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# Using Crack Geometry to Determine Fracture Properties

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**ABSTRACT**
Linear elastic fracture mechanics theory predicts a parabolic crack opening profile. However, direct observation of crack tip shape in-situ for brittle materials is challenging due to the small size of the active crack tip region. By leveraging advances in optical microscopy techniques and using a soft brittle hydrogel material we can measure crack geometry on the micron scale. For glasses and ceramics, expected crack opening displacements are on the order of nanometers. However, for hydrogels, we can achieve crack opening displacements on the order of hundreds of microns or larger while maintaining brittle fracture behavior. Knowing the elastic properties, we can use crack geometry to calculate the stress intensity factor, K, and energy release rate, G, during propagation. Assuming the gel is hyperelastic, we can also approximate the size of the nonlinear region ahead of the crack tip. Geometric measurement of fracture properties eliminates the need to measure complex boundary and loading conditions allowing us to explore new methods of inducing crack propagation. Further, this allows us to define measures of fracture resistance in materials that do not fit the traditionally defined theories of fracture mechanics.

**Keywords:** Brittle Fracture, Confocal Microscopy, Crack Geometry, Polymers, Soft Gels

## INTRODUCTION

With advances in fields such as wearable devices and implants, soft robotics, and additive manufacturing, the structural performance of soft materials has become of great interest [1-3]. Materials that are mechanically compatible with biological tissues can help improve comfort of wearable devices and reduce inflammation, scarring, and rejection of implants [4]. Soft actuation and reducing joint friction are critical topics in robotics development [5]. Additionally, customized scaffolds, with the porosity to provide critical nutrients, are being used to grow cells into simple organs, where it has been shown that the scaffold's stiffness has a strong impact on cell growth [6-8].

In all these fields, the mechanical properties of soft polymers are of great interest. However, conventional gels exhibit very low toughness and brittle fracture properties, making them too delicate for most of these high demand applications [9-11]. While brittle fracture is well understood within the linear elastic regime for crystalline materials such as metals and ceramics, the microstructure of polymers is significantly different [12,13]. Polymers consist of chains of monomers, typically hundreds to hundreds of thousands of monomer units in a chain, where strong covalent bonds form the backbone and weaker hydrogen bonds between chains combine with physical entanglement to form the polymer [14]. Crosslinking, such as in many polymer hydrogels, can provide further strong covalent bonds to form a networked polymer system. This means that mechanisms of polymer failure are significantly different from metals and ceramics. Where there are identifiable slip and cleavage planes in the crystalline structure of metals and ceramics, even networked polymers show complex chain interactions such as chain stretching and pull-out before the strong bonds between monomer units are affected and fracture occurs [15,16].

Many polymers exhibit features of brittle fracture: No observable permanent deformation and smooth fracture surfaces perpendicular to the loading direction [17]. However, traditional measures of fracture properties were developed based on crystalline metals and ceramics. When applying these measures to polymers or other amorphous materials like glasses, the theories behind these measures no longer have the same meaning. However, the concept of fracture toughness as a measure of the energy needed to create new surfaces within the material is still applicable.

## BACKGROUND

Linear elastic fracture mechanics (LEFM) theory predicts a parabolic crack opening profile described as

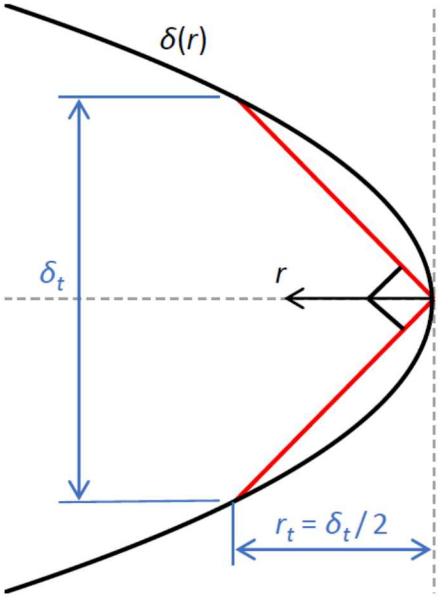


Figure 1: Parabolic crack opening profile with crack tip opening displacement, $\delta_t$, determined by the intersection of perpendicular lines from the crack tip with the crack edges

$$\delta(r) = \frac{8K}{\sqrt{2\pi}E\lambda}\sqrt{r} \tag{1}$$

$$where\ \lambda = \begin{cases} 1 & for\ plane\ stress, \\ \frac{1}{1-\nu^2} & for\ plane\ strain, \end{cases} \tag{2}$$

