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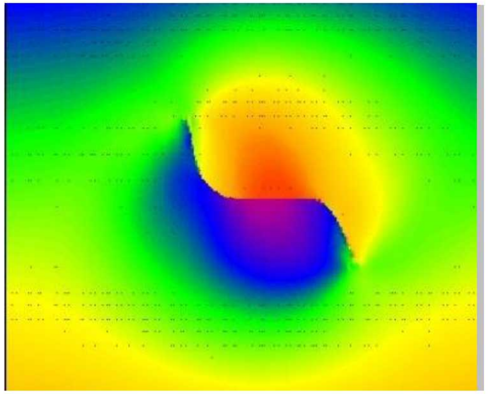

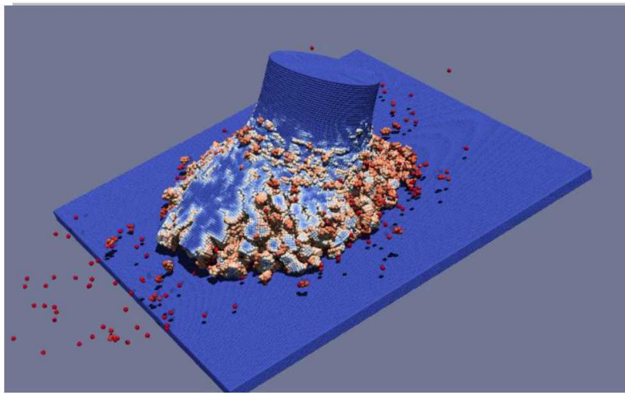

Nonlocality in peridynamics

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Workshop on Experimental and Computational Fracture Mechanics

Baton Rouge, LA, February 26, 2020

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

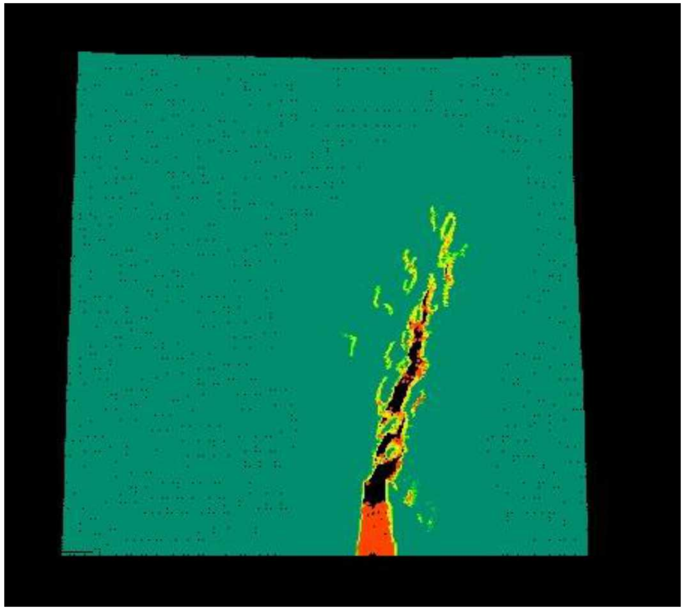
Outline

- Nonlocality
  - It’s not as weird as everybody thinks
- Peridynamics background
  - All-in on nonlocality
- Can nonlocality be derived or observed?
  - Long-range forces
  - Smoothed degrees of freedom (homogenization)
  - Multiple pathways for flux
  - Wave dispersion

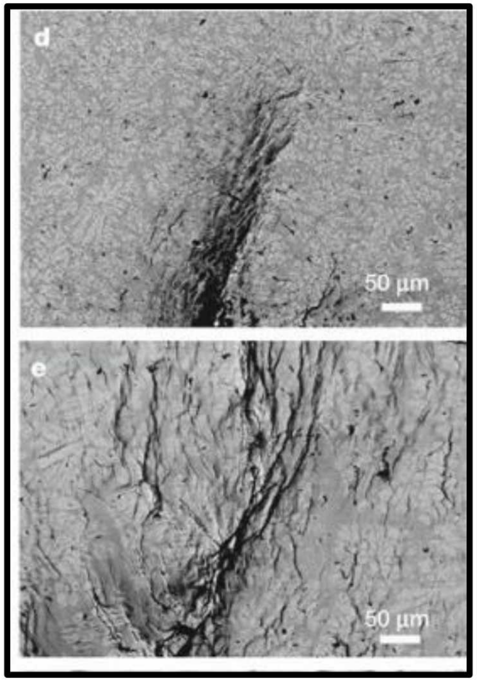
Do we ask too much of the local theory of continuum mechanics?

What peridynamics seeks to accomplish

- Treat material points on or off of evolving discontinuities with the same equations.
- Include long-range forces in the basic equations.
- Fit all this into a thermodynamic framework that's consistent with the mechanics.



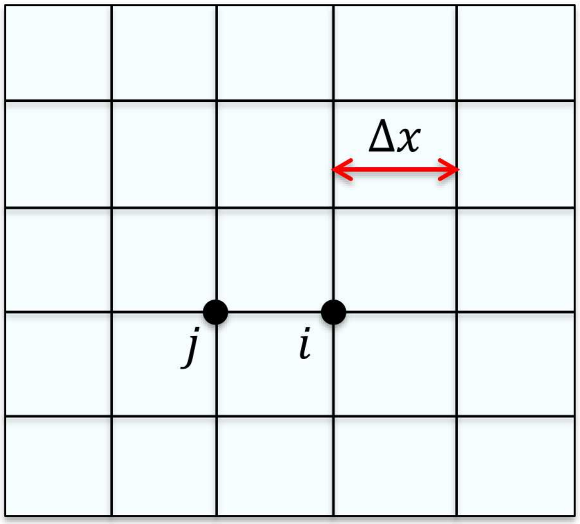
Peridynamic simulation



Metallic glass crack tip*

*Hofmann et al, Nature (2008)

Discretized numerical methods are nonlocal



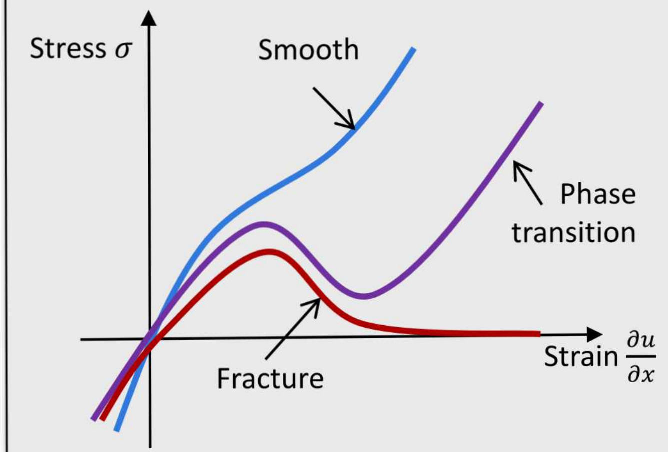
- Node i interacts directly with node j through the finite element equations.
- Interaction is across a finite distance Δx.
- This is a form of nonlocality.
    - Notwithstanding that the result converges to the local result as $\Delta x \to 0$.

Local PDEs get themselves into trouble

- Classical (Cauchy) PDE:

$$\rho \ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma} \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \right) + \mathbf{b}.$$

- Many material models $\boldsymbol{\sigma}(\cdot)$ evolve into deformations that are incompatible with the fundamental assumptions.
  - Phase boundaries, shock waves, cracks, ...
- Can't directly treat some important physical effects.
  - Wave dispersion, surface energy, microstructure evolution, long-range forces, ...
- People often take drastic measures if they want to work with this PDE.
  - Element deletion, ...



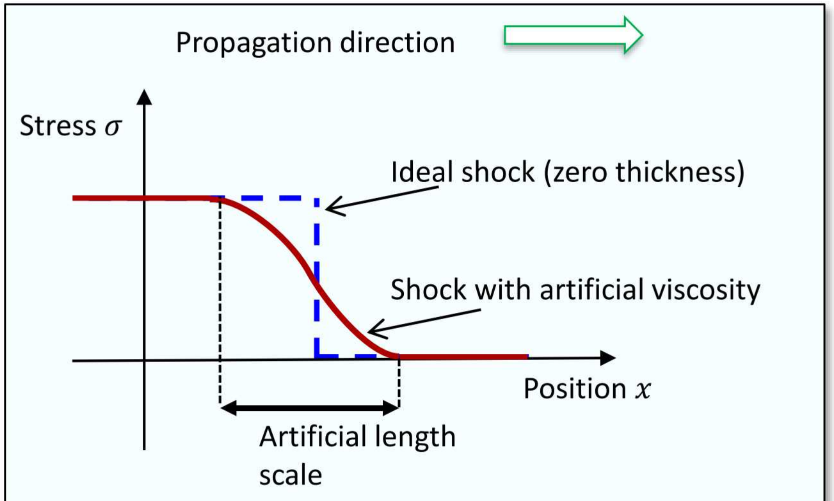
Material models and the features they "try" to predict

These drastic measures often involve nonlocality

- Example: Artificial viscosity spreads out a shock wave and dissipates energy.

$$\rho\ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma}\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right) + \gamma(\nabla \cdot \dot{\mathbf{u}})^2 + \mathbf{b}.$$

- It avoids the need to apply jump conditions across an ideal shock.
- It allows converntional discretization to be used "within" a shock.
- By spreading out a shock it introduces a length scale.
- This is a type of nonlocality.



FEM codes spread out a shock wave over ~6 elements

- J. Von Neumann & R. D. Richtmyer, *J. Appl. Phys*. 21 (1950). 232

Peridynamics goes all-in on nonlocality

Classification of some theories with respect to local/nonlocality:

PDEs with no length scale:
- Classical continuum mechanics

PDEs with a length scale:
- Micropolar
- Mindlin
- Kroner
- Eringen
- Phase field
- Nonlocal damage
- Plate & shell theories
- Gradient theories

Full nonlocality:
- Kunin
- Peridynamics

- Every fundamental relation in peridynamics is nonlocal in space:
  - Transport
  - Conservation
  - Material models

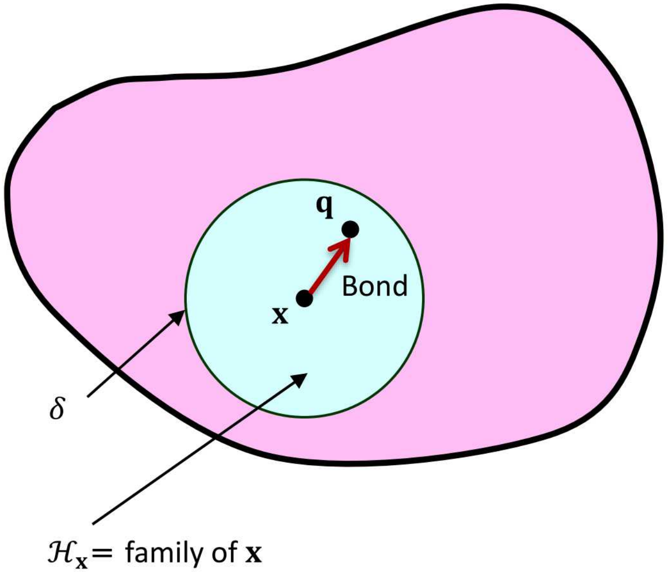
Peridynamic* momentum balance

- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.

Peridynamic equilibrium equation

$$\int_{\mathcal{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{q}, \mathbf{x}) \, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}) = 0$$

$\mathbf{f}$ = bond force density (from the material model, which includes damage)



- If $\mathbf{f}$ satisfies $\mathbf{f}(\mathbf{x}, \mathbf{q}) = -\mathbf{f}(\mathbf{q}, \mathbf{x})$ for all $\mathbf{x}, \mathbf{q}$ then linear momentum is conserved.

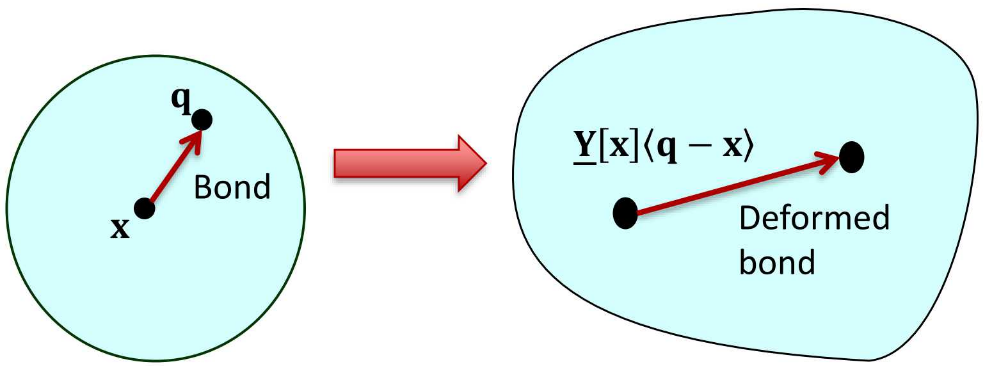
- SS, JMPS (2000)

* Peri (near) + dyne (force)

Formalism for nonlocal interactions: States

- A *state* is a mapping whose domain is all the bonds ξ in a family.

$$\underline{\mathbf{A}}\langle\xi\rangle = \text{something} \qquad \forall \xi \in \mathcal{H}.$$

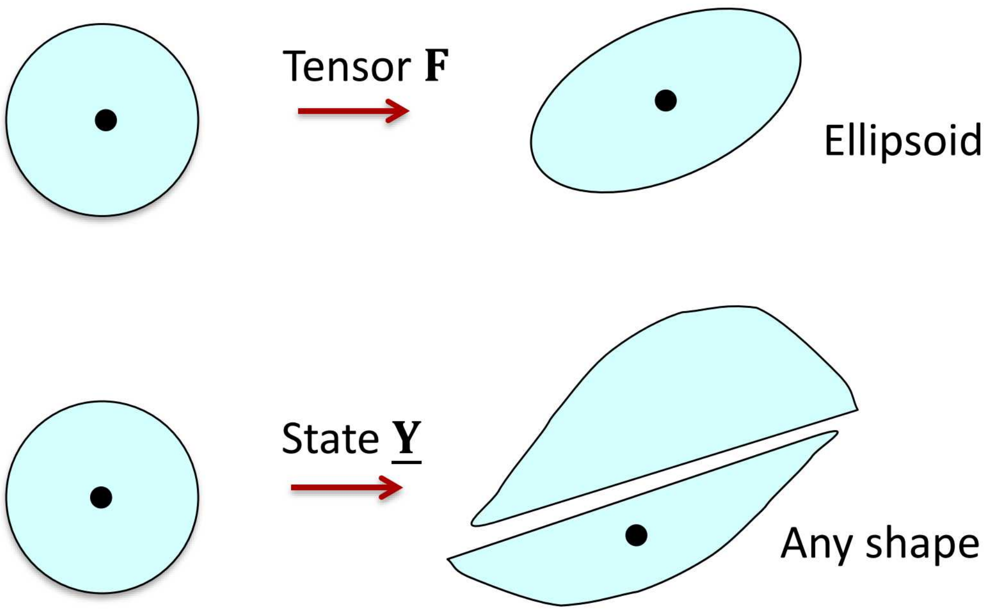


- Deformation state...

$$\underline{\mathbf{Y}}[\mathbf{x}]\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x}) = \text{deformed image of the bond}$$

States: Nonlocal analogues of second order tensors

- Classical theory uses tensors (linear mappings from vectors to vectors).
- Peridynamics uses states (nonlinear mappings from vectors to vectors).



