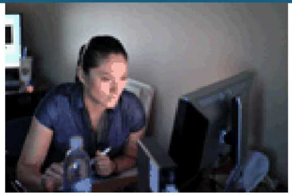
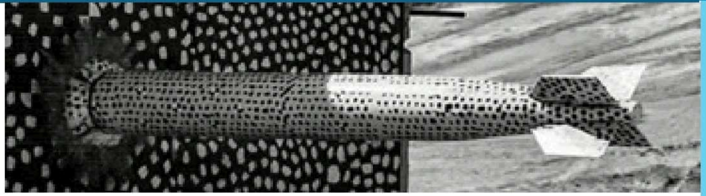
SAND2020-1066C

Improved Expansion Results using Regularized Solutions





Presented By:

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February XX, 2020

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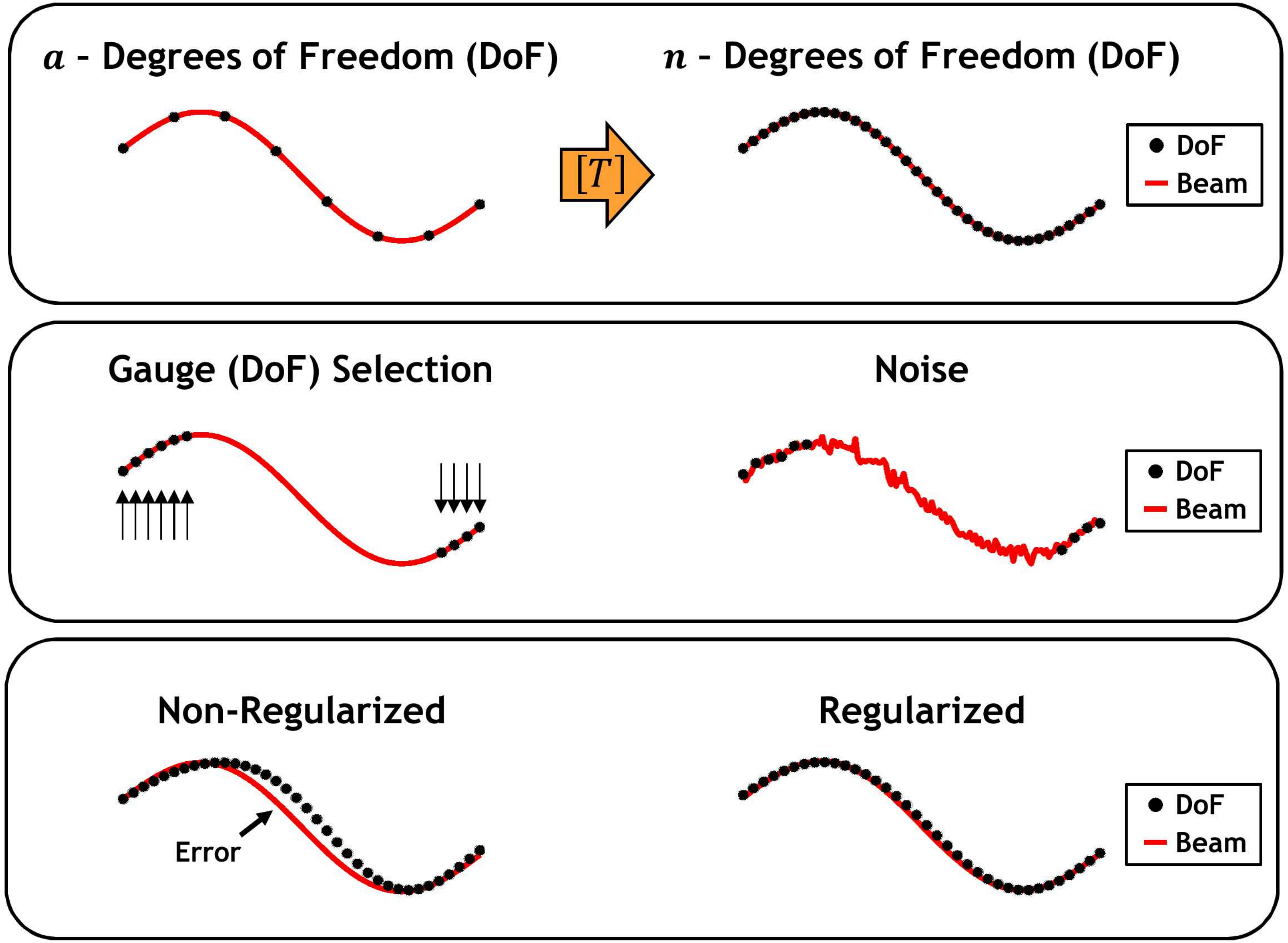
Outline

- Introduction
- Theory
- Expansion Sensitivity
  - Effects of Gauge Placement
  - Effects of Noise
- Regularized Expansion
- Conclusions



Introduction

Definition:

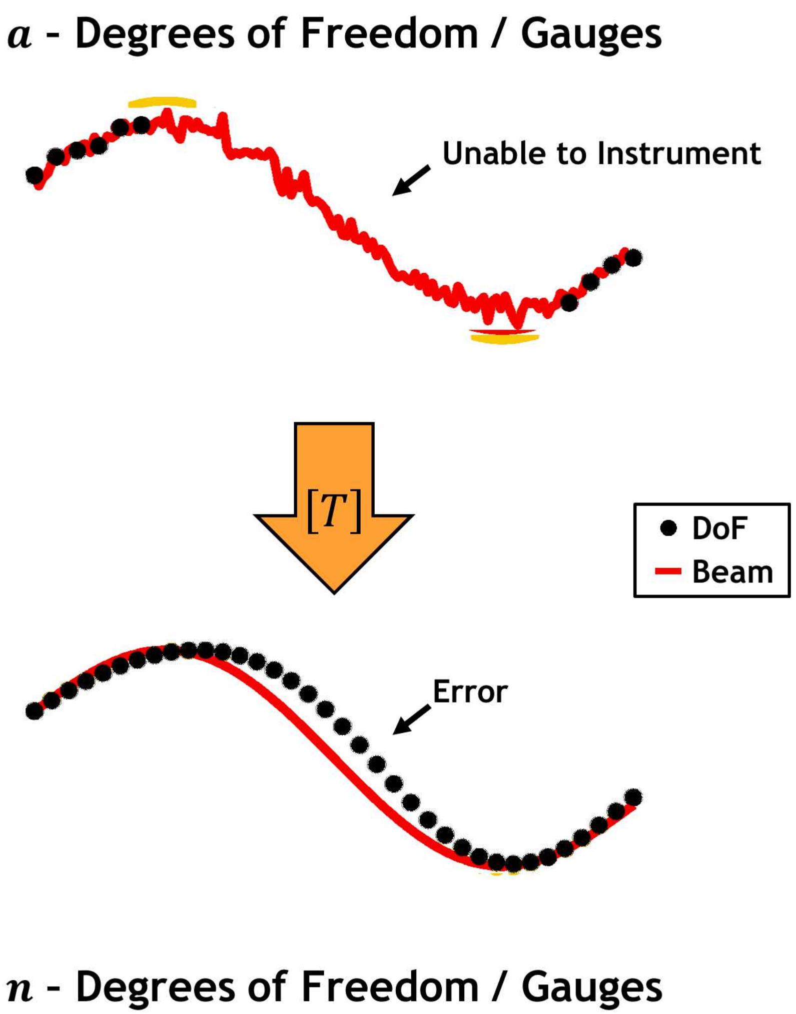
- Expansion is a technique used to project a reduced set of measured, or simulated, responses to a larger set of responses.

Applications:

- Obtain system responses at unmeasured locations
- Test data validation by comparing measured and expanded responses

Challenges:

- Expansion techniques are sensitive to:
  - Selection of Degrees of Freedom (DoF) / Gauge placement
  - Noise
- All factors are inevitable when considering measured data



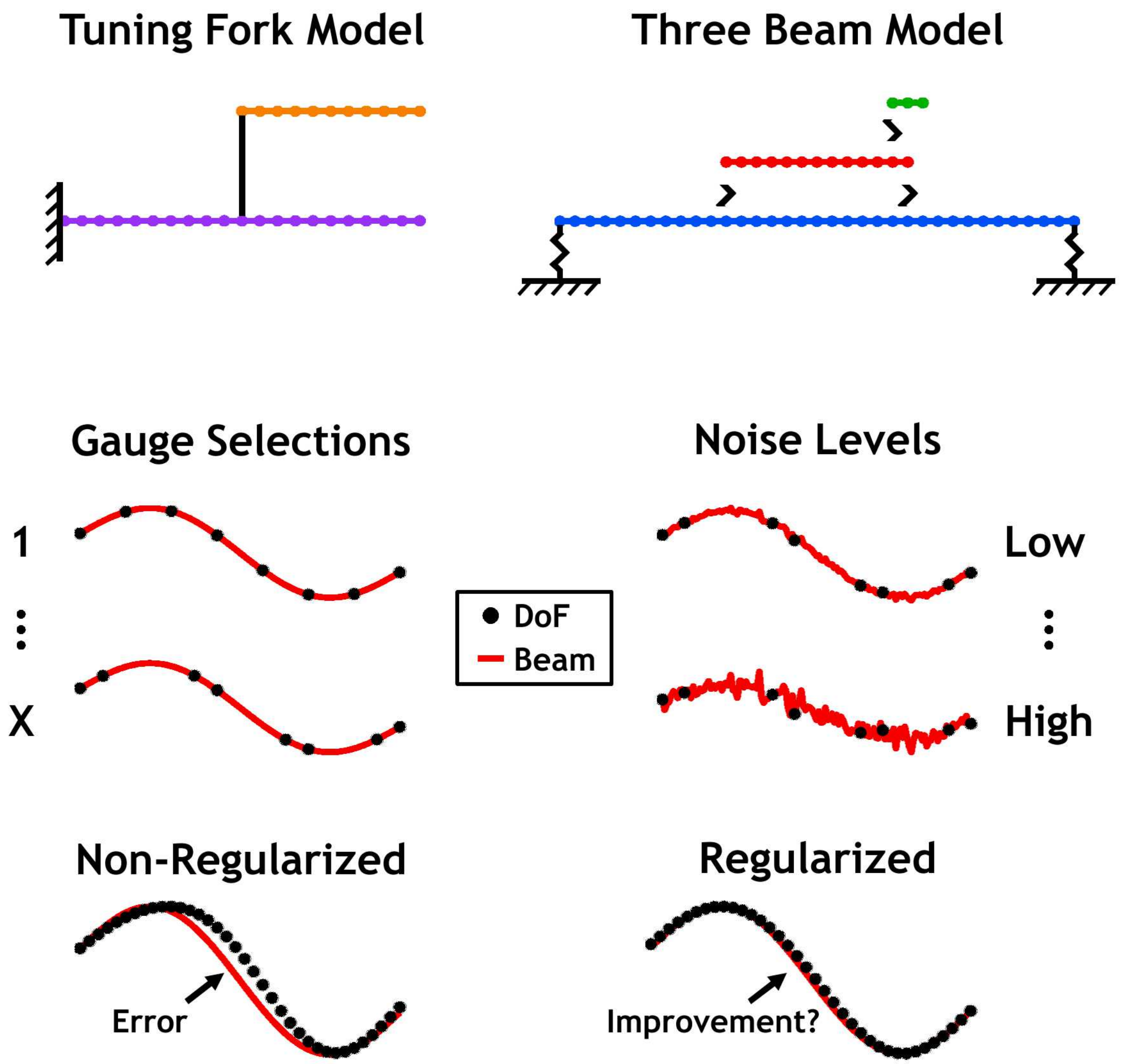
Introduction

Objectives:

- Demonstrate the sensitivity of gauge placement and noise on expansion results
- Investigate the use of regularization in the expansion process to enhance the accuracy of results

Approach:

- Apply expansion techniques to two beam models, one simple and one complex
- Compare the quality of expansion results from different levels of noise and gage configurations
- Compare the quality of expansion results when regularization was and was not used



