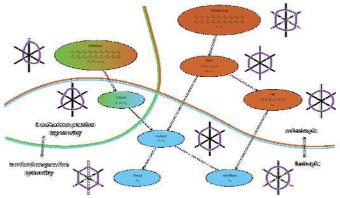
SAND2020-1532PE

# Verification of Material Models for Modeling and Simulation

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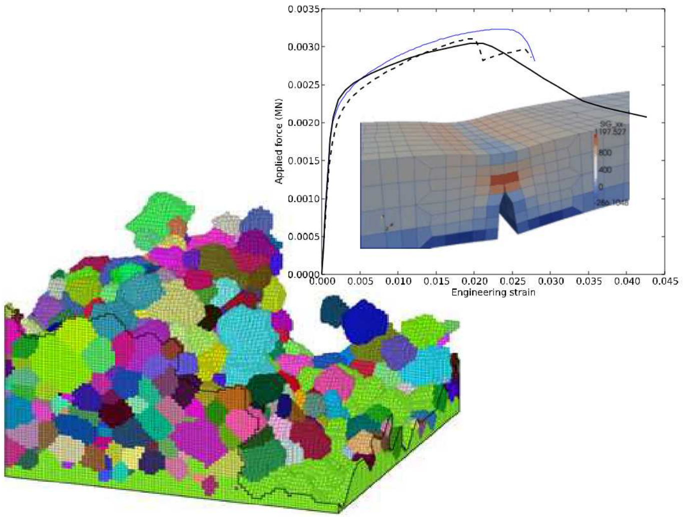
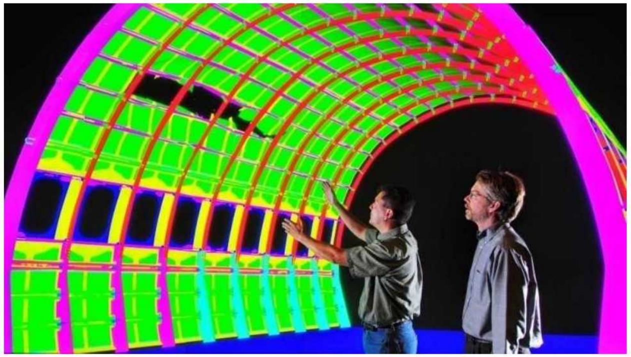
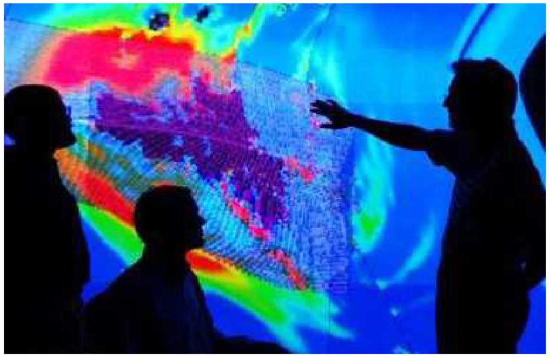
# Modeling and Simulation

Enormous progress in computational mechanics over the past 3 decades.

- Computer architectures
- Geometric details
- Scalable algorithms
- Multiphysics simulation codes
- Physics in computational models



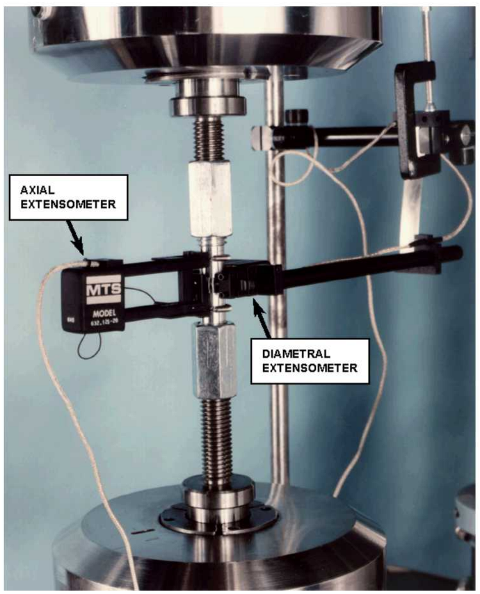





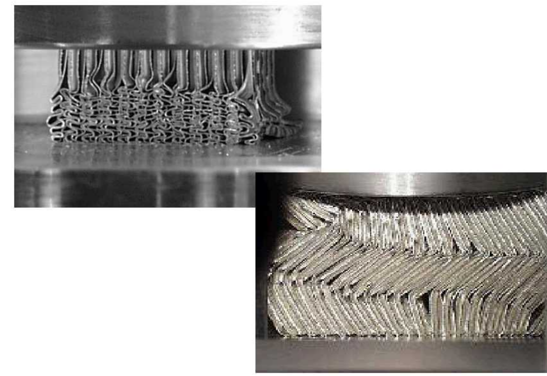
***Solving previously intractable problems***

# Material Behavior

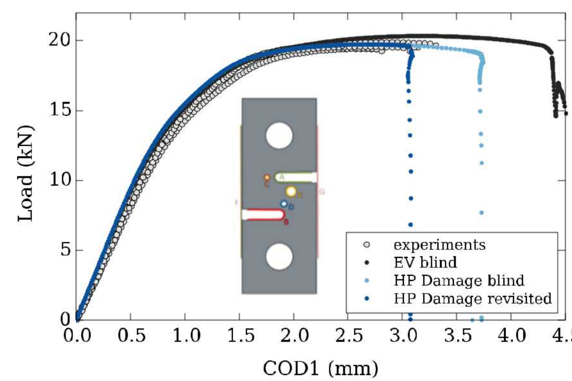
- Understanding and modeling material behavior is at the core of solid mechanics simulations