Peridynamic vs. local equations

- Structurally similar but with states instead of local operators.

Relation	*Peridynamic theory*	*Standard theory*
Kinematics	$\underline{\mathbf{Y}}\langle \mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \dfrac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \int_{\mathcal{H}} \Big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\Big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle \mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle \mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle \mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative)	$\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient)
First law	$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} + q + r$	$\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$

$$\underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle \boldsymbol{\xi}\rangle \cdot \dot{\underline{\mathbf{Y}}}\langle \boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$

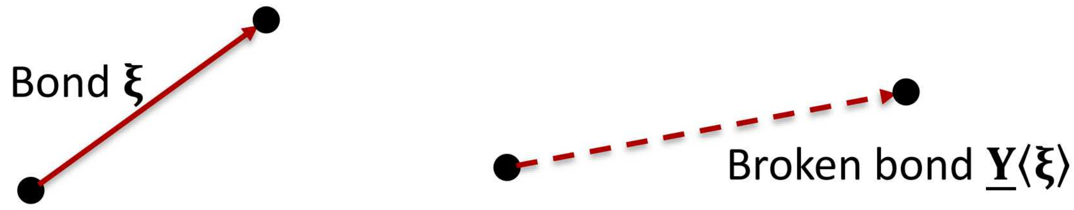
Damage

- Damage is usually treated through *bond breakage*.
- After a bond ξ *breaks* according to some criterion, it no longer carries any force.
- Typical breakage criterion: prescribed *critical bond strain* s_0:

$$s = \frac{|\underline{\mathbf{Y}}\langle\xi\rangle| - |\xi|}{|\xi|} \qquad \text{bond strain.}$$

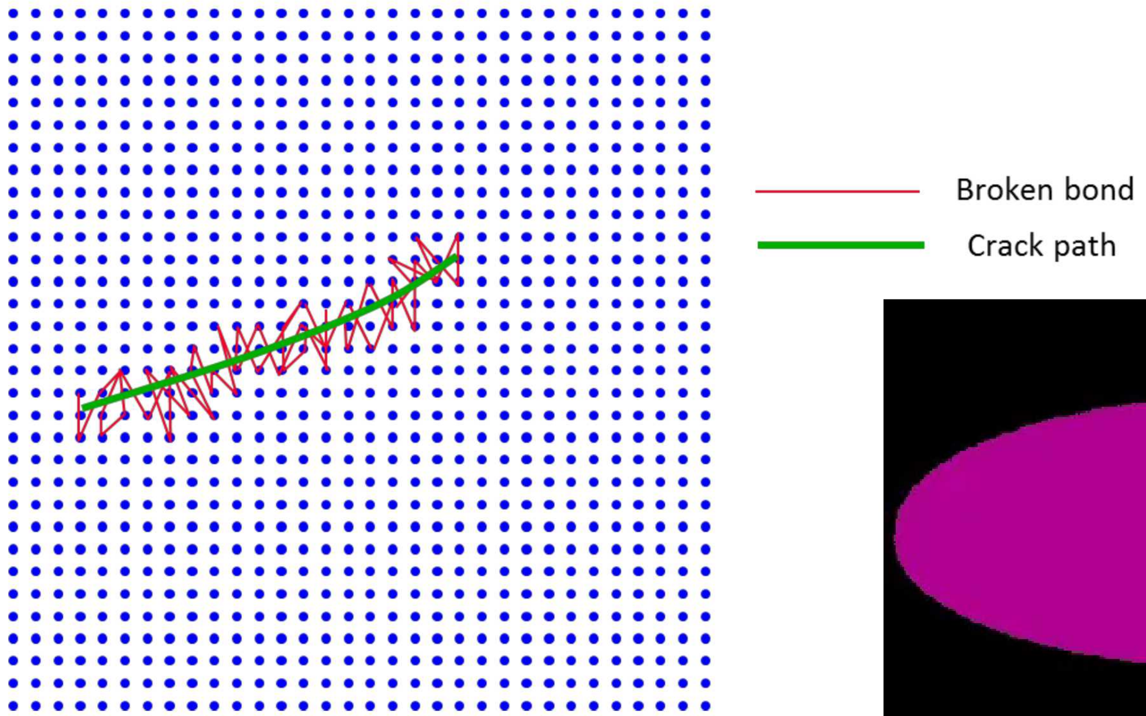
$$s >= s_0 \text{ at some time } t_0$$

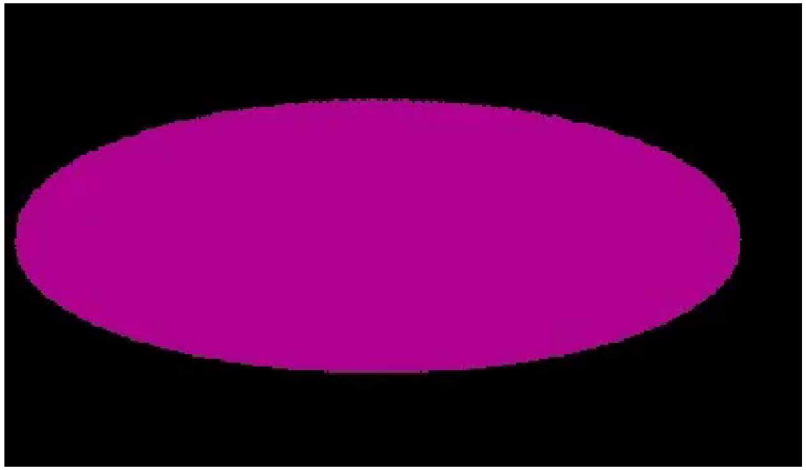
means the bond remains broken for all $t \geq t_0$.



Autonomous crack growth

- Cracks do what they want (grow, arrest, branch, curve, oscillate, …)





- SS & Askari, *Computers and Structures* (2005)

Many validation studies have been done

- First issue of the new *Journal of Peridynamics and Nonlocal Modeling* had a review article by Diehl on published validation to date:

Journal of Peridynamics and Nonlocal Modeling
https://doi.org/10.1007/s42102-018-0004-x

REVIEWS

A Review of Benchmark Experiments for the Validation of Peridynamics Models

Patrick Diehl[1] · Serge Prudhomme[2] · Martin Lévesque[1]

Received: 2 November 2018 / Accepted: 25 December 2018

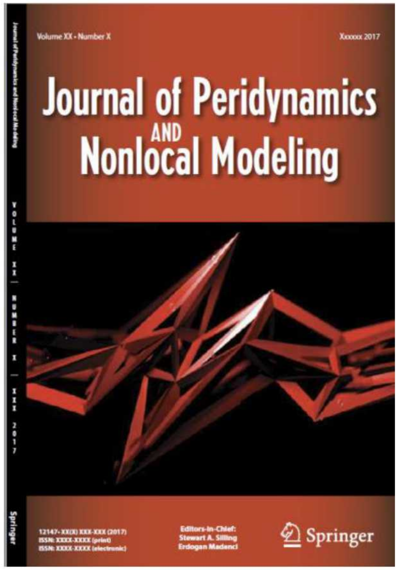


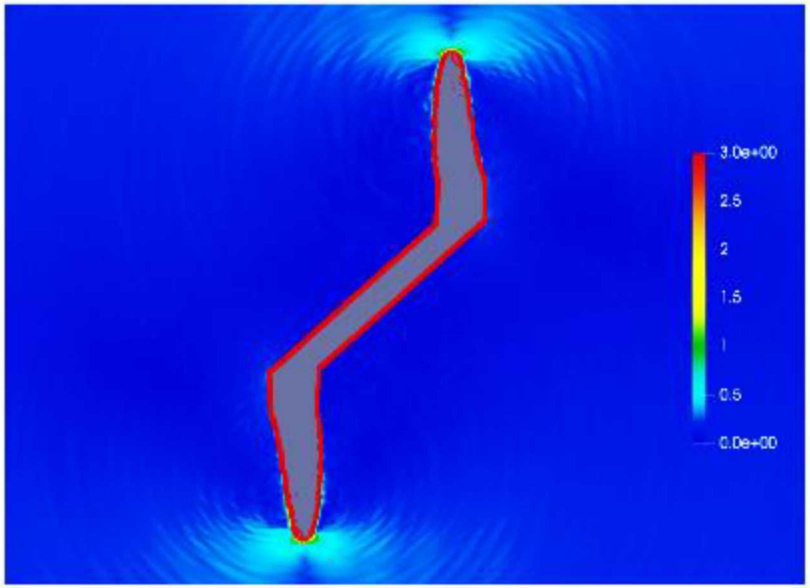
Table 3 Applications of bond-based and state-based peridynamics for the comparison with experimental data

Material	Mechanical test	B	S	Exp	Sim
Composite	Flexural test with an intial crack	✓		[75]	[2]
Composite	Damage growth prediction (six-bolt specimen)	✓		[120]	[96]
Composite	Damage prediction (center-cracked laminates)	✓		[6, 12, 69, 134]	[70]
Composite	Dynamic tension test (prenoteched rectangular plate)	✓		[12, 65]	[58]
Steel	Crack growth (Kalthoff-Winkler)	✓	✓	[66–68]	[3, 52, 114, 144]
Aluminum/Steel	Fracture (compact tension test)	✓		[9, 77, 89, 91]	[135, 141, 142]
Aluminum	Taylor impact test		✓	[4, 21]	[3, 43, 45]
Aluminum (6061-T6)	Ballistic impact test		✓	[132]	[127]
Concrete	Lap-splice experiment	✓		[48]	[48]
Concrete	3-point bending beam	✓	✓	[19, 63]	[7, 51]
Concrete	Failure in a Barazilian disk under compression		✓	[51]	[54]
Concrete	Anchor Bolt Pullout	✓		[128]	[83]
Glass	Dynamic crack propagation (prenotched thin rectangular plate)	✓		[15, 36, 100]	[2, 53, 144]
Glass	Impact damage with a thin polycarbonate backing	✓		[8, 20, 40]	[59]
Glass	Single crack paths (quenched glass plate)	✓		[13, 103, 136]	[71]
Glass	Multiple crack paths (quenched glass plate)	✓		[102, 137]	[71]
Glass	Crack tip propagation speed	✓		[15]	[52, 53, 144]
PMMA	Fast cracks in PMMA	✓		[39]	[2]
PMMA	Tensile test	✓		[124]	[32]
Soda-lime glass	Impact on a two-plate system	✓		[16, 130]	[130]

Legend: B refers to bond-based peridynamics, S refers to state-based peridynamics, Exp to experimental data, and Sim to simulation

Peridynamics converges as the horizon → 0

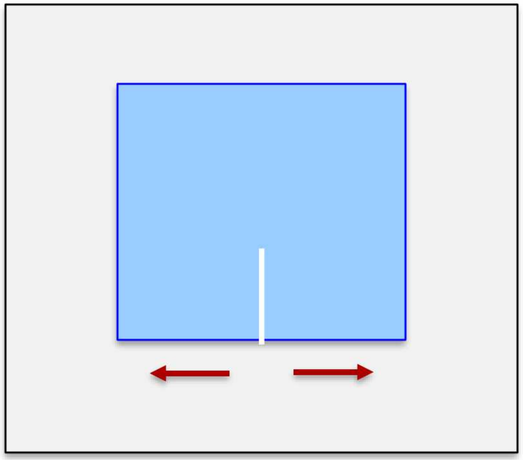
- Linear peridynamics converges to Navier equations of linear elasticity.
- Linear or nonlinear material models converge to a stress-strain relation.
- Problems with nonconvex elastic peridynamic models can converge to nonlinear elasticity with Griffith cracks.



