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Ice Sheet Intialization

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Center for Computing Research

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SciDAC

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Ice Sheet Initialization

Finding the initial/present-day thermo-mechanical state of the ice sheet and estimate the unknown/poorly known model parameters

A "good" initial state should be "consistent" with

- present day observations (ice geometry, surf velocity, surf. air temp., surf/basal mass balance),
- trends (mainly thickness time derivative),
- ice sheet model (thermo-mechanical model)

Fields to estimate

- Parameters of basal boundary conditions (basal friction, or better parameters of the basal hydrology model)
- bed topography,
- rehology parameters (stiffening factor, Glen's law exponent)
- Geothermal heat flux
- ...

Ice Sheet Initialization

Main issues/topics not addressed in this talk:

- Availability/reliability of observations data (ideally all the data are unbiased and should come with some measurement of associated uncertainty, at least RMS errors / variance)
- Probabilistic interpretation (Bayesian inference, Uncertainty quantification..). Focusing only on "deterministic" initialization.
- Model reduction
- Machine learning! (other than PDE-constrained optimization)

Ice Sheet Modeling

Ice momentum equations

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

with:

$$\sigma = 2\mu \mathbf{D} - pI, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Nonlinear viscosity:

$$\mu = \frac{1}{2}\alpha(T)\,|\mathbf{D}(\mathbf{u})|^{\frac{1}{n}-1}, \quad n \geq 1, \quad (\text{tipically } n \simeq 3)$$

Viscosity is singular when ice is not deforming

Stiffening/Damage factor

$$\mu^*(x,y,z) = \phi(x,y)\,\mu(x,y,z)$$

ϕ : stiffening factor that accounts for modeling errors in rheology

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Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

- **Model for the ice sheet evolution**
  (thickness evolution equation)

$$\frac{\partial H}{\partial t} = \tau_{\mathrm{mb}} - \nabla \cdot (\bar{\mathbf{u}} H)$$

$$\tau_{\mathrm{mb}} : SMB + BMB$$

$$\bar{\mathbf{u}} = \frac{1}{H} \int_z \mathbf{u}\, dz$$

- **Enthalpy equation**

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q}(h) + \mathbf{u} \cdot \nabla h = \tau : \dot{\epsilon}$$

enthalpy flux

Initialization

PDE-constrained Optimization (often introduced in a Bayesian Framework) is nowadays the most used method for large scale problems (order of million parameters)

Alternatives:
- ad-hoc methods
- Unscented Kalman filter
  - Pros: provides probabilistic framework
  - Cons: expensive, the number of forward solves needed is the same as the number of parameters
- Bayesian Calibration
  - Pros: provides probabilistic framework
  - Cons: unfeasible due to curse of dimensionality (at least in the vanilla/ “brute force” formulation)

(Early) Bibliography

- *Arthern, Gudmundsson*, J. Glaciology, 2010
- *Price, Payne, Howat and Smith*, PNAS, 2011
- *Petra, Zhu, Stadler, Hughes, Ghattas*, J. Glaciology, 2012
- *Pollard DeConto*, TCD, 2012
- W. J. J.Van Pelt et al., The Cryosphere, 2013
- *Morlighem et al.* Geophysical Research Letters, 2013
- *Goldberg and Heimbach*, The Cryosphere, 2013
- Brinkerhoff and Johnson, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014
- *Perego, Price, Stadler,* Journal of Geophysical Research, 2014
- *Goldberg et al.*, The Cryosphere Discussions, 2015

Initialization

(PDE-constrained) Optimization, often introduced in a Bayesian framework, is nowadays the most used method for large scale problems (order of million parameters)

Two main flavors of Optimization:

- Transient optimization: data are assimilated in time (e.g. Goldberg/Heimbach)
- Steady optimization: data are assumed to be acquired simultaneously and we work on a "steady state" version of the model where tendencies are assumed to be known or negligible.

Here we focus on the second approach. The first one is more powerful/flexible but also more expensive/complex to implement

(Early) Bibliography

- *Arthern, Gudmundsson*, J. Glaciology, 2010
- *Price, Payne, Howat and Smith*, PNAS, 2011
- *Petra, Zhu, Stadler, Hughes, Ghattas*, J. Glaciology, 2012
- *Pollard DeConto*, TCD, 2012
- W. J. J.Van Pelt et al., The Cryosphere, 2013
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- *Goldberg and Heimbach*, The Cryosphere, 2013
- Brinkerhoff and Johnson, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014
- *Perego, Price, Stadler,* Journal of Geophysical Research, 2014
- *Goldberg et al.*, The Cryosphere Discussions, 2015

PDE-Constrained Optmization

Estimate basal friction matching obs. velocity

Optimization problem 1:

find β that minimize the functional $\mathcal{J}$

$$\mathcal{J}(\beta) = \int_\Omega \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds \qquad \text{surface velocity mismatch}$$

$$+\mathcal{R}(\beta) \qquad \text{regularization terms.}$$

subject to ice sheet model equations (FO or Stokes)

$\mathbf{u}$: computed depth averaged velocity
β: basal sliding friction coefficient
$\mathcal{R}(\beta)$ regularization term

Temperature is given.

PDE-Constrained Optmization

Estimate basal friction and stiffening matching obs. velocity

Optimization problem 2:

find β and ϕ that minimize the functional $\mathcal{J}$

$$\mathcal{J}(\beta, \phi) = \int_\Omega \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds$$

surface velocity mismatch

$$+ \int_\Omega \frac{1}{\sigma_\phi^2} |\phi - 1|^2 \, ds$$

stiffening factor mismatch

$$+\mathcal{R}(\beta, \phi)$$

regularization terms.

subject to ice sheet model equations (FO or Stokes)

$\mathbf{u}$: computed depth averaged velocity
ϕ: stiffening factor
β: basal sliding friction coefficient
$\mathcal{R}(\beta, \phi)$ regularization term

Temperature is given.

MPAS-Albany Landice model (MALI)

Algorithm and Software

ALGORITHM	SOFTWARE TOOLS
Linear Finite Elements on test/hexas	Albany
Quasi-Newton optimization (L-BFGS)	ROL
Nonlinear solver (Newton method)	NOX
Krylov linear solvers/Prec	AztecOO/ML, Belos/MueLu
Automatic differentiation	Sacado

***MPAS:** Model for Prediction Across Scales, fortran finite volume library:*

- *works on Voronoi Tessellations*
- *conservative Lagrangian schemes for advecting tracers*
- *evolution of ice thickness*

Albany: C++ finite element library built on Trilinos to enable multiple capabilities:

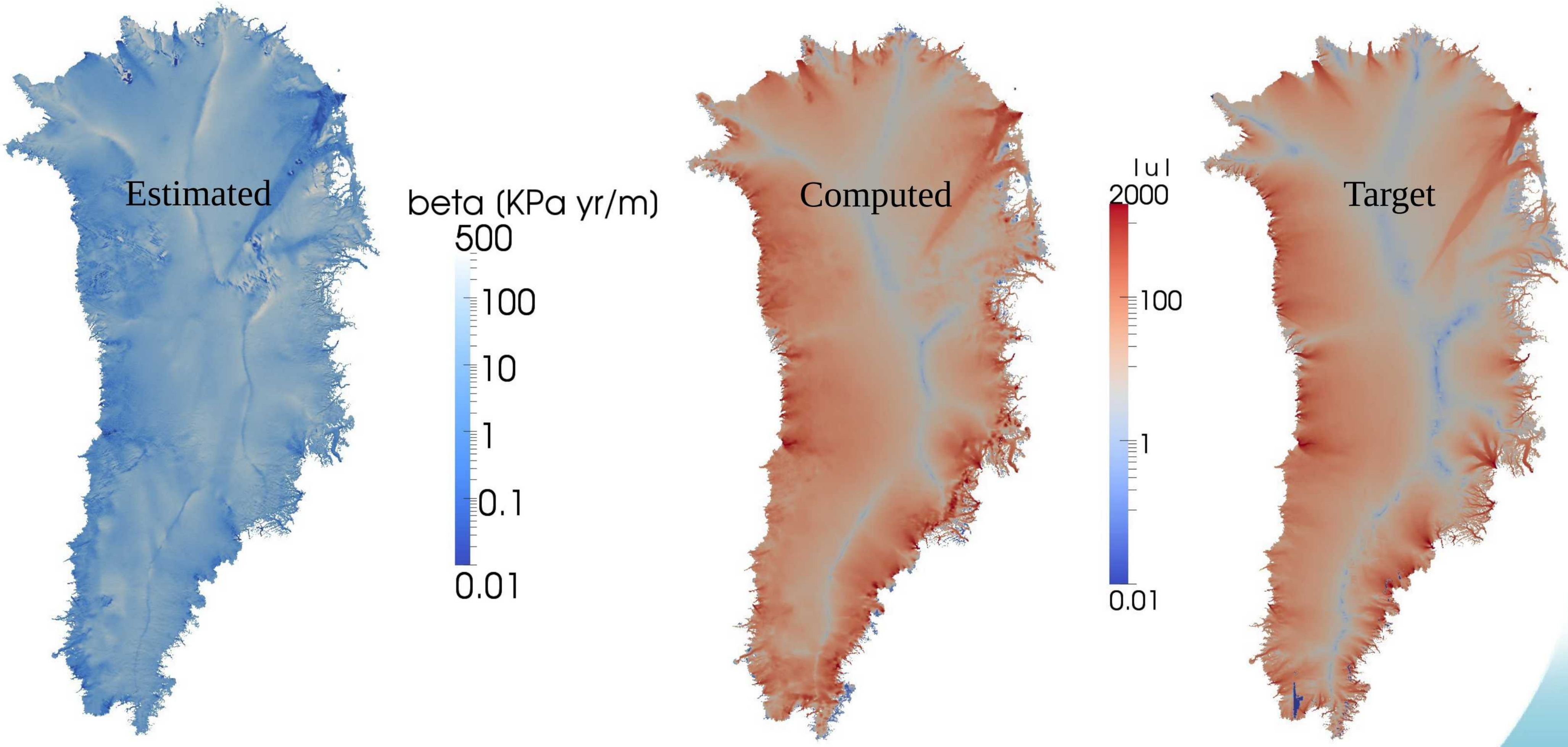
- Jacobian/adjoints assembled using automatic differentiation (Sacado).
- nonlinear and parameter continuation solvers (NOX/LOCA)
- large scale PDE constrained optimization (Piro/ROL)
- linear solver and preconditioners (Belos/AztecOO, ML/MeuLu/Ifpack)

Hoffman, et al. GMD, 2018
Tuminaro, Perego, Tezaur, Salinger, Price, SISC, 2016.
Tezaur, Perego, Salinger, Tuminaro, Price, Hoffman, GMD, 2015
Perego, Price, Stadler, JGR, 2014

Greenland Inversion

velocity mismatch only, tuning basal friction

Inversion with 1.6M parameters



Basal friction coefficient (kPa yr/m)

surface velocity magnitude (m/yr)

Geometry (Morlighem et al., Nature Geo., 2014)