Courtesy of B. Antoun, Sandia National Laboratories



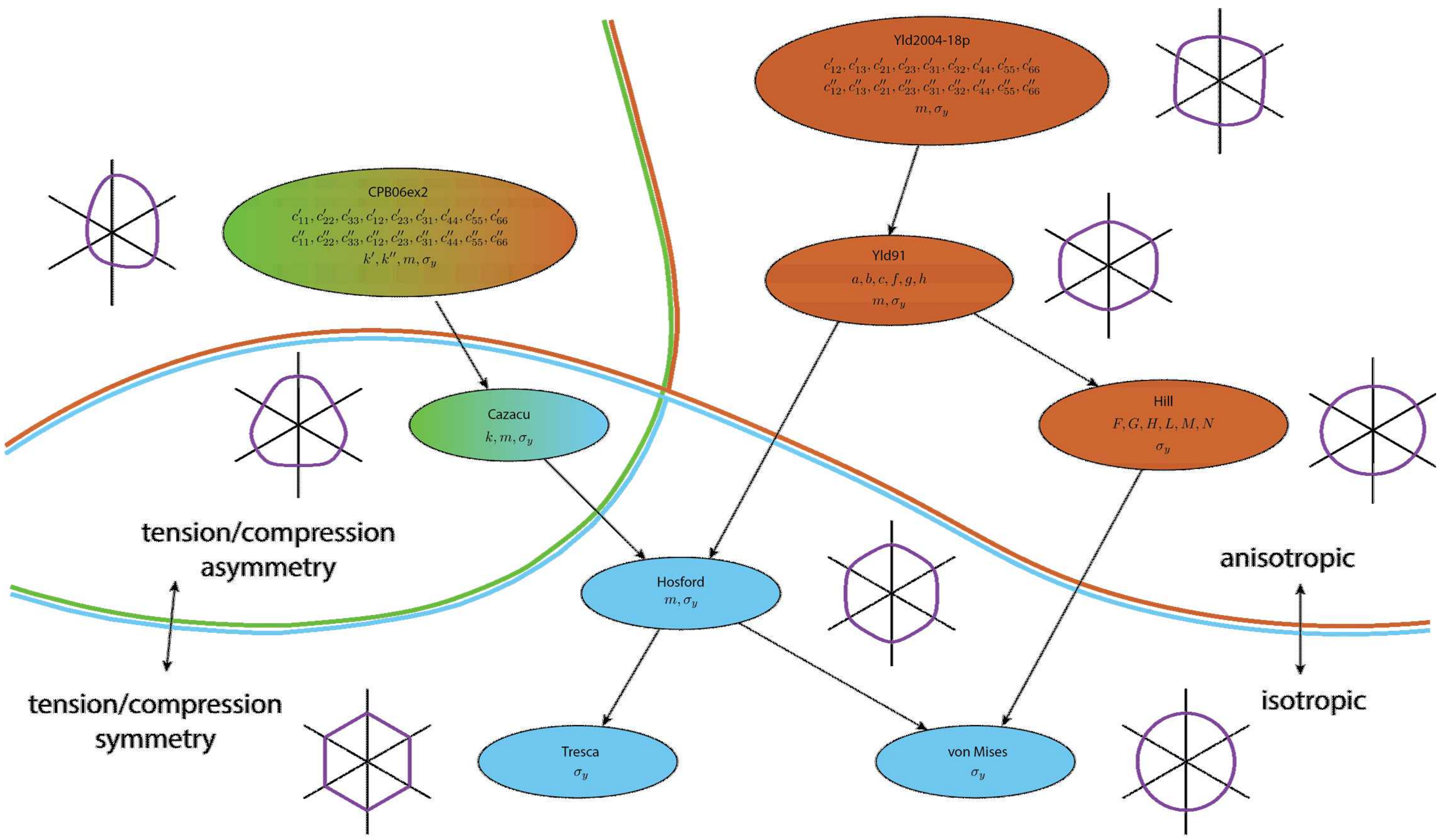
Courtesy of W. Y. Lu, Sandia National Laboratories




Karlson et al., 2016, *Int. Jrnl. Frac.*, 198: 179-195

# Motivation

A family of yield surfaces implemented in Sierra/SolidMechanics provides the basis for a flexible and **reliable** family of plasticity models

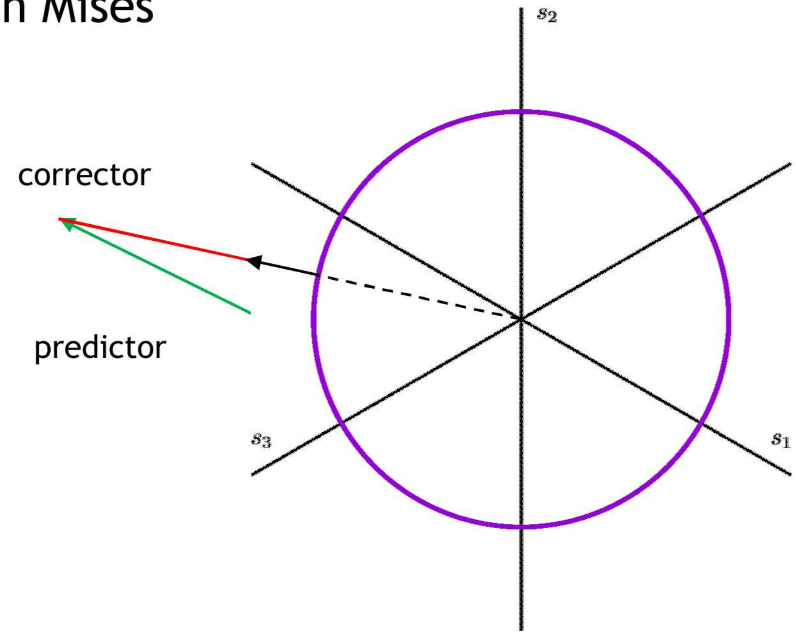


# Return Mapping

Predictor: $\boldsymbol{\sigma}^{\text{tr}} = \boldsymbol{\sigma}_n + \mathbb{C} : \Delta\varepsilon$

