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Component and Circuit Performance Measurements for Coherent Radar – A Systems Perspective

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Abstract

A useful performance metric for Intelligence, Surveillance, and Reconnaissance (ISR) radar systems is the Impulse Response (IPR). This is true for a fidelity metric for the signal channel, as well as a stability measure across multiple pulses. The IPR represents performance with respect to both amplitude and phase modulations of the transfer function for components, circuits, subassemblies, and even the looped radar hardware. The proper IPR performance specification limits will depend on radar operating mode. Generally, it will be the intersection of the strictest requirements.

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Acronyms and Definitions

1-D, 2-D, 3-D	1-, 2-, 3-Dimesional
ADC	Analog-to-Digital Converter
AESA	Active Electronically-Steered Array
AGC	Automatic Gain Control
AM	Amplitude Monopulse
AWGN	Additive White Gaussian Noise
CPI	Coherent Processing Interval
DFT	Discrete Fourier Transform
DMTI	Dismount Moving Target Indicator
DSP	Digital Signal Processing
DWS	Digital Waveform Synthesizer
EMI	Electromagnetic Interference
GMTI	Ground Moving Target Indicator
IP_3	Third Order Intercept Point
IIP_3	Input IP_3 power level
IPR	Impulse Response
ISAR	Inverse Synthetic Aperture Radar
ISL	Integrated Sidelobe
ISR	Intelligence, Surveillance, and Reconnaissance
I/Q	In-phase and Quadrature
LFM	Linear Frequency Modulated
MWAS	Maritime Wide-Area Search
OIP_3	Output IP_3 power level
PEC	Phase Error Correction
PLL	Phase-Locked Loop
PM	Phase Monopulse
PRF	Pulse Repetition Frequency
PSF	Point Spread Function
RCS	Radar Cross Section
RX	Receive or Receiver
SAR	Synthetic Aperture Radar
SNR	Signal-to-Noise Ratio
STALO	Stable Local Oscillator
STC	Sensitivity-Time-Control
TOI	Third Order Intercept
TX	Transmit or Transmitter
WGN	White Gaussian Noise

Foreword

This report details the results of an academic study. It does not presently exemplify any modes, methodologies, or techniques employed by any operational system known to the authors.

Classification

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

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1 Introduction and Background

The purpose of radar is to make echo measurements and deduce characteristics of the intended scattering environment. This requires generating and transmitting some known signal, and receiving echoes of that transmitted signal. However, any transmitted signal will have undergone some processing between generation and transmission, and any received echo signal will undergo processing prior to its analysis. Some of the processing may be analog, and some Digital Signal Processing (DSP). Every component or circuit through which the signal passes will in fact process the signal. The overall processing will have some desired and intended effects, and likely some undesired, unintended, and perhaps unknown, effects. In any case, the signal being analyzed is a blending of the radar's received echo, and alterations imparted by the processing, i.e. passing through components and circuits in the overall signal path.

Ideally, any radar processing is perfect in the sense of completely intended and understood. Alas, the ideal is never quite achieved, generally only approximated to some degree of accuracy and precision. The quality of a radar system is often commensurate with how close the actual signal processing is compared to the desired ideal signal processing. We note that high-dynamic-range radar modes like Synthetic Aperture Radar (SAR) and Ground-Moving Target Indicator (GMTI) radar are especially disposed to revealing subtle anomalies in signal characteristics. Our focus herein is principally airborne Intelligence, Surveillance, and Reconnaissance (ISR) radar systems, although the principles are often much more broad.

There are two main aspects to the "goodness" of a radar.

1. Within a single pulse -- the accuracy and precision with which a received signal represents the expected echo. This includes how well a signal passes without unintended distortion. Recall that for amplifiers a distortionless component imparts only gain and delay. This is generally a signal fidelity measure of the components and circuits.

2. Across multiple pulses -- the degree which a signal response is repeatable from one pulse to the next. That is, the degree to which echo signals from two distinct pulses, with identical inputs, through the same components and circuits, configured identically, yield identical outputs. This is generally a stability measure of the components and circuits. It is particularly important to coherent range-Doppler radar systems.

Clearly this is a two-dimensional (2-D) analysis. We desire both high fidelity and stability.

What follows in this report is an amalgamation of observations, thoughts, and techniques employed in determining the goodness of radar components, circuits, and subassemblies, from the viewpoint of a radar systems analyst/engineer.

Useful References

We offer the following reports as relevant background for the discussion that follows.

SAND2007-5042 discusses the nature of a 2-D IPR for Synthetic Aperture Radar (SAR).[1]

SAND2012-10688 discusses radar passband characteristics for a transmitter.[2]

“Things are not always what they seem.”
-- Phaedrus

For radar designers, this might be translated to
“The circuit you have isn’t the circuit you think you have.”

2 Impulse Response

The conventional assumption for radar is that a target echo is perfect in the sense of replicating exactly the incident signal, without distortion, or even delay beyond its range from the radar. This gives rise to the somewhat mythical concept of a "point target." A point target is defined to be a radar reflector with a precise single-point location in space, yet having a Radar Cross Section (RCS) that is measurable, and typically isotropic. It is distortionless, and consequently has infinite frequency response. The target point has area, but no physical size, hence the characteristics of an impulse. Such a target can only be approximated in practice, but sufficiently so for it to be a useful concept.

A radar's ability to precisely measure the echo delay, and hence range, to the point target is tantamount to its response to the impulse target, termed its Impulse Response (IPR). This is a fundamental goodness measure of radar performance; in a sense a cleanliness measure of the radar's response. In some communities this is also known as a Point Spread Function (PSF). In subsequent sections we will apply the IPR measure to linearity as well as stability.

Appendix A discusses IPR in more detail, and relates it to concepts of correlation and matched filtering.

Even with an impulse target, a radar will interrogate that target with a band-limited signal, and the radar receiver will have its own band-limited ability to process the echoes. These bandwidth limitations will likewise affect the precision with which a time-delay, and hence range, can be measured by the radar, by 'smearing' the echo response. They will limit the ability to resolve range.

They will further impart processing sidelobes to the signal being assessed. These sidelobes can mask low-level echo signal characteristics. The usual remedy is to employ window taper functions to reduce sidelobe levels, but at the expense of somewhat wider mainlobe response, i.e. resolution of the impulse response.[3] This is illustrated in Figure 1.

The next question is then "How do we specify an IPR as a requirement?"

First, it behooves us to understand the nature of an IPR, especially given that a window taper function is employed. Important regions of the IPR are displayed in Figure 2, and are described in detail in the prior report.[3] A good IPR specification will address each of these major regions. However, a good IPR specification must have in mind the window taper function that will be employed in the data analysis. Even a perfect component or circuit, where the IPR is calculated with a different window taper function, may very well fail an otherwise reasonable IPR specification. An IPR specification should match a specific window taper function.

A reasonable specification criteria might be to limit measured IPR data to have a mainlobe with −3 dB width no wider than 10% greater than the ideal window taper response, −18 dB width no wider than three times the ideal window taper response's −3 dB width, sidelobes be no greater than 10 dB above the sidelobes for the ideal window taper response, and 10 dB above the desired noise floor due to time-base phase-noise desired limits.

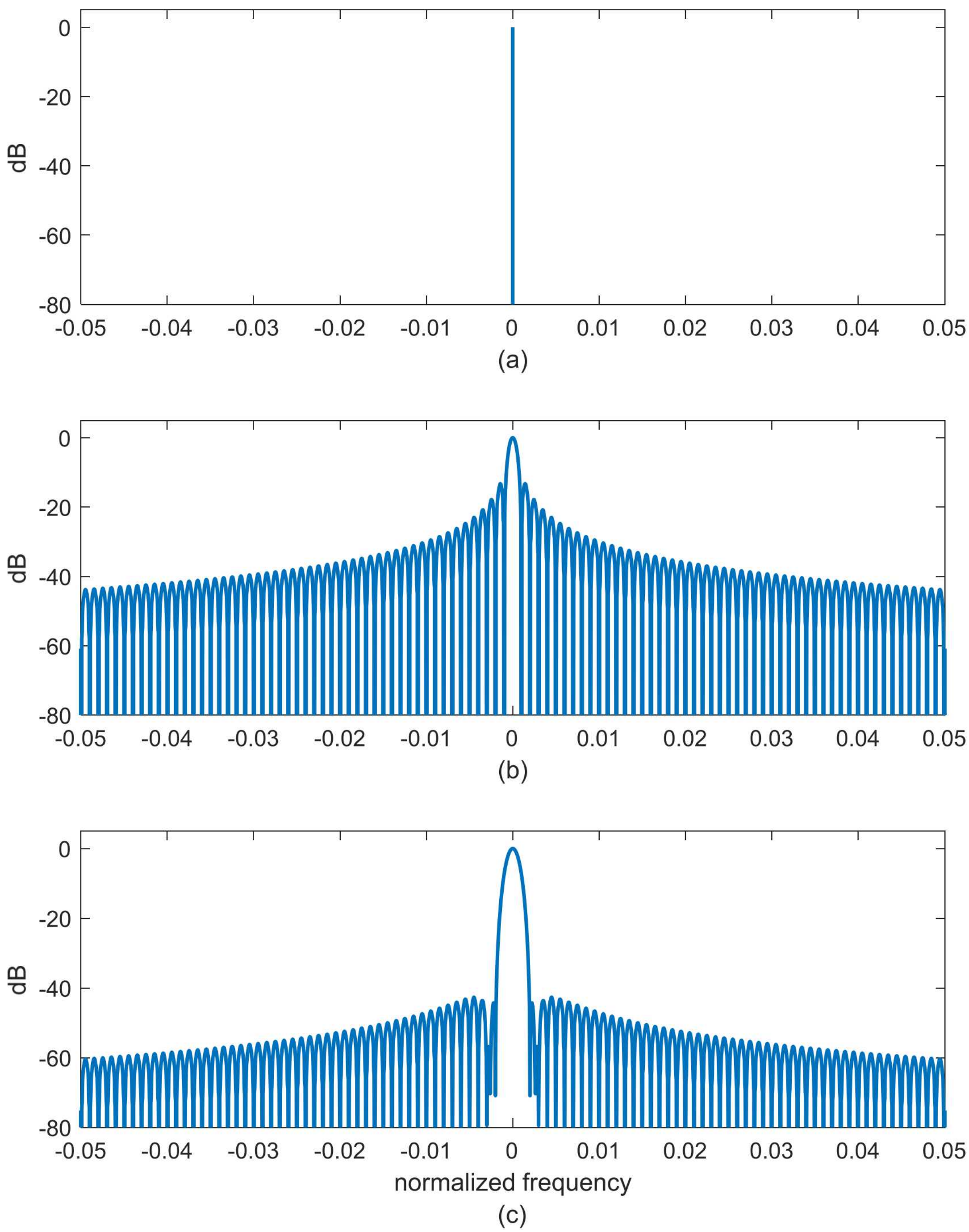

Figure 1. (a) impulse input to radar band-limited processing, (b) result of impulse passing through ideal but band-limited filter, and (c) result of impulse passing through band-limited processing with Hamming taper.

