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SAND2020-1121C

Designing an Optimized Fixture for the BARC Hardware Using a Parameterized Model

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Date: 02/11/2020

IMAC 2020

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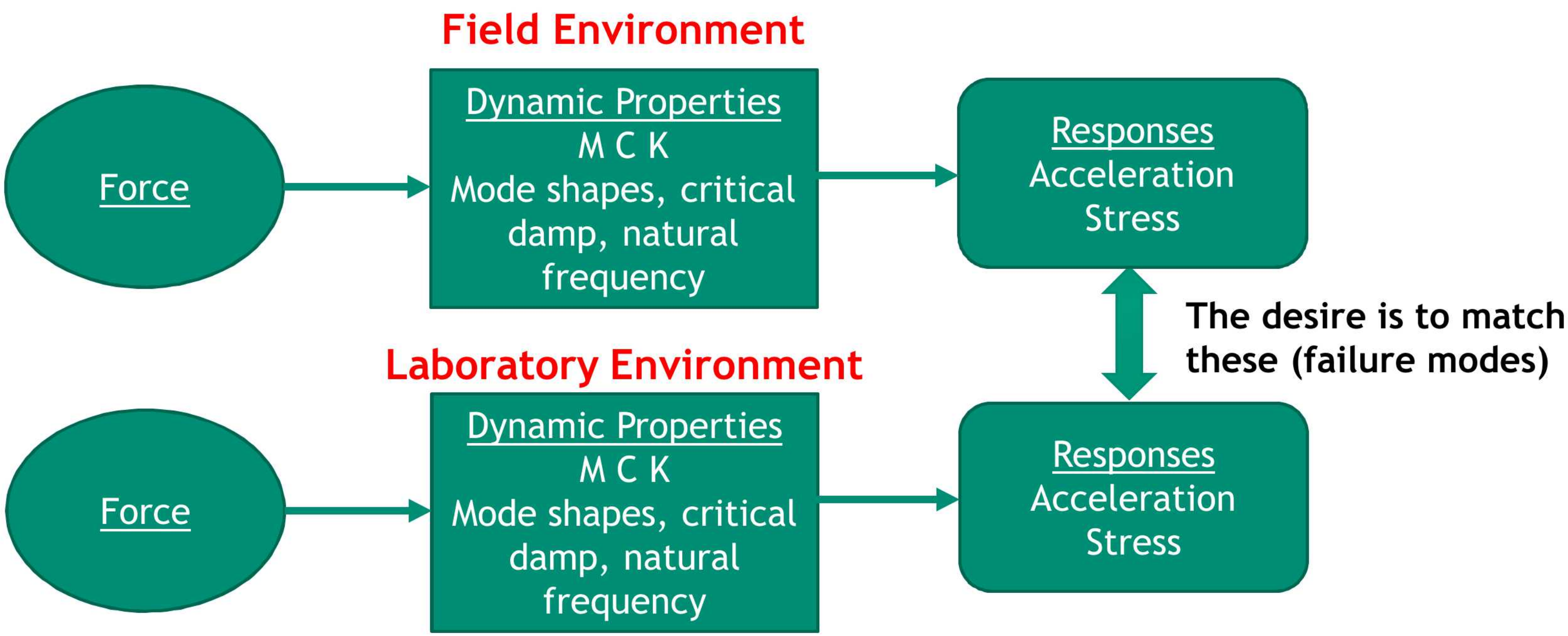
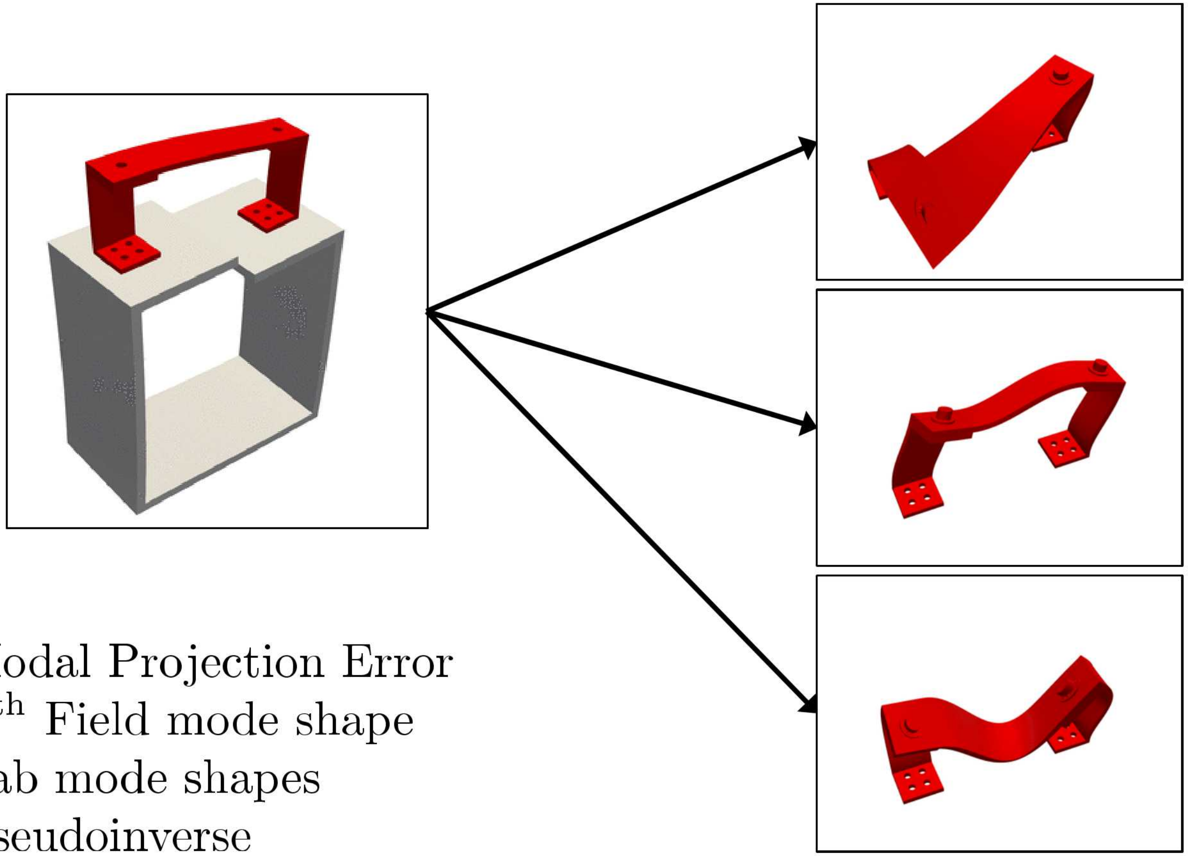
What makes a good test fixture?