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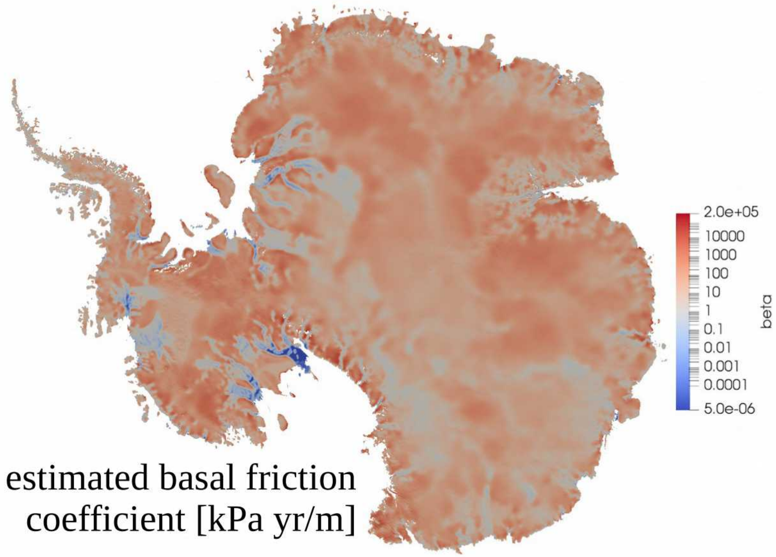
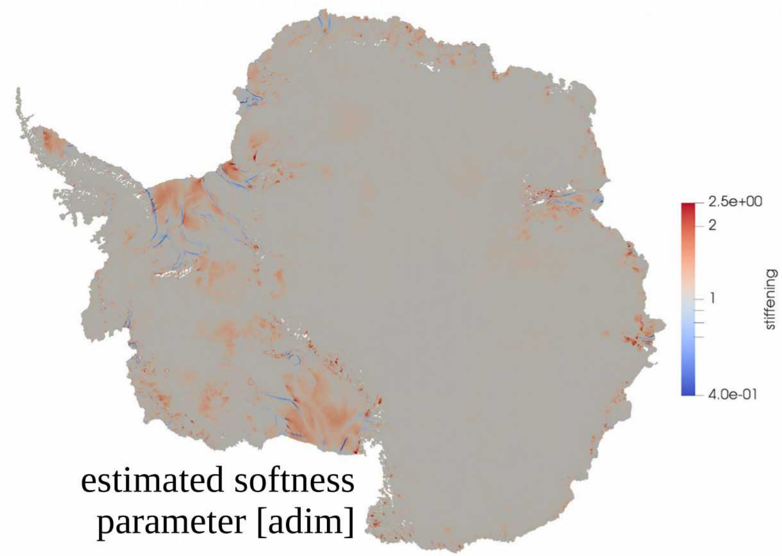
Antarctica Inversion

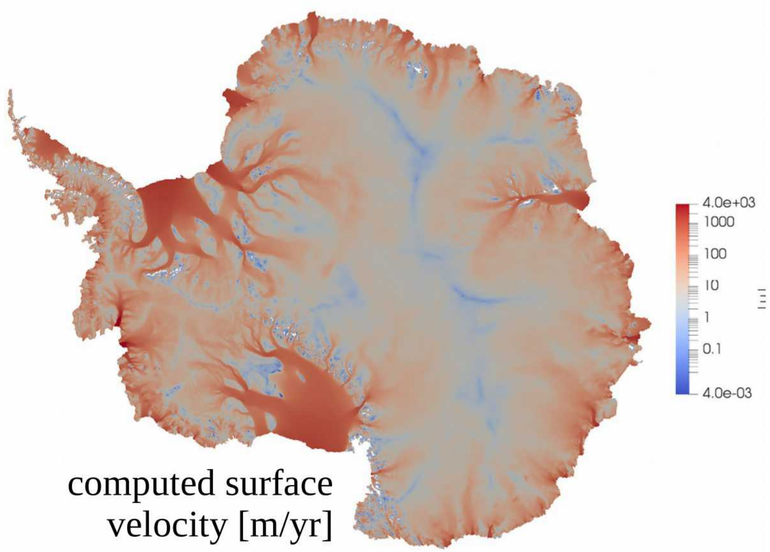
velocity and stiffening mismatches, tuning basal friction and stiffening



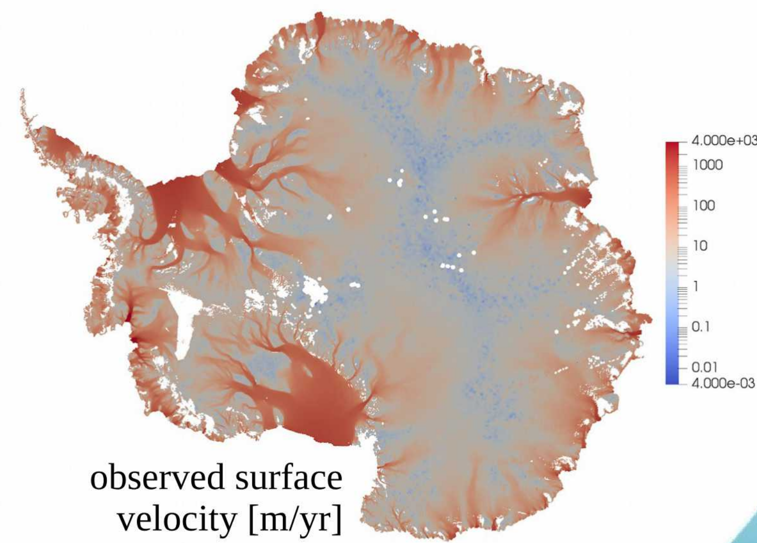
estimated basal friction coefficient [kPa yr/m]



estimated softness parameter [adim]



computed surface velocity [m/yr]



observed surface velocity [m/yr]

simulation details	
#parameters: 2.5M	#cores: 8640
#unknowns: 30M	#nodes:180
machine: Edison (NERSC)	#hours:18

Shortcomings of current optimization approach

velocity and stiffening mismatches, tuning basal friction and stiffening

Main issues with the proposed optimization approach:

1. initial state **does not match** observed **thickness tendencies**

2. intial state **is not consistent** with **temperature**

3. basal friction field **is not steady** in time

- As soon as we start evolving the ice sheet in time, we experience fast unphysical transients (mainly because of 2.) and the modeled dynamics won't be accurate, especially in the medium term (50-100 years).

- Typically one "spins up" the model for O(100-1000) years, but this can lead to an initial state that is far form the present day one.

- Time response of ice to temperature changes is of the order of several thousands of years. In order to obtain a self consistent initial state, temperature model is typically spun up for $O(10^4)$ years.

Today we focus on 1. and 2. To address 3. a subglacial hydrology model is needed.

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Deterministic Inversion

PDE-constrained optimization problem: cost functional

Optimization problem 3:

find β and H that minimize the functional* $\mathcal{J}$

$$\mathcal{J}(\beta, H) = \int_\Omega \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds$$

surface velocity mismatch

$$+ \int_\Omega \frac{1}{\sigma_\tau^2} \left| \nabla \cdot (\bar{\boldsymbol{u}} H) - \tau_{\text{mb}} + \left\{ \frac{\partial H}{\partial t} \right\}^{\text{obs}} \right|^2 ds$$

mismatch with climate forcing and thickness rate of change

$$+ \int_\Omega \frac{1}{\sigma_H^2} |H - H^{obs}|^2 \, ds$$

thickness mismatch

$$+\mathcal{R}(\beta, H)$$

regularization terms.

subject to ice sheet model equations (FO + **Enthalpy** solver)

$\bar{\boldsymbol{u}}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_{mb}: surface and basal mass balance
$\mathcal{R}(\beta)$ regularization term

With an <u>implicit steady-state</u> temperature model, coupled with the flow model, it is possible to obtain a self-consistent state in one shot.

Steady State Enthalpy Model

Steady-state Enthalpy equation reads:

$$\nabla \cdot \mathbf{q}(h) + \mathbf{u} \cdot \nabla h = \tau : \dot{\epsilon}$$

	cold ice $h < h_m$	temperate ice $h \geq h_m$
T	$T = T_0 + \frac{1}{\rho c} h$	$T = T_m$
ϕ	0	$\frac{1}{\rho_w L}(h - h_m)$

$$\mathbf{q} = \begin{cases} -\frac{k}{\rho c} \nabla h & \text{cold}(h < h_m) \\ -\frac{k}{\rho c} \nabla h_m + \rho_w L \, \mathbf{j}(h) & \text{temperate} \end{cases}$$

total enthalpy flux

$$\mathbf{j}(h) = \frac{1}{\eta_w} k_0 \left(\frac{h - h_m}{\rho c} \right)^{\gamma} (\rho_w - \rho) \mathbf{g}$$

gravity driven water flux
(Hewitt, Schoof)

Stefan's condition at the bed

$$\mathrm{m} = \mathrm{G} + \tau_b \cdot \mathbf{u} - k \nabla T \cdot \mathbf{n}$$

melting rate geothermal heat flux frictional heating

At surface elevation:

$$\mathrm{T} = \mathrm{T}_{\mathrm{air}}$$

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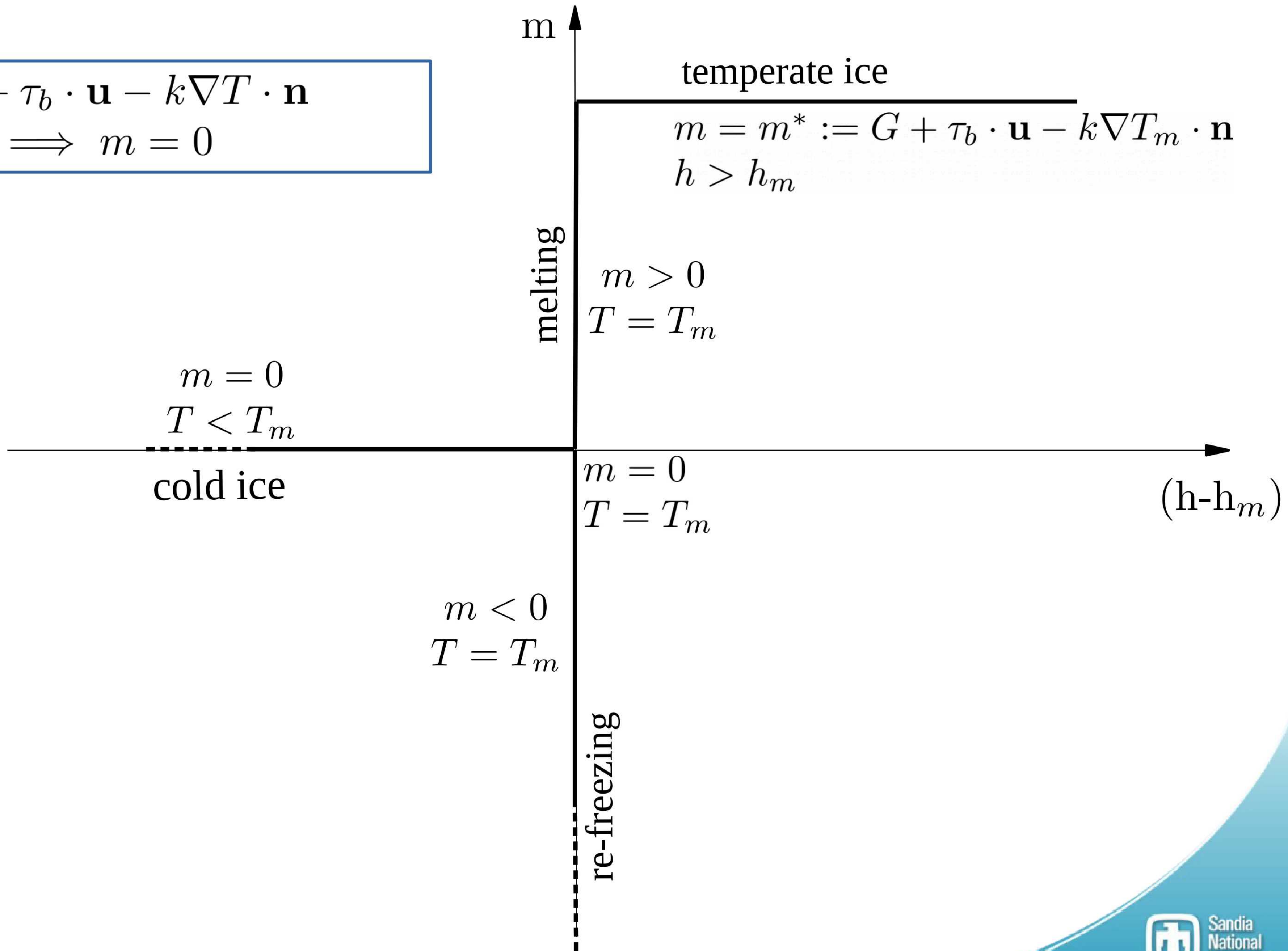
Melting/Enthalpy graph at the bed interface