$E$ is Young's modulus, $\nu$ is Poisson's ratio, $r$ is the horizontal distance along the crack surface away from the tip, and $K$ is the stress intensity factor [17]. If we define $\delta_t$ as the crack opening where perpendicular lines from the vertex of the parabola (crack tip) intersect with the parabolic profile (crack edges), we can define $r_t = \delta_t/2$ as indicated in Fig. 1 [18,19]. This allows us to determine $K$ in terms of elastic constants and a measure of the crack tip opening displacement (CTOD, $\delta_t$)

$$K = \frac{\sqrt{\pi\delta_t}E\lambda}{4}. \tag{3}$$

For small scale yielding, the energy release rate is then

$$G = \frac{K^2}{E\lambda} = \frac{\pi\delta_t E\lambda}{16}. \tag{4}$$

Many soft materials, including the gel used in this study, could be considered hyperelastic, which means there is a nonlinear region near the crack tip in which the parabolic crack profile would be altered. Geubelle and Knauss [20] estimate the size of this nonlinear region to be

$$r_{NL} = \frac{1}{3\pi}\frac{J}{\mu} \tag{5}$$

where $\mu$ is the shear modulus which can be related to $E$ and $\nu$ for a linear elastic material, and $J$ is Rice's J-integral which corresponds to the energy release rate as $J = G$ for a linear elastic material [17,19]. Thus, using Eq 4, $r_{NL}$ is found to depend only on the Poisson's ratio (compressibility) and $\delta_t$:

$$r_{NL} = \frac{1+\nu}{24}\lambda\delta_t. \tag{6}$$

For an incompressible hyperelastic gel, $\nu = 0.5$ so for plane stress $r_{NL} = \delta_t/16$ and for plane strain $r_{NL} = \delta_t/12$. In both cases, the nonlinear region is significantly smaller than the crack tip opening displacement, so assuming LEFM holds in determining the CTOD is appropriate.

## SUMMARY OF EXPERIMENT AND RESULTS

Due to the high water content of many hydrogels (often > 90%) and their very low toughness, it can be challenging to conduct traditional fracture tests on these materials. We also conduct *in-situ* confocal microscope imaging on the sub-millimeter scale to understand both crack geometry and the damage processes occurring in the polymer network, introducing further experimental challenges. For these reasons, we use a fracking inspired method to induce stable, slow crack propagation in a thin, confined disk of hydrogel. The experimental methods are described in detail in [21].

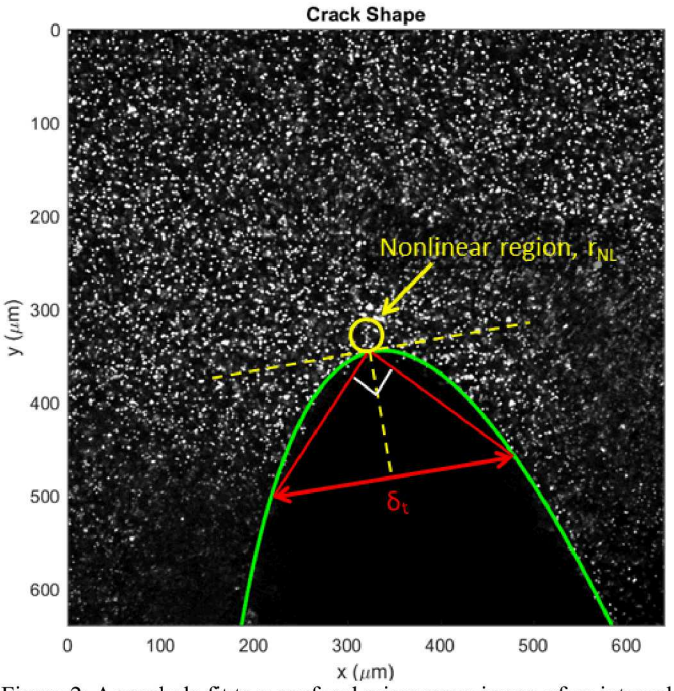

Figure 2: A parabola fit to a confocal microscope image of an internal plane of the gel indicating the measured CTOD (mean value, 339 $\mu m$) and estimated size of the nonlinear region (mean value, 28 $\mu m$)

A second order polynomial (parabola) fits the crack shapes observed in the experiments very well, as shown in Fig. 2. Details of the calculated fracture properties of this gel using this method to induce crack propagation can be found in [21]. For a propagating crack in this gel, which has a Young's Modulus of approximately 78 $kPa$, the mean CTOD ($\delta_t$) is 339 $\mu m$ corresponding to a mean SIF ($K$) and ERR ($G$) of 850 $Pa\sqrt{m}$ and 6.9 $J/m^2$ respectively. The estimated size of the nonlinear region near the crack tip ($r_{NL}$) is 28 $\mu m$ which corresponds to about 22 pixels in the captured images, roughly the size of an appropriate digital image correlation subset for this speckle pattern. Errors in these calculated values are comparable to the errors reported for more traditional fracture experiments in similar soft gel materials [9].