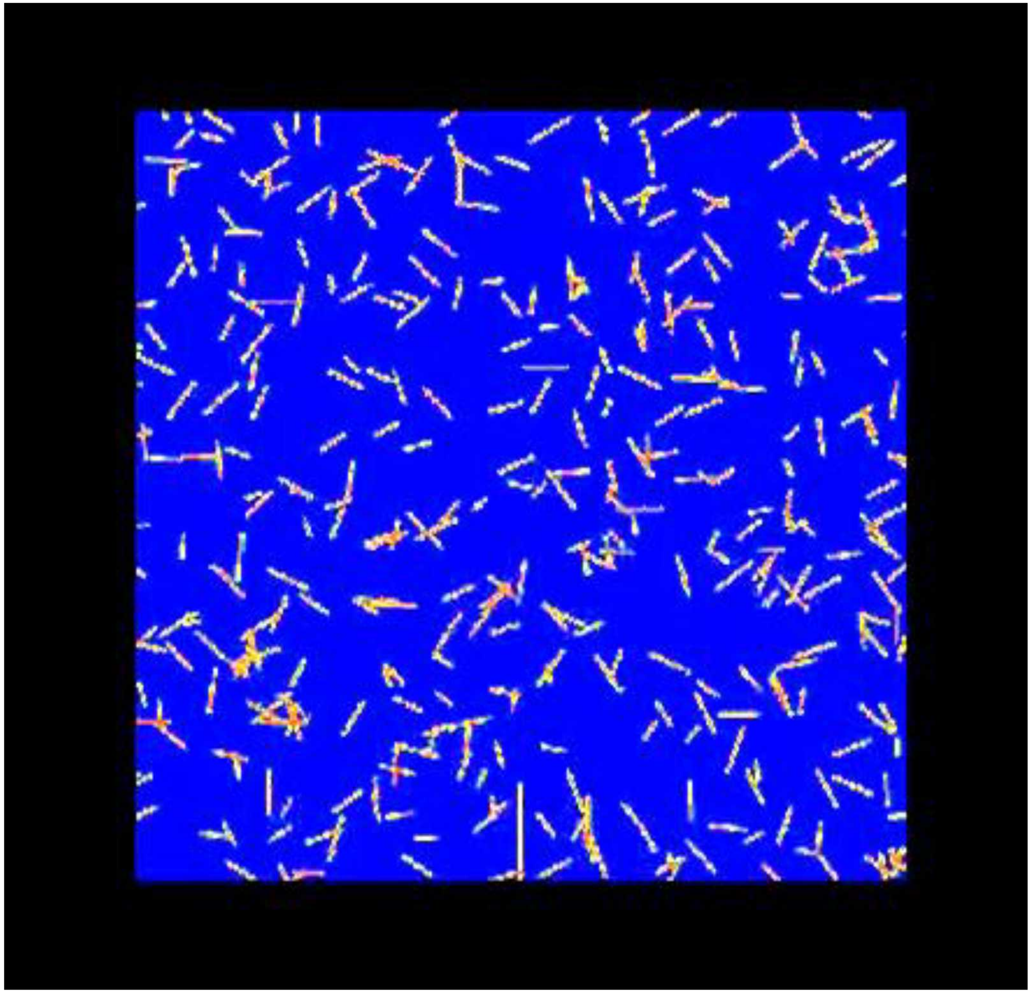
Angled crack growth simulated with a nonconvex peridynamic material model*

- E. Emmrich & O. Weckner, *Communications in Mathematical Sciences* (2007).
- F. Bobaru et al., Int. *Journal for Numerical Methods in Engineering* (2009).
- T. Mengesha, & Q. Du, *Journal of Elasticity* (2014).
- S.S. & R. B. Lehoucq, *Journal of Elasticity* (2008).
- P. Seleson & D.J. Littlewood, *Computers & Mathematics with Applications* (2016).
- *R. P. Lipton, R. B. Lehoucq, & P.K. Jha, *Journal of Peridynamics and Nonlocal Modeling* (2019).

Example: Fracture in a brittle plate with a lot of defects

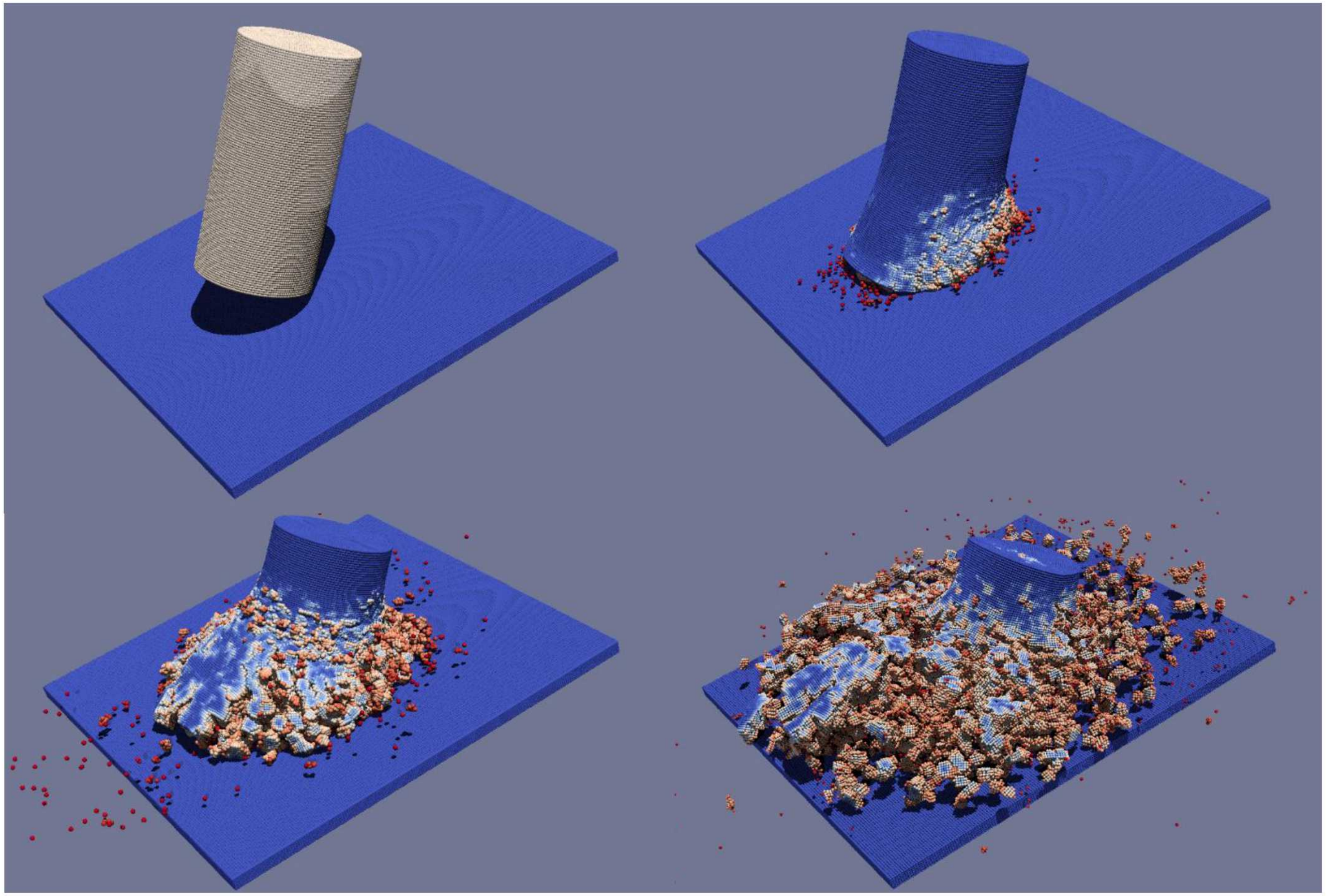


VIDEO



Example: Fragmentation due to impact

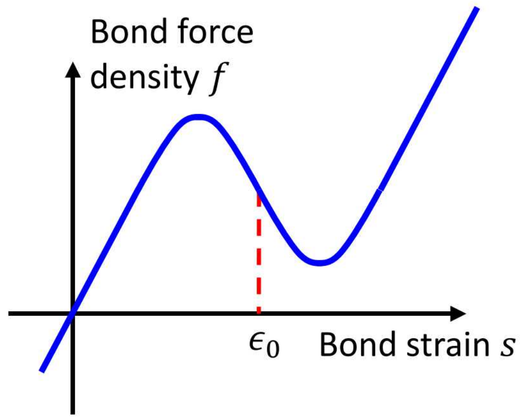
- Brittle cylinder vs. rigid plate at 1km/s.

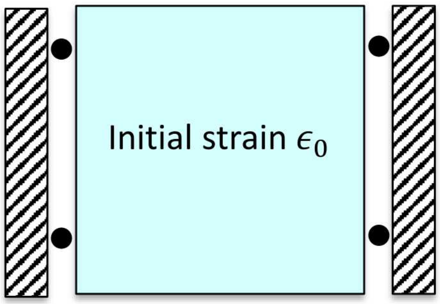


Colors show damage

Example: Microstructure evolution

- Plate with ends fixed. Global strain ϵ_0 is in the unstable part of the material model.
- Complex microstructure appears at first, then simplifies.
- Driving force is the energy stuck in a phase boundary.



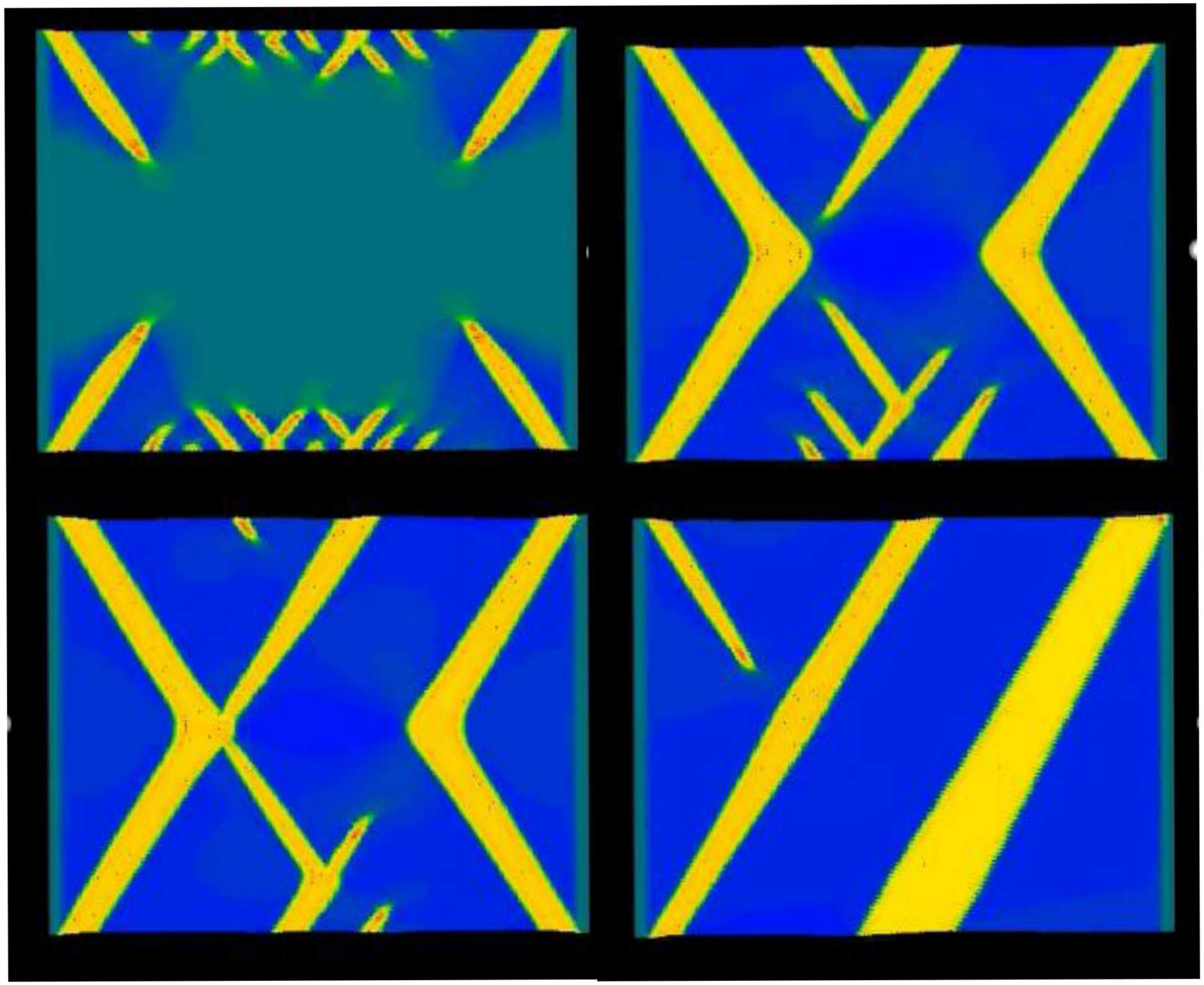


VIDEO

Colors show bond strain

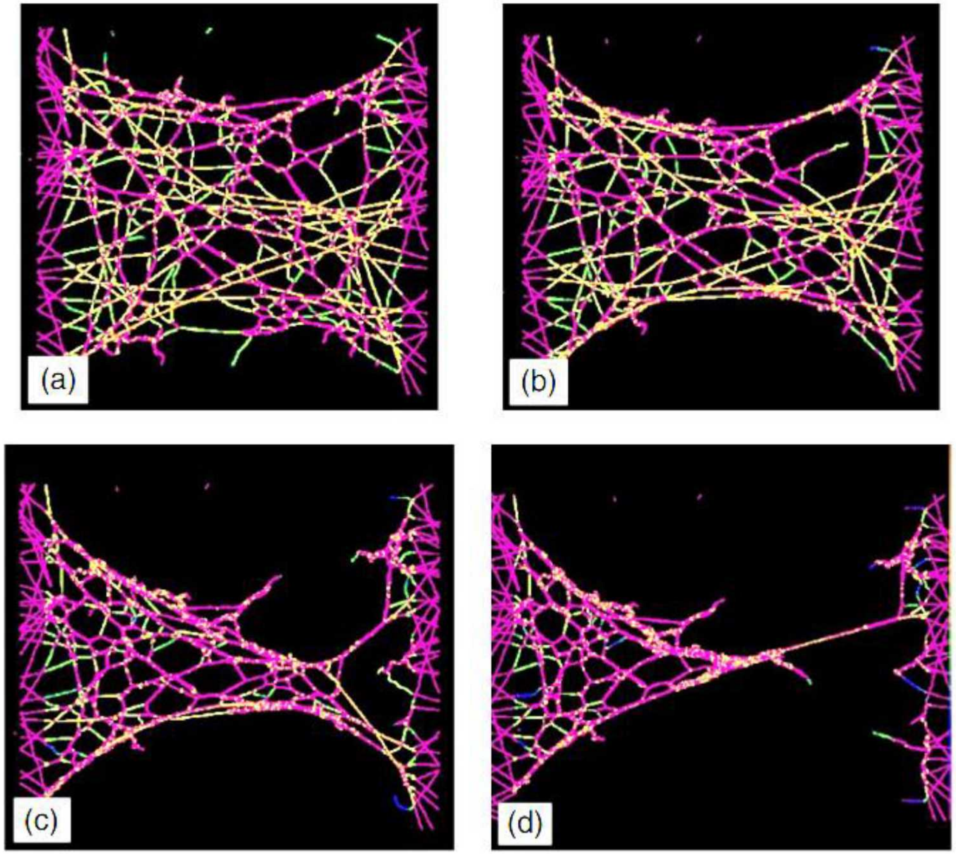
Example: Microstructure evolution

Colors show bond strain



Straightforward case for nonlocality: When there really are long-range forces

- Fracture of nanofiber network held together by Van der Waals forces.



F. Bobaru, *Modelling and Simulation in Materials Science and Engineering* 15, no. 5 (2007): 397.

Smoothing the smallest scale degrees of freedom results in nonlocality

- Try to approximate known, small-scale response (e.g. molecular motion) by a continuous variable, yet retain realistic behavior.
- How to make the connection?
- One approach: Smooth out the small-scale degrees of freedom.
- Example:
  - Heterogeneous infinite bar.

- Small-scale model (local):

$$\rho(x)\ddot{u}(x,t) = \sigma'(x,t) + b(x,t)$$

where ρ=density, u=displacement, σ=stress, and b=body force density.

