



Embracing Unidentifiability in Bayesian Modal Calibration with Modularization

Introduction

- Bayesian model calibration (BMC) refers to coupling of limited experimental data with an expensive computer simulator [1].

$$y(x_i) = \eta(x_i, \theta) + \delta(x_i) + \epsilon_i$$

- Computer simulator takes a high dimensional set of inputs known as calibration parameters.
- Goal: Use Bayesian inference to learn the posterior distribution of the physical parameters.

Physical parameter inference

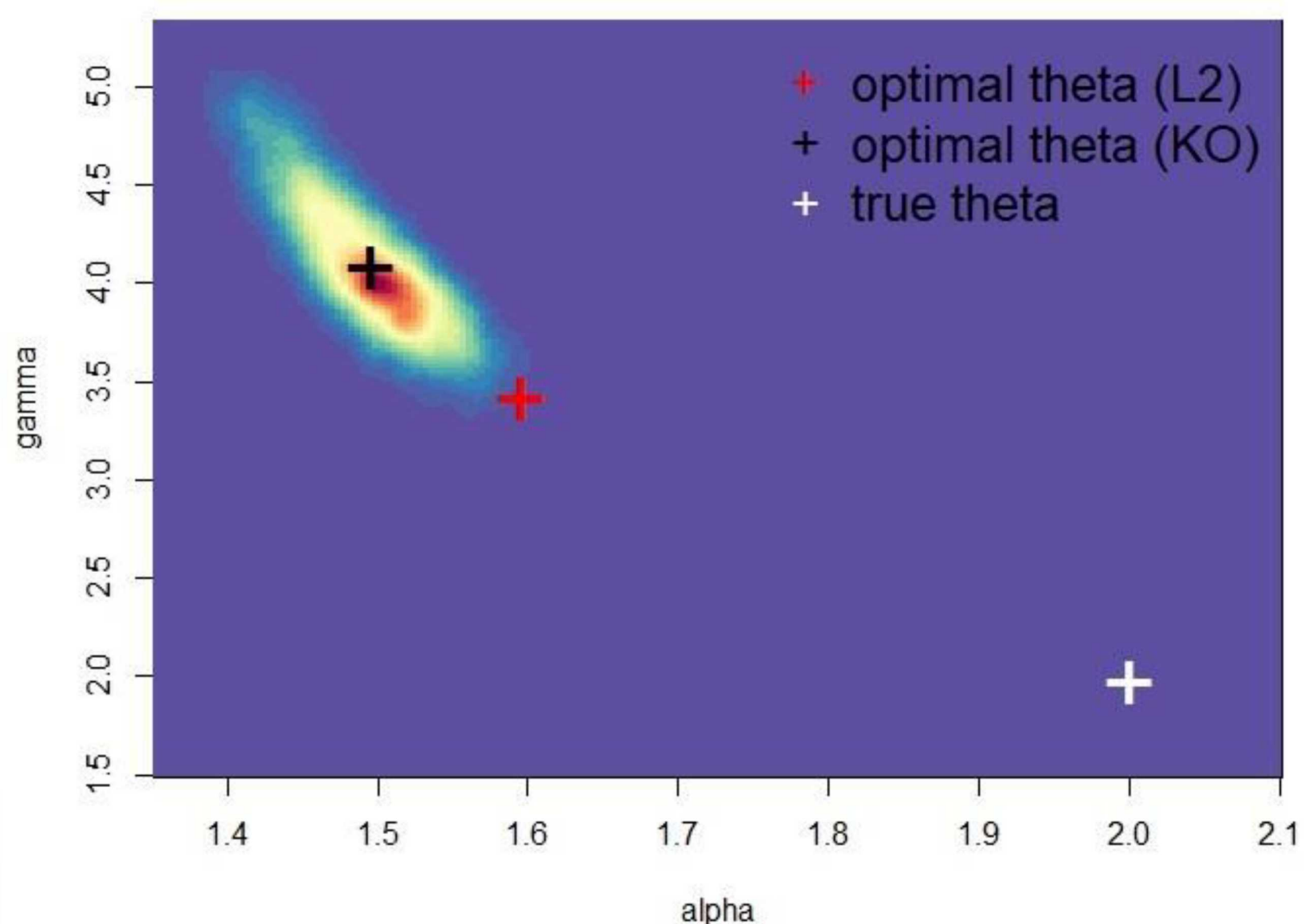
- Consider the simple machine [2].

$$\zeta(x) = \frac{2x}{1 + x/10}$$

- The efficiency is 2 (physical parameter).
- Model discrepancy: Use a piecewise linear simulator with slope α when $x < \gamma$ and slope β when $x \geq \gamma$.
- Optimal parameter values minimize a loss function [3]

$$\mathcal{L}(\zeta(\cdot), \eta(\cdot, \theta)).$$

- Optimal parameter values are often not the same as the “true” parameter values.



