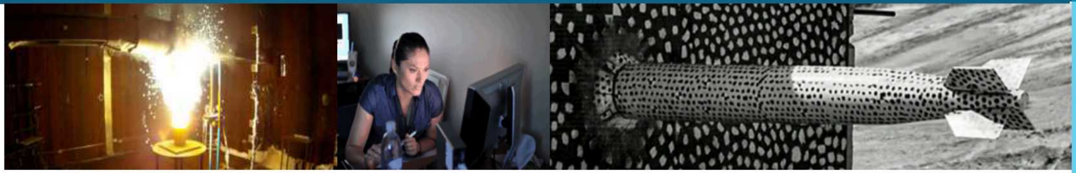
SAND2020-1386C

Nonlinear Forced Response Synthesis with Quasi-static Modal Analysis



Presented at IMAC XXXVIII on February 12, 2020 at 11:00 am

Presented by

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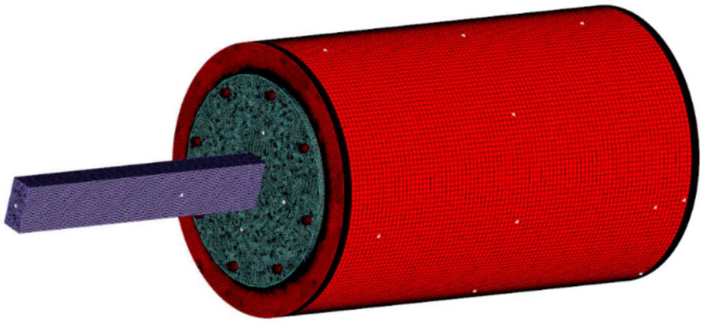
Motivation: Transfer Function Analysis

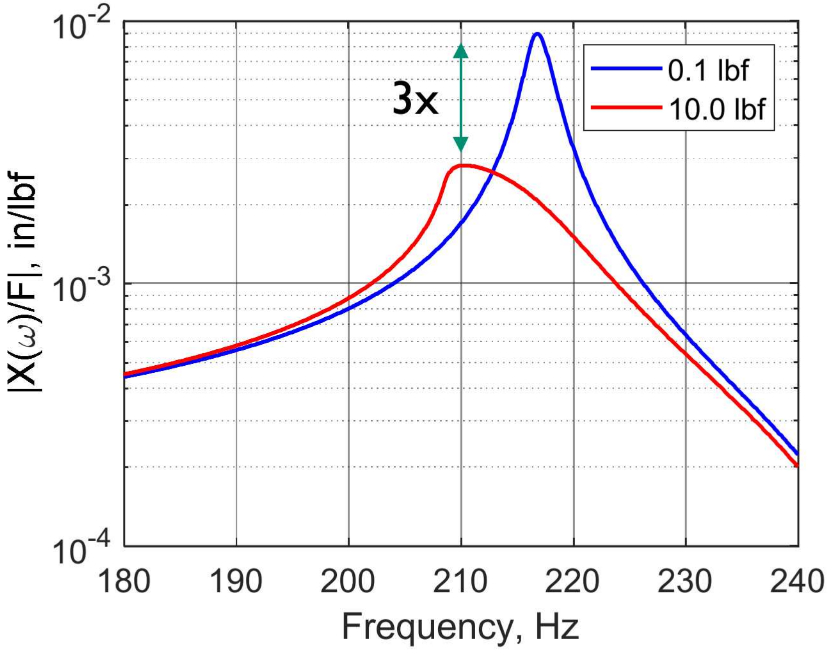
Transfer function analyses rely on linear operators that scale response to force magnitude

- Nonlinearity complicates the use of transfer functions about an operating load



Mechanical interfaces in assembled structures lead to nonlinear frictional energy loss

- Increased effective damping
- Decreased effective stiffness



Use of linear(ized) frequency response functions (FRFs) may lead to **over-** or **under**testing of structure

- Overly designed system that might compromise mission objectives
- Unexpected failures in qualification and field testing

Nonlinear Forced Response

Frequency Response Function: determines the steady-state periodic response of a **linear system** to a harmonically driven input force at various frequency lines

Nonlinear Forced Response: determines the steady-state periodic* response of a **nonlinear system** to a harmonically driven input force at various frequency lines