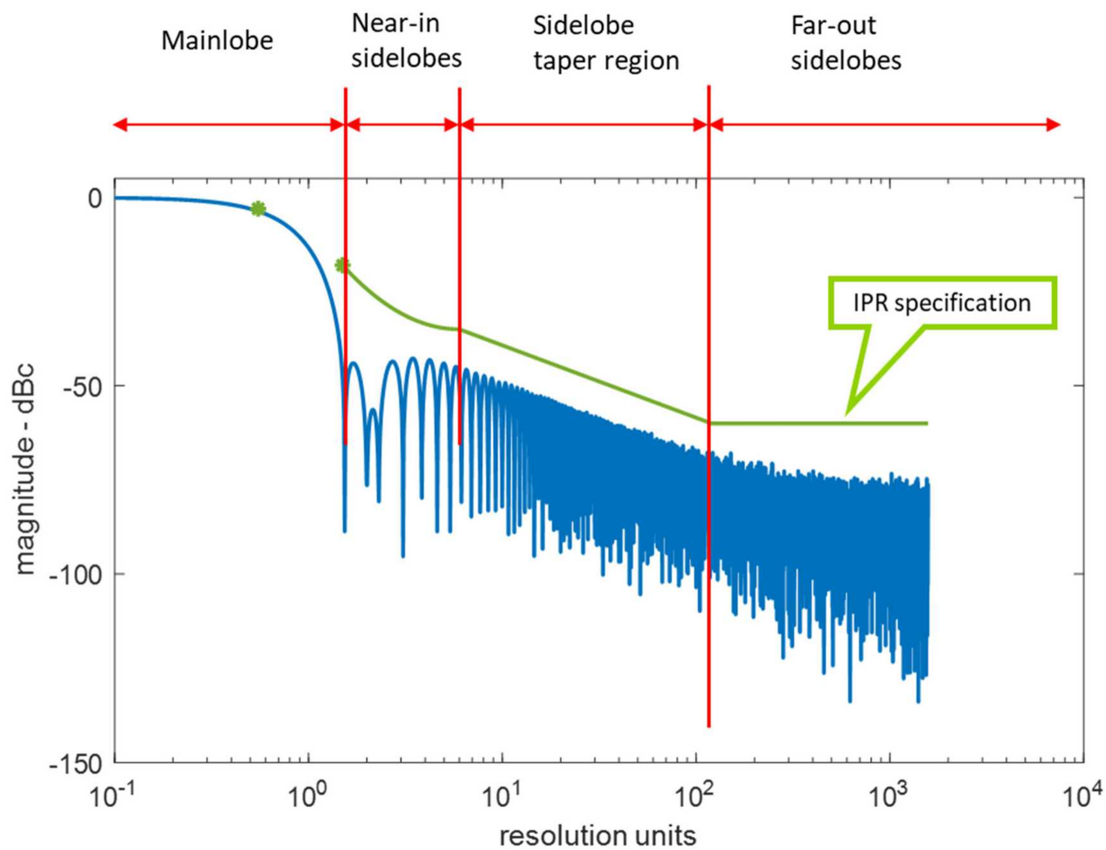

Figure 2. Major regions of the IPR of a window taper function. The window whose IPR is displayed is a 4096-point Hamming window, but quantized in magnitude to 128 different levels. Only the positive normalized time offsets of the IPR are displayed with a logarithmic axis. Boundaries between major sidelobe regions are somewhat squishy.

Figure 2 also illustrates an example IPR specification, which is mathematically described as follows.

$$\text{IPR\_limit} = \begin{cases} -3 & u = 0.55 \\ 17\left(1 - \dfrac{\log_{10}(u/1.5)}{0.6021}\right)^2 - 35 & 1.5 \le u \le 6 \\ -25\left(\dfrac{\log_{10}(u/6)}{1.301}\right) - 35 & 6 \le u \le 120 \\ -60 & 120 \le u \end{cases}, \text{ in units of dB,} \tag{1}$$

where

$$u = \text{resolution units (i.e. normalized to the ideal } -3 \text{ dB mainlobe width).} \tag{2}$$

A more general treatment of IPR specification development is given in Appendix B.

IPR Specification

Given that we know what an IPR specification might look like, a reasonable question becomes "What should the IPR specification be?" As is typical in radar system engineering, this might be an easy question to ask, but a rather difficult question to answer. We start by stating that the "right" IPR specification is probably radar mode dependent. That is, what is right for one mode, is probably not the best for another mode. Consequently, for a multi-mode radar, the overall requirement might be the harshest specification over all modes.

We next comment on some specific radar modes. We will discuss the IPR for the entire signal path, noting that component specifications should be somewhat lower yet to provide margin and allow for accretion. The radar modes we discuss below are meant to be merely representative.

Synthetic Aperture Radar (SAR)

The essential function of SAR is generally to map clutter in a scene of interest. The output is an image; a range-Doppler image. The background against which objects of interest are characterized is usually other clutter that is often well above the noise floor, but not always. Low-level far-out sidelobes are usually only visible for very bright specular reflectors and against low-reflectivity clutter such as still water, smooth concrete, etc., and in shadow areas.

In addition to clutter regions, it is also often of interest to characterize regions of low, or even no clutter. Consequently, an important parameter for SAR is the overall Integrated Sidelobe (ISL) ratio. This is the amount of "spilling" of sidelobe energy to be expected into dark clutter regions and characterized the available contrast between dark regions and adjacent brighter clutter. Furthermore, high-performance SAR is often operated with very fine resolutions, implying wide-bandwidth signals. Single-pulse bandwidths of 1800 MHz to support 0.1-m range-resolution is not uncommon.[4]

A typical window taper function that balances resolution with sidelobe suppression for SAR images is a −35 dB Taylor window ($\bar{n} = 4$). Anecdotal subjective evidence suggests that low-level far-out sidelobes should be below on the order of −60 to −70 dBc.

Ground-Moving Target Indicator (GMTI) Radar

This includes Dismount Moving Target Indicator (DMTI) Radar. The essential function of GMTI/DMTI radar is to detect moving targets of interest. The output is typically detection reports that specify map locations. The background against which reflecting targets are detected is typically the noise floor of the range-Doppler map, either outside of, or after suppression of stationary clutter. However, residual clutter effects may still remain. Suggested DMTI requirements are discussed and recommended in an earlier paper by Doerry, et al.[5]

Since the targets of interest for GMTI are vehicles, and then typically only their location and velocity, range resolution for GMTI radar is typically much coarser than for SAR, often from 3 m to 10 m or so. Exceptions exist for specialized modes like High-Range-Resolution (HRR) modes. DMTI radar, intended for detecting humans, are often finer resolution than GMTI, perhaps as fine as 0.3 m or so, but still often not as fine as the finest-resolution SAR modes.

A typical window taper function for GMTI/DMTI radar modes places a greater premium on reduced sidelobes rather than mainlobe width (for resolution). Resolution is still somewhat of a concern mainly because a wider mainlobe also often correspond to a slight reduction in Signal-to-Noise Ratio (SNR), but sidelobes are still the principal concern owing to their contribution to false alarms. Consequently, a typical window taper function for GMTI/DMTI might limit near-in sidelobes to −50 to −70 dBc, and low-level far-out sidelobes to below on the order of −60 to −70 dBc.

Maritime Wide-Area Search (MWAS) Radar

The essential function of MWAS radar is to detect stationary and moving maritime targets of interest, typically ships on the open sea. The output is typically detection reports that specify map locations, although pre-detection echo data is also often displayed on a map for human analysis. Since the sea clutter reflectivity may vary widely, depending on sea-state and radar interrogation geometry, the background against which reflecting targets are detected may be sea clutter, or may be the noise floor of the system. A primer addressing detection of maritime targets is given in an earlier report.[6]

Since the targets of interest for MWAS are often comparatively large sea vessels, and then typically mainly their location in a fairly large range-swath, range resolution for MWAS radar is typically much coarser than for even GMTI, often from 10 m to 100 m or so. Exceptions exist here, too, for specialized modes like HRR modes. However, the Radar Cross Section (RCS) of sea vessels of interest might vary 60 dB or more. Clearly, we desire sidelobes of a larger vessel to not hide the signature of a nearby smaller vessel.

Much like GMTI/DMTI radar modes, a typical window taper function for MWAS radar modes also places a greater premium on reduced sidelobes rather than mainlobe width (for resolution). Resolution is still somewhat of a concern mainly because a wider mainlobe also often correspond to a slight reduction in Signal-to-Noise Ratio (SNR), but sidelobes are still the principal concern owing to their contribution to false alarms. Consequently, a typical window taper function for MWAS might limit near-in sidelobes to −50 to −70 dBc, and low-level far-out sidelobes to below on the order of −60 to −70 dBc.

Maritime Inverse-SAR (ISAR)

Inverse-SAR (ISAR) is essentially SAR of a moving object such as a ship on the sea, presumed to be a rigid body. Different than MWAS, ISAR intends to form an image of the vessel, usually a mapping of specular reflectors on the vessel, often termed a point cloud. The surrounding sea clutter is usually ignored; not the object of attention. Focusing the target vessel is data-driven, and the difficult part of the overall processing. Resolutions are typically from fine to moderate; from 0.3 m to perhaps 3 m in range, implying bandwidths to perhaps 600 MHz or so.

Consequently, a typical window taper function will be more interested in near-in sidelobes than far-out sidelobes, with near-in sidelobes limited to perhaps −40 dB or so, such as with perhaps a Hamming window.

“All generalizations are false, including this one.”
-- Mark Twain

3 Single-Pulse Channel Fidelity

In a typical radar system, an interrogation signal is first generated, then up-converted to some desired radar band, amplified, and transmitted. Echo energy is then received, amplified, down-converted to facilitate easier processing, and typically digitized for subsequent data analysis. We do not intend to trivialize the design and construction of high-performance radar hardware, and fully acknowledge that on both the transmitter side as well as the receiver side, circuits are generally composed of multiple stages of amplifiers, attenuators, filters, mixers, limiters, splitters, frequency multipliers, circulators, isolators, etc. Characteristics of such components are discussed in any of a number of texts and papers, a text by Pozar being one example.[7]

Nevertheless, it is essential that the signal data analysis is able to separate signal characteristics of the underlying waveform from the modulations imposed upon the echoes by the target scene or environment. This means that the various components and circuits through which the signal passes, from generation to analysis, are well understood as to their effects on the eventual echo signal. The signal path is often referred to as a signal "channel."

Several descriptors of specific channel fidelity characteristics are commonly used, and are important to us. Among them is "linearity." We are mindful that the definition of a linear system is one in which superposition holds. That is, for

$$\begin{aligned} x_1(t) &= \text{input signal \#1}, \\ x_2(t) &= \text{input signal \#2}, \end{aligned} \tag{3}$$

and if each is applied to a system described by an operator $g\{\ \}$ such that

$$\begin{aligned} y_1(t) = g\{x_1(t)\} &= \text{output signal \#1}, \\ y_2(t) = g\{x_2(t)\} &= \text{output signal \#2}, \end{aligned} \tag{4}$$

then a linear system must satisfy superposition, namely

$$\alpha y_1(t) + \beta y_2(t) = g\{\alpha x_1(t) + \beta x_2(t)\}, \tag{5}$$

for constants α and β.

In any case, there are a number of useful radar components and circuits through which a signal might pass that are not strictly linear. This can be quite fine, if in fact the circuit was designed to be so.

Time-Invariance is the property that if some input signal yields an output signal as

$$y(t) = g\{x(t)\}, \tag{6}$$

then a delayed input signal yields an output signal with the same delay

$$y(t-\tau)=g\{x(t-\tau)\}, \tag{7}$$

for some arbitrarily chosen delay τ. A system that is both linear and time-invariant is termed a Linear Time-Invariant (LTI) system.