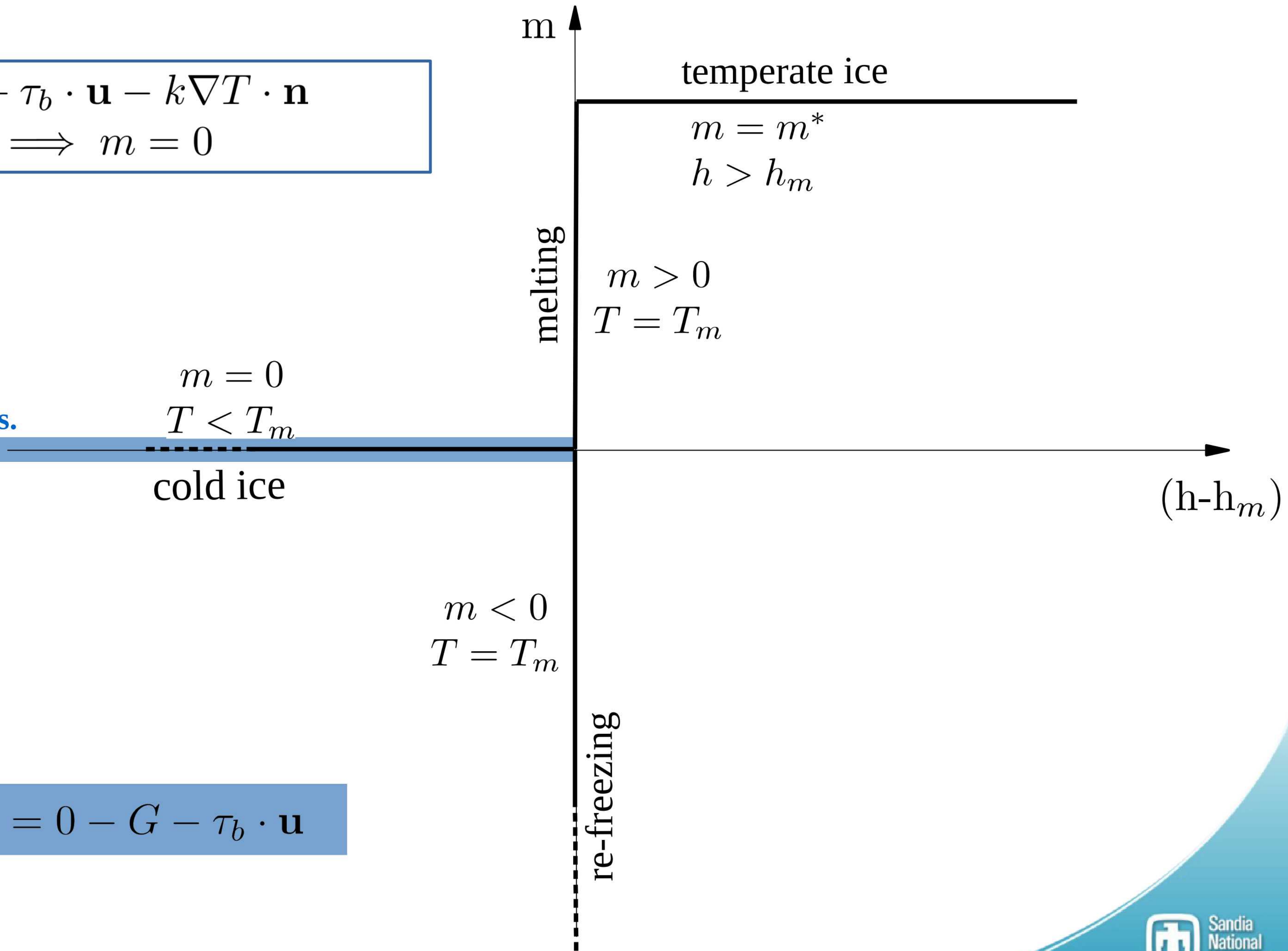
$$m = G + \tau_b \cdot \mathbf{u} - k\nabla T \cdot \mathbf{n}$$
$$T < T_m \implies m = 0$$



Melting/Enthalpy graph at the bed interface

$$m = G + \tau_b \cdot \mathbf{u} - k\nabla T \cdot \mathbf{n}$$
$$T < T_m \implies m = 0$$

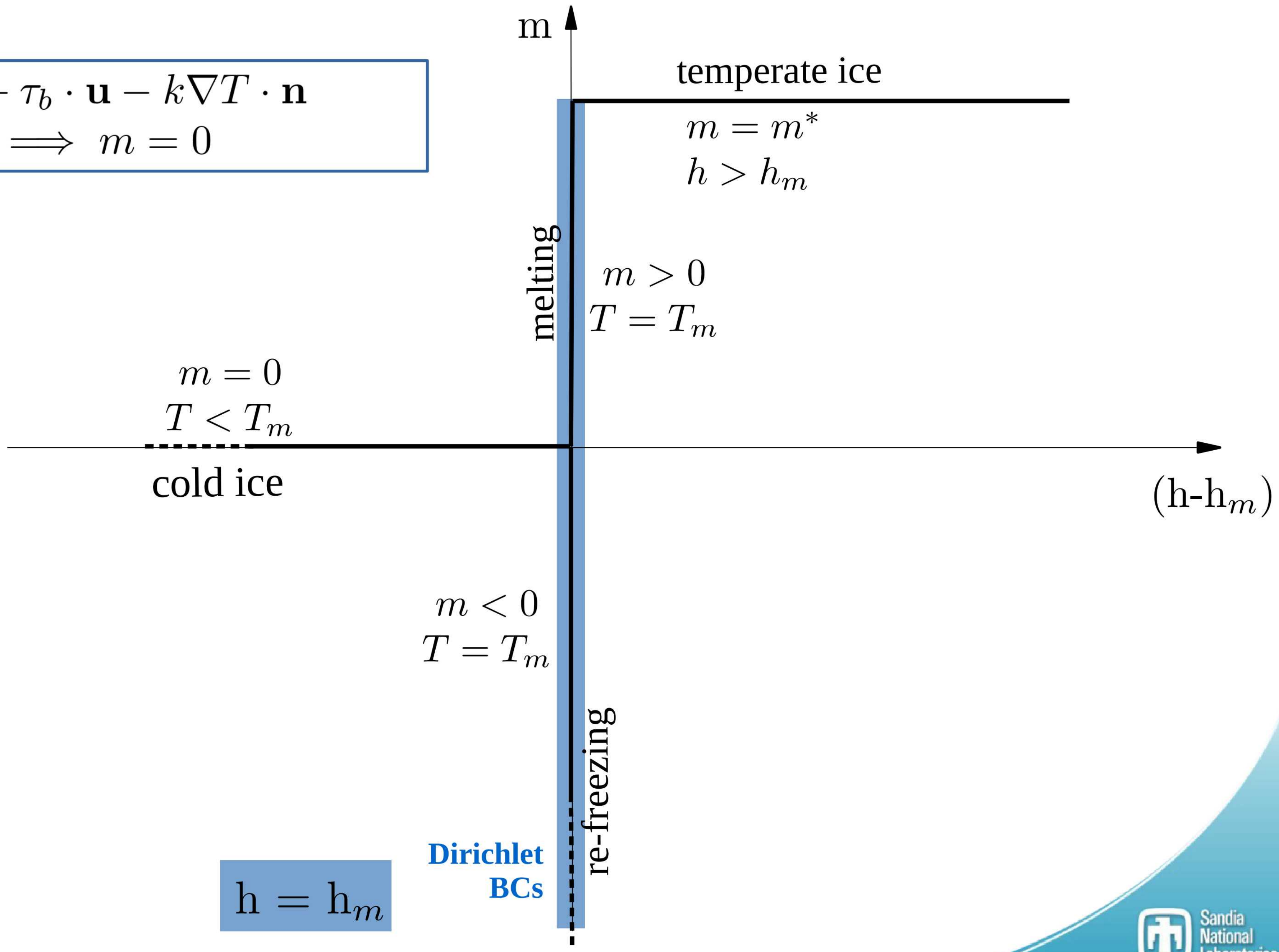
Neumann BCs.



$$-\frac{k}{\rho c}\nabla h \cdot \mathbf{n} = 0 - G - \tau_b \cdot \mathbf{u}$$

Melting/Enthalpy graph at the bed interface

$$m = G + \tau_b \cdot \mathbf{u} - k\nabla T \cdot \mathbf{n}$$
$$T < T_m \implies m = 0$$



