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Using Modal Projection Error to Predict Success of a Six Degree of Freedom Shaker Test

Tyler Schoenherr, Janelle Lee

Sandia National Laboratories

Justin Porter

Rice University

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Motivation / Goals of the Research

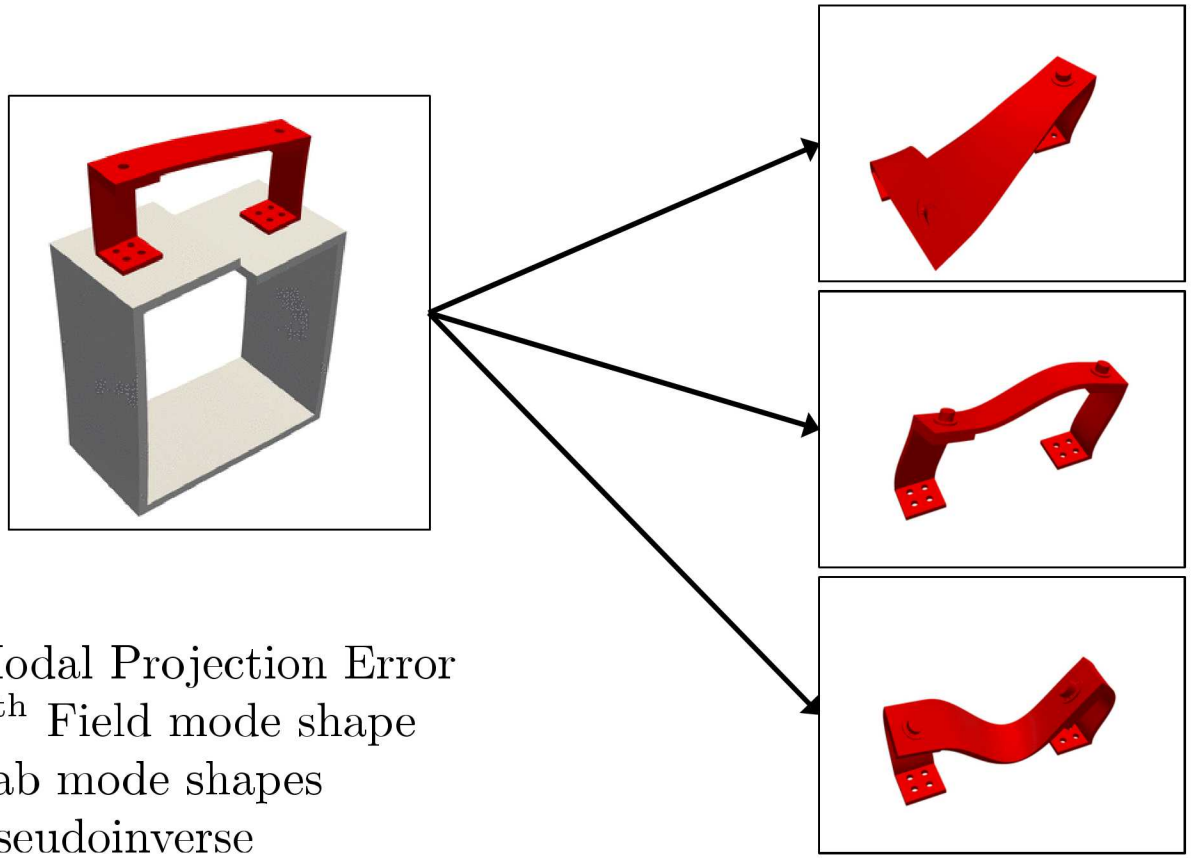
- Component level testing is typically done on shaker tables with "rigid" fixtures.
- How do we know if the right modes can be excited and, therefore, the right stress distributions?
- Can we examine the mode shapes of the field and laboratory configurations to determine the success of the laboratory?

Goal: With a (flawed) model, predict if a 6 DOF shaker test with rigid fixture will be successful in reproducing the field environment.

- Use modal projection error as the quantity of interest

Explanation of the Modal Projection Error

$$MPE = \Psi_n^2 = 1 - \bar{\phi}_{Fn}^{+} \boldsymbol{\phi}_L \boldsymbol{\phi}_L^{+} \bar{\phi}_{Fn}$$



Ψ_n^2 = Modal Projection Error
$\bar{\phi}_{Fn}$ = n$^{\text{th}}$ Field mode shape
$\boldsymbol{\phi}_L$ = Lab mode shapes
$^{+}$ = Pseudoinverse