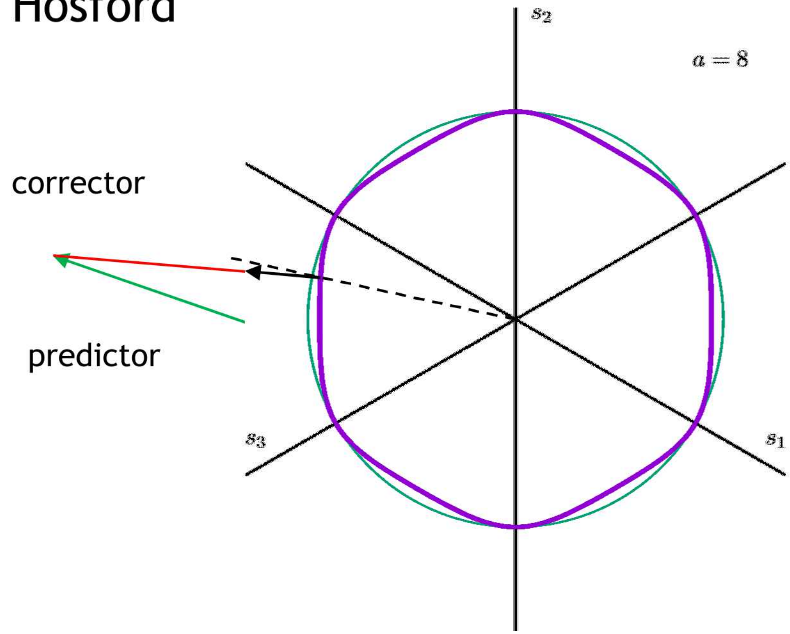
Corrector: $\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{\text{tr}} - \mathbb{C} : \Delta\bar{\varepsilon}^p$

## von Mises




Radial Return

## Hosford

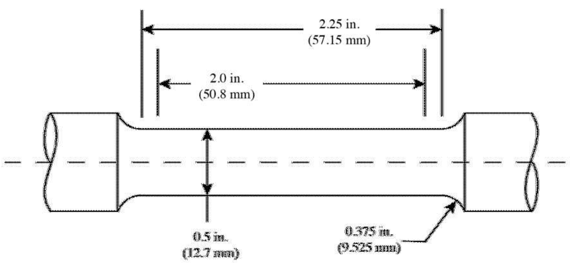


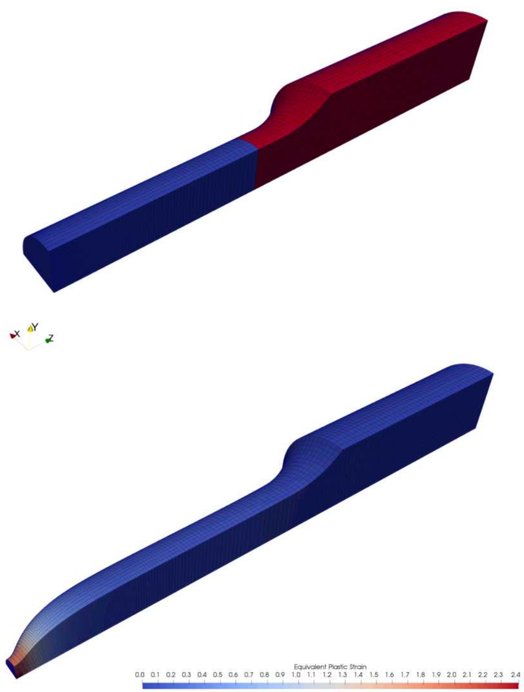

$$R^{\text{tr}} = \frac{\phi(\boldsymbol{\sigma}^{\text{tr}})}{\bar{\sigma}}$$

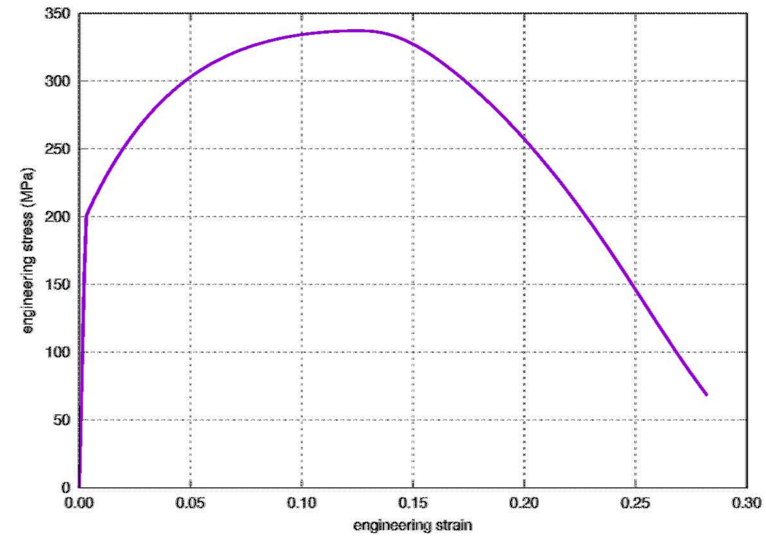
magnitude of trial stress (predictor)

