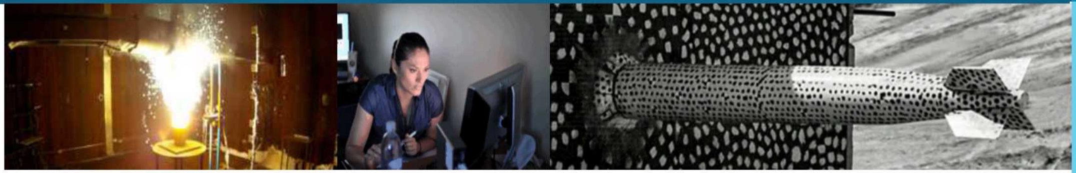
SAND2020-1190C

Using the Modal Craig-Bampton Procedure for the Test Planning of a Six-Degree-of-Freedom Shaker



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Testing with a 6-DOF shaker shows potential and using modal analysis can better inform these tests

6-DOF shakers show promise for vibration testing:

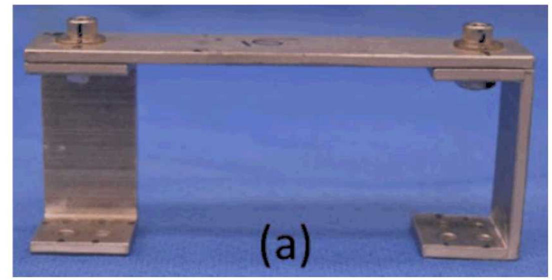
- Enables simultaneous multi-axis testing rather than testing each axis individually
- Can potentially reproduce the responses/stresses obtained in the field environment provided the boundary conditions between the next-level assembly and the test fixture are similar
- Focus here is on reproducing the response of base-mounted components/payloads on a shaker with a rigid fixture

How can we incorporate modal analysis to better inform shaker tests?

- Use the Modal Craig-Bampton procedure that requires the modal parameters from a modal test of the test article on the fixture
- Use this procedure to transform the free-free modes to a set of fixed-base and rigid-body modes that simulates the boundary conditions on a shaker table
- Identify how the test article's modes respond to the shaker's rigid-body inputs
- Predict shaker inputs required to replicate the field environment

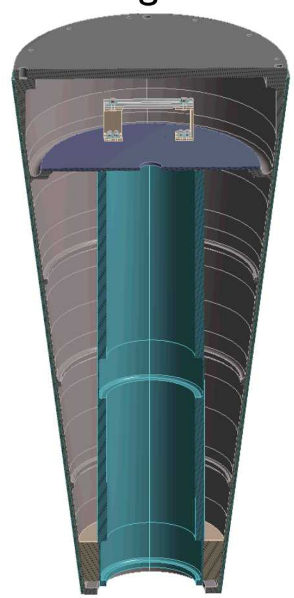
Experimental data obtained from an acoustic environment provided the reference response measurements for this study

Removable Component (RC)

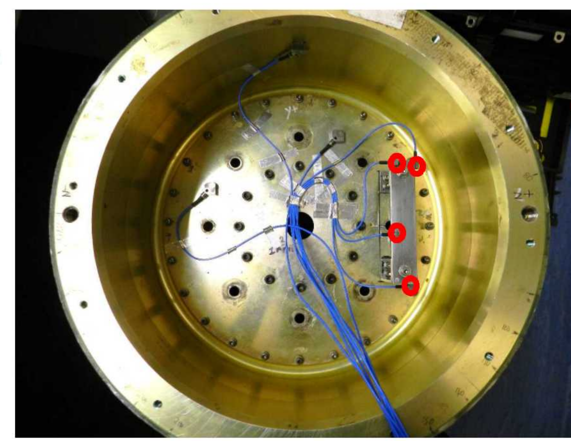


Modal Analysis Test Vehicle (MATV)

Diagram



Top-down view



Acoustic Test Details:

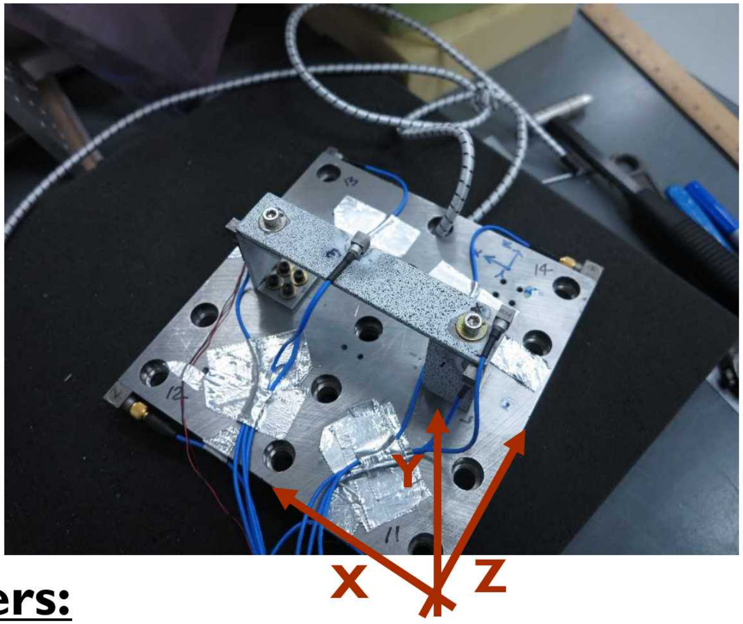
- Component of interest was the Removable Component (RC), a test article developed for the dynamics environment community
- The RC was mounted in the Modal Analysis Test Vehicle (Hardware developed by developed by the Atomic Weapons Establishment, AWE, UK)
- The MATV was subjected to an acoustic environment with a sound-pressure level (SPL) up to 147 dB
- Response measurements taken on the RC using 4 triaxial accelerometers

Mayes et al. (2019), "Predicting system response at unmeasured locations," *International Modal Analysis Conference.*

A modal test of the RC on the fixture provided the required modal parameters

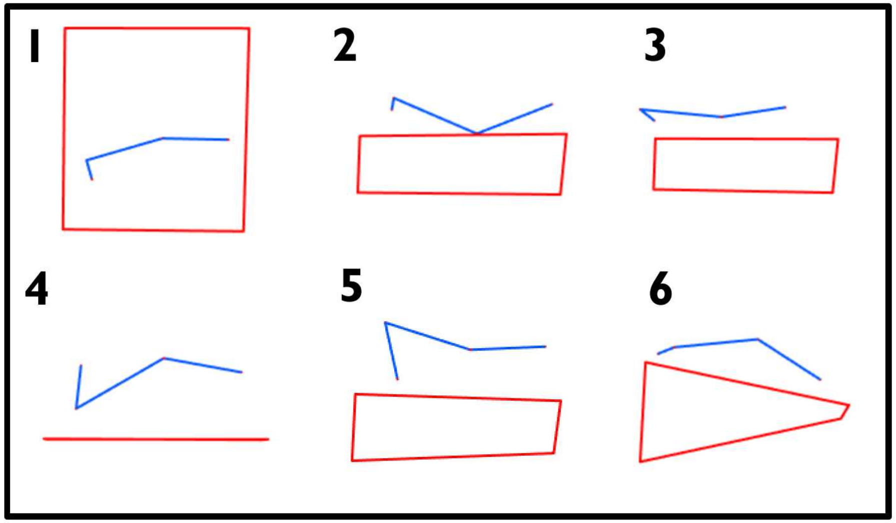
Modal Test of the RC on the fixture:

- Performed the modal test by suspending the RC/Fixture with bungee cords and impacting with modal hammer at various locations
- Constructed rigid-body modes analytically with component geometries and measured masses



Modal Parameters:

Mode	f_n [Hz]	ζ [%]
1	389.8	0.19
2	1044	0.49
3	1140	0.83
4	1648	0.97
5	1827	0.33
6	2013	0.16



A transformation to fixed-base/rigid-body motion enables a replication of the inputs provided by a shaker

Begin with the modal equations from the free-free modal test:

$$\begin{bmatrix} \mathbf{x}_{\mathrm{RC}} \\ \mathbf{x}_{\mathrm{F}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{\mathrm{RC}} \\ \boldsymbol{\Phi}_{\mathrm{F}} \end{bmatrix} \mathbf{q}$$