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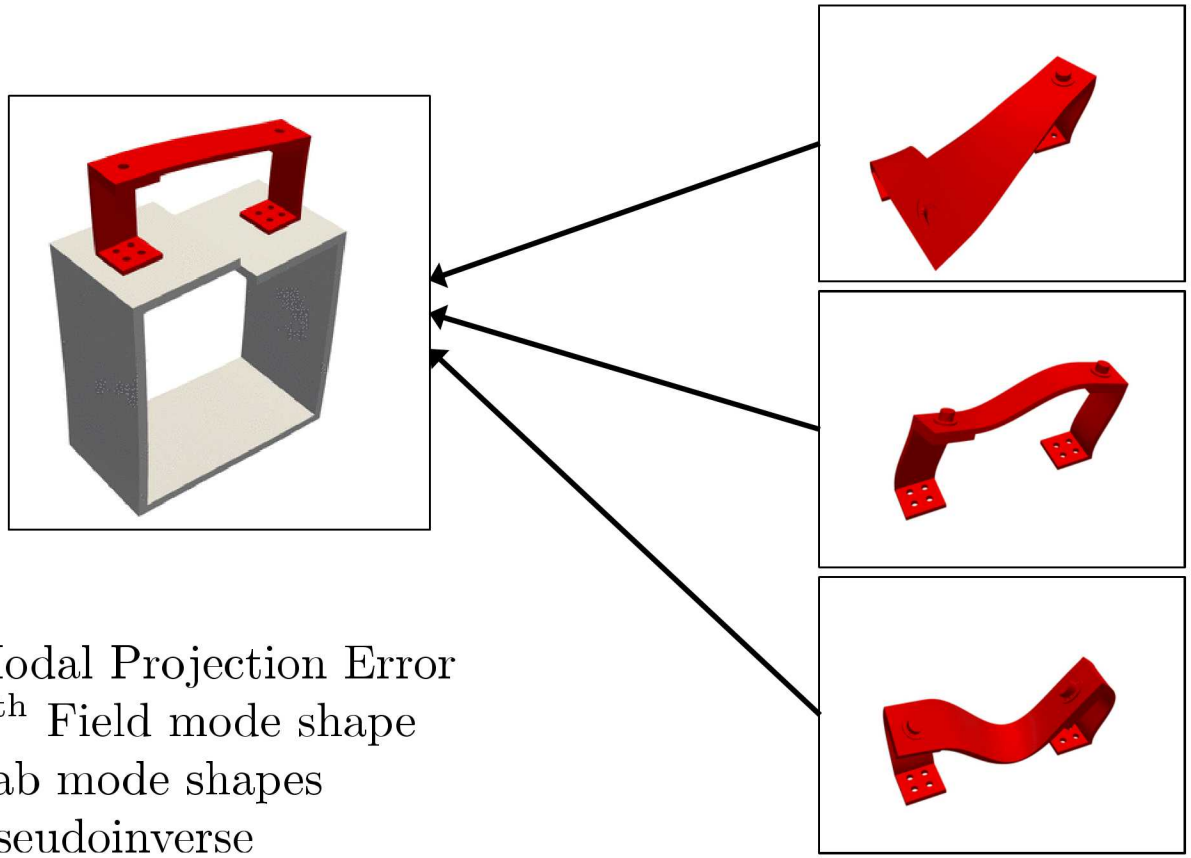

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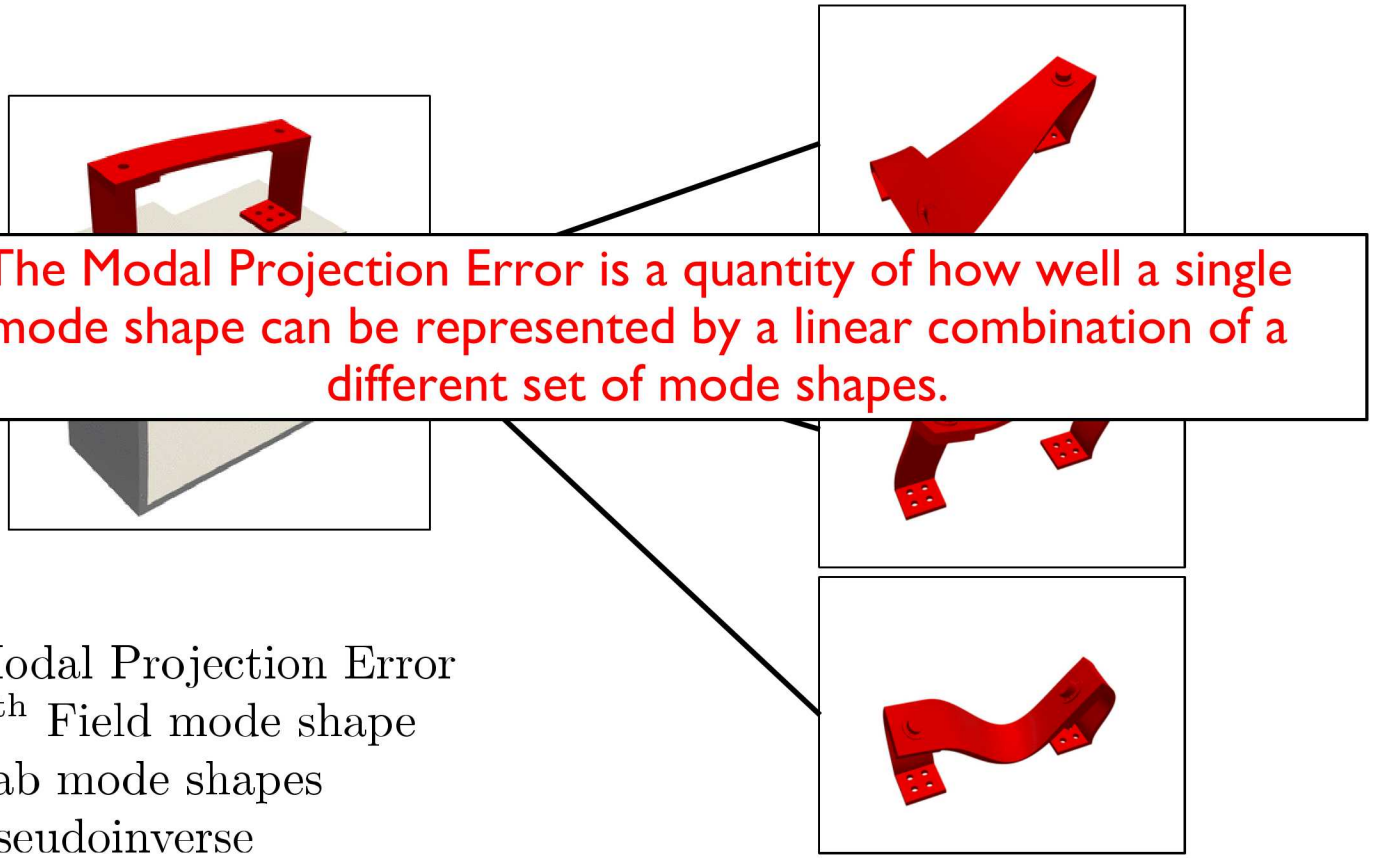

