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Shape-Constrained Input Estimation for Multi-Shaker Vibration Testing

PRESENTED BY

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Outline

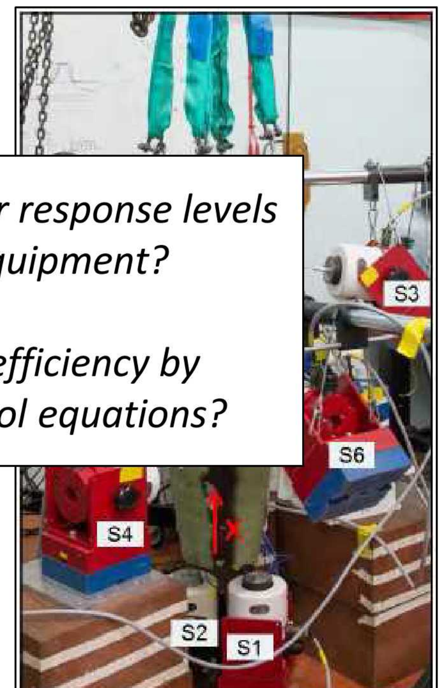
- Motivation: Efficient MIMO Testing
- MIMO Input Estimation Theory
- Shape-Constrained Input Estimation
- Demonstration on Example System
- Comparison of Constraint Vectors
- Effects of the Number of Constraint Vectors
- Comparison vs. Standard Input Estimation
- Conclusions

Motivation: Efficient Multi-Shaker MIMO Vibration Testing

- Recent examples tests showed that complex vibration response can be accurately replicated in the lab using multiple modal shakers and MIMO control schemes
- Shaker capabilities appear to be a limiting factor in scaling up this technique for larger structures and aggressive environments

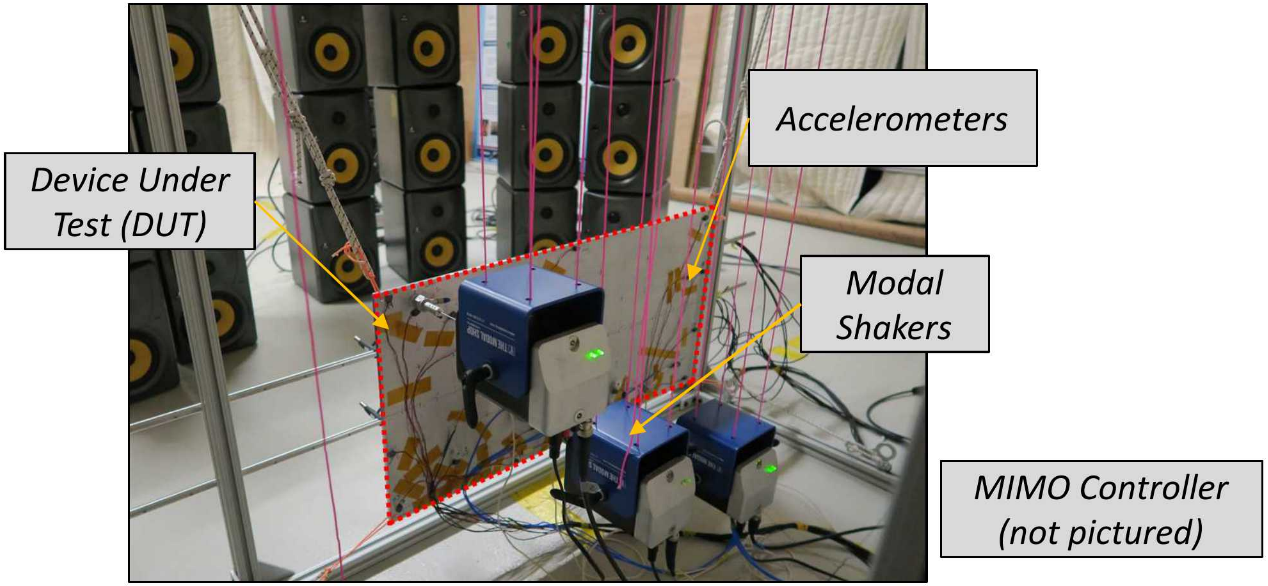
Can we achieve higher response levels from existing equipment?