Another important concept is one of a "distortionless" component or circuit. Distortionless has the specific meaning of a component imparting only gain and delay. Otherwise would be to distort the signal. That is, if a system exhibits the behavior

$$y(t)=g\{x(t)\}=\kappa\, x(t-\tau), \tag{8}$$

for arbitrary constants κ and τ, then the system is distortionless. Note that $\kappa<1$ implies a loss, or attenuation. Recall that a constant time delay equates to a linear phase shift with frequency.

As with linear components and circuits, there are a number of useful radar components and circuits through which a signal might pass that are not strictly distortionless. Again, this can be quite fine, if in fact the circuit was designed to be so. Many are.

Even where we might require LTI and/or distortionless systems, it frequently suffices for this behavior to exist over a limited frequency band of interest. Furthermore, it also generally suffices for this behavior to exist over a limited dynamic range of inputs.

We note that filters are by definition not distortionless, although they might exhibit distortionless behavior over limited frequency bands. Furthermore, all components exhibit some degree of limited frequency response, i.e. filtering.

In the subsequent analysis, we generally will measure "goodness" by inputting a perfect signal, and examining the output for imperfections; departures from the ideal response. Generally, the perfect input signal will be constant-amplitude sinusoids swept or indexed over frequency.

3.1 LTI Components (amplifiers, attenuators, filters)

Processing with this class of components can be described by convolution, where

$$y(t)=h(t)*x(t)=\int_{-\infty}^{\infty} h(t-\tau)x(\tau)\,d\tau=\int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau, \tag{9}$$

We identify

$h(t)$ = the IPR of this component, and

$H(f)=\Im\{h(t)\}$ = the Transfer Function of this component. (See Appendix A.) (10)

Recall that for these components, superposition holds. This means that the behavior of complex sums of signals can be deduced from the behavior of the component to individual perhaps simpler signals, like single-frequency sinusoids. We like this.

To identify the IPR, it is often more convenient to first determine $H(f)$, and then calculate $h(t)$ by Inverse Fourier Transform. Of course, the way $H(f)$ is determined is by applying a sinusoidal $x(t)$ with specific amplitude and frequency, and measuring the amplitude and phase of the resulting output $y(t)$. Doing this for a span of *f*, with careful attention to relative amplitude and phase, yields $Y(f)$ that also equates to $H(f)$.

Both amplitude and phase need to be measured as a function of frequency. Mechanisms for doing so include the following.

1. Network Analyzer capable of measuring Scattering-parameters (S-parameters), in particular the S21 forward-gain parameter or equivalent.

2. Employing a Linear-FM (LFM) chirp signal for $x(t)$. Such a chirp has a linear relationship between time *t* and frequency *f*. In this case

   $$y(t) \approx H(\gamma t), \tag{11}$$

   where γ = chirp rate.

   A caveat is that slower chirps are better, that is, chirps with large time-bandwidth products.

Wideband vs. Narrowband Signals

Note that any anomalies in the IPR function $h(t)$ also implies anomalies in the transfer function $H(f)$. However the significance of any anomalies in $H(f)$ depends on what portion of the component's passband the input signal $x(t)$ is exciting. This is obviously much more likely for a wideband input signal than a narrowband input signal.

As example, for a radar using stretch processing, where a LFM signal is de-chirped prior to subsequent processing, a target echo may be wideband prior to dechirping, but narrowband after dechirping. Of course, the terms wideband and narrowband are highly subjective.

Consequently, it is how a particular component is employed that will impact how the component's IPR affects the overall system's IPR.

Analog-to-Digital Converter (ADC)

We note that an Analog-to-Digital Converter (ADC) is generally expected to be linear. However, for modern high-performance radar systems, some attention generally needs to be paid to ensure adequate ADC performance in this regard.[8] This is often best handled by good ADC selection during the design process. This selection should be handled with great care.

Digital-to-Analog Converters (DAC)

Modern high-performance radar systems often generate their intended waveform digitally, and then convert it to an analog signal using a Digital-to-Analog Converter (DAC), which is also generally expected to be linear, that is, faithfully reproducing the intended waveform. However, due to the quantized output, a DAC has inherently nonlinear characteristics. The quantization error is furthermore highly correlated with the signal being generated. This nature generates harmonics and mixing products generally described as "spurious signals" or simply "spurs." While some of these are out-of-band and can be mitigated with filtering, other spurs are in-band and more problematic.

As the DAC input signal is swept in frequency, as with a LFM chirp waveform, we will observe at the output that in-band spurs may sweep at the same rate, or at different rates that are multiples of the signal sweep rate, and/or even in the opposite direction at multiple rates. Of these, those that sweep in the same direction and at the same rate as the input chirp are the ones that are of most concern, as these will be passed and compressed by the range-processing correlators in the receiver.

As with ADC performance, this is best handled by good DAC selection during the design process. However, verification can be accomplished by spectral analysis while frequency of a test signal is slowly swept. More complicated waveforms will tend to decorrelate the quantization noise so that it manifests as a low-level noise floor in an IPR.

3.2 Nonlinear Components – Non-Frequency-Translation

We limit our attention here to a class of components that exhibits the following properties.

1. An input signal with specific frequency f_0 will output a signal with the same specific frequency f_0. Although harmonics might be generated, the principal output frequency of interest is the same as the input frequency f_0.
2. The instantaneous amplitude of the output signal $y(t)$ is not proportional to the instantaneous amplitude of the input signal $x(t)$. Consequently, superposition does not hold.

Examples of such components are limiters, and compressive amplifiers, such as might be found in the power amplifier stages of a radar transmitter. Efficient transmit power generation is of particular interest to us, so consequently we will limit our subsequent analysis for this section to power amplifiers operating in compression. We will model such an amplifier as having signal-level dependent gain. That is, assuming a real-valued input

$$y(t) = A(x(t))\,x(t), \tag{12}$$

where

$$A(z) = \text{nonlinear function of argument } z. \tag{13}$$

A reasonable model for the relationship between input $x(t)$ and output $y(t)$ might be a sigmoid function. We will model $A(z)$ as a normalized

$$A(z) = \frac{\tanh(z)}{z}. \tag{14}$$

This allows the output to be modelled as

$$y(t) = \tanh(x(t)) . \tag{15}$$

This relationship is plotted in Figure 3. We note that for small signals, $y(t) \approx x(t)$, but for larger signals, $y(t)$ is limited to ± 1. This is easily extensible to other small-signal gains and large-signal gain limits. Very importantly, zero-crossings are unaffected. Expanding Eq. (15) into a Taylor series allows the approximation over a limited range of inputs as

$$y(t) \approx x(t) - \frac{x^3(t)}{3} + \frac{2x^5(t)}{15} - \frac{17x^7(t)}{315} + \dots . \tag{16}$$

We observe that along with passing the input signal, odd harmonics of the input signal are also generated, as we might expect nonlinearities to do. Typically, in radar circuits, when harmonics are generated, they are almost immediately mitigated with filters. Even a compressive power amplifier feeding an antenna will have to contend with the limiting frequency response of the antenna.

Nevertheless, with respect to how an input signal transmogrifies to an output signal, it is highly input-signal dependent. To make further analysis tractable, we will limit ourselves to input signals that are constant-amplitude sinusoids, intentionally modulated only in phase/frequency. An example is a LFM chirp, although other waveforms with these characteristics can be formed as well.

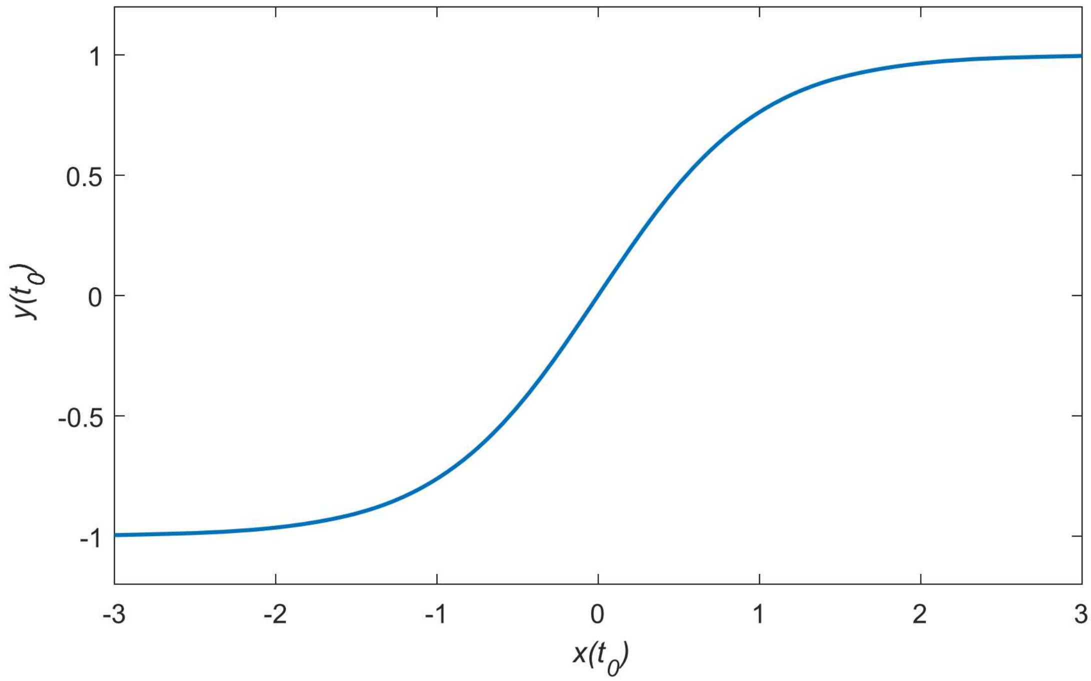

Figure 3. Sigmoid function modelling gain for an amplifier operating in compression. Specifically, this illustrates a hyperbolic-tanget relationship.

We will assume the output is heavily compressed and filtered with a well-behaved and well-characterized filter to retain only the fundamental frequency, retaining negligible harmonic content. This has the added benefit of mitigating any amplitude modulation on the input signal. We expect the output signal then retains the phase of the input signal, but adopts a nearly constant amplitude characteristic due to compression. Departures from this are problematic for us. In any case, the relevant signal to be analyzed for an IPR is the output signal, perhaps filtered, when the amplifier is input with a constant amplitude sinusoid over a span of frequencies.

Consequently, determining the IPR for this class of components is similar to that of the linear components of the previous section, differing only in that care must be taken that the amplifier input level is sufficient to drive the amplifier into compression, representative of how it will be used in the actual radar circuits.

If the amplifier will be operated with different input drive levels, the so too should any IPR testing be engaged with different input drive levels.

3.3 Mixers & Multipliers

This class of components are inherently designed to alter the frequency content of a signal. A mixer intends to simply translate a signal in frequency. A multiplier essentially intends to mix a signal with itself, thereby doubling, tripling, etc., a signals phase/frequency and bandwidth.

Mixer

A mixer may be modelled as a multiplication of a signal with a Local Oscillator (LO) sinusoidal signal, such as

$$y(t) = x(t) \times \left[2\cos(2\pi f_c t) \right]. \tag{17}$$

The factor "2" is merely a convenience for analysis. If the intent is to increase the output frequency, then this is up-conversion, or modulation. If the intent is to decrease the output frequency, then this is down-conversion, or demodulation.