- Material model:

$$\sigma(x,t) = E(x)u'(x,t)$$

where E=Young's modulus.

Define a smoothed displacement field

- Let $w(z)$ be a smoothing function on $z \in [-\epsilon, \epsilon]$, $\int w = 1$, $w(-z) = w(z)$.
- Define the smoothed displacement field $\bar{u}$ by

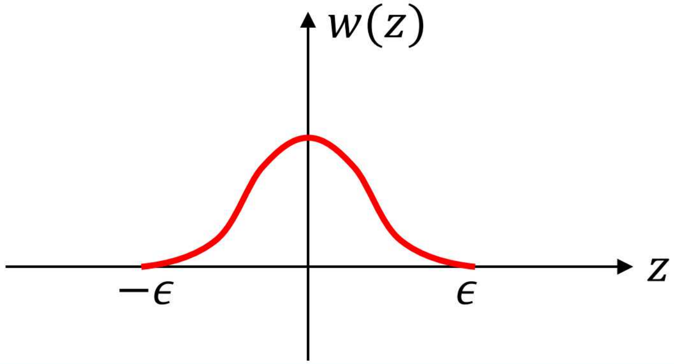
$$\bar{u}(x,t) = \frac{1}{\bar{\rho}(x)} \int_{-\infty}^{\infty} w(p-x)\rho(p)u(p,t)\, dp, \qquad \bar{\rho}(x) := \int_{-\infty}^{\infty} w(p-x)\rho(p)\, dp$$

- Recall

$$\rho(x)\ddot{u}(x,t) = \sigma'(x,t) + b(x,t).$$

- Multiply through by w and integrate, find that

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = \int_{-\infty}^{\infty} w(x-p)\sigma'(p,t)\, dp + \bar{b}(x,t), \qquad \bar{b}(x,t) := \int_{-\infty}^{\infty} w(x-p)b(p,t)\, dp$$



Evolution equation for smoothed DOFs

- Recall

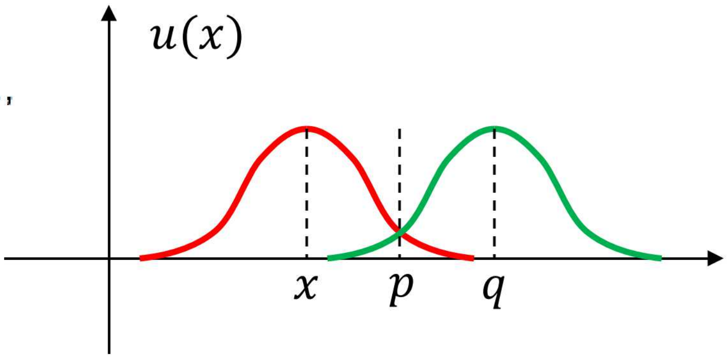
$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = \int_{-\infty}^{\infty} w(x-p)\sigma'(p,t)\, dp + \bar{b}(x,t).$$

- Integrate by parts (surprise!):

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = -\int_{-\infty}^{\infty} w'(x-p)\sigma(p,t)\, dp + \bar{b}(x,t).$$

- Starting to look nonlocal.
- Let q be defined so that p is halfway between q and x, i.e.,

$$p = \frac{x+q}{2}.$$



- Then

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = -\frac{1}{2}\int_{-\infty}^{\infty} w'\left(\frac{q-x}{2}\right)\sigma\left(\frac{q+x}{2},t\right)\, dp + \bar{b}(x,t).$$

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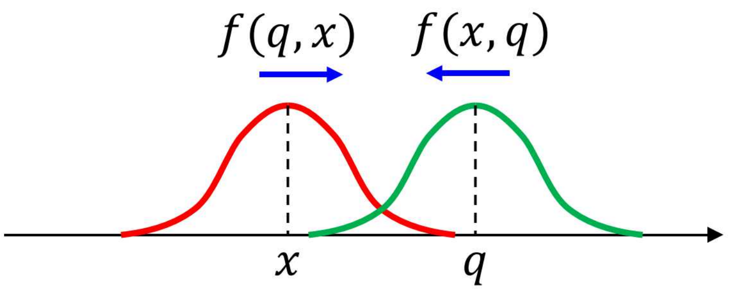
Evolution equation is nonlocal

- Recall

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = -\frac{1}{2}\int_{-\infty}^{\infty} w'\left(\frac{q-x}{2}\right)\sigma\left(\frac{q+x}{2},t\right)\,dq + \bar{b}(x,t).$$

- Now define the *pairwise bond force density* by

$$f(q,x) = -\frac{1}{2}w'\left(\frac{q-x}{2}\right)\sigma\left(\frac{q+x}{2},t\right)$$



and define the *horizon* by

$$\delta = 2\epsilon.$$

- We now have

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = \int_{x-\delta}^{x+\delta} f(q,x)\,dq + \bar{b}(x,t).$$

- Observe that f has the required symmetry

$$f(x,q) = -f(q,x).$$

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Need a material model in terms of the smoothed DOFs

- Unfortunately we don't know σ.
- One possibility is to back out u' from the Fourier transform using the convolution theorem:

$$\mathcal{F}\{\bar{u}\} = \mathcal{F}\{w\}\mathcal{F}\{u\} \quad \Longrightarrow \quad u = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{\bar{u}\}}{\mathcal{F}\{w\}}\right\}$$

hence

$$\sigma(x) = E(x)\frac{d}{dx}\mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{\bar{u}\}}{\mathcal{F}\{w\}}\right\}.$$

- This is too much work!
- Instead come up with a nonlocal material model.

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Bond-based heterogeneous material model

- Observe that in equilibrium with $b \equiv 0$ and fixed stress σ_0,

$$u_0(x) = \int_0^x \frac{\sigma_0}{E(z)}\, dz.$$

- From this compute the smoothed displacements:

$$\bar{u}(x) = \int_{-\epsilon}^{\epsilon} w(\zeta) u_0(x+\zeta)\, d\zeta.$$

- Define a nonlocal material model by (omit t):

$$f(q,x) = C(q,x)(\bar{u}(q) - \bar{u}(x)), \qquad C(q,x) := \frac{\sigma_0 w'((q-x)/2)}{\bar{u}_0(q) - \bar{u}_0(x)}.$$

- This exactly reproduces the local result for equilibrium with $b \equiv 0$.
- (But not in general.)

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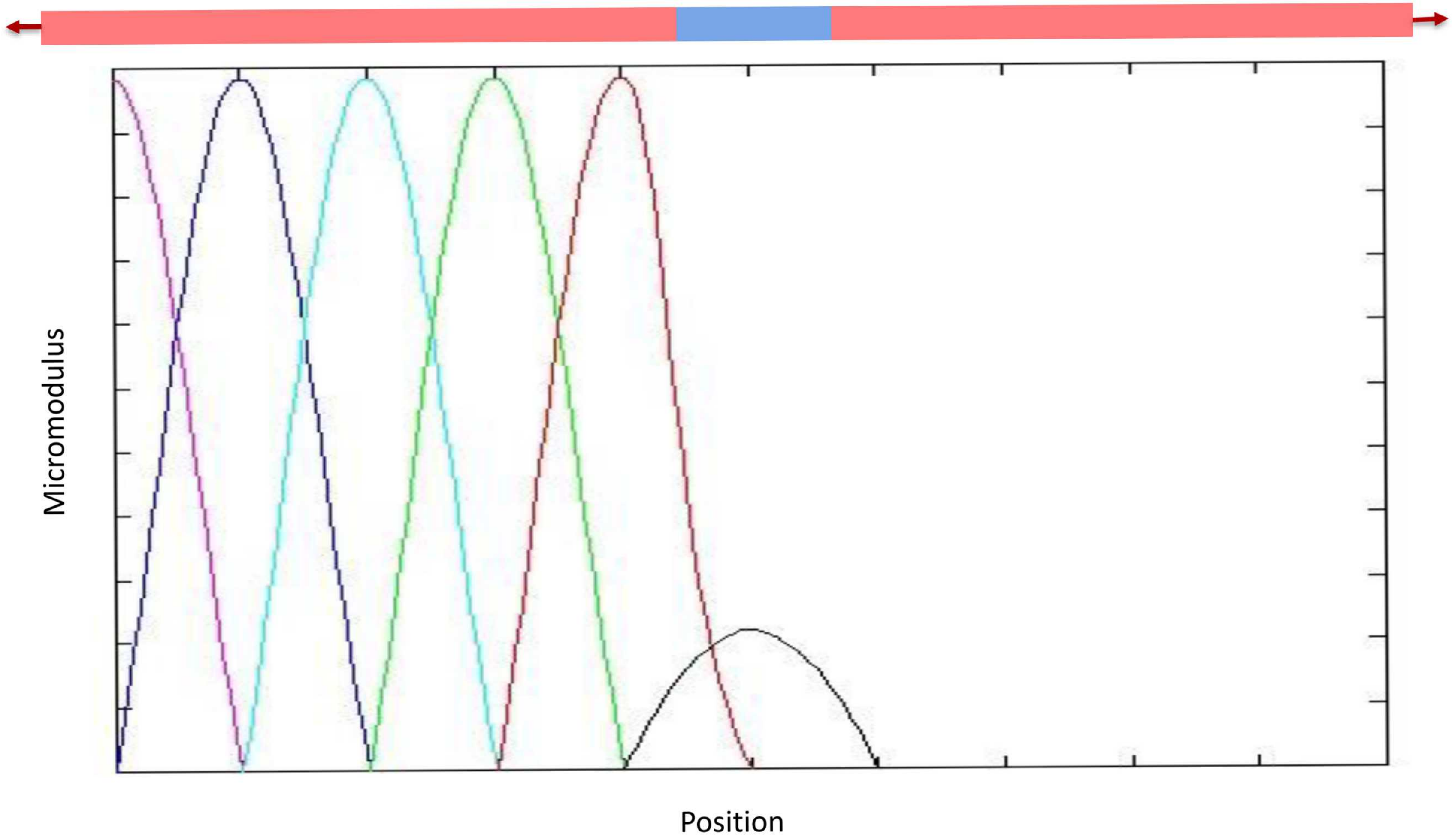
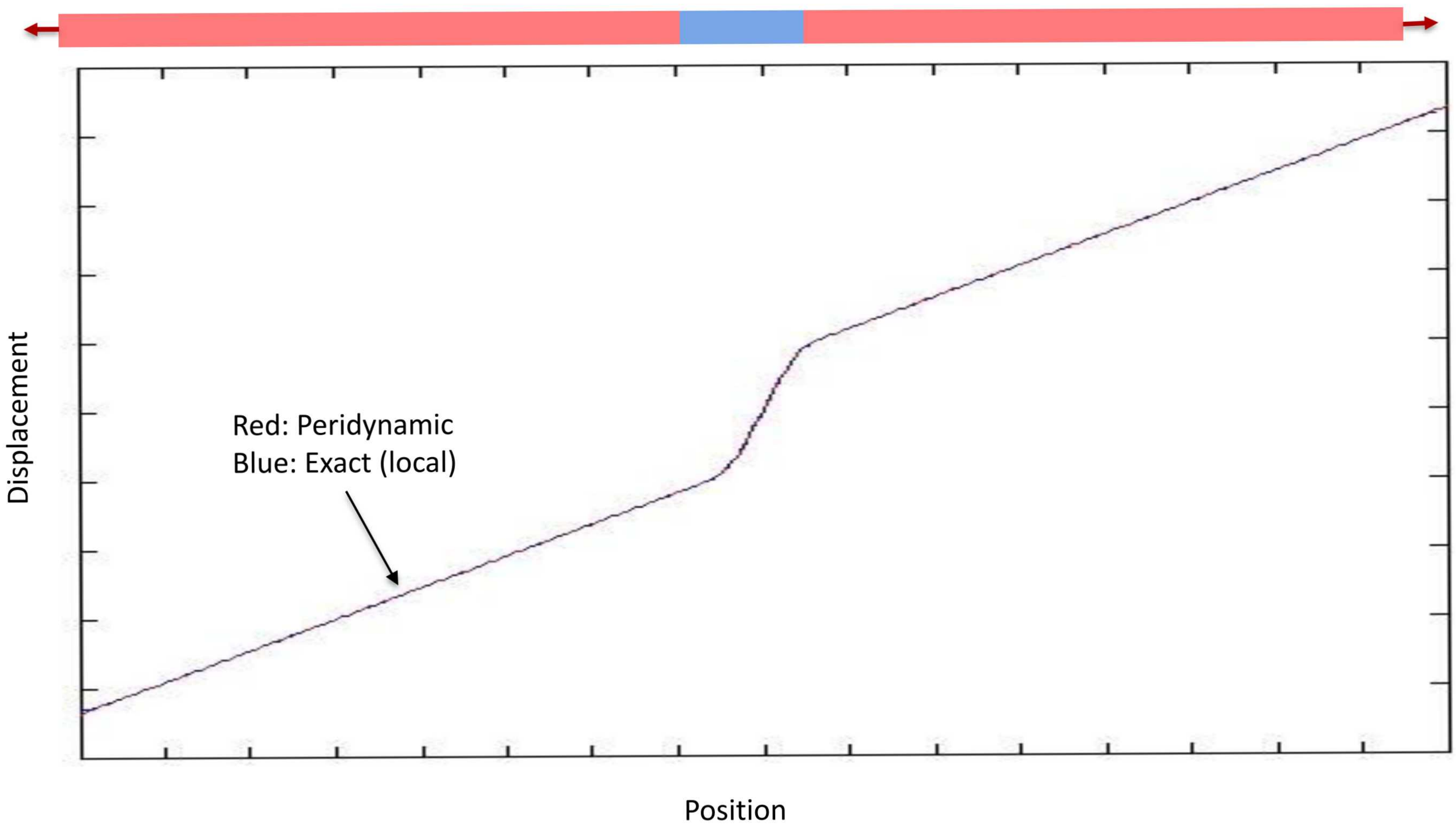
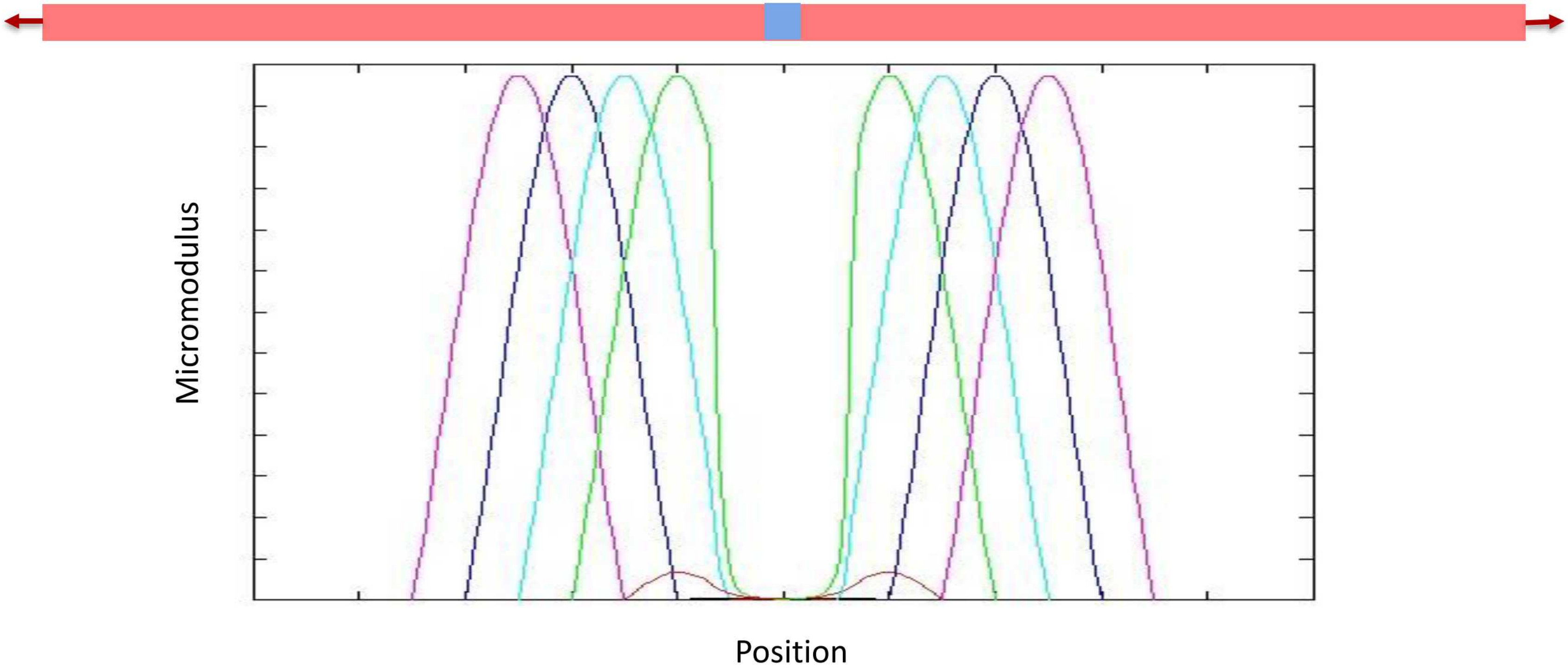
Bar with a soft spot: Micromodulus