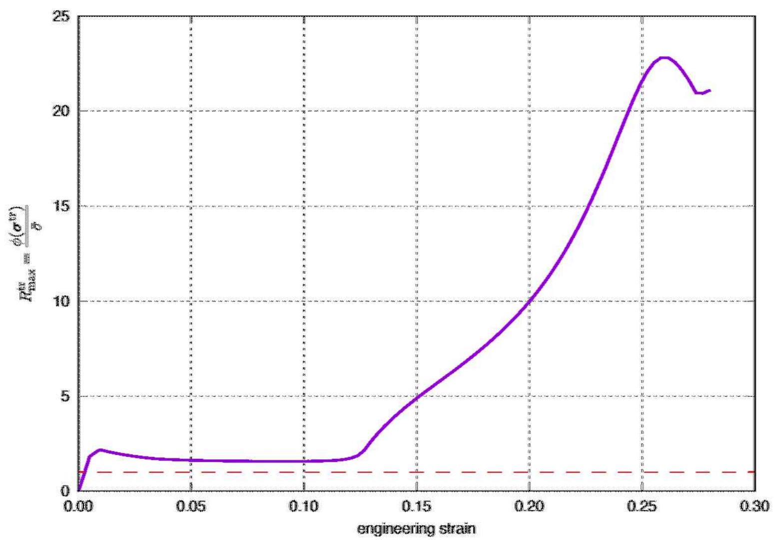
# Return Mapping Algorithm – Testing

ASTM E8 specimen



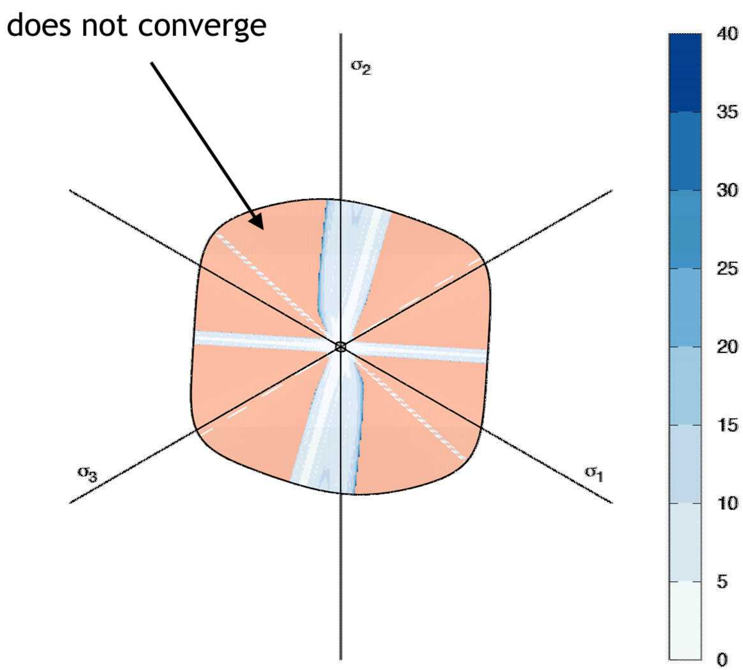





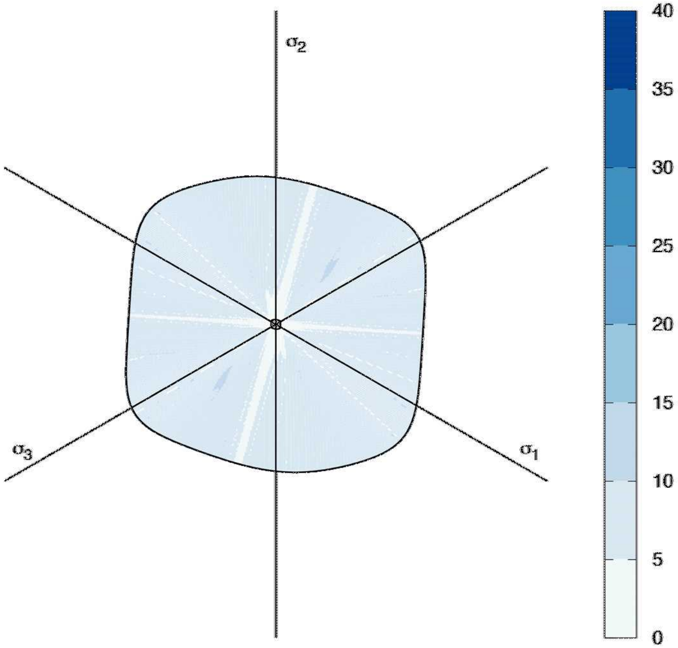



# Performance of Yield Surface Models

Yld2004-18p*




Newton-Raphson




Line Search

* F. Barlat et. al., *Int. J. Plast.*, **19** (2005) 1009-1039

# Verification

"The process of verification assesses the fidelity of the computational model to the mathematical model." *

Four approaches:

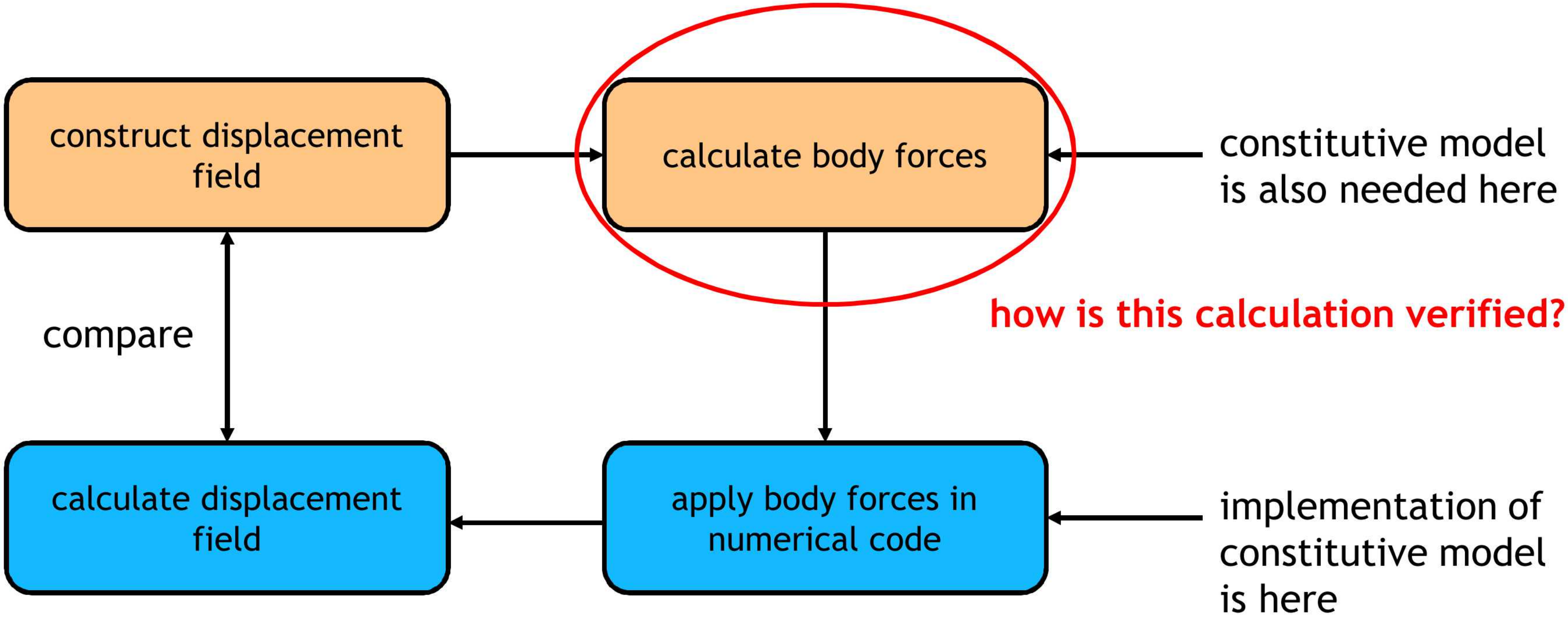
- *Analytical Solutions* – difficult to find
- *Method of Manufactured Solutions* – forcing function depends on material model
- *Numerical Benchmark Solutions* – semi-analytical, code-to-code
- *Consistency Tests* – "complementary to the other types of algorithm tests"

