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Exploring Uncertainties in Multi-Input-Multi-Output (MIMO) Testing

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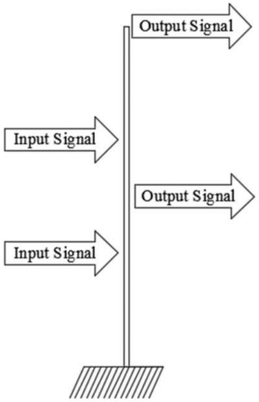

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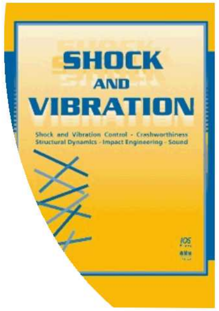
SCHOOL OF ENGINEERING

Contents

1. Introduction

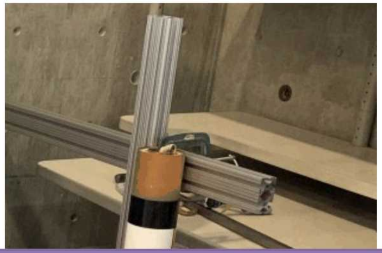
MIMO Background

- MIMO is an integral part of our engineering community today.
- It came to practice in the 60s as an accepted tool in laboratory settings (possibly somewhat developed before.)
- Most recently, the SAVIAC MIMO Recommended Practice Committee has begun discussing how to standardize best practices for MIMO experiments, which could become a guideline or standard, as there currently is none, which can make it difficult to establish what makes a good experiment.



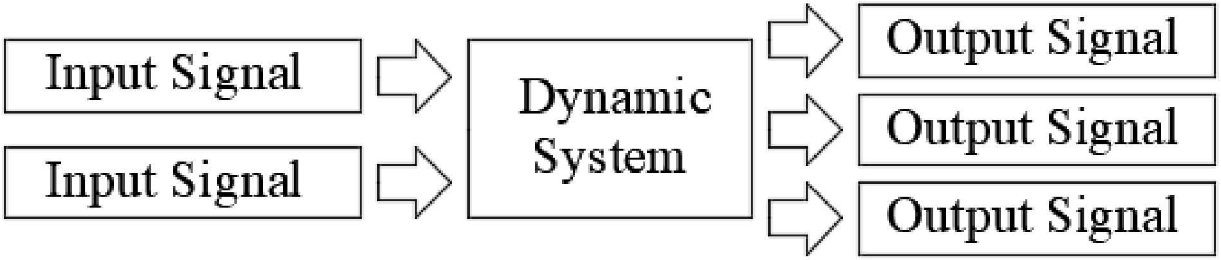
Challenges in Dynamics Testing

- Aerodynamics

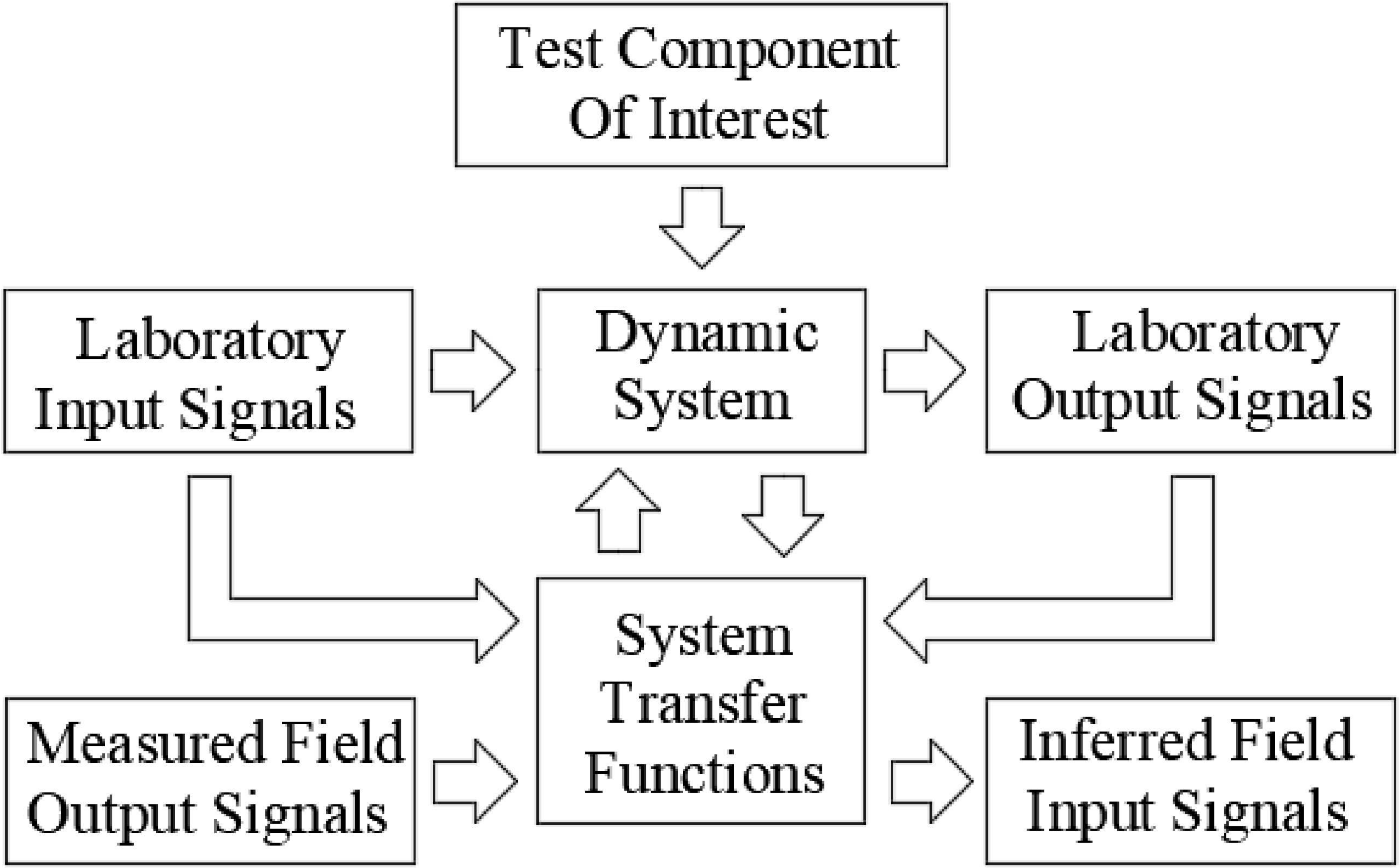


Good data is an essential foundation to any experiment. An understanding of how to accurately and efficiently perform MIMO tests improves the design, safety, operations, and durability of dynamic systems.