Melting/Enthalpy graph at the bed interface

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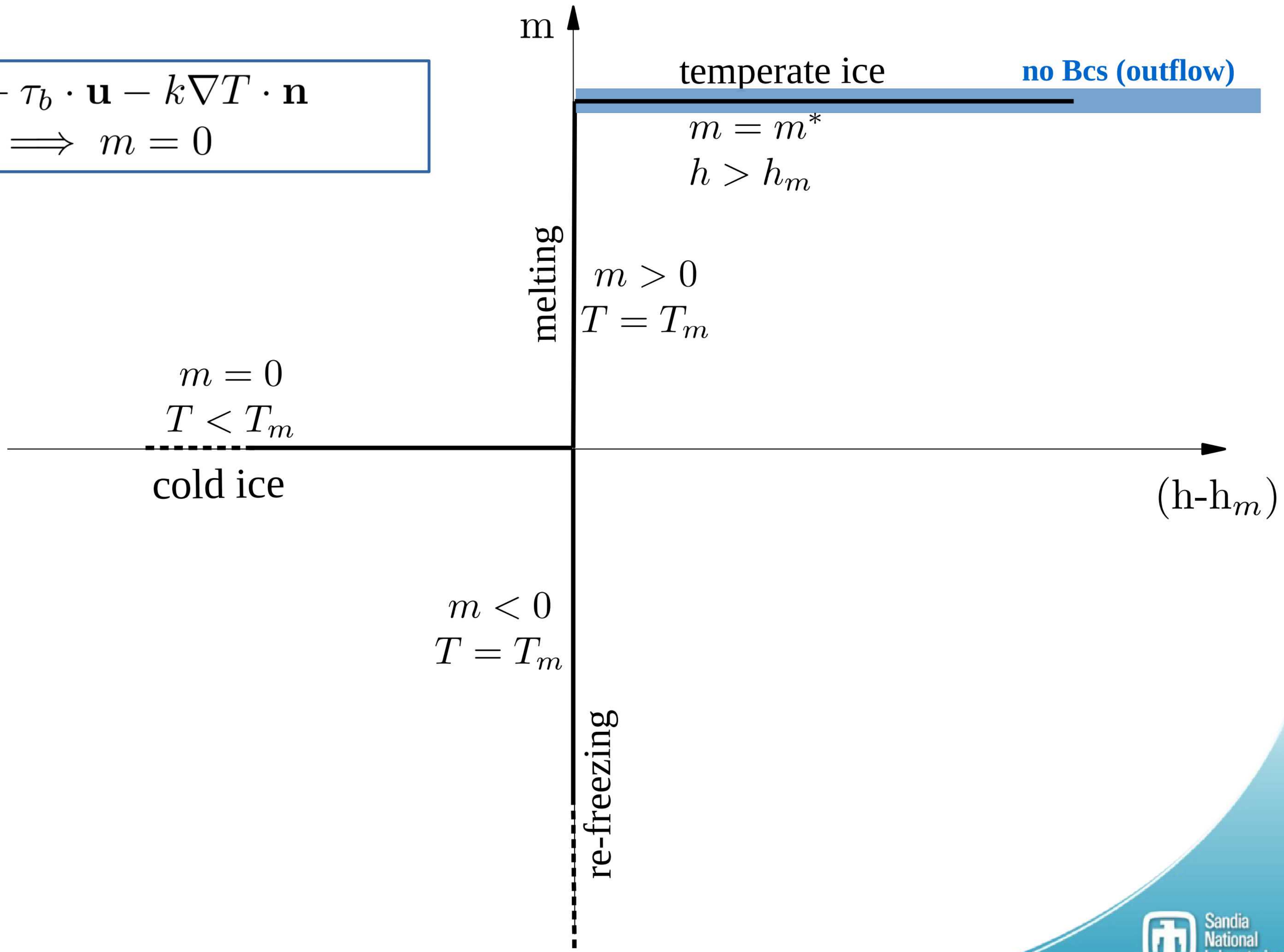
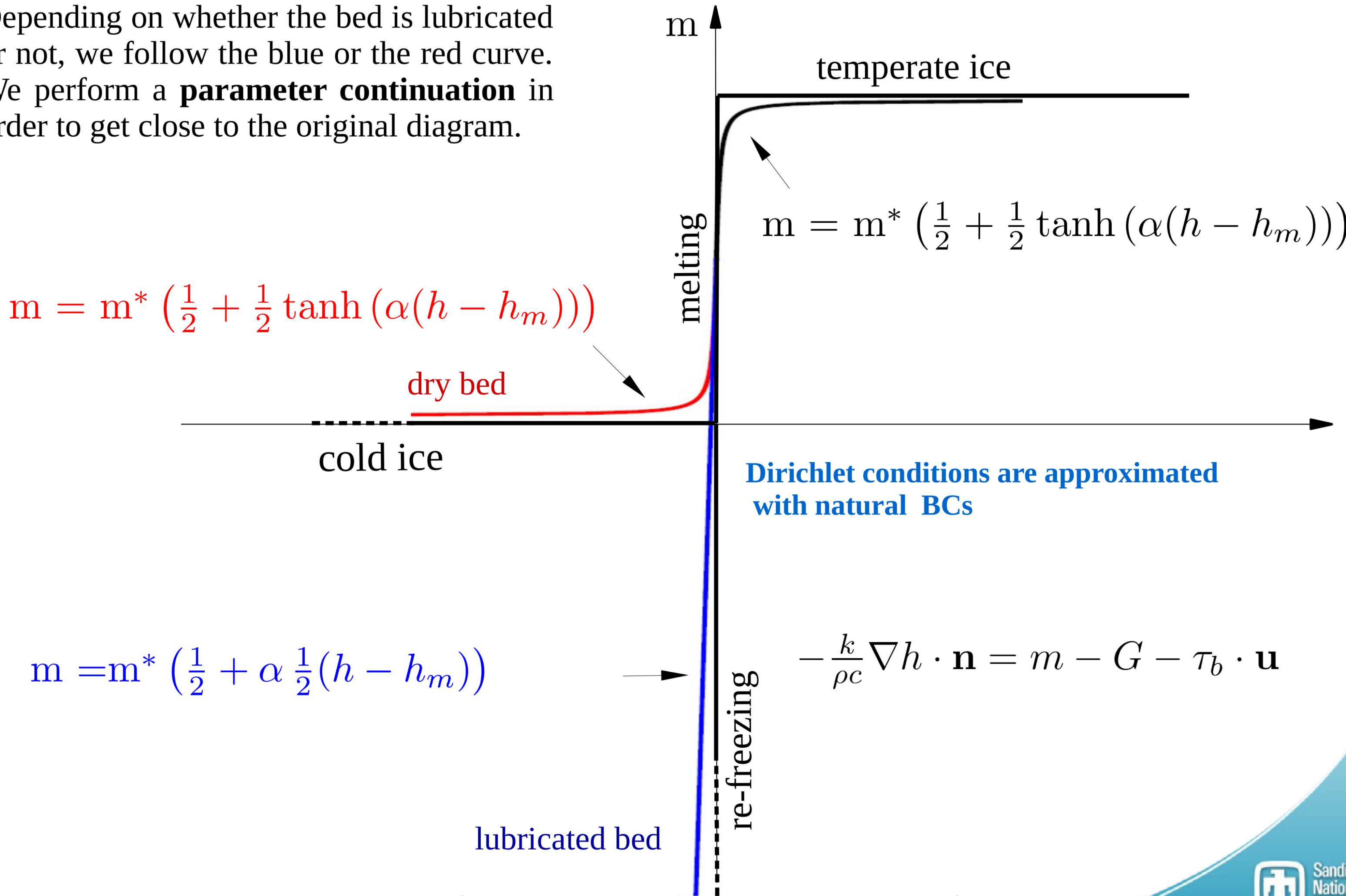


Approximation/smoothing of the enthalpy/melting graph

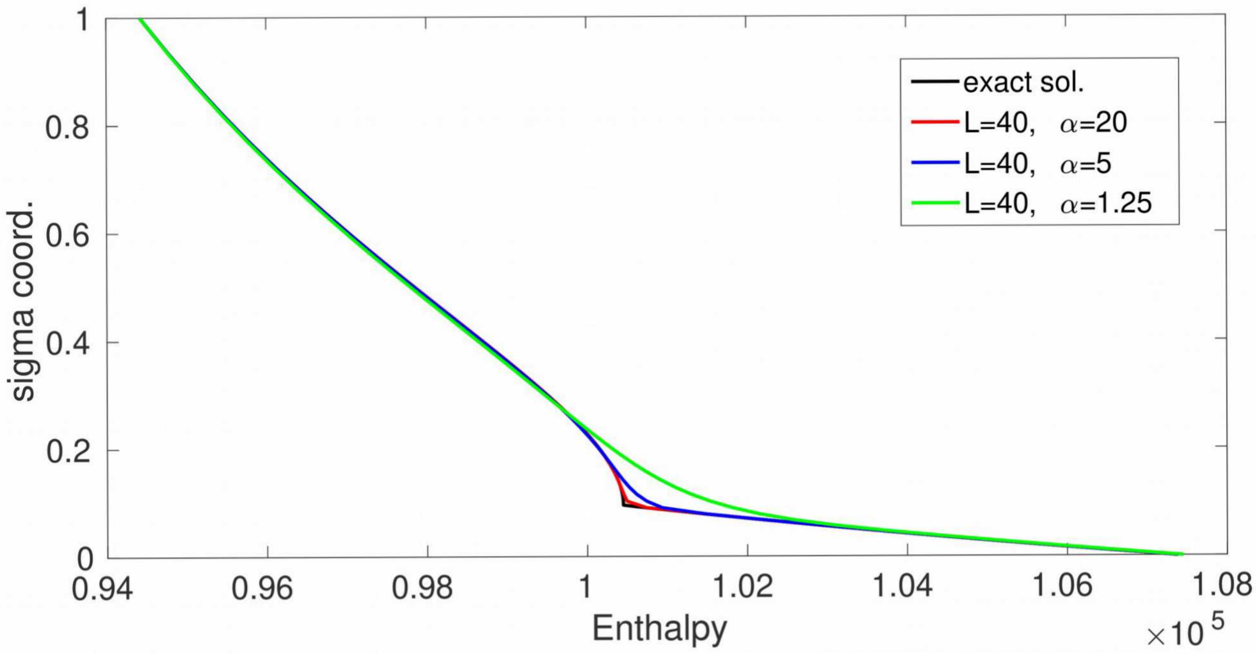
Depending on whether the bed is lubricated or not, we follow the blue or the red curve. We perform a **parameter continuation** in order to get close to the original diagram.

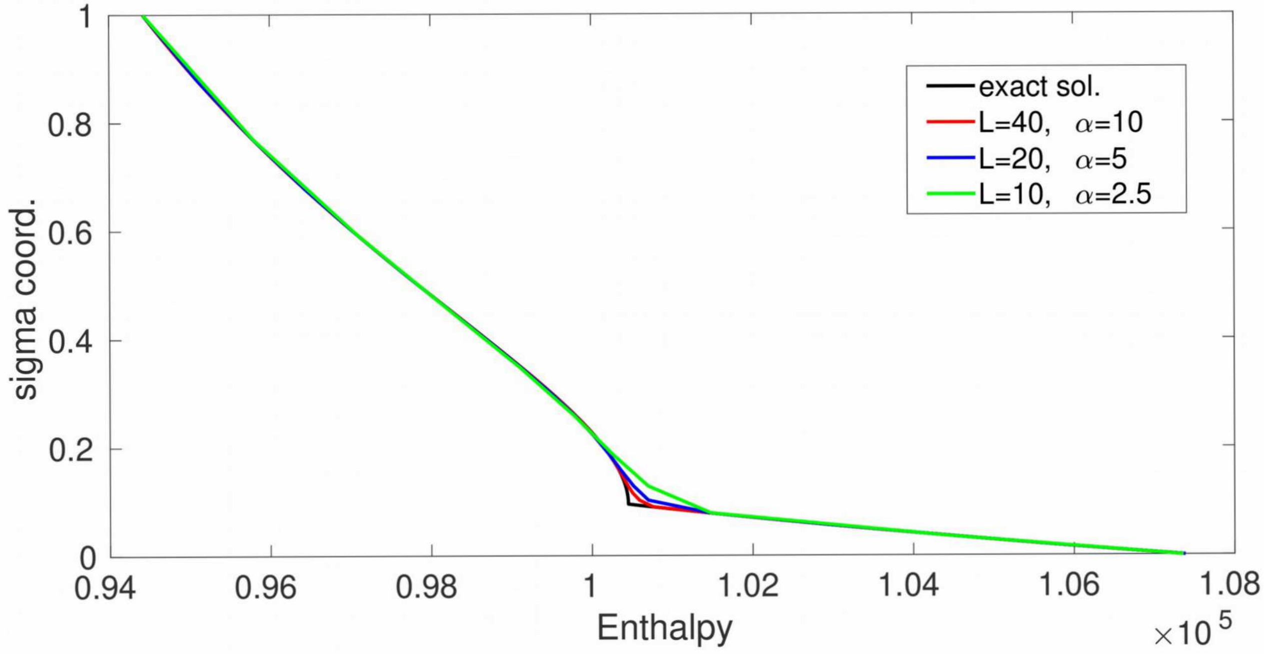


α: continuation parameter (e.g. α goes form 1e-3 to 1e3)