Bar with a weak spot: Displacement



Bar with a very weak spot: Micromodulus shows broken bonds



- The heterogeneous peridynamic material model zeroes out the micromodulus for bonds crossing the crack.
- Bond breakage!

What the preceding analysis shows

- Using smoothed displacements results in a nonlocal evolution law.
- This evolution law is peridynamics provided a material model in terms of $\bar{u}$ is defined.
- The micromodulus is determined by:
  - The small-scale (local) material model and heterogeneity.
  - The smoothing function w.
- A nonlocal concept of damage (bond breakage) emerges naturally when the original problem contains a crack.

A hint of unexpected behavior

- Recall

$$\bar{u}(x) = \int w(x-p)u(p)\,dp.$$

- Fourier transform of any function v:

$$v^*(k) = \mathcal{F}\{v(x)\} = \int_{-\infty}^{\infty} e^{-ikx} v(x)\,dx.$$

- Convolution theorem

$$\bar{u}^* = w^* u^*$$

so that formally we can derive the small-scale displacements from any given $\bar{u}$:

$$u(x) = \mathcal{F}^{-1}\left\{\frac{\bar{u}^*}{w^*}\right\}.$$

A hint of unexpected behavior, ctd.

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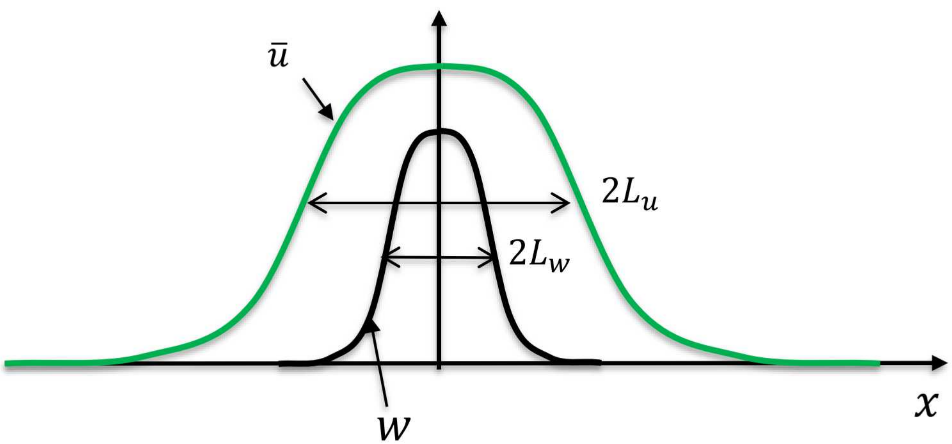
- Can we arbitrarily prescribe $\bar{u}$?
- Suppose w and $\bar{u}$ are both Gaussians:

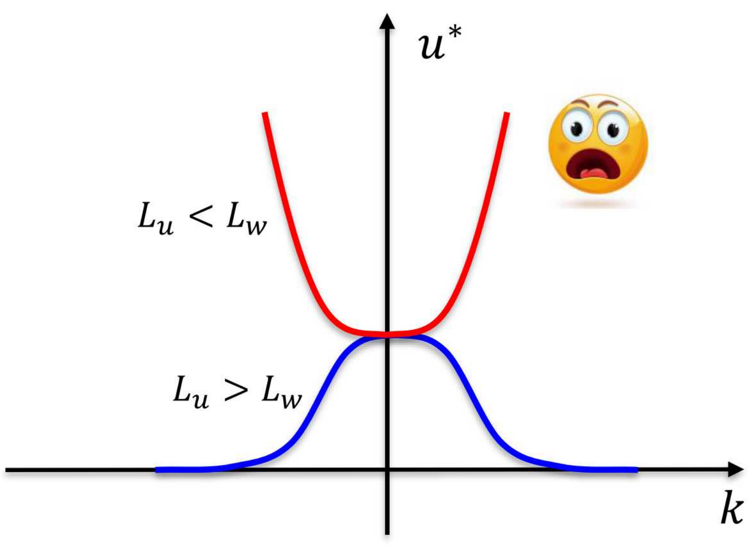
$$\bar{u}(x) = e^{-(x/L_u)^2}, \qquad w(x) = e^{-(x/L_w)^2}.$$

- Then

$$u^*(k) = \frac{\bar{u}^2(k)}{w^*(k)} = \sqrt{\frac{L_u}{L_w}}\, e^{\pi^2(L_w^2 - L_u^2)k^2}$$

- Bad news if $L_u < L_w$!



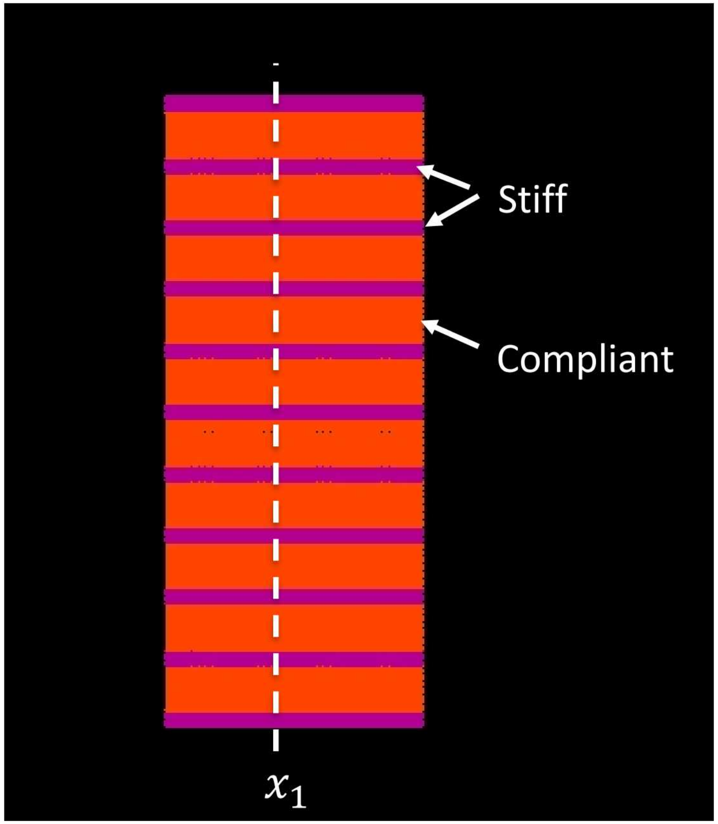


Can nonlocality be observed experimentally in elastostatics?

- Consider a 2D composite composed of alternating layers of stiff and compliant material.
- Smoothed DOF is the average x displacement along a vertical line.

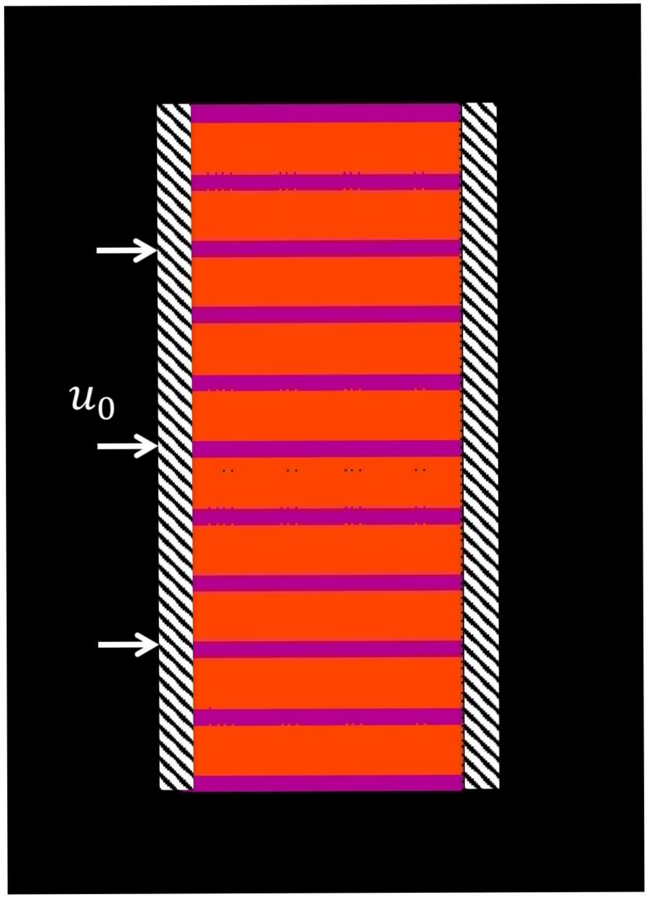
$$\bar{u} = \frac{1}{L}\int_0^L u_1 \, dx_2$$

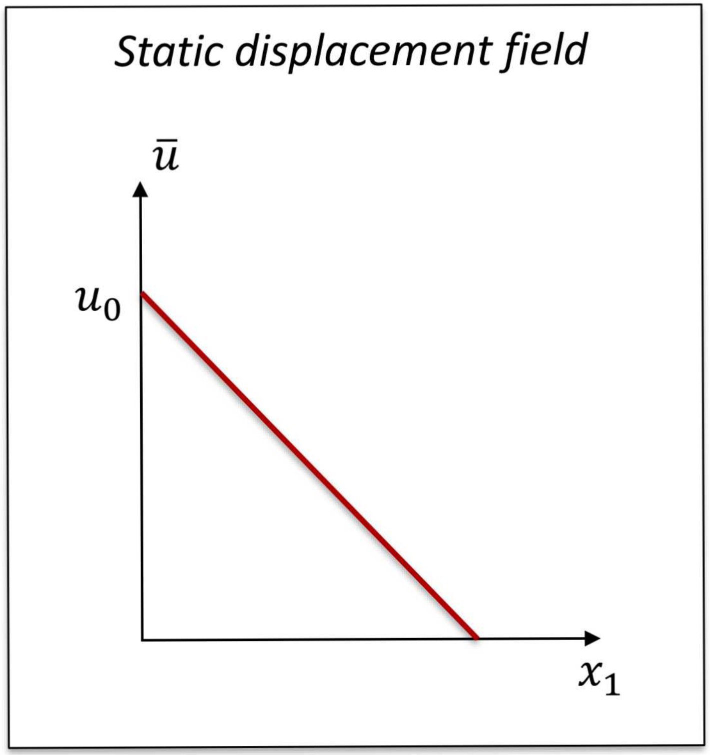
- We will examine "seemingly" 1D deformations.



Static Dirichlet problem for a composite

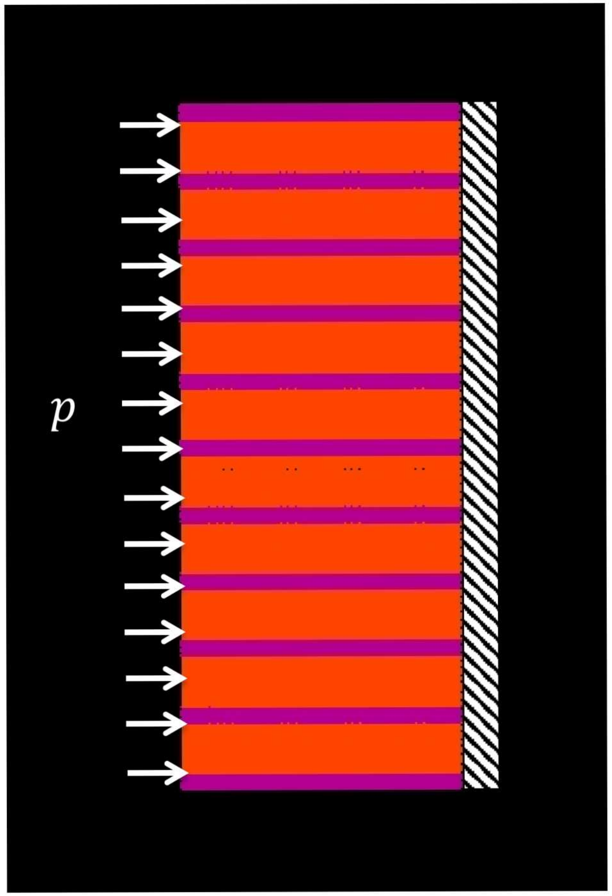
- Solve for the 2D displacements in the local theory.
- Both phases deform the same way.
- No surprises (yet).