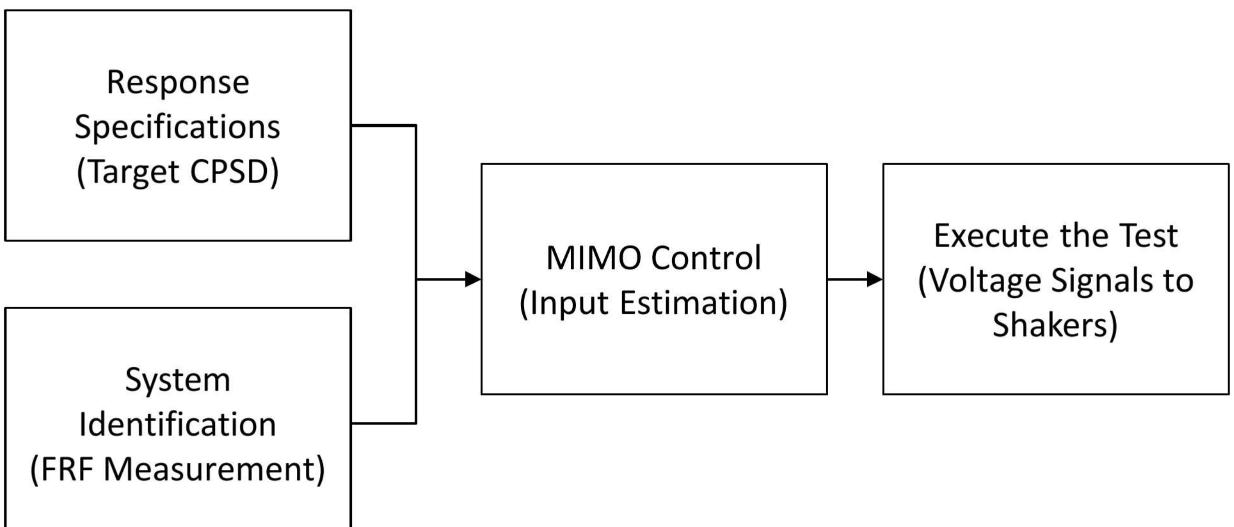
Can we increase efficiency by changing the control equations?



[1]R. L. Mayes and D. P. Rohe, "Physical Vibration Simulation of an Acoustic Environment with Six Shakers on an Industrial Structure," in Proceedings of IMACXXIV, the 34th International Modal Analysis Conference, 2016.
[2]P. M. Daborn, "Scaling up of the Impedance-Matched Multi-Axis Test (IMMAT) Technique," in Proceedings of IMAC XXXV, the 35th International Modal Analysis Conference, Garden Grove, CA, 2017

