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Expansion Methods Applied to Internal Acoustic Problems

PRESENTED BY

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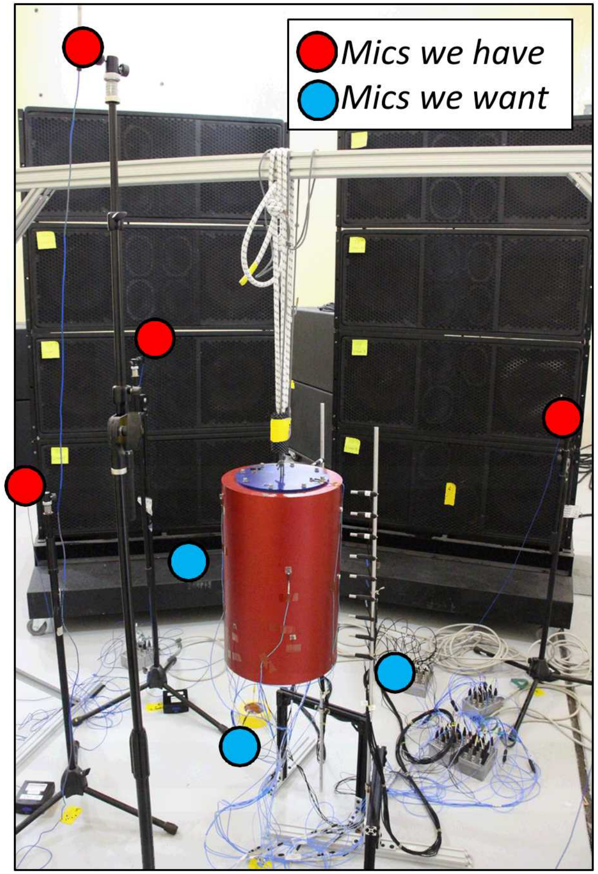
Outline

- Motivation
- Expansion Theory
- Small Domain (SEREP Expansion)
- Large Domain (Transmissibility Expansion)
- Conclusions

Motivation: Obtaining More Pressure Information From Acoustic Experiments

- Typical acoustic environment tests have limited microphone measurements
- Sparse pressure measurements limit the usefulness of test measurements
- Full-field measurements are useful for:
  - Estimating pressure at unmeasured locations
  - Field visualization
  - Field assessment (i.e. diffuse or other shape)
  - Generating loads for models in a V&V effort

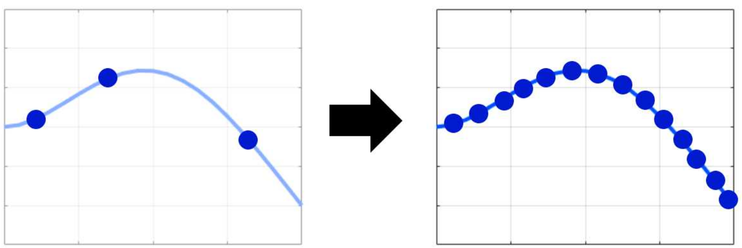


Expansion, Generally

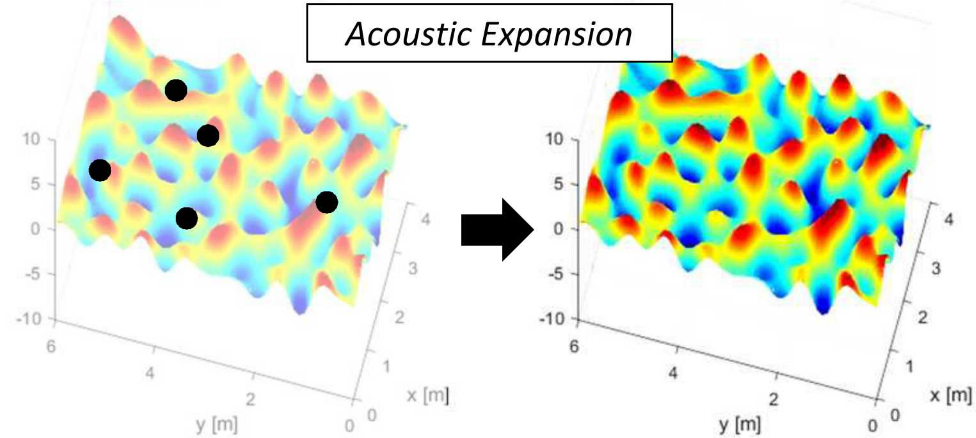
- Take a small set of measurements and project them to a larger set

$$\{x_n\} = [T_{na}]\{x_a\}$$

Structure Expansion



Acoustic Expansion



- Recent uses:
  - Estimate response at un-measured locations on a rocket payload
    - B. R. Zwink, D. G. Tipton, B. C. Owens, R. L. Mayes and M. Khan, "Flight environments demonstrator: Part II - Ground trials of a sounding rocket experiment for characterization of flight," in *Proceedings of IMAC XXXVII, the 37th International Modal Analysis Conference*, 2019
  - Estimate strain using strain mode shapes and measured response at sparse locations
    - B. Witt, D. Rohe and T. Schoenherr, "Full-field strain shape estimation from 3D SLDV," in *Proceedings of IMAC XXXVII, the 37th International Modal Analysis Conference*, 2019.