- In structural dynamics, the response is a sum of the component's mode shapes (modal superposition). These set of shapes are defined by the component and its boundary conditions (the test fixture).
- How do we know if a test fixture will allow us to observe the desired response of the unit under test? How well does the mode shapes of the laboratory span the space of the mode shapes of the field?



Explanation of the Modal Projection Error

$$MPE = \Psi_n^2 = 1 - \bar{\phi}_{Fn}^{+} \boldsymbol{\phi}_L \boldsymbol{\phi}_L^{+} \bar{\phi}_{Fn}$$



Ψ_n^2 = Modal Projection Error
$\bar{\phi}_{Fn}$ = $\mathrm{n^{th}}$ Field mode shape
$\boldsymbol{\phi}_L$ = Lab mode shapes
$^{+}$ = Pseudoinverse

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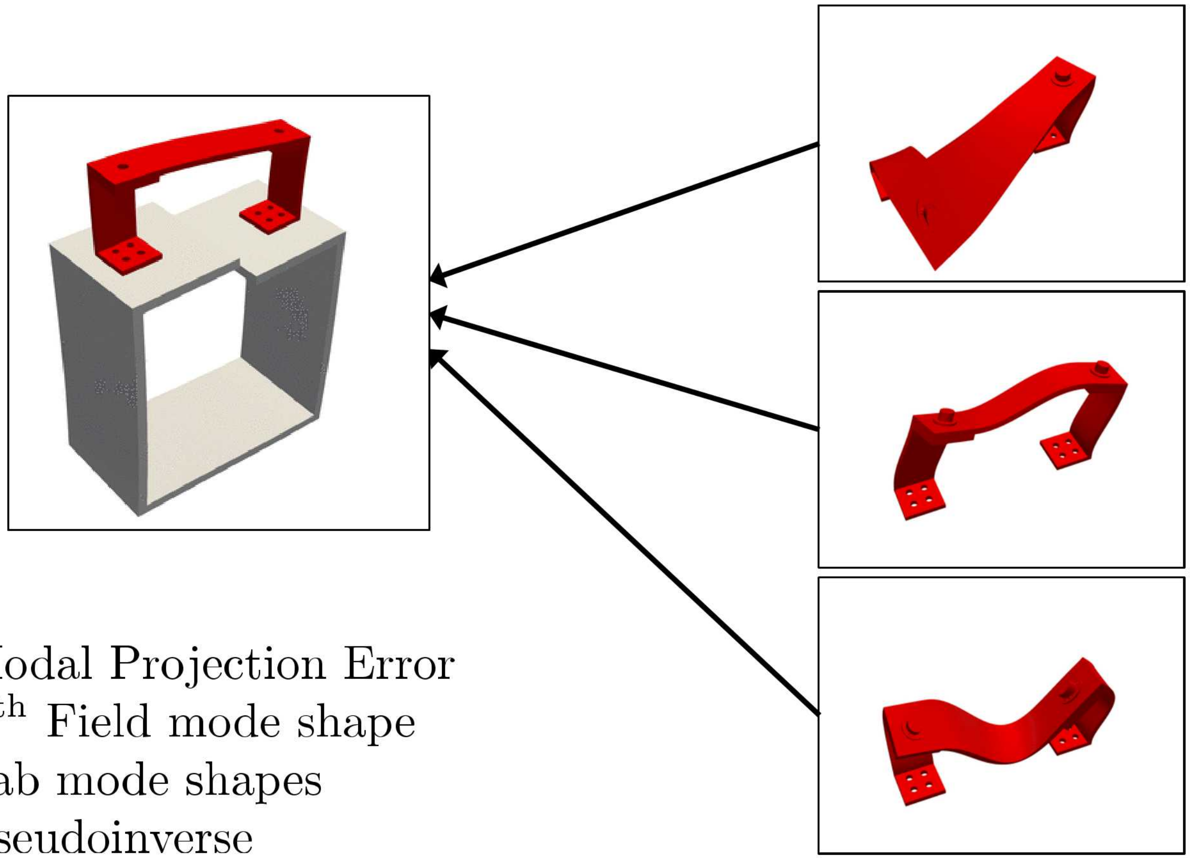