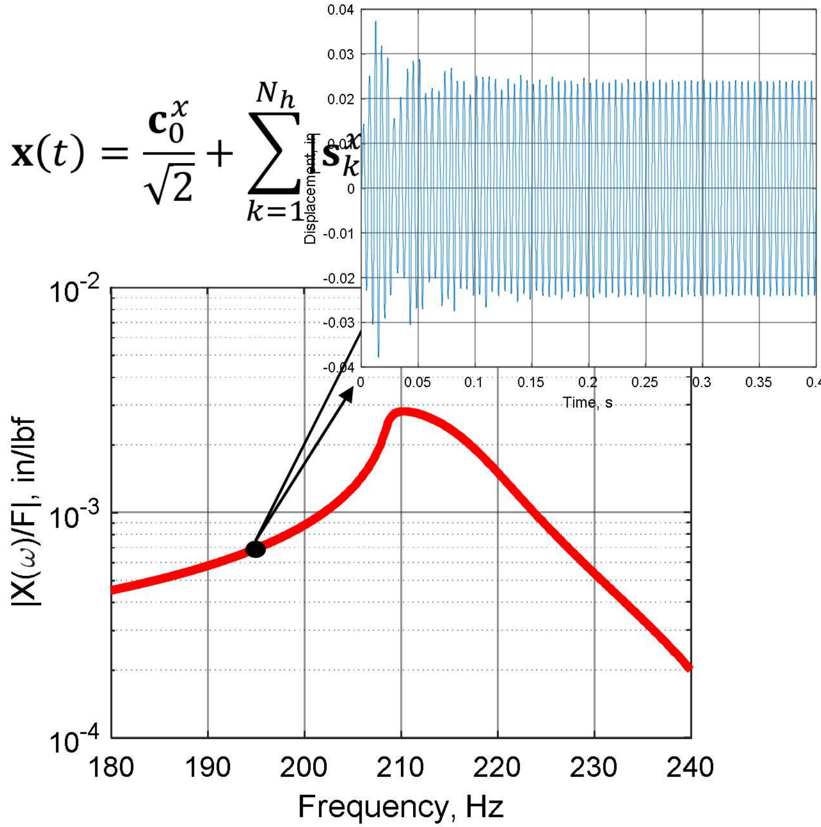
Two common approaches to calculate the nonlinear steady-state response:

1.) Direct time integration to reach steady-state conditions

2.) Path-following continuation

- 2a.) Shooting method
- 2b.) Multi-harmonic balance

$$\mathbf{x}(t) = \frac{\mathbf{c}_0^x}{\sqrt{2}} + \sum_{k=1}^{N_h} [\mathbf{s}_k^x$$

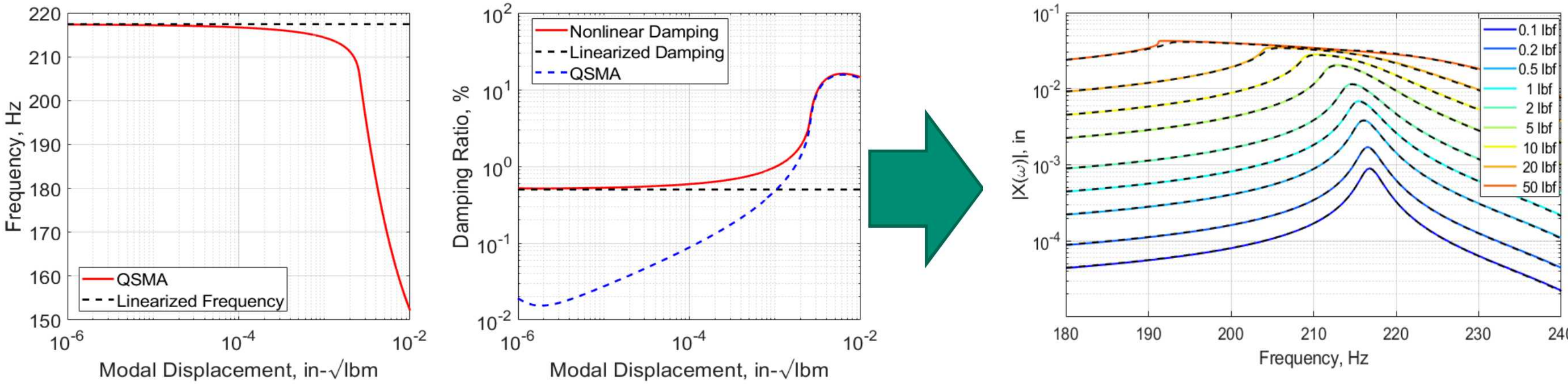


Nonlinear Forced Response

Single, nonlinear resonant mode approximation by Szemplinska-Stupnicka [1] and later extended by Krack et al. [2] using complex nonlinear modes without internal resonances

- Weakly nonlinear systems such that modal coupling terms negligible
- Modes well-separated by frequency

Objective: use single, nonlinear resonant mode approximation to synthesize the nonlinear forced response curves using nonlinear frequency and damping curves from quasi-static modal analysis – applicable to jointed structures with weak damping nonlinearity



[1] W. Szemplińska-Stupnicka, "The modified single mode method in the investigations of the resonant vibrations of non-linear systems," *Journal of Sound and Vibration,* vol. 63, no. 4, pp. 475-489, 1979.