"With the ever-increasing complexity in CSM [computational solid mechanics] models, ***especially constitutive models***, the task of verification becomes more difficult because of a lack of relevant analytical solutions." *

* Guide for Verification and Validation in Computational Solid Mechanics - ASME V&V 10-2006 (reaffirmed 2016)

# Verification

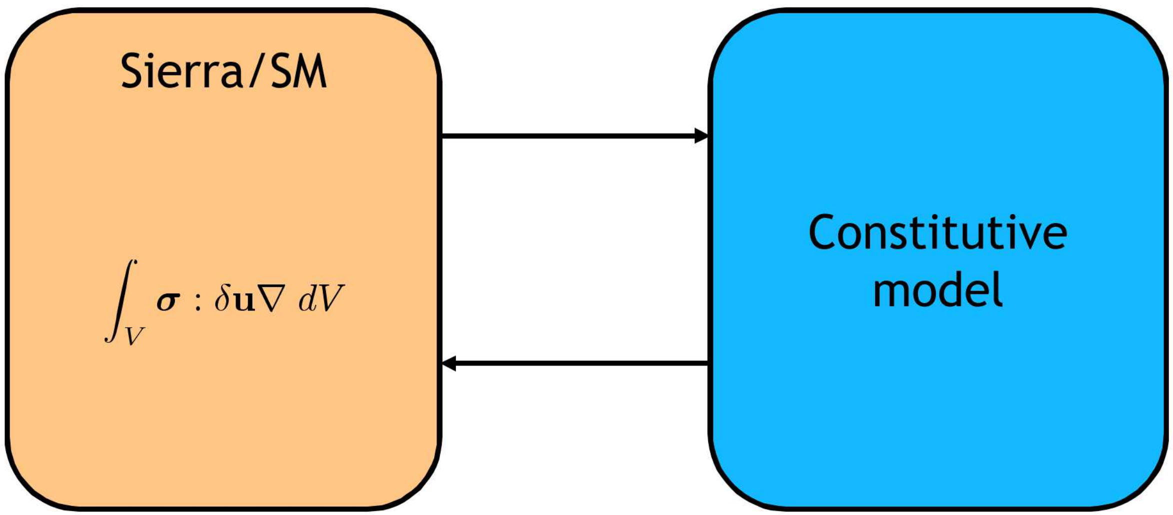
- Method of Manufactured Solutions (MMS)
  - Standard and effective method for verification of solid mechanics codes
  - Difficult to use for nonlinear, path dependent material models



# Verification

- Material Point Driver (MPD)
  - Code that exercises **only** the material model

$$\sigma = f(\varepsilon) \qquad \dot{\sigma} = f(\dot{\varepsilon})$$



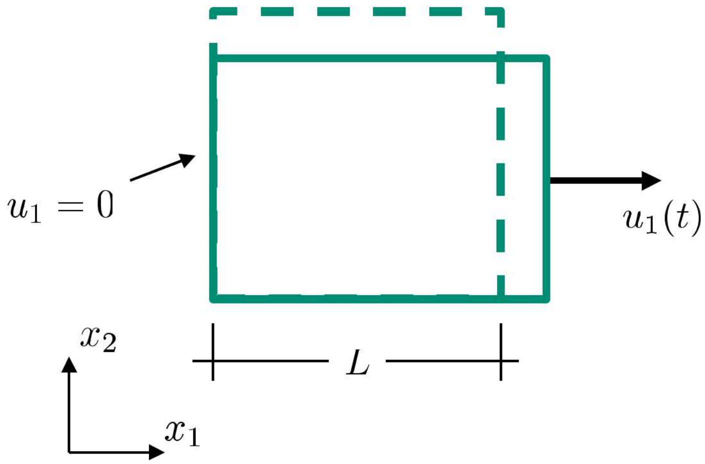
# Verification

- Use a Sierra/SolidMechanics as MPD
  - Find a solution you can quantify
  - Carefully construct boundary/initial conditions
  - **Document and peer review**

- Derive stress/strain paths to get the “correct” result
  - Strain paths
    - Uniaxial strain
    - Simple shear
    - ***Pure shear***
  - Stress paths
    - ***Uniaxial stress***
    - ***Pure shear***
    - Biaxial stress