Kleiner at al. Benchmark B

convergence w.r.t. smoothing parameter and number of vertical layers

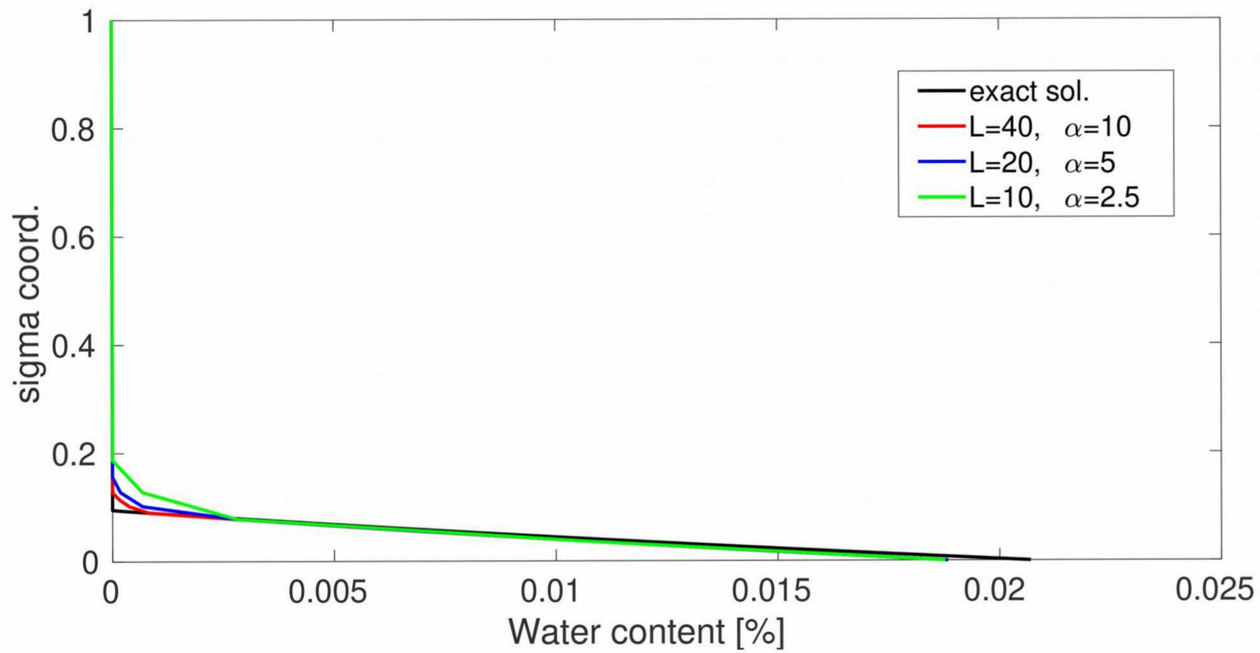


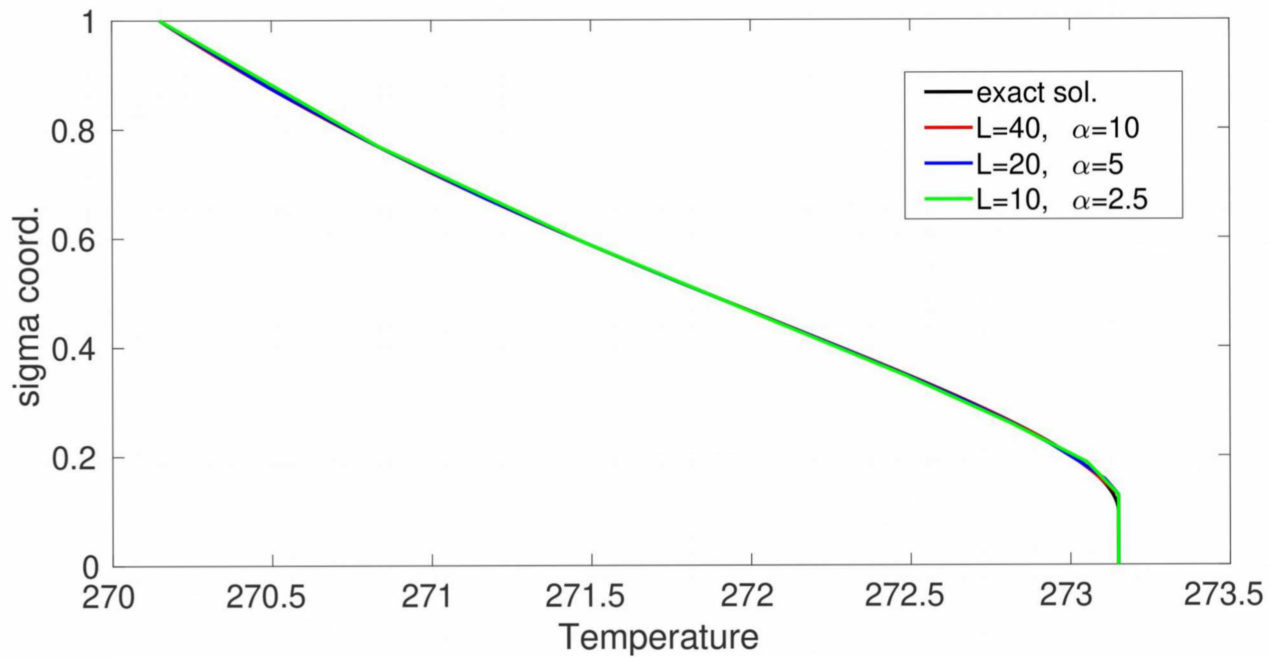


Setting:
prescribed ice velocity
Top Temp = 270.15 K
Diffusivity temperate ice : 0
L vertical layers (denser at bed)
Heat Flux = 0
Frictional heat neglected.

Kleiner at al. Benchmark B

convergence w.r.t. smoothing parameter and number of vertical layers

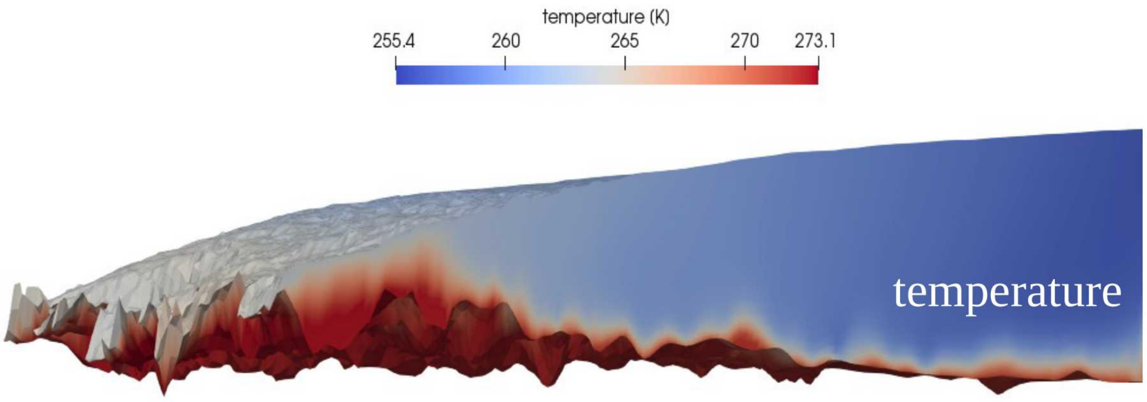


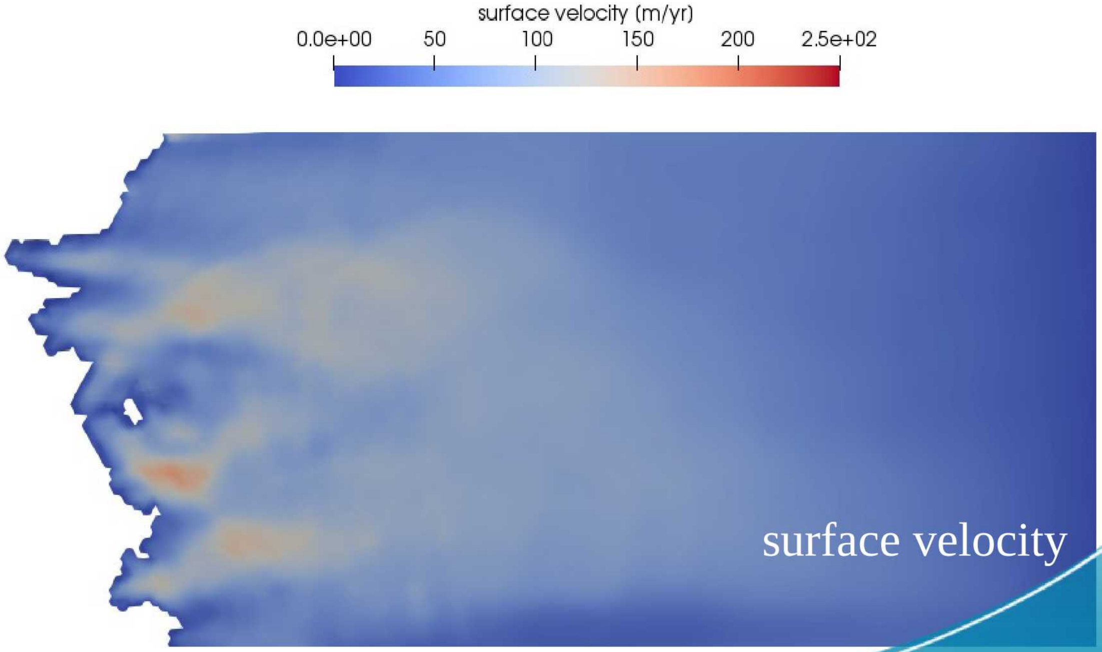


Setting:
prescribed ice velocity
Top Temp = 270.15 K
Diffusivity temperate ice : 0
L vertical layers (denser at bed)
Heat Flux = 0
Frictional heat neglected.

Realistic Geometry:
Isunnguata Sermia glacier from Western Greenland

Solution obtained after calibrating the basal friction and the bed topography with a PDE-constrained optimization approach where the constraint is the coupled enthalpy/velocity system and the cost functional is the mismatch with observed surface velocity, thickness, and tendencies





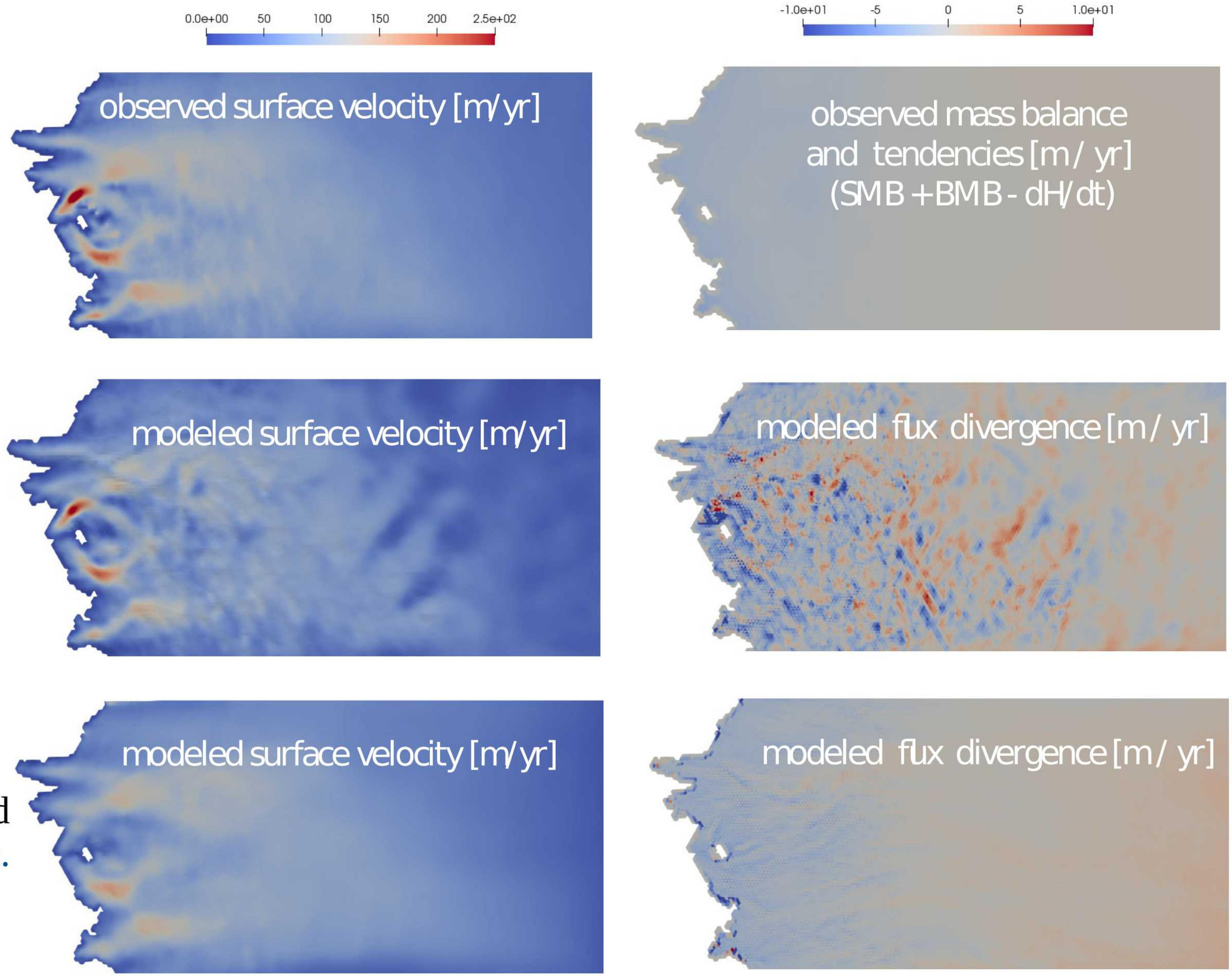
Initialization:

