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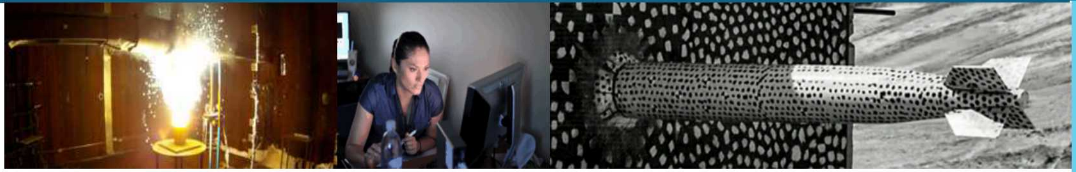
A Bayesian Approach for Identifying the Spatial Correlation of Acoustic Loads During Vibroacoustic Testing



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Correlating model and experiment requires the correct loads

For the random field generated during vibroacoustic testing, model/test correlation requires identifying the acoustic pressure power-spectral density (PSD) matrix to generate the response:

Response PSD:

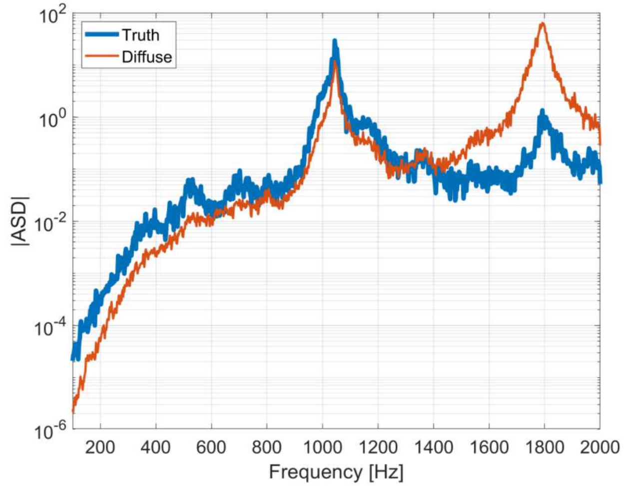
$$\mathbf{S}_{xx}(\omega) = \mathbf{H}_{xf}(\omega)\mathbf{S}_{ff}(\omega)\mathbf{H}_{xf}(\omega)^H$$

Input PSD:

$$\mathbf{S}_{ff}(\omega) = \begin{bmatrix} S_{11}(\omega) & \cdots & S_{1N}(\omega) \\ \vdots & \ddots & \vdots \\ S_{N1}(\omega) & \cdots & S_{NN}(\omega) \end{bmatrix}$$

Some approaches to build the input PSD matrix include

- Uncorrelated Inputs: diagonal terms from measured pressure levels, off-diagonal terms are zero
- Diffuse Field: diagonal terms from measured pressure levels, off-diagonal terms from *sinc* function



Incorrect loading can significantly degrade response predictions!

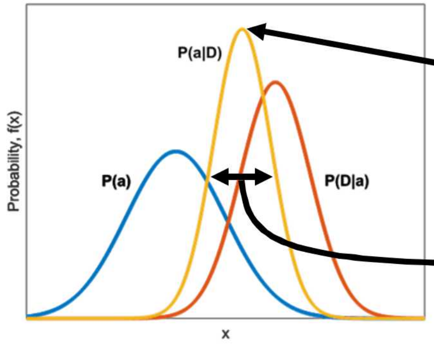
Bayesian inference relies on Bayes' Theorem to estimate the unknown variables and also quantifies uncertainty

Bayes' Theorem: $\mathrm{p}(\mathbf{a}|D) \propto \mathrm{p}(D|\mathbf{a})\ \mathrm{p}(\mathbf{a})$

Prior probability $\mathrm{p}(\mathbf{a})$: represents knowledge of unknown variables before collecting any data

Likelihood (Evidence) $\mathrm{p}(D|\mathbf{a})$: represents the probability of the measured data given a set of the unknown variables

Posterior probability $\mathrm{p}(\mathbf{a}|D)$: represents the updated probability of the unknown variables given the measured data



Maximum a posteriori (MAP) estimate: mode of the posterior distribution

Uncertainty estimate: variance of the posterior distribution

- Several recent studies utilize Bayesian inference for inverse problems in acoustics and structural dynamics: Zhang (2012 JSV), Antoni (2012 JASA), Pereira (2015 AA), Aucejo (2016 MSSP), Faure (2017 MSSP)
- This work follows the framework set forth by Pereira (2015 AA), but with several differences:
  - Allows for inclusion of the measured input levels from microphone measurements
  - Estimates/quantifies the uncertainty of unmeasured structural locations
  - Estimates PSDs when dealing with random signals

Bayesian Inference leads to an estimate of the unknown forces given the measured structural responses

Response at each freq. is a combination of deterministic and probabilistic components

$$\mathbf{x}(\omega) = \mathbf{H}(\omega)\mathbf{f}(\omega) + \mathbf{n}(\omega)$$

$\mathbf{x}$: Vector of the response measurements

$\mathbf{H}$: Matrix of known transfer functions

$\mathbf{f}$: Vector of unknown forces

$\mathbf{n}$: Vector of the measurement/model errors that is normally distributed with zero mean and variance σ_n^2

Likelihood function:

$$\mathrm{p}(\mathbf{x}|\mathbf{f}, \sigma_n^2) \sim N_c(\mathbf{Hf}, \sigma_n^2\mathbf{I}) = \frac{1}{\pi^{N_o}(\sigma_n^2)^{N_o}} \exp\left[-\frac{1}{\sigma_n^2}(\mathbf{x} - \mathbf{Hf})^H(\mathbf{x} - \mathbf{Hf})\right]$$

Prior for the unknown forces:

$$\mathrm{p}\left(\mathbf{f}\middle|\sigma_f^2\right) \sim N_c\left(\mathbf{0}, \sigma_f^2\boldsymbol{\Sigma}_f\right) = \frac{1}{\pi^{N_i}(\sigma_f^2)^{N_i}\left|\boldsymbol{\Sigma}_f\right|} \exp\left[-\frac{1}{\sigma_f^2}\mathbf{f}^H\boldsymbol{\Sigma}_f^{-1}\mathbf{f}\right]$$

Apply Bayes' Theorem to estimate forces:

$$\mathrm{p}(\mathbf{f}|\mathbf{x}, \sigma_n^2, \sigma_f^2) \propto \mathrm{p}(\mathbf{x}|\mathbf{f}, \sigma_n^2)\ \mathrm{p}\left(\mathbf{f}\middle|\sigma_f^2\right) \sim N_c\left(\hat{\mathbf{f}}, \hat{\mathbf{C}}_{ff}\right)$$

MAP Estimate: $\hat{\mathbf{f}} = \left(\mathbf{H}^H\mathbf{H} + \tau^2\boldsymbol{\Sigma}_f^{-1}\right)^{-1}\mathbf{H}^H\mathbf{x}$

Uncertainty: $\hat{\mathbf{C}}_{ff} = \sigma_n^2\left(\mathbf{H}^H\mathbf{H} + \tau^2\boldsymbol{\Sigma}_f^{-1}\right)^{-1}$

Regularization Param: $\tau^2 = \frac{\sigma_n^2}{\sigma_f^2}$

Estimate the unmeasured responses:

$$\mathrm{p}\left(\mathbf{x}_*\middle|\mathbf{x}, \sigma_n^2, \sigma_f^2\right) = \int \mathrm{p}(\mathbf{x}_*|\mathbf{f})\mathrm{p}\left(\mathbf{f}\middle|\mathbf{x}, \sigma_n^2, \sigma_f^2\right)\mathrm{d}\mathbf{f} \sim N_c\left(\hat{\mathbf{x}}_*, \hat{\mathbf{C}}_{**}\right)$$

MAP Estimate: $\hat{\mathbf{x}}_* = \mathbf{H}_*\hat{\mathbf{f}}$

Uncertainty: $\hat{\mathbf{C}}_{**} = \mathbf{H}_*\hat{\mathbf{C}}_{ff}\mathbf{H}_*^H$

Pereira et al. (2015), "Empirical Bayesian regularization of the inverse acoustic problem," *Applied Acoustics*.