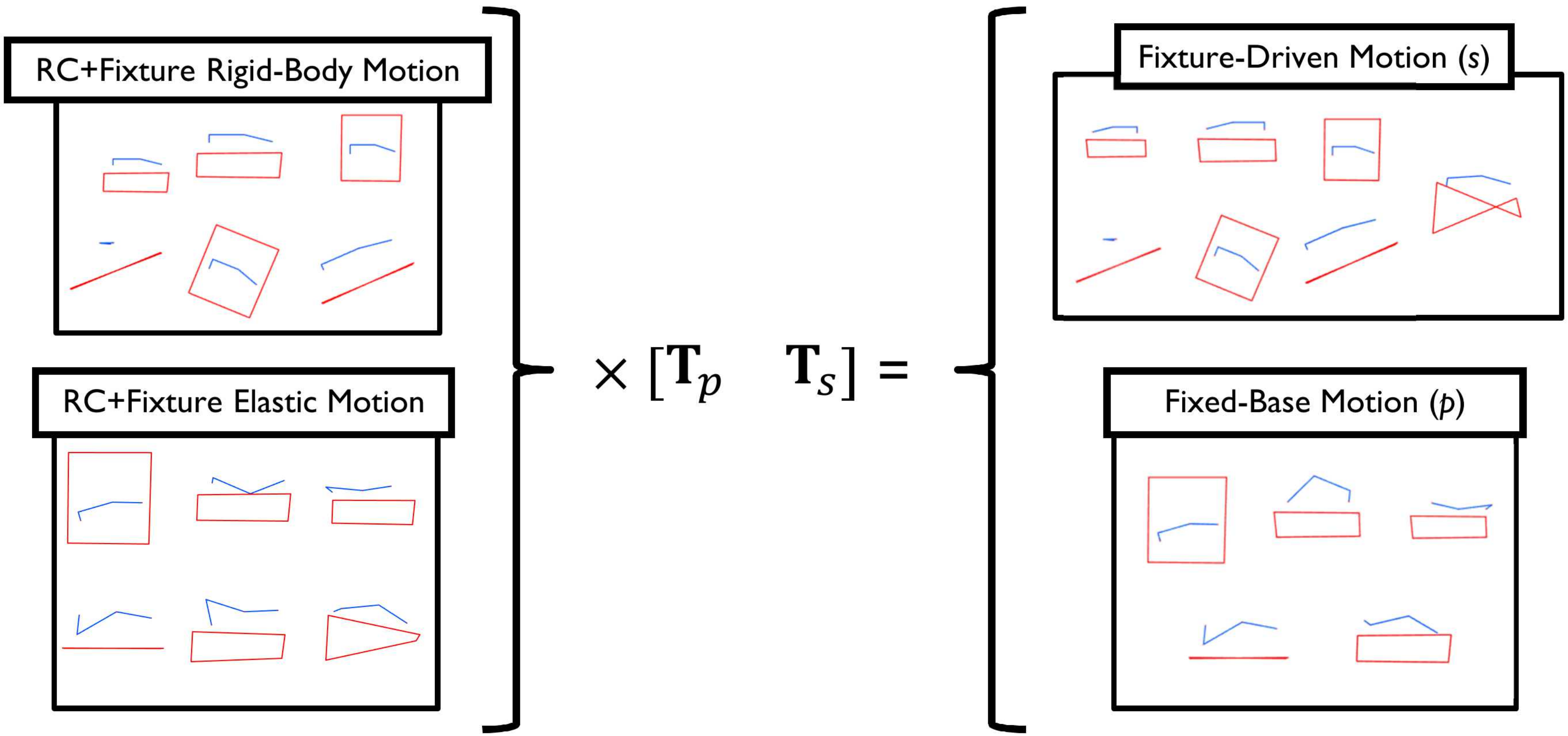
$$\left[[\boldsymbol{\omega}_{\mathrm{free}}^2] + i\omega[2\boldsymbol{\zeta}_{\mathrm{free}}\boldsymbol{\omega}_{\mathrm{free}}] - \omega^2 \mathbf{I}_q \right] \mathbf{q} = \mathbf{0}$$