Isunnguata Sermia glacier from Western Greenland

Observations
(target)

Basic optimization
(calibrate basal friction to match obs. sfc. velocity)

Improved optimization
(calibrate basal friction and thickness to match obs. sfc. velocity and tendencies)

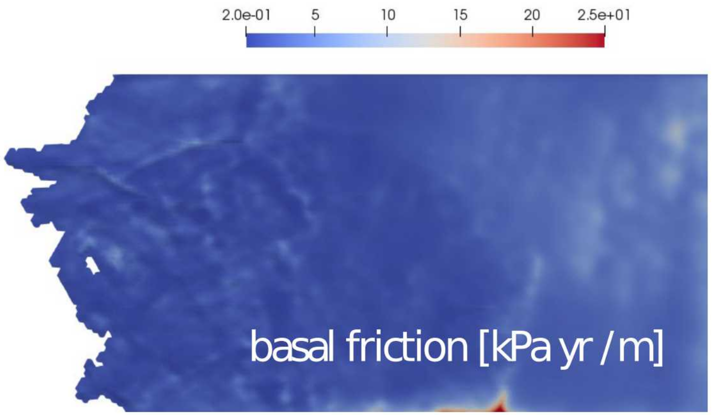


Initialization:

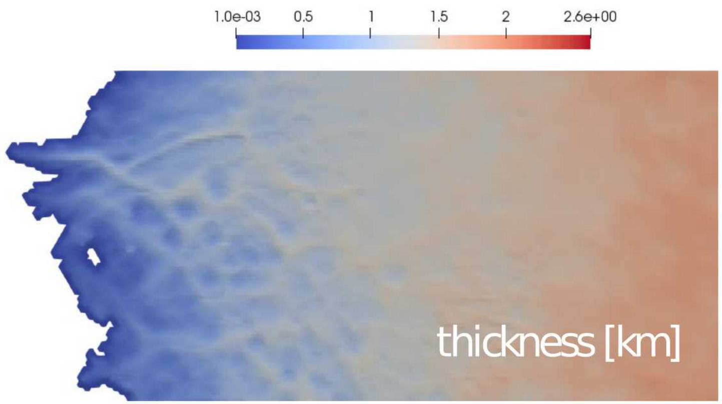
Isunnguata Sermia glacier from Western Greenland

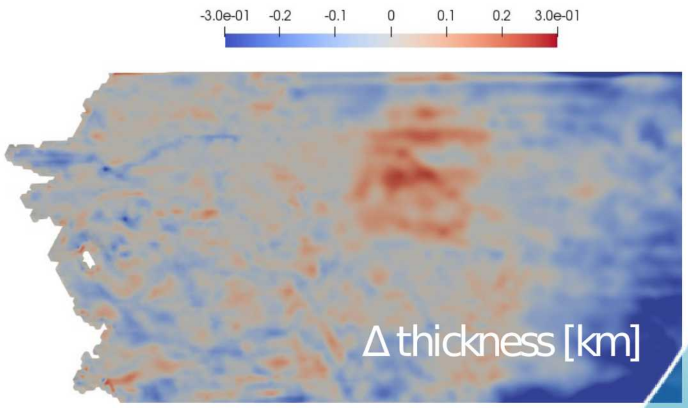
Fields estimated with improved optimization:

- basal friction:



- thickness (bed topography):





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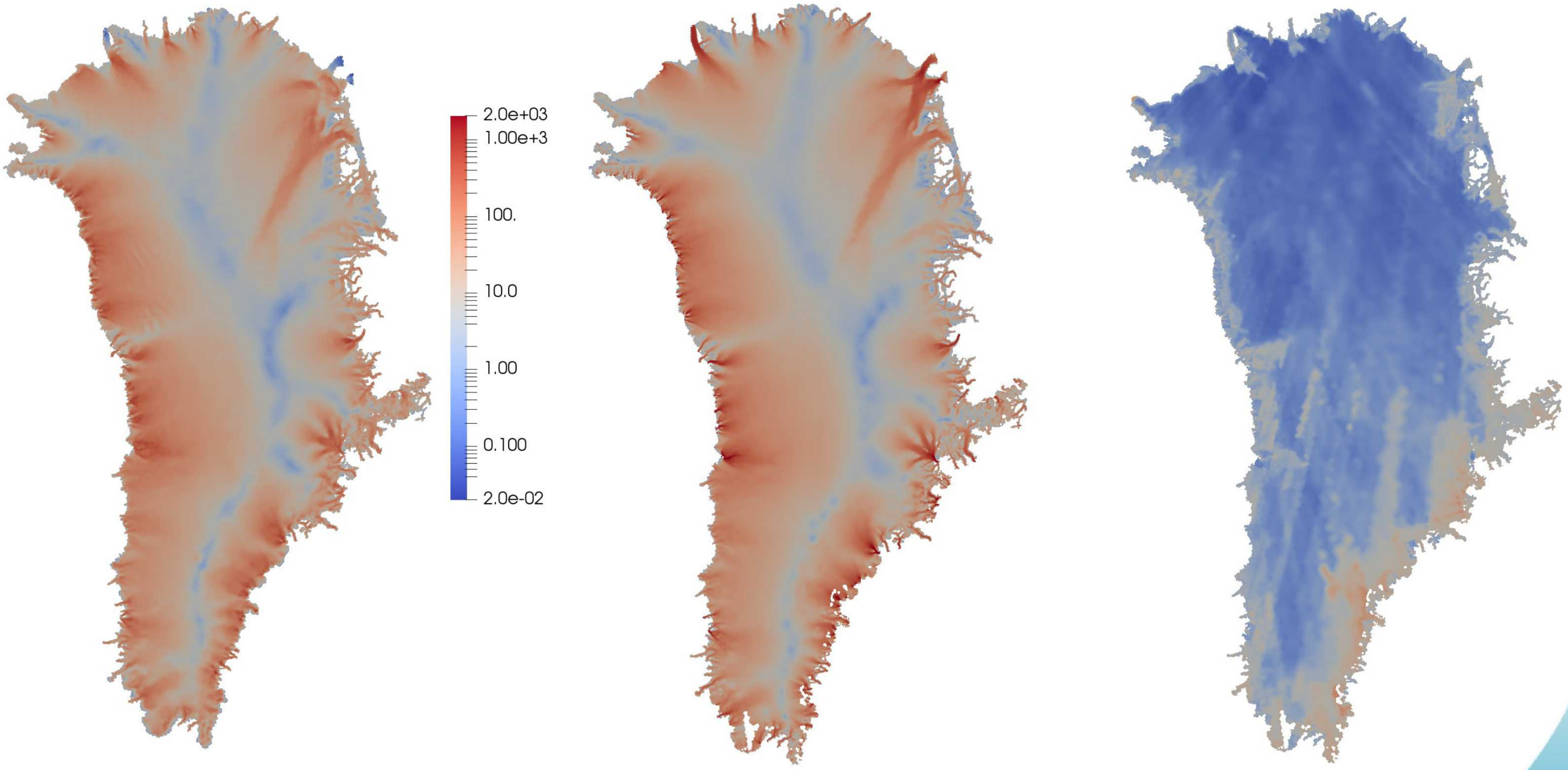
Greenland Ice Sheet Initialization

estimating basal friction matching obs. velocity. Self consistent with temperature



computed surf. velocity [m/yr]

obs surf. velocity [m/yr]

obs velocity RMS error [m/yr]