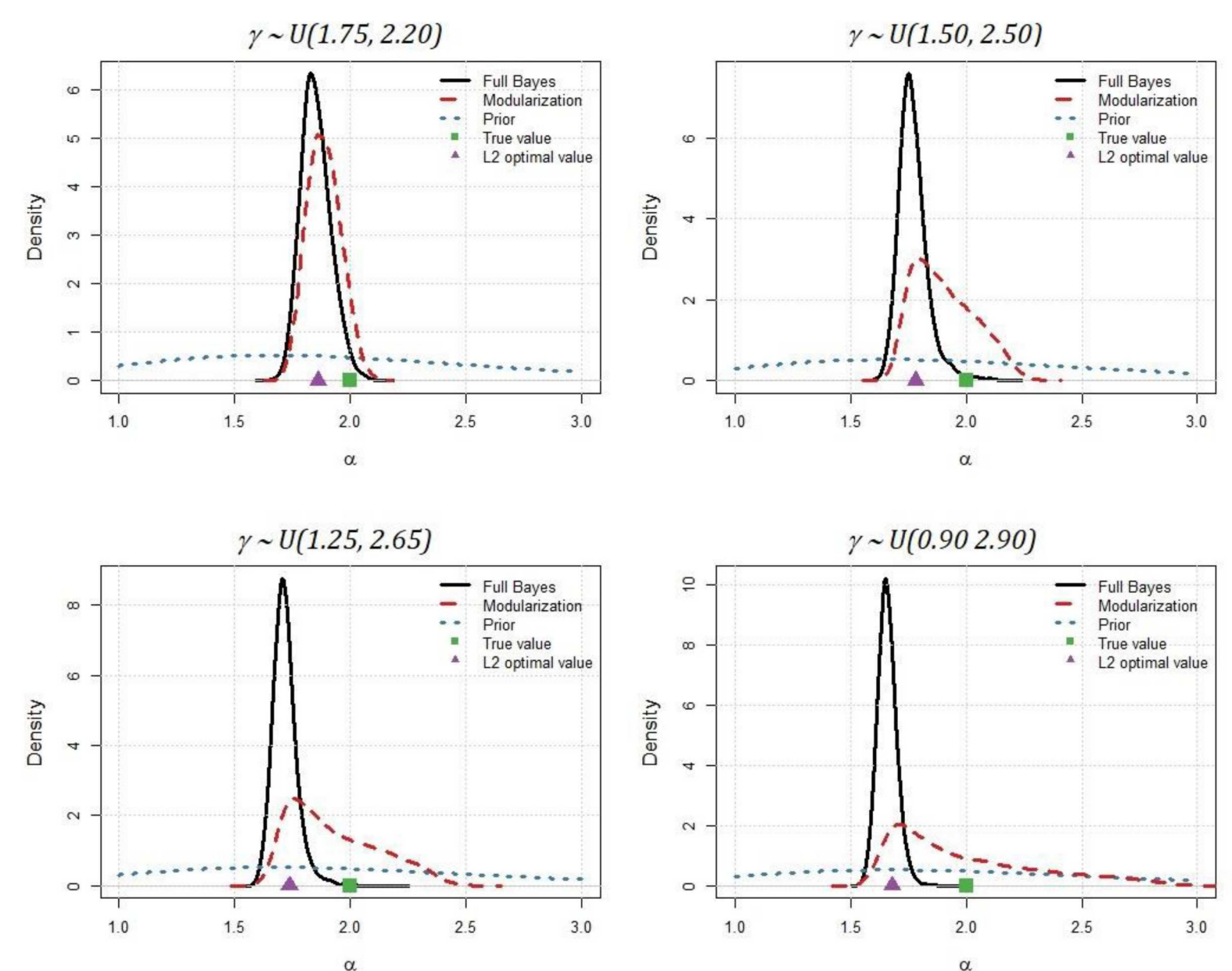
Modularization

- Sometimes physical parameters α and nuisance parameters γ are inherently jointly unidentifiable.