# Uniaxial Stress – Rate Dependent Plasticity

$$\sigma_{11} \neq 0 \quad ; \quad \sigma_{ij} = 0 \text{ otherwise}$$




drive boundary condition with plastic strain rate

hardening model

$$u_1(t) = \left[\exp\left(\frac{\bar{\sigma}\left(\bar{\varepsilon}^p(t)\right)}{E\phi(\bar{\boldsymbol{\sigma}})} + \bar{\varepsilon}^p(t)\frac{\partial \phi}{\partial \sigma_{11}}\right) - 1\right] L$$

yield surface

initial condition

$$\sigma_{11} = \sigma_y g(\dot{\bar{\varepsilon}}^p)$$

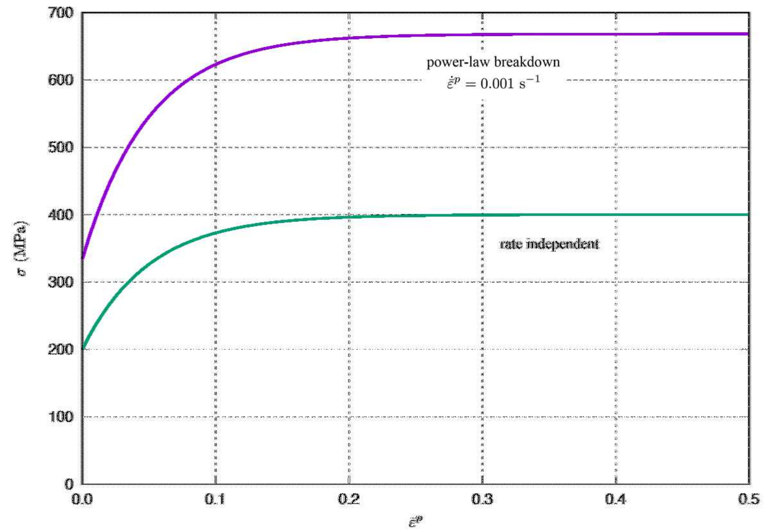
rate dependent multiplier

# Uniaxial Stress

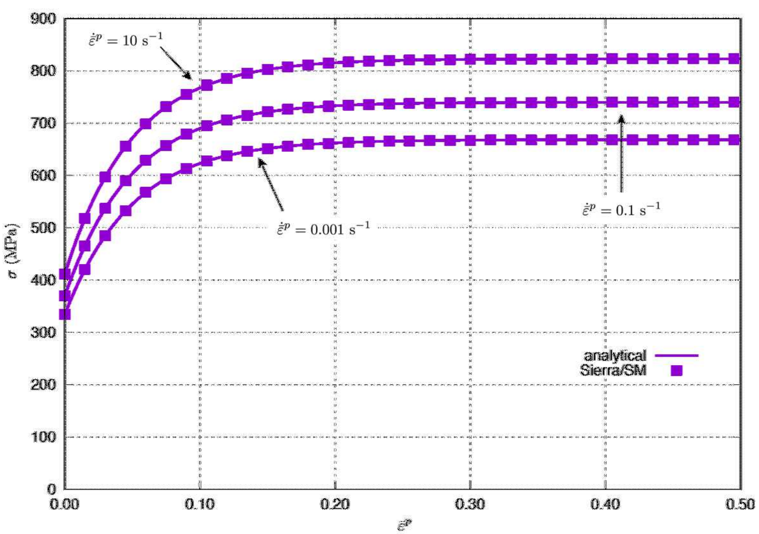
$$\bar{\sigma} = \left[\sigma_y + A\left(1 - \exp(-b\bar{\varepsilon}^p)\right)\right] g(\dot{\bar{\varepsilon}}^p)$$

$$g(\dot{\bar{\varepsilon}}^p) = 1 + \sinh^{-1}\left[\left(\frac{\dot{\bar{\varepsilon}}^p}{g}\right)^{1/m}\right]$$

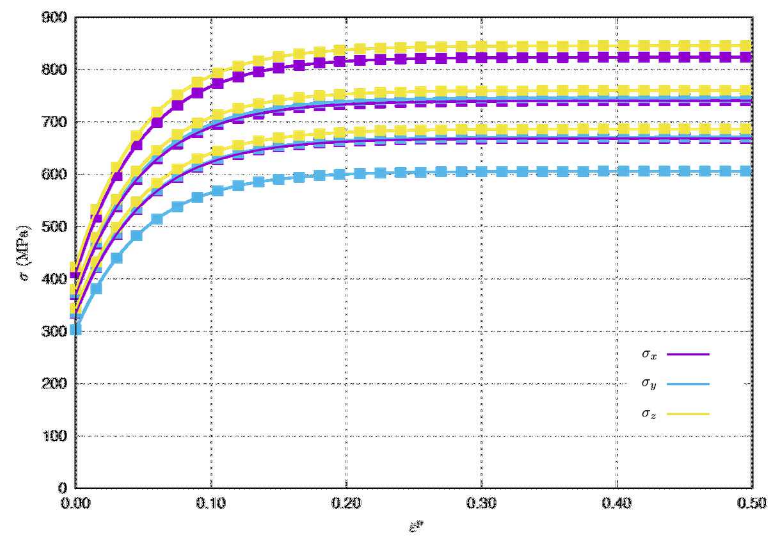
$$\sigma_y = 200 \text{ MPa}$$
$$A = 200 \text{ MPa}$$
$$b = 20$$
$$g = 0.210 \text{ s}^{-1}$$
$$m = 16.4$$