Expansion for Acoustic Problems

- Traditionally, expansion has been used to expand structural response
  - Measure acceleration at some sparse locations
  - Use expansion techniques to estimate the response at other locations
- This works because the system being expanded is a nice linear system defined by modes or FRFs
  - Note: Expansion techniques have been shown to be effective at expanding mildly nonlinear systems as well
- Many acoustic problems or domains are nice and linear as well
  - So, expansion should work
  - Measure pressure at some sparse locations
  - Use pressure mode shapes or FRFs as the expansion basis functions

Expansion Theory

Magic With Math

Expansion for Different Quantities

- Generally: $\{x_n\} = [T_{na}]\{x_a\}$

- Time Histories: $\{x_n(t)\} = [T_{na}]\{x_a(t)\}$

- Linear Spectra: $\{X_n(\omega)\} = [T_{na}]\{X_a(\omega)\}$

- CPSDs: $[S_{nn}(\omega)] = [T_{na}]\ [S_{aa}(\omega)]\ [T_{na}]^T$

Basic Process:

1. *Create transformation matrix,* $[T_{na}]$
2. *Response measurements at a-DOF,* $\{x_a\}$
3. *Perform expansion to estimate response at n-DOF,* $\{x_n\}$

Expansion Theory - SEREP

- SEREP = System Equivalent Reduction Expansion Process
- Sparse set of measurements: $\{x_a(t)\}, \{X_a(\omega)\}$
- System mode shapes: $[U_{a,k}], [U_{n,k}]$
- Transformation matrix: $[T_{na}] = [U_{n,k}][U_{a,k}]^{+}$
- Expand to a larger set: $\{X_n(\omega)\} = [T_{na}]\{X_a(\omega)\}$
- Note:
  - $[T_{na}]$ is a constant so SEREP can expand time or frequency domain quantities or even mode shapes
  - The number of a-DOF must be larger than the number of modes

J. O'Callahan, P. Avitabile and R. Riemer, "System Equivalent Reduction Expansion Process (SEREP)," in Proceedings of the 7th International Modal Analysis Conference (IMAC VII), Las Vegas, NV, 1989.

Expansion Theory - Transmissibility

- FRFs with a common reference: $[H_{ai}]\,,[H_{ni}]$
- FRFs at measurement DOF: $[H_{ai}] = [S_{ai}][S_{ii}]^{+}$
- FRFs at expanded DOF: $[H_{ni}] = [S_{ni}][S_{ii}]^{+}$
- Transmissibility: $[T_{na}] = [H_{ni}][H_{ai}]^{+}$
- Expand to a larger set: $\{X_n\} = [T_{na}]\{X_a\}$
- Note:
  - $[T_{na}]$ is a function of frequency so it can only be used to expand frequency domain quantities, not time data or mode shapes
  - Obtain FRFs with a model or with measurements
  - Example: Roving microphone for pre-test measurement of $[H_{ni}]$

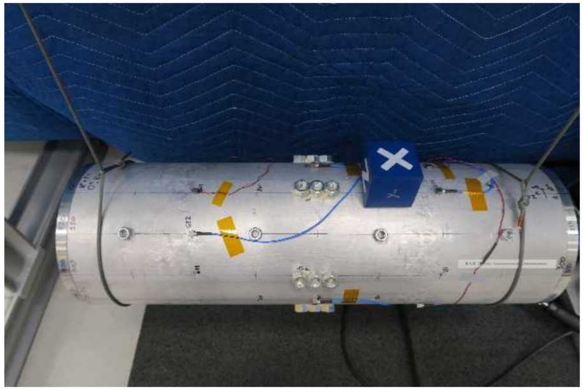
Acoustic Expansion Demonstration: Small Domain