For random signals, the Bayesian-based force estimation also extends to power-spectral densities

Can work with measured response PSD matrix ($\mathbf{S}_{xx}$) rather than any specific realization of $\mathbf{x}$:

$$\mathrm{p}(\mathbf{x}|\mathbf{S}_{xx}) \sim N_c(0,\mathbf{S}_{xx})$$

where $\mathbf{S}_{xx}$: Measured response PSD matrix

Can estimate input PSD matrix by marginalizing over all possible values of the $\mathbf{x}$:

$$\mathrm{p}(\mathbf{f}|\sigma_n^2,\sigma_f^2) = \int \mathrm{p}(\mathbf{f}|\mathbf{x},\sigma_n^2,\sigma_f^2)\mathrm{p}(\mathbf{x}|\mathbf{S}_{xx})\mathrm{d}\mathbf{x} \sim N_c\left(0,\hat{\mathbf{S}}_{ff}\right)$$

Input PSD Matrix Estimate:

$$\hat{\mathbf{S}}_{ff} = \left[\left(\mathbf{H}^H\mathbf{H} + \tau^2\mathbf{\Sigma}_f^{-1}\right)^{-1}\mathbf{H}^H\right]\mathbf{S}_{xx}\left[\left(\mathbf{H}^H\mathbf{H} + \tau^2\mathbf{\Sigma}_f^{-1}\right)^{-1}\mathbf{H}^H\right]^H + \sigma_n^2\left(\mathbf{H}^H\mathbf{H} + \tau^2\mathbf{\Sigma}_f^{-1}\right)^{-1}$$

Can estimate the unmeasured response PSD matrix by also marginalizing over all possible values of $\mathbf{x}$:

$$\mathrm{p}\left(\mathbf{x}_*\middle|\sigma_n^2,\sigma_f^2\right) = \int \mathrm{p}(\mathbf{x}_*|\mathbf{x},\sigma_n^2,\sigma_f^2)\mathrm{p}(\mathbf{x}|\mathbf{S}_{xx})\mathrm{d}\mathbf{x} \sim N_c\left(0,\hat{\mathbf{S}}_{xx}\right)$$

Response PSD Matrix Estimate:

$$\hat{\mathbf{S}}_{**} = \mathbf{H}_*\hat{\mathbf{S}}_{ff}\mathbf{H}_*^H$$

Identifying the regularization parameter reduces to a one-dimensional optimization problem

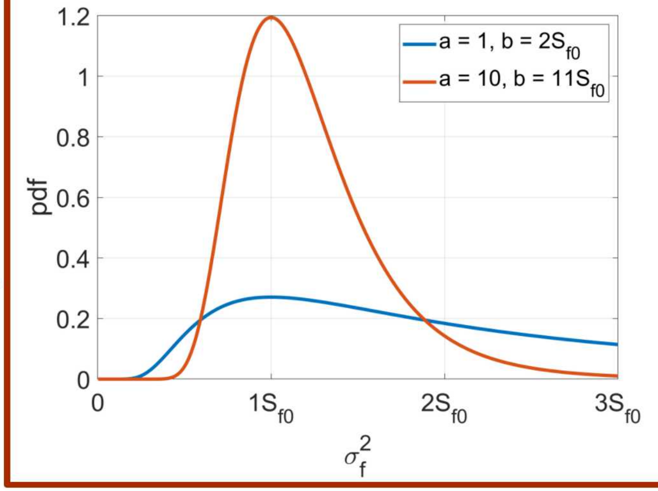
Identify the hyperparameters σ_n^2 and σ_f^2 using a second application of Bayes' Theorem:

Marginal Likelihood: $\mathrm{p}(\mathbf{x}|\sigma_n^2,\sigma_f^2) = \int \mathrm{p}(\mathbf{x}|\mathbf{f},\sigma_n^2)\mathrm{p}(\mathbf{f}|\sigma_f^2)\mathrm{d}\mathbf{f}$

Hyperparameter priors:

Noise variance:
$$\mathrm{p}(\sigma_n^2) \propto \frac{1}{(\sigma_n^2)^{\alpha_n+1}}\exp\left(-\frac{\beta_n}{\sigma_n^2}\right)$$

Input variance:
$$\mathrm{p}(\sigma_f^2) \propto \frac{1}{\left(\sigma_f^2\right)^{\alpha_f+1}}\exp\left(-\frac{\beta_f}{\sigma_f^2}\right)$$



Apply Bayes' Theorem and marginalize out σ_f^2 to obtain the posterior distribution of τ^2:

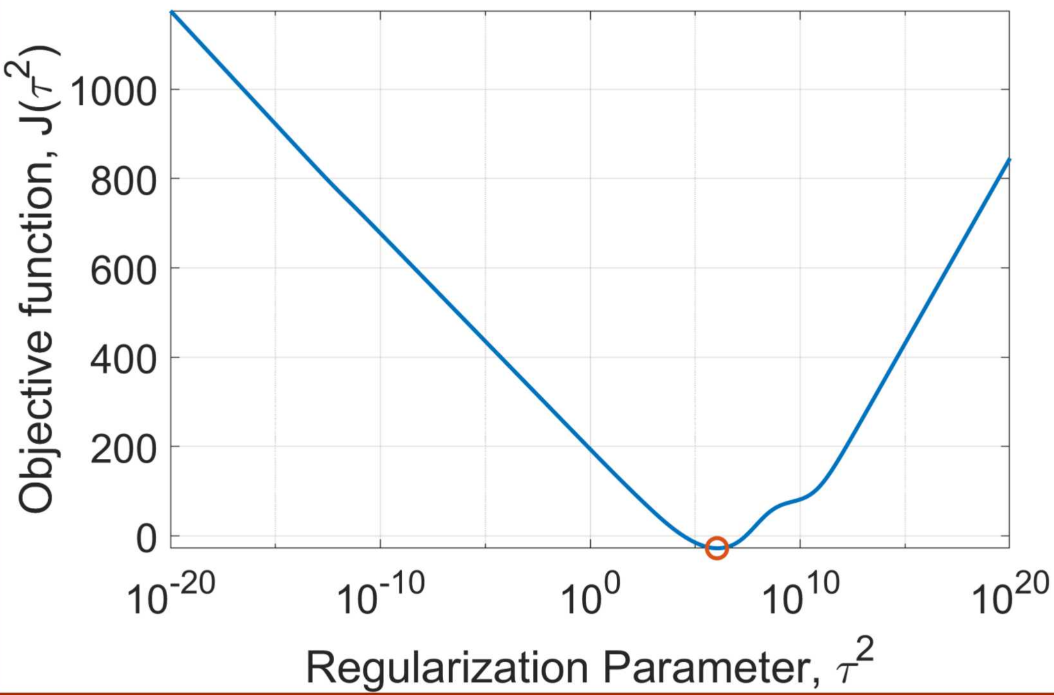
$$\mathrm{p}(\tau^2|\mathbf{x}) \propto \left[(\tau^2)^{\alpha_n+1}\left(\sum_{i=1}^{N_o}\frac{\mathbf{u}_i^H\mathbf{S}_{xx}\mathbf{u}_i}{s_i^2+\tau^2}+\frac{\beta_f}{N_R}+\frac{1}{\tau^2}\frac{\beta_n}{N_R}\right)^{N_oN_R+\alpha_n+\alpha_f}\prod_{i=1}^{N_o}\left(s_i^2+\tau^2\right)^2\right]^{-1}$$

Optimize τ^2 by minimizing the negative natural logarithm of this posterior distribution:

$$\hat{\tau}^2 = \operatorname{argmin}\ \frac{\alpha_n+1}{N_R}\ln(\tau^2) + \left(N_o+\frac{\alpha_n+1}{N_R}+\frac{\alpha_f+1}{N_R}\right)\ln\left(\hat{\sigma}_f^2\right) + \sum_{i=1}^{N_o}\ln\left(s_i^2+\tau^2\right) - \frac{2}{N_R}\ln\left(\hat{\sigma}_f^2\right)$$

$$\hat{\sigma}_f^2 = \frac{1}{N_o+\frac{\alpha_f+1}{N_R}}\sum_{i=1}^{N_o}\frac{\mathbf{u}_i^H\mathbf{S}_{xx}\mathbf{u}_i}{s_i^2+\tau^2}+\frac{\beta_f}{N_R}$$