The Modal Projection Error is a quantity of how well a single mode shape can be represented by a linear combination of a different set of mode shapes.

Ψ_n^2 = Modal Projection Error
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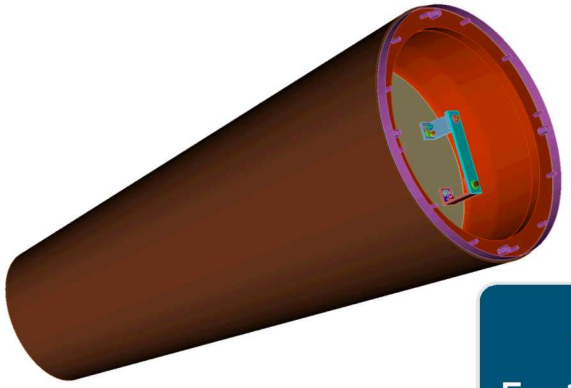
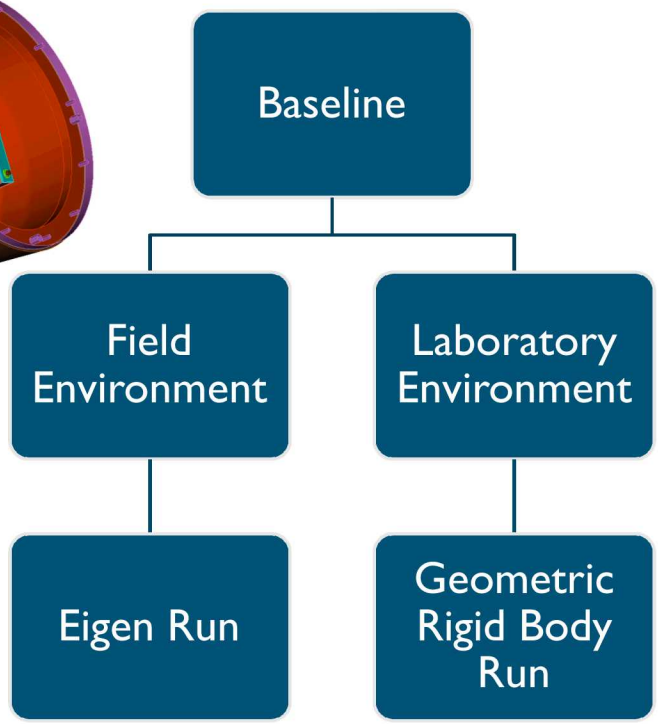
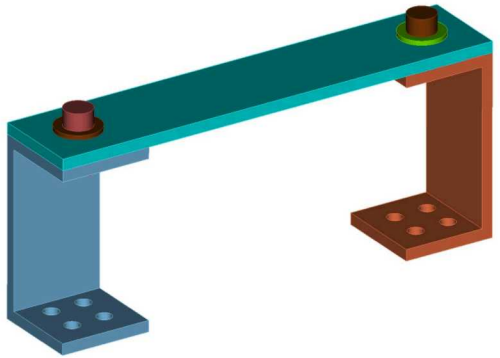
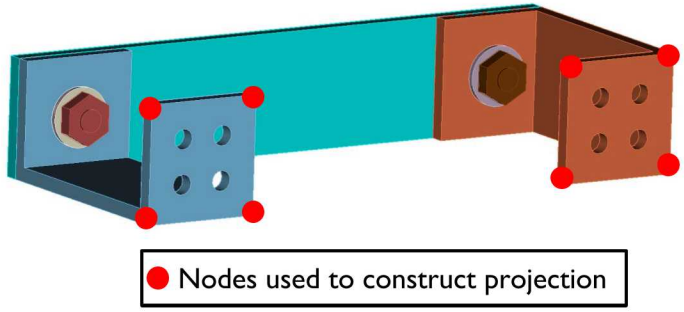
Process Proposal

- (Modal Analysis Test Vehicle) MATV FEM was used to generate field configuration mode shapes of the Removable Component (RC).
- The removable component was isolated for the laboratory environment and 'attached' to a rigid fixture. The resulting mode shapes were only rigid body modes.
- Only the nodes on the base of the RC were used in the projection error.