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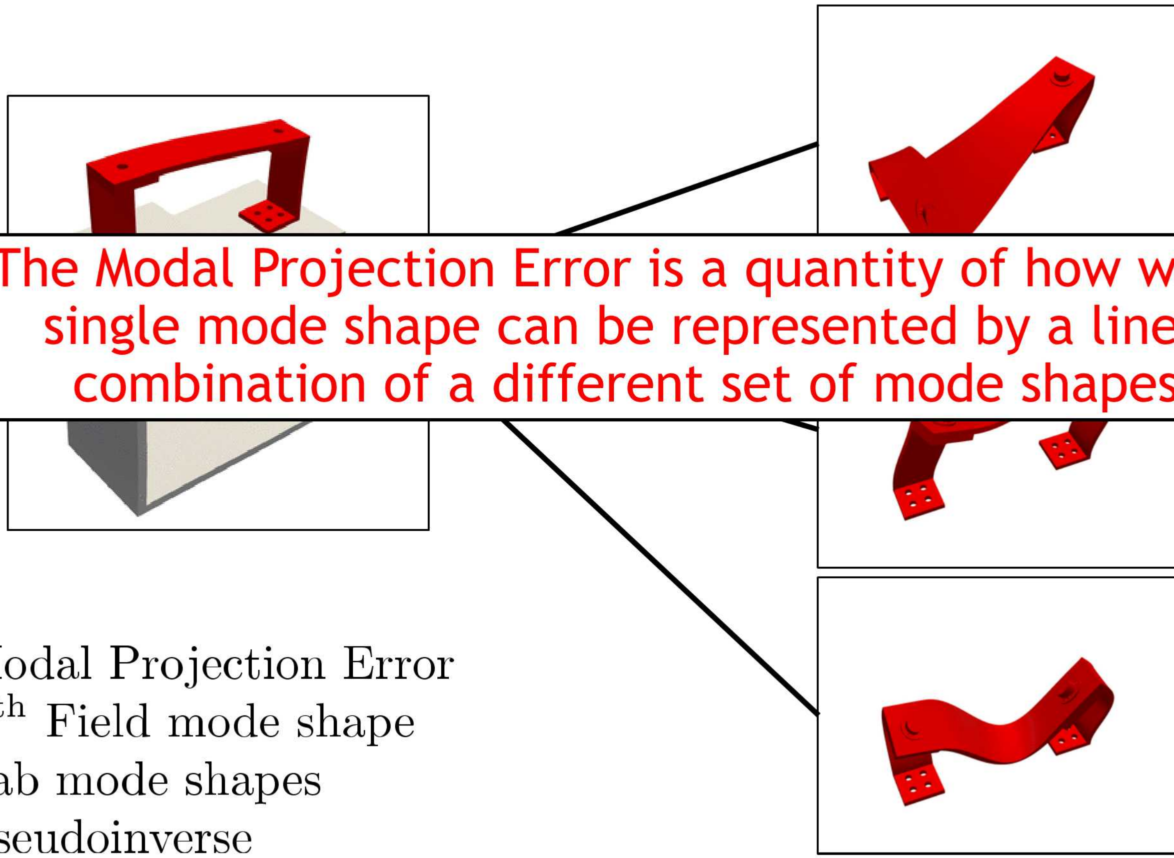


