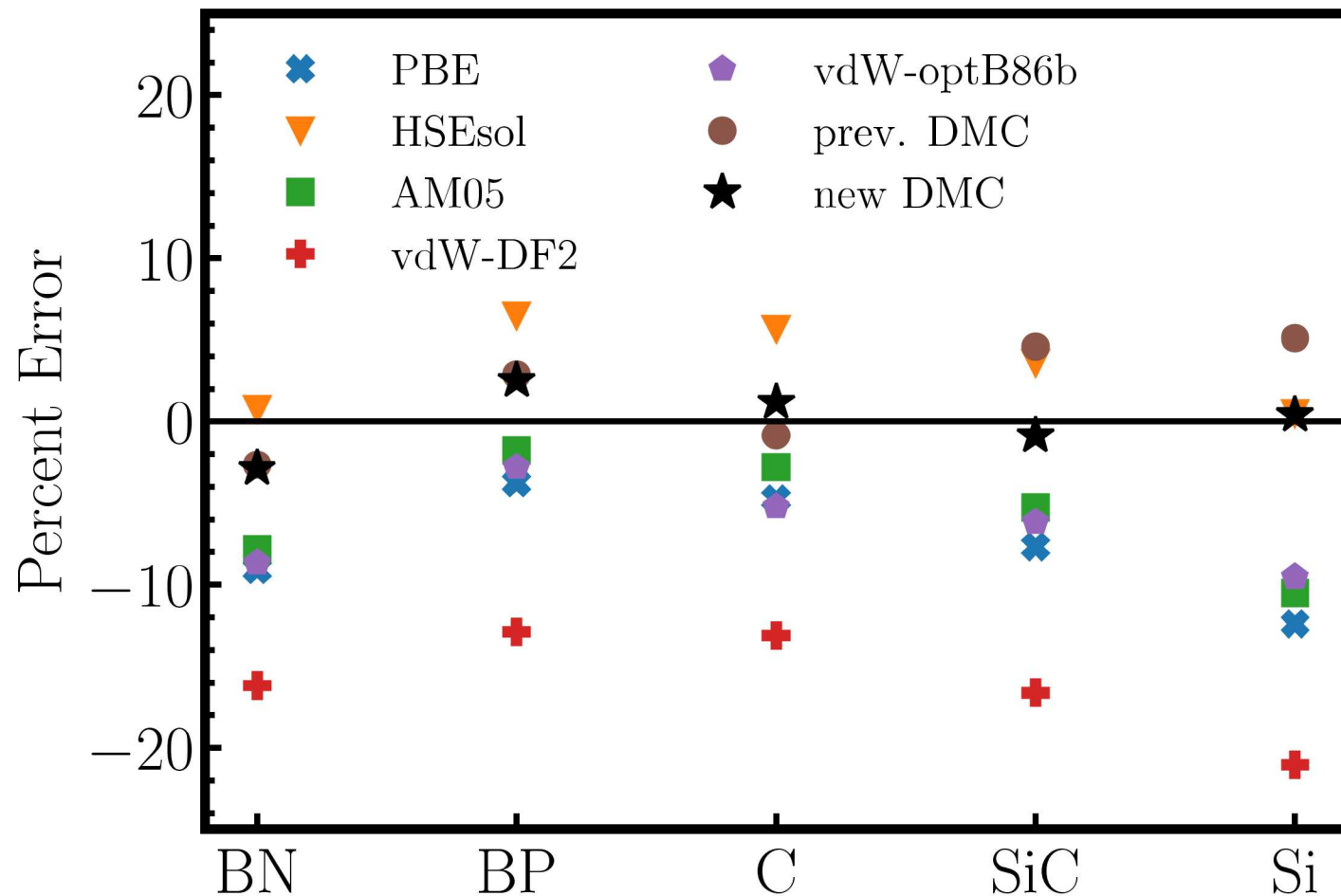
- Previous DFTs up to 5% MARE for this subset