- Automotive

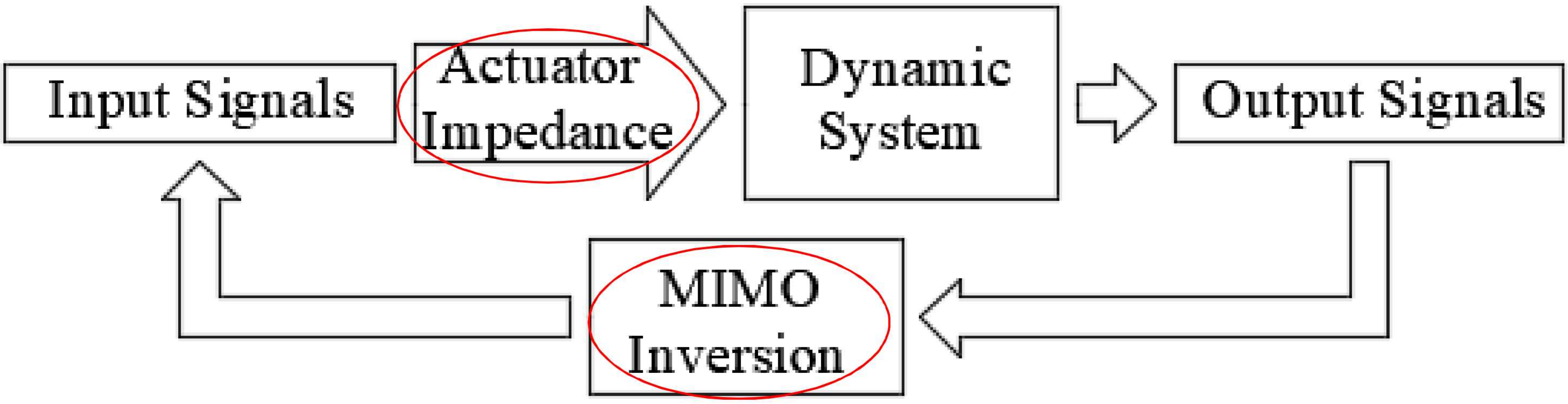


Experimental Determination of Input Loads



2. Methodology

Tests Performed

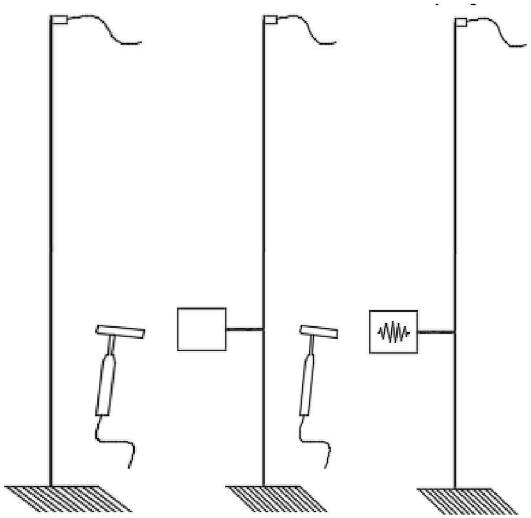


Two primary case studies are examined, each investigating a critical step in the MIMO process.

- First is the manner of actuating the member, as this changes the dynamic properties.
- Second is the inversion process used to derive inputs from measured outputs.

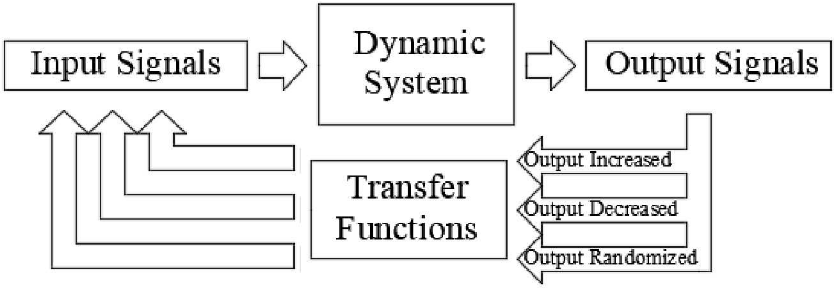
Experimental and Simulation Approaches

Experimental work: Actuator Effects



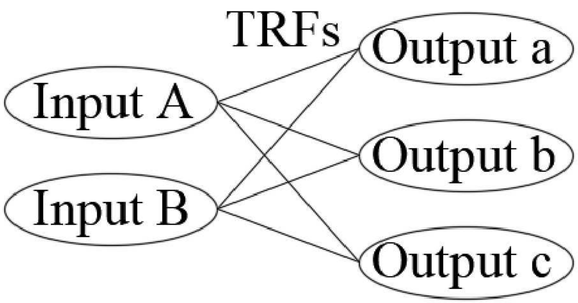
- To investigate the effects of actuator attachment on data collection, three experiments were run with three different actuator scenarios.
- Transfer functions were found for each case, and these were used to compare dynamic properties.

Simulation work: Inversion Process



- A virtual two input two output experiment was simulated
- Artificial error was added to the measured output
- The effects of the error in the input estimation was quantified.

Transfer Function Calculation



Transfer functions (TRFs) relate the input at one location to the output at another. This function was generated from the test data using the equation (frequency domain):

$$TF = \frac{cpsd(In,Out)}{asd(In)}$$

where

- TF = Transfer Function of one output with respect to one input,
- In = Measured Input Signal,
- Out = Measured Output Signal,
- $cpsd$ = Cross-power spectral density of two variables,
- asd = Auto-spectral density of a variable.

Example of a MIMO System

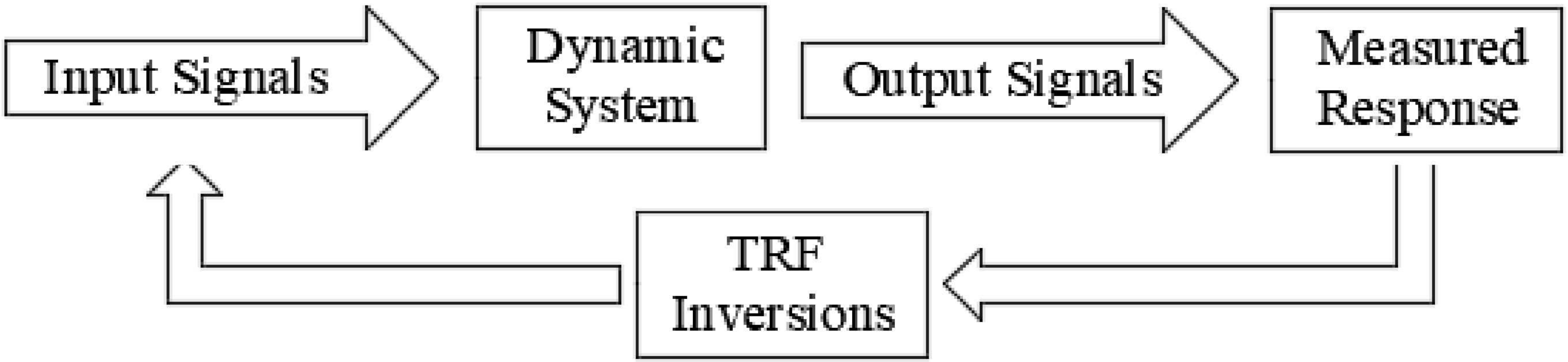
A two input by three output MIMO system in matrix form:

$$\begin{bmatrix} TF_{1,1} & TF_{1,2} \\ TF_{2,1} & TF_{2,2} \\ TF_{3,1} & TF_{3,2} \end{bmatrix} * \begin{bmatrix} Input\ 1 \\ Input\ 2 \end{bmatrix} = \begin{bmatrix} Output\ 1 \\ Output\ 2 \\ Output\ 3 \end{bmatrix}$$

$TF_{A,B}$ = Transfer Function of output A with respect to input B,
$Input\ A$ = Input signal A in power spectral density domain,
$Output\ A$ = Output signal A in power spectral density domain,

* denotes matrix multiplication.