Transform →

$$\mathbf{q} = [\mathbf{T}_p \quad \mathbf{T}_s] \begin{Bmatrix} \mathbf{p} \\ \mathbf{s} \end{Bmatrix}$$

p: Fixed-base modes that simulate the response on a rigid fixture

s: Fixture-driven motion



Modal Craig-Bampton procedure transforms free-free modes to a set of fixed-base modes (p) + fixture-driven modes (s)

Fixture-Driven Modal Transformation:

Equate the motion at the fixture in combined system to that of the fixture only:

$$\mathbf{x}_{\mathrm{F}} \approx \boldsymbol{\Phi}_{\mathrm{F}}\mathbf{q} \approx [\boldsymbol{\Psi}_{\mathrm{F,RB}}\ \boldsymbol{\Psi}_{\mathrm{F,E}}]\mathbf{s}$$

where

$\boldsymbol{\Psi}_{f,RB}$: Rigid-body motion of fixture calculated analytically

$\boldsymbol{\Psi}_{f,E}$: Elastic motion of fixture – without model, can use the dominant singular vectors of the SVD of the measured fixture+RC elastic mode shapes

$$\mathbf{q} \approx \underbrace{\boldsymbol{\Phi}_f^{+}[\boldsymbol{\Psi}_{\mathrm{F,RB}}\ \boldsymbol{\Psi}_{\mathrm{F,E}}]}_{\mathbf{T}_s}\mathbf{s}$$

Fixed-Base Modal Transformation:

Constrain the fixture motion to zero: $\mathbf{x}_{\mathrm{F}} \approx \boldsymbol{\Phi}_{\mathrm{F}}\mathbf{q} \approx [\boldsymbol{\Psi}_{\mathrm{F,RB}}\ \boldsymbol{\Psi}_{\mathrm{F,E}}]\mathbf{s} = \mathbf{0}$

Rewrite constraint equation as

$$\underbrace{[\boldsymbol{\Psi}_{\mathrm{F,RB}}\ \boldsymbol{\Psi}_{\mathrm{F,E}}]^{+}\boldsymbol{\Phi}_{\mathrm{F}}}_{\mathbf{B}_m}\mathbf{q} = \mathbf{0} \xrightarrow{\text{Transform}} \mathbf{q} = \mathbf{L}_m\boldsymbol{\eta}, \quad \mathbf{L}_m = \mathrm{null}(\mathbf{B}_m)$$

Inserting this transformation into free-free modal equations results in a coupling between coordinates

$$\mathbf{L}_m^T\left[[\boldsymbol{\omega}_{\mathrm{free}}^2] + i\omega[2\boldsymbol{\zeta}_{\mathrm{free}}\boldsymbol{\omega}_{\mathrm{free}}] - \omega^2\mathbf{I}_q\right]\mathbf{L}_m\boldsymbol{\eta} = \mathbf{0}$$

The eigensolution of these equations generates the fixed-base frequencies ω_{fix}^2 and eigenvectors $\boldsymbol{\Gamma}$

$$\boldsymbol{\eta} = \boldsymbol{\Gamma}\mathbf{p} \longrightarrow \mathbf{q} = \underbrace{\mathbf{L}_m\boldsymbol{\Gamma}}_{\mathbf{T}_p}\mathbf{p}$$

Mayes et al. (2015), "A modal Craig-Bampton substructure for experiments, analysis, and control specifications," *International Modal Analysis Conference.*

The transformed modal model can estimate the required 6-DOF shaker inputs and the corresponding test response

Transforming the free-free modal equations results in a coupling between the p and s modes:

$$\left[\begin{bmatrix}[\boldsymbol{\omega}_{\text{fix}}^2] & \mathbf{K}_{ps} \\ \mathbf{K}_{ps}^T & \mathbf{K}_{ss}\end{bmatrix} + i\omega\begin{bmatrix}\mathbf{T}_p^T[2\boldsymbol{\zeta}_{\text{free}}\boldsymbol{\omega}_{\text{free}}]\mathbf{T}_p & \mathbf{C}_{ps} \\ \mathbf{C}_{ps}^T & \mathbf{C}_{ss}\end{bmatrix} - \omega^2\begin{bmatrix}\mathbf{I}_p & \mathbf{M}_{ps} \\ \mathbf{M}_{ps}^T & \mathbf{M}_{ss}\end{bmatrix}\right]\begin{bmatrix}\mathbf{p} \\ \mathbf{s}\end{bmatrix} = \begin{bmatrix}\mathbf{0} \\ \mathbf{0}\end{bmatrix}$$