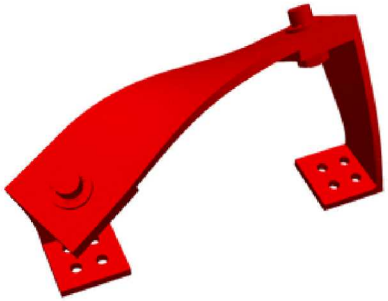
Laboratory Mode Shapes

Mode 7

Mode 8

Mode 9

Mode 10



Mode 11

Mode 12

Field Mode Shapes

Mode 7

Mode 8

Mode 9

Mode 10

Mode 11

Mode 12

RC Active Mode Shapes in Field Configuration

Mode 7

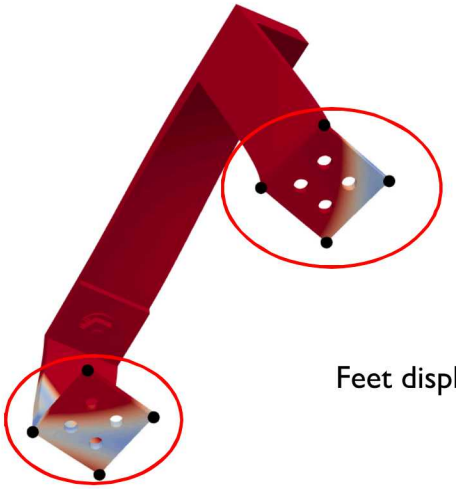
Mode 12

Initial Modal Projection Error Results

- The modal projection error between the field configuration and component level configuration was calculated.
- The rigid body modes of the field configuration were perfectly projected to by the rigid body modes of the component configuration.
- The error for the elastic modes of the field configuration was high (>5%) for almost all of the modes. This was not expected due to first mode of the field being essentially a FBM of the component.
- Using only the connection degrees of freedom was a bad idea.

$$\Psi_n^2 = \begin{pmatrix} -8.9e-14\ \% \\ -4.4e-14\ \% \\ -2.2e-14\ \% \\ 0\ \% \\ 1.1e-8\ \% \\ 2.4e-8\ \% \\ 11.0\ \% \\ 17.0\ \% \\ 34.0\ \% \\ 5.0\ \% \\ 11.0\ \% \\ 18.0\ \% \\ 77.0\ \% \\ 27.0\ \% \\ 2.1\ \% \\ 1.9\ \% \\ 74.0\ \% \\ 24.0\ \% \\ 0.87\ \% \\ 0.86\ \% \end{pmatrix}$$

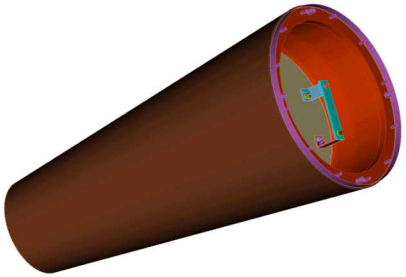
Rigid body modes exhibit a low modal projection compared to flexible modes

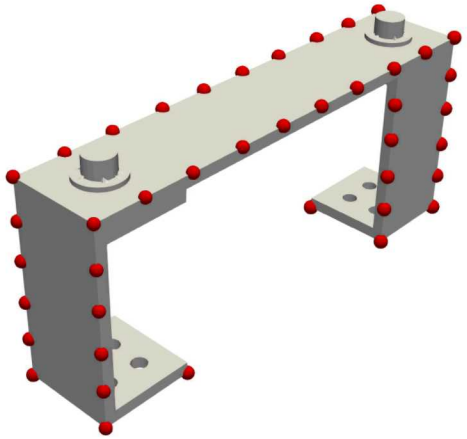


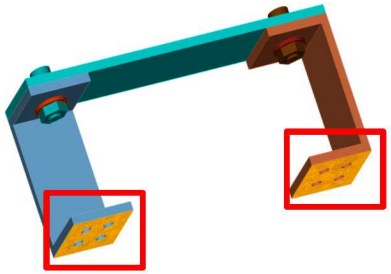
Feet displacement of a field elastic mode