Identifying the regularization parameter reduces to a one-dimensional optimization problem



$$\hat{\tau}^2 = \text{argmin} \; \frac{\alpha_n+1}{N_R}\ln(\tau^2) + \left(N_o + \frac{\alpha_n+1}{N_R} + \frac{\alpha_f+1}{N_R}\right)\ln(\hat{\sigma}_f^2) + \sum_{i=1}^{N_o}\ln\left(s_i^2+\tau^2\right) - \frac{2}{N_R}\ln(\hat{\sigma}_f^2)$$

$$\hat{\sigma}_f^2 = \frac{1}{N_o + \frac{\alpha_f+1}{N_R}}\sum_{i=1}^{N_o}\frac{\mathbf{u}_i^H \mathbf{S}_{xx}\mathbf{u}_i}{s_i^2+\tau^2} + \frac{\beta_f}{N_R}$$

Numerical simulation of a direct-field acoustic test provides initial validation

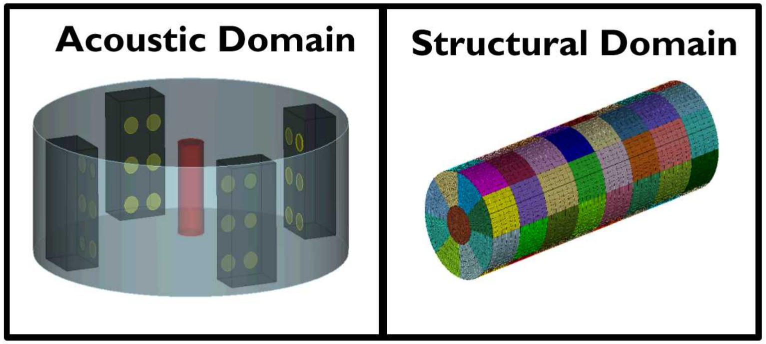
Direct-field acoustic test (DFAT) setup:

- Used 4 speaker clusters to excite the cylinder with uncorrelated white noise
- Defined structural inputs as point forces acting at the center surface patches using the distributed pressures

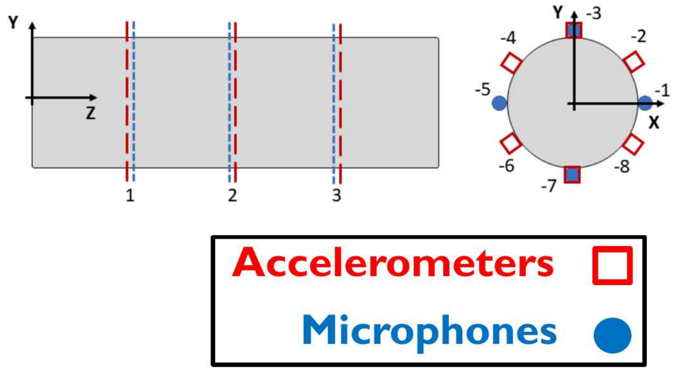
Simulated measurements:

- 18 accels located at 6 circumferential locations and 3 axial locations used for estimation procedure
- 12 microphones located at 4 circumferential locations and 3 axial locations used for tuning parameters of the σ_f^2 prior
- Time-domain measurements polluted by measurement noise with a SNR =15 dB

DFAT Simulation Setup



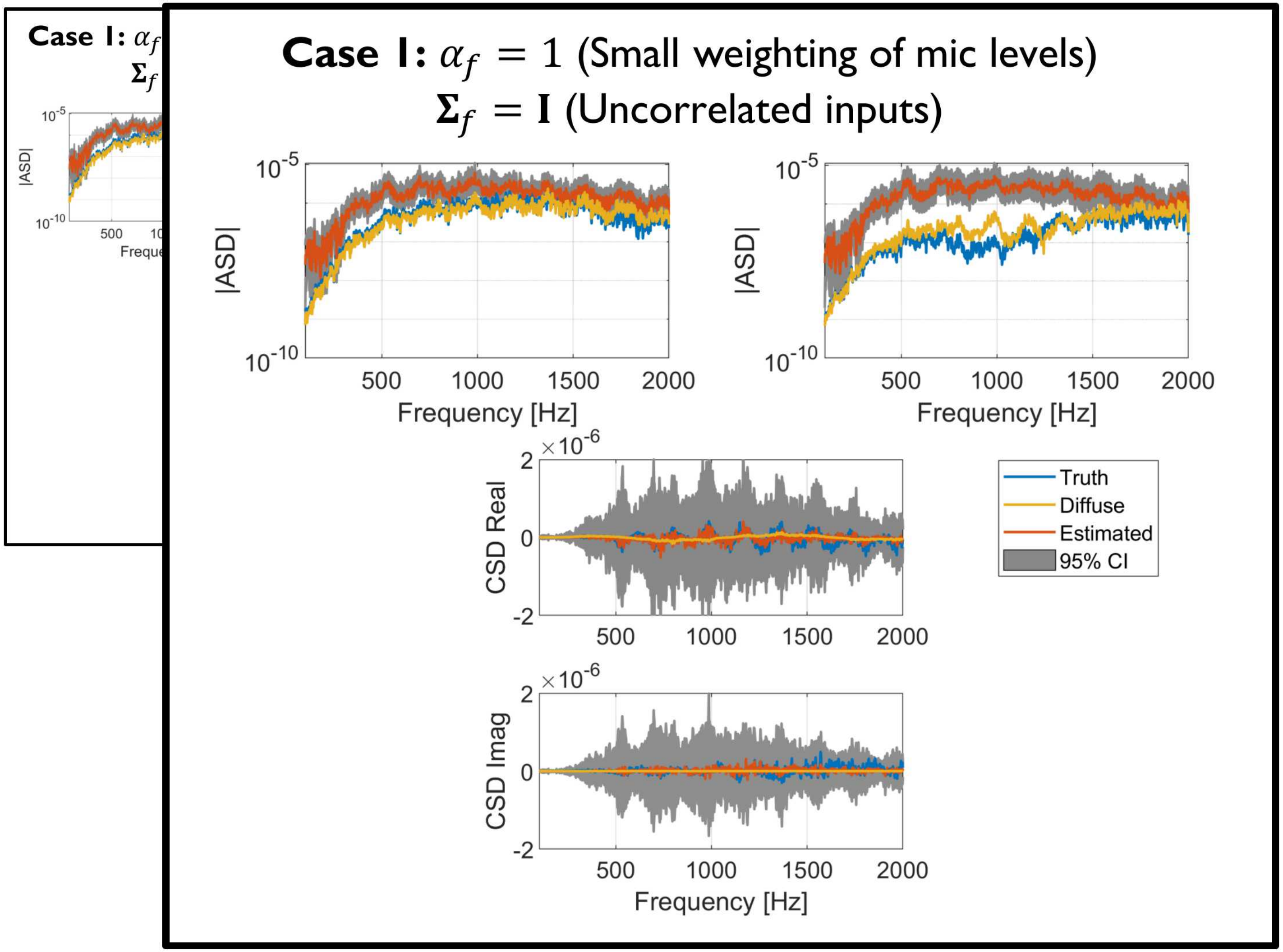
Measurement Locations



Four cases examined:

1) $\alpha_f = 1$ (Small weighting of mic levels), $\boldsymbol{\Sigma}_f = \mathbf{I}$ (Uncorrelated inputs)
2) $\alpha_f = 100$ (Larger weighting of mic levels), $\boldsymbol{\Sigma}_f = \mathbf{I}$ (Uncorrelated inputs)
3) $\alpha_f = 1$ (Small weighting of mic levels), $\boldsymbol{\Sigma}_f =$ Correlated in axial, diffuse around circumference
4) $\alpha_f = 100$ (Larger weighting of mic levels), $\boldsymbol{\Sigma}_f =$ Correlated in axial, diffuse around circumference