From the first row, the prescribed fixture motion drives the p modes:

$$\mathbf{p} = \underbrace{-\left[[\boldsymbol{\omega}_{\text{fix}}^2] + i\omega\mathbf{T}_p^T[2\boldsymbol{\zeta}_{\text{free}}\boldsymbol{\omega}_{\text{free}}]\mathbf{T}_p - \omega^2\mathbf{I}_p\right]^{-1}\left[\mathbf{K}_{ps} + i\omega\mathbf{C}_{ps} - \omega^2\mathbf{M}_{ps}\right]}_{\mathbf{H}_{ps}}\mathbf{s}$$

Only include rigid-body fixture motion

Consequently, the s modes also drive the physical response:

$$\mathbf{x}_{\text{RC}} = \underbrace{\boldsymbol{\Phi}_{\text{RC}}(\mathbf{T}_p\mathbf{H}_{ps} + \mathbf{T}_s)}_{\mathbf{H}_{xs}}\mathbf{s}$$

A least-squares fit estimates the input PSD and corresponding response PSD:

$$\mathbf{S}_{xx} = \mathbf{H}_{xs}\mathbf{S}_{ss}\mathbf{H}_{xs}^H \rightarrow$$

Estimated Input

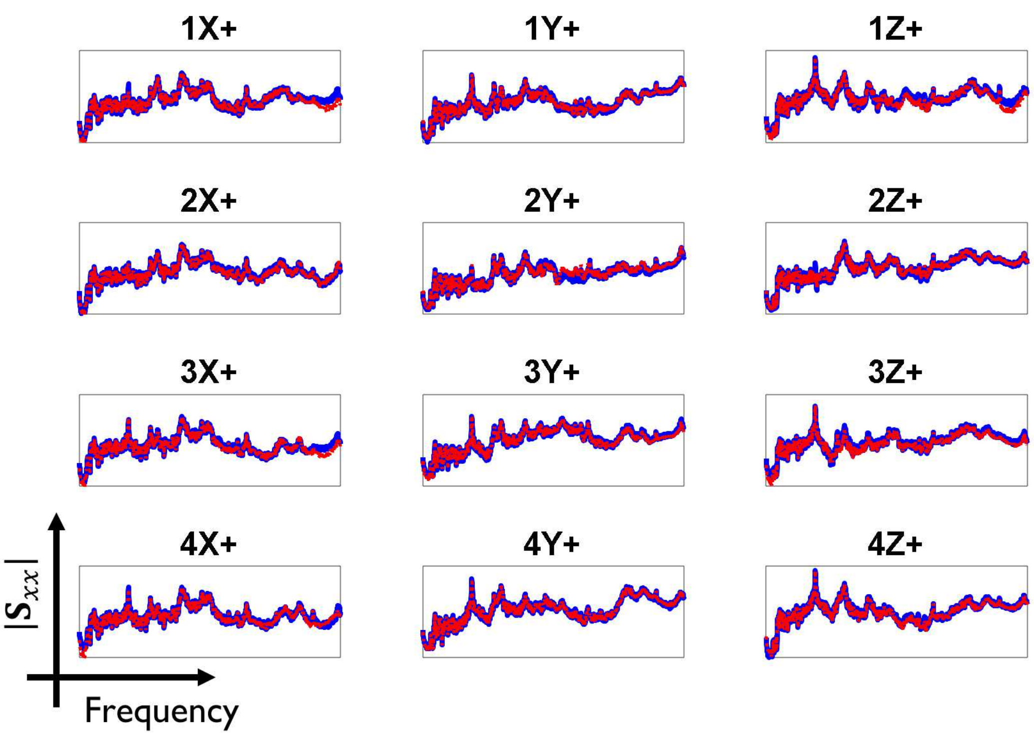
$$\hat{\mathbf{S}}_{ss} = (\mathbf{H}_{xs})^+\mathbf{S}_{xx}(\mathbf{H}_{xs}^H)^+ \rightarrow$$