[2] M. Krack, L. Panning-von Scheidt, and J. Wallaschek, "A method for nonlinear modal analysis and synthesis: Application to harmonically forced and self-excited mechanical systems," *Journal of Sound and Vibration,* vol. 332, no. 25, pp. 6798-6814, 2013.

Nonlinear Resonant Mode Approximation

Modal Frequency Response Function (FRF) for a linear system

$$Q_k(\omega) = \frac{\boldsymbol{\phi}_k^{\mathrm{T}}\mathbf{F}}{-\omega^2+\omega_k^2+2\mathrm{i}\zeta_k\omega_k\omega}$$

Reconstructing FRF in the physical domain as a linear superposition

$$\mathbf{X}(\omega) = \sum_{k=1}^{m} \mathbf{X}_k(\omega) = \sum_{k=1}^{m} \boldsymbol{\phi}_k Q_k(\omega)$$

Adapt the modal FRF to a nonlinear oscillator [1]

$$Q_r(\omega, Q_r) = \frac{\boldsymbol{\phi}_r^{\mathrm{T}}\mathbf{F}}{-\omega^2 + \omega_r^2(|Q_r|) + 2\mathrm{i}\zeta_r(|Q_r|)\omega_r(|Q_r|)\omega}$$

Synthesize the nonlinear forced response (NLFR) as [2]

$$\mathbf{X}(\omega) = \boldsymbol{\phi}_r Q_r(\omega, Q_r) + \sum_{k\neq r}^{m} \boldsymbol{\phi}_k Q_k(\omega)$$

[1] W. Szemplińska-Stupnicka, "The modified single mode method in the investigations of the resonant vibrations of non-linear systems," *Journal of Sound and Vibration,* vol. 63, no. 4, pp. 475-489, 1979.
[2] M. Krack, L. Panning-von Scheidt, and J. Wallaschek, "A method for nonlinear modal analysis and synthesis: Application to harmonically forced and self-excited mechanical systems," *Journal of Sound and Vibration,* vol. 332, no. 25, pp. 6798-6814, 2013.

Solve NLFR: Newton-Raphson Algorithm

Setup the complex residual equation

$$R^*(\omega, Q_r) = (-\omega^2 + \omega_r^2(|Q_r|) + 2\mathrm{i}\zeta_r(|Q_r|)\omega_r(|Q_r|)\omega)Q_r - \boldsymbol{\phi}_r^{\mathrm{T}}\mathbf{F}$$

Rearrange to be written in terms of real and imaginary parts

$$Q_r = Q_r^r + \mathrm{i}Q_r^i$$

$$\mathbf{R}(\omega, Q_r) = \begin{Bmatrix} R^r \\ R^i \end{Bmatrix} = \begin{Bmatrix} -\omega^2 Q_r^r - 2\zeta_r(|Q_r|)\omega_r(|Q_r|)\omega Q_r^i + \omega_r^2(|Q_r|)Q_r^r - \boldsymbol{\phi}_r^{\mathrm{T}}\mathbf{F} \\ -\omega^2 Q_r^i + 2\zeta_r(|Q_r|)\omega_r(|Q_r|)\omega Q_r^r + \omega_r^2(|Q_r|)Q_r^i \end{Bmatrix}$$

Damped Newton-Raphson algorithm to iteratively minimize the l^2 norm of the residual at a discrete frequency, ω