- Introduction
- **Theory**
- Expansion Sensitivity
  - Effects of Gauge Placement
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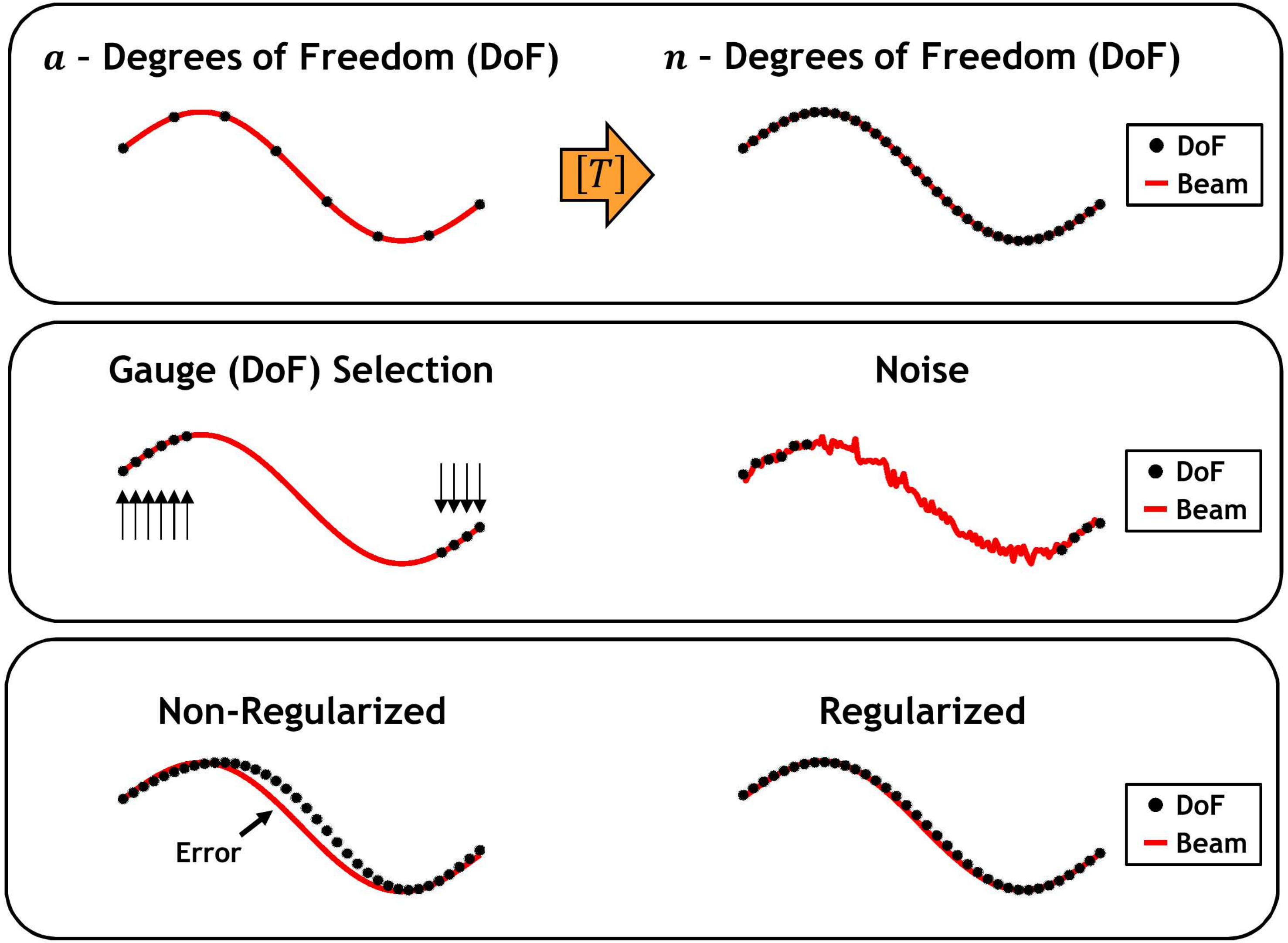


Theory – Expansion

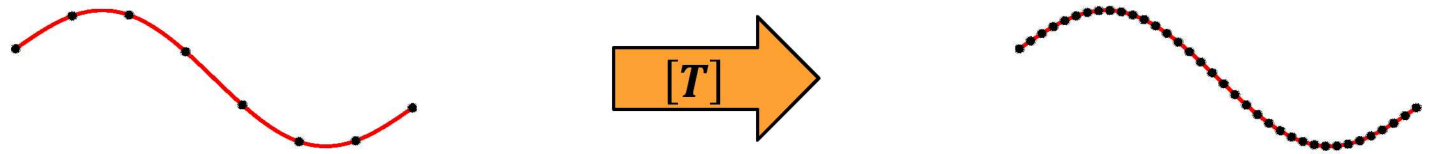
System Equivalent Reduction and Expansion Process (SEREP):

Expansion:

$[T]\{x_a(t)\} = \{x_n(t)\}$

a - Degrees of Freedom (DoF)

n - Degrees of Freedom (DoF)



Transformation Matrix:

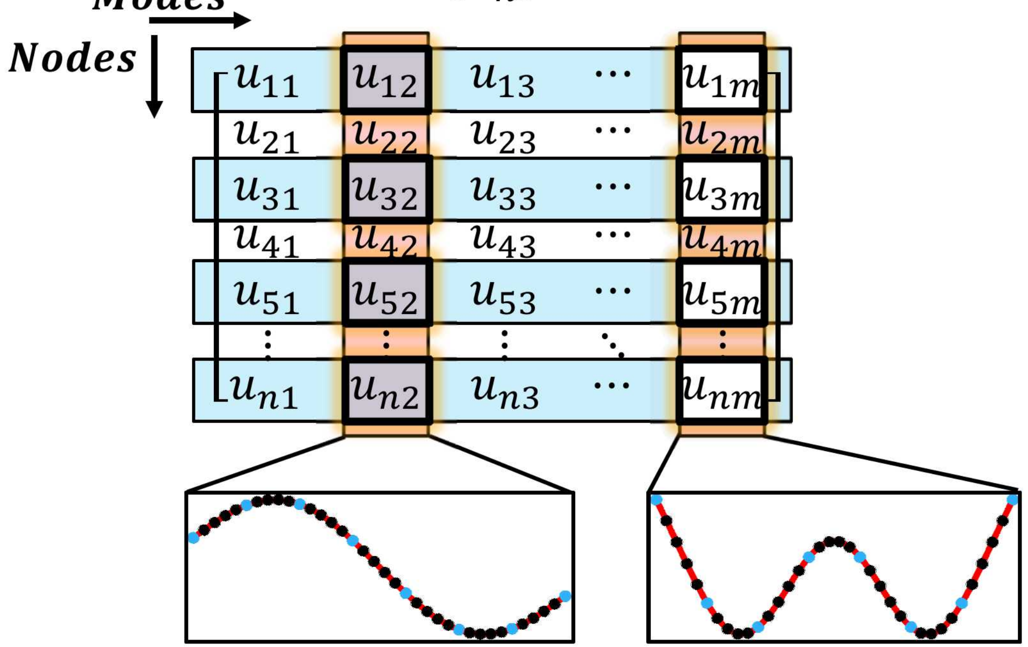
$[T] = [U_n]\boxed{[U_a]^+}$

Generalized Inverse

$[U_a]^+ = ([U_a]^T[U_a])^{-1}[U_a]^T$

$[U_n]$

$[U_a]^+$



Theory - Regularization

[1]

Definition:

- A technique used to slightly perturb a matrix to better condition it for inverse problems.
- Condition number is the ratio of the largest and smallest singular values of a matrix
  - Sensitivity of an inverse problem to small perturbations
  - ⇧ $Cond(A)$ → ⇧ Sensitivity

$$Cond(A) = \frac{max(S_\Sigma)}{min(S_\Sigma)}$$

Well-Conditioned Matrix Example - $\boldsymbol{Cond(A) = 23.7}$

$$[A] \quad * \quad [X] = \quad [B]$$

Baseline

$$\begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 110 \\ 66 \\ 47 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19.0 \\ 22.0 \\ -35.0 \end{bmatrix}$$

Noisy

$$\begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 112 \\ 67 \\ 47 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20.0 \\ 22.0 \\ -37.0 \end{bmatrix}$$

$$Error = \begin{bmatrix} 1.0 \\ 0.0 \\ -2.0 \end{bmatrix} = \begin{bmatrix} 5.3 \\ 0.0 \\ 5.7 \end{bmatrix} \%$$

$$Noise = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Theory - Regularization

Well-Conditioned Matrix Example - $\boldsymbol{Cond(A) = 23.7}$

$$[A] \quad * \quad [X] = \quad [B]$$

Baseline

$$\begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 110 \\ 66 \\ 47 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19.0 \\ 22.0 \\ -35.0 \end{bmatrix}$$

Noisy

$$\begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \color{red}{112} \\ \color{red}{67} \\ 47 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20.0 \\ 22.0 \\ -37.0 \end{bmatrix}$$

$$Error = \begin{bmatrix} 1.0 \\ 0.0 \\ -2.0 \end{bmatrix} = \begin{bmatrix} 5.3 \\ 0.0 \\ 5.7 \end{bmatrix} \%$$

Ill-Conditioned Matrix Example - $\boldsymbol{\color{red}{Cond(A) = 524.1}}$

$$[A] \quad * \quad [X] = \quad [B]$$

Baseline

$$\begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & \color{red}{1.5} \\ \color{red}{2.0} & \color{red}{1.5} & \color{red}{1.2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 110 \\ 66 \\ 47 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4.0 \\ 42.0 \\ -20.0 \end{bmatrix}$$

Noisy

$$\begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & \color{red}{1.5} \\ \color{red}{2.0} & \color{red}{1.5} & \color{red}{1.2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \color{red}{112} \\ \color{red}{67} \\ 47 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.0 \\ 62.0 \\ -40.0 \end{bmatrix}$$

$$Error = \begin{bmatrix} -3.0 \\ 20.0 \\ -20.0 \end{bmatrix} = \begin{bmatrix} -75.0 \\ 47.6 \\ 100.0 \end{bmatrix} \%$$

Theory - Regularization

Tikhonov Regularization

$$[U_a]^+ = ([U_a]^T[U_a] + \lambda^2[I])^{-1}[U_a]^T$$

Baseline

$$\begin{array}{ccc} [A] & [B] & [X] \\ \begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.5 \\ 2.0 & 1.5 & 1.2 \end{bmatrix}; & \begin{bmatrix} 110 \\ 66 \\ 47 \end{bmatrix} \rightarrow & \begin{bmatrix} 4.0 \\ 42.0 \\ -20.0 \end{bmatrix} \end{array}$$

Noisy

$$\begin{array}{cccccc} [A] & [B] & & [X] & [X] & [X] \\ \begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.5 \\ 2.0 & 1.5 & 1.2 \end{bmatrix}; & \begin{bmatrix} 112 \\ 67 \\ 47 \end{bmatrix} & \rightarrow & \begin{bmatrix} 1.0 \\ 62.0 \\ -40.0 \end{bmatrix} & \begin{bmatrix} 2.1 \\ 55.8 \\ -34.0 \end{bmatrix} & \begin{bmatrix} 4.3 \\ 43.8 \\ -22.4 \end{bmatrix} \\ & & \boldsymbol{\lambda}: & \mathbf{0.000} & \mathbf{0.006} & \mathbf{0.012} \\ & & \textbf{\textit{Condition Number}}: & \mathbf{524.1} & \mathbf{491.2} & \mathbf{420.4} \\ & & \%\ \textbf{\textit{Error}}: & \begin{bmatrix} -75.0 \\ 48.0 \\ 100.0 \end{bmatrix} & \begin{bmatrix} -47.2 \\ 32.8 \\ 70.0 \end{bmatrix} & \begin{bmatrix} 6.6 \\ 4.2 \\ 12.0 \end{bmatrix} \end{array}$$

Singular Value Decomposition Regularization

$$svd([U_a]) \rightarrow [L_\Sigma][S_\Sigma][R_\Sigma]$$

$$\Downarrow$$

$$[S_\Sigma] \rightarrow [S_\Sigma']$$

$$\Downarrow$$

$$[U_a'] = [L_\Sigma][S_\Sigma'][R_\Sigma]^T$$

$$\Downarrow$$

$$[U_a']^+ = ([U_a']^T[U_a'])^{-1}[U_a']^T$$

$$[S_\Sigma'] = \begin{bmatrix} 8.450 & 0 & 0 \\ 0 & 0.734 & 0 \\ 0 & 0 & \boxed{0.016} \end{bmatrix}$$