Estimated Response

$$\hat{\mathbf{S}}_{xx} = \mathbf{H}_{xs}\hat{\mathbf{S}}_{ss}\mathbf{H}_{xs}^H$$

6-DOF shaker able to control the RC to match the response measured in the acoustic environment

ASDs for each response

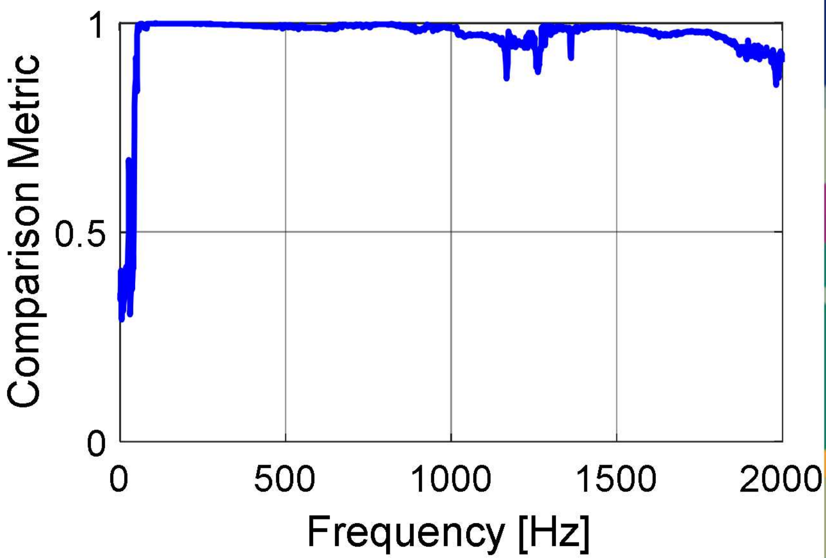


The 6-DOF shaker accurately reproduces the response in all directions and at all frequencies