In the frequency domain, this becomes

$$Y(f) = X(f) * \left[\delta(f - f_c) + \delta(f + f_c) \right] = X(f - f_c) + X(f + f_c). \tag{18}$$

The signal has been translated by the LO frequency (both positive and negative). If $x(t)$ is itself a sinusoid, that is

$$\begin{aligned} &x(t) = 2\cos(2\pi f_0 t), \text{ with} \\ &X(f) = \delta(f - f_0) + \delta(f + f_0), \end{aligned} \tag{19}$$

then the output spectrum is

$$Y(f) = \delta(f - f_c - f_0) + \delta(f - f_c + f_0) + \delta(f + f_c - f_0) + \delta(f + f_c + f_0). \tag{20}$$

The desired sideband(s) are then generally selected by filtering.

Interestingly, superposition holds for Eq. (17), making it linear, but not time-invariant.

Mixers might in fact be (and often are) created using non-linear components, via intermodulation, followed by suitable filtering. Such components are able to take additive interference and convert it to a multiplicative modulation. This can be both good and bad, depending on our intent.

We further observe that if $x(t)$ in fact were an impulse, then $X(f)$ would be a constant, and a frequency-shifted constant is just the same constant. The inverse Fourier Transform of this output constant is just an impulse again. For our purposes, we are interested in imperfections in the output $y(t)$ of the mixer generated by a perfect input $x(t)$.

So, determining the IPR of a mixer then becomes a process similar, but with differences, to previous components. Namely, precisely controlled constant-amplitude sinusoids are applied to the input of the mixer, and output amplitude and phase are measured as a function of output frequency. Over a span of input frequencies, $Y(f)$ can be determined. Therefrom $y(t)$ becomes the IPR of the mixer. It is imperative that if different frequency components are measured at different times, then precise control over the LO signal needs to be maintained with respect to amplitude, frequency, and phase.

Another technique to determine the IPR of a mixer under test might be to employ a second well-characterized mixer at either the input or the output of the mixer under test. It would be fed the same LO signal, but with output product selected, i.e. filtered, so that overall combined mixers are a LTI system. As such, the overall IPR can be determined, and the well-characterized mixer characteristics subtracted from the result to ascertain the IPR of the mixer under test.

Frequency Multiplier

A frequency multiplier is essentially a mixer, where the signal is mixed with itself. It may be described mathematically as

$$y(t) = 2^{n-1}\left[x(t)\right]^n , \tag{21}$$

where n is typically a low-order integer, such as 2 or 3. The scaling by a power of two normalizes the output.

Typically, in a radar system it is employed to multiply the bandwidth of a radar waveform prior to its transmission.

Consider a sinusoidal waveform that is phase modulated, described by

$$x(t) = \cos\left(\Phi(t)\right), \tag{22}$$

where

$$\Phi(t) \text{ = the phase modulation.} \tag{23}$$

Now consider a frequency-doubler, so that

$$y(t) = 2\cos\left(\Phi(t)\right)^2 . \tag{24}$$

The output may be expanded using trigonometric identities to

$$y(t) = 1 + \cos\left(2\Phi(t)\right). \tag{25}$$

Note that we now have a term where we have doubled the phase function, including its time-derivative, otherwise known as frequency. Any bandwidth has also been doubled. More generally, for some power *n*, the output will have a term $\cos\left(n\,\Phi(t)\right)$.

In any case, a constant-amplitude input sinusoid, when varied over frequency, should yield a constant amplitude output sinusoid, with appropriate phase/frequency multiplication. This is the ideal behavior.

Normally, a frequency multiplier is used to increase the bandwidth of a phase/frequency modulated waveform, such as a LFM chirp. Consequently, any amplitude variations in the output can be suppressed with down-stream limiters or compressive amplifiers. Of greater interest, and more problematic, is any phase modulations added by the multiplier.

Similar to a mixer, the IPR of a frequency multiplier may be determined with precisely controlled constant-amplitude sinusoids applied to the input of the multiplier, and output amplitude and phase measured as a function of output frequency. Over a span of input frequencies, $Y(f)$ can be determined. Therefrom $y(t)$ becomes the IPR of the mixer.

We stipulate that frequency multipliers may have an impact that exacerbates other non-ideal behavior in the signal path. For example, spurious signals generated by the DAC in the exciter will be elevated by 6 dB for every doubling of the frequency. Consequently, spur levels for the DAC and more generally the exciter, need to be kept in mind with any subsequent frequency multiplication.

3.4 Cables and Connectors

Often overlooked is the fact that cables and connectors are, and should be considered, precision components and not merely RF hookup wires. The cables are transmission lines, and connectors offer the possibility of transmission-line impedance discontinuities, as so too might damage to a cable (e.g. kinks, bends, frayed shielding, etc.). The usual deleterious effect is a return-loss issue, ultimately resulting in second-time-around echoes of a signal.

A cable with connectors might be considered a linear component. The second-time-around echo manifests as a one-sided bump in the IPR of a cable. This can be decomposed into precisely balanced amplitude and phase modulations. If such a signal is then routed through a compressive amplifier, then the balance between amplitude and phase modulation will be altered, and the single sidelobe on one side of the IPR mainlobe will bifurcate into two sidelobes, one on either side of the mainlobe.

To minimize performance degradation, both cables and connectors need to be properly rated for the frequencies they are to carry.

3.5 Subassemblies

More important than any individual component's performance is the performance of the overall radar signal path. We distinguish the overall radar signal path by dividing it into two main parts

1. Transmit (TX) signal-chain subassembly, and
2. Receive (RX) signal-chain subassembly

Of course, these subassemblies may be tested together in an overall loop test.

TX Signal-Chain Subassembly

The TX signal-chain begins with the waveform generation component or circuit, often called the exciter, and ends at the TX antenna.

Classical monostatic radars (those that provide their own transmit signals) have generally employed high-power amplifiers in their final stages. Although exceptions exist, these have generally been compressive amplifiers. Consequently, waveforms have been favored that employ phase/frequency modulation, with amplitudes generally desired and expected to be constant. A LFM chirp is an example of such a waveform.

Since phase/frequency modulations are desired to be accurate and precise, so that the receiver may later employ matched-filter processing or equivalent, any phase anomalies are particularly problematic, as they manifest as transmitted by the antenna. That is, it is the cumulative IPR at the antenna that is of concern to us for this subassembly. For compressive final power amplifiers, and otherwise well-designed circuits, the IPR response at this point is due to the accumulated phase anomalies from all upstream components, but largely the amplitude anomalies of the final power amplifier itself.

However, a perfectly good radar can be built with a linear final power amplifier. If upstream components are also linear, then waveforms may be employed that also employ amplitude modulation, either in place of, or along with, any phase/frequency modulation. Some "noise" radars operate in this manner.

Nevertheless, since a TX subassembly is made up of a sequence of components and circuits, we acknowledge that any individual component might be able to meet some IPR specification, but the overall subassembly might not. Alternately, individual components might not meet some IPR specification, but errors might cancel so that the subassembly does perform adequately. In the end, we are interested in the overall radar performance, measured as its overall system IPR.

RX Signal-Chain Subassembly

The RX signal-chain begins at the RX antenna, and in modern high-performance radar systems ends somewhere after the echo signals have been digitized. It may or may not include some preliminary DSP. It often assumes that echo signals have been down-converted to baseband.

We generally wish to separate superposed echo signals as received at the antenna in the subsequent processing. This means that superposition needs to hold for the RX signal chain, at least adequately so, such that subsequent DSP algorithms can be optimally employed. That is, the RX signal chain needs to generally be linear.

However, that is not to say that nonlinear components and circuits might not be useful under certain circumstances. Such components and circuits might include Automatic Gain Control (AGC), Sensitivity-Time-Control (STC), and even nonlinear detectors, such as magnitude or square-law detectors, etc. However, when these components and circuits are used, their impact on RX signal-chain IPR need to be assessed, and evaluated compared to desired and expected behaviors.

As with the TX signal-chain, since a RX subassembly is made up of a sequence of components and circuits, we acknowledge that any individual component might be able to meet some IPR specification, but the overall subassembly might not. Alternately, individual components might not meet some IPR specification, but errors might cancel so that the subassembly does perform adequately. Again, in the end, we are interested in the overall radar performance, measured as its overall system IPR.

<u>Loop Test</u>

Of course, transmitters and receivers do not operate in a vacuum. The overall signal path is from exciter via a target reflector to the RX output, subsequent DSP notwithstanding. If the receiver receives an echo that is consistent with a point target reflector, then the RX output should represent the IPR of the entire signal path, through both the TX signal-chain and the RX signal-chain. Such a point target reflector may be simulated in several ways, including

1. A delay-line – a hardware transmission line with some useful delay. This could be RF/microwave cabling, or an optical delay line, or something similar.

2. A test range – a usually ground-based test range (outdoor, or indoor) facility that allows the radar to transmit and receive delayed echoes. Indoor ranges are usually microwave anechoic chambers, with transponders that incorporate delay lines.[9]

3. Flight testing – using canonical reflectors readily identified in the radar echo data. This is probably the best end-to-end testing, albeit probably the most expensive way to test.[10]

The presumption in all cases is that the whatever generates the echo is an ideal point reflector. Consequently, any specific testing components' characteristics which might affect or alter the IPR need to be carefully assessed and compensated in the overall system IPR measurements. That is, non-ideal testing-hardware's effects need to be subtracted from the test data.

3.6 Additional Comments

Here we offer several ancillary comments, in no particular order.

Amplitude vs. Phase IPRs

An IPR is affected by anomalous deviations in both amplitude characteristics as well as phase characteristics. It is often useful to understand whether an IPR anomaly comes from the amplitude characteristic or from a phase characteristic. Consider decomposing the output data spectrum to

$$\begin{aligned} |Y(f)| &= \text{magnitude of } Y(f), \text{ and} \\ \angle Y(f) &= \text{phase of } Y(f). \end{aligned} \tag{26}$$

We may calculate the IPR contributions of these components as

$$\begin{aligned} \Im\{|Y(f)|\} &= \text{IPR contribution from the magnitude of } Y(f), \text{ and} \\ \Im\left\{e^{j\angle Y(f)}\right\} &= \text{IPR contribution from the phase of } Y(f). \end{aligned} \tag{27}$$

These decompositions may help in finding the source of an anomaly, and/or devising a correction or compensation for the anomaly.

Waveform Generation

As previously stated, modern high-performance radar systems often generate their intended waveform digitally, and then convert it to an analog signal using a DAC. This allows relatively easy reprogrammability of the waveform, and potentially improved waveform parameter control. The expectation is that the digital waveform so generated is faithful to our intent, and the DAC is adequately linear to likewise produce the analog waveform consistent with our intent. The DAC was discussed previously.

The ability of the digital waveform to match our intent depends on the digital architecture and implementation of the Digital Waveform Synthesizer (DWS). An inability of the DWS to create the waveform that we intend constitutes an undesired modification to the waveform, or a reduction in fidelity, just as if it were modulated or otherwise altered by a downstream component or signal in the channel. For example, this might be the case with an excessive amount of waveform quantization, as might result from an inadequate number of bits in the DWS internal digital circuits.

As with ADC and DAC performance, DWS fidelity is best handled with good design. Inadequate fidelity will result in spurious signals, and perhaps a low-level noise floor apparent in the IPR.

Phase Equalization

Frequency-modulated chirp signals, whether LFM chirps or nonlinear chirps, offer the nice feature that an instantaneous frequency is associated with any particular time location in the waveform. This allows the possibility of incorporating a frequency-dependent phase correction to the signal at its generation.[11] This Phase Error Correction (PEC) can compensate for any phase errors downstream in the signal path, thereby equalizing the overall signal path.