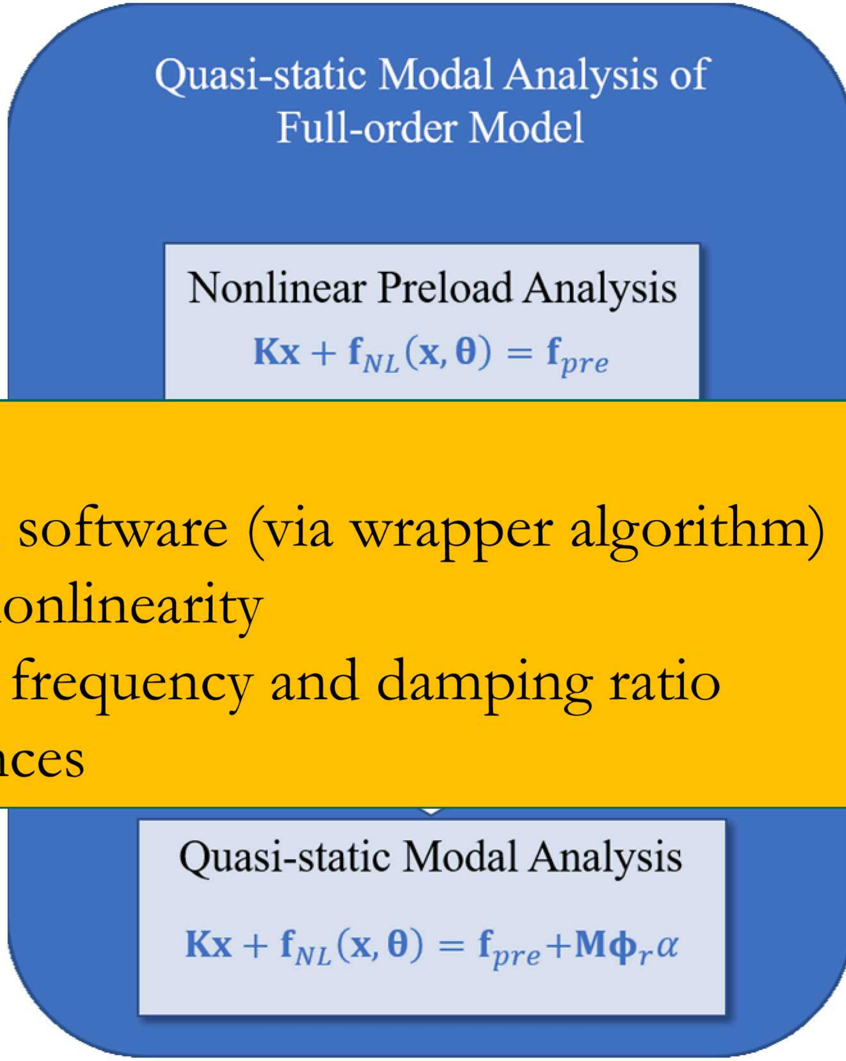
$$Q_{r,n+1} = Q_{r,n} - \varepsilon \left[\frac{\partial \mathbf{R}}{\partial Q_r}\right]_n^{-1} \mathbf{R}(\omega, Q_r) \quad \text{where} \quad 0 < \varepsilon \leq 1$$

$$\frac{\|\mathbf{R}(\omega, Q_r)\|}{\|\boldsymbol{\phi}_r^{\mathrm{T}}\mathbf{F}\|} \leq \delta_{rel}$$

Quasi-static Modal Analysis

Estimate modal amplitude dependent natural frequencies, $\omega_r(|Q_r|)$, and damping ratios, $\zeta_r(|Q_r|)$, for:

- High-fidelity finite element models with frictional contact [1]
- Reduced models with 4-parameter Iwan elements [2]



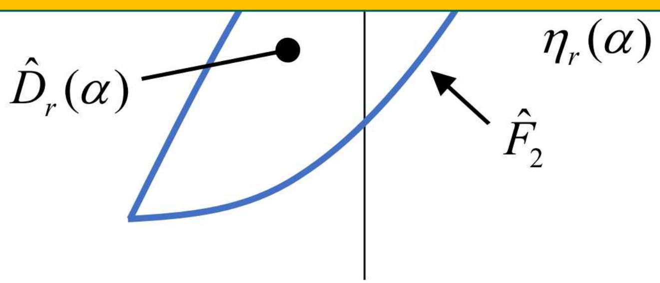
Advantages:

1.) Accessible to commercial FEA software (via wrapper algorithm)