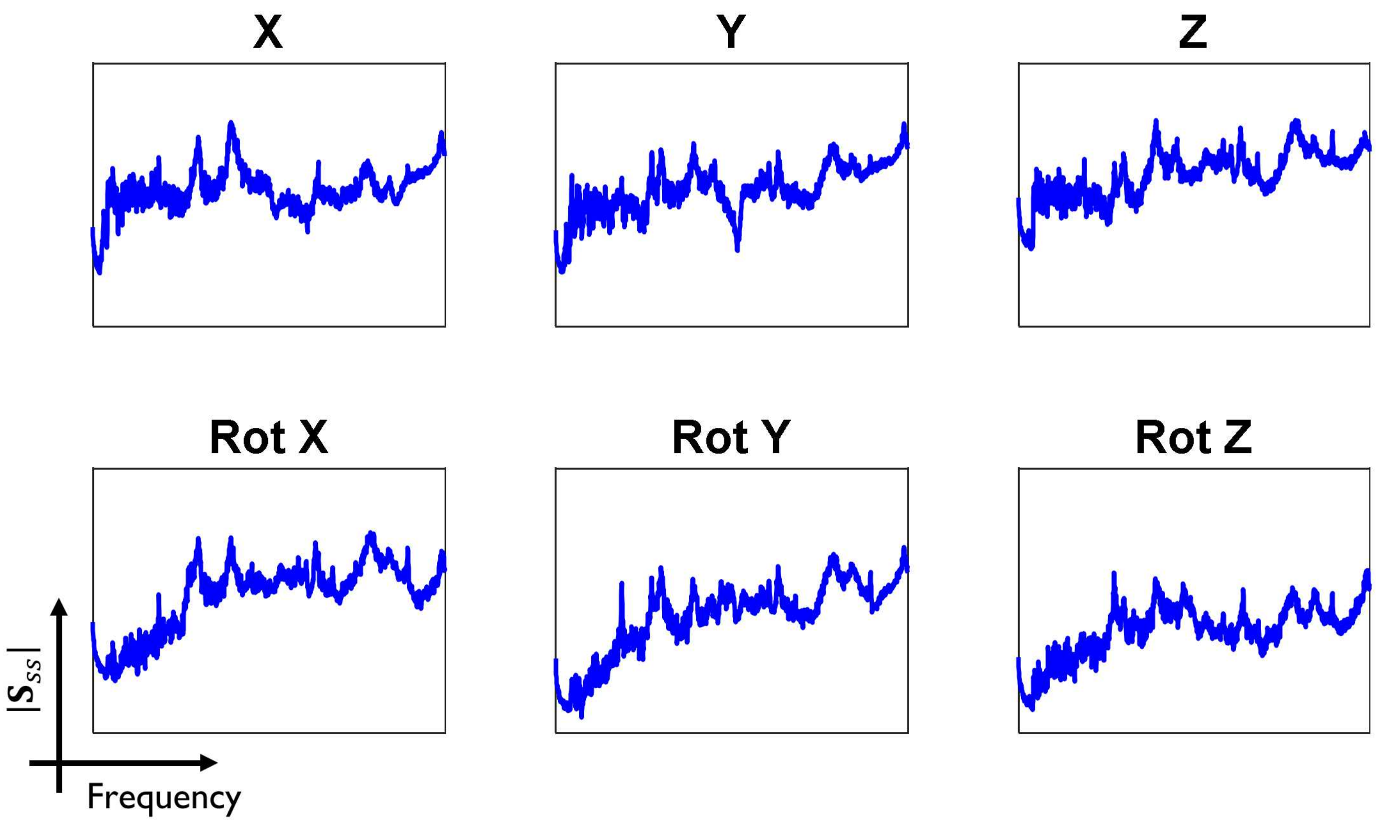
Comparison Metric

(Calculated for all responses at each frequency line)

$$R = \frac{|\mathrm{tr}(\hat{\mathbf{S}}_{xx}^{H}\mathbf{S}_{xx})|^2}{\mathrm{tr}(\mathbf{S}_{xx}^{H}\mathbf{S}_{xx})\mathrm{tr}(\hat{\mathbf{S}}_{xx}^{H}\hat{\mathbf{S}}_{xx})}$$



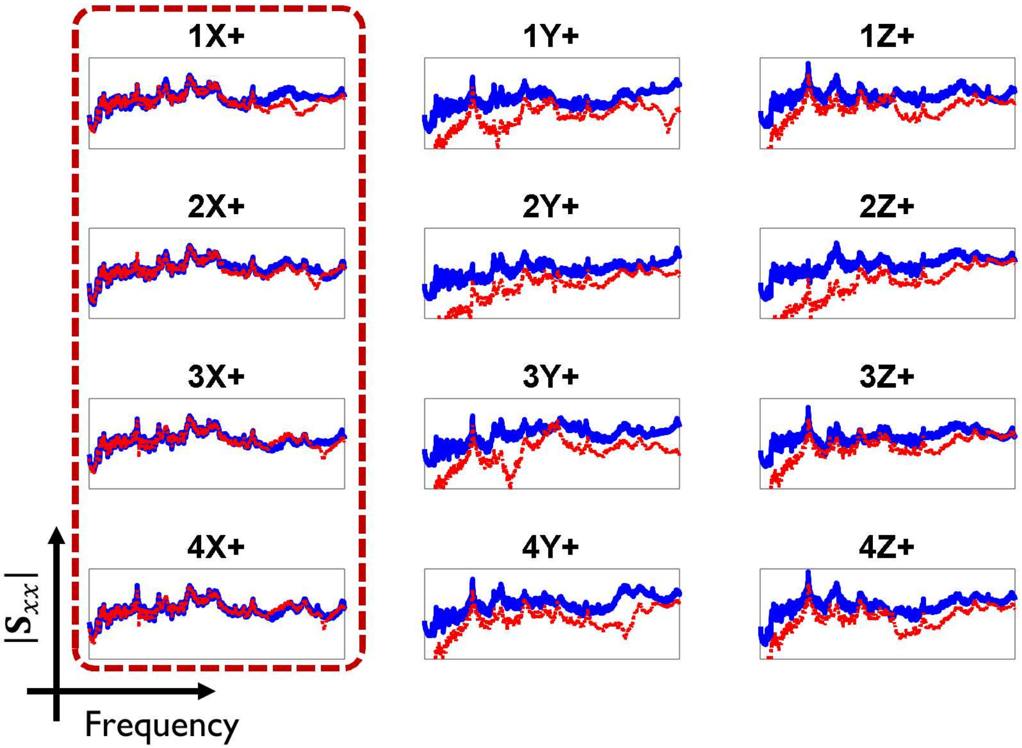
This approach also enables an estimation of the 6-DOF rigid-body inputs