MIMO Inversion



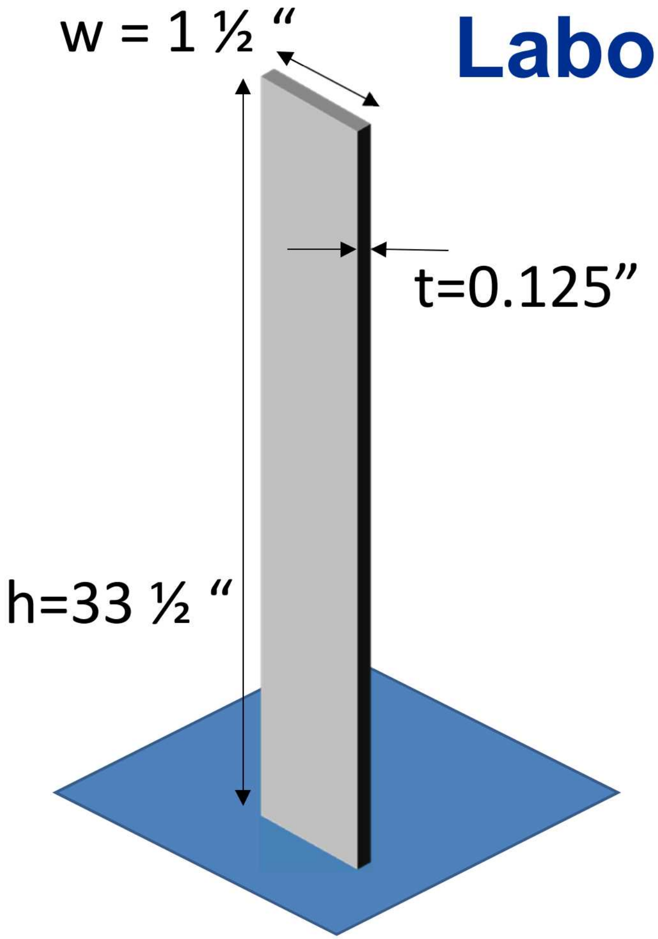
Inferring inputs to a system based upon measured responses is done with the following example equation:

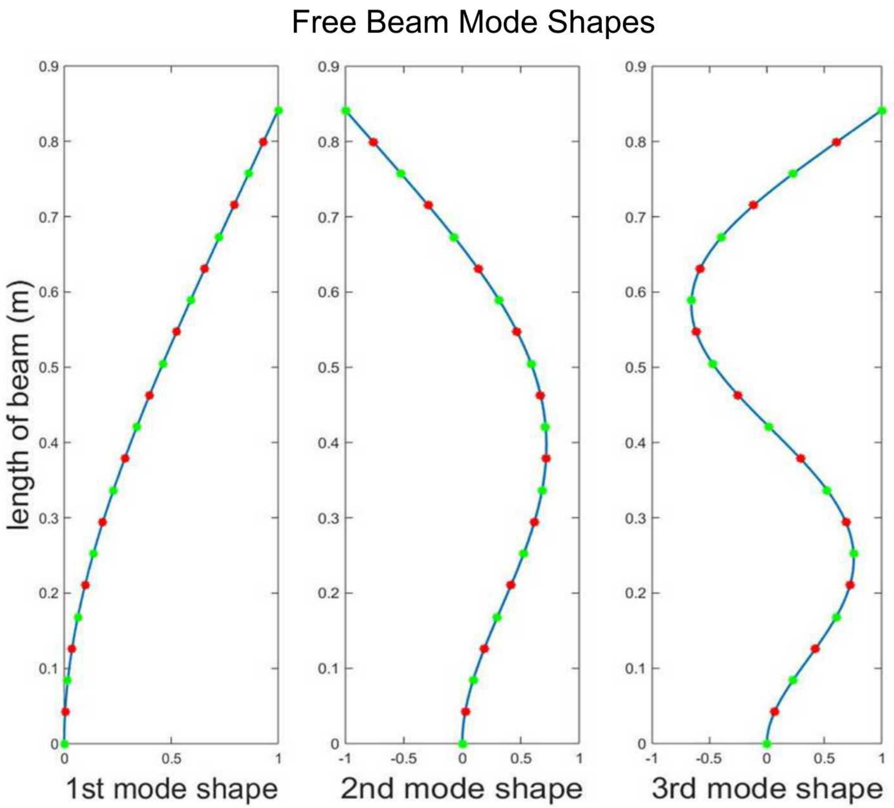
$$[In]_{2x1} = [TF]_{3x2}{}^{+}[Out]_{3x1}$$

In = Input Signal Matrix,
TF = Transfer Function Matrix,
Out = Output Signal Matrix,
$^{+}$ denotes the Moore-Penrose Pseudo-Inverse of a matrix

3. Actuation Method Test

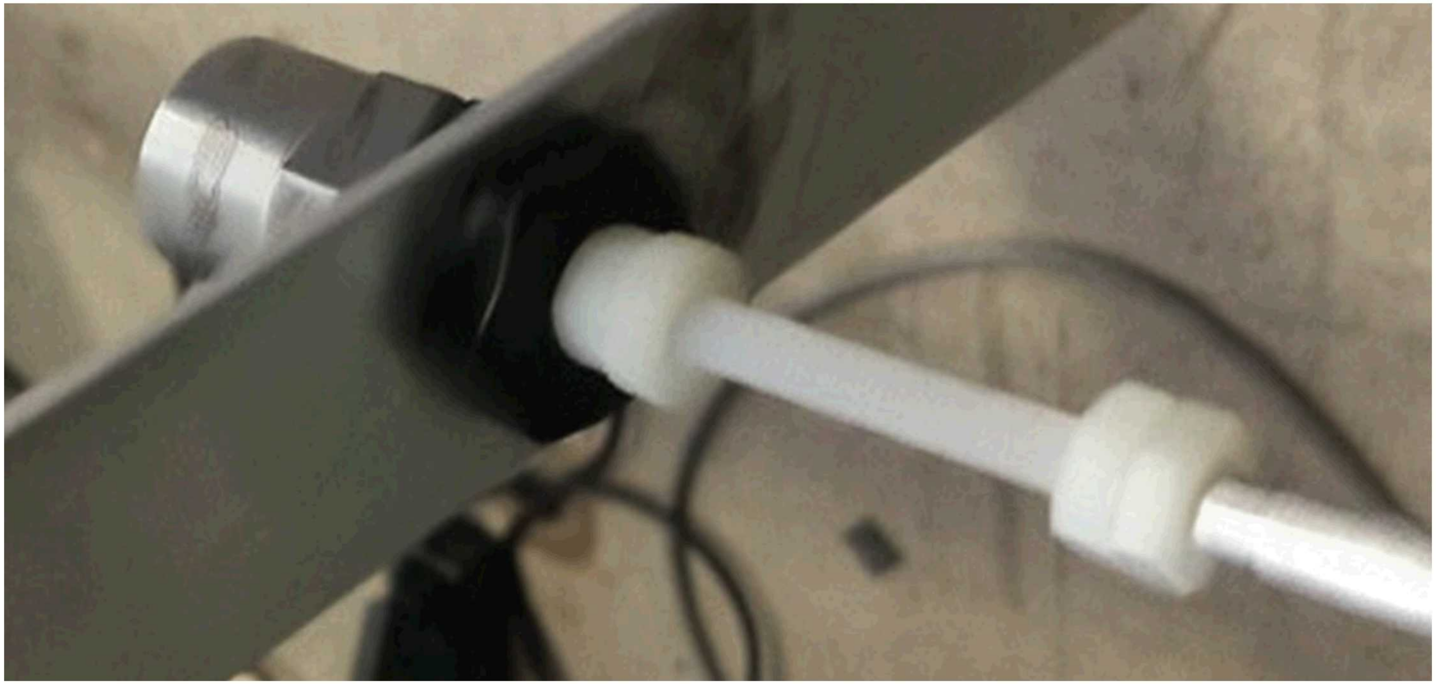
Laboratory Specimen





- The tests performed all examine a steel cantilever beam affixed to the ground.
- The acceleration response is measured at the tip.
- Input is applied from the 1/3rd mark.
- Transfer functions relating these two locations used to quantify dynamic properties.