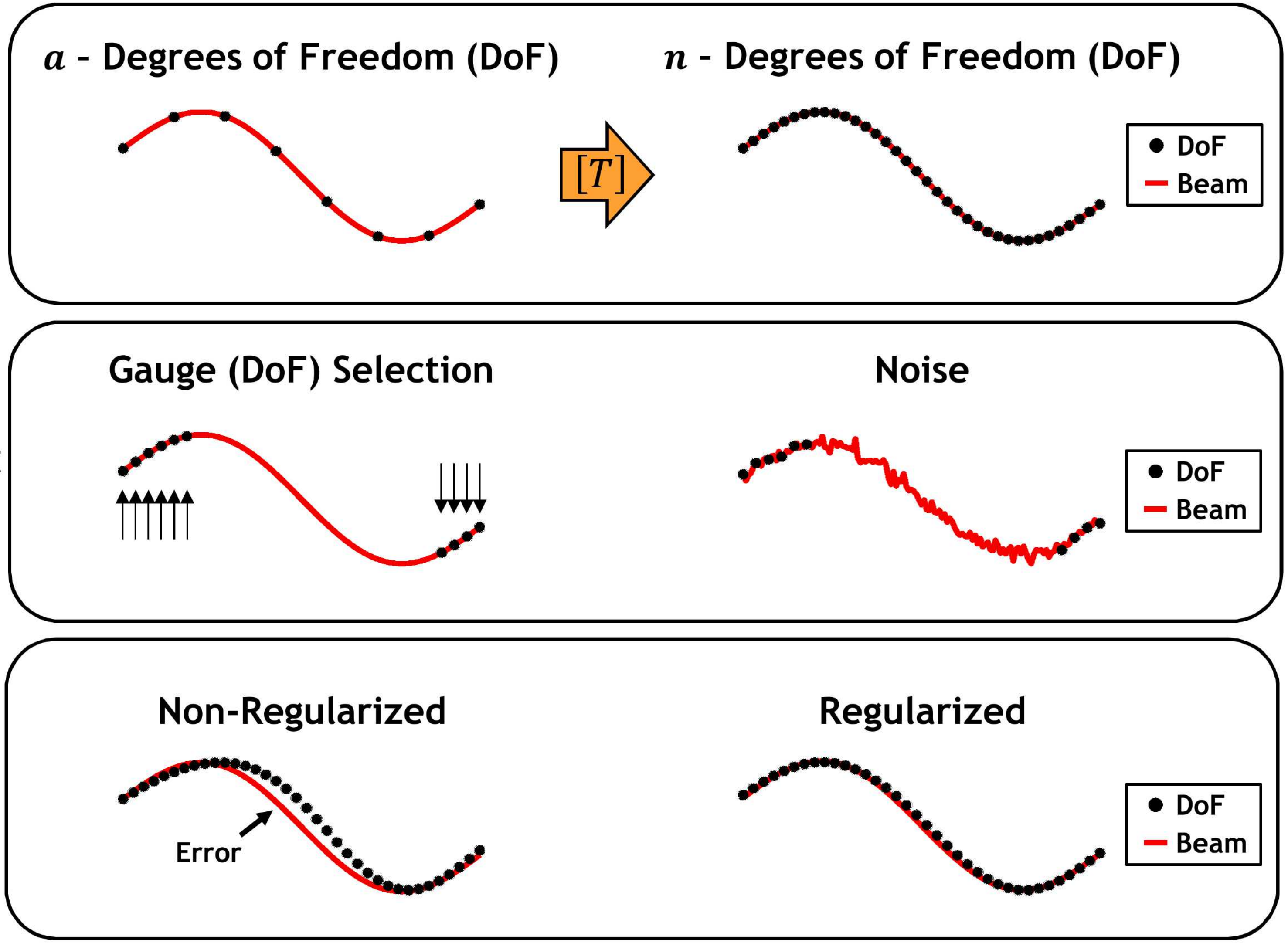
Increase $S_\Sigma(3,3)$

Noisy

$$\begin{array}{cccccc} [A] & [B] & & [X] & [X] & [X] \\ \begin{bmatrix} 6.0 & 3.0 & 2.0 \\ 3.0 & 2.0 & 1.5 \\ 2.0 & 1.5 & 1.2 \end{bmatrix}; & \begin{bmatrix} 112 \\ 67 \\ 47 \end{bmatrix} & \rightarrow & \begin{bmatrix} 1.0 \\ 62.0 \\ -40.0 \end{bmatrix} & \begin{bmatrix} 2.8 \\ 51.8 \\ -30.2 \end{bmatrix} & \begin{bmatrix} 4.6 \\ 42.1 \\ -20.8 \end{bmatrix} \\ & & \boldsymbol{S_\Sigma(3,3)}: & \mathbf{0.016} & \mathbf{0.020} & \mathbf{0.026} \\ & & \textbf{\textit{Condition Number}}: & \mathbf{524.1} & \mathbf{420.0} & \mathbf{320.0} \\ & & \%\ \textbf{\textit{Error}}: & \begin{bmatrix} -75.0 \\ 48.0 \\ 100.0 \end{bmatrix} & \begin{bmatrix} -29.6 \\ 23.4 \\ 51.0 \end{bmatrix} & \begin{bmatrix} 14.1 \\ 0.2 \\ 3.9 \end{bmatrix} \end{array}$$

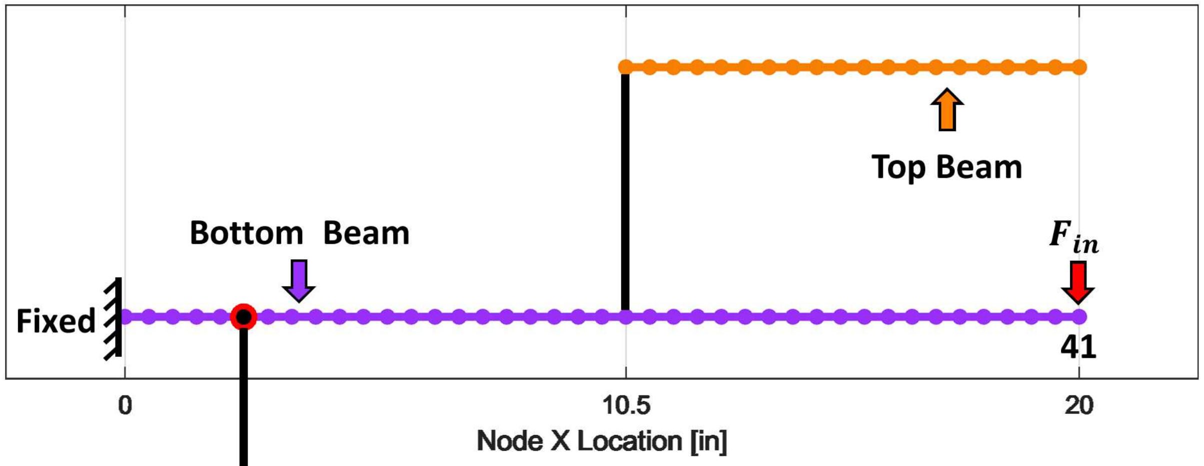
Expansion Sensitivity

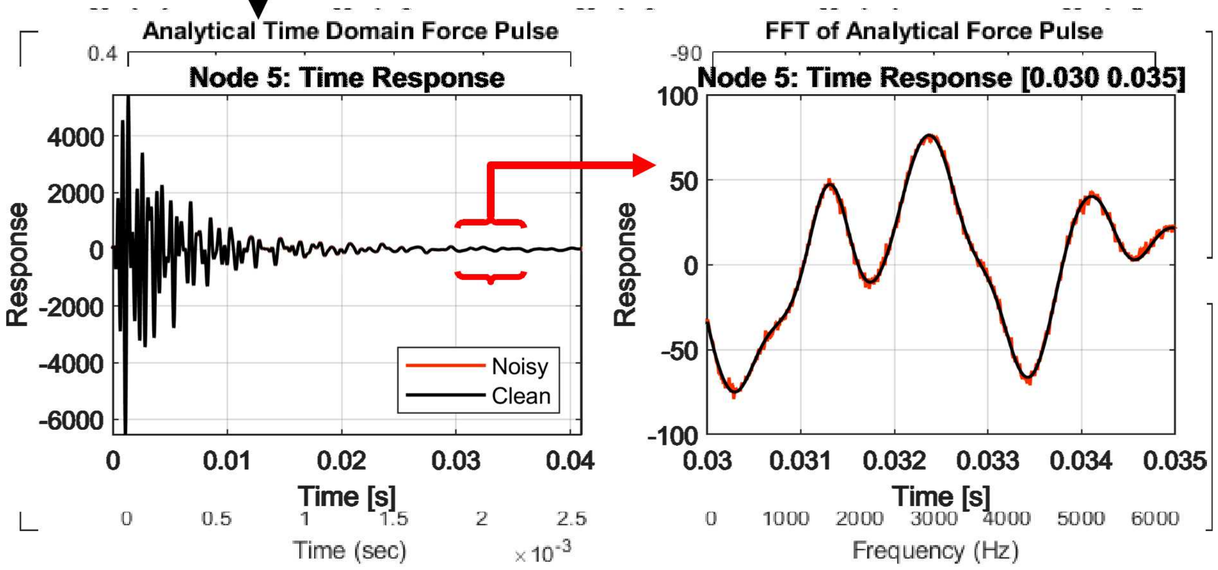
- Introduction
- Theory
- **Expansion Sensitivity**
  - **Effects of Gauge Placement**
  - **Effects of Noise**
- Regularized Expansion
- Conclusions



Expansion Sensitivity – Gage Placement

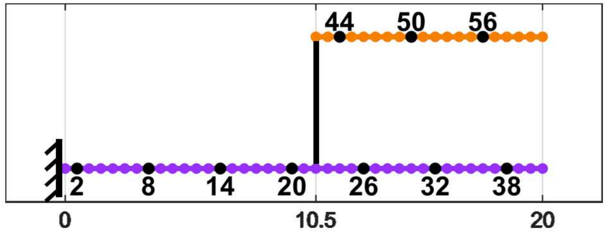
Overview of Tuning Fork Model



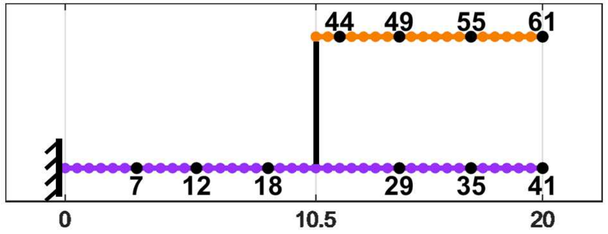


Overview of Test Cases

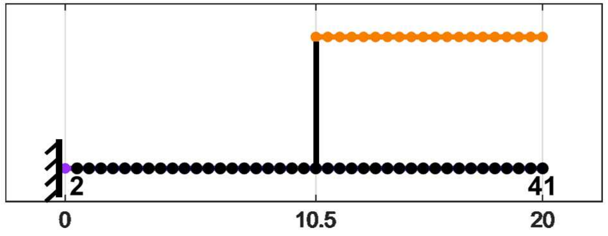
1. 10 DoF Evenly-Spaced



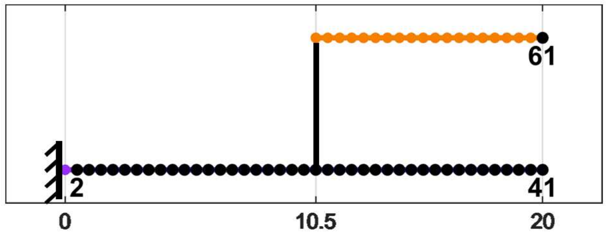
2. 10 DoF Optimized (EIM)



3. 40 DoF Bottom Beam

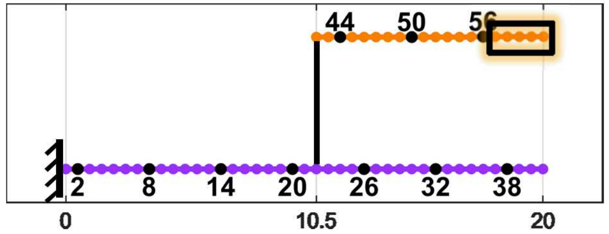


4. 41 DoF 40-Bottom Beam 1-Top Beam



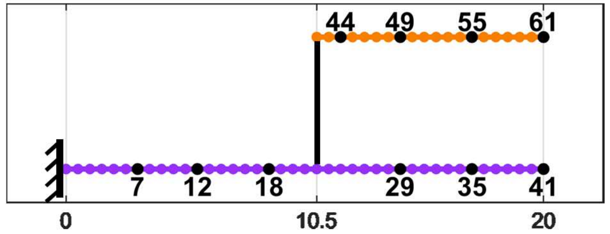
Expansion Sensitivity – Gage Placement

1. 10 DoF Evenly-Spaced



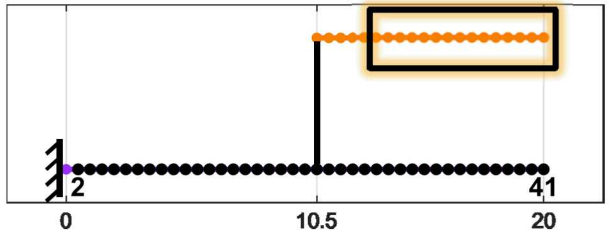
Condition #: 90

2. 10 DoF Optimized (EIM)