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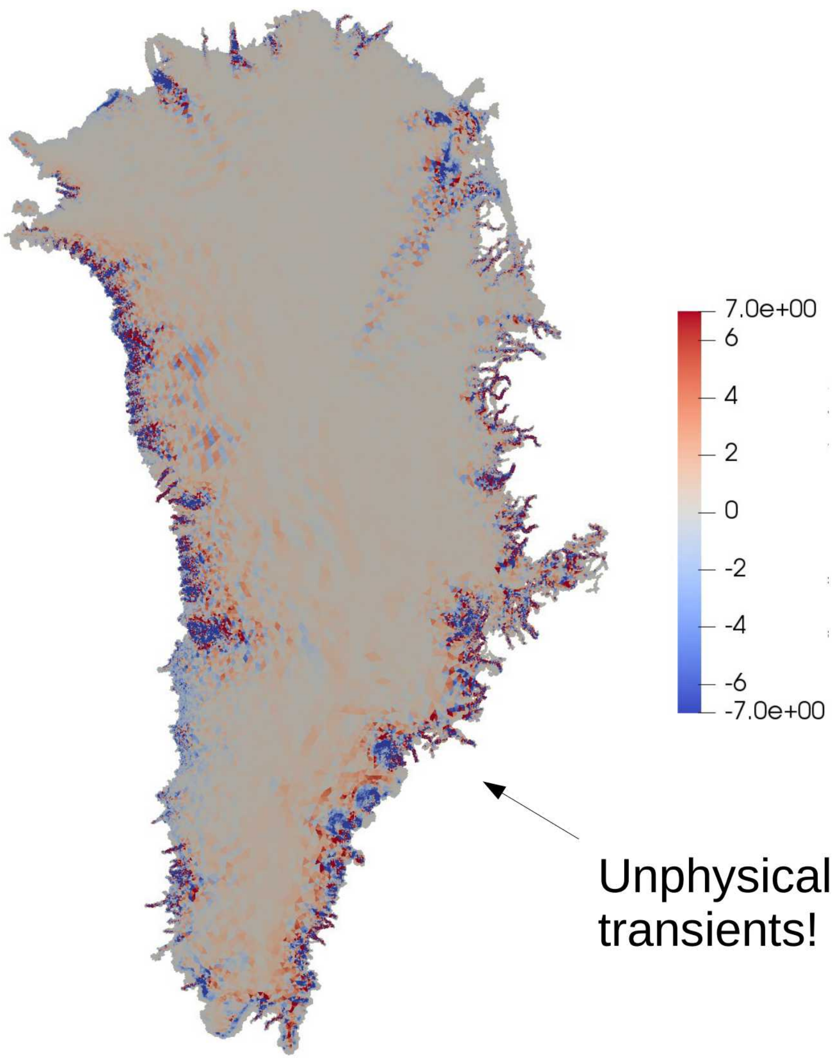
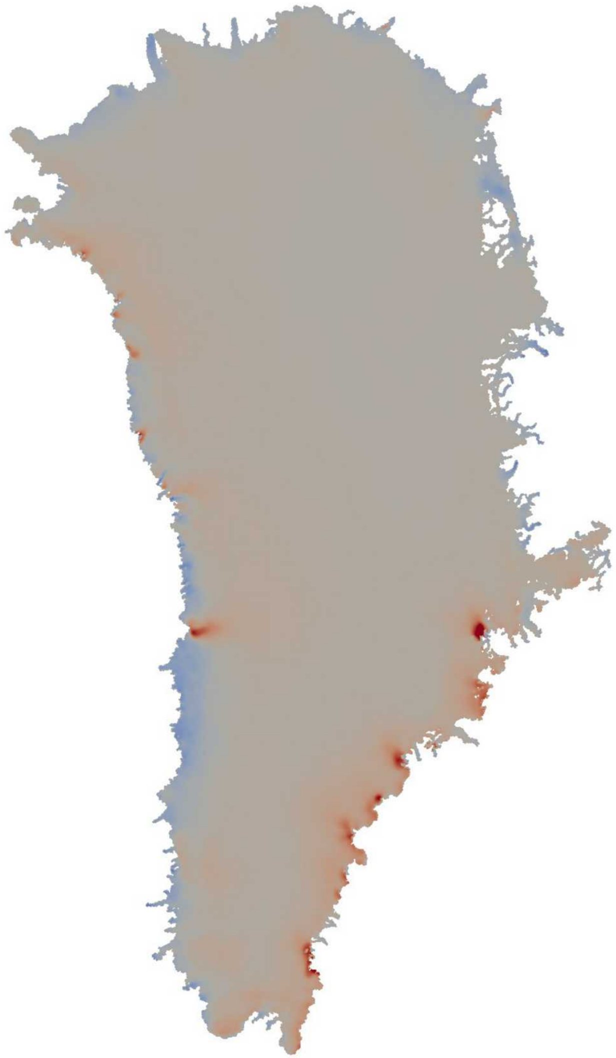
Greenland Ice Sheet Initialization

estimating basal friction matching obs. velocity. Self consistent with temperature



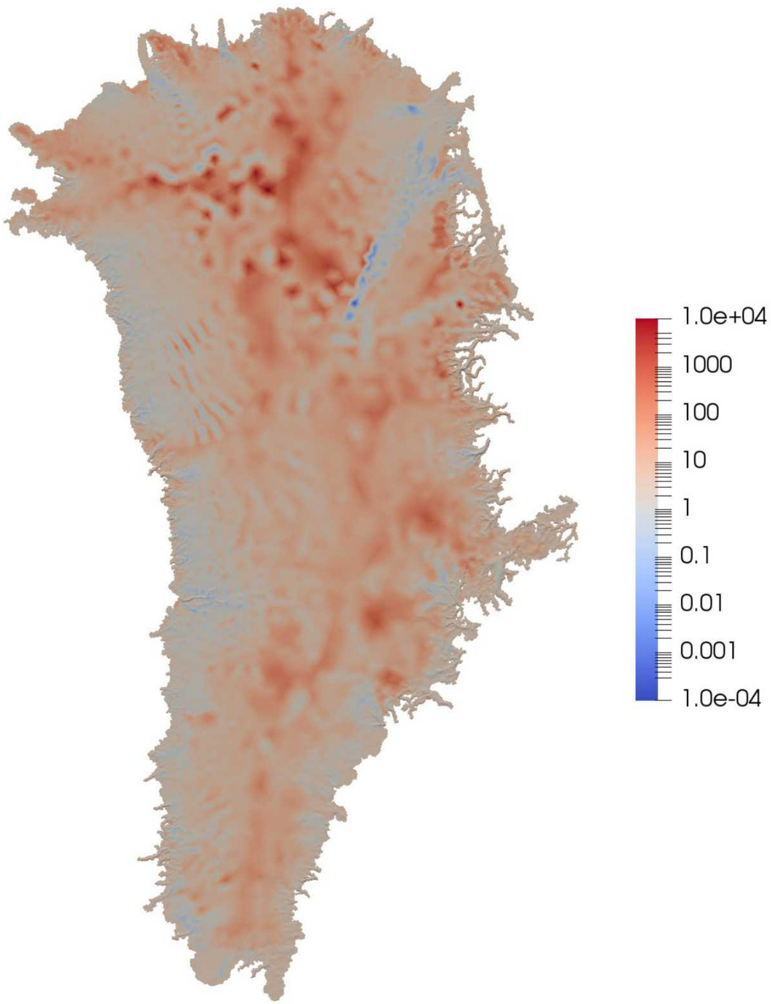
computed flux divergence [m/yr]



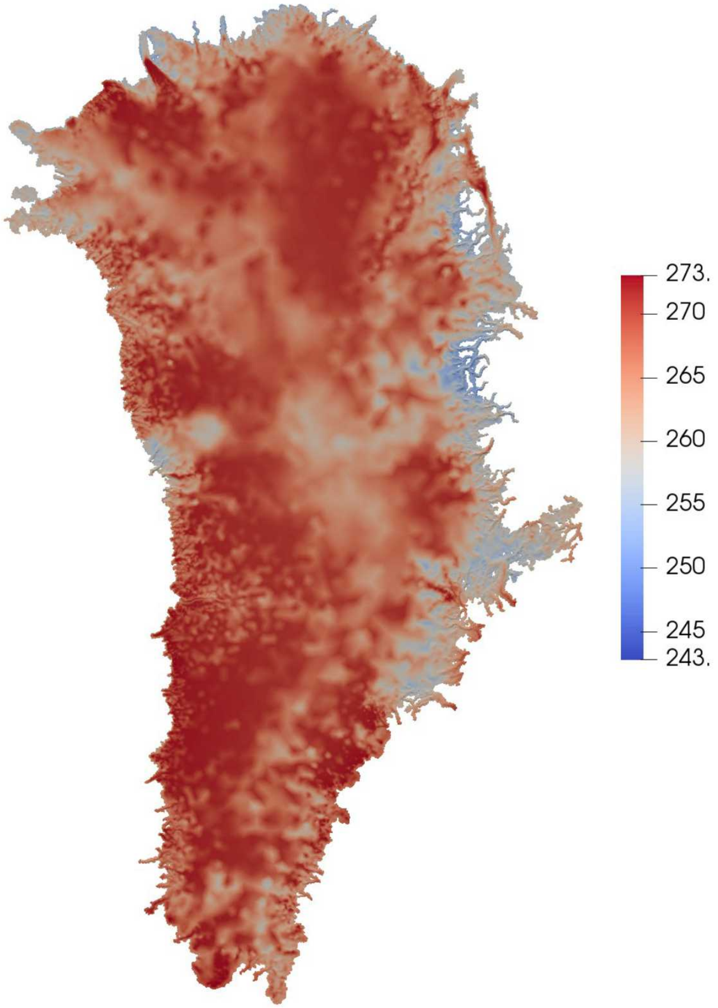
observed apparent mass balance [m /yr]

Greenland Ice Sheet Initialization

estimating basal friction matching obs. velocity. Self consistent with temperature



basal friction [kPa yr/m]



basal temperature [K]

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Initialization

How to choose weights and regularization terms?

Functional of Optimization problem 1:

$$\mathcal{J}(\beta) = \int_\Omega \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds \qquad \text{surface velocity mismatch}$$

$$+\mathcal{R}(\beta) \qquad \text{regularization terms.}$$

If RMS are available, the weights of mismatch terms are chosen based on the standard deviation, otherwise…. expert knowledge!

Possible regularizations:

$$\mathcal{R}(\beta) = \frac{\alpha}{2} \int_\Omega |\nabla \beta|$$

Total variaton

$$\mathcal{R}(\beta) = \frac{\alpha}{2} \int_\Omega |\nabla \beta|^2$$

Gradient squared

$$\mathcal{R}(\beta) = \frac{\alpha}{2} \int_\Omega (-\gamma \Delta \beta + \delta \beta)^2, \qquad \mu \beta + \frac{\partial \beta}{\partial \mathbf{n}} = 0$$

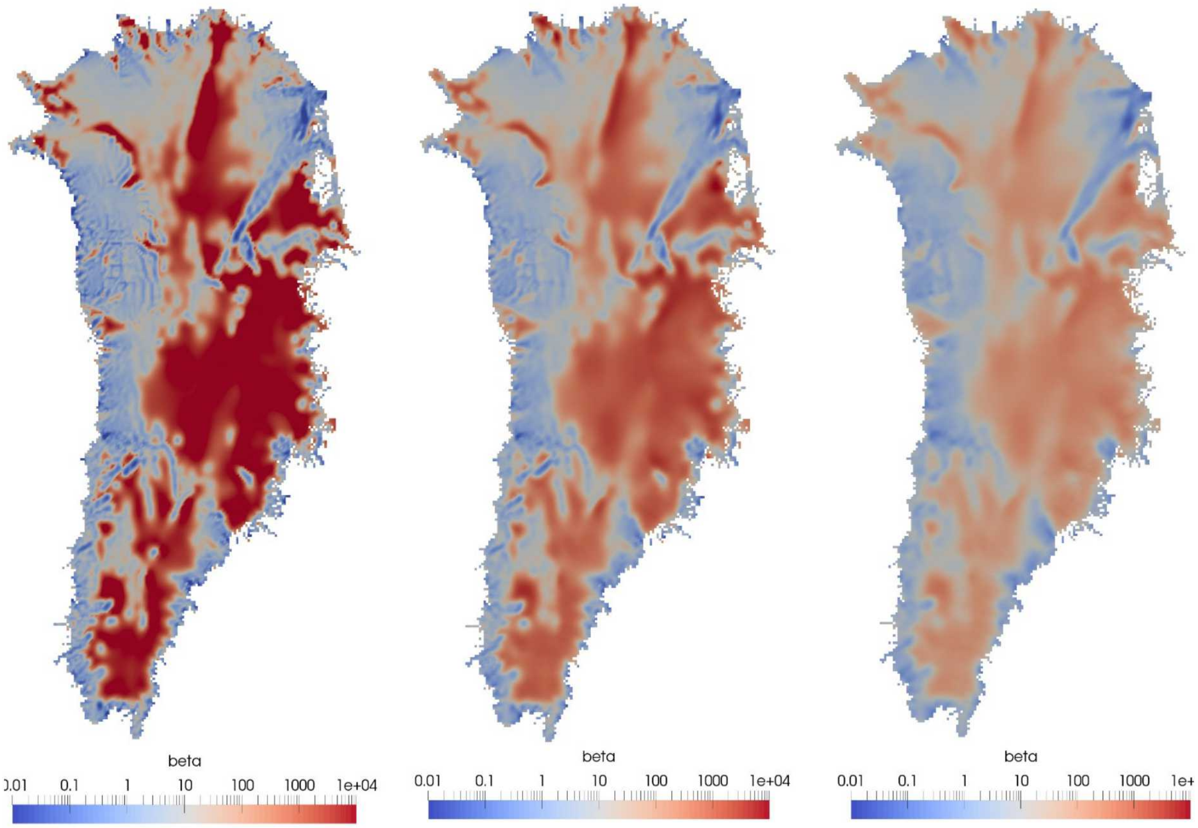
Laplacian squared regularization with Robin boundary conditions

Yes, but how do I choose the regularization parameters?