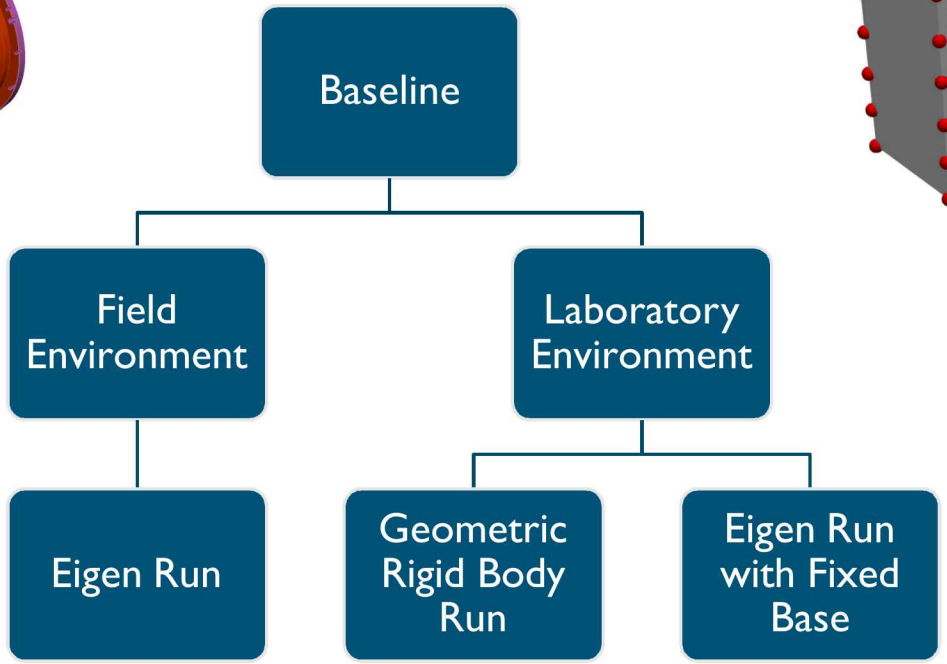
Revised Computation of Modal Projection Error

- More DOFs were added to the modal projection error analysis that span the space of the component rigid and fixed base modes
- Because more DOFs were added, the component configuration modes needed to be augmented with fixed base mode shapes.









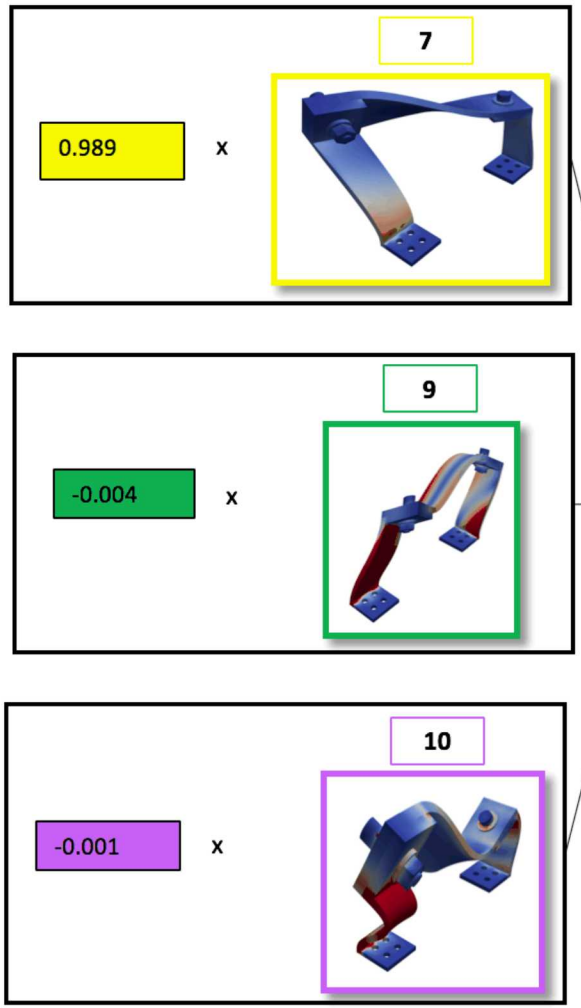
Stress Superposition for Field Mode 7

Laboratory Mode Shapes

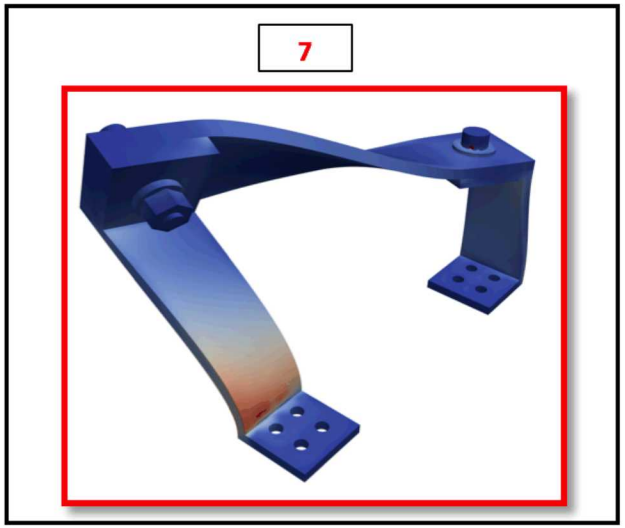
$\bar{q}_L =$

Field Mode Shapes	7	8	9	10
5	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000
7	0.989	0.000	-0.004	-0.001
8	-0.001	0.000	-0.001	0.000
9	-0.005	0.000	-0.001	0.000

Laboratory Mode Shapes



Field Mode Shape



Modal Projection Error

$$\Psi_n^2 = \begin{pmatrix} 1.5e-8\ \% \\ 7.4e-9\ \% \\ 2.9e-9\ \% \\ 5.4e-7\ \% \\ 1.1e-7\ \% \\ 2.0e-7\ \% \\ 7.6e-5\ \% \\ 1.1\ \% \\ 0.16\ \% \\ 0.22\ \% \\ 0.96\ \% \\ 0.57\ \% \\ 1.9\ \% \\ 1.7\ \% \\ 0.58\ \% \end{pmatrix}$$