Proposed procedure able to estimate the input force PSDs



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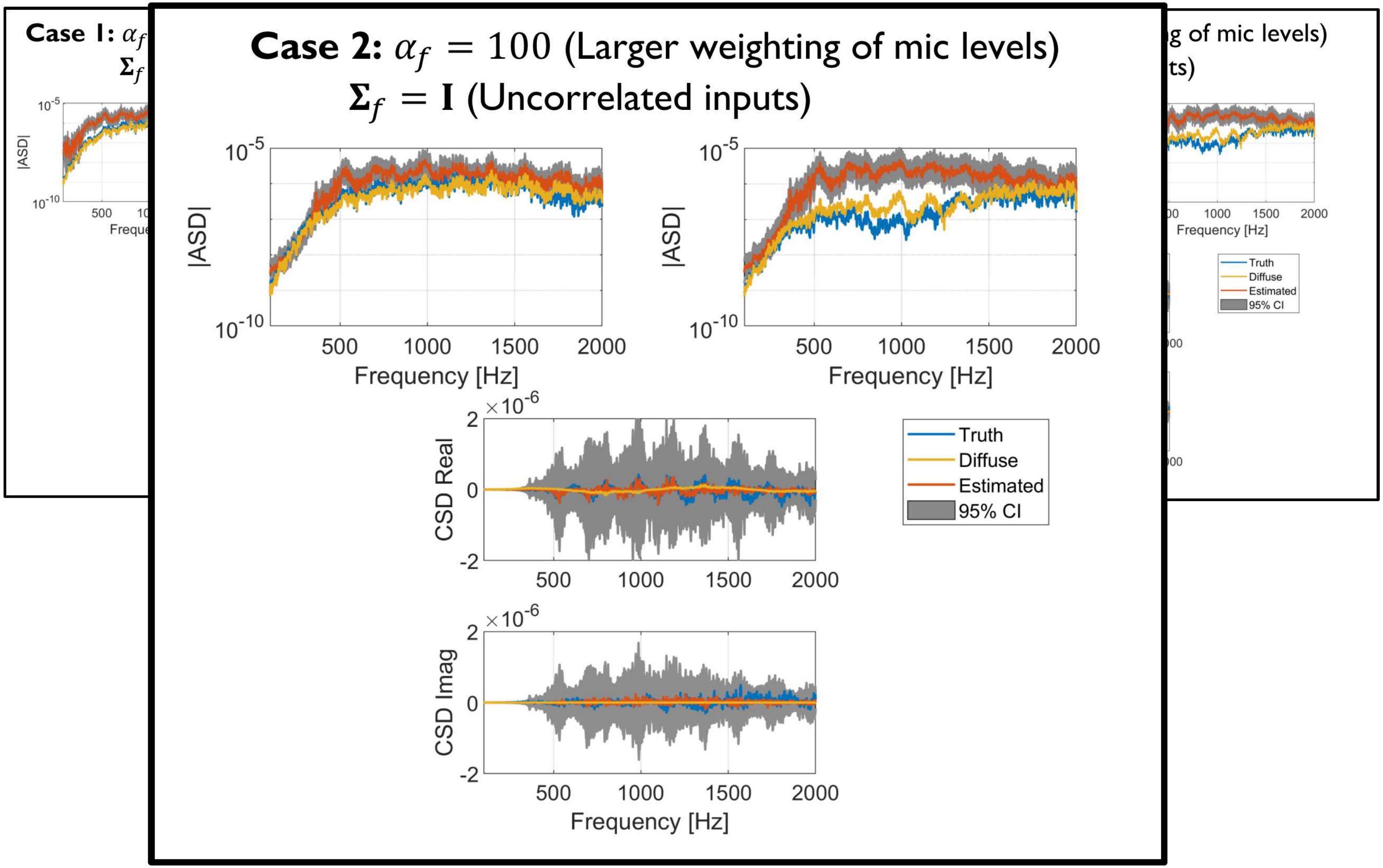

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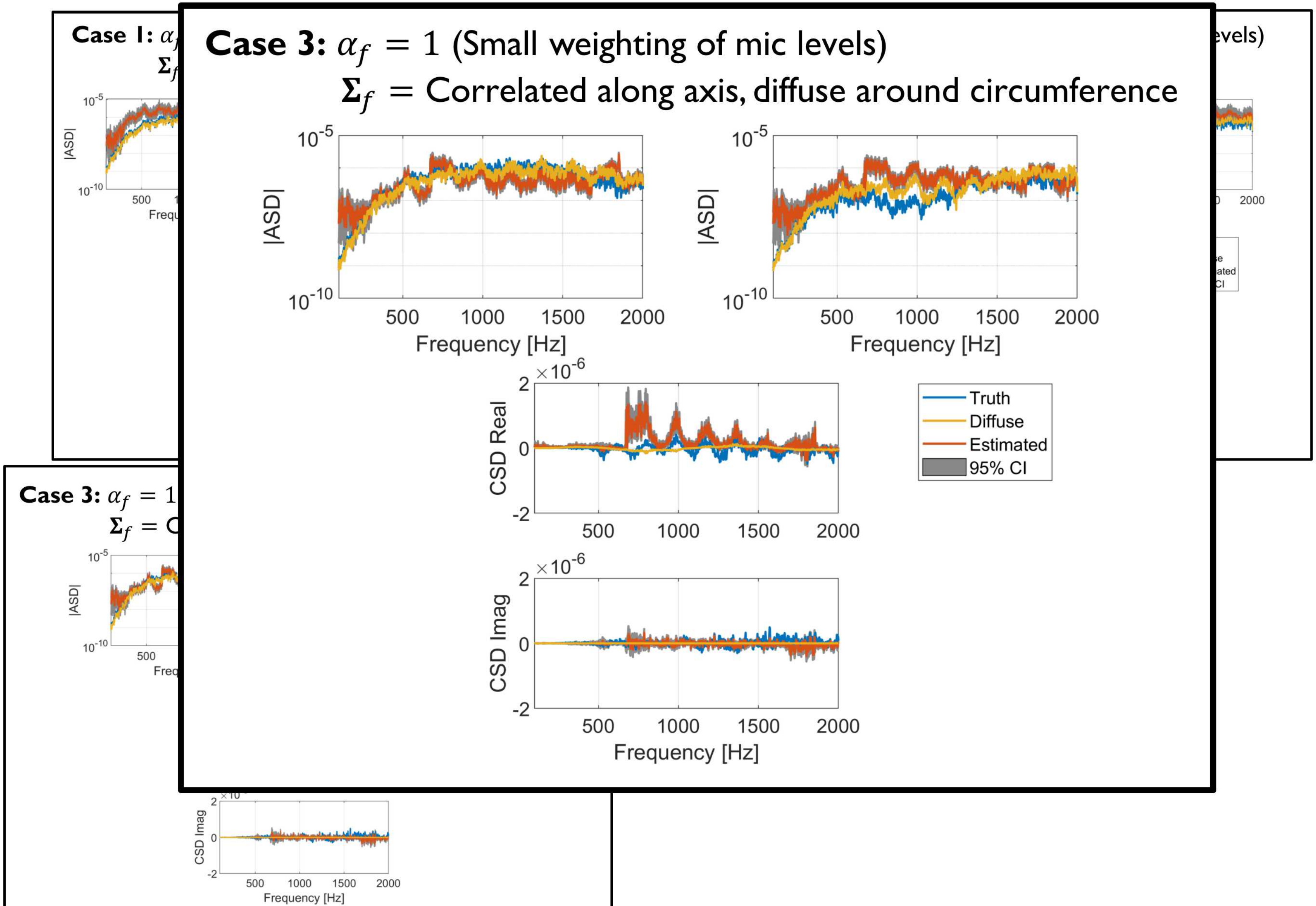