A Typical Multi-Shaker Vibration Test



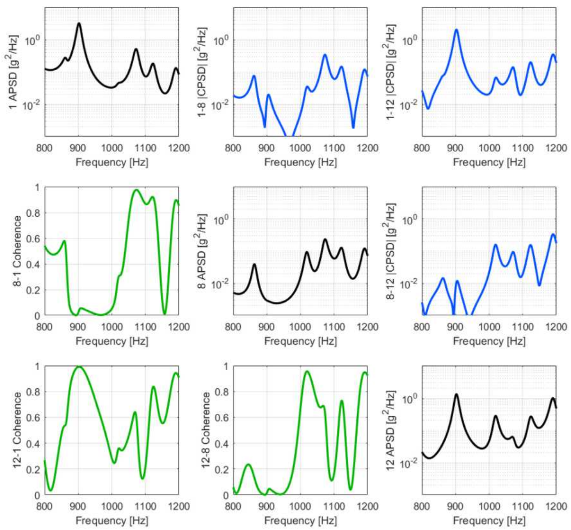


MIMO Input Estimation Theory

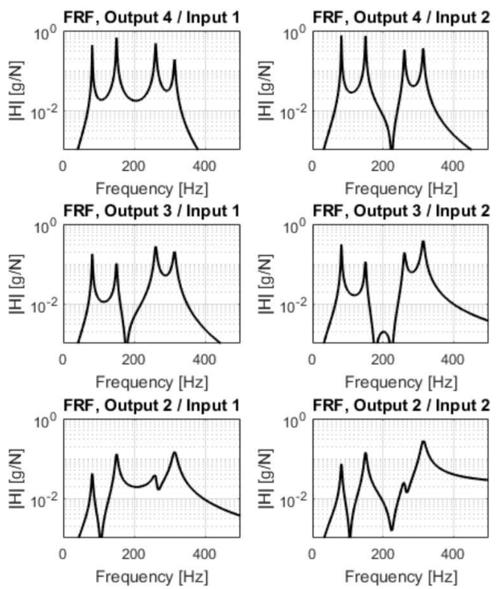
The math behind the test

MIMO Linear System

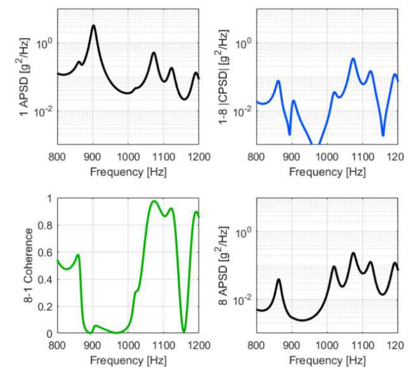
$$[S_{yy}] = [H_{yx}][S_{xx}][H_{yx}]^H$$



Response CPSD



System FRF



Input CPSD

MIMO Input Estimation Theory

	Forward Problem:	**Inverse Problem:**
Linear	$\{X_y\} = [H_{yx}]\{X_x\}$	$\{X_x\} = [H_{yx}]^{+}\{X_y\}$
Power	$[S_{yy}] = [H_{yx}][S_{xx}][H_{yx}]^{H}$	$[S_{xx}] = [H_{yx}]^{+}[S_{yy}][H_{yx}]^{+H}$

Given inputs, get outputs

Pseudo-inverse solution results in a least squares solution

Least squares match to the target response, regardless of the input requirements

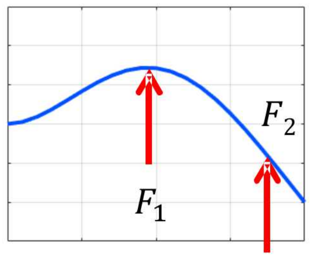
$\{X_x\}$ = Nx1, Input Linear Spectra
$\{X_y\}$ =Mx1, Output Linear Spectra
$[H_{yx}]$ =MxN, FRF Matrix
$[\cdot]^{+}$ = pseudo-inverse
$[\cdot]^{H}$ = Hermitian

Shape-Constrained Input Estimation

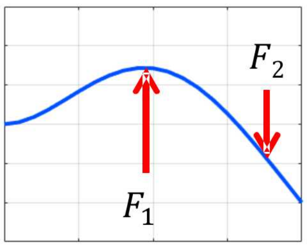
Do the math differently

Modification of the MIMO Input Estimation Equation: Shape-Constrained Input Estimation

- Motivating Questions:
  - *Can we pre-define the shaker relationships to increase their efficiency?*
  - *Can we take advantage of the inherit system dynamics?*
  - *Can we avoid issues like forces increasing when more shakers are used?*
- Rationale:
  - *Enforcing an input pattern similar to the dominant system modes should efficiently excite the structure (shake it how it wants to respond)*
  - *Pre-defining the relationships between shakers should avoid problems where the solution "blows up" or where shakers "fight each other"*



Inefficient, Uncoordinated Inputs



Efficient, Coordinated Inputs

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = [C]\hat{F}$$

Prescribed F_1, F_2 Input Relationship

Modification of the MIMO Input Estimation Equation: Shape-Constrained Input Estimation

- Approach:
  - *Apply a constraint matrix to the columns (inputs) of the FRF matrix in the input estimation equation*
  - *Populate the constraint matrix with vectors that are similar to system modes*

1. Standard Input Estimation Equation

$$[S_{xx}] = [H_{yx}]^{+}[S_{yy}][H_{yx}]^{+H}$$

2. Form Constrained FRF Matrix

$$[\widehat{H}_{yx}] = [H_{yx}][C]$$

3. Estimate Inputs (Constrained Set)

$$[\hat{S}_{xx}] = [\widehat{H}_{yx}]^{+}[S_{yy}][\widehat{H}_{yx}]^{+H}$$

4. Convert Inputs to Full Set

$$[S_{xx}] = [C][\hat{S}_{xx}][C]^{H}$$

Modification of the MIMO Input Estimation Equation: Shape-Constrained Input Estimation