* Greatest three q_L values are shown
* Scaling is colored according to VonMises stress

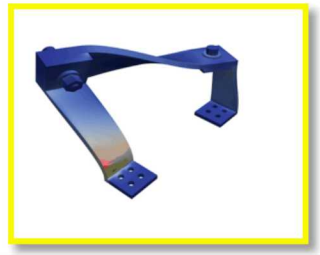
Stress Superposition for Field Mode 12

Laboratory Mode Shapes

$\bar{q}_L =$

Field Mode Shapes	7	8	9	10	11	12
10	-0.009	-0.001	0.000	0.000	0.000	0.000
11	0.001	0.000	0.003	-0.001	0.002	0.000
12	-0.073	-0.003	0.019	0.001	-0.005	0.000
13	0.000	0.000	-0.001	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	-0.001	0.000

Laboratory Mode Shapes

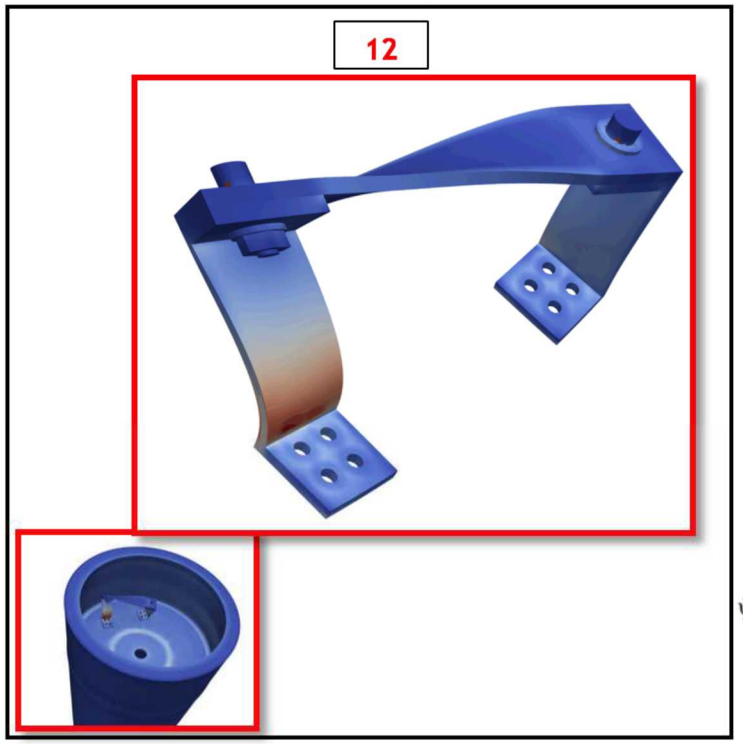
-0.073 x 

-0.028 x 

-0.039 x 

Σ

Field Mode Shape



Modal Projection Error

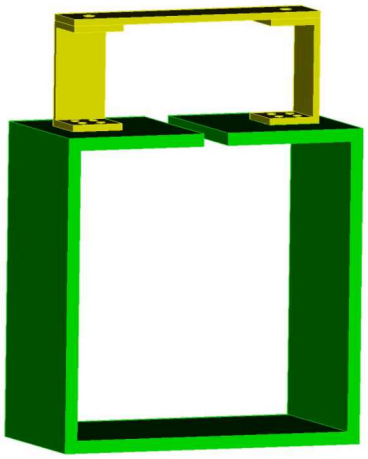
$\Psi_n^2 =$

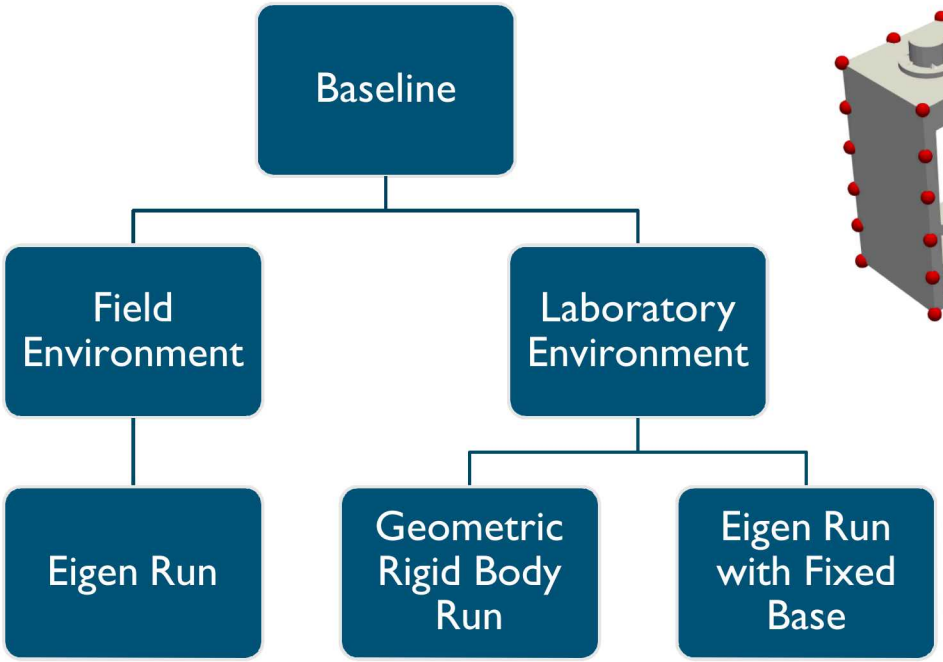
1.1 %
0.16 %
0.22 %
0.96 %
0.57 %
1.9 %
1.7 %
0.58 %