Actuation Connection to Specimen

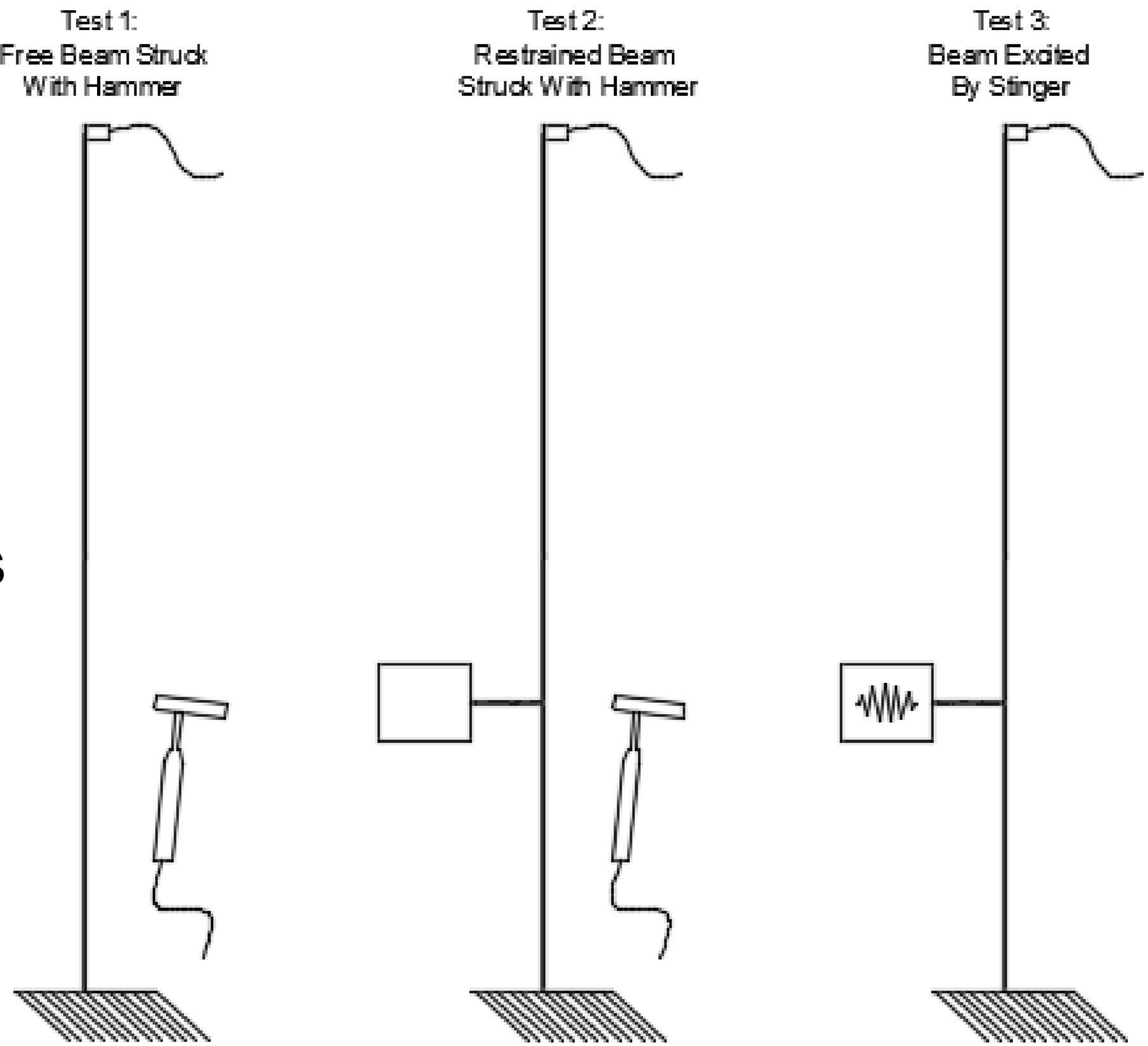


Objective: To quantify how the addition of the actuator stinger changes the system's dynamic properties.

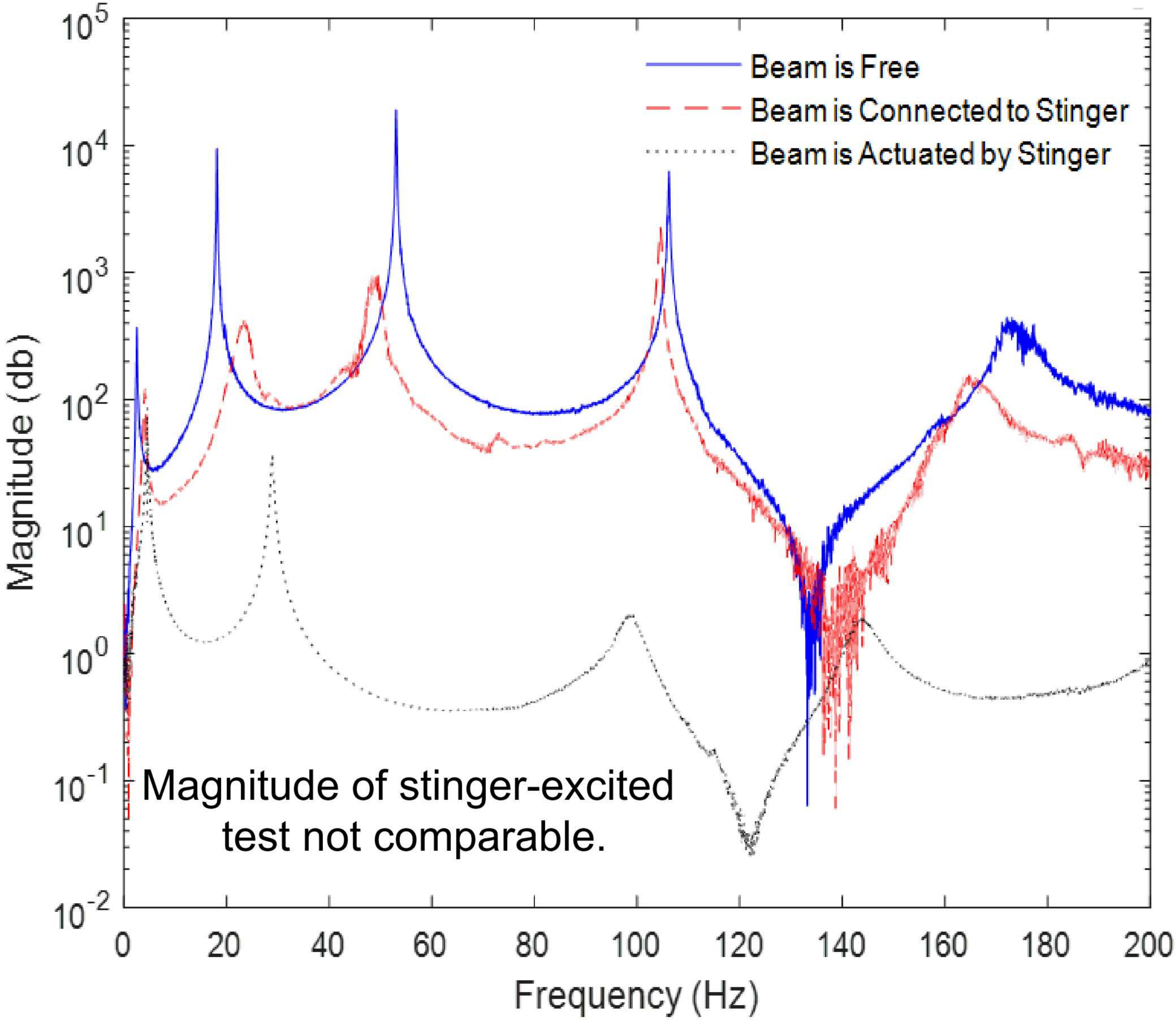
Actuator Setups

To investigate actuator impact, TRFs were generated by:

- Striking with an impact hammer
- Striking with an impact hammer while the beam was restrained by an exciter at that location
- Actuating the beam via the exciter.