- Approach:
  - *Apply a constraint matrix to the columns (inputs) of the FRF matrix in the input estimation equation*
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1. Standard Input Estimation Equation

$$[S_{xx}] = [H_{yx}]^{+}[S_{yy}][H_{yx}]^{+H}$$

2. Form Constrained FRF Matrix

$$[\widehat{H}_{yx}] = [H_{yx}][C]$$

Constraint matrix reduces the column dimension (space) of the FRF matrix

3. Estimate Inputs (Constrained Set)

$$[\hat{S}_{xx}] = [\widehat{H}_{yx}]^{+}[S_{yy}][\widehat{H}_{yx}]^{+H}$$

4. Convert Inputs to Full Set

$$[S_{xx}] = [C][\hat{S}_{xx}][C]^{H}$$

$$[H_{yx}] = MxN$$

$$[C] = Nx\widehat{N}$$

$$\widehat{N} < N$$

$$[\widehat{H}_{yx}] = Mx\widehat{N}$$

Vectors in the Constraint Matrix

- $[C]$ contains a set of $\hat{N}$ constraint vectors

Mode Shape Constraints:

$$[C] = [\{U_1\}\{U_2\}]$$

- *Best mode for constraint depends on the pattern of the response*
- *Need to replace vectors with new ones throughout the frequency range*

Singular Vector Constraints:

$$[C] = [\{V_{\Sigma 1}\}\{V_{\Sigma 2}\}]$$

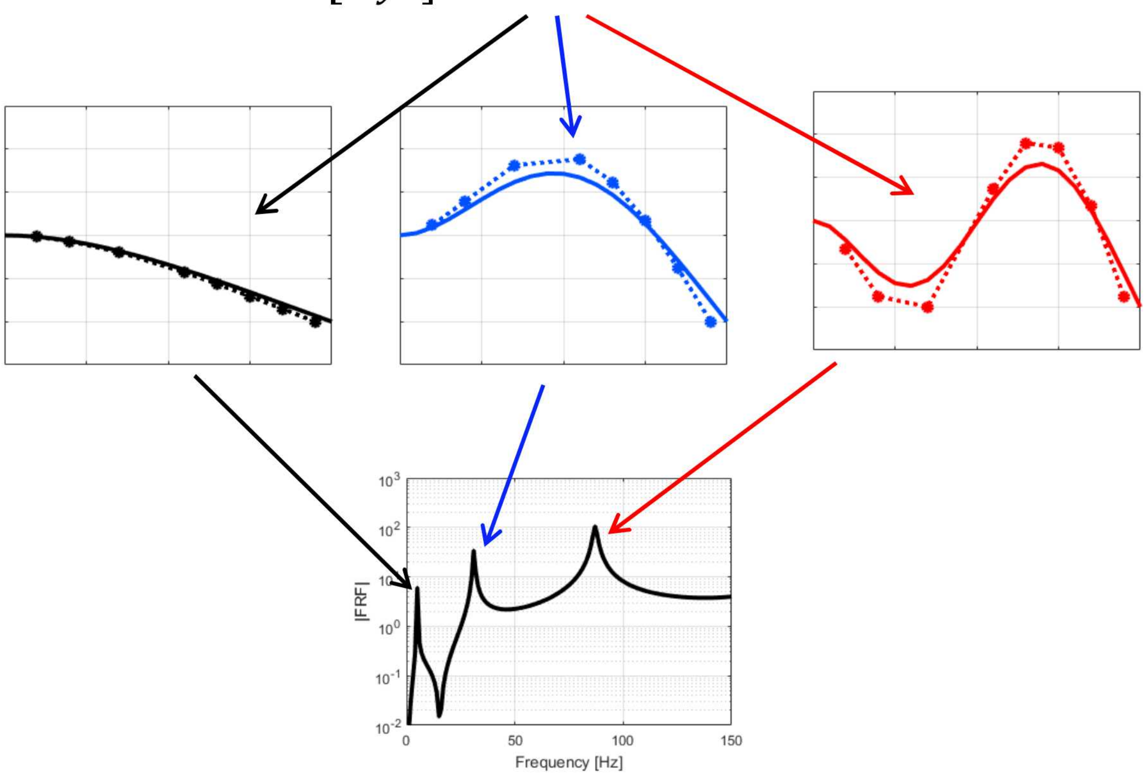
$$[H_{yx}] = [U_\Sigma][S_\Sigma]\,[V_\Sigma]^H$$