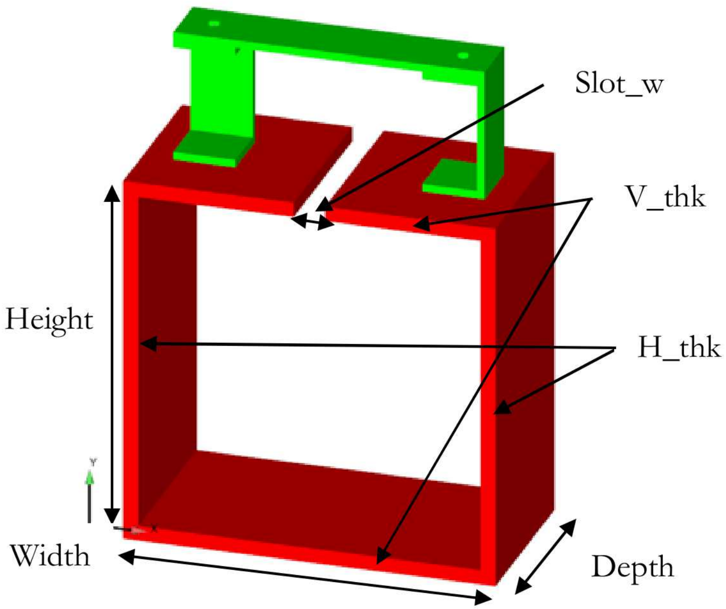
The Modal Projection Error is a quantity of how well a single mode shape can be represented by a linear combination of a different set of mode shapes.

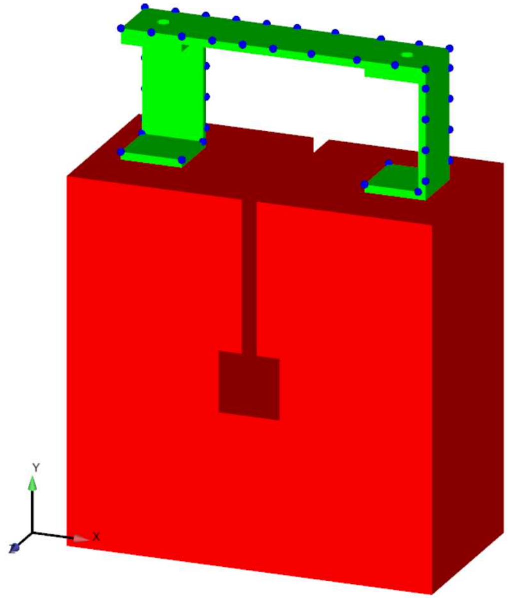
Ψ_n^2 = Modal Projection Error
$\bar{\phi}_{Fn}$ = n^{th} Field mode shape
$\boldsymbol{\phi}_L$ = Lab mode shapes
$^{+}$ = Pseudoinverse

Case Study – Optimization on Modal Projection Error the BARC Assembly

- Six parameters of the system were modified and optimized to reproduce the original (or reference) system.
- The objective function was the averaged the MPE of the first 15 elastic modes of the Removable Component.
- The number of modes included in the MPE calculation greatly affected the optimization and result (15 modes converged, 8 modes almost converged)

	Reference	Initial Cond.	Optimized
H_thk	0.238 in	2.25 in	0.235 in
V_thk	0.238 in	2.25 in	0.239 in
Slot_w	0.5 in	0.25 in	0.50 in
Height	5.98 in	6 in	5.92 in
Width	5.98 in	6 in	5.99 in
Depth	3 in	3 in	3.00 in





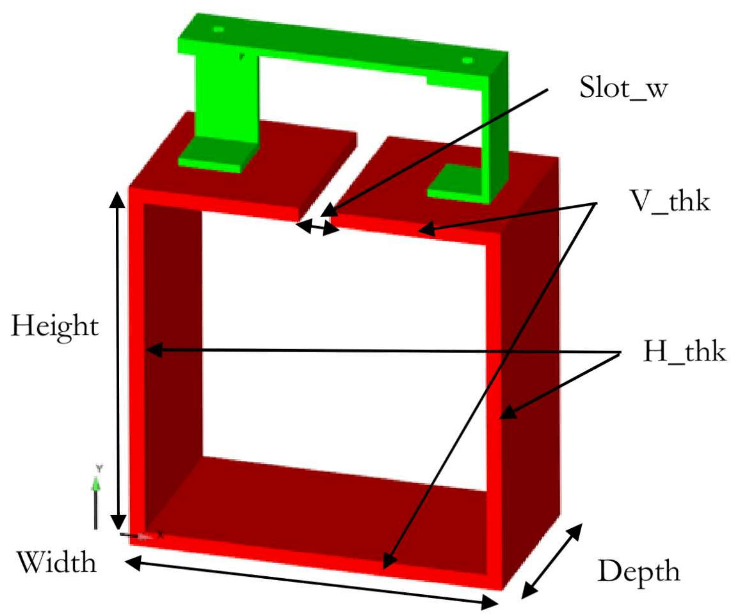
Case Study – Non-Trivial Optimization on Modal Projection Error the BARC Assembly