Actuator Attachment Effects



- Applying the stinger dampened system and shifted peaks.
- Peak shifts were more pronounced when the exciter ran, and the fourth mode was removed.

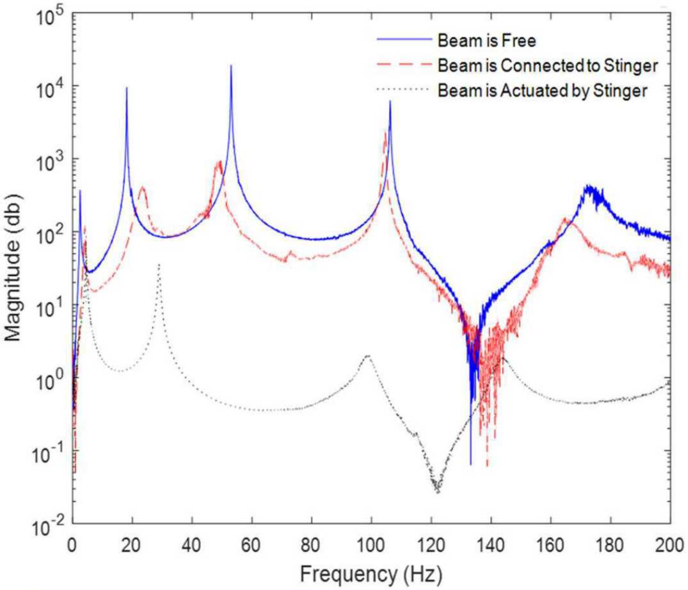
Actuator Attachment Effects

Mode Frequencies (Hz)

Mode	1	2	3	4	5
Free	2.5	17.9	52.9	106.2	172.5
Attached	4.2	23.3	48.8	104.6	165
Excited by Stinger	4.6	28.8		98.8	143.8

Mode Frequency Percent of Free Beam's

Mode	1	2	3	4	5
Attached	168	130.2	92.2	98.5	95.6
Excited by Stinger	184	160.9	??	93	83.4

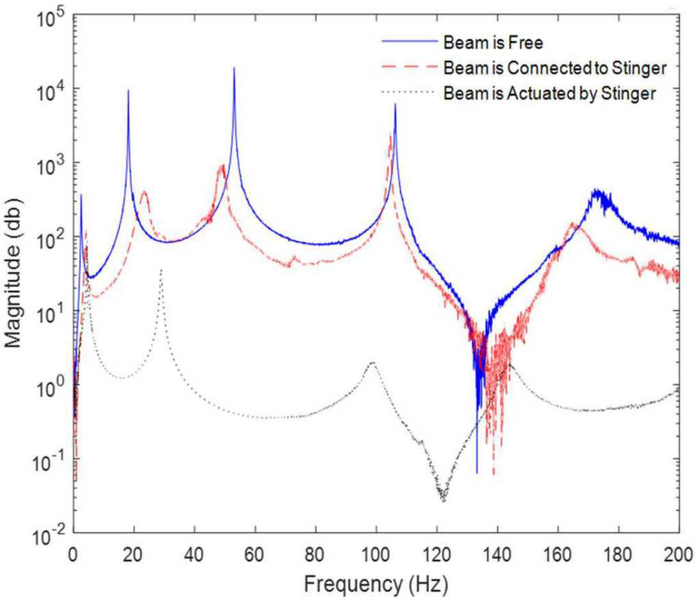


- Attaching the stinger clustered modes in toward around 30 Hz.
- This trend was emphasized when the actuator excited the beam.
- The third mode's absence could then be due to converging with the second mode based on experimental data.

Actuator Attachment Effects

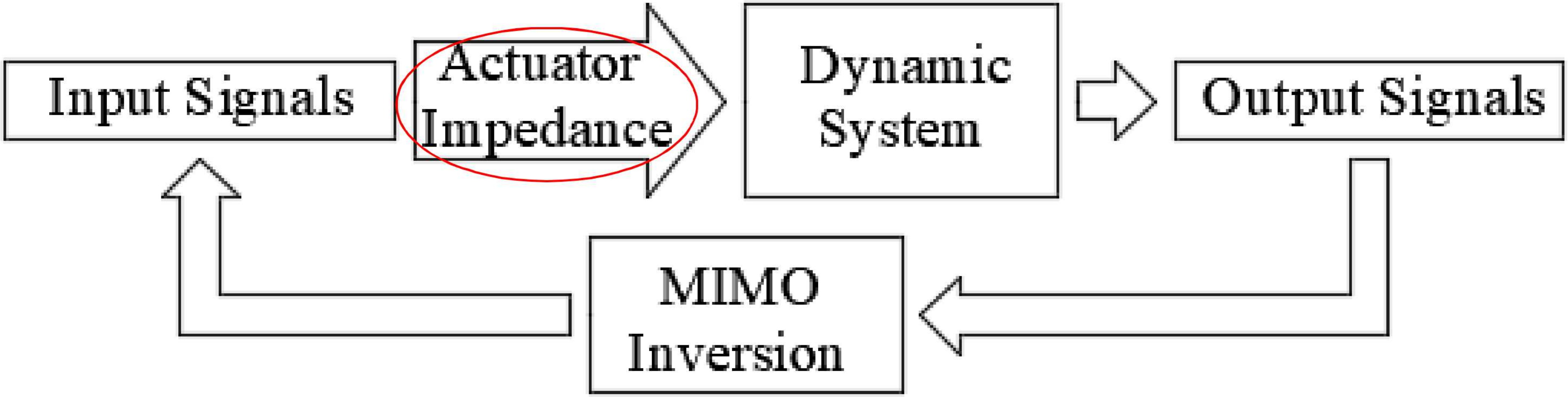
Approximate Damping (%)

Mode	1	2	3	4	5
Free	14	2.2	0.9	0.3	1.6
Attached	12.5	6.8	2.5	0.3	1.7
Excited by Stinger	4.2	1.2	**??**	1.9	2.2



- Attaching the stinger generally increased the damping of the modes.
- For modes 1 and 2 the actuator exciting the beam brought damping down.
- Conversely, modes 4 and 5 were increased.
- Once more, the third mode is a point of interest.