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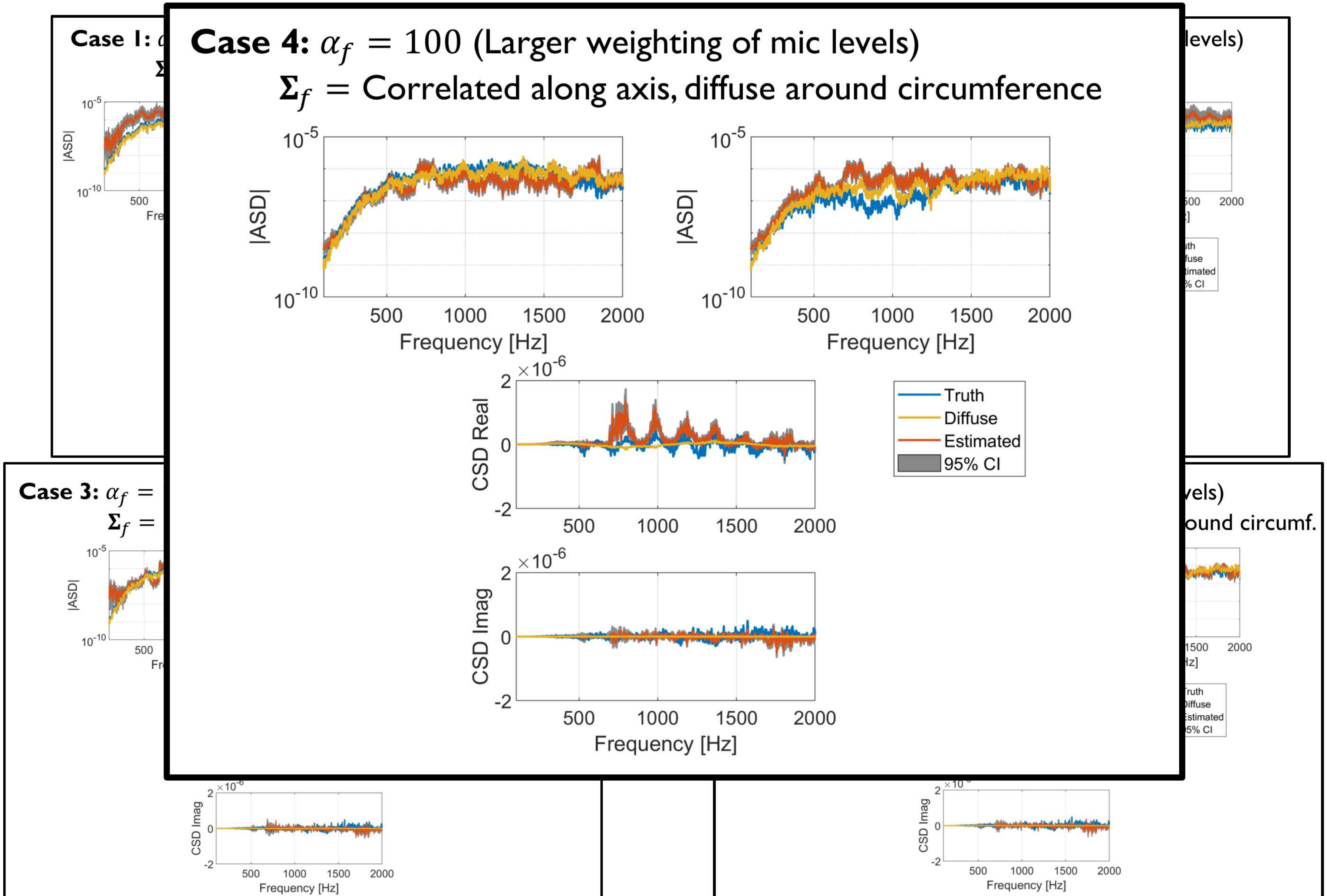
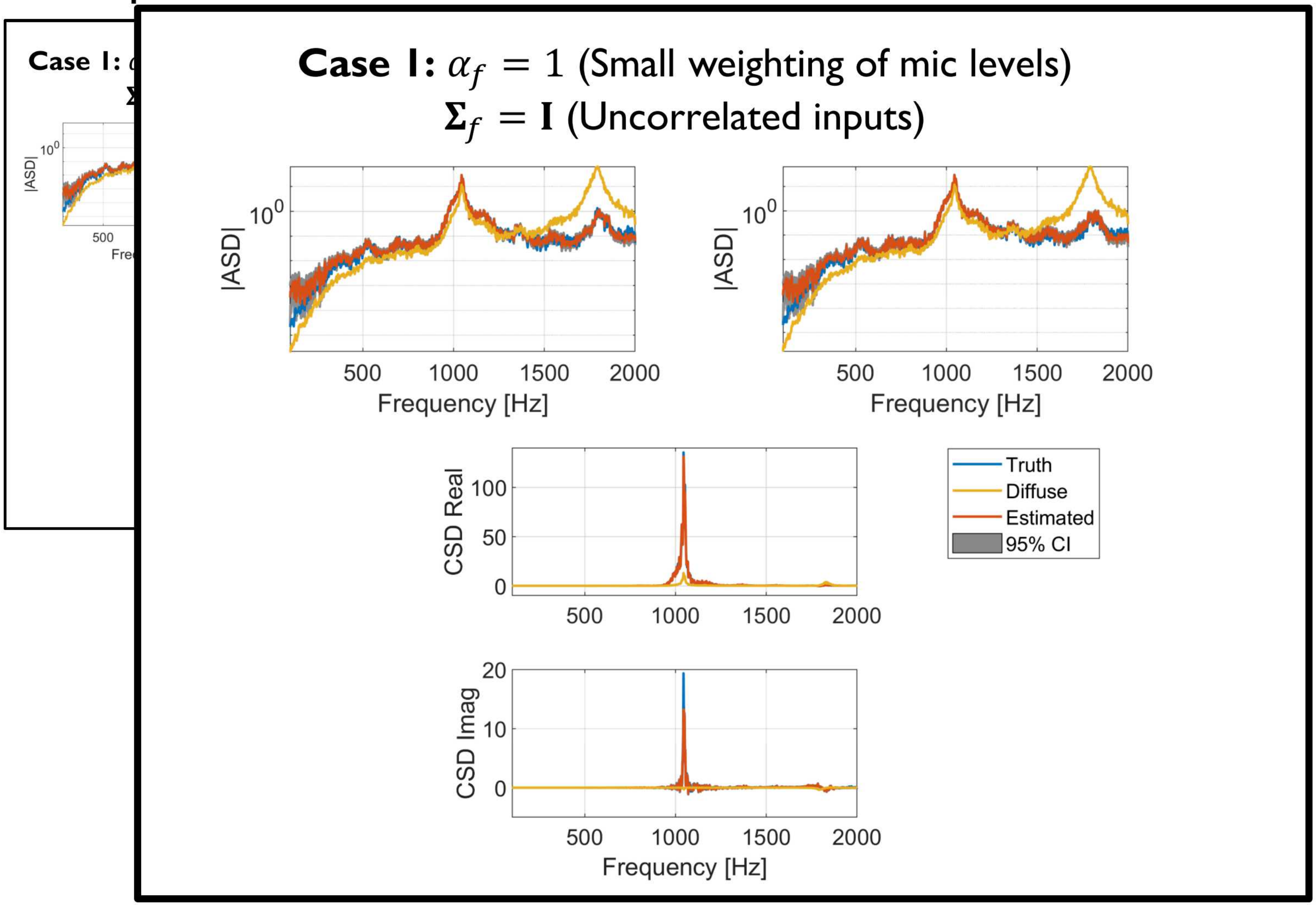


Proposed procedure accurately reproduces the structural response, even at unmeasured locations