Interference

Heretofore we have discussed what are essentially passband characteristics of components and circuits that are inherent properties of the devices themselves; what really amounts to things like phase and amplitude ripple and other similar effects.

We now briefly address a new class of anomalies, namely undesired interference signals and their effects on the radar echo signals in which we are interested. Electromagnetic Interference (EMI) signals propagate into the signal path via undesired coupling from somewhere else, and fall under the general heading of "susceptibility," both conducted and radiated. Such signals might manifest as

1. Additive signals that add to the echo signals in the desired signal path, or

2. Multiplicative signals that modulate the echo signals in the desired signal path. The modulation might be Amplitude Modulation (AM) or Phase Modulation (PM).

These modulations might be synchronous, or asynchronous. A common point of entry into the signal path is via the power supply lines, although coupling between circuit board traces or elements is also not uncommon. The first line of defense against such susceptibility is good circuit design practices. A good reference for this is a report by Dudley.[12] Another common source for errors are additive biases that are not mitigated, such as a DC bias in a baseband amplifier, and leakage of mixer products, etc.

Both additive and multiplicative interference will be apparent in the IPR measurements. However, multiplicative interference signals will typically track signal levels, and additive interference signals generally will not. However, additive interference signals will manifest even when no legitimate echo signals are present.

AM to PM Conversion

It is well-known that non-linear components can convert AM signals into PM signals.[13] An extension of this is that additive signals can become multiplicative signals via intermodulation products in non-linear components.

Inadequate Filtering

Many components and circuits generate undesired byproducts that we desire to eliminate with filtering. For example, nonlinear components generate harmonics, and mixers in practice will generate image frequencies, and often leak some level of LO signals into their output. If filtering is inadequate, the vestiges of these undesired components will remain in the echo signal path, corrupting the IPR of the net circuit.

The answer to inadequate filtering is to get better filtering, even if this calls for a redesign of the overall frequency plan, i.e. the selection of LO frequencies, etc.

Channel Balance

We state without elaboration that in multiple-channel radar systems, it is often not good enough to have independently acceptable IPR performance for the individual channels. In addition, the channels must match each other to some degree of tolerable error.[14]

One area for which we will elaborate somewhat is that often a baseband radar signal will have In-phase and Quadrature (I/Q) components, which constitute independent channels for a common input signal. When I/Q components are inadequately balanced, errors will be generated that can interfere with IPR quality.[15]

Vibration Effects

Some components, notably those that might employ crystals and ceramics, exhibit piezoelectric effects, where their electrical characteristics are influenced by mechanical stresses, such as those introduced by vibration. This means that the IPR under vibration might not be the same IPR in a static environment, with vibration typically inducing degraded performance.

4 Pulse-to-Pulse Stability

We employ the word "stability" not in the sense of control systems, but rather as a synonym for predictability, constancy, and preciseness over time, specifically on a pulse-to-pulse basis.

Our expectation is that a radar system is time-invariant. That is, consider two pulses with waveforms that are identical in all respects except their time of generation. If each generates a target echo with the same target, and same target geometry, and are received by the same receiver configured identically for both pulses, then the receiver output should be identical for both pulses. We might only make allowances for the fundamental thermal noise floor, but anything above this basic noise floor should be identical.

Any pulse-to-pulse differences in receiver outputs for otherwise identical inputs, is attributed to a lack of constancy of the radar system itself. This is what we term to be a lack of stability of the system.

Imparting differences on the otherwise identical pulses constitutes a modulation of the pulses. That is, some aspect of the received pulse's waveform is caused to change from one pulse to the next. The modulation might be amplitude, phase, frequency, or even time.

Our measure of pulse waveform similarity between two pulses is coherence, which is essentially a normalized cross-correlation measure. For a group of pulses (more than two) generated at some Pulse Repetition Frequency (PRF), our measure becomes the Discrete Fourier Transform (DFT) across the pulses. This dimension across pulses is typically referred to as the Doppler dimension in pulse-Doppler radars, but may also be referred to as the azimuth dimension for SAR, or the velocity or range-rate dimension for GMTI radar and similar systems.

To maximize the SNR, we typically use the range-compressed pulses. A DFT across truly identical range-compressed pulses will yield a 2-dimensional range-Doppler image with a nominal point-like response that ideally exhibits a perfect IPR in the Doppler dimension centered on the point response. Any pulse-to-pulse modulation will yield an imperfect IPR, with broadened mainlobe and/or elevated sidelobes in the Doppler dimension. Nevertheless, the overall IPR for such a range-Doppler image is in fact a 2-dimensional function. Although we often just concern ourselves with 1-dimensional "cuts" in cardinal directions, it behooves us to look for anomalies even in the larger image, and not just in the cardinal, or principal axes' cuts.

We now examine several classes of pulse-to-pulse stability degradation sources. Our measure of goodness in all cases is still an IPR generated by a DFT in the Doppler dimension, on range-compressed data, employing suitable window taper functions.

4.1 Time-Base Phase Noise

Radar corresponds range to an echo time-delay. A radar measures time-delay in terms of cycles of an internal clock, usually derived from a master oscillator whose frequency is presumed known to a high degree of accuracy and precision. Any fluctuations in the frequency of the time-base manifest as a modulation of the reference time of the radar itself. These modulations of the time-base are indistinguishable from range variations, which of course also manifest a range-rate, and hence Doppler.

Jitter in the radar time-base is classically termed "phase-noise," and is detailed in a previous report.[16] In a well-designed radar system, careful attention must be paid to the selection of a master oscillator to manifest adequate phase-noise performance. Some other components often considered for frequency generation in Stable Local Oscillator (STALO) circuits are also known to exacerbate phase noise, notably Phase-Locked Loop (PLL) circuits and components, and if employed, must be done so judiciously. Please read this last sentence again.

Phase noise in the radar's time-base or STALO circuits will normally manifest as a low-level elevated far-out sidelobes in the Doppler dimension from a legitimate signal, although it may extend somewhat in the range dimension, too.

Since the master oscillator frequency is typically determined by a piezoelectric crystal, phase noise under vibration must be considered to gauge the overall quality of the radar performance.

4.2 EMI Susceptibility

As mentioned in an earlier section, circuit susceptibility to conducted or radiated EMI can induce both additive interference as well as modulations of a legitimate signal. Recall that a common point of entry into the signal path is via the power supply lines, although coupling between circuit board traces or elements is also not uncommon.

If this interference varies from pulse to pulse, for example if it was nonsynchronous with the radar PRF, then we might it expect it to manifest being displaced and/or smeared in the Doppler dimension.

Pulse-to-pulse modulations will manifest in the Doppler direction as sidelobes from a legitimate signal. The modulations may induce mainlobe broadening and/or elevated sidelobes in the Doppler dimension IPR of a point target via either phase modulation or amplitude modulation, or both.

Additive signals may appear anywhere in a range-Doppler map or image. They will not necessarily be limited to the same range-line as a legitimate signal.

4.3 Dynamic Stability

Heretofore we have discussed multiple pulses with identical input signals at the exciter, expecting identical output signals from the receiver. However, it is also true that if we alter the input signal in some known and precise manner, then the receiver output should also be altered in a predictable way, with accuracy and precision. This lets us evaluate radar performance with dynamic input conditions.

A particularly useful dynamic input signal is to impart a rolling phase shift to the exciter waveform on a pulse-to-pulse basis. This adds a faux Doppler shift to the data, shifting the location of the point target response's IPR peak in the Doppler dimension in the range-Doppler image. If the exciter waveform retains high fidelity, and the signal path remains high-quality, then a Doppler-dimension cut of the IPR should retain an ideal shape, except shifted in Doppler to a position that can be precisely and accurately calculated.

While true for the main desired echo signal of a point target response, spurious energy generated by system nonlinearities may not shift identically, and can appear anywhere in the range-Doppler image. In fact, new spurious signals may even appear. These spurious responses will not be limited to principal axes' cuts, and may be considered as off-axis sidelobe energy. ADC anomalies are well-known to cause this.[8,15]

But, as stated, this is "if" the exciter waveform retains high fidelity.

However, if the DWS itself is unable to create the desired pulse-to-pulse changing waveform with adequate fidelity, that is, with adequate accuracy and precision, then for a changing waveform, so too will the errors in the resulting signal change on a pulse-to-pulse basis. This means that a pulse-to-pulse variation in error will result, impacting the Doppler IPR.

Such pulse-to-pulse waveform variations may arise for a variety of reasons, including real-time motion compensation of radar waveforms, and error mitigation techniques and algorithms. Another source of potential pulse-to-pulse net signal variations might be if the signal channel changes on a pulse-to-pulse basis, for example if an Active Electronically-Steered Array (AESA) antenna needed to alter its pointing during a collection of pulses, i.e. a Coherent Processing Interval (CPI).

Even if not Electronically-Steered, Active Array antennas, with many parallel amplifiers and radiating elements, need to be carefully assessed over the frequency band of the radar signals they are intended to process. For example, different gain/phase functions between these parallel elements over frequency might cause the beam to "wiggle" during the pulse, thereby modulating the received echo signal to the detriment of the radar signal's quality.

“Nothing in the affairs of men is worthy of great anxiety.”
-- Plato

5 Other Factors

Here we address some additional factors and concepts related to the performance of radar components, circuits, subassemblies, and systems.

5.1 Noise Figure

The unavoidable bane of any radar system is system noise. In particular, we are referring to what is often termed "thermal noise" but encompasses not just thermal emissions from the radar's receive-antenna's field of view, but also the random noise generated in the receiver components themselves, including ADC quantization noise, and accounted for by the system noise figure. This noise is modelled as Additive White Gaussian Noise (AWGN). This is discussed in detail in an earlier report.[17]

This system thermal noise manifests as a basic noise floor to any signal. The only way to overcome it is with integration gain achieved by coherently combining multiple copies of the signal of interest. This might be necessary to observe low-level features in the IPR, especially in the far-out sidelobe region.

5.2 Component Linearity

If driven hard enough, even the most linear component will eventually exhibit nonlinear behavior. This is especially recognized for active components like amplifiers, etc. In fact, it is useful for a circuit designer to understand when an otherwise linear component just begins to exhibit some degree of non-linear behavior, and have a parameter or other figure-of-merit to help them appreciate when to expect this. Several measures of component non-linearity are in common usage. We discuss a pair of them here.

We note that usually of greater concern than single-frequency harmonics, are intermodulation products of multiple tones, or more complex signals.[7]

1-dB Compression Point

As an amplifier's input signal increases in amplitude, there comes a point after which its gain diminishes from a linear input versus output relationship. When the expected gain decreases by 1 dB from the expected, then this is termed the 1-dB compression point.

Figure 4 illustrates the 1-dB compression point for the gain function illustrated in Figure 3.