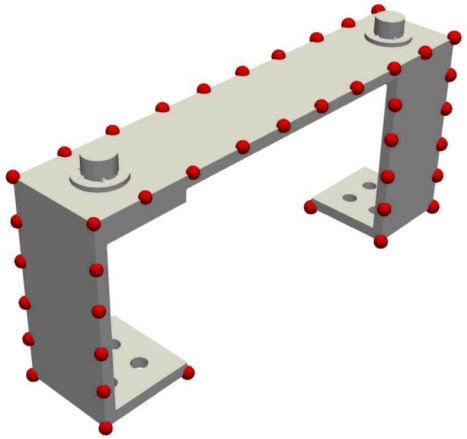
* Greatest three q_L values are shown
* Scaling is colored according to VonMises stress

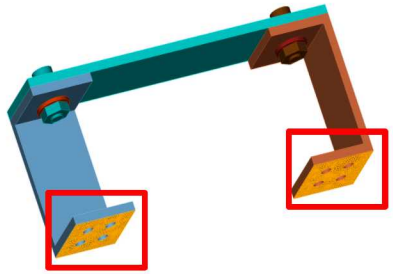
Alternate Field System, BARC

- Box Assembly with Removable Component (BARC) was used to generate the field environment mode shapes
- The BARC was designed so that the RC would not represent its field motion when attached to a rigid fixture.

