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A Current and Charge Integral Equation for Dielectric Regions in the Time Domain

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***Abstract*—Emerging classical and quantum applications require computational electromagnetics methods that can efficiently analyze complex structures over wide bandwidths, including down to very low frequencies. This work begins to address these needs by presenting a type of charge and current integral equation that has been formulated in the time domain and is applicable to dielectric regions. This system introduces charge densities as unknowns in addition to the current densities, resulting in a system that does not exhibit a low frequency breakdown. An appropriate marching-on-in-time discretization scheme is discussed so that stable and accurate results can be achieved down to very low frequencies. Numerical results are shown to verify the accuracy and stability of this formulation.**

I. Introduction

A growing number of classical and quantum physics applications require the analysis of complex structures over wide bandwidths, such as performing signal integrity studies and analyzing atom-photon interactions in circuit quantum electrodynamics [1]. To address these needs, it is desirable to have CEM methods that use simple discretizations and do not exhibit low frequency breakdown effects. Methods operating in the time domain are of interest to analyze nonlinear components and cover wide bandwidths in a few simulations.

One formulation that can potentially meet these needs is the current and charge integral equation (CCIE) of [2]. This work develops for the first time a type of CCIE in the time domain for dielectric regions. The resulting time domain integral equation (TDIE) is found to be accurate down to very low frequencies. We provide guidelines on basis functions to be used in a marching-on-in-time (MOT) discretization of the presented equations to achieve a system with improved stability in practice. The stability and accuracy of the system is demonstrated through numerical results.

II. Formulation

The dominant source of low frequency instability in EM integral equations is the hypersingular integral operator found in many formulations [2]. The simplest way to alleviate these effects is to introduce the charge densities as unknowns to decompose the hypersingular operator into two components that can be stably discretized. This allows for a simple discretization, in contrast to other methods that use loop-tree decompositions of the current density [3].

For dielectric regions, introducing the charge densities results in four unknown surface sources to be solved for. Following the general approach of the CCIE, we consider a single equation related to each of the boundary conditions on EM fields and fluxes at an interface [2]. Integral equations for the exterior and interior regions are combined following a Müeller-type approach, giving an overall system of

$$\hat{n}\times\big[\mathbf{E}_e\{\mathbf{J},\mathrm{d},\mathbf{M}\}(\mathbf{r},t)\big]\times\hat{n} - \hat{n}\times\big[\mathbf{E}_i\{\mathbf{J},\mathrm{d},\mathbf{M}\}(\mathbf{r},t)\big]\times\hat{n} = \hat{n}\times\big[\mathbf{E}_{\mathrm{inc}}(\mathbf{r},t)\big]\times\hat{n}, \;\; \mathbf{r}\in S, \quad (1)$$

$$\hat{n}\cdot\mathbf{D}_e\{\mathbf{J},\mathrm{d},\mathbf{M}\}(\mathbf{r},t) - \hat{n}\cdot\mathbf{D}_i\{\mathbf{J},\mathrm{d},\mathbf{M}\}(\mathbf{r},t) = \hat{n}\cdot\mathbf{D}_{\mathrm{inc}}(\mathbf{r},t), \;\; \mathbf{r}\in S, \quad (2)$$

$$\hat{n}\times\big[\mathbf{H}_e\{\mathbf{J},\mathbf{M},\mathrm{b}\}(\mathbf{r},t)\big]\times\hat{n} - \hat{n}\times\big[\mathbf{H}_i\{\mathbf{J},\mathbf{M},\mathrm{b}\}(\mathbf{r},t)\big]\times\hat{n} = -\hat{n}\times\big[\mathbf{H}_{\mathrm{inc}}(\mathbf{r},t)\big]\times\hat{n}, \;\; \mathbf{r}\in S, \quad (3)$$

$$\hat{n}\cdot\mathbf{B}_e\{\mathbf{J},\mathbf{M},\mathrm{b}\}(\mathbf{r},t) - \hat{n}\cdot\mathbf{B}_i\{\mathbf{J},\mathbf{M},\mathrm{b}\}(\mathbf{r},t) = -\hat{n}\cdot\mathbf{B}_{\mathrm{inc}}(\mathbf{r},t), \;\; \mathbf{r}\in S, \quad (4)$$

where $\mathbf{J}=\hat{n}'\times\mathbf{H}$, $\mathbf{M}=\mathbf{E}\times\hat{n}'$, $\mathrm{d}=\hat{n}'\cdot\mathbf{D}$, $\mathrm{b}=-\hat{n}'\cdot\mathbf{B}$, and the notation on the left hand side of each equation emphasizes that these are functionals evaluated at position $\mathbf{r}$ and time t produced by the surface sources in the brackets. Additionally, the subscripts e and i denote that material properties should be selected from the exterior or interior region, respectively.

The exact form of the functionals given in (1) to (4) are

$$\hat{n}\times\big[\mathbf{E}_j\{\mathbf{J},\mathrm{d},\mathbf{M}\}(\mathbf{r},t)\big]\times\hat{n} = \mu_j\mathcal{T}_j\{\partial_t\mathbf{J}\}(\mathbf{r},t) - \epsilon_j^{-1}\nabla\mathcal{T}_j\{\mathrm{d}\}(\mathbf{r},t) \pm \frac{1}{2}\hat{n}\times\mathbf{M}(\mathbf{r},t) - \mathcal{K}_j\{\mathbf{M}\}(\mathbf{r},t), \quad (5)$$