- *Singular vectors automatically change shape with frequency*
- *Right singular vectors are associated with the input DOF*
- *Orthonormal form of the singular vector matrices means this is equivalent to singular value truncation regularization*

Singular Vector Shapes

- Different vector or shape at each frequency line
- Near mode frequencies, singular vector shapes look like mode shapes

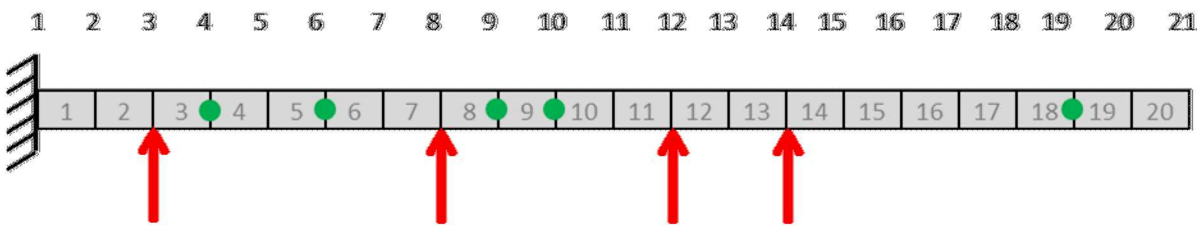
$$[H_{yx}] = [U_\Sigma][S_\Sigma]\,[V_\Sigma]^H$$



Demonstration on Example System

See how it works

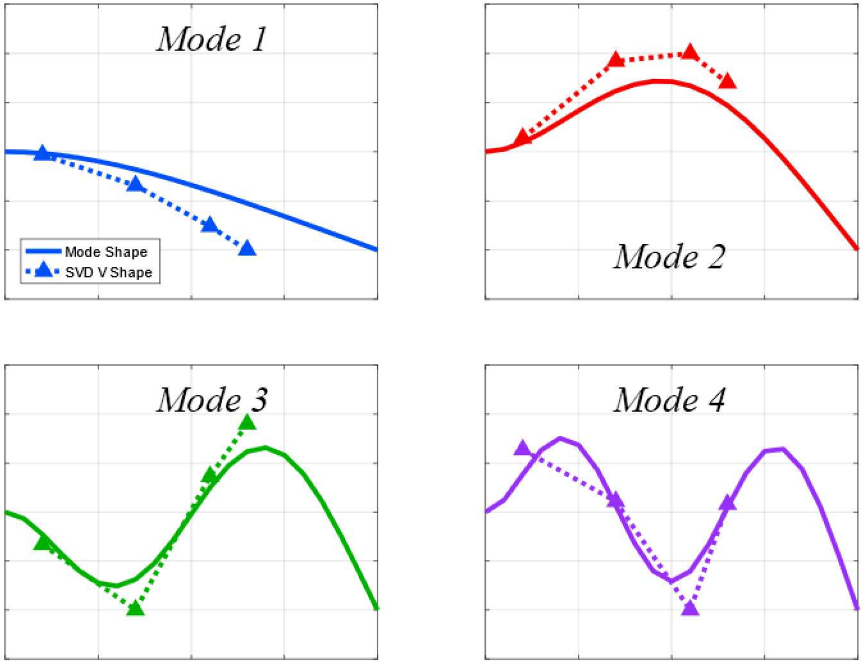
Example System: Cantilever Beam



- 20 Elements, 21 nodes, 42 DOF (vertical displacement & rotation)
- 5 output DOF
- 2 configurations:
  - Field: Random (uncorrelated) forces on all displacement DOF (random acoustic load). Used to generate the target response CPSD matrix
  - Lab: 4 forces (shakers) at arbitrary displacement DOF. Provides the lab FRF matrix

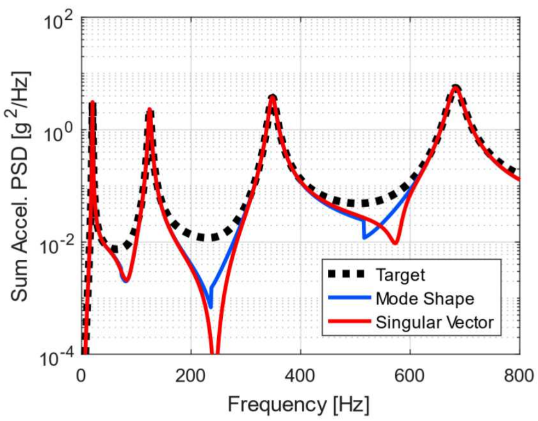
Similarity of Mode Shapes and Right Singular Vectors