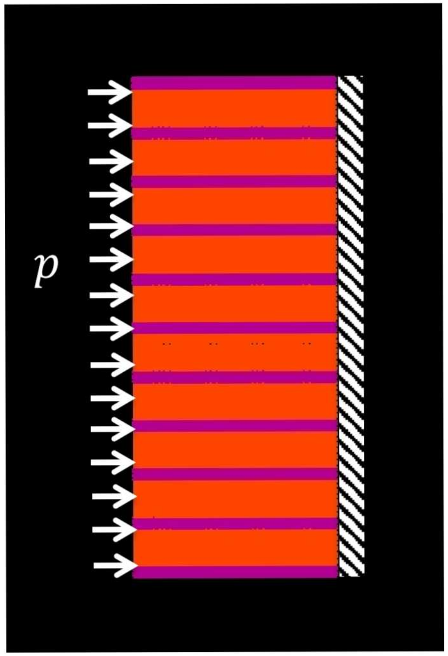
Now consider a mixed Dirichlet/Neumann static problem

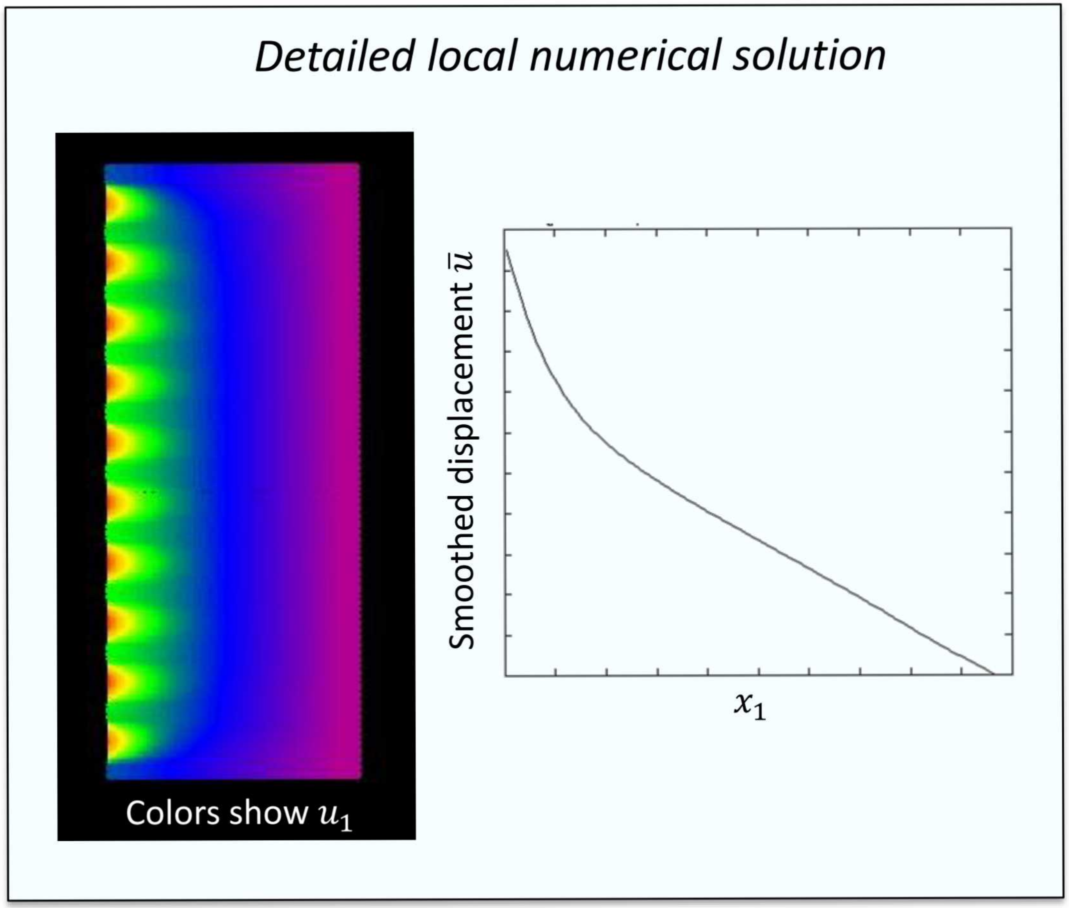
- Apply a constant traction p along the left surface.
- Still using 2D local theory.
- Should we still expect $\bar{u}$ to vary linearly with x_1?



Smoothed DOFs show interesting features

- A detail computational model shows complex behavior near the left edge.
- Smoothing this solution results in nonlinear $\bar{u}(x_1)$.





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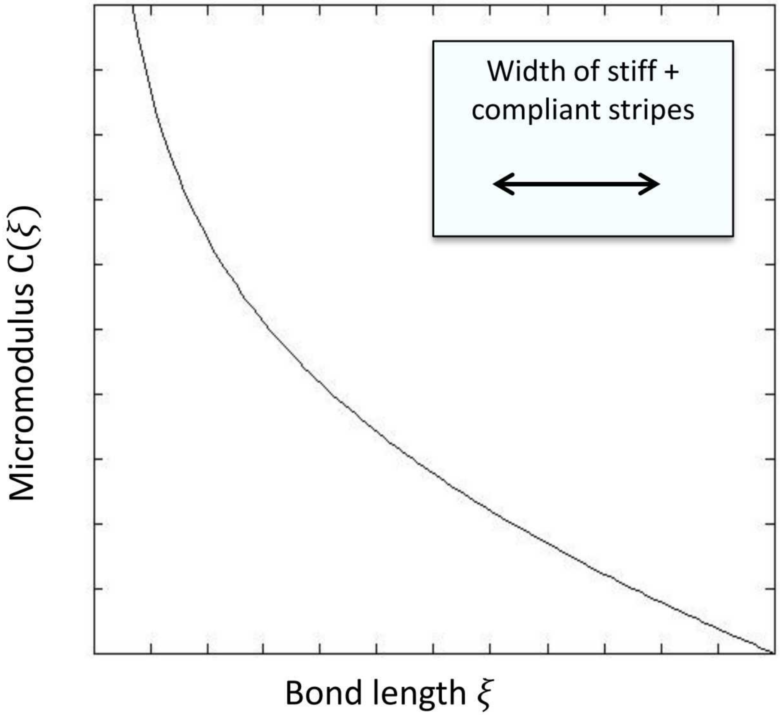
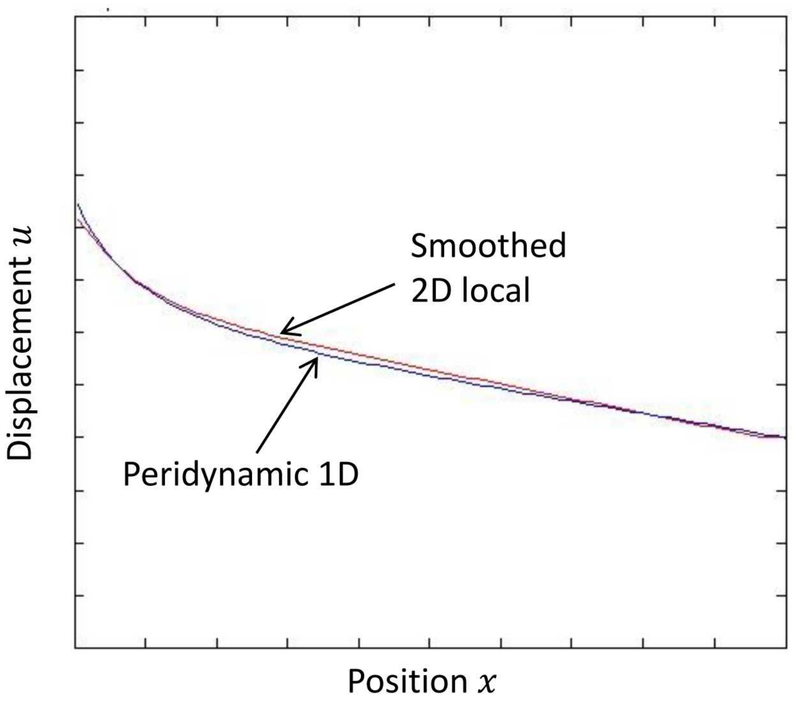
Nonlocality helps reproduce response near loaded boundary

- Tune a 1D peridynamic microelastic material model.
- Try to reproduce the behavior seen in the detailed 2D local solution..

Peridynamic material model

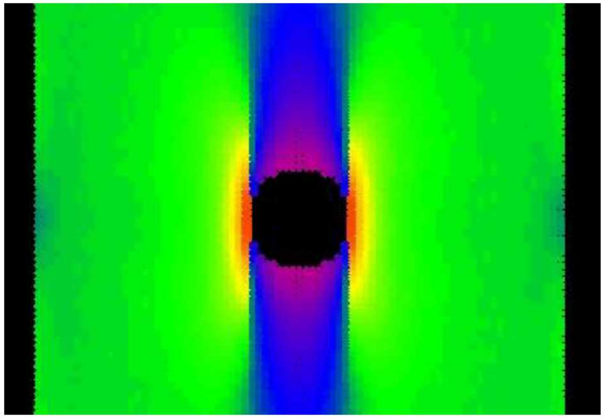


Predicted displacement

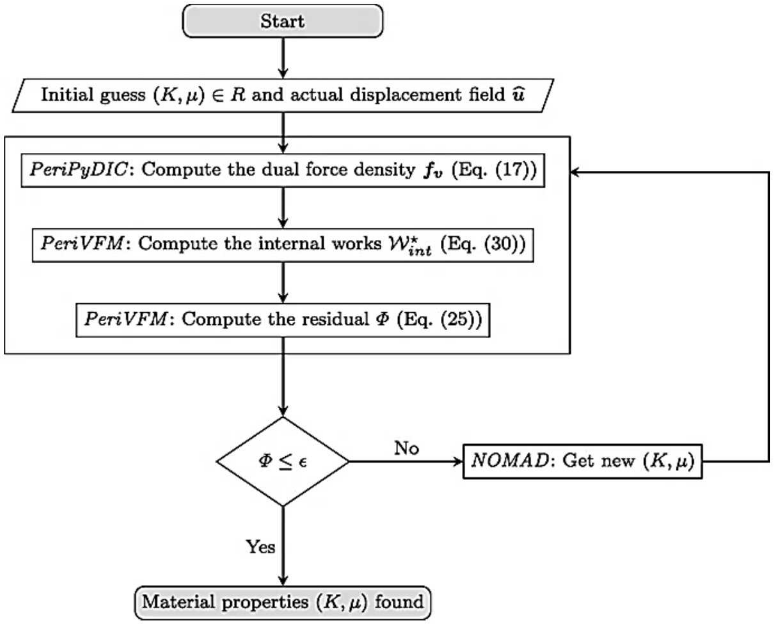


Nonlocal material parameters can be derived from static full-field data

- Digital image correlation (DIC).
- Virtual field method (VFM).
- Electronic speckle pattern interferometry (ESPI).



Strain concentration in a composite is influence by nonlocal length scales

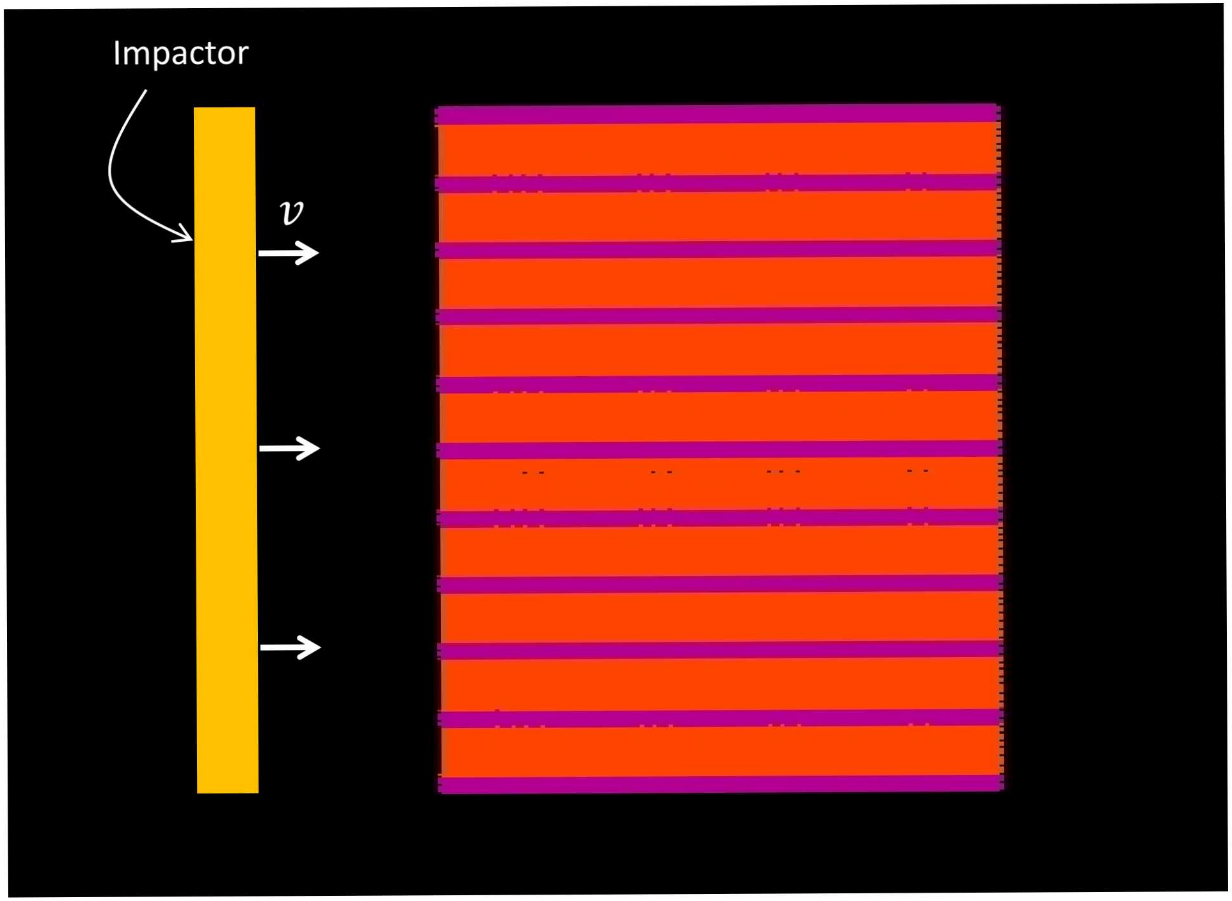


Flow chart for obtaining PD parameters by VFM*.