2.) Only valid for weak damping nonlinearity

3.) Captures amplitude dependent frequency and damping ratio

4.) Ignores/filters internal resonances

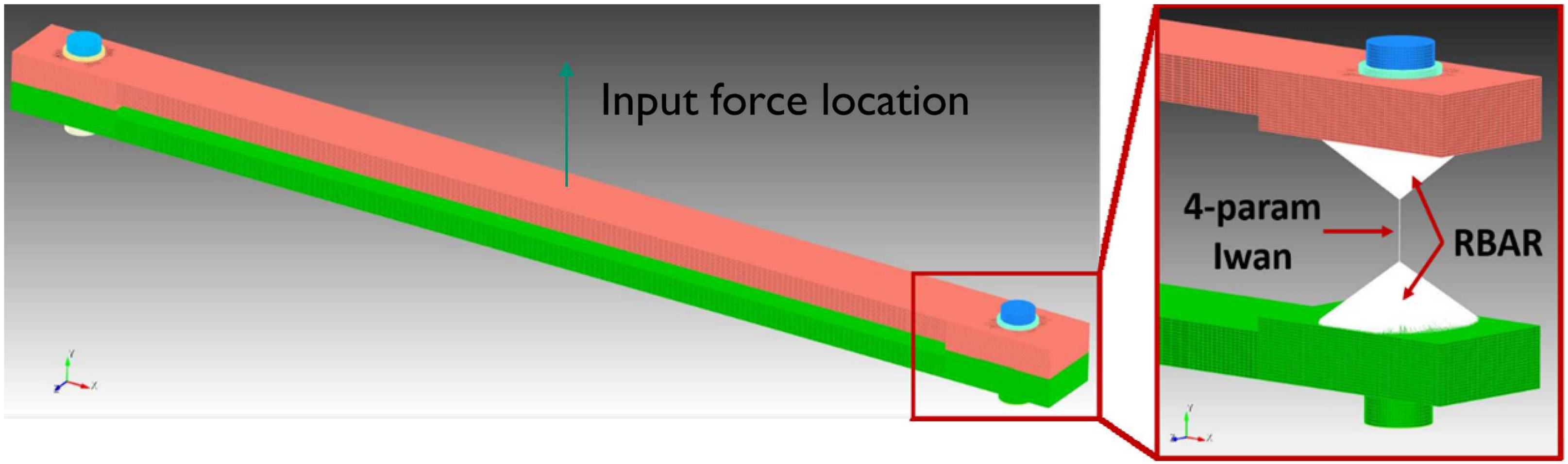


[1] E. A. Jewell, M. S. Allen, and R. M. Lacayo, "Predicting Damping of a Cantilever Beam with a Bolted Joint Using Quasi-Static Modal Analysis," presented at the ASME 2017 International Design Engineering Technical Conferences IDETC/CIE, Cleveland, OH, Aug. 6-9, 2017.

[2] R. M. Lacayo and M. S. Allen, "Updating structural models containing nonlinear Iwan joints using quasi-static modal analysis," *Mechanical Systems and Signal Processing*, vol. 118, pp. 133-157, 2019/03/01/ 2019.

Demonstration of algorithm

C-Beam Assembly with Whole Joints



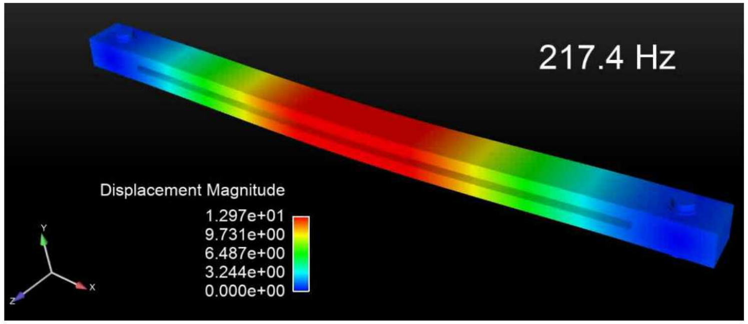
Demonstrate the accuracy of the approach on a reduced order model with two, four-parameter Iwan elements to describe the joint forces

Total DOF: 25 fixed-interface modes, and 27 static constraint modes (52 total DOF)

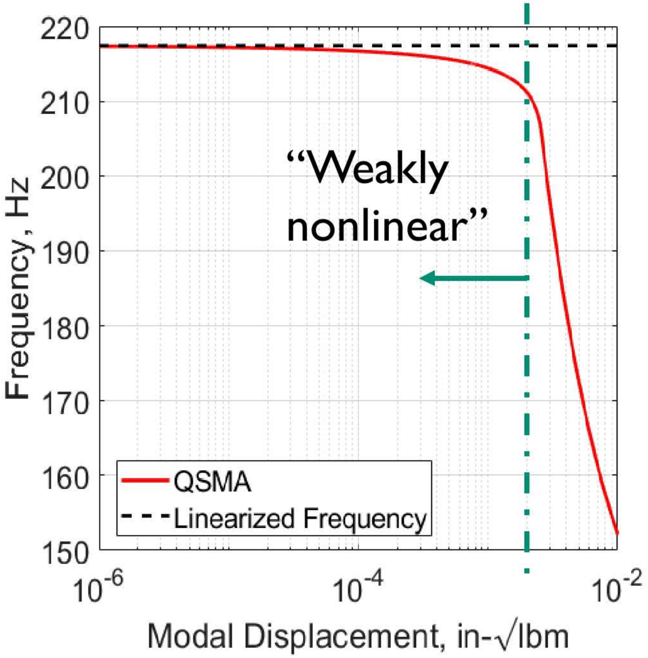
Whole Joint: Linear springs except for x-displacement direction (Iwan) to target first mode