3rd-Order Intercept

If we assume that a component's gain can be adequately described by a 3rd-order polynomial, such as

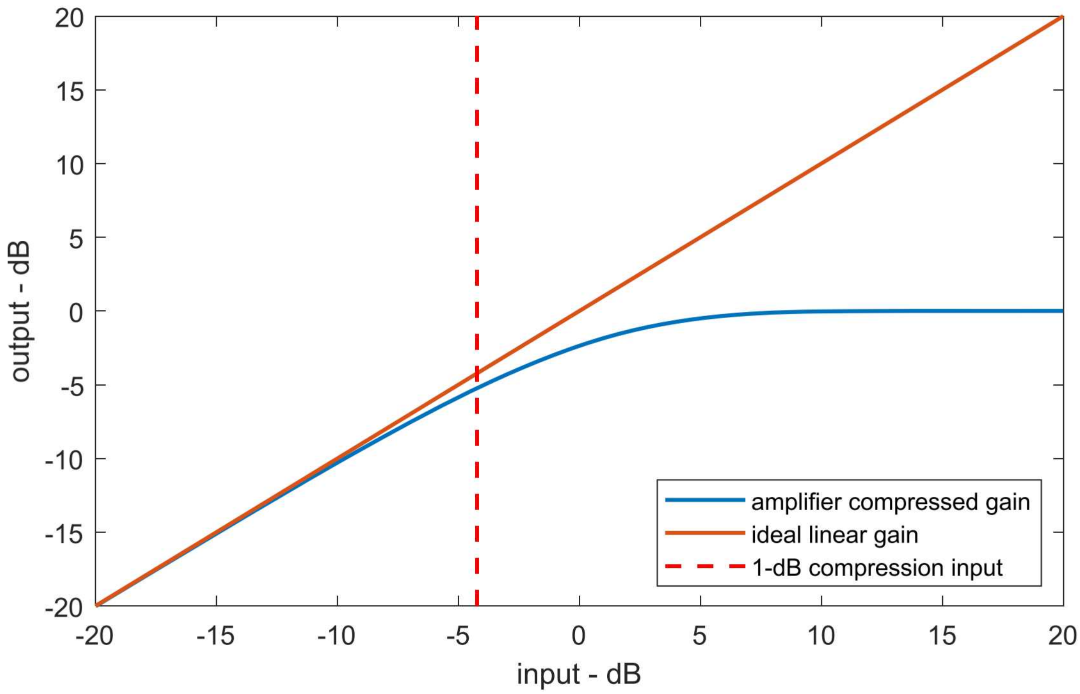

Figure 4. 1-dB compression point for gain function illustrated in Figure 3.

$$y(t) \approx g_1 x(t) - g_3 x^3(t), \tag{28}$$

where

$$g_i = \text{gain factor for the } i^{\text{th}} \text{ power of the input signal.} \tag{29}$$

Now consider that the input signal is a sinusoid of the form

$$x(t) = v_m \cos(2\pi f_m t), \tag{30}$$

where

v_m = amplitude of input signal, and
f_m = frequency of input signal. (31)

Combining Eq. (30) with Eq. (28) yields

$$y(t) \approx \left[g_1 v_m - \frac{3}{4} g_3 v_m^3 \right] \cos(2\pi f_m t) - \frac{g_3 v_m^3}{4} \cos(6\pi f_m t). \tag{32}$$

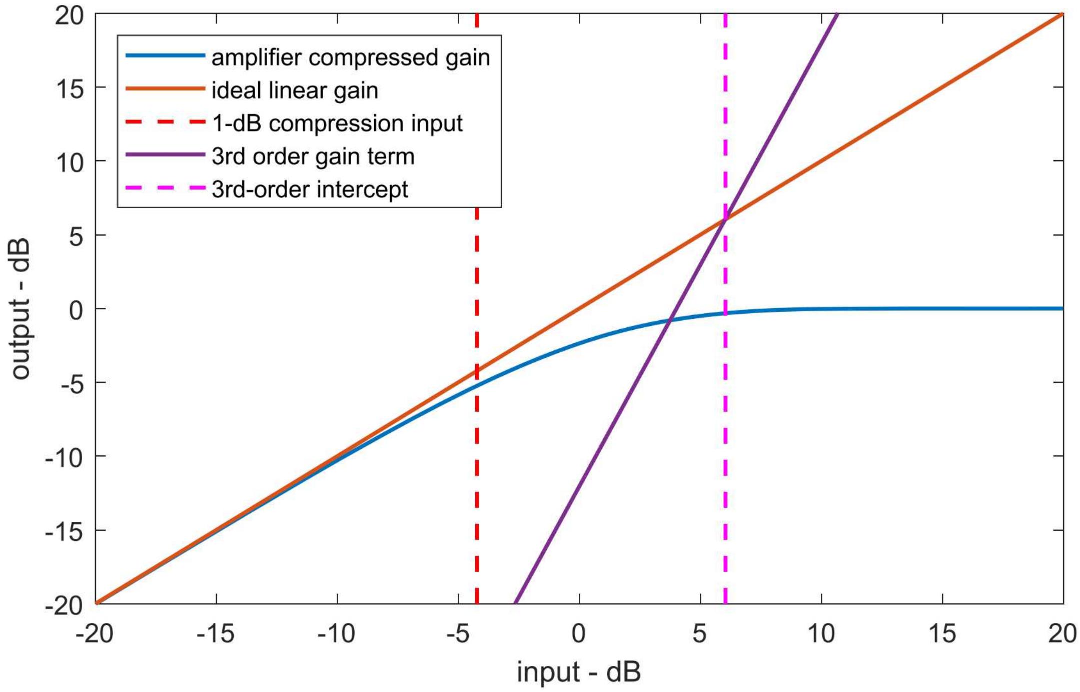

Figure 5. 3rd-order intercept point for gain function illustrated in Figure 3.

The 3rd-order intercept (TOI) point (IP_3) is defined when the coefficient of $\cos\left(2\pi f_m t\right)$ is zero, signifying that the 3rd-order harmonic component is equal in power to the input signal. The input amplitude corresponding to IP_3 is

$$v_{m,IP_3} = \sqrt{\left(\frac{4}{3}\right)\frac{g_1}{g_3}}\,. \tag{33}$$

For the gain function in Figure 3, the IP_3 is illustrated in Figure 5. Specific power levels are defined as

IIP_3 = input power level corresponding to IP_3, and
OIP_3 = output power level corresponding to IP_3. (34)

Note that a common rule-of-thumb is that IP_3 occurs about 10 dB beyond the 1-dB compression point.

We also note that other intercept points might be calculated for other powers of the input signal, but linearity of otherwise linear components are usually limited by the 3rd-order distortion.

5.3 Dynamic Range

A signal path generally has some noise floor, and also some maximum signal level that exhibits adequate fidelity with respect to linearity, etc. The difference in these levels is the dynamic range of the channel, usually expressed as a ratio, more often in dB.

A common choke point for dynamic range in the receiver is the ADC.[18] In general, we desire the dynamic range of the channel to be limited by the ADC rather than any prior analog components. Once the signal becomes data, then any further dynamic range limitations are a function of firmware and software.

This suggests that signal fidelity measurements, in the form of IPR, should occur at perhaps several different signal levels.

6 Conclusions

We offer and repeat some key points.

- The goodness of a radar system is measured in two dimensions,
  1. The fidelity of the radar signal path; the radar channel, and
  2. The stability of the channel from pulse to pulse.

- The radar channel may by design contain a variety of components, linear, non-linear, time-invariant, non-time-invariant, etc.

- An excellent measure of goodness is the Impulse Response (IPR). This is true for a component, circuit, subassembly, or the overall signal path. It is also true as a stability measure.

- We may calculate contributions to the IPR separately from phase modulations than from amplitude modulations. This can help us identify the source of anomalous behaviors.

- A common source of undesired signal modulation is EMI susceptibility, often via the power connection of active components, but not exclusively so.

- Nonlinear components often have the ability to turn additive interference signals into multiplicative phase modulations.

- Performance criteria for components should be stricter than for subassemblies.

- Laboratory measurements should be held to a higher standard (more stringent specification limits) than flight data.

- The "right" IPR specification depends on the modes with which the radar intends to operate. Consequently, a multi-mode radar needs to meet performance expectations for the harshest of all modes.

and finally, we cannot stress these points enough…

- Minimally acceptable threshold test criteria should ***never*** be confused with design criteria.

- Minimally acceptable performance today may prohibit what you can do tomorrow.

“It is not down on any map; true places never are.”
— Herman Melville, Moby-Dick or, the Whale

Appendix A – Impulse Response, Convolution, and Related Concepts

The concept of IPR has its roots in Linear Time-Invariant (LTI) System theory, where a component's, or circuit's response can be characterized by a convolution operation. That is, for

$$x(t) = \text{input signal}, \tag{A1}$$

the output of the component or circuit, hereafter in this section termed "system," with this input is characterized as

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau, \tag{A2}$$

where the system's nature is described by the function $h(t)$.

If the input signal is an impulse, defined to be the Dirac delta function, then we identify

$$x(t) = \delta(t), \tag{A3}$$

and the system output becomes the response to this input impulse, calculated to be

$$y(t) = h(t), \tag{A4}$$

hence the name "Impulse Response."

Such systems may also be described in the frequency domain by calculating their Fourier Transforms. We employ the Fourier Transform definition for the IPR as

$$H(f) = \Im\{h(t)\} = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt, \tag{A5}$$

and the Inverse Fourier Transform as

$$h(t) = \Im^{-1}\{H(f)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(f)e^{+j2\pi ft}df. \tag{A6}$$

Note that f is the frequency variable, typically with units Hz. With $h(t)$ identified as the IPR, then $H(f)$ is termed the Transfer Function.

We may use shorthand notation to relate $h(t)$ to $H(f)$ with

$$h(t) \Leftrightarrow H(f). \tag{A7}$$

In a similar manner, input and output signals also have Fourier Transforms which we identify with

$$x(t) \Leftrightarrow X(f), \text{ and}$$
$$y(t) \Leftrightarrow Y(f). \tag{A8}$$

These spectra are related by

$$Y(f) = H(f)X(f). \tag{A9}$$

Correlation and Matched Filters

Very much related to convolution are the concepts of correlation and matched filters.

The Matched Filter is optimum for maximizing Signal to Noise Ratio (SNR) in the presence of Additive White Gaussian Noise (AWGN). Its derivation can be found in numerous sources in the open published literature. The matched filter implements a cross correlation operation. The basic mechanics for implementing this function are discussed below.

Cross Correlation

Radar pulse compression is accomplished by making a similarity measure of an input signal with the expected pulse response. We identify the similarity measure for this purpose as the cross correlation operation of an input $x(t)$ with a reference signal $g(t)$ as

$$y(t) = \mathrm{xcorr}(g(t), x(t)) = \int_{-\infty}^{\infty} g^*(-u)x(t-u)\,du \tag{A10}$$

where

$$x(t) = \text{input signal to be processed, and}$$
$$g(t) = \text{reference signal.} \tag{A11}$$

Similarly, the cross correlation can be written as any of the following,

$$y(t) = \int_{-\infty}^{\infty} g^*(u)x(u+t)\,du, \text{ or} \tag{A12}$$

$$y(t) = \int_{-\infty}^{\infty} g^*(u-t)x(u)\,du. \tag{A13}$$

In addition, we observe that

$$y(t-a)=\int_{-\infty}^{\infty} g^*(u-t)x(u-a)\,du. \tag{A14}$$

We recall that cross correlation is related to convolution as

$$y(t)=\text{xcorr}\left(g(t),x(t)\right)=g^*(-t)*x(t) \tag{A15}$$

where, as is convention, the asterisk as operator implies convolution, and the asterisk as superscript implies complex conjugate. In terms of their Fourier Transforms, cross correlation implies

$$Y(f)=G^*(f)X(f) \tag{A16}$$

where we identify the Fourier Transform pairs

$$x(t)\Leftrightarrow X(f),$$
$$g(t)\Leftrightarrow G(f), \text{ and}$$
$$y(t)\Leftrightarrow Y(f). \tag{A17}$$

Matched Filter

A filter $h(t)$ that provides the same result as correlation with $g(t)$ has the form

$$h(t)=g^*(-t) \tag{A18}$$

or, in the frequency domain

$$H(f)=G^*(f) \tag{A19}$$

where we identify the Fourier Transform pair

$$h(t)\Leftrightarrow H(f). \tag{A20}$$

Consequently, the filtering operation is given in the time domain as

$$y(t)=h(t)*x(t) \tag{A21}$$

and in the frequency domain as

$$Y(f)=H(f)X(f). \tag{A22}$$

Since $g(t)$ represents the desired response, then $h(t)$ is the matched filter for the desired signal. Since they provide equivalent results, we will henceforth use the terms matched filter interchangeably with correlation.