- L. Toubal, M. Karama, & B. Lorrain, *Composite structures*, (2005).
- D. Turner, B.Van Bloemen Waanders, & M. Parks *J. Mechanics of Materials and Structures* (2015).
- D. Turner, *J. Engineering Mechanics* (2015).
- *Delorme, R., Diehl, P., Tabiai, I. et al., *J Peridyn Nonlocal Model* (2020)

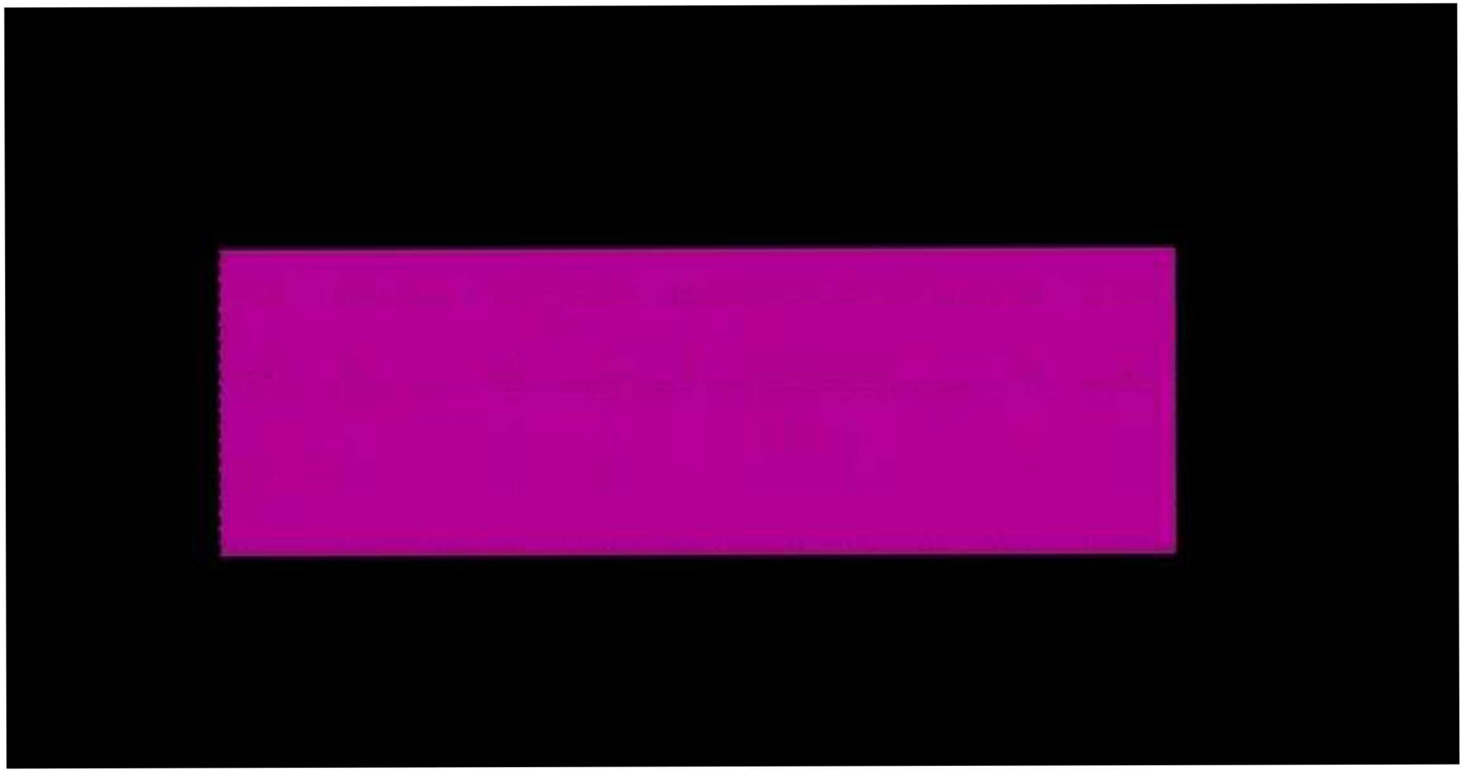
Dynamics: impact problem

- Impactor strikes the composite edge-on.



Dynamics: impact problem video

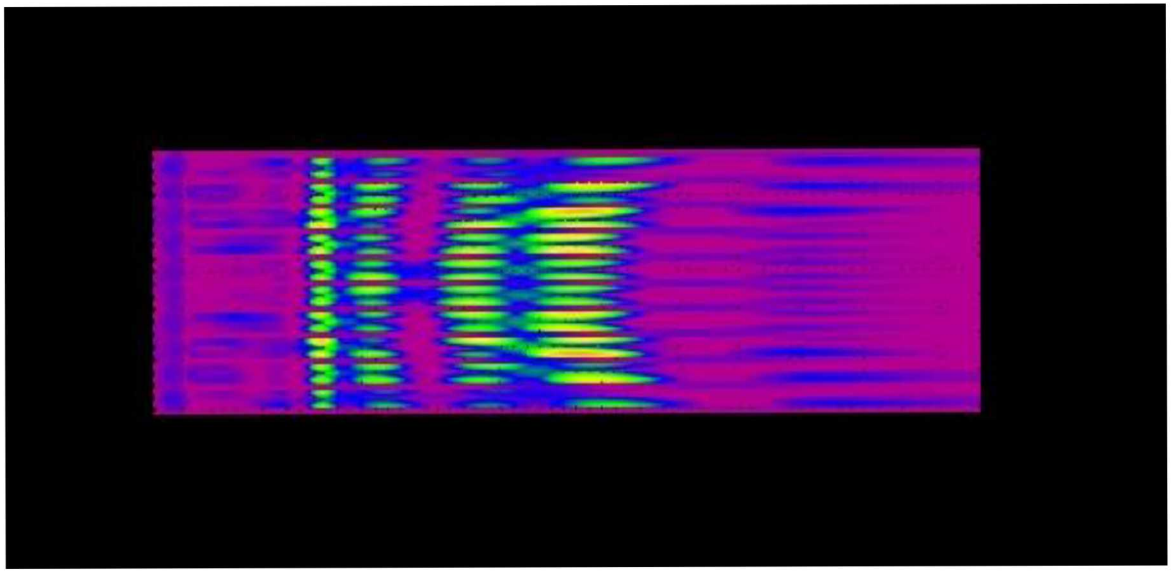
- Detailed 2D local simulation.
- Complex wave structure is created in the composite.



Colors show maximum principal strain

Dynamics: impact problem

- Detailed 2D local simulation.
- Complex wave structure is created in the composite.

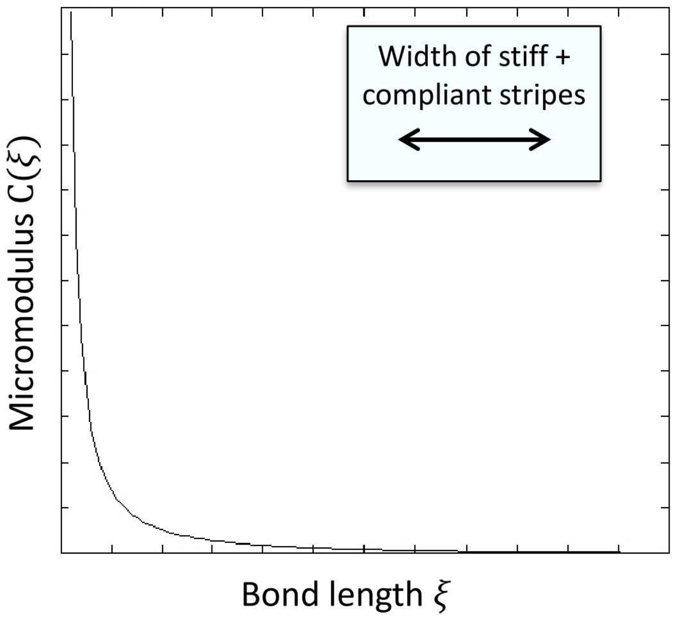


Colors show maximum principal strain

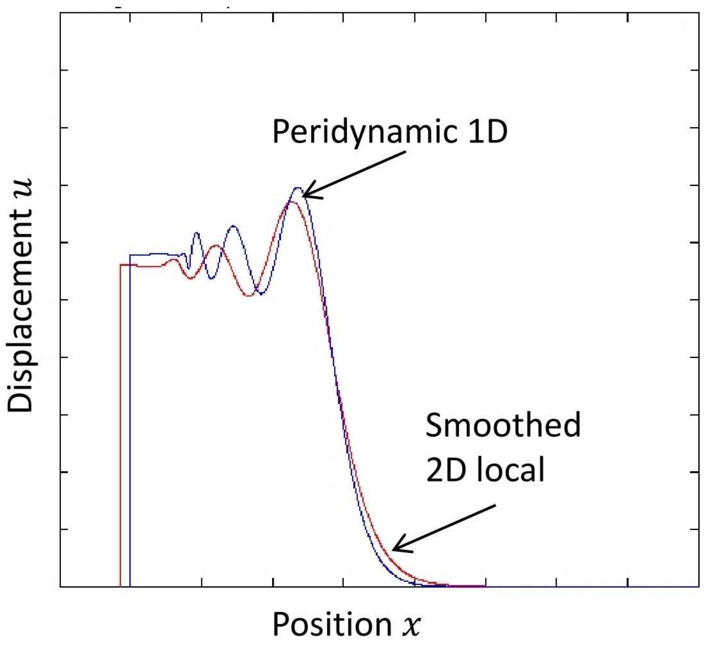
Nonlocality helps predict the dispersive nature of waves in the composite

- After smoothing the displacement along vertical lines, the complex wave structure is manifested as dispersion.
- A 1D peridynamic model (after tuning of the micromodulus) reproduces some of these features.

Peridynamic material model

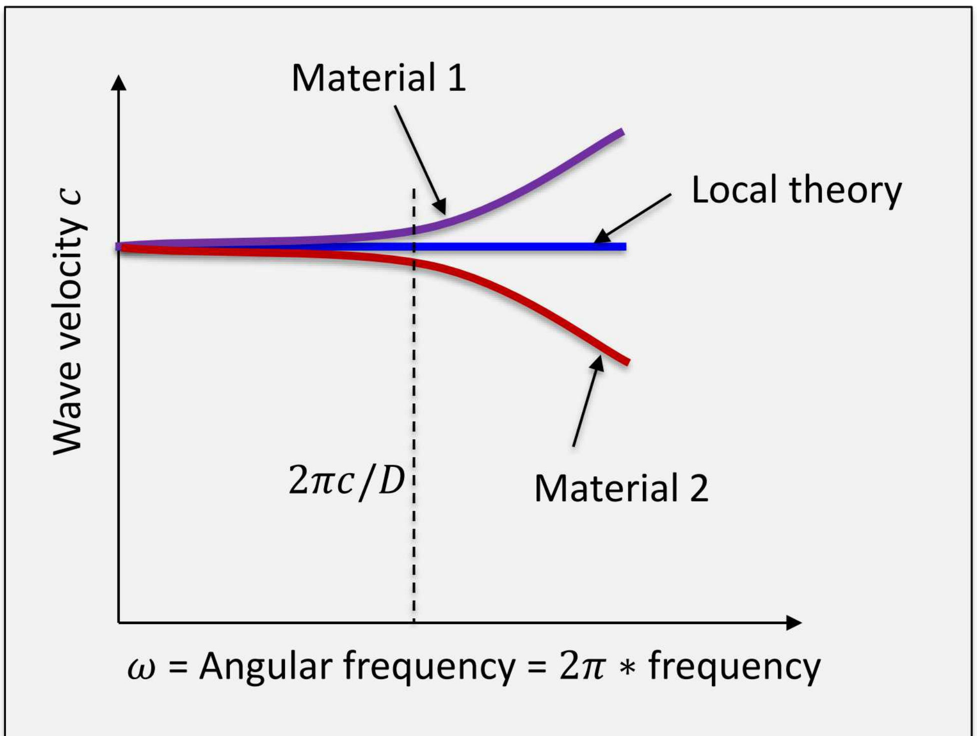


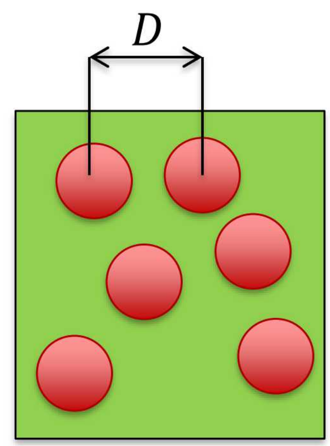
Wave structure



Wave dispersion

- All real solids exhibit dispersion for sufficiently short wavelengths.
- The wavelength depends on the microstructure and composition.
    - Dispersion starts to appear for **wavelengths < microstructure size**.
    - This implies that nonlocality is required to predict dispersion.





Microstructure

Wave dispersion in linear peridynamics

- Equation of motion with $b \equiv 0$:

$$\rho \ddot{u}(x,t) = \int_{-\delta}^{\delta} C(\xi)(u(x+\xi,t) - u(x,t))\, d\xi$$

- Look for plane wave solutions of the form

$$u(x,t) = e^{i(kx - \omega t)}$$

where k=wavenumber and ω=angular frequency.