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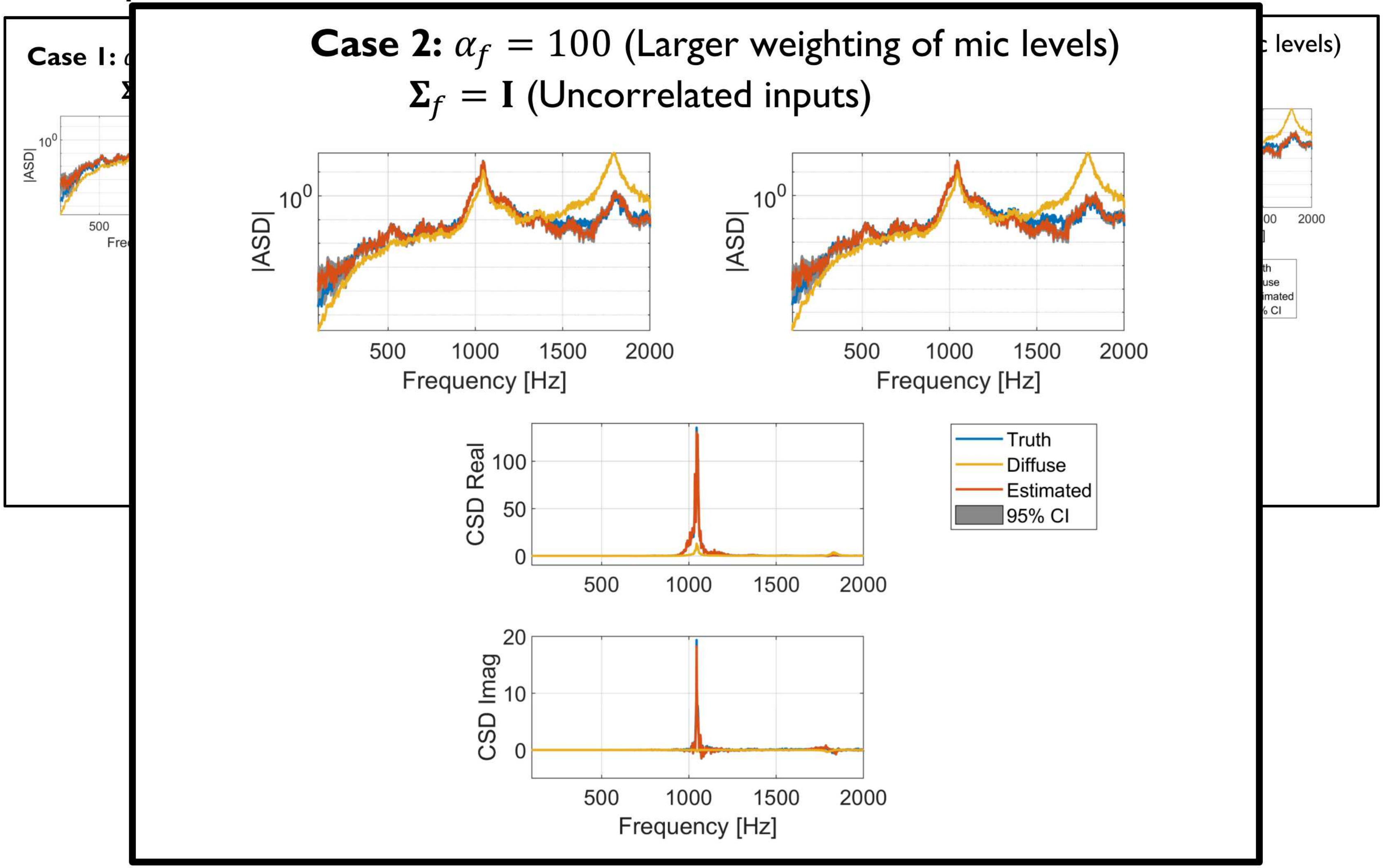

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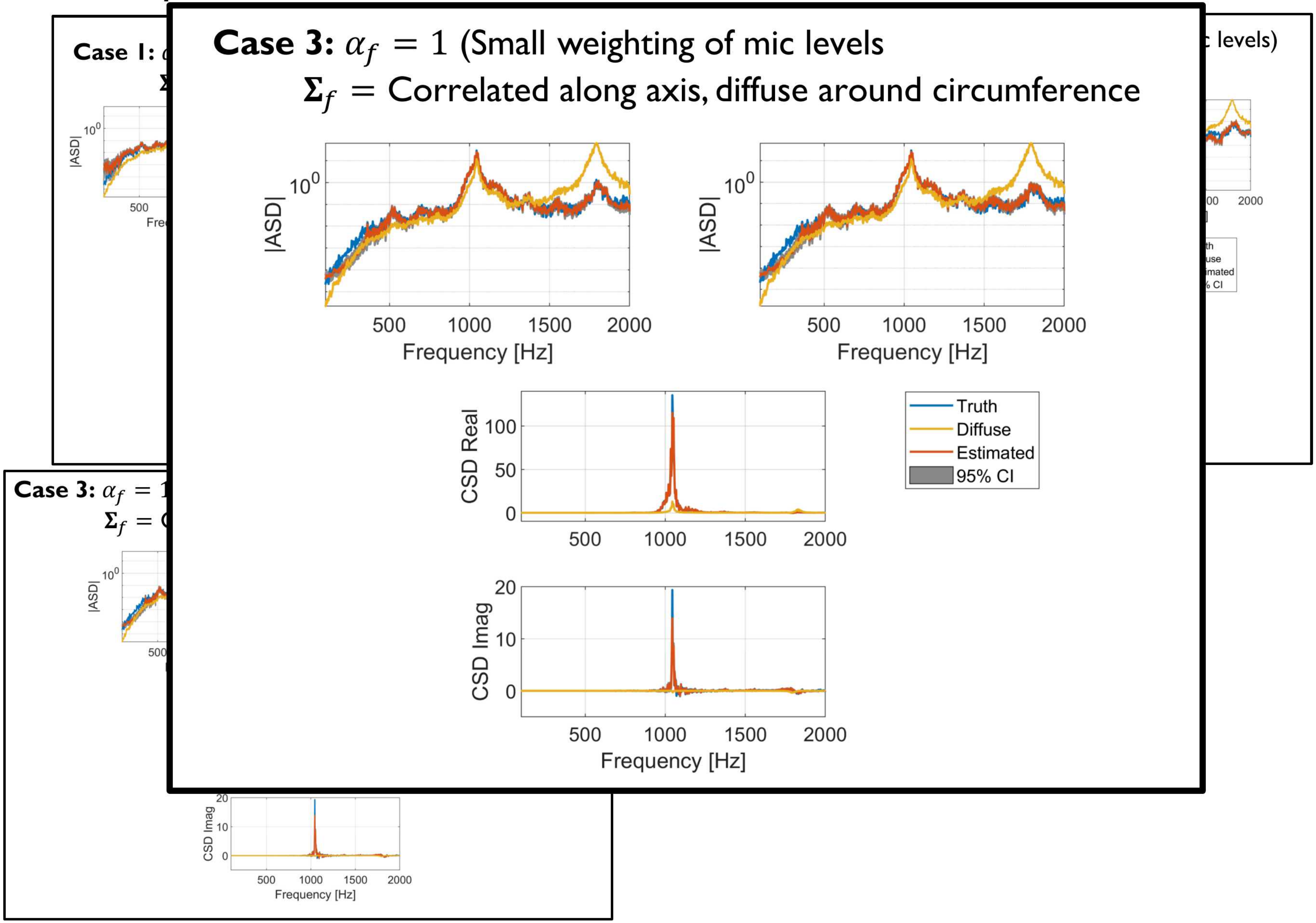
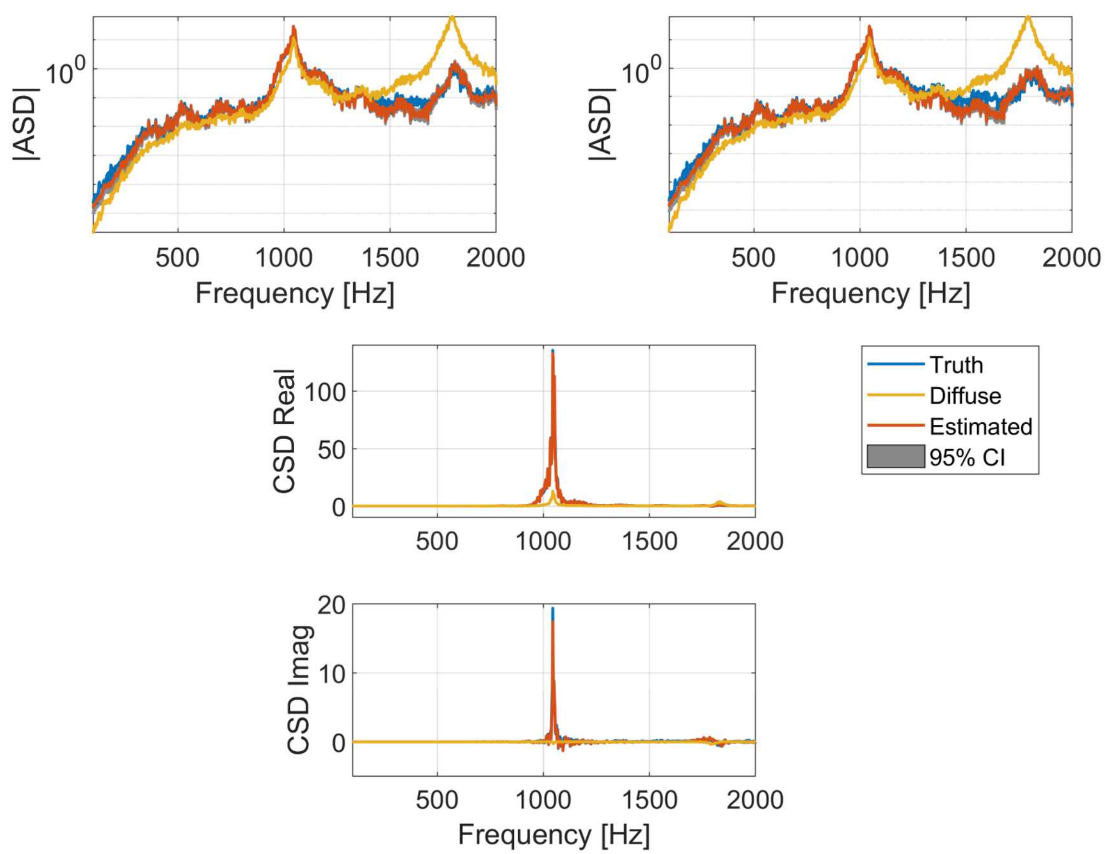