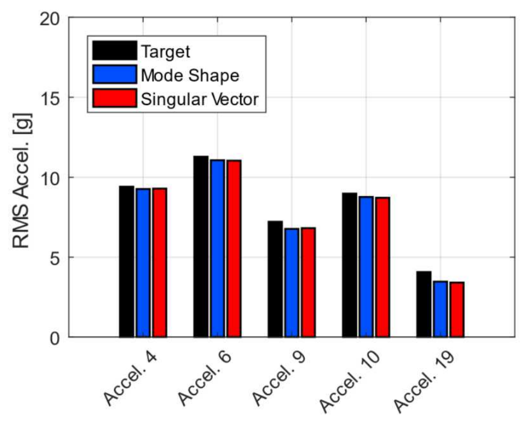
- Examining the shape of the right singular vectors at mode frequencies
  - Shown as signed magnitude for simplicity

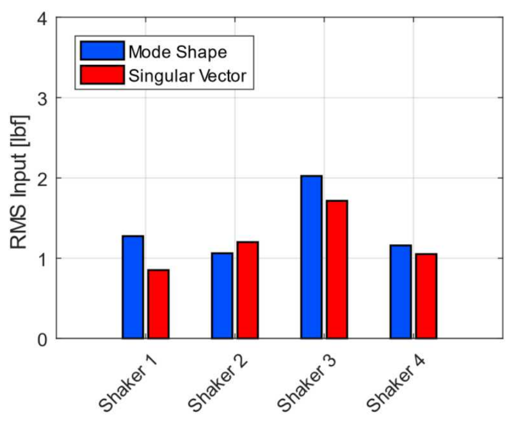


Comparison of Modes vs. Singular Vectors as Constraints

- Similar performance in this case
- Details:
  - Single constraint vector
  - Mode vector chosen as mode nearest each frequency line



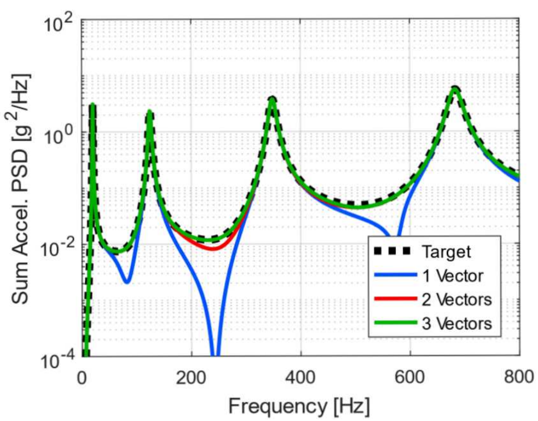


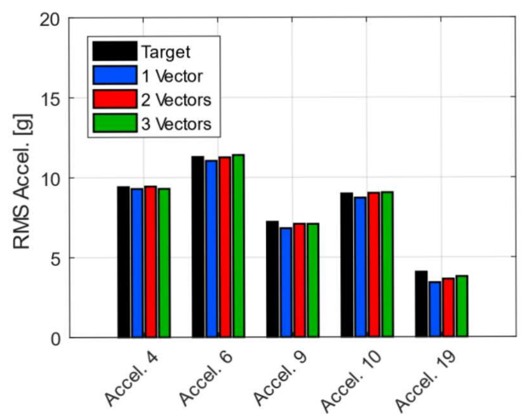


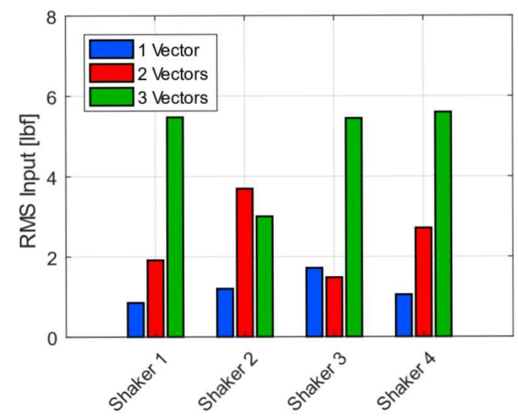
Singular vectors perform well, and don't require a secondary test, curve fitting process, etc.
Singular vectors come from simple decomposition of the FRF matrix which is already measured in a MIMO test