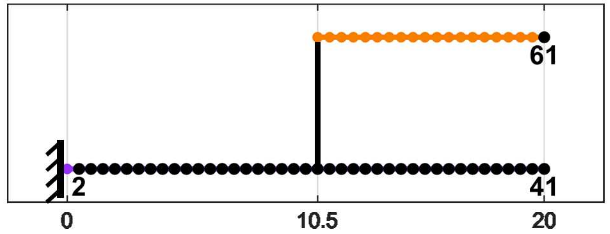
Condition #: 3.2

3. 40 DoF Bottom Beam

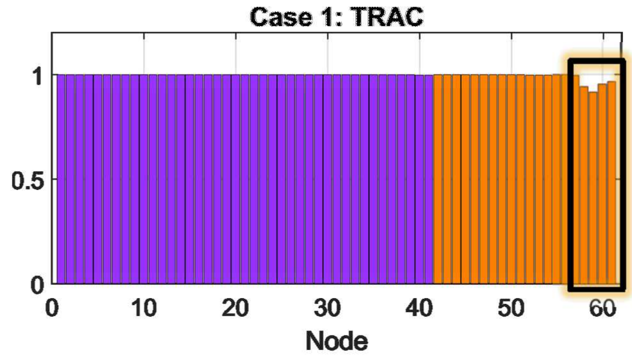


Condition #: 13,040

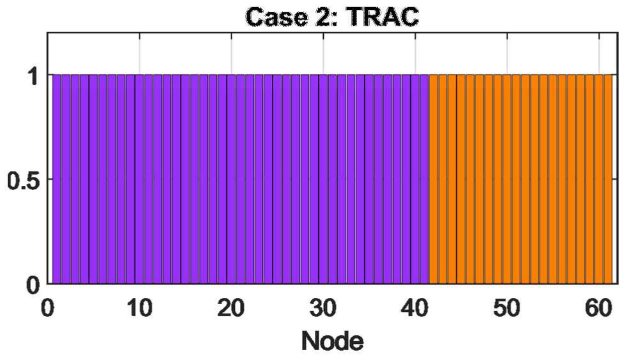
4. 41 DoF 40-Bottom Beam 1-Top Beam



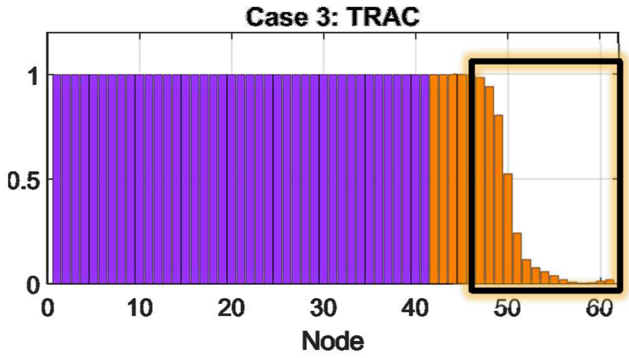
Condition #: 230



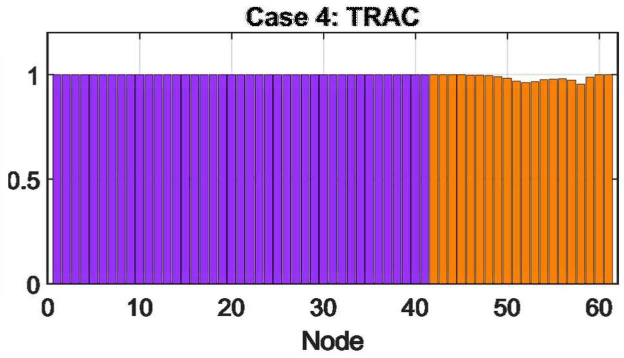
Mean TRAC: 0.99



Mean TRAC: 1.00



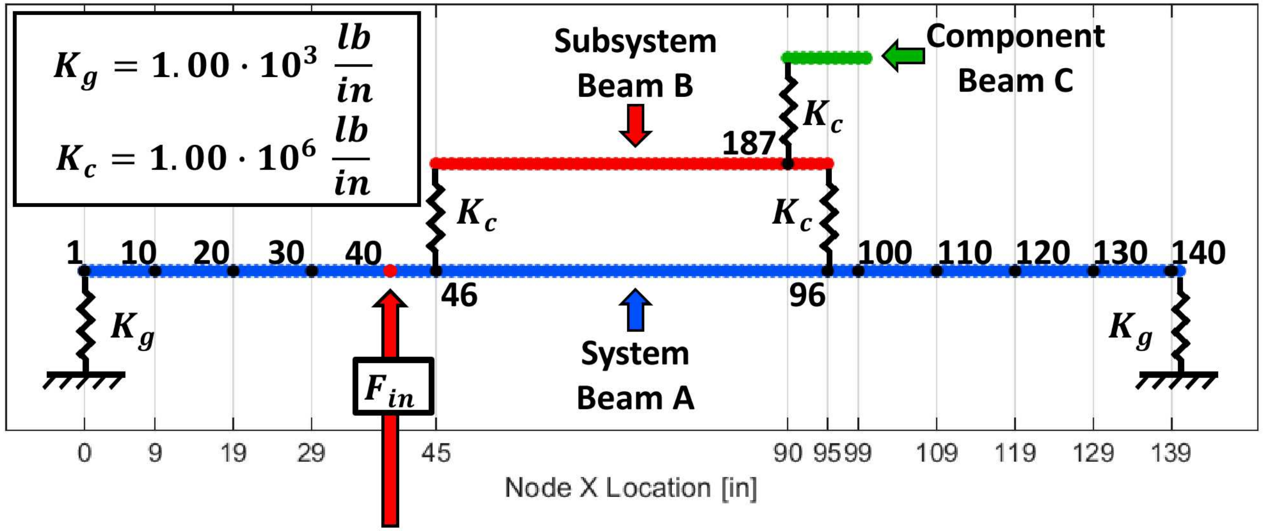
Mean TRAC: 0.82

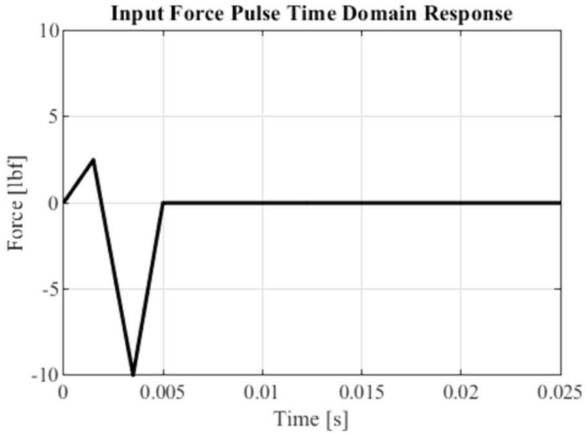


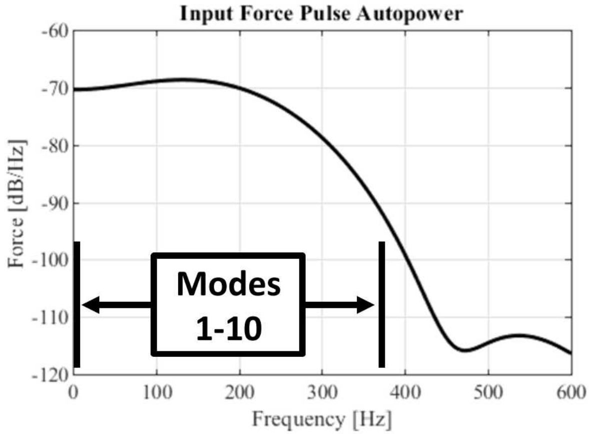
Mean TRAC: 0.99

Expansion Sensitivity – Noise

Overview of Three Beam System Model



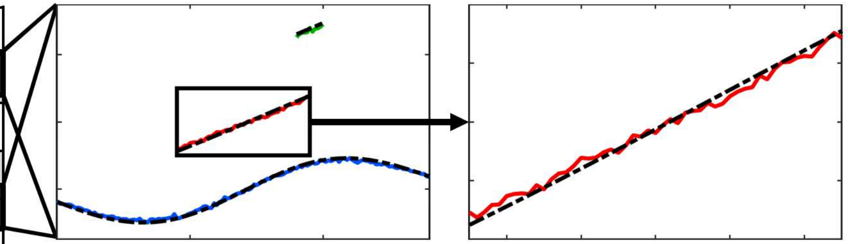




Overview of Test Cases

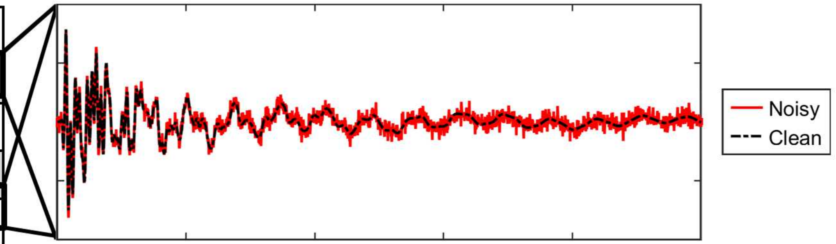
Mode Shape Expansion Noise Cases

Case	*Amplitude*
1	0.10
2	0.20
3	0.40
4	0.80



Time Domain Expansion Noise Cases

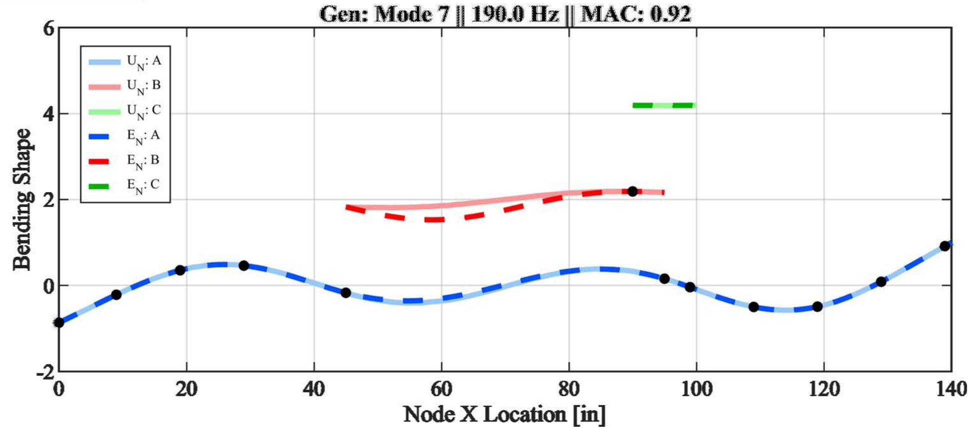
Case	*Amplitude*
1	0.005
2	0.010
3	0.020
4	0.040