Matched Filter Output Characteristics

Recall the output of a matched filter, when input with the signal to which it is matched, is the autocorrelation of the signal. Recall also that the autocorrelation function is related to the energy spectrum of the desired signal via a Fourier Transform. We now define

$$S(f - f_0) = \text{radar signal energy spectrum of the desired signal} \tag{A23}$$

where $S(f)$ is real, band-limited, and even. The autocorrelation function can then be written as

$$R(\tau) = Q(\tau)\exp(j2\pi f_0 \tau) \tag{A24}$$

where $Q(\tau)$ is also real-valued and even. $Q(\tau)$ defines the shape of the matched filter output. Furthermore, for signals of interest to us, $Q(\tau)$ exhibits a single main lobe. The phase within the mainlobe, however, depends on delay τ and is proportional to signal center frequency f_0.

Of significance to Doppler processing, subtle changes in range between radar and target causes a subtle change in the echo delay time of the pulse, which causes a noticeable phase rotation in the output of a filter matched to a particular delay. This is essential to range-Doppler processing, including SAR processing.

LFM Chirp Waveform

A common signal for radar systems is the Linear Frequency Modulated (LFM) chirp. Such a waveform, with reasonably high time-bandwidth product, will exhibit a nearly rectangular energy spectrum. That is, for a LFM chirp with characteristic

$$x(t) = \text{rect}\left(\frac{t}{T}\right)\exp j\left\{\frac{\gamma}{2}t^2\right\}, \tag{A25}$$

where pulse parameters are

$$\begin{aligned} T &= \text{the chirp duration, or pulsewidth, and} \\ \gamma &= \text{the chirp rate,} \end{aligned} \tag{A26}$$

and the rect( ) function is defined as

$$\text{rect}(z) = \begin{cases} 1 & |z| \le 1/2 \\ 0 & \textit{else} \end{cases}. \tag{A27}$$

We identify the energy spectrum of the chirp as effectively

$$S(f) = \frac{T}{B_c} \text{rect}\left(\frac{f}{B_c}\right), \tag{A28}$$

where spectrum parameters are

$$B_c = \frac{\gamma T}{2\pi} = \text{chirp bandwidth in Hz } (B_c > 0). \tag{A29}$$

Consequently, a matched filter output will have the shape in the vicinity of its mainlobe

$$Q(\tau) = T \,\text{sinc}(B_c \tau), \tag{A30}$$

where we define the function

$$\text{sinc}(z) = \frac{\sin(\pi z)}{\pi z}. \tag{A31}$$

Some observations include

- The width of the matched filter output is defined by the chirp bandwidth B_c.
- The matched filter output has some fairly significant sidelobes.

Sidelobe Control

Sidelobes can be reduces somewhat by additional filtering of the output of the matched filter. We define this filter characteristic generically for now as

$$w(t) = \text{sidelobe filter IPR}. \tag{A32}$$

We also identify the Fourier Transform pair

$$w(t) \Leftrightarrow W(f). \tag{A33}$$

In particular, for LFM chirp waveforms, such filtering is customary. Typical filters have the properties

$$w(t) = \text{even, and real, and}$$
$$W(f) = \text{even, and real}. \tag{A34}$$

With foresight, we identify from these properties that

$$w(t) = w^*(-t) \text{ , and}$$
$$W(f) = W^*(-f). \tag{A35}$$

Hence, we redefine the output signal of interest as the matched filter output with this additional filtering, namely

$$y(t)=\big(h(t)*x(t)\big)*w(t)\,. \tag{A36}$$

We do note that convolution is commutative and associative. Consequently

$$y(t)=\big(h(t)*w(t)\big)*x(t)=\big(w(t)*h(t)\big)*x(t)\,. \tag{A37}$$

This implies that any sidelobe control can be 'built-in' to the matched filter itself. In the frequency domain, the new output is

$$Y(f)=H(f)W(f)X(f)\,. \tag{A38}$$

Some observations include

- By employing the sidelobe control filter, the combined filter will be perturbed somewhat from the matched filter, and will no long be strictly matched to the signal of interest. This will result in a slight degradation in performance with respect to maximizing SNR, but is generally believed worth the sidelobe reduction.
- The frequency-domain sidelobe filter transfer function $W(f)$ is often referred to as the "aperture taper", or the "window function".

For cross-correlation, we identify

$$\begin{aligned} y(t)&=\mathrm{xcorr}\big(g(t),x(t)\big)*w(t)=\mathrm{xcorr}\big(h^*(-t),x(t)\big)*w(t) \\ &=g^*(-t)*w(t)*x(t) \end{aligned} \tag{A39}$$

But, we recall

$$y(t)=h(t)*w(t)*x(t)\,. \tag{A40}$$

Consequently,

$$y(t)=\mathrm{xcorr}\big(h^*(-t)*w^*(-t),x(t)\big)=\mathrm{xcorr}\big(g(t)*w^*(-t),x(t)\big)\,. \tag{A41}$$

But, for real and even $w(t)$, this reduces to

$$y(t)=\mathrm{xcorr}\big(g(t)*w(t),x(t)\big)\,. \tag{A42}$$

This implies that the sidelobe filter can again be incorporated into the correlation kernel.

We pause now to mention that the use of Non-Linear FM chirps would allow sidelobe control without the need for additional sidelobe filters.[19,20]

Appendix B – Creating an IPR Specification

We will assume that a "perfect" signal channel is constant (flat) over some passband of interest. We model this with the transfer function

$$H(f) = \text{rect}\left(\frac{f - f_0}{B}\right), \tag{B1}$$

where

$$\text{rect}(z) = \begin{cases} 1 & |z| \le 1/2 \\ 0 & else \end{cases},$$

f_0 = passband center frequency, and

B = passband bandwidth. (B2)

The impulse response for this channel, absent any window taper functions for sidelobe control, is

$$h(t) = B\,\text{sinc}(Bt)\,e^{j2\pi f_0 t}, \tag{B3}$$

where

$$\text{sinc}(z) = \frac{\sin(\pi z)}{\pi z}. \tag{B4}$$

We note that the impulse response in Eq. (B3) has relatively high sidelobes which might mask more subtle IPR anomalies of which we wish to be aware. Consequently, employing window taper functions will push down the sidelobes to potentially reveal the more subtle IPR characteristics at the expense of a broadening of the mainlobe. A previous report discusses this in great detail.[3]

In any case, any IPR specification needs to both be able to reveal any subtle IPR anomalies, but also to not "fail" a normal sidelobe level inherent to the window function employed. Consequently, an IPR specification needs to take into account the window function employed to assess the IPR.

We adopt the following principles for developing an IPR specification.

1. Some mainlobe broadening should be tolerated, but not very much. Something on the order of 10% broadening at the –3 dBc level of the mainlobe might be reasonable. Thereafter, at the –18 dBc level, perhaps a broadening of three times the nominal (perfect) –3 dBc level.

2. Some sidelobe elevation should be tolerated, but not very much. Something on the order of no more than about 10 dB above the perfect sidelobe envelope might be reasonable down to some tolerable lower limit.

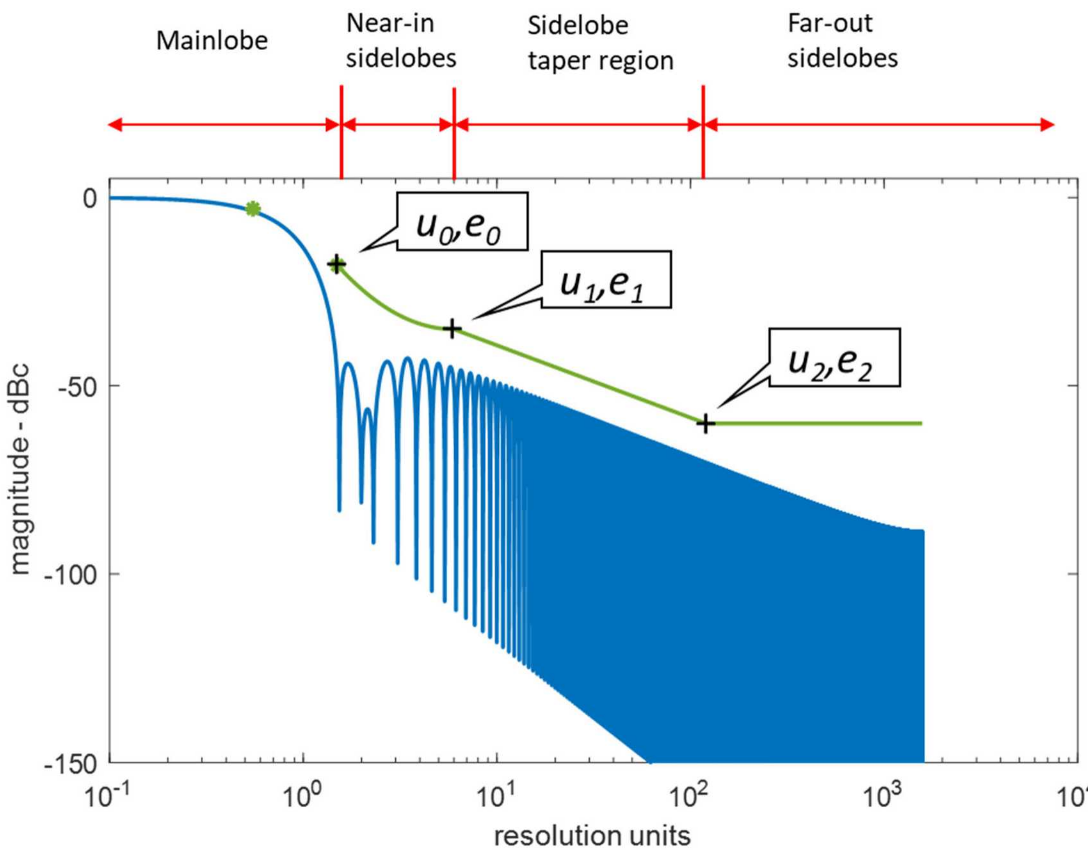

Figure 6. Hamming window taper function (4096 points) with notional sidelobe limit specification and breakpoints identified. Only positive offsets from mainlobe peak are plotted.

Accordingly, we define some breakpoints as indicated in Figure 6, further identified as follows

u_0, u_1, u_2 = breakpoints in resolution units, and

e_0, e_1, e_2 = corresponding sidelobe level in dBc. (B5)

We note that a resolution unit is the –3 dBc width of a perfect mainlobe. In addition, for the near-in sidelobe region we define

m = decay exponent. (B6)

Given these breakpoints and decay exponent, we set the IPR limit as

$$\text{IPR\_limit} = \begin{cases} (e_0 - e_1)\left(1 - \dfrac{\log_{10}(u/u_0)}{\log_{10}(u_1/u_0)}\right)^m + e_1 & u_0 \le u < u_1 \\ (e_2 - e_1)\left(1 - \dfrac{\log_{10}(u/u_1)}{\log_{10}(u_2/u_1)}\right) + e_1 & u_1 \le u < u_2 \\ e_2 & u_2 \le u \end{cases} \tag{B7}$$

For the Hamming window taper function, the notional IPR limit specification pictured in Figure 6 uses the following breakpoints and decay exponent.