Effects of the Number of Constraint Vectors

- 1, 2, or 3 right singular vectors in the constraint matrix
- Number of constraint vectors balances response accuracy & input force
- More vectors = improved response accuracy
- Fewer vectors = reduced input force requirement







The number of constraints is now a knob for the test engineer to turn to tailor the input estimation solution to the objectives of a given test

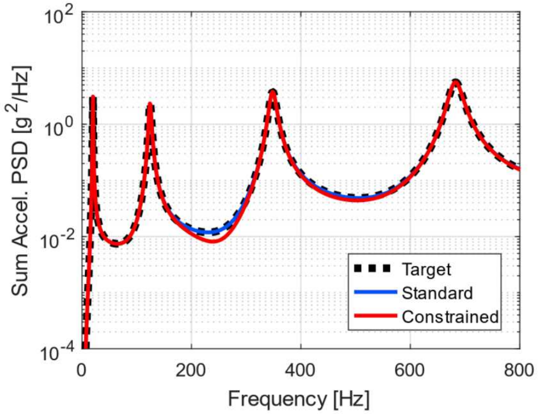
Comparison with Standard Input Estimation

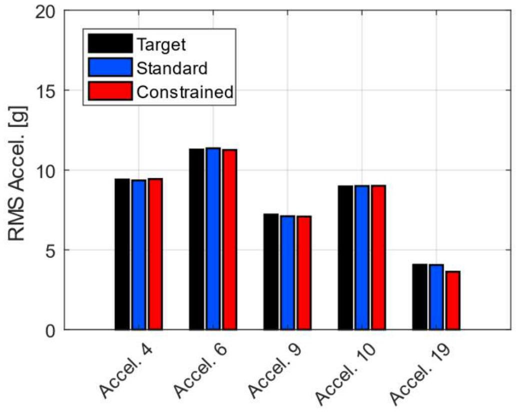
$$[S_{xx}] = [H_{yx}]^{+}[S_{yy}][H_{yx}]^{+H}$$

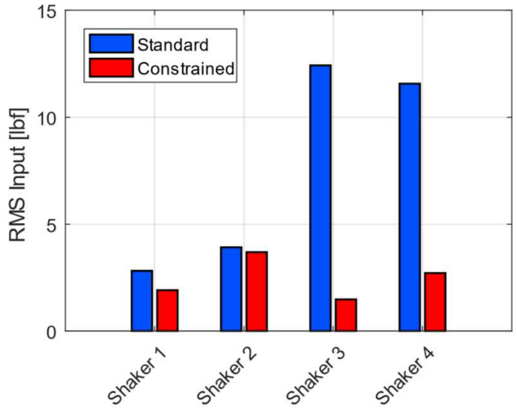
vs.

$$[\hat{S}_{xx}] = \left[[H_{yx}][C]\right]^{+}[S_{yy}]\left[[H_{yx}][C]\right]^{+H}$$

- Shape-constrained input estimation shows similar response accuracy but reduced input force requirements in this case
- Errors only in the regions between peaks in the response







Constraining inputs to a chosen pattern improves the efficiency (response/input) of the MIMO test

Conclusions

Conclusions

- Increasing efficiency (response/input) will expand the use of multi-shaker vibration testing
- Shape-constrained input estimation utilizes the system dynamics to enforce a pattern of the inputs via a constraint matrix applied to the FRF matrix
- Utilizing right singular vectors for constraints is simple, cheap, and effective
- Results show good accuracy with reduced input force requirements

- Future Work:
  - Explore vector selection methods[1]
  - Determine how it works for problems with many shakers[1]
  - Apply to various models and experiments to assess accuracy and force reduction trends in general

[1]R. Schultz and P. Avitabile, "Application of an automatic constraint shape selection algorithm for input estimation," in Proceedings of IMAC XXXVIII, the 38th International Modal Analysis Conference, 2020

Shape-Constrained Input Estimation for Multi-Shaker Vibration Testing

Ryan Schultz and Peter Avitabile

Thank you to my advisor & co-author, Dr. Peter Avitabile

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