Expansion Sensitivity – Noise

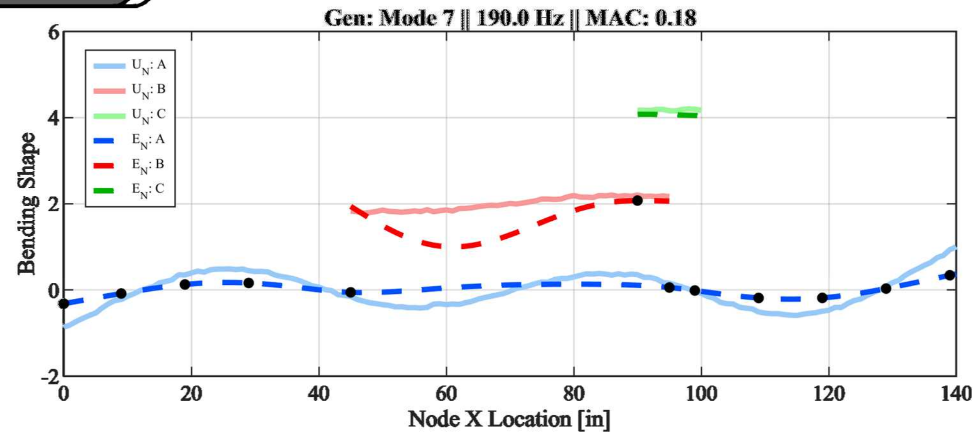
Mode Shape Expansion

Noise 0.10



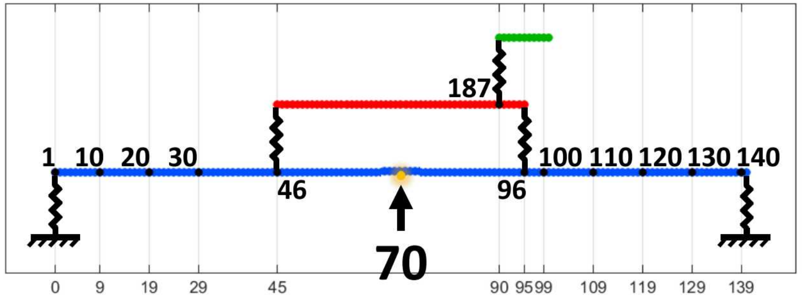
MAC: 0.92

Noise 0.80

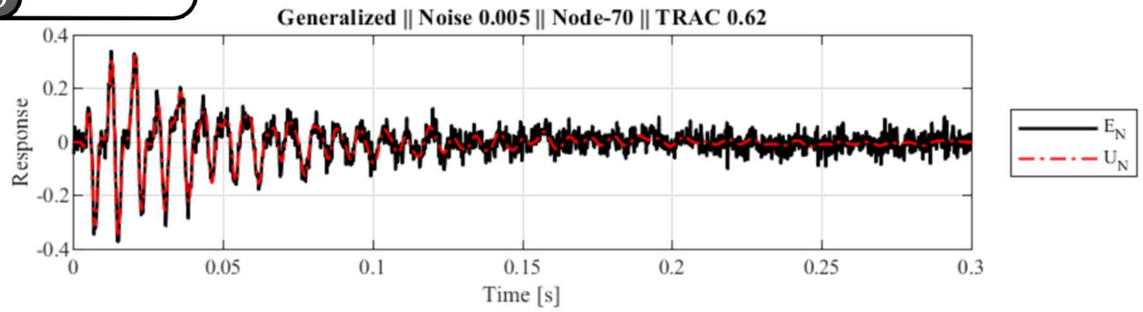


MAC: 0.18

Time Domain Expansion

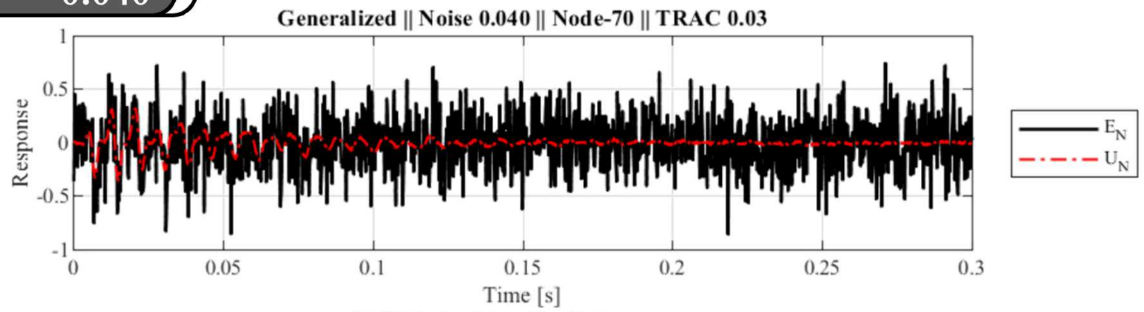


Noise 0.005



TRAC: 0.62

Noise 0.040



TRAC: 0.03

Regularized Expansion

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  - Effects of Noise
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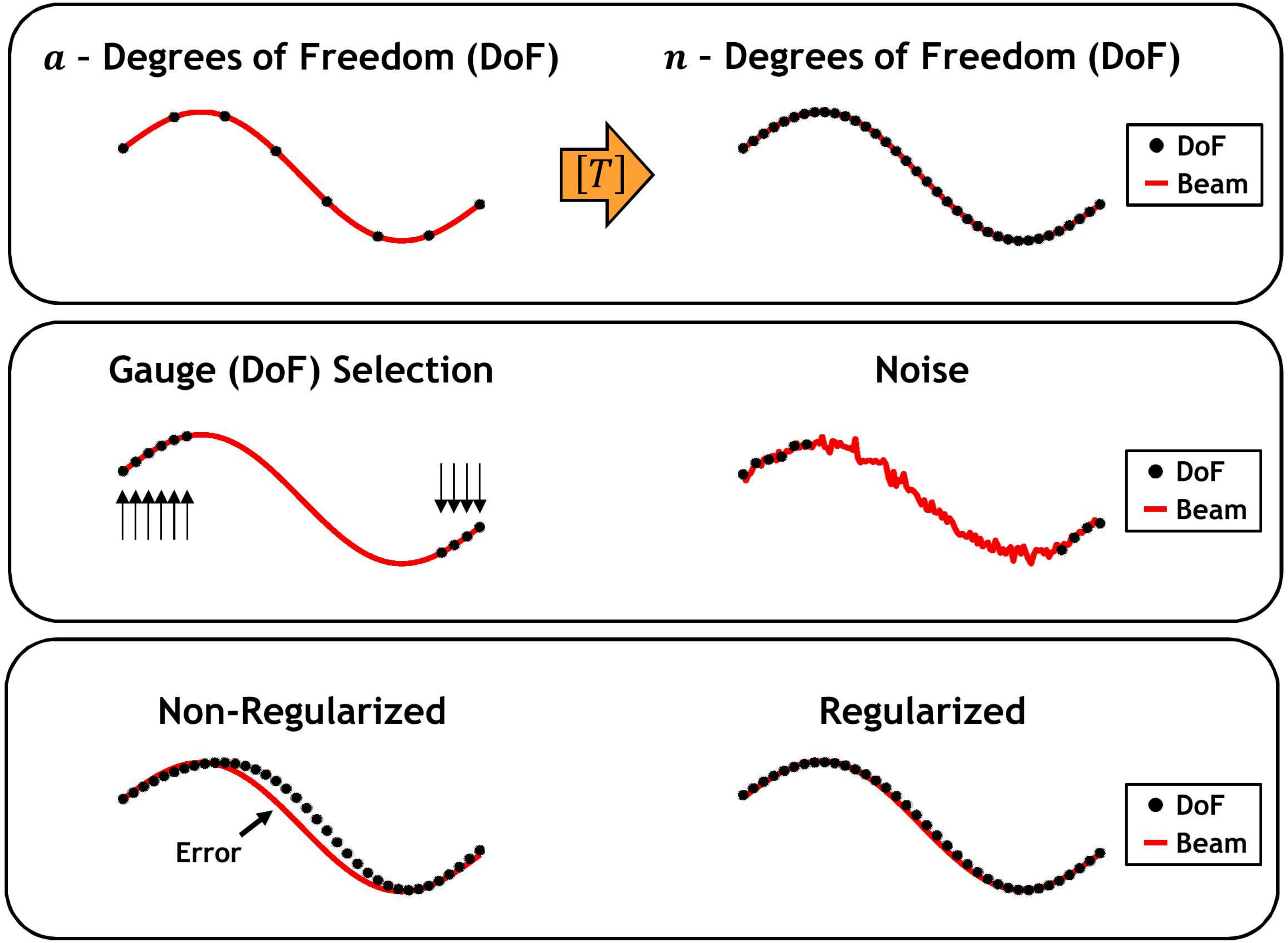
- Conclusions

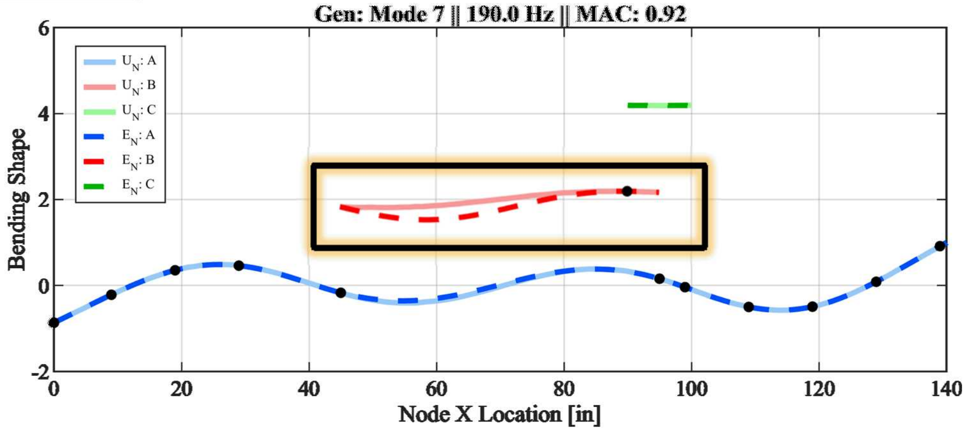


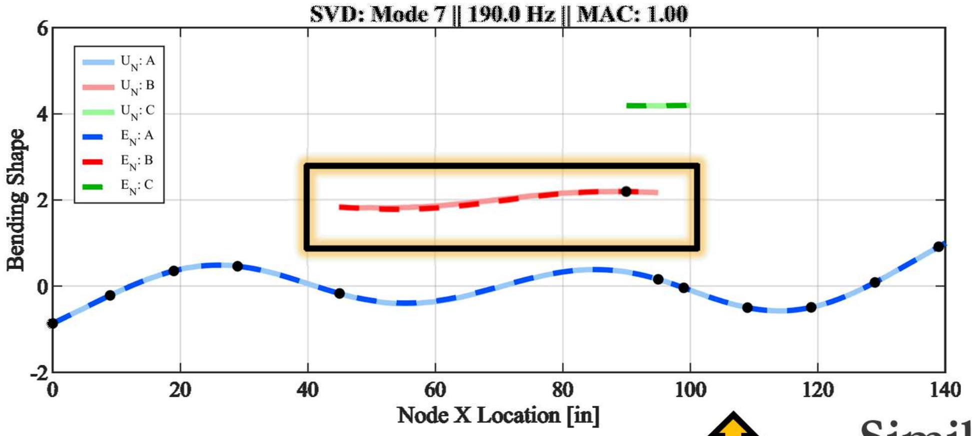
Regularized Expansion - Mode Shapes (Shape Vector)

Non-Regularized Expansion Results

Regularized Expansion Results

Noise
0.10

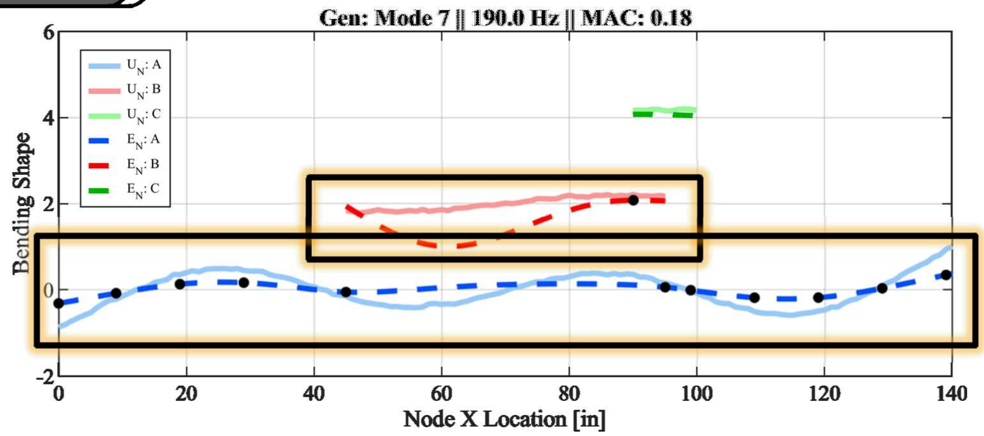


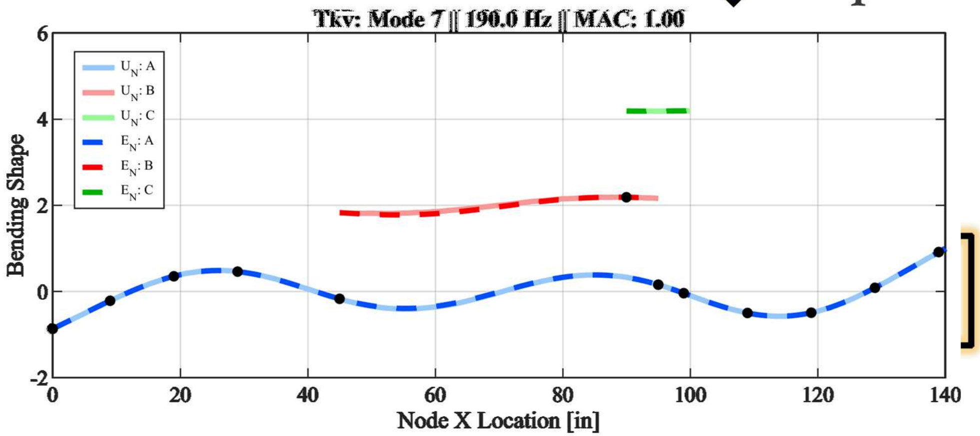


MAC: 0.92 → **+0.08** → MAC: 1.00

Similar Improvement

Noise
0.80



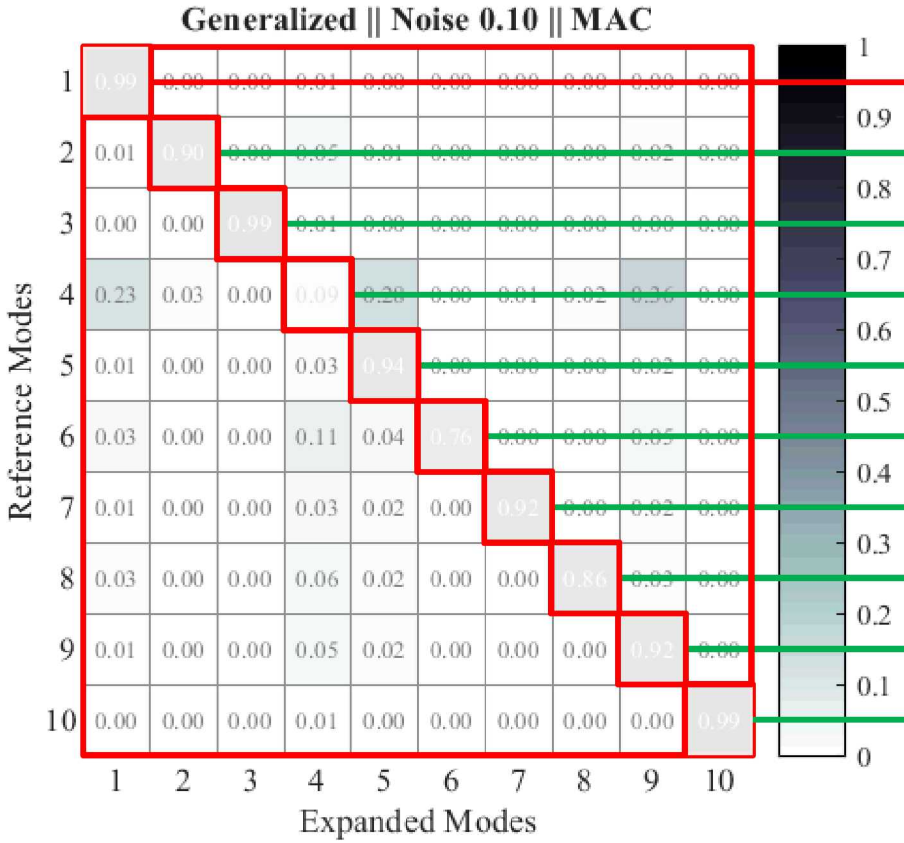


MAC: 0.18 → **+0.81** → MAC: 0.99

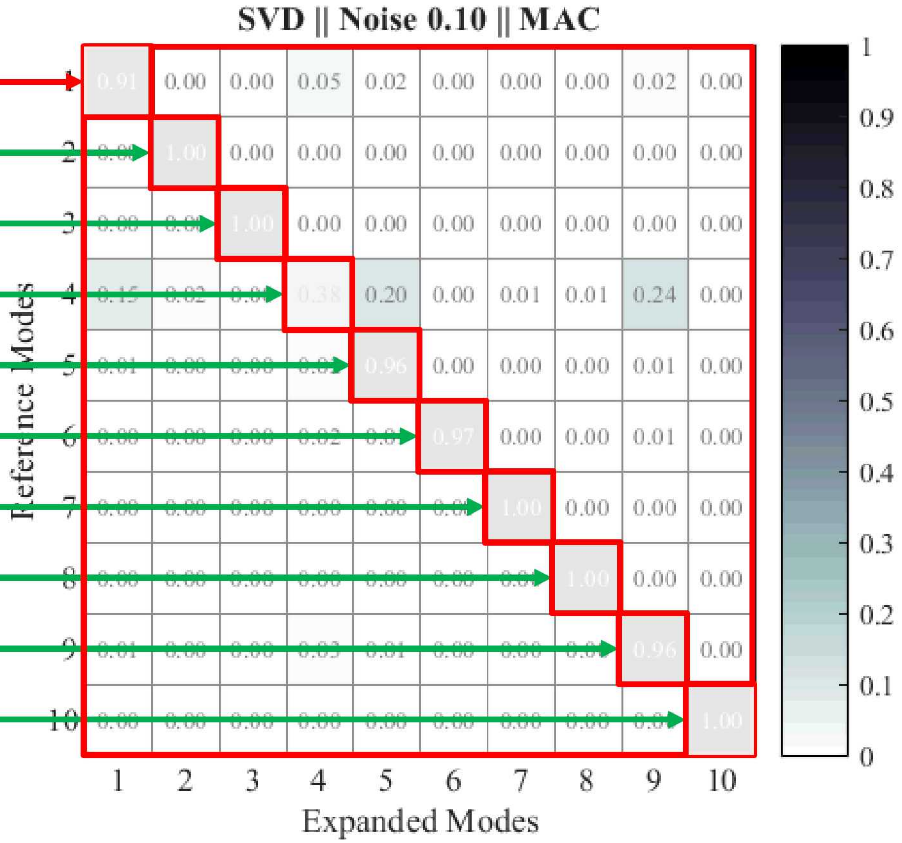
Regularized Expansion - Mode Shapes (MAC)