- MARE(%):
Old DMC: 1.8 New DMC: 0.6

- Using more accurate ECPs leads to 3x reduction in MARE

Preliminary DMC Results

Bulk Modulus



$$E(V) = E_0 + \frac{2B_0V_0}{(B'_0 - 1)^2} \left[2 - \left(5 + 3 \left(\frac{V}{V_0} \right)^{1/3} (B'_0 - 1) - 3B'_0 \right) \right] \times \exp \left(-\frac{3}{2} (B'_0 - 1) \left[\left(\frac{V}{V_0} \right)^{1/3} - 1 \right] \right)$$

- Previous DFTs up to 15% MARE for this subset

- MARE(%)
Old DMC: 3.2 New DMC: 1.5

- Using more accurate ECPs leads to 2x reduction in MARE

Conclusions

- Factor of +2x improvement to EOS by using correlated ECPs in QMC calculations
- Finite-size correction schemes allow for accurate EOS without needing large supercells or costly extrapolations
- Nexus workflow to be released as a reference to reproduce all work, test future methodological advances, learning script, etc.

To Do:

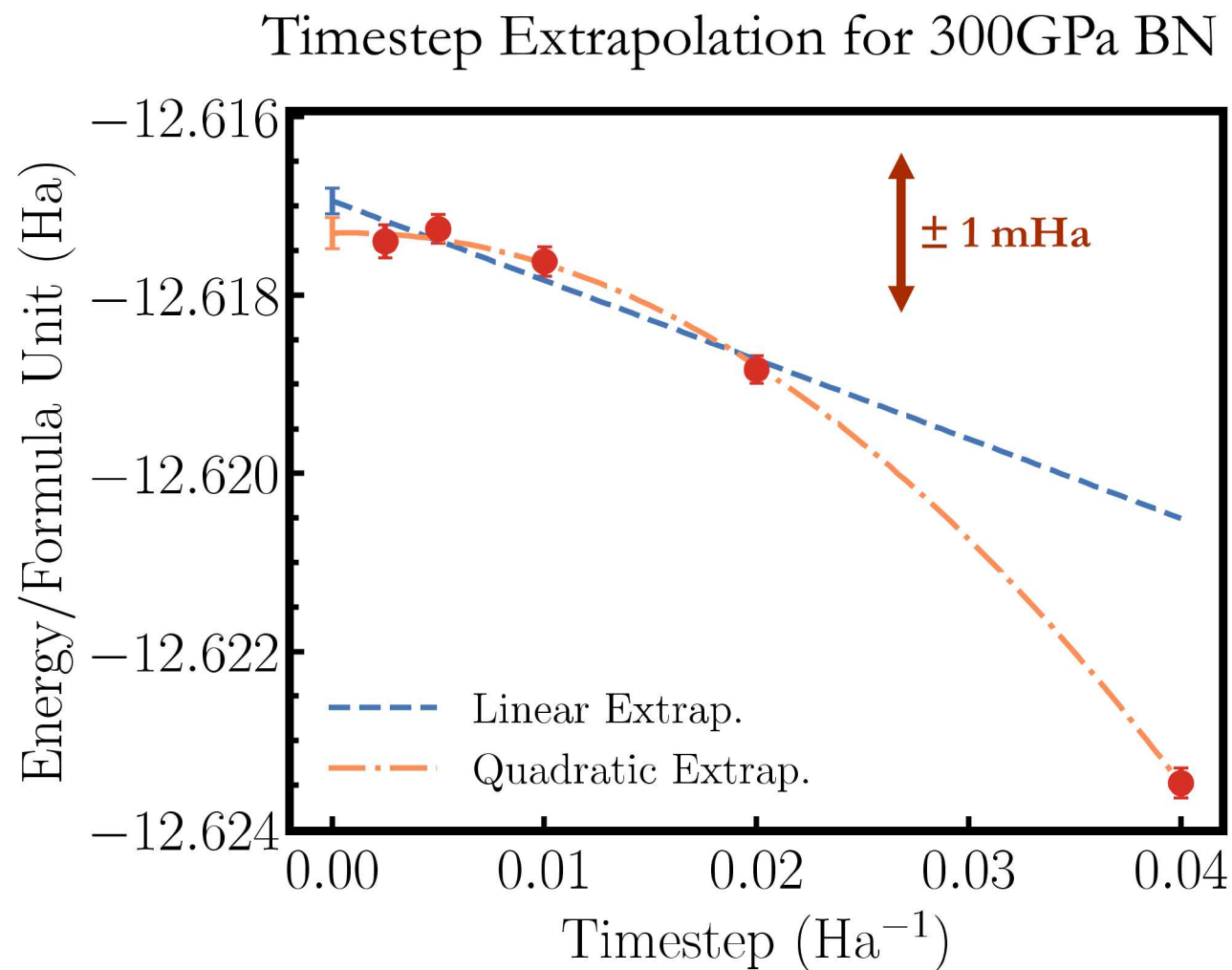
- AFQMC cold curves still ongoing
- Investigate more sophisticated KE corrections (long ranged Jastrow corrections from Holzmann *et al.*, PRB **94**, 035126 (2016))
- FS corrections for AFQMC, basis set corrections AFQMC, comparison of DMC to AFQMC