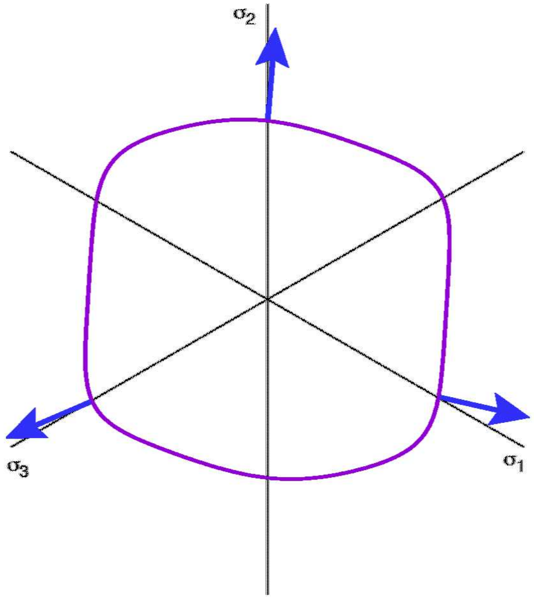

## Hosford (isotropic)

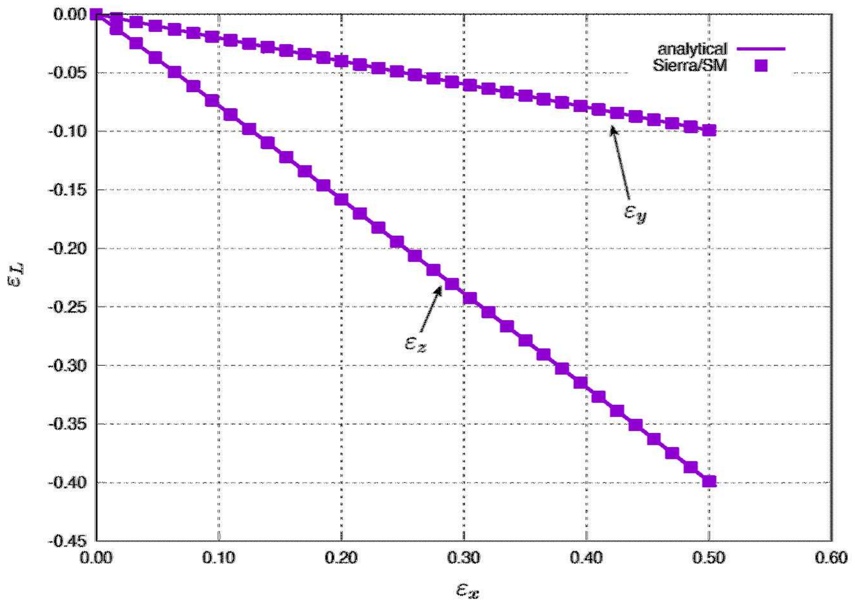


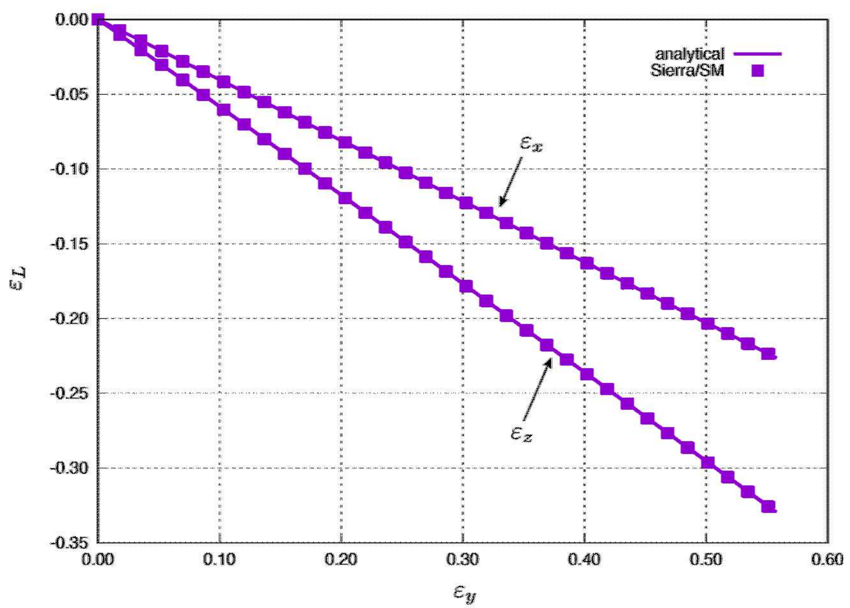

## Yld2004-18p (anisotropic)

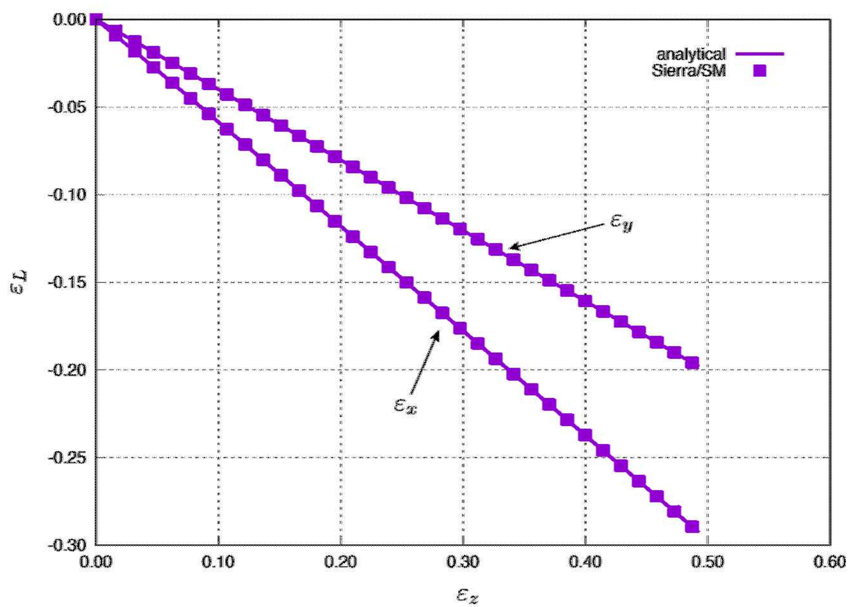


# Lateral Strains for Anisotropy



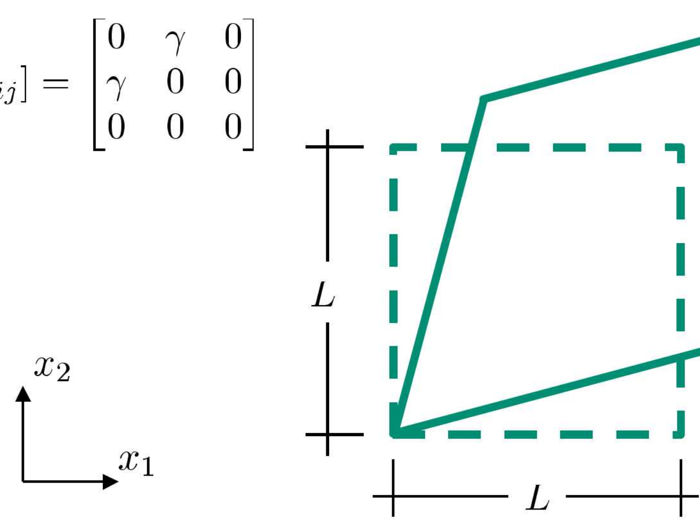



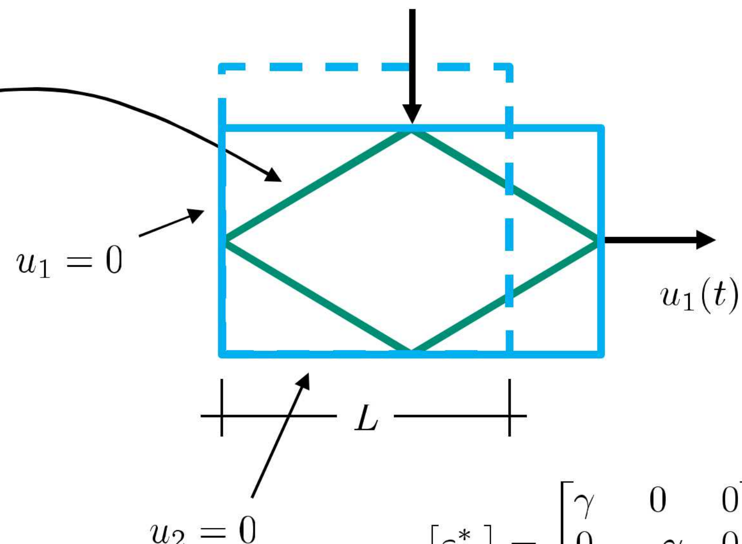



# Pure Shear

$$[\varepsilon_{ij}] = \begin{bmatrix} 0 & \gamma & 0 \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$




Rotate 45°




$$[\varepsilon^*_{ij}] = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

hardening model

$$\gamma(t) = \frac{\bar{\sigma}(\bar{\varepsilon}^p(t))}{2\mu\,\phi(\bar{\boldsymbol{\tau}})} + \bar{\varepsilon}^p(t)\frac{\partial\phi}{\partial\sigma_{12}}$$

yield surface

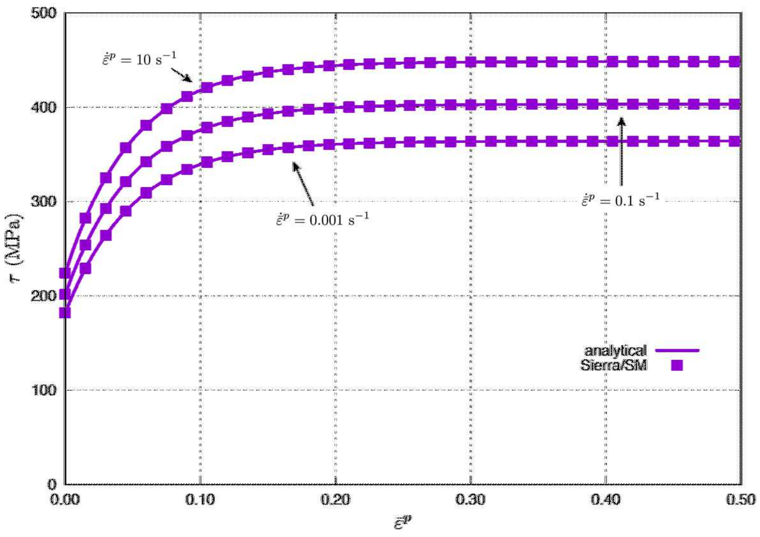
$$\lambda(t) = \exp(\gamma(t))$$