Noise
Low 0.1 High

Non-Regularized Expansion Results



Regularized Expansion Results



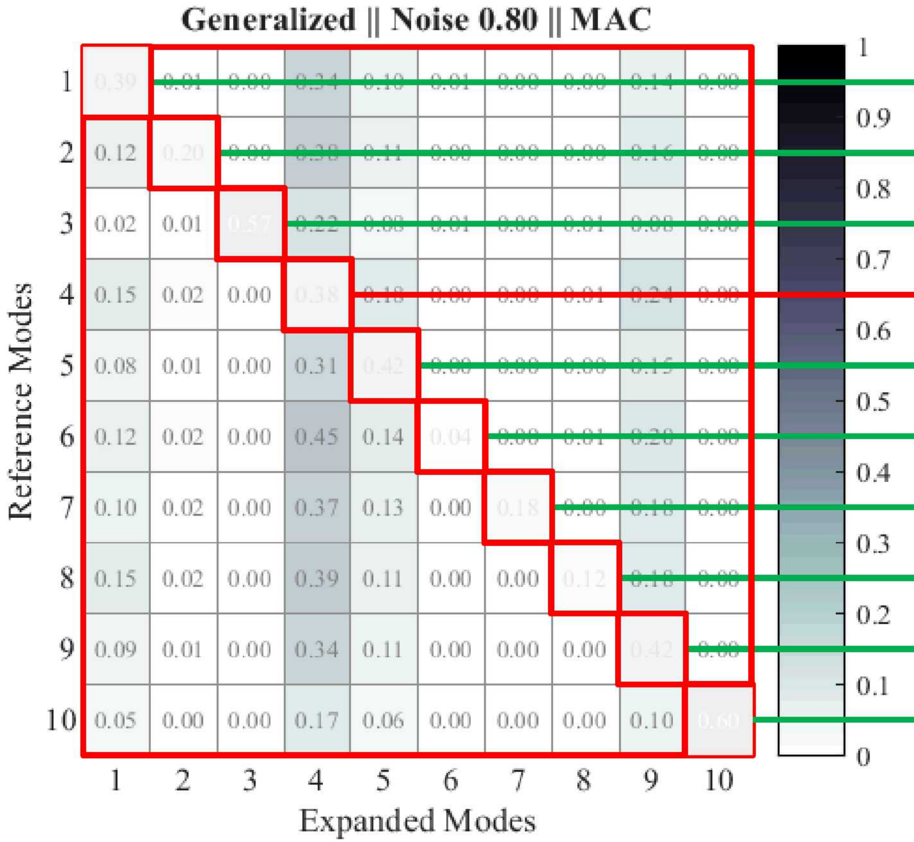
−0.08
+0.10
+0.01
+0.29
+0.03
+0.20
+0.08
+0.13
+0.04
+0.01

Mean Diagonal:	**0.84**	**+0.08**	**Mean Diagonal:**	**0.92**
Mean Off-Diagonal:	**0.02**	**−0.01**	**Mean Off-Diagonal:**	**0.01**

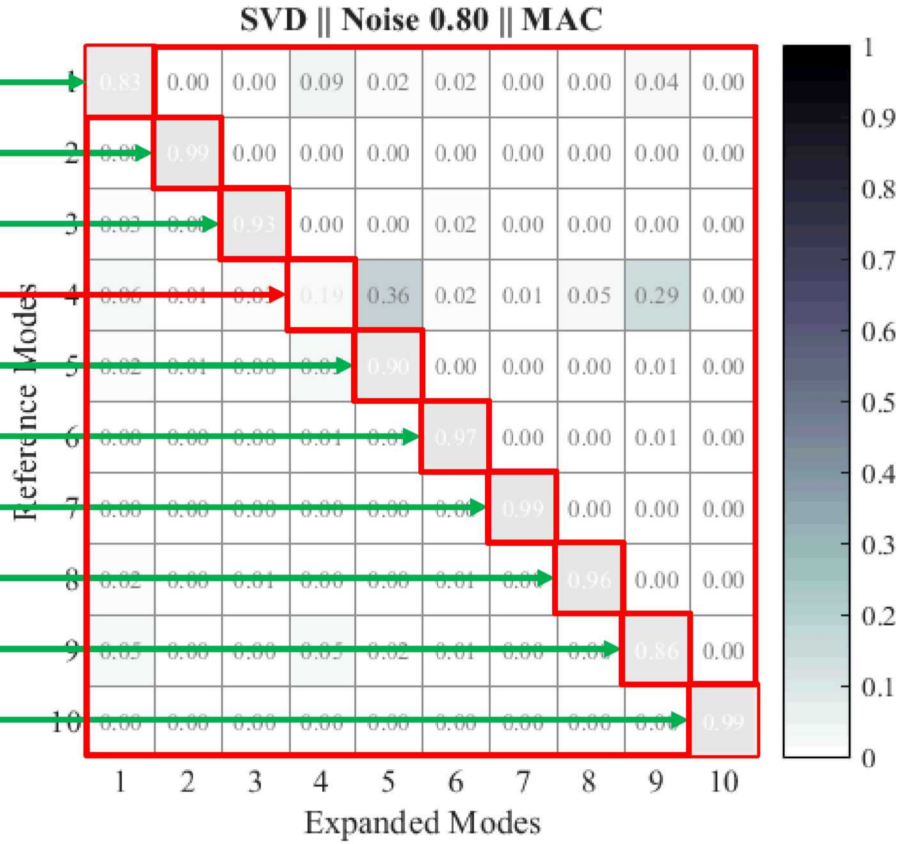
Regularized Expansion - Mode Shapes (MAC)

Noise
Low 0.8 High

Non-Regularized Expansion Results



Regularized Expansion Results

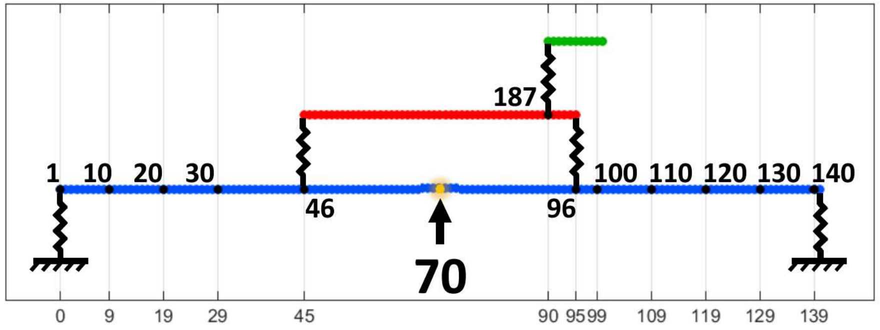


+0.43
+0.79
+0.37
−0.19
+0.48
+0.93
+0.82
+0.84
+0.44
+0.39

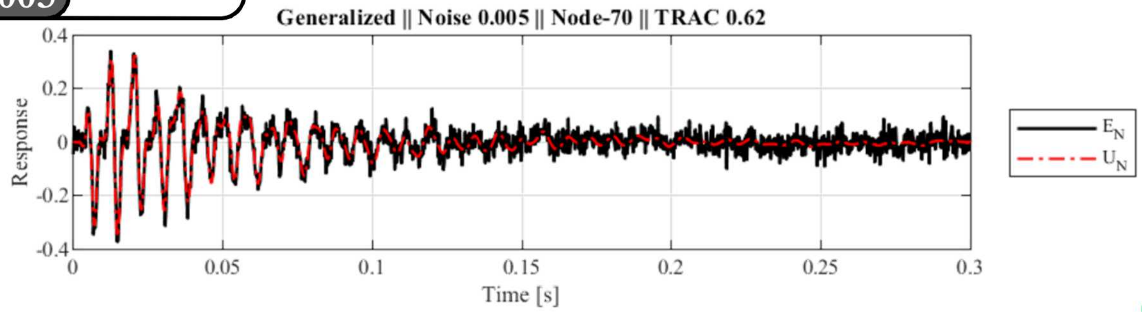
	Non-Regularized	Change	Regularized
Mean Diagonal:	0.33	+0.53	0.86
Mean Off-Diagonal:	0.07	−0.05	0.02

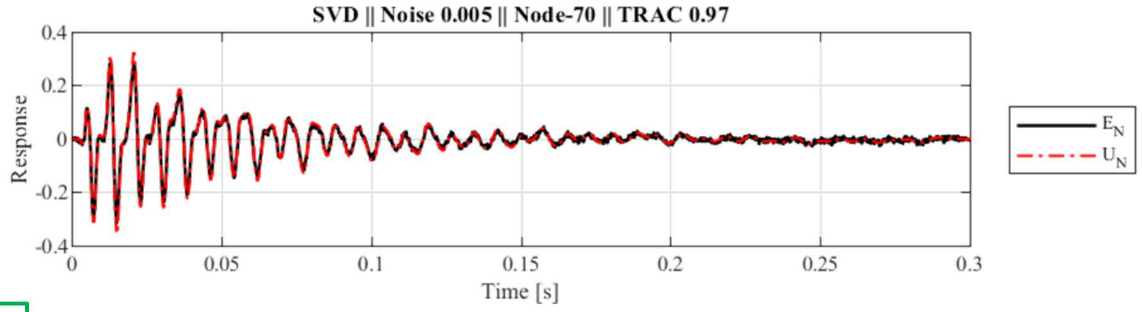
Regularized Expansion – Time Domain (Time History)