Some features of crack renucleation in this networked polymer hydrogel were observed at the submillimeter scale such as intact ligaments and indicators of local toughening. However, to better understand nonlinear processes in this gel, higher resolution imaging of the crack tip nonlinear region is needed.

In this study we were able to use crack geometry and the polymer's material properties to determine traditional measures of fracture toughness without needing to define and measure complex boundary conditions. We were also able to determine the scale at which we would expect nonlinear processes to affect the crack.

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**REFERENCES**

[1] Baldi F, Bignotti F, Peroni I, Agnelli S, Ricco T. "On the measurement of the fracture resistance of polyacrylamide hydrogels by wire cutting tests." *Polymer Testing* 31(3):455-465, 2012.

[2] Gong JP, Osada Y. "Soft and wet materials: From hydrogels to biotissues." In: Cloitre M (ed) High Solid Dispersions, Springer Berlin Heidelberg, Berlin, Heidelberg, pp 203-246, 2010.

[3] Long R, Hui CY. "Fracture toughness of hydrogels: measurement and interpretation." *Soft Matter* 12(39):8069-8086, 2016.

[4] Moshayedi P, Ng G, Kwok JCF, Yeo GSH, Bryant CE, Fawcett JW, Franze K, Guck J. "The relationship between glial cell mechanosensitivity and foreign body reactions in the central nervous system." *Biomaterials* 35(13):3919-3925, 2014.

[5] Coyle S, Majidi C, LeDuc P, Hsia KJ. “Bioinspired soft robotics: Material selection, actuation, and design.” *Extreme Mechanics Letters* 22:51-59, 2018.
[6] Engler AJ, Sen S, Sweeney HL, Discher DE. “Matrix elasticity directs stem cell lineage specification.” *Cell* 126(4):677-689, 2006.
[7] Whang M, Kim J. “Synthetic hydrogels with stiffness gradients for durotaxis study and tissue engineering scaffolds.” *Tissue Engineering and Regenerative Medicine* 13(2):126-139, 2016.
[8] Yeung T, Georges PC, Flanagan LA, Marg B, Ortiz M, Funaki M, Zahir N, Ming W, Weaver V, Janmey PA. “Effects of substrate stiffness on cell morphology, cytoskeletal structure, and adhesion.” *Cell Motility* 60(1):24-34, 2005.
[9] Agnelli S, Baldi F, Bignotti F, Salvadori A, Peroni I. “Fracture characterization of hyperelastic polyacrylamide hydrogels.” *Engineering Fracture Mechanics* 203:54-65, 2018.
[10] Nilsson F. “A tentative method for determination of cohesive zone properties in soft materials.” *International Journal of Fracture* 136(1):133-142, 2005.
[11] Tanaka Y, Shimazaki R, Yano S, Yoshida G, Yamaguchi T. “Solvent effects on the fracture of chemically crosslinked gels.” *Soft Matter* 12(39):8135-8142, 2016.
[12] Ahmed EM. “Hydrogel: Preparation, characterization, and applications: A review.” *Journal of Advanced Research*, 6(2):105–121, 2013.
[13] Denisin AK, Pruitt BL. “Tuning the range of polyacrylamide gel stiffness for mechanobiology applications.” *ACS Applied Materials & Interfaces,* 8(34):21893-21902, 2016.
[14] Odian G. *Principles of Polymerization*. Wiley InterScience electronic collection. Wiley, 2004.
[15] Brighenti R, Spagnoli A, Carpinteri A, Artoni F. “Notch effect in highly deformable material sheets.” *Thin-Walled Structures*, 105:90–100, 2016.
[16] Naficy S, Brown HR, Razal JM, Spinks GM, Whitten PG. “Progress toward robust polymer hydrogels.” *Australian Journal of Chemistry*, 64(8):1007–1025, 2011.
[17] Anderson TL. *Fracture Mechanics: Fundamentals and Applications, Third Edition*. CRC Press, 2005.
[18] Rice JR. “A path independent integral and the approximate analysis of strain concentration by notches and cracks.” *Journal of Applied Mechanics,* 35(2):379-386, 1968.
[19] Shih CF. “Relationships between the J-integral and the crack opening displacement for stationary and extending cracks.” *Journal of the Mechanics and Physics of Solids,* 29(4):305- 326, 1981.
[20] Geubelle PH, Knauss WG. “Finite strains at the tip of a crack in a sheet of hyperelastic material: I. Homogeneous case.” *Journal of Elasticity*, 35:61-98, 1994.
[21] Mac Donald KA, Ravichandran G. “Crack propagation and renucleation in soft brittle hydrogels”. *International Journal of Fracture,* 2020. In press.