9 Backup: Timestep Bias

Both DMC and AFQMC use a Trotter decomposition to approximate the imaginary-time projector

For each system, we perform a timestep extrapolation of energies at 300GPa and ambient conditions to converge energy to <1 mHa/Formula Unit

In each system and method, we find a 0.01 Ha^{-1} was sufficient for convergence. This is used for the entire $E(V)$ curve.



Backup: One-Body Finite-Size Bias

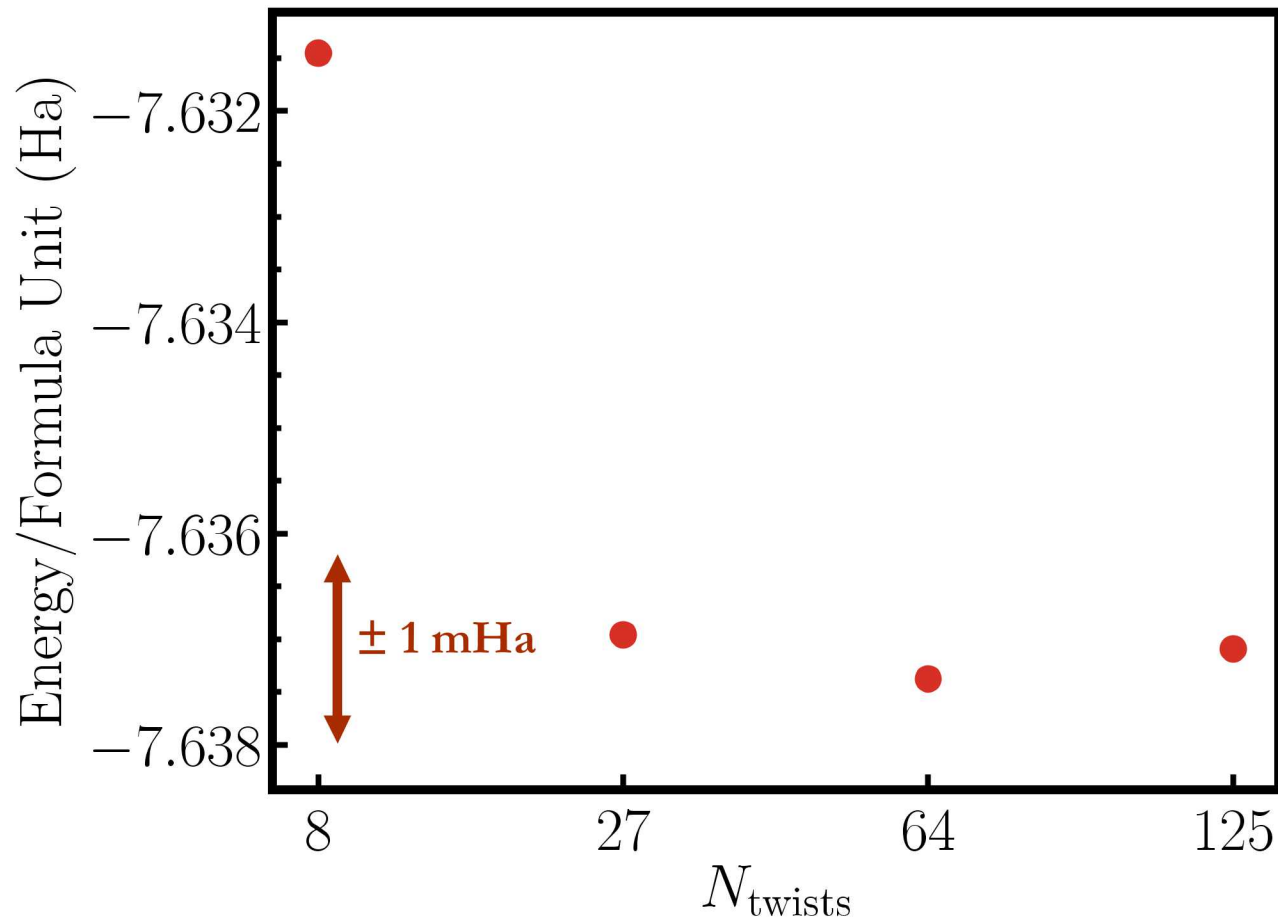
1b FS effects are dominated by shell effects, sub-leading order come from long-range behavior of Jastrow