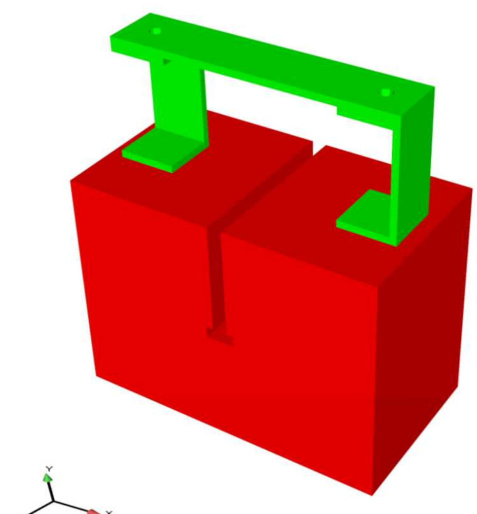
- Six parameters of the system were modified and optimized to reproduce a limited system where the height was not allowed to be larger than 4 in.

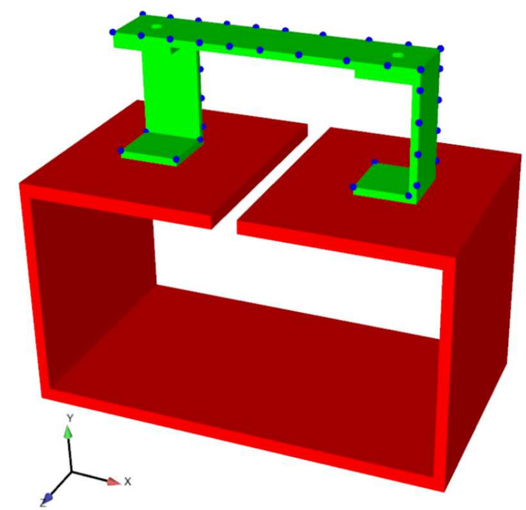
	Reference	Initial Cond.	Optimized
H_thk	0.238 in	2.5 in	0.18 in
V_thk	0.238 in	2.5 in	0.23 in
Slot_w	0.5 in	0.25 in	0.41 in
Height	6 in	3 in	3.98 in
Width	6 in	6 in	7.05 in
Depth	3 in	3 in	3.91 in

- The objective function was the averaged the MPE of the first 8 elastic modes of the Removable Component. Then restarted to an objective function of only the first two elastic modes.

- Compared optimized fixture to the initial condition of the optimization as comparison of using a “rigid” fixture.