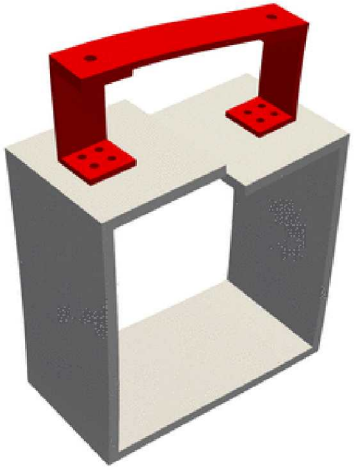




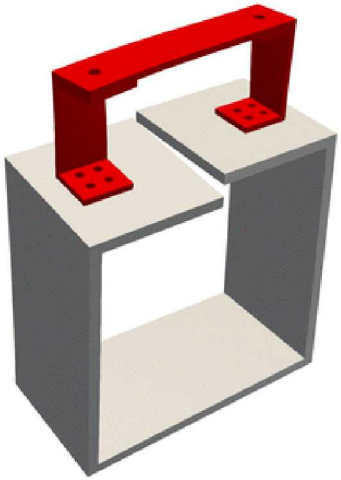




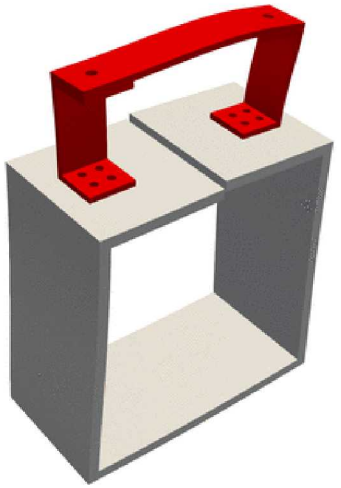
Mode Shapes of the BARC System



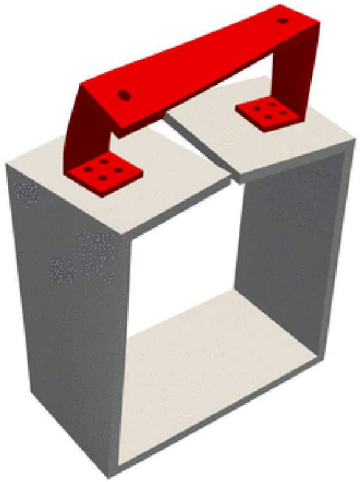
Mode 7



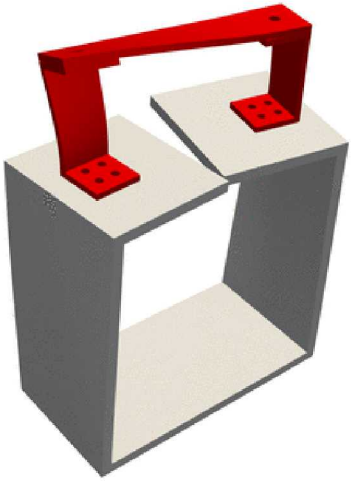
Mode 8



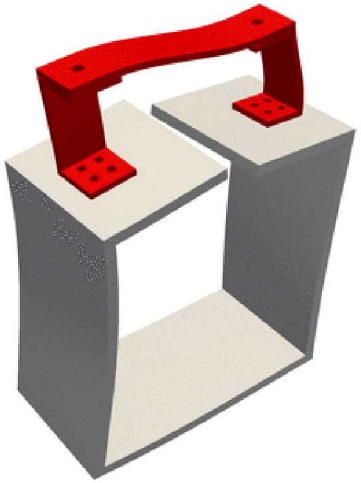
Mode 9



Mode 10



Mode 11



Mode 12

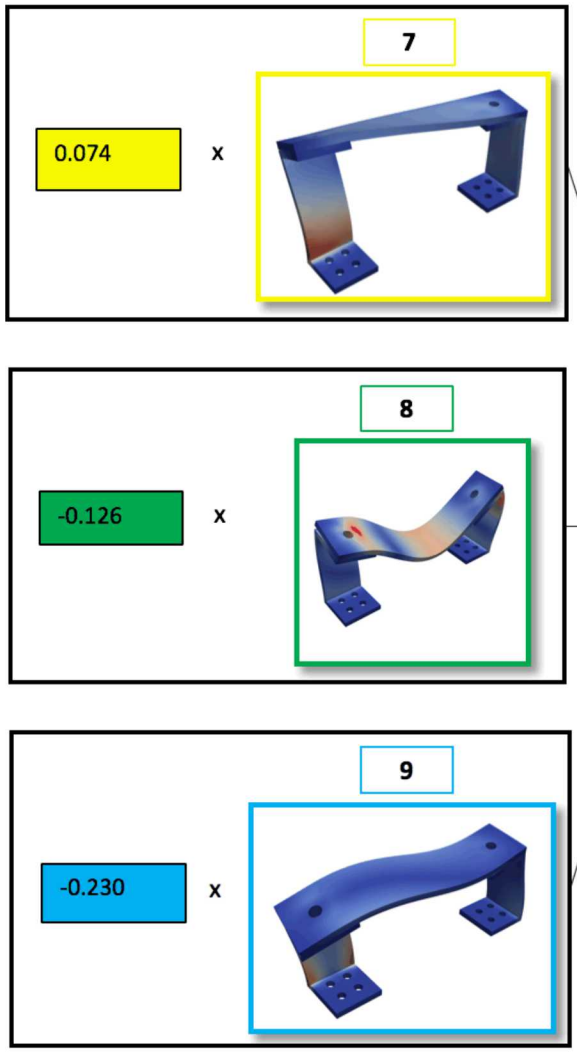
Stress Superposition for Field Mode 9

Laboratory Mode Shapes

Field Mode Shapes	7	8	9	10
7	0.033	-0.091	0.299	0.008
8	0.073	-0.049	0.027	-0.028
9	0.074	-0.126	-0.230	0.041
10	-0.744	0.021	0.132	-0.019
11	0.752	0.019	0.023	0.000

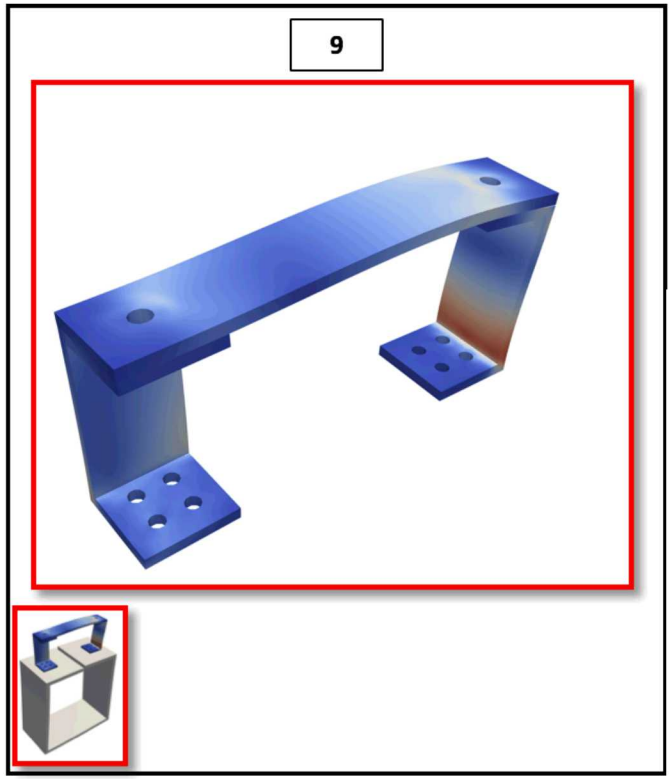
$$\bar{q}_L =$$

Laboratory Mode Shapes



Σ

Field Mode Shape



Modal Projection Error

$$\Psi_n^2 = \begin{pmatrix} -2.4e-13\ \% \\ -8.9e-14\ \% \\ 1.7e-13\ \% \\ -2.7e-13\ \% \\ 0 \\ -6.7e-14\ \% \\ 6.8\ \% \\ 8.8\ \% \\ 37.0\ \% \\ 2.1\ \% \\ 4.1\ \% \\ 4.4\ \% \\ 16.0\ \% \\ 26.0\ \% \\ 1.9\ \% \end{pmatrix}$$

* Scaling is colored according to VonMises stress

Summary

- The Modal Projection Error is a quantity of interest that can be used to predict failure of a 6 DOF test.
  - Success of a test is defined here by being able to reproduce the mode shapes of the field to the same levels and, therefore, the same stresses.
- Using full field motion from the FEA is important when calculating the MPE.