Leading order corrections are addressed by **twist-averaging**. Similar to BZ sampling in DFT.

$$E_{\text{TA}} = \frac{1}{N_{\text{tw}}} \sum_{\mathbf{k}_s}^{N_{\text{tw}}} E_{\text{QMC}}(\mathbf{k}_s)$$

In each system and method, we find a 4x4x4 twist grid sufficient to converge the energies to <1mHa/Formula Unit.

Twist Averaging for 300 GPa SiC in AFQMC

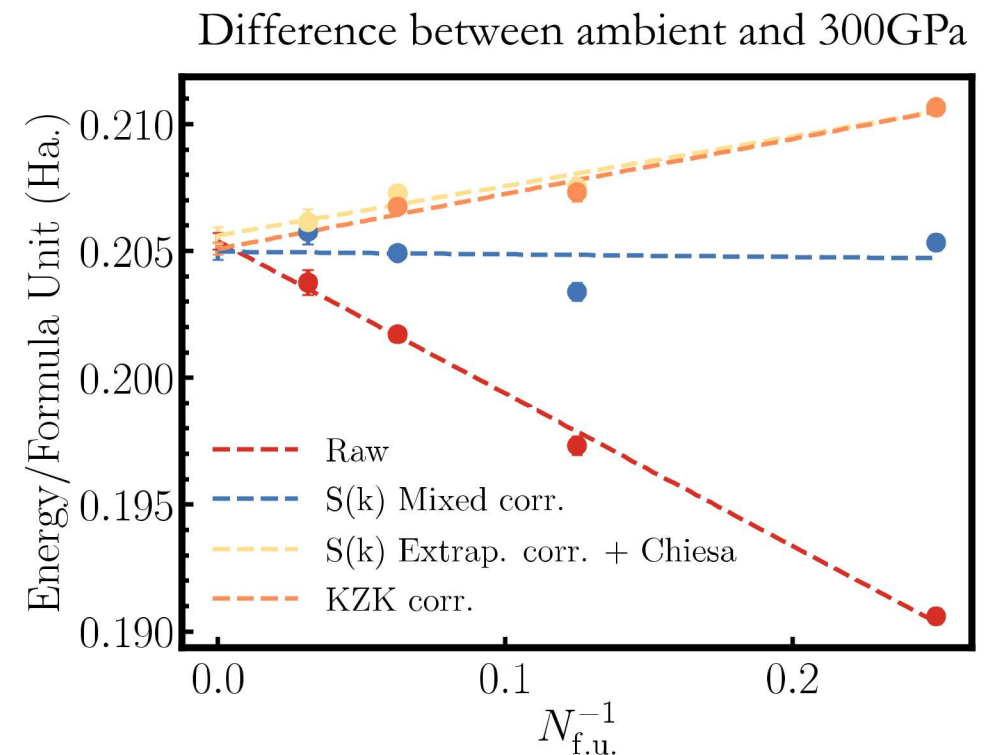
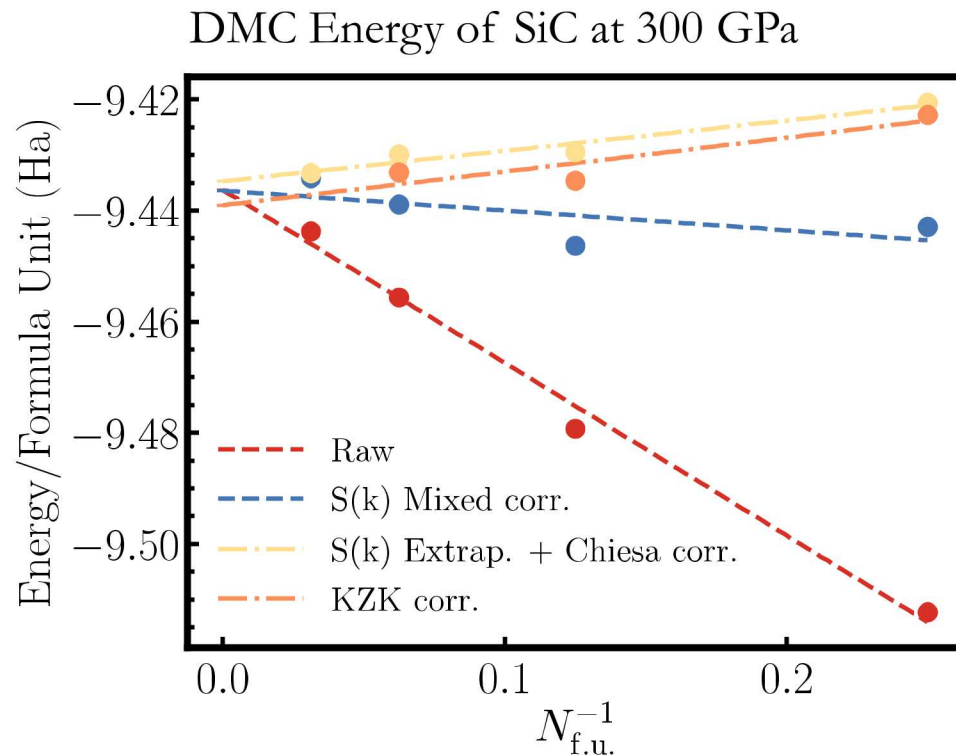


Backup: Two-Body Finite-Size Bias

In a finite simulation cell under PBC, artificial repetition of XC hole introduces large error into the potential (and kinetic) energies...must be extrapolated or corrected.

Simple $1/N$ extrapolation requires many simulation cells which is costly.

FS corrections: KZK (DFT-based total energy), $S(k)$ (potential energy), Chiesa (kinetic energy)



Backup: AFQMC Basis Set Convergence

Whereas DMC works in \mathbb{R}^{3N} configuration space, AFQMC works in 2nd quantization, i.e. determinant space. Plagued by finite basis set error.

Here, we use Kohn-Sham basis for AFQMC, following Ma *et al.*, PRL **114**, 226401 (2015)

For E(V) curves, interested in converging *relative* energies