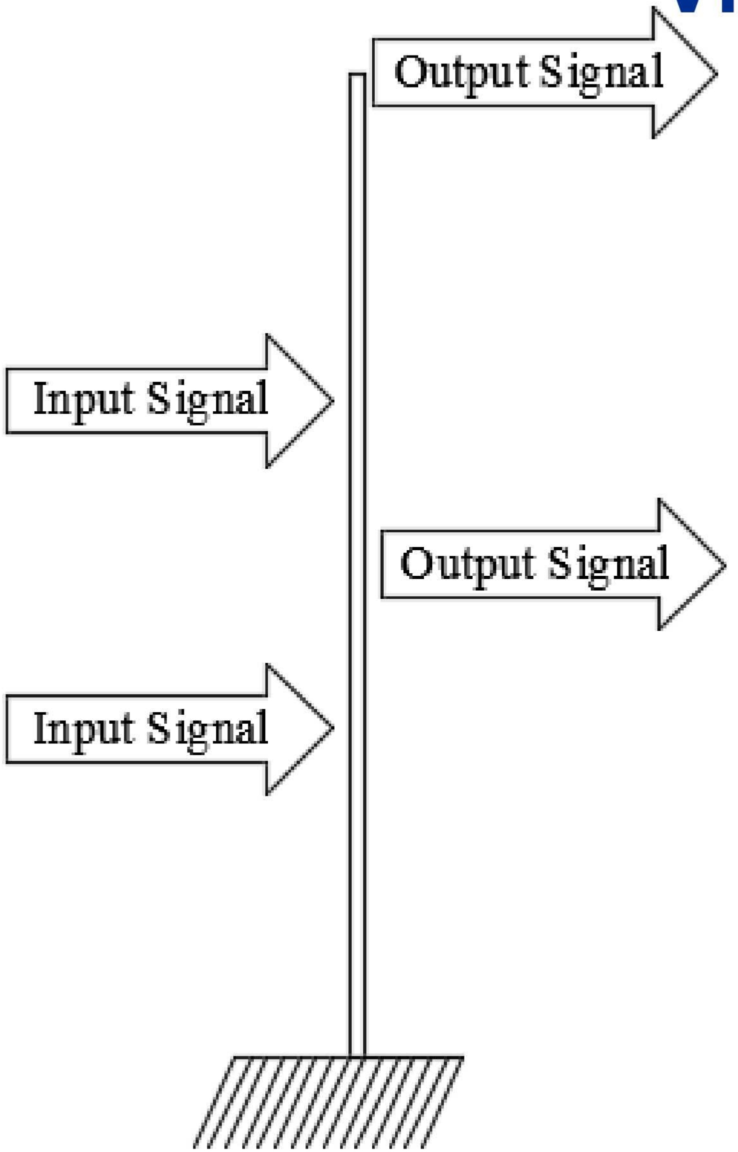
Actuation Method Test Conclusions



- The manner of actuating a structure can have pronounced effects on determining its dynamic properties.
- It is important to understand how the system is changed by simply trying to induce a vibration, and how that impact results.
- Future stages of this research will collect experimental data using a 3D camera to better quantify the effect of the actuator.

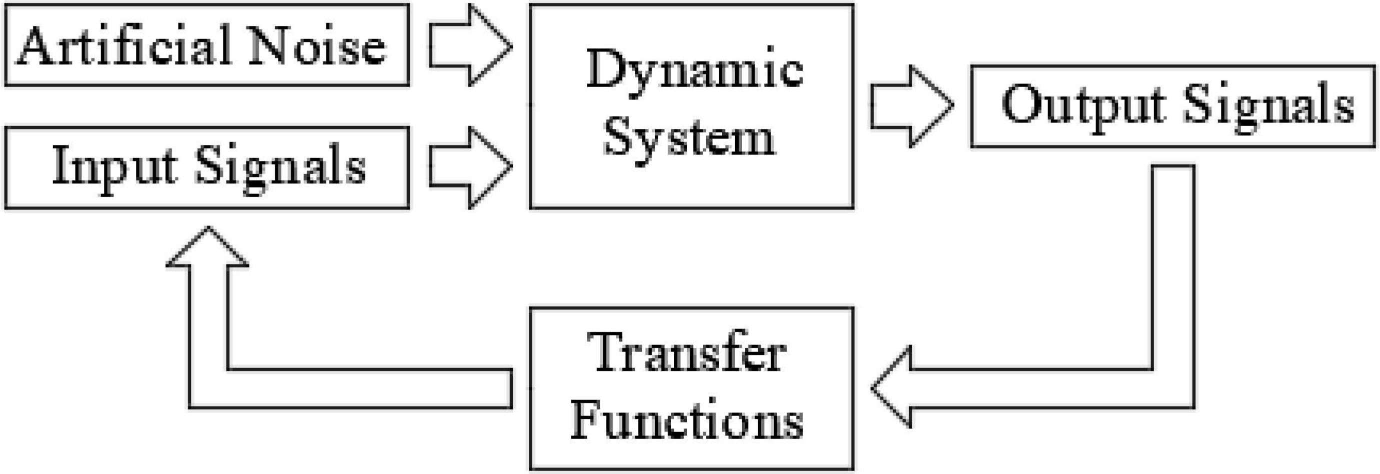
4. MIMO Inversion Test

Virtual Model



- The system was modeled as a free Euler-Bernoulli cantilever beam.
- Inputs were placed at 1/3rd and 2/3rd the height.
- Output responses were taken at the middle and tip of the beam.
- Band limited white noise between 0 and 200 Hz was generated as input.

MIMO Inversion Test Methods



- To control all factors, this test was performed virtually modeling a simple cantilever beam.
- Transfer Functions were derived from the state space model of a two input- two output system.
- An input was generated, and an artificial error was injected into the resultant output. From this, the MIMO inversion process was performed.