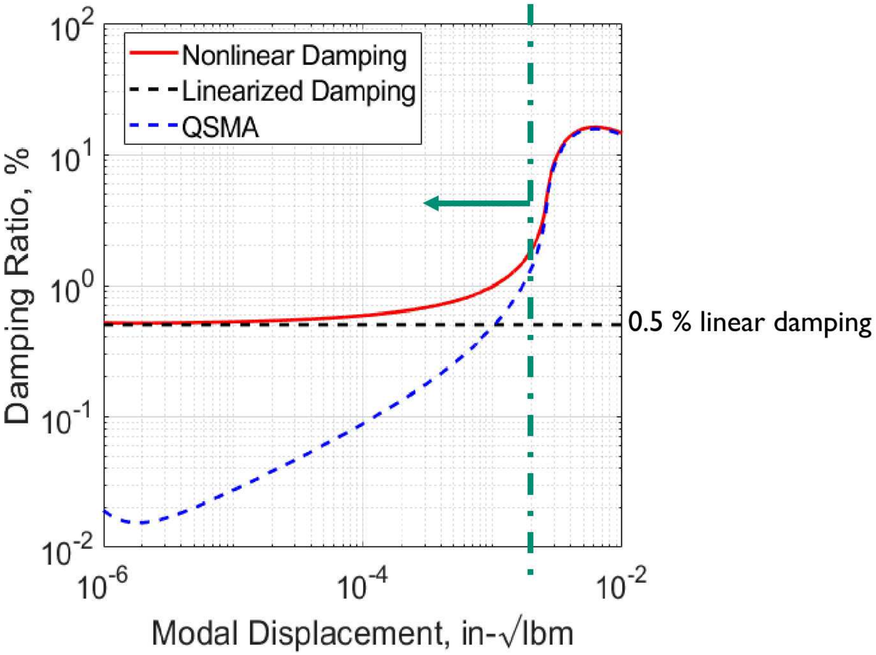
Parameter	Value
K_T	5.125E+06 lbf/in (897.5E+06 N/m)
F_S	1086 lbf (4831 N)
χ	-0.5714
β	1.783E-03