Accurately reproducing the measured response requires inputs in all directions, including rotations

This approach applies not only to a 6-DOF shaker, but also a traditional SDOF shaker

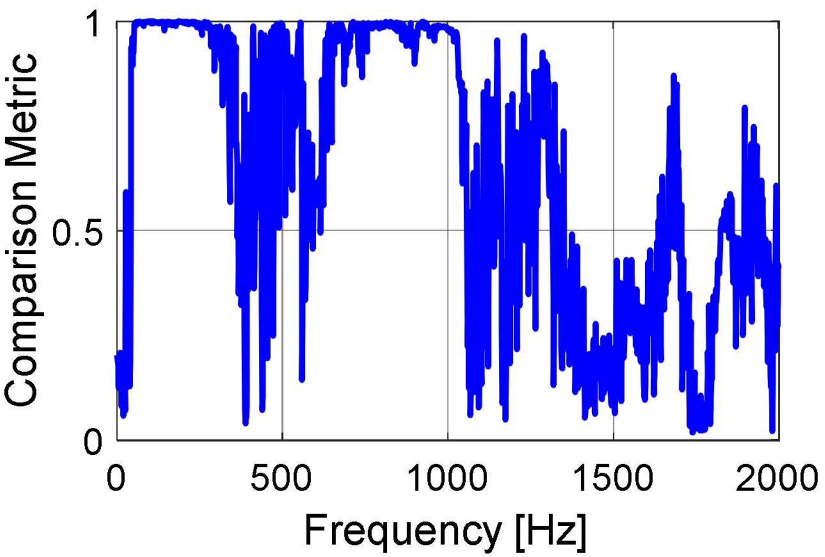
ASDs for each response



$\mathbf{S}_{xx}$: Acoustic Test
$\hat{\mathbf{S}}_{xx}$: SDOF Simulation (*x* dir)

Shaker constrained to motion in the *x* direction

Targeting responses in the *x* direction can accurately reproduce these responses, especially at lower frequencies



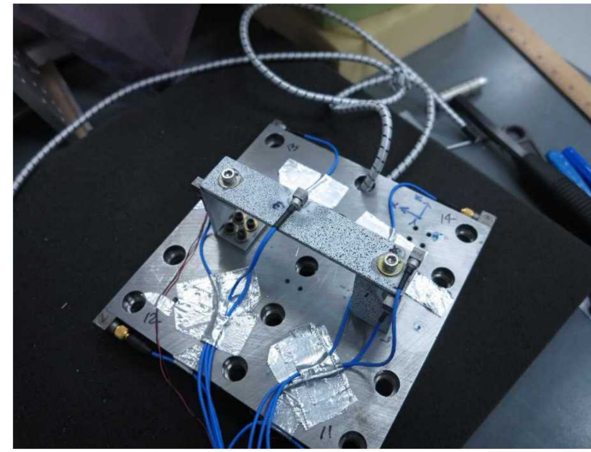
Comparison Metric
(Only calculated for the *x* responses at each frequency line)

$$R = \frac{|\mathrm{tr}(\hat{\mathbf{S}}_{xx}^H \mathbf{S}_{xx})|^2}{\mathrm{tr}(\mathbf{S}_{xx}^H \mathbf{S}_{xx})\mathrm{tr}(\hat{\mathbf{S}}_{xx}^H \hat{\mathbf{S}}_{xx})}$$

Modal analysis can aid in planning shaker tests

Modal Craig-Bampton Procedure:

- Transforms the modes from a free-free modal test of component on the fixture to a set of fixed-base and fixture-driven modes
- Enables an assessment of the feasibility of the shaker/rigid fixture to match the measured response
- Enables an estimate of the shaker inputs required to best match the measured environmental response



Shaker Performance Predictions:

- The 6-DOF shaker can successfully control the RC on a rigid fixture to reproduce the measured environmental response in all directions simultaneously
- A traditional SDOF shaker can also control the RC to reproduce the measured environmental response in a single direction