$$\hat{n}\cdot\mathbf{D}_j\{\mathbf{J},\mathrm{d},\mathbf{M}\}(\mathbf{r},t) = \mu_j\epsilon_j\hat{n}\cdot\mathcal{T}_j\{\partial_t\mathbf{J}\}(\mathbf{r},t) \pm \frac{1}{2}\mathrm{d}(\mathbf{r},t) + \mathcal{N}_j\{\mathrm{d}\}(\mathbf{r},t) + \epsilon_j\hat{n}\cdot\nabla\times\mathcal{T}_j\{\mathbf{M}\}(\mathbf{r},t), \quad (6)$$

$$\hat{n}\times\big[\mathbf{H}_j\{\mathbf{J},\mathbf{M},\mathrm{b}\}(\mathbf{r},t)\big]\times\hat{n} = -\epsilon_j\mathcal{T}_j\{\partial_t\mathbf{M}\}(\mathbf{r},t) + \mu_j^{-1}\nabla\mathcal{T}_j\{\mathrm{b}\}(\mathbf{r},t) \pm \frac{1}{2}\hat{n}\times\mathbf{J}(\mathbf{r},t) - \mathcal{K}_j\{\mathbf{J}\}(\mathbf{r},t), \quad (7)$$

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$$\hat{n}\cdot\mathbf{B}_j\{\mathbf{J},\mathbf{M},\mathrm{b}\}(\mathbf{r},t) = -\mu_j\epsilon_j\hat{n}\cdot\mathcal{T}_j\{\partial_t\mathbf{M}\}(\mathbf{r},t)$$
$$\mp\frac{1}{2}\mathrm{b}(\mathbf{r},t) - \mathcal{N}_j\{\mathrm{b}\}(\mathbf{r},t) + \mu_j\hat{n}\cdot\nabla\times\mathcal{T}_j\{\mathbf{J}\}(\mathbf{r},t), \quad (8)$$

which make use of the following integral operators:

$$\mathcal{T}_j\{p\}(\mathbf{r},t) = \int_S \frac{p(\mathbf{r}',\tau_j)}{4\pi R}dS', \quad (9)$$

$$\mathcal{K}_j\{\mathbf{q}\}(\mathbf{r},t) = \int_S \hat{R}\times\left[\frac{\mathbf{q}(\mathbf{r}',\tau_j)}{4\pi R^2} + \frac{\partial_t\mathbf{q}(\mathbf{r}',\tau_j)}{4\pi R c_j}\right]dS', \quad (10)$$

$$\mathcal{N}_j\{p\}(\mathbf{r},t) = \int_S \hat{n}\cdot\hat{R}\left[\frac{p(\mathbf{r}',\tau_j)}{4\pi R^2} + \frac{\partial_t p(\mathbf{r}',\tau_j)}{4\pi R c_j}\right]dS'. \quad (11)$$

Note that when there is a choice of sign in (5) to (8), the top choice is selected for the exterior region.

III. Discretization & Numerical Results

Selecting a temporal basis function from an appropriate Sobolev space is essential to achieving a stable MOT discretization of systems combining vector and scalar TDIEs [4]. For the TDIEs introduced in this work, an appropriate temporal basis function is a triangle function. Methods for determining this can be found in [4].

The spatial functions used in the discretization of (1) to (4) are selected to conform to the spatial Sobolev spaces of these TDIEs. These functions are defined on the primal mesh, and a dual mesh formed by the barycentric refinement of the primal mesh. In particular, $\mathbf{J}$ is discretized with RWG functions, $\mathbf{M}$ with Buffa-Christiansen (BC) functions [5], d with a piecewise constant function defined over triangles, and b with piecewise constant functions supported on all dual mesh triangles attached to a primal mesh node. RWG functions are used to test (1) and BC functions to test (3). To have a square matrix system that conforms to the Sobolev spaces of these TDIEs, (4) is tested with pyramid functions (i.e., equal to one at a node and linearly go to zero at surrounding nodes) defined on the primal mesh. Finally, the scalar function defined in [5] that is composed of a linear combination of pyramid functions defined over the dual mesh is used to test (2).

The stability and accuracy of this method down to very low frequencies are shown with a numerical example. The incident field is a plane wave, with temporal dependence given by a modulated Gaussian pulse. The scattering from a 1 meter radius sphere with a relative permittivity of 2.56 is calculated for a sequence of incident pulses with progressively lower frequencies. For each pulse, the bandwidth is half the center frequency. The relative error compared to the analytical solution is shown in Fig. 1, where it is seen that a stable level of accuracy is achieved. For comparison, the results for traditional methods are also shown, which are seen to exhibit catastrophic low frequency breakdowns.

Unfortunately, the system formulated in this work has not been found to be stable at higher frequencies where wave physics effects become more important. However, for these situations, standard integral equations such as the PMCHWT or Müeller integral equations can be used.

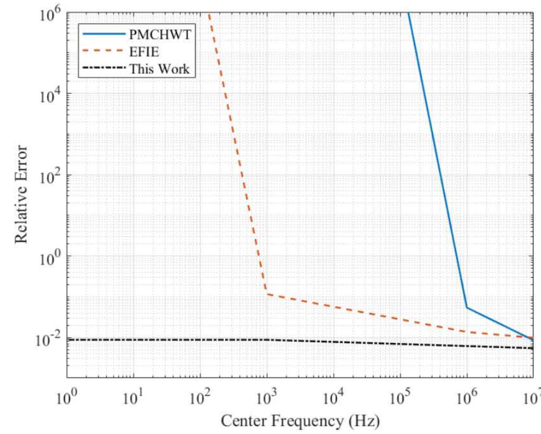

Fig. 1: Relative error in the RCS at low frequencies for the PMCHWT, EFIE, and the formulation of this work.

IV. Conclusion

This work presented a CCIE in the time domain that is applicable to dielectric regions. These new equations are accurate and stable down to very low frequencies, as demonstrated through numerical results. Future work will focus on determining the source of instability for this system at middle frequencies in an effort to achieve a system stable over all frequency regimes.

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