Mode 1: Quasi-static Modal Analysis



$$|Q_1| = 2 \cdot 10^{-3}\ in - \sqrt{lbm}$$



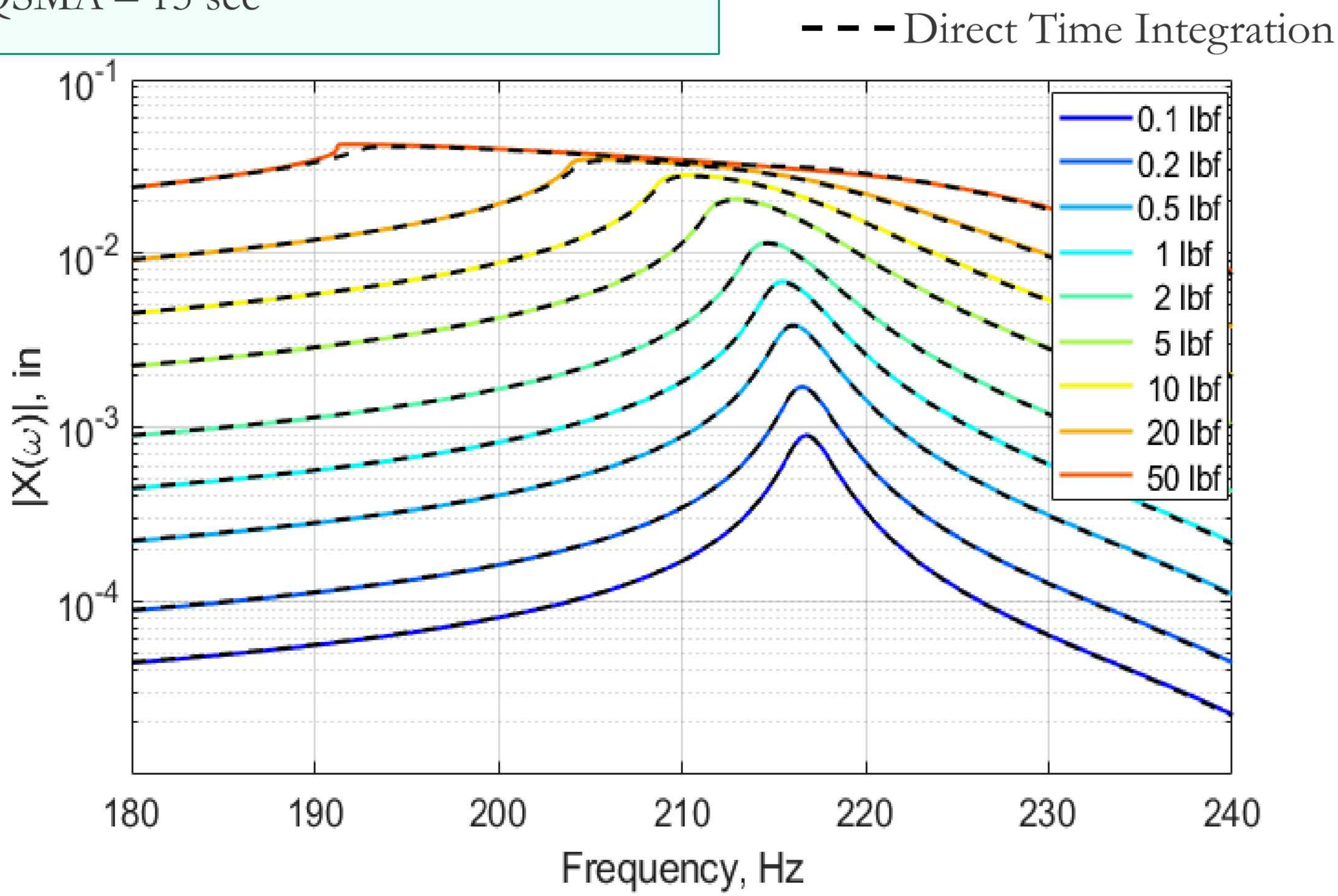


$$\zeta_r(q_r) = \frac{D(\alpha)}{2\pi\left(q_r(\alpha)\omega_r(q_r)\right)^2} + \zeta_{0,r}$$

Nonlinear Forced Response

*Direct time integration – 4.7 hours (non-optimal)

*NLFR w/ QSMA – 15 sec



Direct time integration: integrated over 250 cycles per frequency line to reach steady-state

NLFR Correlation

Modified Frequency Response Assurance Criterion (mFRAC):

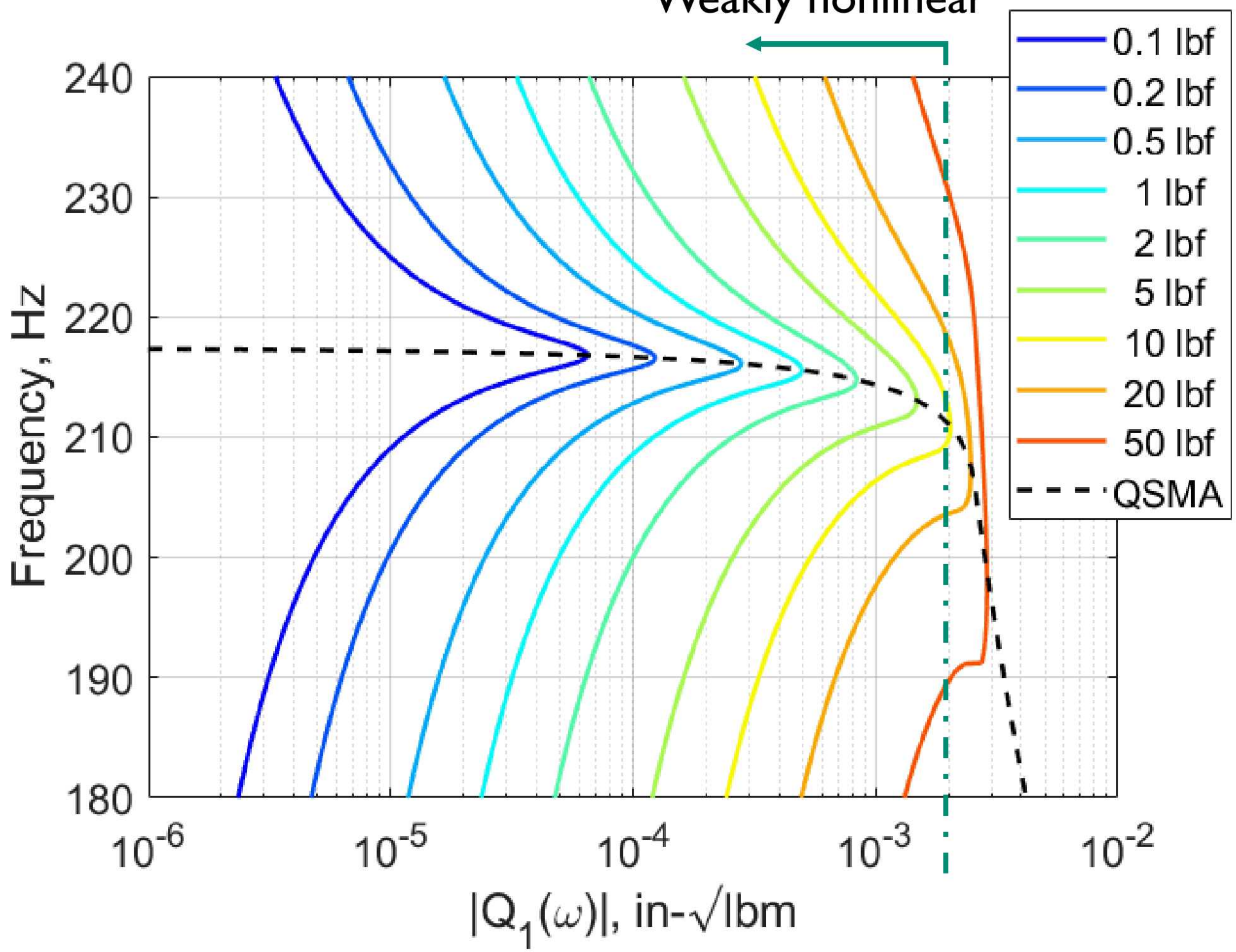
$$mFRAC = \frac{\sum_{\omega=\omega_1}^{\omega_2} |X_{\boldsymbol{i}}|_{NLFR}(\omega)|X_{\boldsymbol{i}}|_{DT}(\omega)}{\sum_{\omega=\omega_1}^{\omega_2} |X_{\boldsymbol{i}}|_{NLFR}(\omega)|X_{\boldsymbol{i}}|_{NLFR}(\omega)\sum_{\omega=\omega_1}^{\omega_2} |X_{\boldsymbol{i}}|_{DT}(\omega)|X_{\boldsymbol{i}}|_{DT}(\omega)}$$