$u_0, e_0 = 1.5, -18$,
$u_1, e_1 = 6, -35$,
$u_2, e_2 = 120, -60$, and
$m = 2$. (B8)

Incorporating a 10% broadening of the –3 dBc level of the mainlobe, the total IPR limit specification (in units dBc) becomes

$$\text{IPR\_limit} = \begin{cases} -3 & u = 0.55 \\ 17\left(1 - \dfrac{\log_{10}(u/1.5)}{0.6021}\right)^2 - 35 & 1.5 \le u \le 6 \\ -25\left(\dfrac{\log_{10}(u/6)}{1.301}\right) - 35 & 6 \le u \le 120 \\ -60 & 120 \le u \end{cases}. \tag{B9}$$

This is the limit that was reported in Section 2 of this report. Note that although these limits are written in terms of positive offsets from the mainlobe peak (positive u), they apply to negative offsets as well. That is, we might replace u with $|u|$.

We examine a sampling of other window taper functions in the following pages.

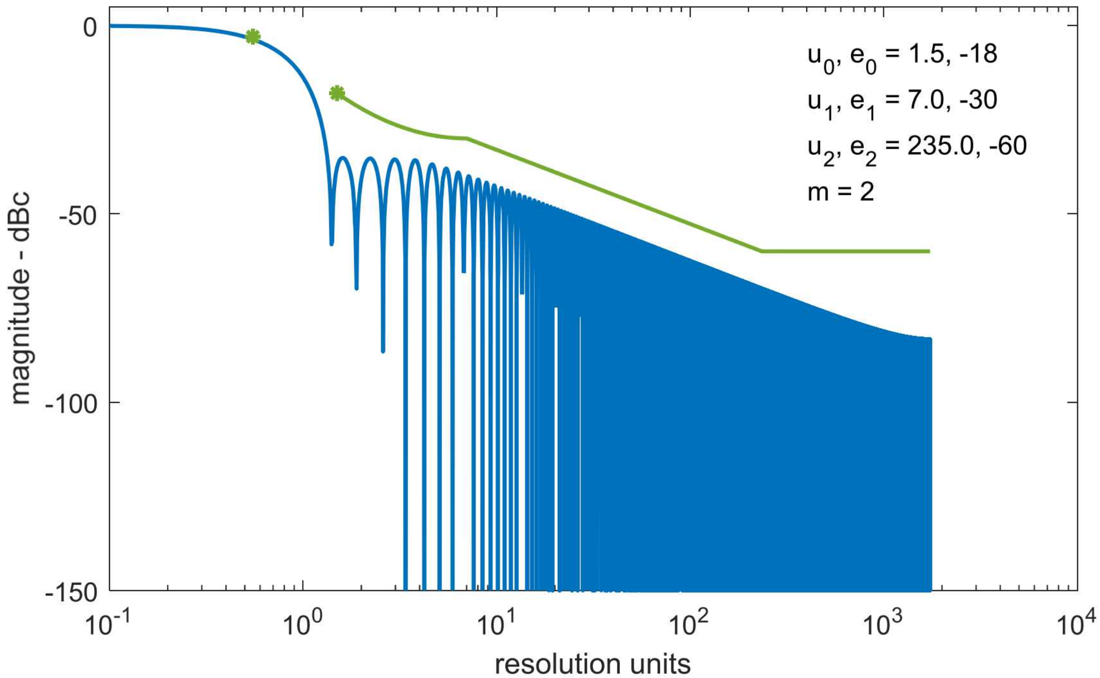

Figure 7. IPR for Taylor window, with –35 dBc sidelobes, and $\overline{n} = 4$.

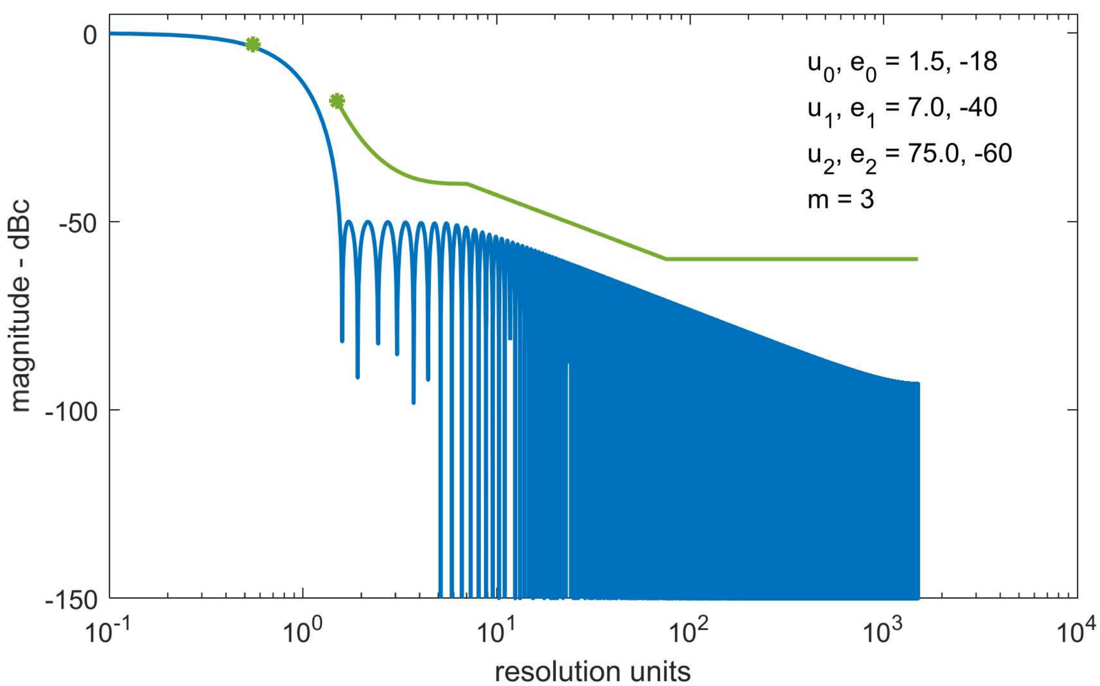

Figure 8. IPR for Taylor window, with –50 dBc sidelobes, and $\overline{n} = 7$.

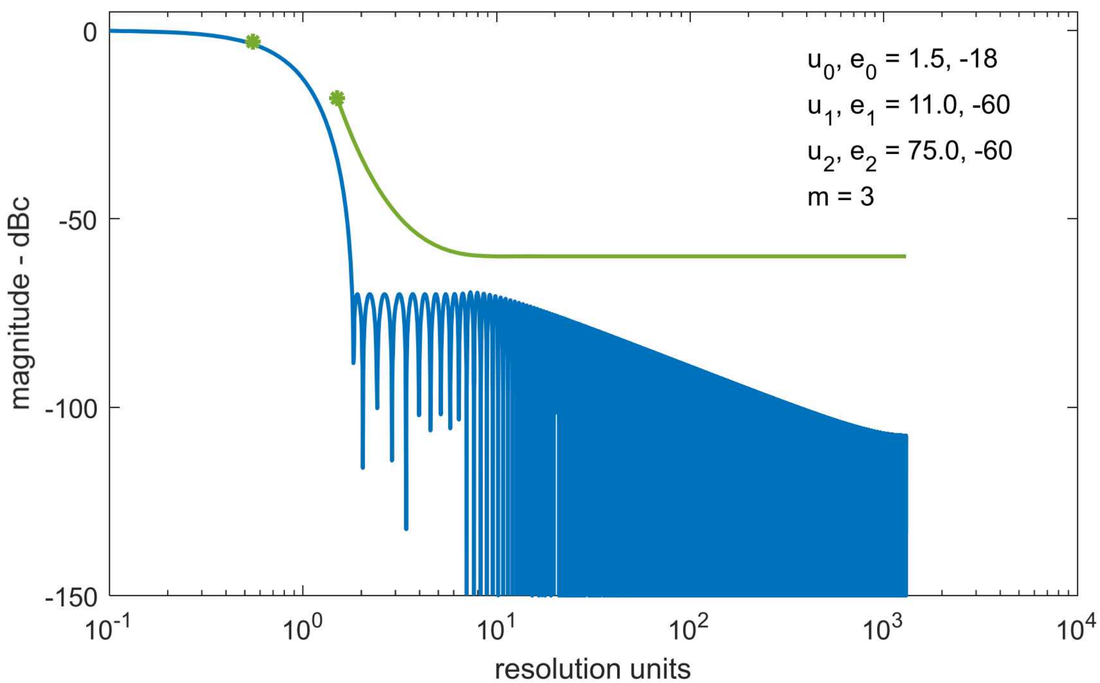

Figure 9. IPR for Taylor window, with –70 dBc sidelobes, and $\overline{n} = 11$.

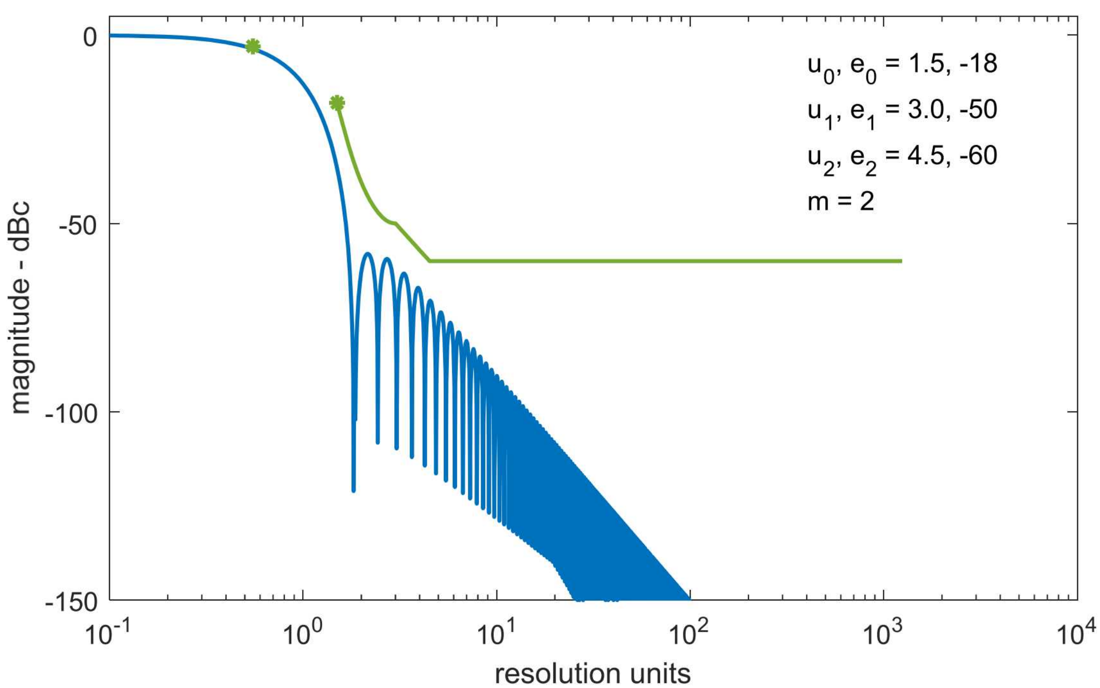

Figure 10. IPR for Blackman window.

"The most interesting information comes from children,
for they tell all they know and then stop."
-- Mark Twain

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