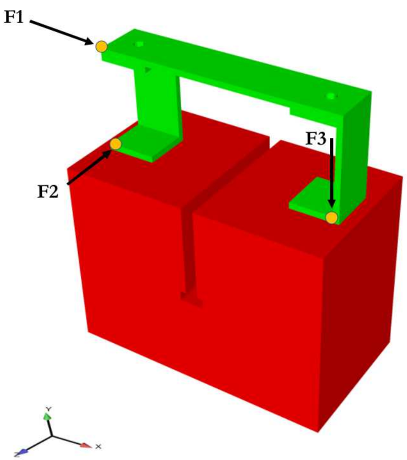


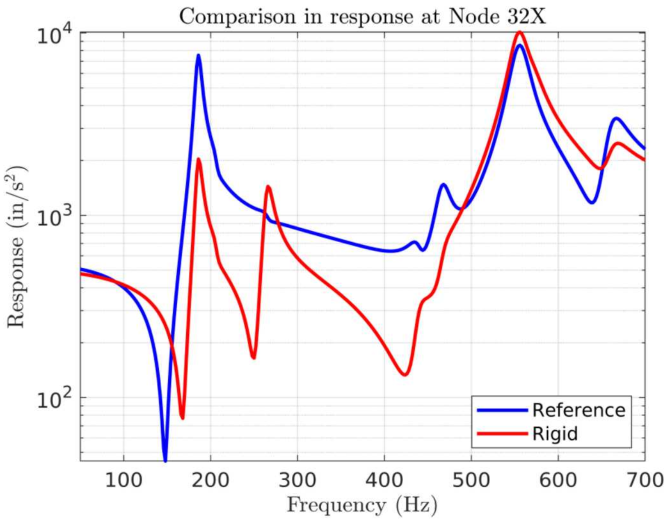


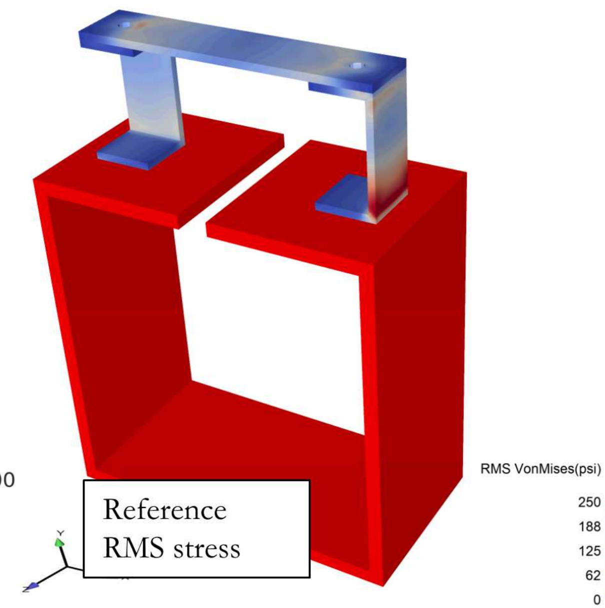


Case Study – Non-Trivial Optimization on Modal Projection Error the BARC Assembly

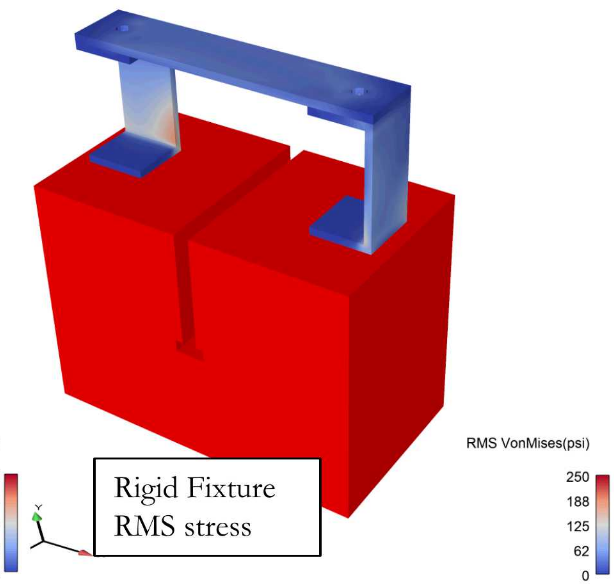
- To compare current methods to ones with the proposed optimized fixture, the reference environment was compared to an environment with a rigid fixture and 3 forcing locations.
  - Locations were selected based on modal activity