Proposed procedure accurately reproduces the structural response, even at unmeasured locations

Case 4: $\alpha_f = 100$ (Larger weighting of mic levels)

$\boldsymbol{\Sigma}_f$ = Correlated along axis, diffuse around circumference



Bayesian-based approach able predict inputs that better reproduce the measured response during vibroacoustic testing

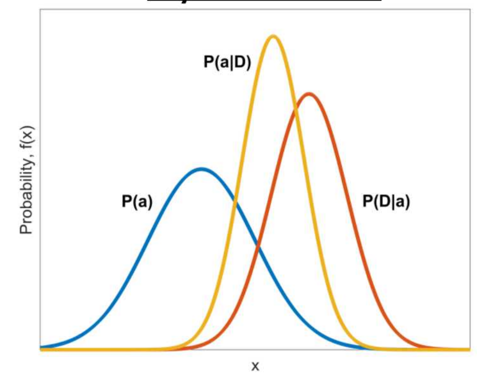
Load estimation strategy developed utilizing Bayesian inference

- Provides point estimates of the unknown forces/unmeasured responses and quantifies uncertainty
- Contains an inherent regularization mechanism in cases of ill-conditioned inversions
- Enables the incorporation of the pressure levels measured during testing through hyperparameter priors
- Extends to PSDs for random inputs/outputs

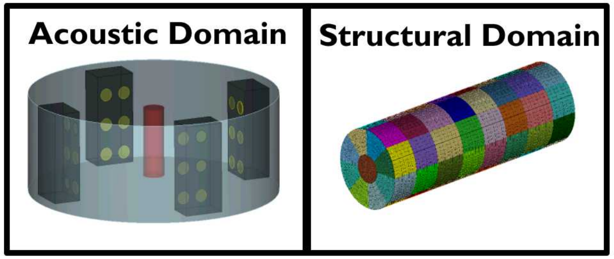
Numerical simulations offered initial validation

- Consisted of speakers exciting a cylindrical test article in a direct-field configuration
- Predicted the levels of the applied loads and also the spatial shape
- Reproduced the test article's response, even at unmeasured locations

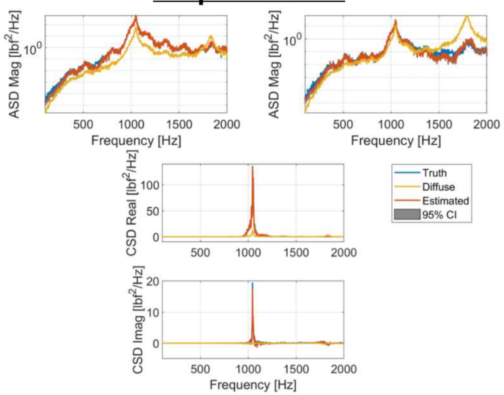
Bayesian Inference



DFAT Simulation Setup

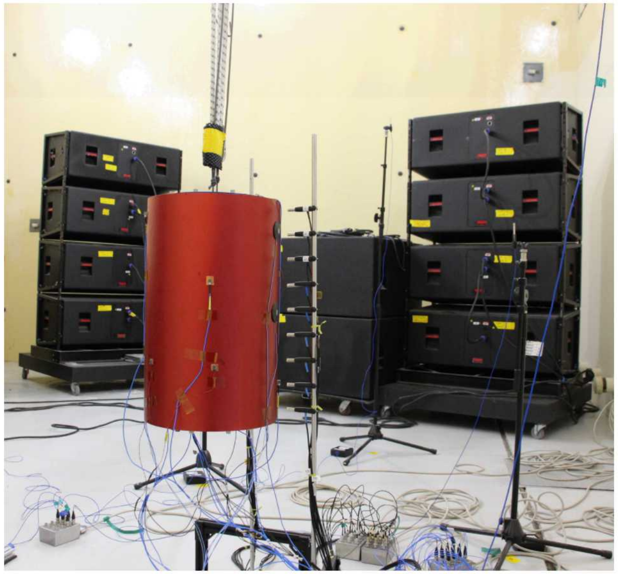


Response PSDs



Ongoing work centers on experimentally validating the proposed load-identification strategy

Reverb-Chamber Test Setup



Perform vibroacoustic test to validate the proposed load estimation strategy:

- Excite the test article with various pressure fields: diffuse, diffuse+direct, direct
- Build FRFs using a calibrated FE model or a hybrid approach where natural freqs. and damping are from modal test and shapes are from FE model

Further refine load estimation strategy:

- Determine sensitivity to incorrect model parameters/incorporate some model updating to better match experimental data
- Incorporate local priors for the unknown force that should offer flexibility for fields with non-uniform pressure levels
- Parameterize the spatial correlation matrix (e.g., using a Gaussian kernel) to better match the field

Bayesian-based approach able predict inputs that better reproduce the measured response during vibroacoustic testing

Load estimation strategy developed utilizing Bayesian inference

- Provides point estimates of the unknown forces/unmeasured responses and quantifies uncertainty
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Numerical simulations offered initial validation

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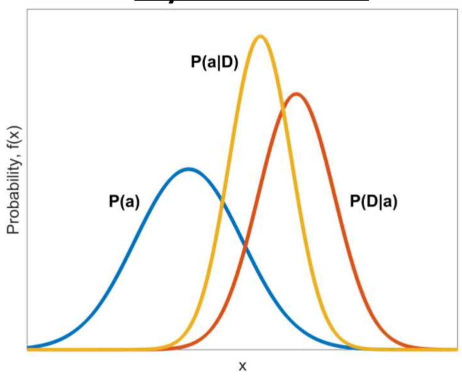
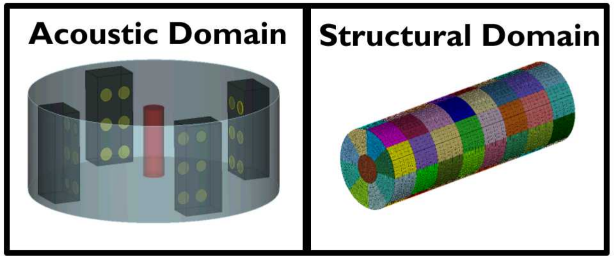
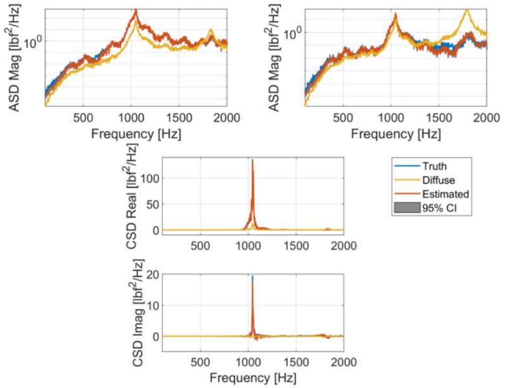
Bayesian Inference