Non-Regularized Expansion Results | **Regularized Expansion Results**



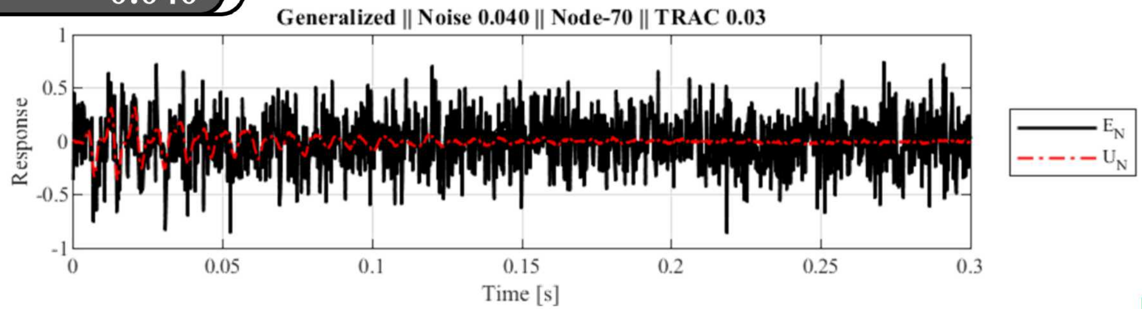
Noise 0.005

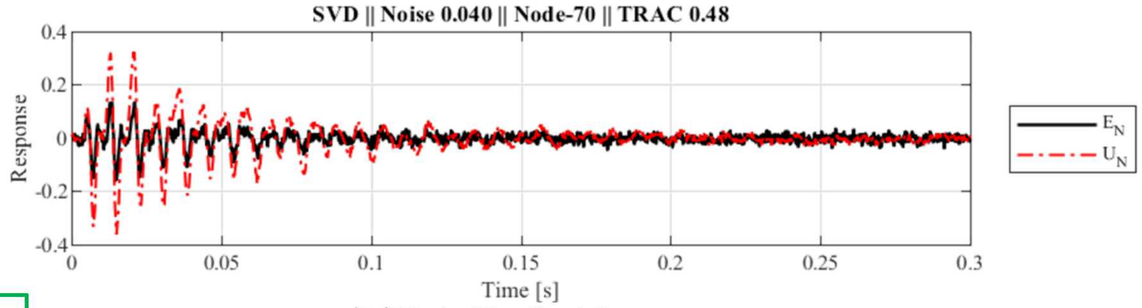




TRAC: 0.62 → **+0.35** → TRAC: 0.97

Noise 0.040

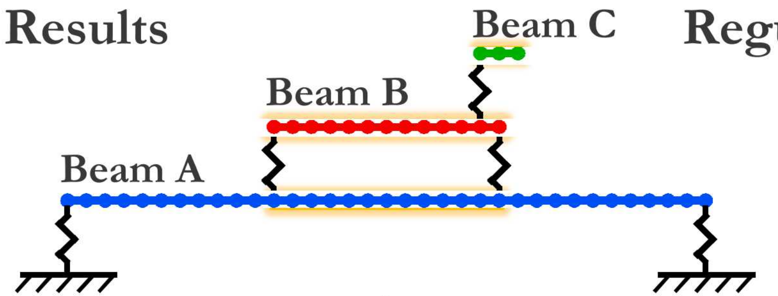




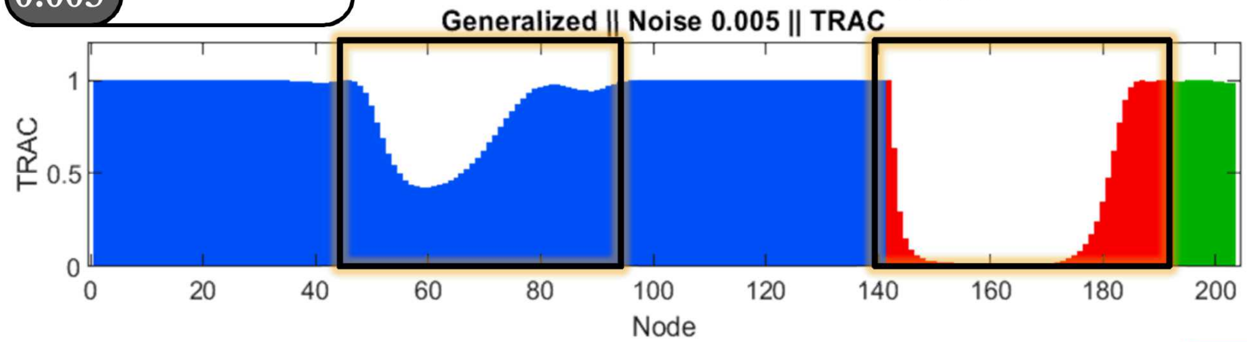
TRAC: 0.03 → **+0.45** → TRAC: 0.48

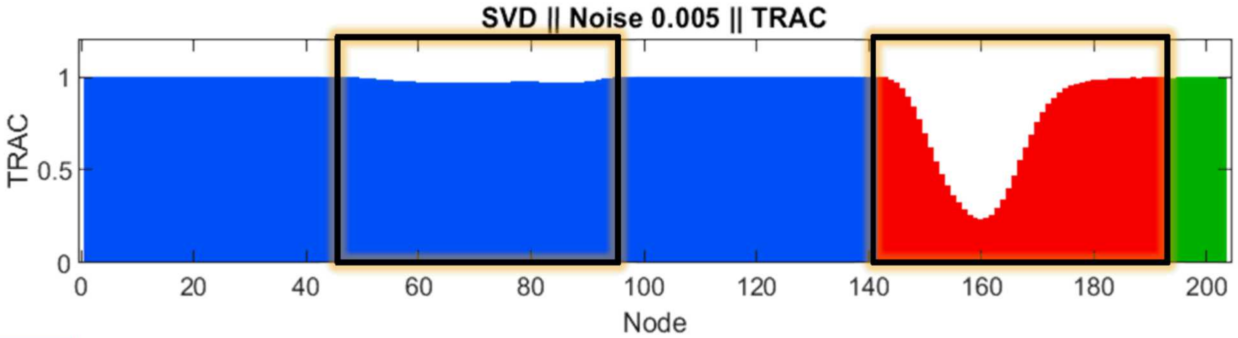
Regularized Expansion – Time Domain (TRAC)

Non-Regularized Expansion Results | **Regularized Expansion Results**



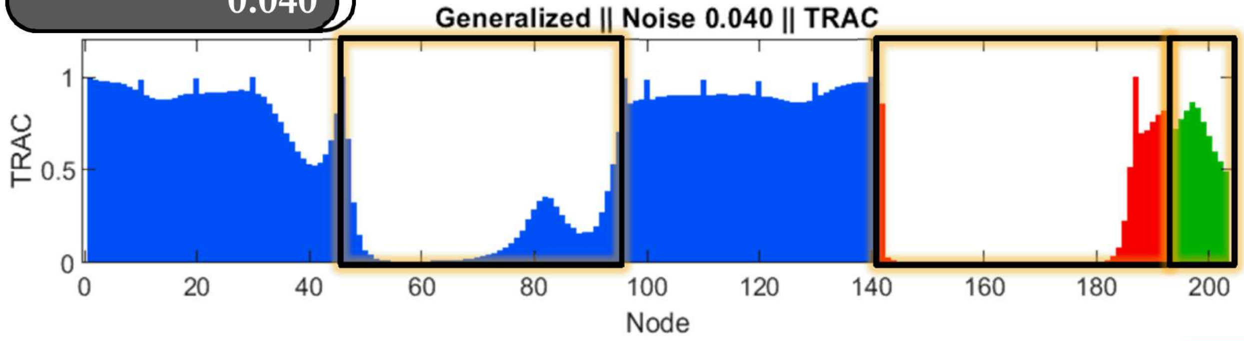
Noise
0.005

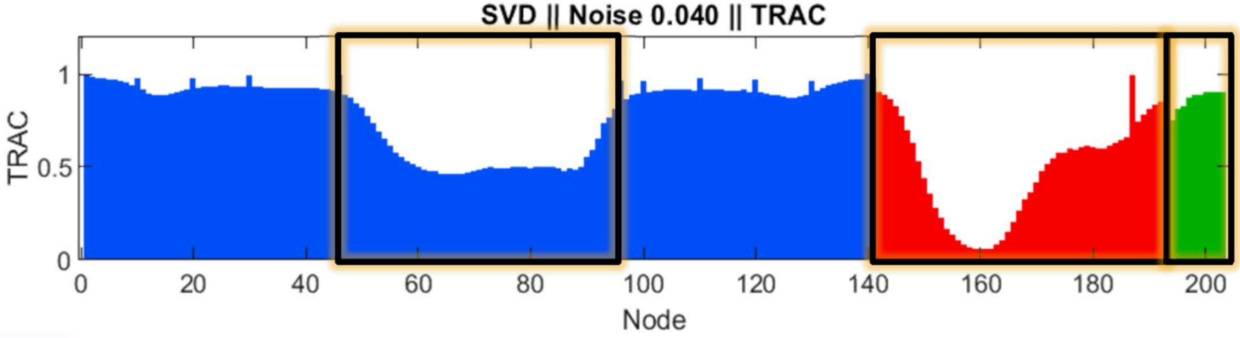




Mean TRAC: 0.76 → **+0.17** → Mean TRAC: 0.93

Noise
0.040

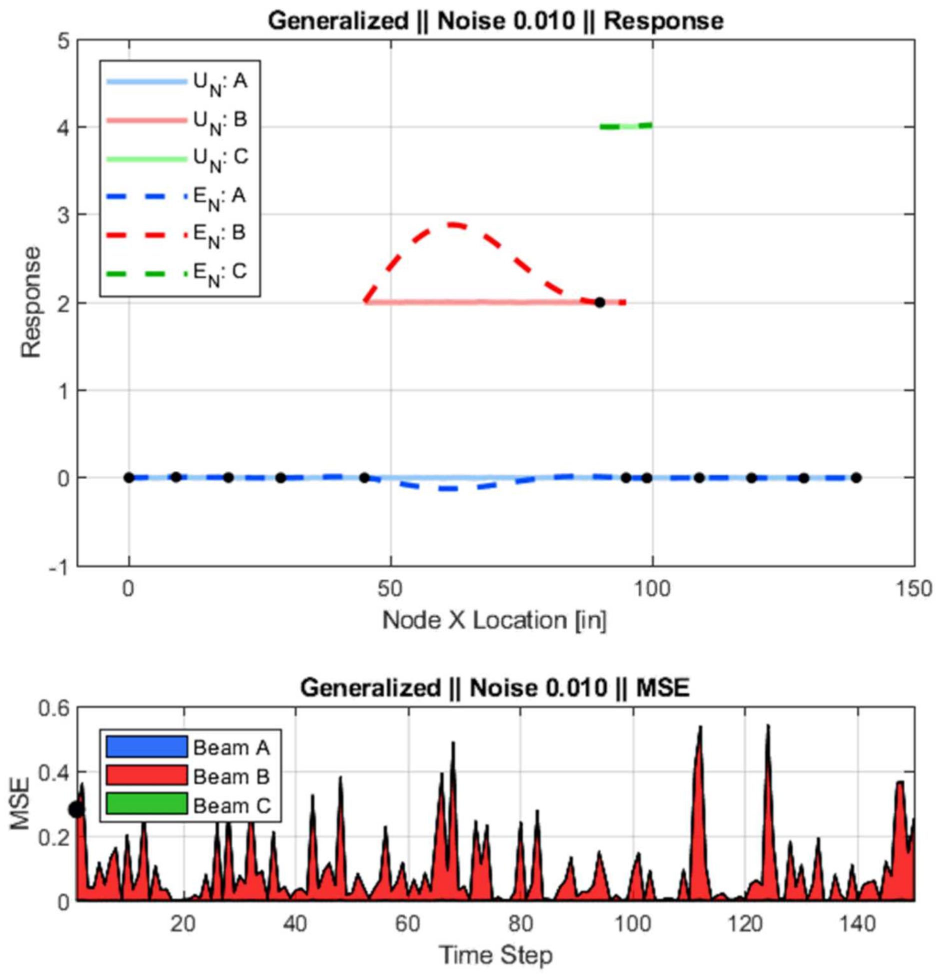




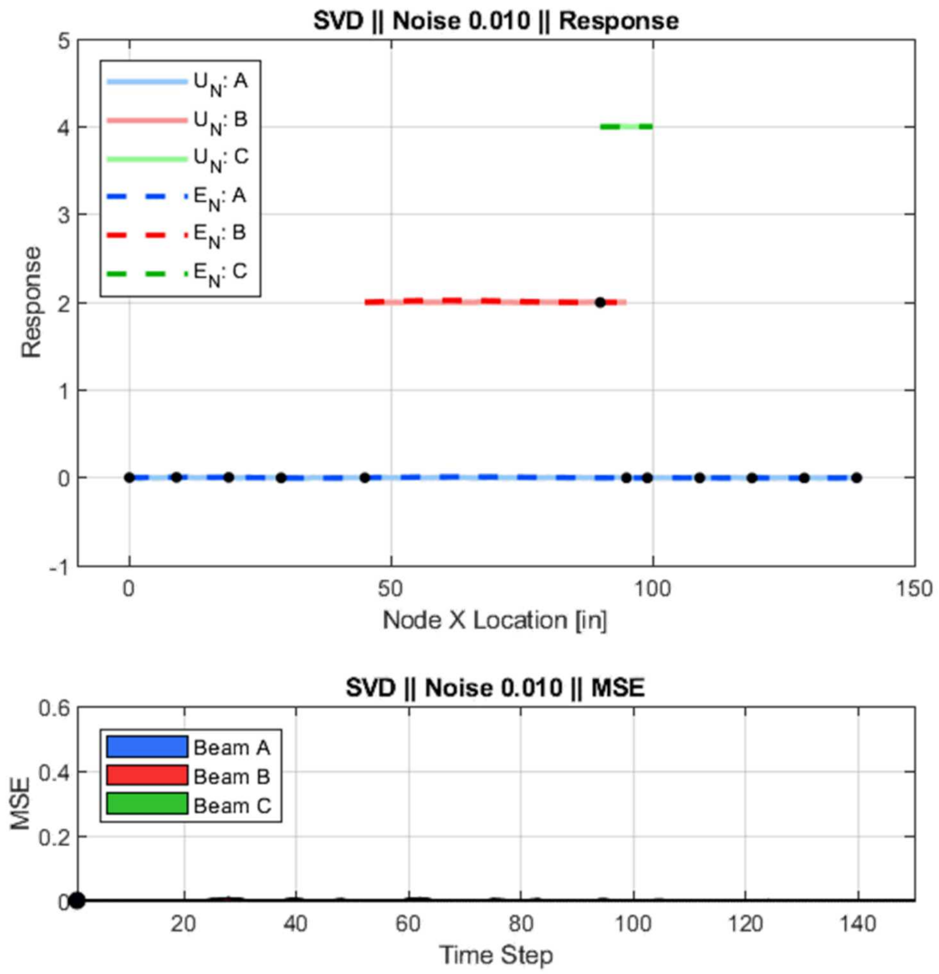
Mean TRAC: 0.51 → **+0.21** → Mean TRAC: 0.72

Regularized Expansion – Time Domain (Animation)