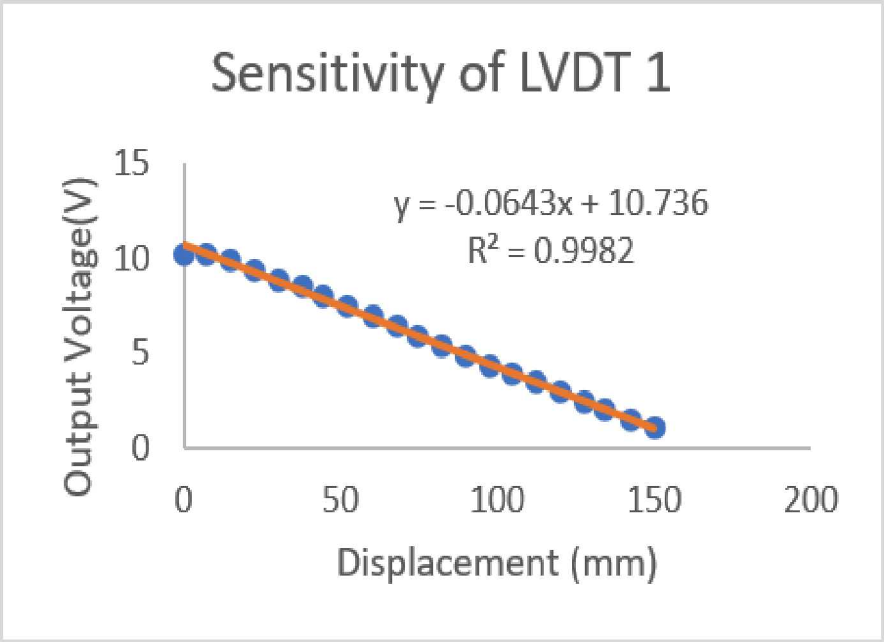
Artificial Error Generation

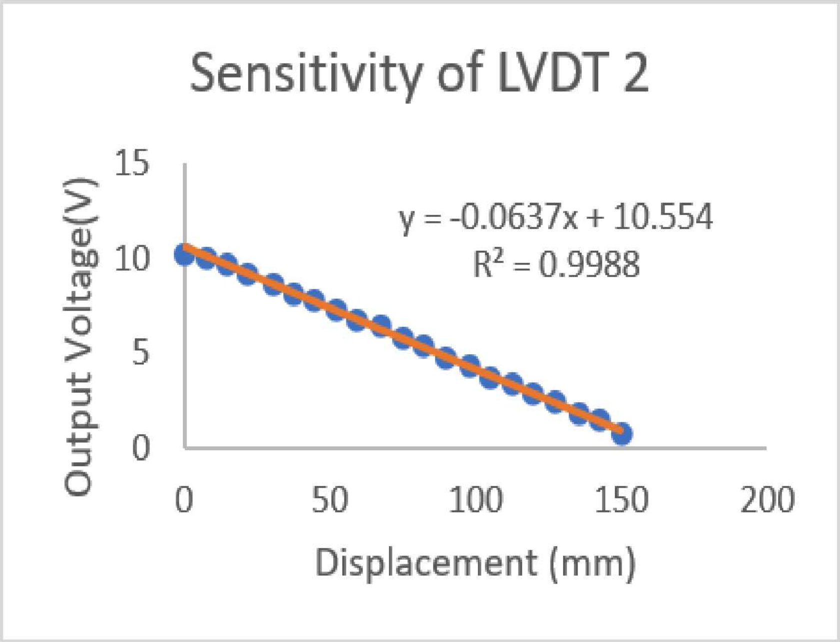
- To estimate an artificial error, a case study was performed with Linear Variable Differential Transformer (LVDT) sensors, as representative of general sensor error.
- A digital micrometer and voltmeter were used to calibrate two LVDT sensors, and compare the manufacturer-given sensitivity of the instrument with the measured ratio of displacement to voltage.

Testing Sensor Accuracy

- The micrometer used in measuring sensitivity had a precision of 0.001 mm, while the voltmeter had a precision of 0.01 V, resulting in a maximum calculation error of 0.1%.

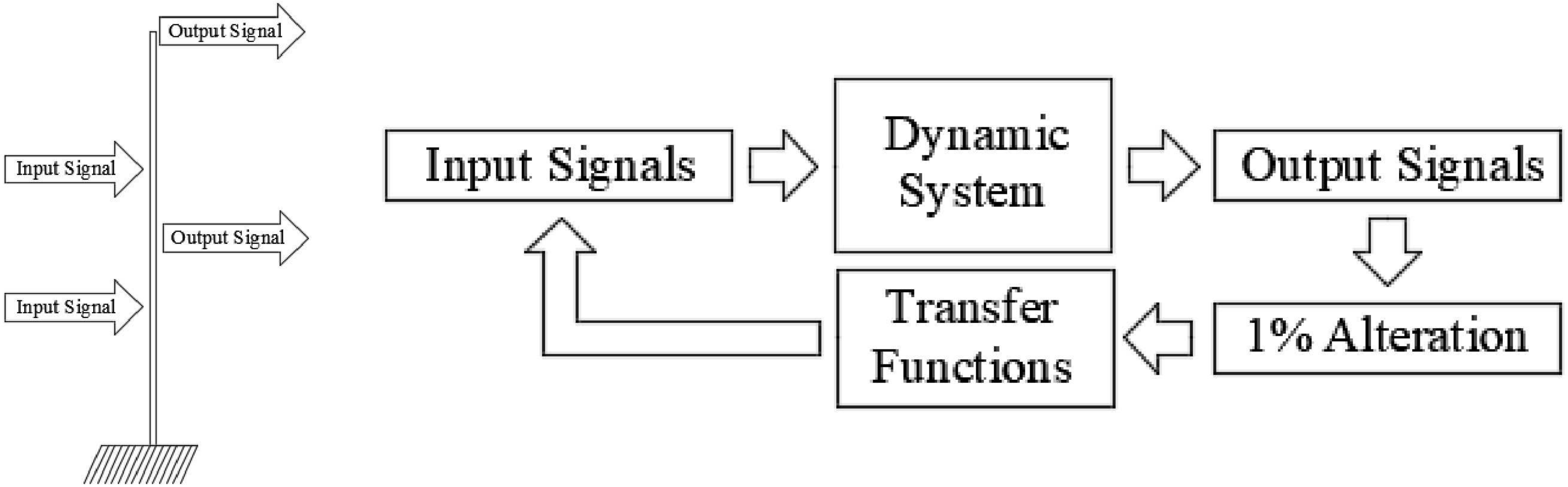
Accuracy of LVDTs





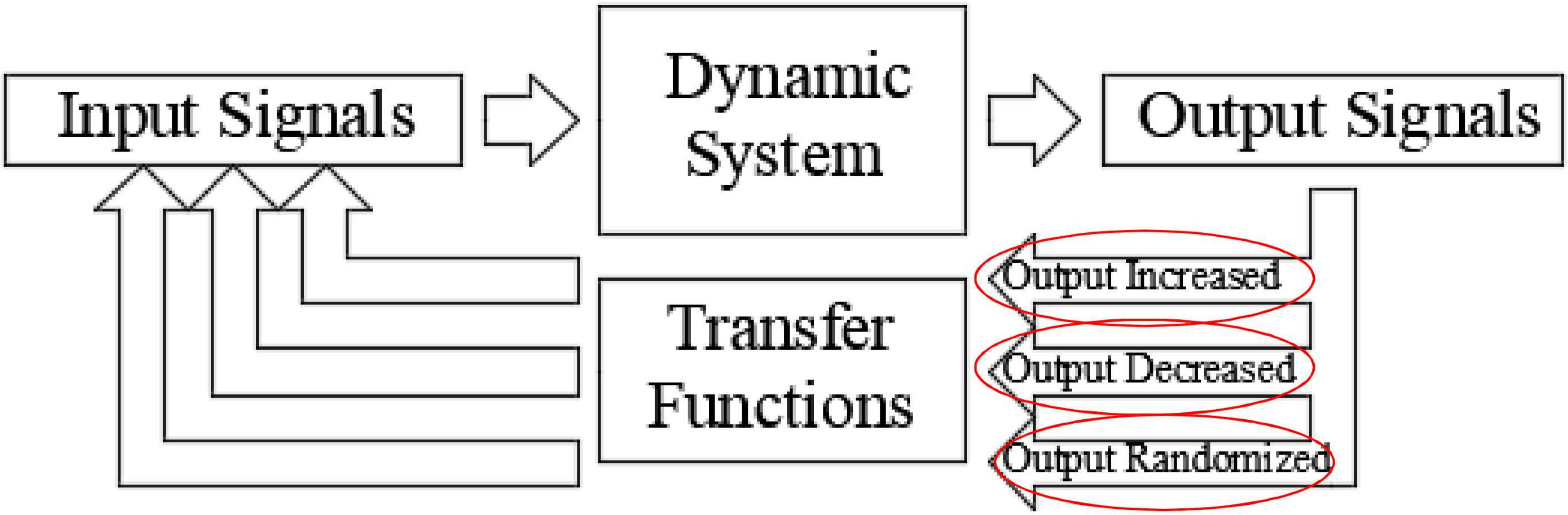
- The measured sensitivities were 64.32 and 63.74 mV/mm, while the manufacturer-given sensitivities were 65.42 and 65.21 mV/mm respectively.
- This is roughly 1% deviation from the manufacturer specifications.

Virtual MIMO Model



This error of 1% was then used as the error artificially injected into the virtual MIMO experiment, which modeled the system shown, a cantilever beam with inputs at 1 and 2/3^{rds} the height respectively, and outputs at the middle and tip of the beam.