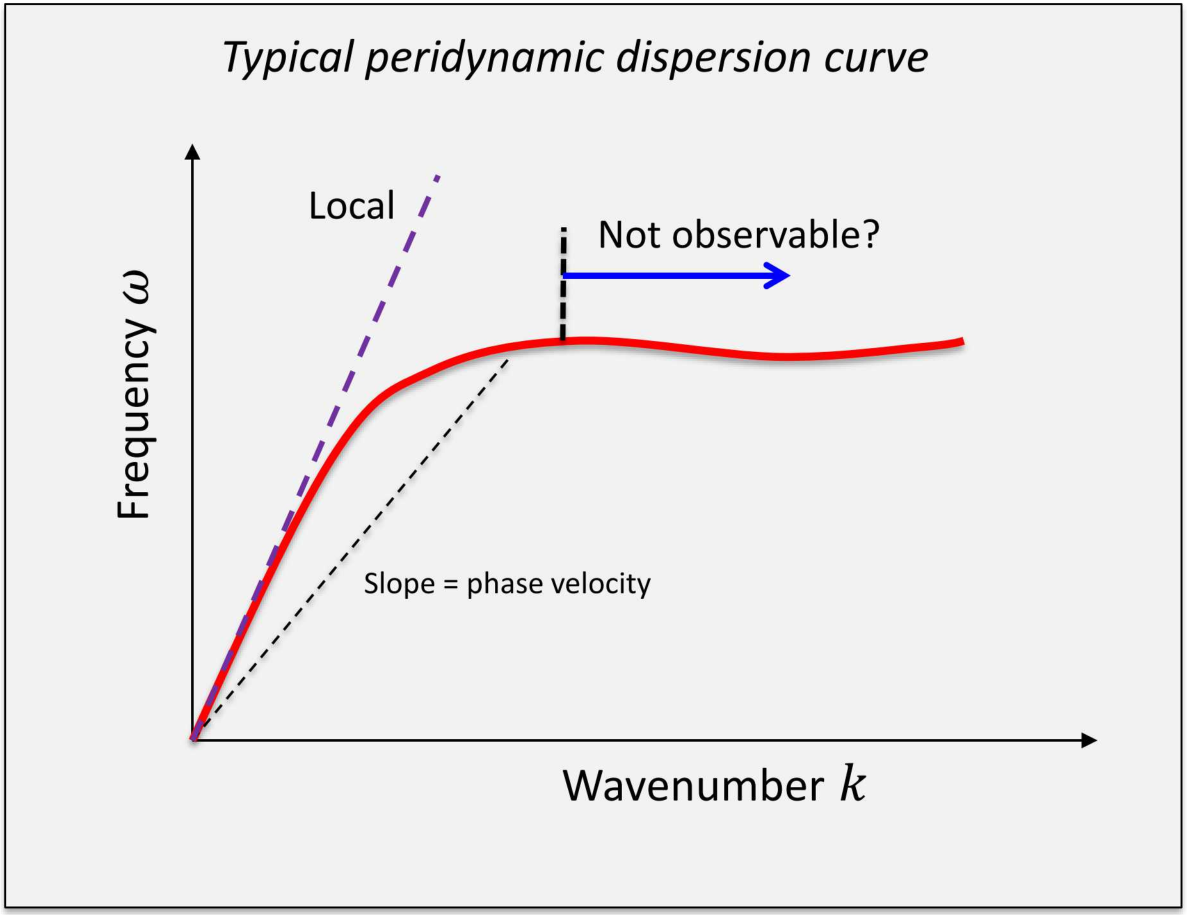
- Condition on ω and k:

$$-\rho\omega^2 = \int_{-\delta}^{\delta} C(\xi) e^{ik\xi}\, d\xi - P, \qquad P := \int_{-\delta}^{\delta} C(\xi)\, d\xi$$

- or in terms of the Fourier transform $C^* = \mathcal{F}\{C\}$,

$$\rho\omega^2(k) = P - C^*(k)$$

Wave dispersion in linear peridynamics



- S. N. Butt, J. J. Timothy, & G. Meschke, *Computational Mechanics* (2017).
- V. S. Mutnuri, USNCCM15 presentation (2019).

Finding peridynamic material properties from measured dispersion data

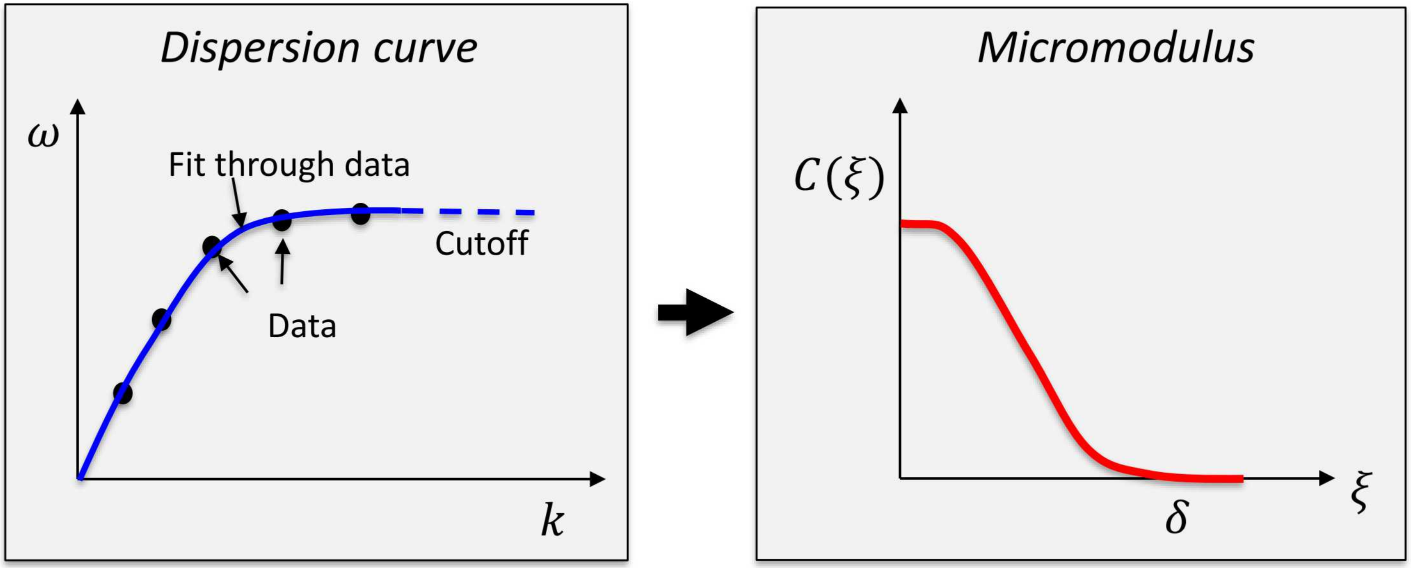
- We found

$$\rho\omega^2(k) = P - C^*(k).$$

- Given measured $\omega_{exper}(k)$, formally solve

$$C(\xi) = \mathcal{F}^{-1}\{P - \rho\omega^2_{exper}(k)\}$$

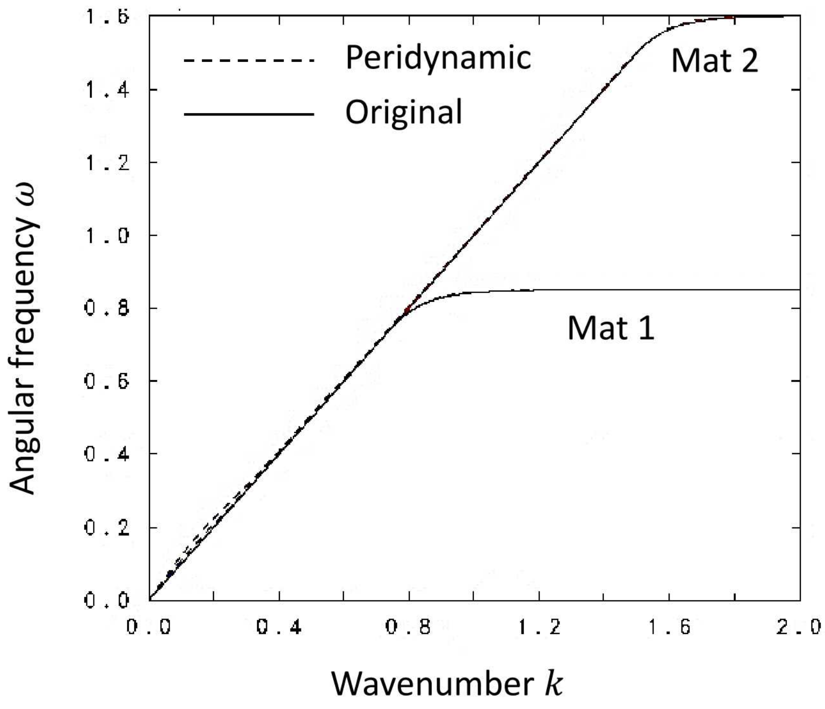
(requires data to be cut off for large k).

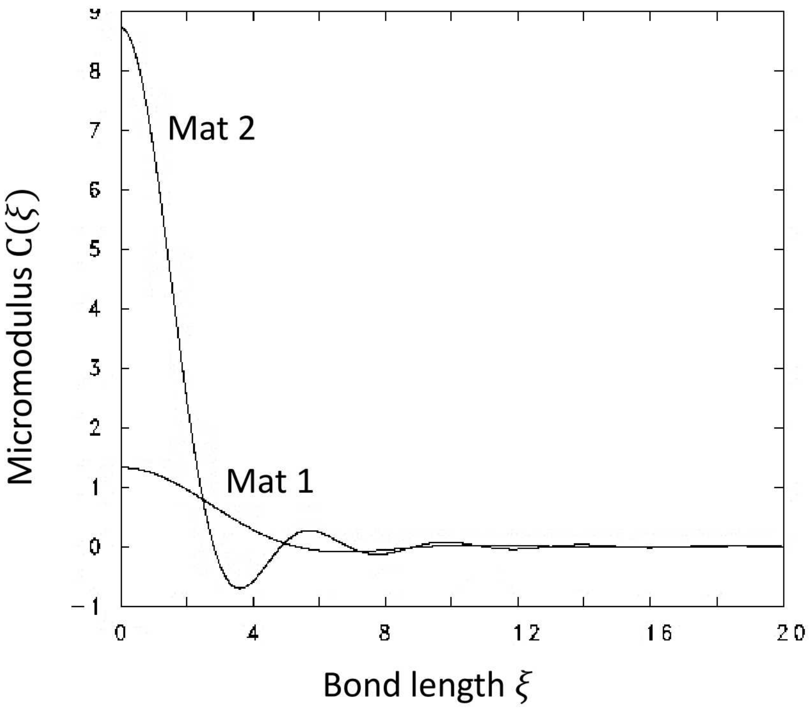


- O. Weckner & S.S., *Int. J. for Multiscale Computational Engineering* (2011).

Higher cutoff frequency leads to narrower micromodulus curve

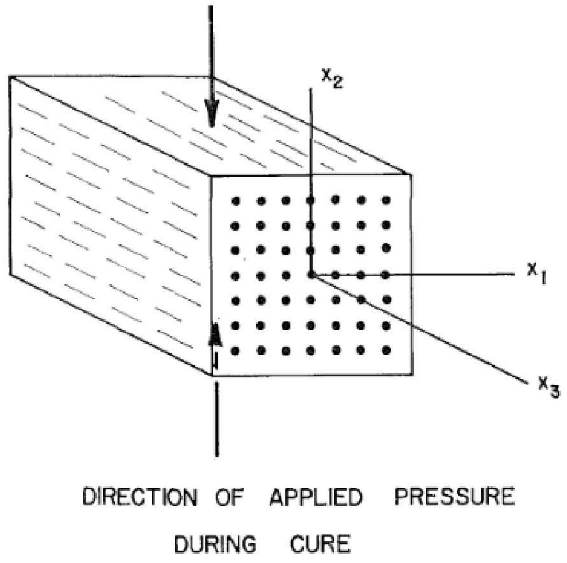
- The limiting case of micromodulus → delta function corresponds to the local theory.

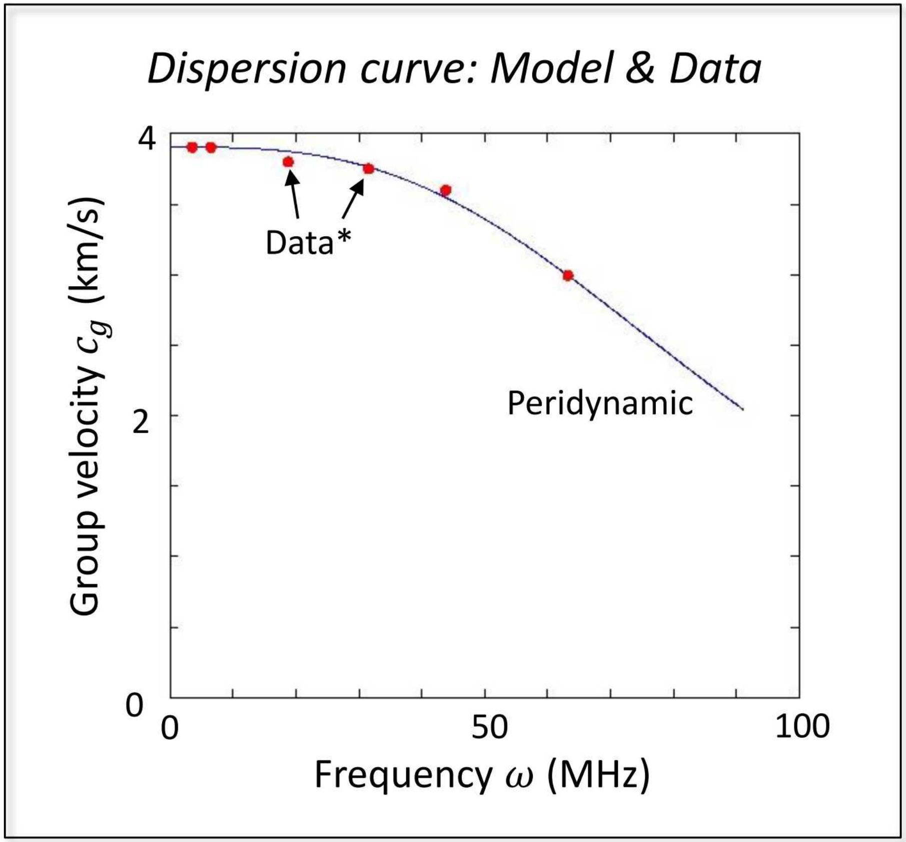




Example: PD model calibrated to a composite dispersion curve

- Boron-epoxy composite.
- Longitudinal waves normal to fibers.
- Compare measured ultrasonic group velocity* with calibrated peridynamic result.





* T. R. Tauchert & A. N. Guzelsu, *J. Applied Mechanics* (1972).

Discussion: Nonlocality in peridynamics

- Nonlocality emerges from how we choose to model a problem.
- Origins
  - Long-range forces
  - Smoothed degrees of freedom
  - Multiple pathways for flux (of momentum, heat, mass, …)
- Consistency
  - Peridynamics uses a consistently nonlocal approach to the evolution of all fields including damage.