Non-Regularized Expansion Results

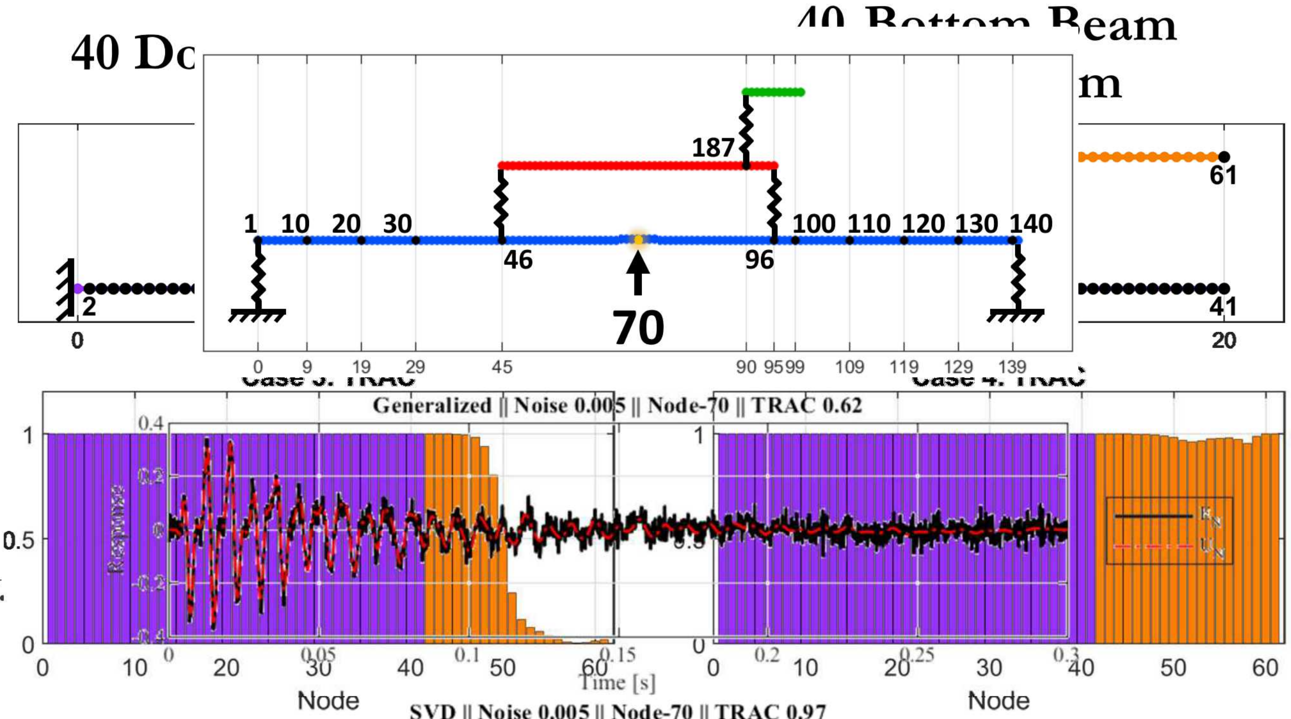


Regularized Expansion Results



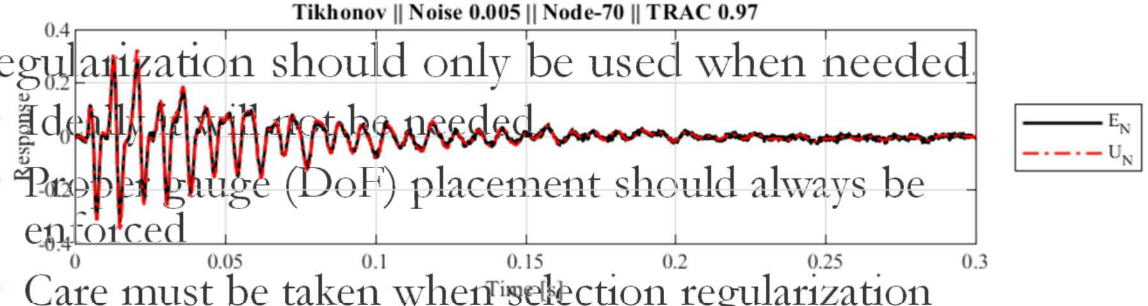
Conclusions

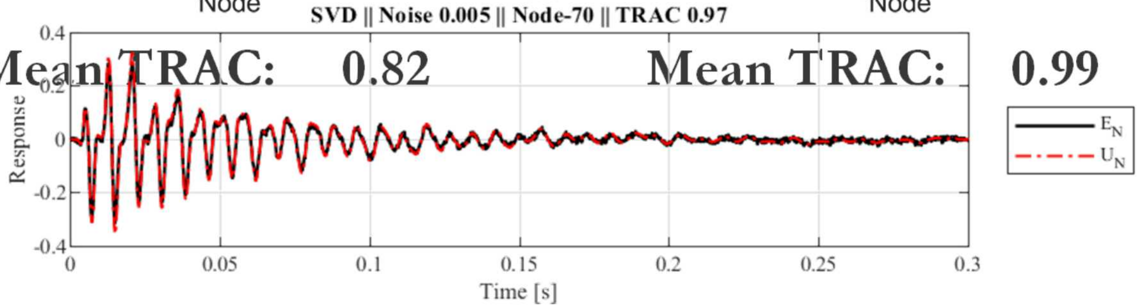
- Expansion was shown to be highly sensitive to:
  - Gauge placement (DoF Selection)
  - Noise



- Regularization was shown to greatly improve the accuracy of the expansion results.

- Tikhonov and SVD regularization performed similarly.





- Regularization should only be used when needed.
  - Ideally it will not be needed
  - Proper gauge (DoF) placement should always be enforced
  - Care must be taken when selection regularization parameters

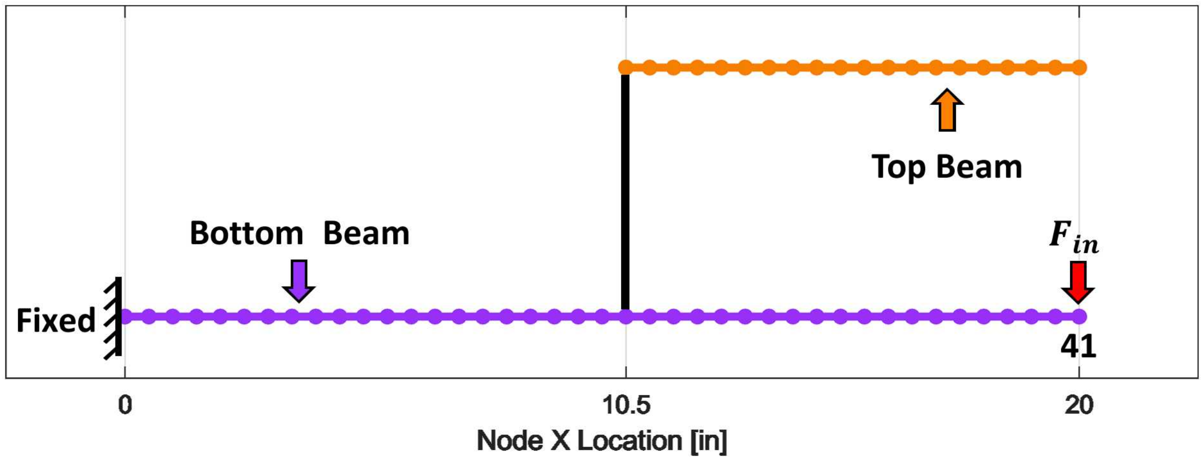
References

[1] Yagle, Andrew E. "Regularized matrix computations." *matrix* 500 (2005): 10.

Backup Slides

Model Parameters

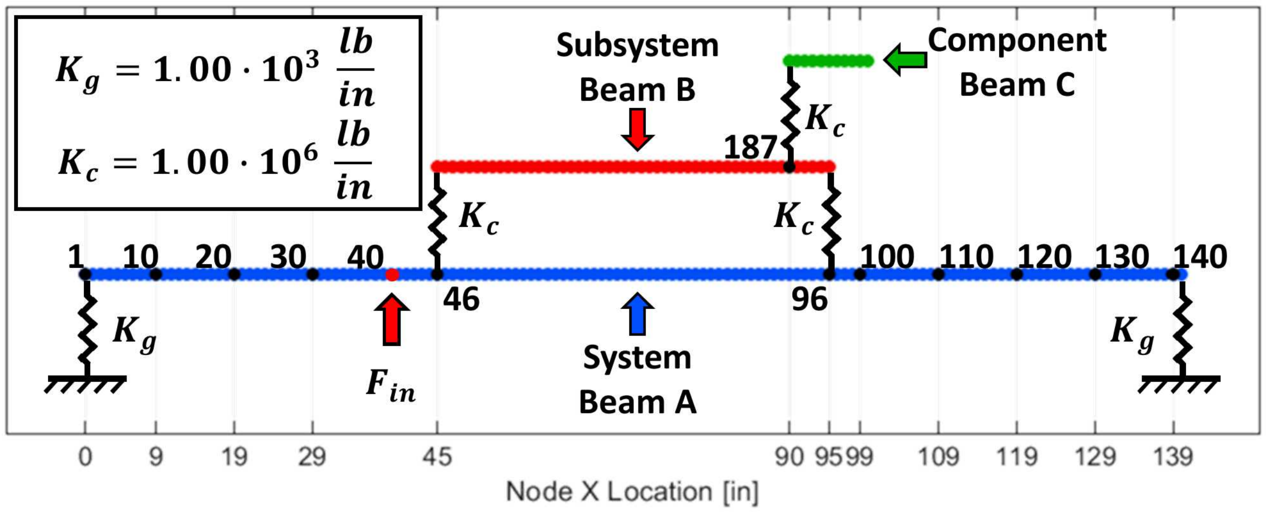
Tuning Fork Model



Overall Properties	Parameter	Value	Units
	Young's Modulus	1.00E+07	psi
	Density	2.59E-04	slug · in^{-3}
	Width	0.50	in
	Height	0.25	in
Base Beam			
	Length	20	in
	Nodes	41	
Top Beam			
	Length	10	in
	Nodes	21	

Parameter	*Value*	*Units*
Sample Rate	100000	Hz
Sample Size	4096	
Sample Time	0.041	s
Pulse Length	0.0005	s
Pulse Amplitude	1	lbf

Three Beam System Model



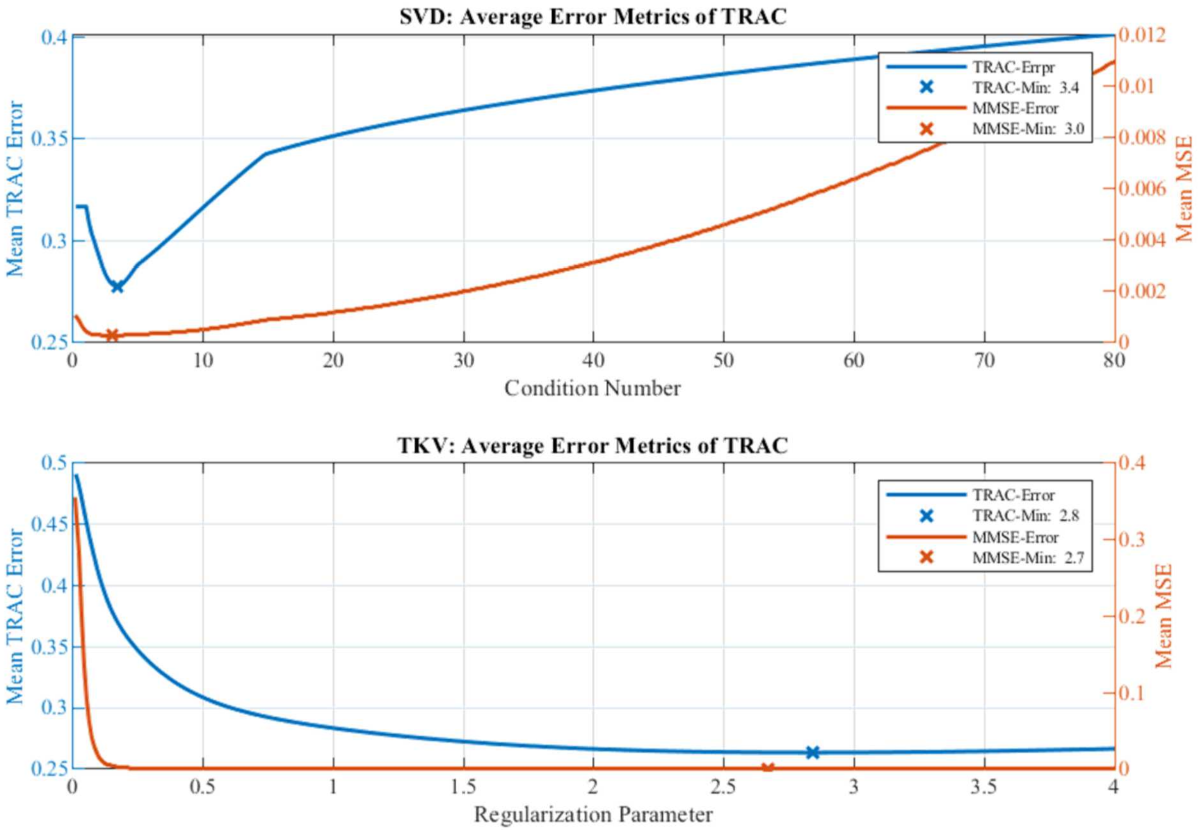
Overall Properties	Parameter	Value	Units
	Young's Modulus	1.00E+07	psi
	Density	2.59E-04	slug · in^{-3}
	Width	3.00	in
	Height	1.50	in
	Wall Thickness	0.188	in
System Beam A			
	Length	140	in
	Nodes	141	
Subsystem Beam B			
	Length	50	in
	Nodes	51	
Component Beam C			
	Length	10	in
	Nodes	11	

Parameter	*Value*	*Units*
Sample Rate	4000	Hz
Sample Size	3000	
Sample Time	0.75	s
Pulse Length	0.005	s
Pulse Amplitude	10	lbf

Regularization Optimization

- Iteratively perform expansion over a range of regularization parameters
- Compute average diagonal MAC / DoF TRAC error
- Parameters that yielded the minimum error was selected

Time Response Expansion Optimization



Mode Shape Expansion Optimization