$$u_1(x_1, x_2; t) = \frac{1}{2}\left[\left(\lambda(t) + \lambda(t)^{-1} - 2\right) x_1 + \left(\lambda(t) - \lambda(t)^{-1}\right) x_2\right]$$

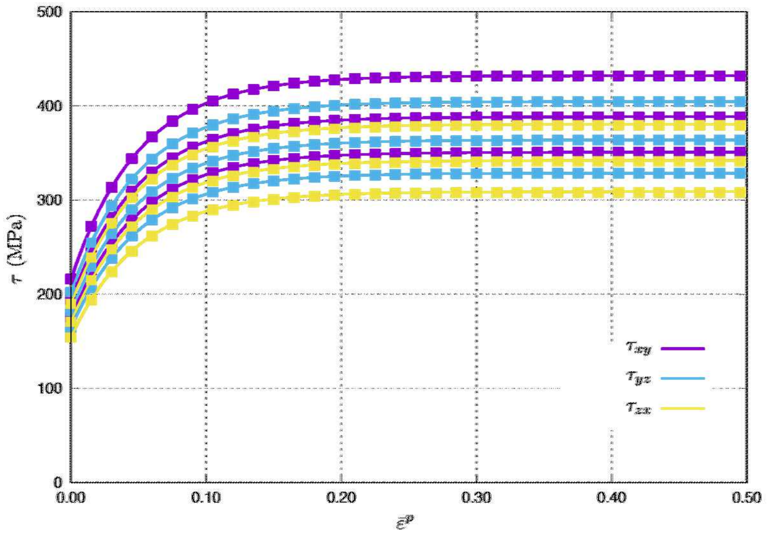
$$u_2(x_1, x_2; t) = \frac{1}{2}\left[\left(\lambda(t) - \lambda(t)^{-1}\right) x_1 + \left(\lambda(t) + \lambda(t)^{-1} - 2\right) x_2\right]$$

# Pure Shear

### Hosford (isotropic)




### Yld2004-18p (anisotropic)




***Very*** complicated boundary/initial conditions give simple results

# Conclusions

- Constitutive models that are used in modeling and simulation to support decision making require <u>extensive</u> verification and testing
- Verification is difficult
  - Show that a model is <u>not</u> verified
  - Test the algorithm -> test the implementation
- Test to fail
  - Avoid positive reinforcement
- Get it right, then make it fast
- Generate a lot of results
- ***Documentation and peer review***