Force Amplitude	**0.1 lbf**	**0.2 lbf**	**0.5 lbf**	**1.0 lbf**	**2.0 lbf**
mFRAC	1.0000	1.0000	1.0000	1.0000	1.0000
Force Amplitude	5.0 lbf	10 lbf	20 lbf	50 lbf	
mFRAC	1.0000	0.9999	0.9991	0.9992	

Frequency Backbone

$|Q_1| = 2 \cdot 10^{-3}\ in - \sqrt{lbm}$

"Weakly nonlinear"



Conclusion

Applied single, nonlinear resonant mode approximation to jointed structure with weak damping nonlinearity

Demonstrated accuracy and efficiency of Quasi-Static Modal Analysis for obtaining nonlinear frequency and damping curves for nonlinear forced response synthesis

Future work will apply this methodology to full-order FEM and extend the modal nonlinear forced response approach to base-excited systems

Any Questions?

Contact information

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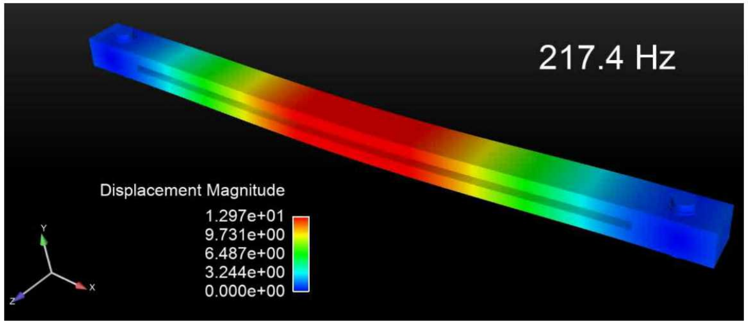

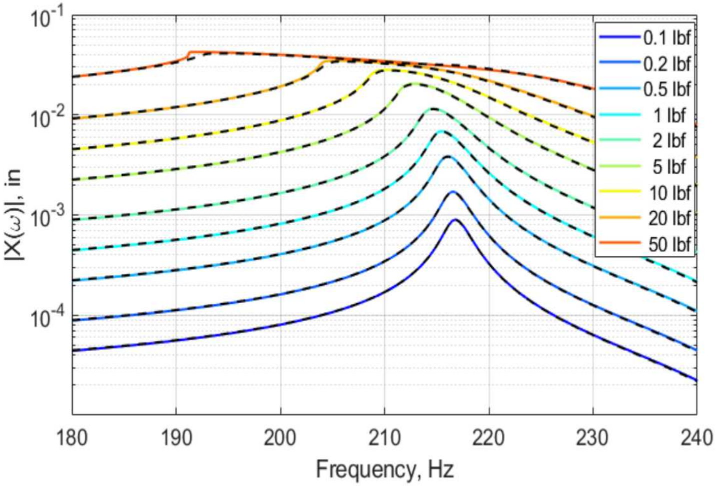

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