- Modularization improves identifiability by forfeiting the ability to learn about nuisance parameters, while rigorously accounting for the associated uncertainty [4].

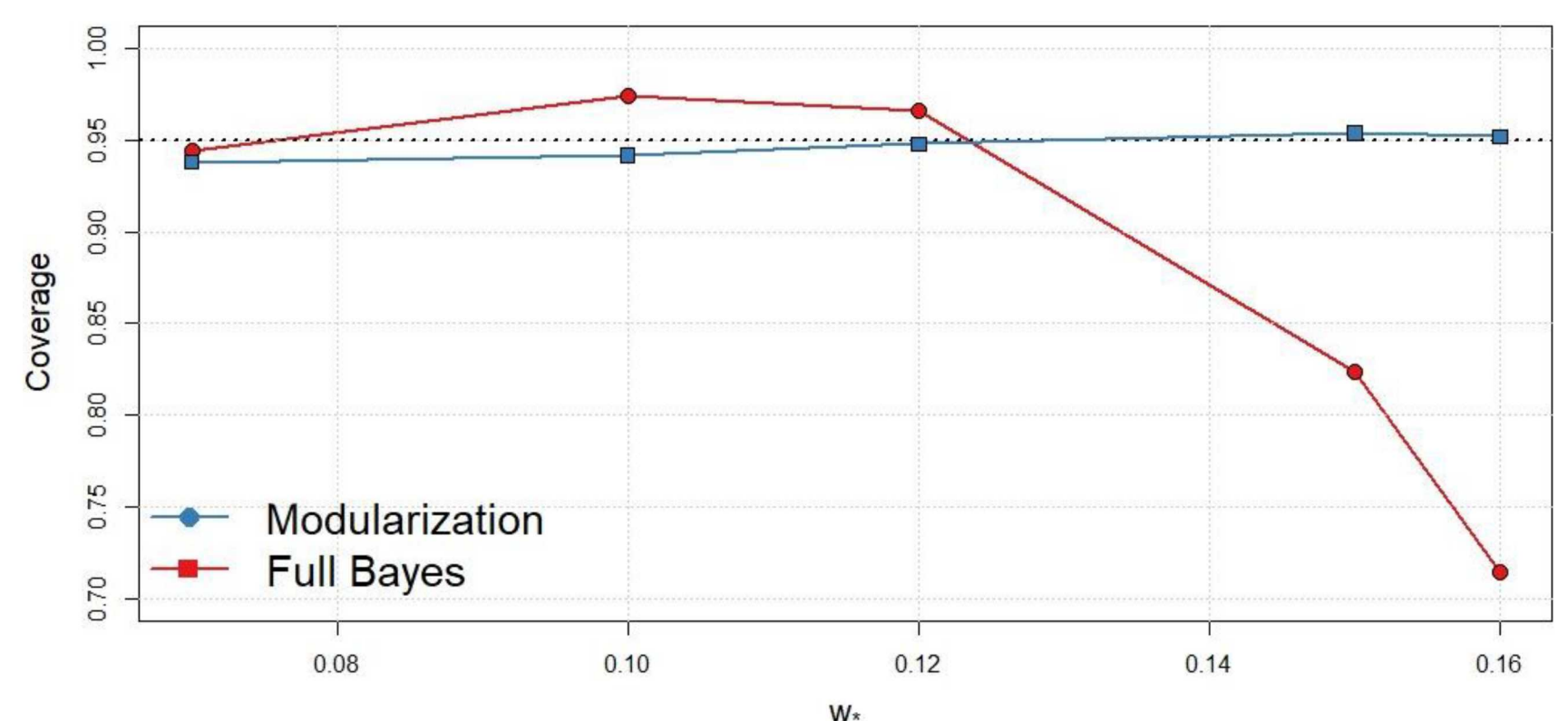
$$\pi_M(\alpha|y) = \int_{\mathbb{R}^q} \pi(\alpha|\gamma, y) \pi(\gamma) d\gamma$$

- The Modularization posterior can be efficiently approximated numerically by emulating the conditional posterior.
- Simple machine example:



Applications

- Borehole function simulations.



- Dynamic material property calibration.