DFAT Simulation Setup



Response PSDs



Extra slides…

Current approaches to correlate models to vibroacoustic experiments incorporate idealized load configurations

For the random field generated during vibroacoustic testing, model/test correlation requires identifying the acoustic pressure power-spectral density (PSD) matrix to generate the response:

Response PSD: $\mathbf{S}_{xx}(\omega) = \mathbf{H}_{xf}(\omega)\mathbf{S}_{ff}(\omega)\ \mathbf{H}_{xf}(\omega)^H$

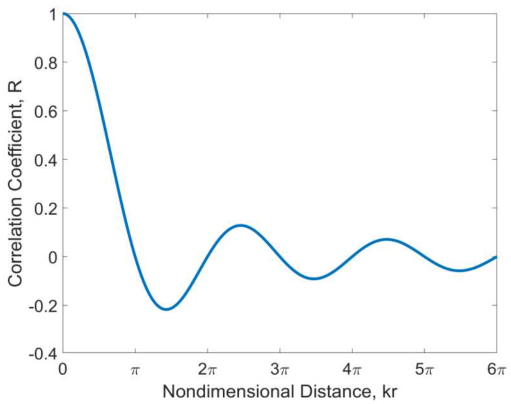
Input PSD: $\mathbf{S}_{ff}(\omega) = \mathrm{E}[\mathbf{f}(\omega)\mathbf{f}(\omega)^H] = \begin{bmatrix} S_{11}(\omega) & \cdots & S_{1N}(\omega) \\ \vdots & \ddots & \vdots \\ S_{N1}(\omega) & \cdots & S_{NN}(\omega) \end{bmatrix}$

A common approach utilizes an ideal diffuse field with a spatial correlation approximated by a sinc function:

$$R_{ij}(\omega) = \frac{\sin k(\omega) r_{ij}}{k(\omega) r_{ij}}$$

$k(\omega)$: wave number

r_{ij}: relative distance between input i and j



$$S_{ij}(\omega) = R_{ij}(\omega)\sqrt{S_{ii}(\omega)S_{jj}(\omega)} \approx R_{ij}(\omega)S_0(\omega)$$

Physical testing can lead to deviations from this idealized field due to scattering effects and test setup factors

DFAT Simulation Setup

Direct-field acoustic test (DFAT) setup:
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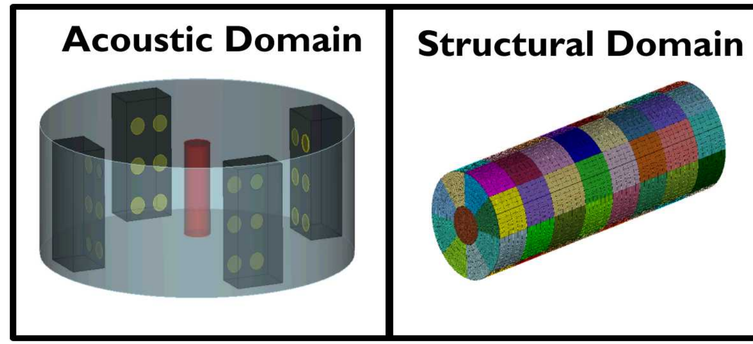
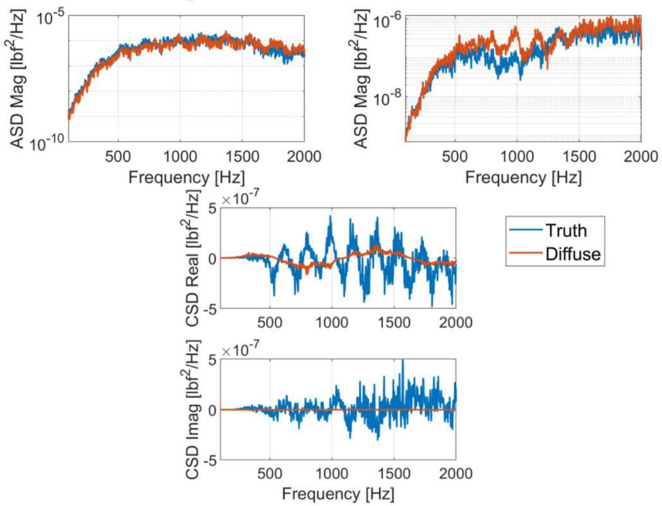
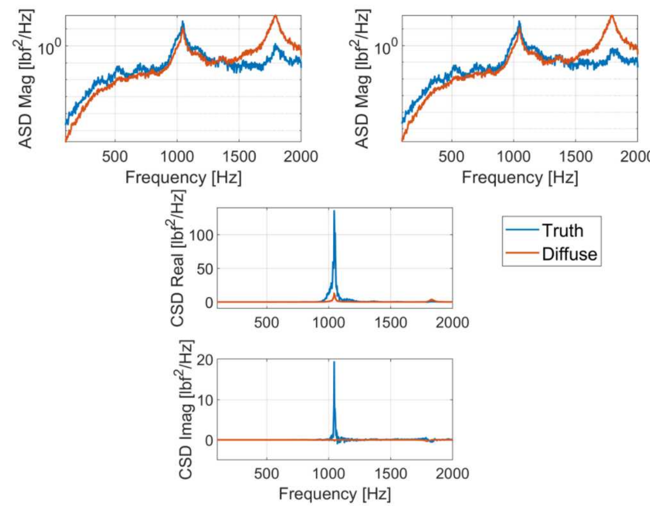


Input Force PSDs



Response PSDs



Incorrect load configuration can significantly degrade response predictions!

Bayesian-based approach contains an intrinsic regularization in the case of an ill-conditioned inversion

Return to the force estimates:

MAP Estimate: $\hat{\mathbf{f}} = \left(\mathbf{H}^H\mathbf{H} + \tau^2\boldsymbol{\Sigma}_f^{-1}\right)^{-1}\mathbf{H}^H\mathbf{x}$

Uncertainty: $\hat{\mathbf{C}}_{ff} = \sigma_n^2\left(\mathbf{H}^H\mathbf{H} + \tau^2\boldsymbol{\Sigma}_f^{-1}\right)^{-1}$

Regularization Param:

$$\tau^2 = \frac{\sigma_n^2}{\sigma_f^2}$$

Perform a singular-value decomposition:

$$\mathbf{H}\boldsymbol{\Sigma}_f^{1/2} = \mathbf{U}\mathbf{S}\mathbf{V}^H$$

The regularized force estimates are

MAP Estimate: $\hat{\mathbf{f}} = \boldsymbol{\Sigma}_f^{1/2}\mathbf{V}\left[\left(\frac{s_i^2}{s_i^2+\tau^2}\right)\frac{1}{s_i}\right]\mathbf{U}^H\mathbf{x}$

Uncertainty: $\hat{\mathbf{C}}_{ff} = \sigma_n^2\boldsymbol{\Sigma}_f^{1/2}\mathbf{V}\begin{bmatrix}\left[\frac{1}{s_i^2+\tau^2}\right] & \mathbf{0} \\ \mathbf{0} & \left[\frac{1}{\tau^2}\right]\end{bmatrix}\mathbf{V}^H\left(\boldsymbol{\Sigma}_f^{1/2}\right)^H$