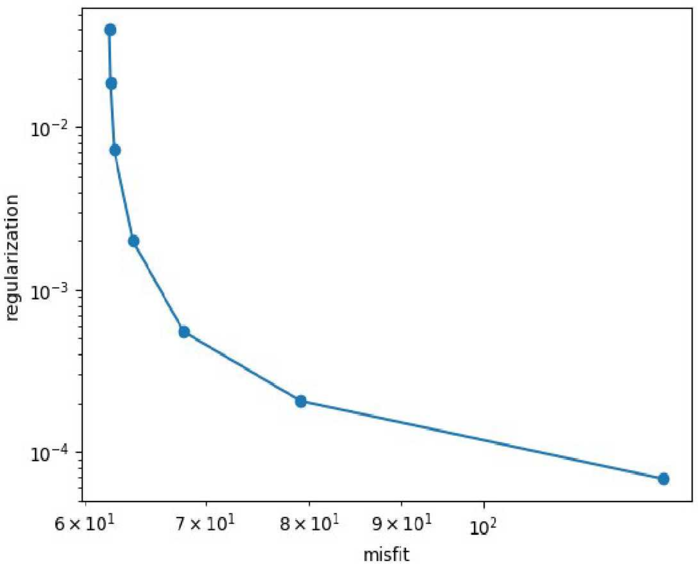
Initialization

How to choose weights and regularization terms?

Estimating Basal friction coefficient [kPa yr /m]
for increasing (from left to right) regularization



L-curve



In principle one can use the L-curve…
However it's not very reliable (L-curve might not look like an L!) and it is very expensive.
Otherwise… use <u>Expert Knowledge</u>!

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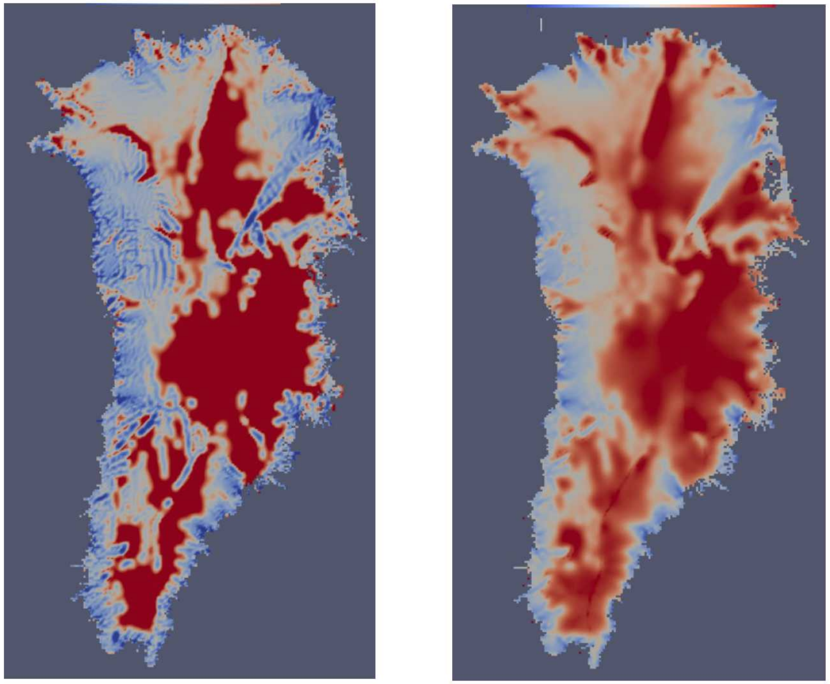
Initialization

Preliminary result inferring the basal hydrology parameter

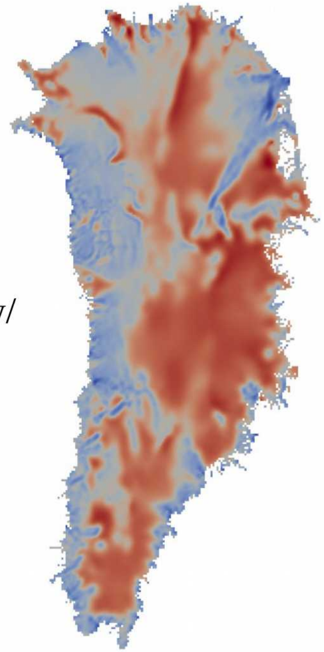
Workflow:

- Compute basal traction solving e.g. Optimization problem 1.
- Estimate hydrology parameters by matching that basal traction (computed using a cavity law) using as the constrain the hydrology model.



Left: target basal friction [kPa yr/m], from FO calibration
Right: basal friction computed w/ calibrated hydrology model

Right: basal friction from FO calibration w/ higher regularization



Discussion

- Initialization need to match trends
- Initialization needs to be consistent with the model used (try not to initialize using Shallow Ice Model and then do the forward run using Stokes + Enthalpy + Hydrology + Calving + …   .. if at all possible)
- Spin up will probably always be needed but we want to minimize that, especially if interested in short term predictions
- What's the minimum set of model equations we want to be consistent with during initialization? Flow model, Enthalpy, basal hydrology…
- Life is hard and so is initialization. Sometimes optimization does not converge.. need to work on solvers, use better algorithms

Thank you!!

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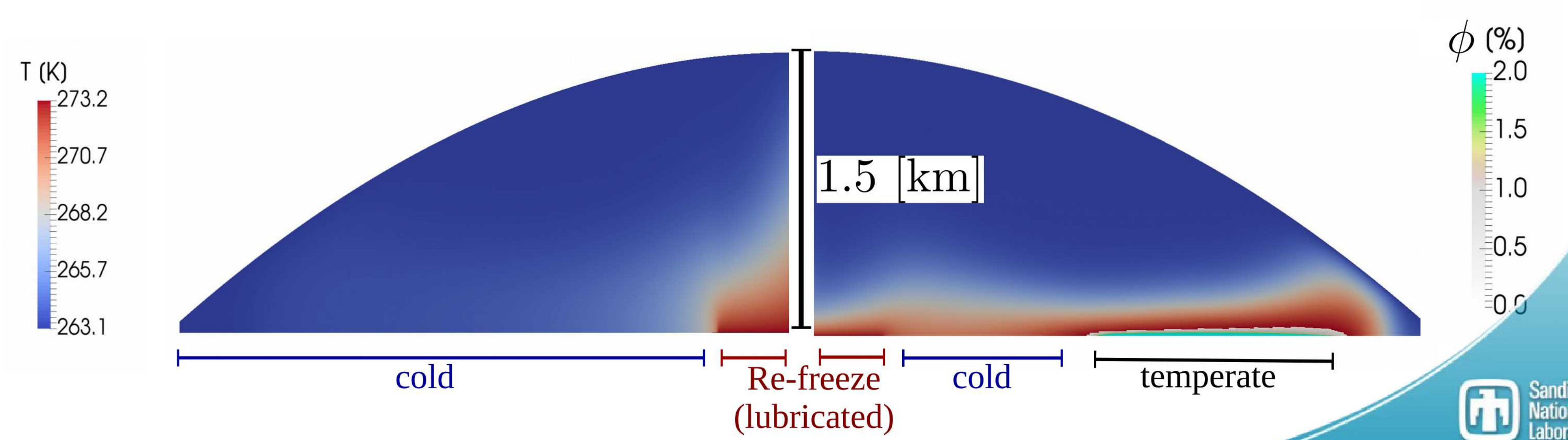
Preliminary Results:

Dome problem: based on Hewitt and Schoof (in preparation)
We explore different scenarios and report, in each picture, the temperature (for cold ice) and porosity (for temperate ice)

Problem 3: Settings
- top surface b.c.: T = -10 C
- no dissipation inside the dome
- bed lubricated near the center of the dome
- basal heat flux = 0.0 [W m^{-2}]
- coupled with FO velocity solver

Problem 4: Settings
- top surface b.c.: T = -10 C
- basal heat flux = 0.01 [W m^{-2}]
- bed lubricated near the center of the dome
- coupled with FO velocity solver

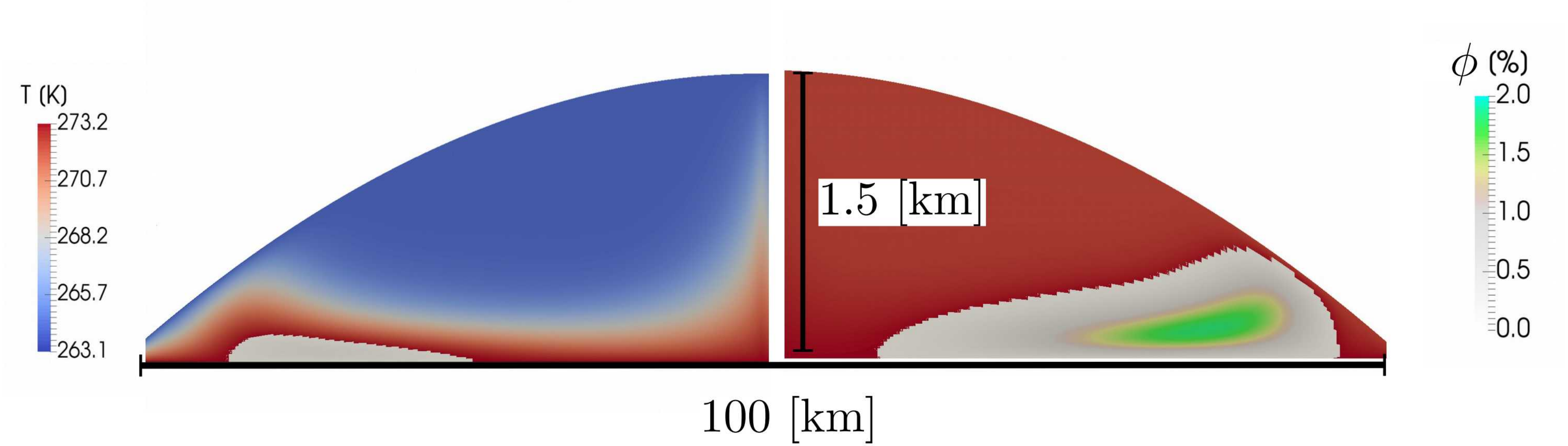


Preliminary Results

Dome problem: (based on Hewitt and Schoof, The Cryosphere, 2016)
We explore different scenarios and report, in each picture, the temperature (for cold ice) and porosity (for temperate ice).

Problem 1: Settings
- top surface b.c.: T = -10 C
- bottom surface b.c.: $h = h_m$
- prescribed SIA velocity profile

Problem 2: Settings
- top surface b.c.: T = -1 C
- bottom surface b.c.: $h = h_m$
- prescribed SIA velocity profile



Initialization

How to choose weights and regularization terms?

$$\mathcal{R}(\beta) = \frac{\alpha}{2}\int_\Omega (-\gamma\Delta\beta + \delta\beta)^2, \qquad \mu\beta + \frac{\partial\beta}{\partial\mathbf{n}} = 0$$

Laplacian squared regularization with Robin boundary conditions

Correction length $l \propto \sqrt{\frac{\gamma}{\delta}}$

$$\Sigma_{\text{prior}} = LL^T$$
$$\beta = \beta_0 + Ln, \qquad n \sim \mathcal{N}(0,1)$$

$l = 28\text{km}$ $\quad$ $l = 57\text{km}$ $\quad$ $l = 89\text{km}$