Error Application



Three error cases were examined

- Sensor overestimates signal (Output increased by 1%).
- Sensor underestimates signal (Output decreased by 1%).
- Noise in signal (Output increased or decreased by up to 1% at each frequency).

MIMO Inversion Overall Error Results

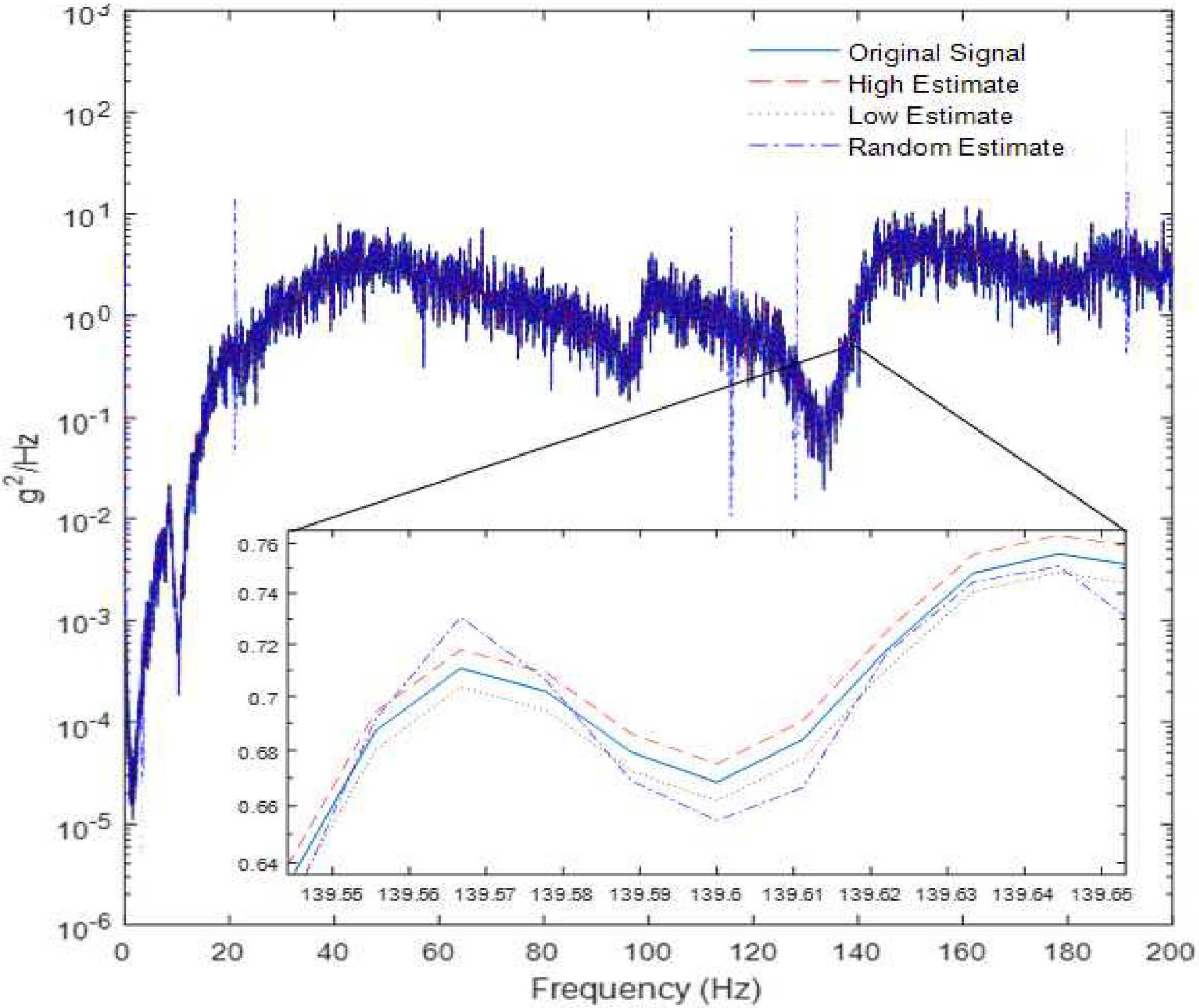
Error is calculated as the following:

$$Error = \log(\frac{Estimated\ Signal}{Measured\ Signal})$$

Error of Estimated Input		
	1/3rd Height Input	2/3rd Height Input
Sensor Underestimates	0.0044	0.0044
Sensor Overestimates	0.0043	0.0043
Sensor Randomly Perturbed	0.0135	0.0142

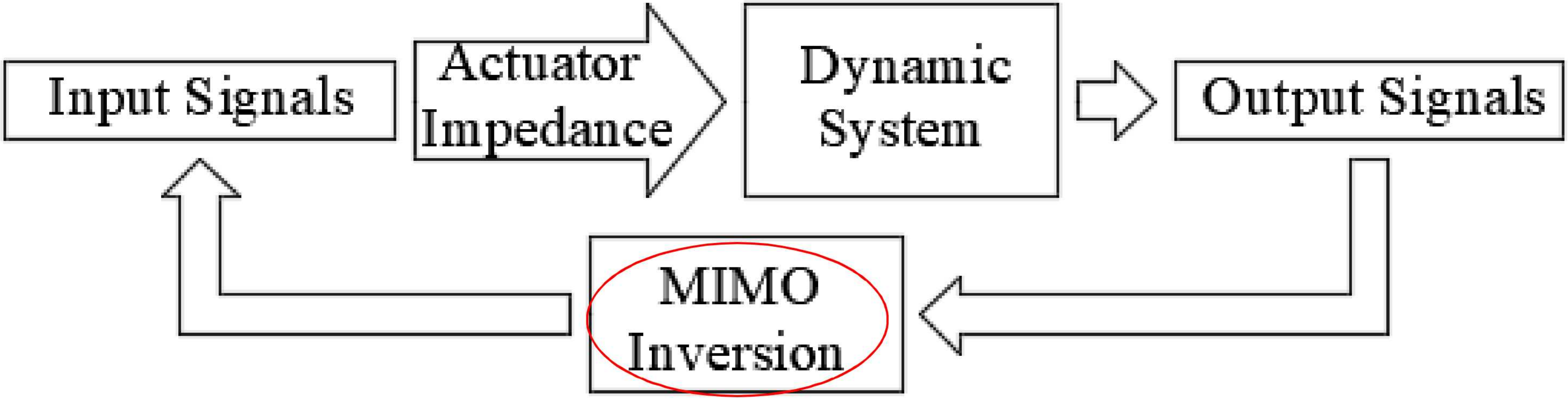
- The table shows the average absolute error of the estimated input for each case. Estimating the input with no signal alteration had no errors.

MIMO Inversion Estimation



- The graph depicts the inputs as under each condition in PSD.
- Constant errors remain linear, with a 1% deviation in estimated input per 1% deviation in output.

MIMO Inversion Test Conclusions



- Errors resulting from the MIMO Inversion process tend to be proportional to the errors in the data used.
- For well-scaled Transfer Function Matrices, no additional error is introduced.
- Calibrating equipment regularly can help limit error propagation and help enhance experiment design.
- Future direction of this research includes parametric study of the different methods in the input estimation

5. Conclusions

Conclusions

- The manner of actuating a structure can have pronounced effects on its dynamic properties. Understanding this effect on an intended test can help improve experiment design,
- A knowledge of mode shapes can help in predicting these effects.
- Calibration of equipment can help maintain accuracy and understand sensor limitations, leading to the design of better tests.
- Error in MIMO inversion is proportional to error in data used.

Acknowledgements

Sandia National Laboratories

Any Questions?