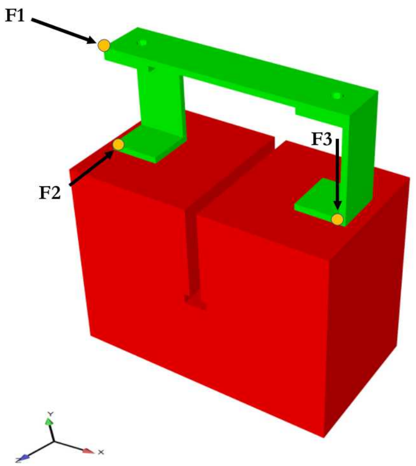
Reference RMS stress

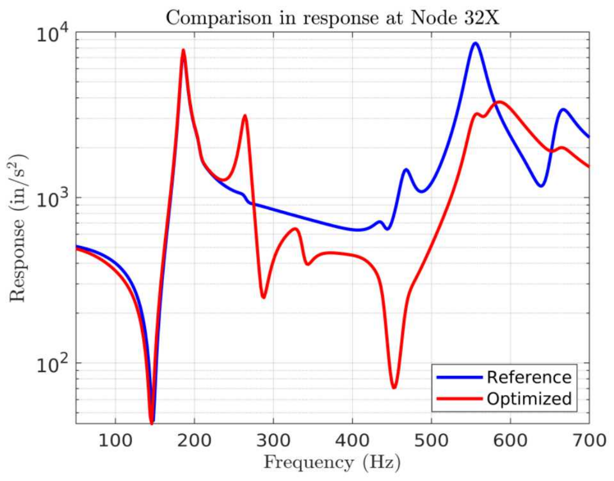


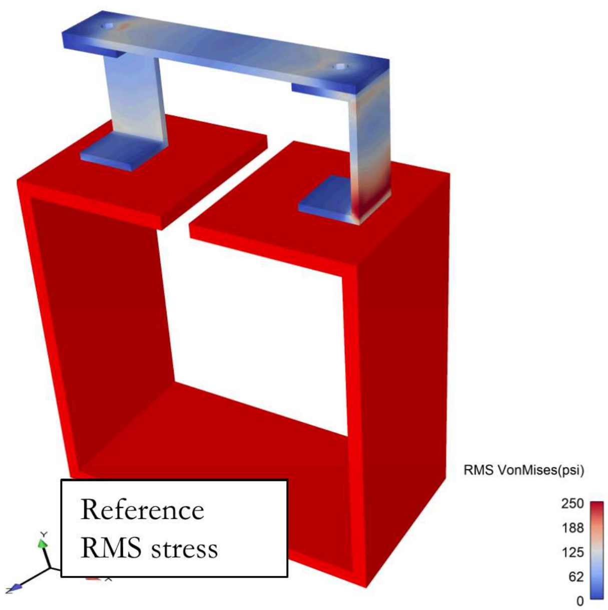
Rigid Fixture RMS stress

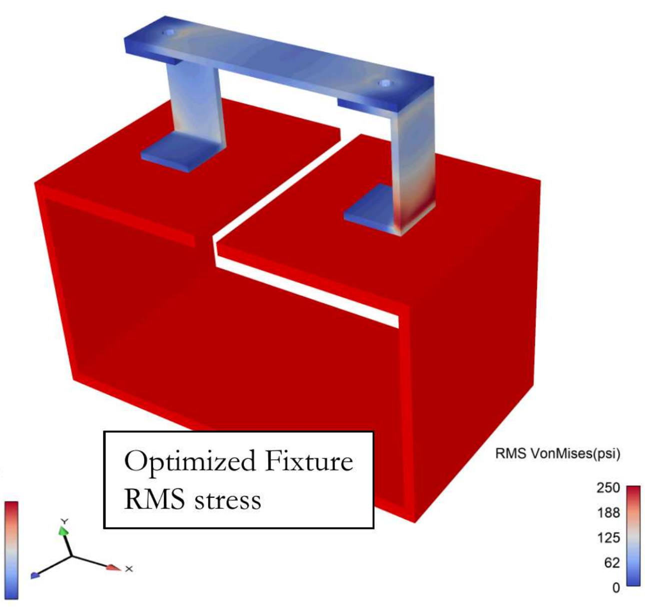
Case Study – Non-Trivial Optimization on Modal Projection Error the BARC Assembly

- The same force locations were used on the optimized fixture.
- Force levels were close to the original force of 1 lb



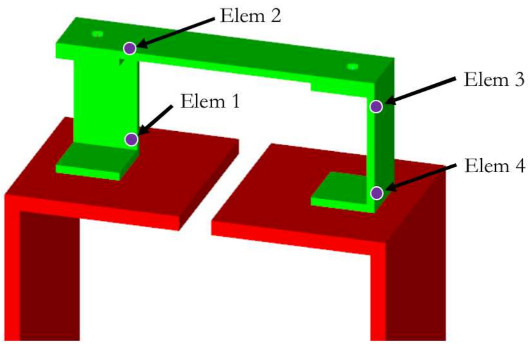


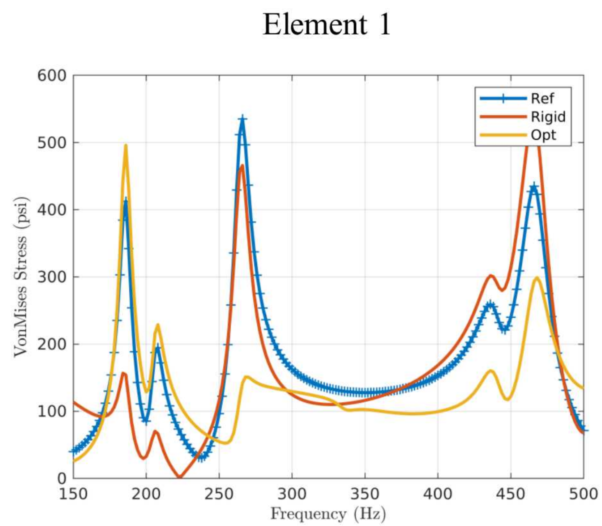


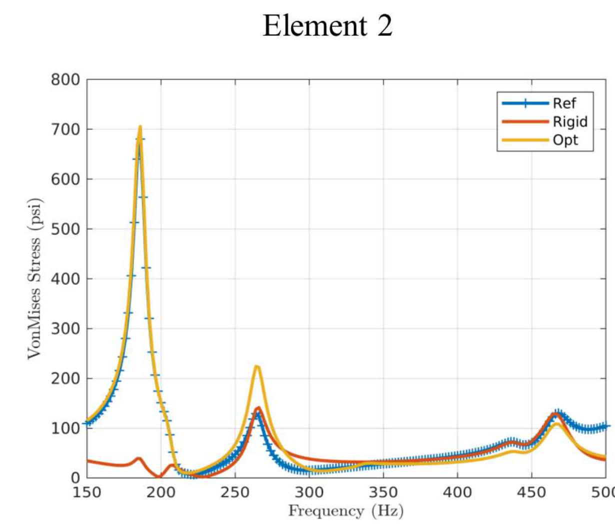


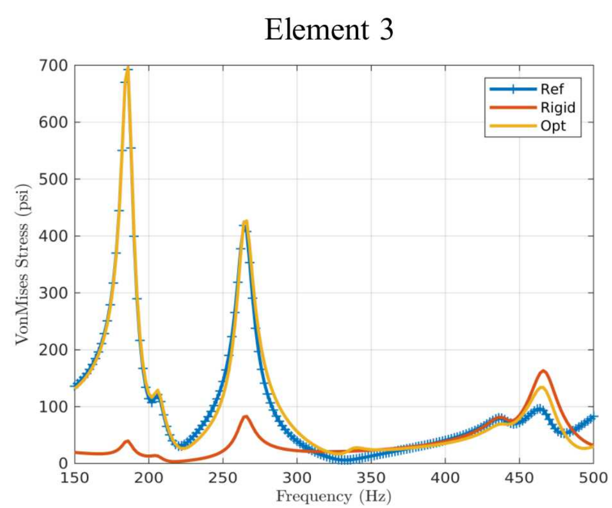
Case Study – Non-Trivial Optimization on Modal Projection Error the BARC Assembly

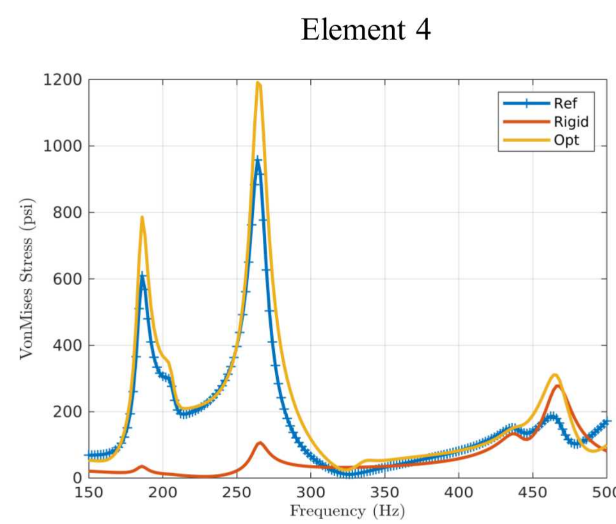
- Stress at elements of interest were examined over the frequency range and compared between configurations.
- Huge improvement for first two modes











Conclusion

- MPE is a new and novel way to optimize a structure as it only focuses on shape space.
- The optimization algorithm was able to converge to the global solution when the global solution was in the design space.
- Matching response at a location is not a good indication of matching stress of the unit under test.
- Although a perfectly impedance matched system is unique (unproven hypothesis), this research showed that a fixture can be designed that partially matches the impedance of the next level of assembly, say for the first couple mode shapes. This allows for these modes to be well represented in a laboratory test.

Derivation of the Modal Projection Error

$\bar{x}_L = \bar{x}_F$	(Eq1) We want the displacement field to be the same between the field and test environments
$x_{iL} \approx \sum_{m=1}^{n} \phi_{imL} q_{mL}$	(Eq2) Modal representation of the displacement during the test environment with a finite number of modes
$x_{iF} \approx \sum_{m=1}^{n} \phi_{imF} q_{mF}$	(Eq3) Modal representation of the displacement during the field environment with a finite number of modes
$\phi_L \bar{q}_L = \phi_F \bar{q}_F$	(Eq4) Equation 1 transformed into truncated modal space
$\bar{q}_L = \phi_L^+ \phi_F \bar{q}_F$	(Eq5) With the modal coordinates known from the field, the motion from the field is projected onto the laboratory mode shape space and the lab modal coordinates are calculated in a least squared solution.

Derivation of the Modal Projection Error

$\bar{q}_L = \boldsymbol{\phi}_L^+ \bar{\phi}_{Fn} q_{Fn}$	(Eq6) It is of interest to determine the error of reconstructing each field mode individually. The modal coordinates for the lab are calculated in a least squared sense.
$\bar{\phi}_{Fn}^+ \boldsymbol{\phi}_L \bar{q}_L = \tilde{q}_{Fn}$ $\bar{\phi}_{Fn}^+ \boldsymbol{\phi}_L \boldsymbol{\phi}_L^+ \bar{\phi}_{Fn} q_{Fn} = \tilde{q}_{Fn}$	(Eq7) With the lab modal coordinates calculated from Eq6, the coordinates are projected back onto the space of the field environment. A reconstructed field modal coordinate is calculated.
$\Gamma_n^2 = \dfrac{\tilde{q}_{Fn}}{q_{Fn}} = \bar{\phi}_{Fn}^+ \boldsymbol{\phi}_L \boldsymbol{\phi}_L^+ \bar{\phi}_{Fn}$ $\Psi_n^2 = 1 - \Gamma_n^2$	(Eq8) The ratio between the reconstructed field coordinate and the original field coordinate is calculated and that can be used to define the modal error term. The error is squared because two projections took place to obtain the value.