Small Domain = Few Modes

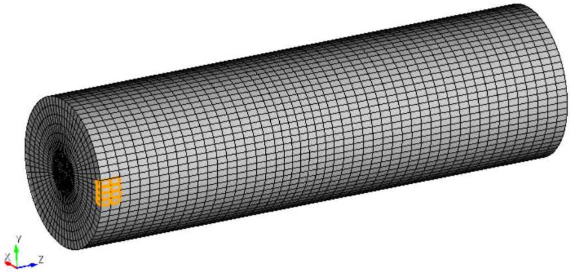
Example Small Domain System

- Small Domain = Few Modes
- *Can we apply SEREP to acoustic problems?*
  - Expand pressure time histories and PSDs

Example System



Cylindrical Shell

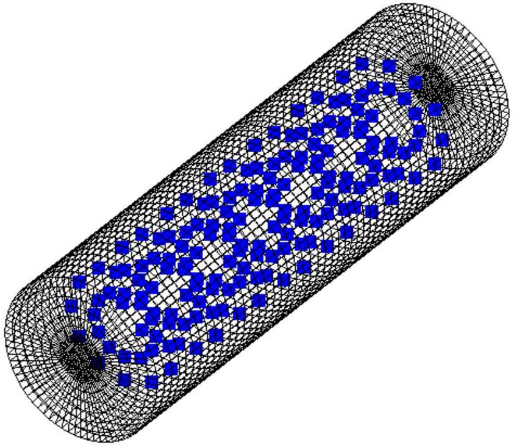


FEM of Internal Cavity

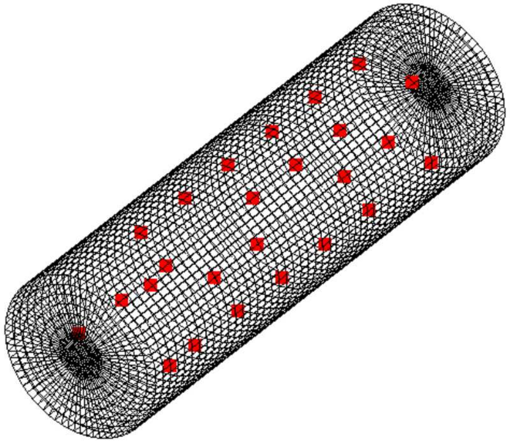
- 8" OD, 7" ID, 24" long aluminum shell
- 20 modes under 2kHz

Example Small Domain System

- 0-2kHz Bandwidth of Interest
- Selected 25 a-DOF from a set of candidate DOF with effective independence
  - 25 A-DOF > 20 modes
  - Effective independence ensures a well-formed $\left[U_{a,k}\right]$ shape matrix



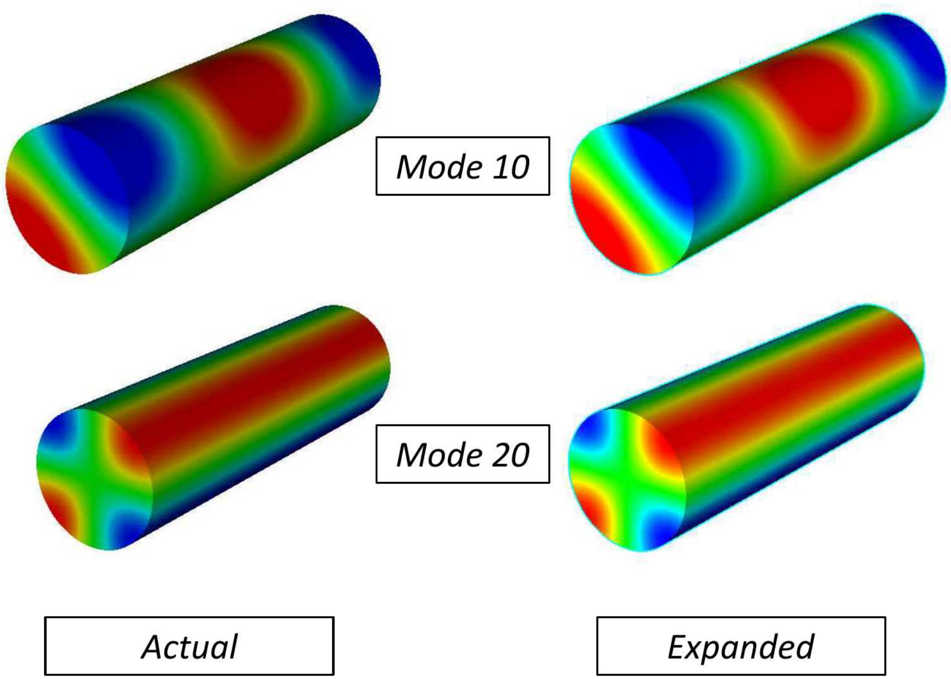
Candidate a-DOF



25 Chosen a-DOF

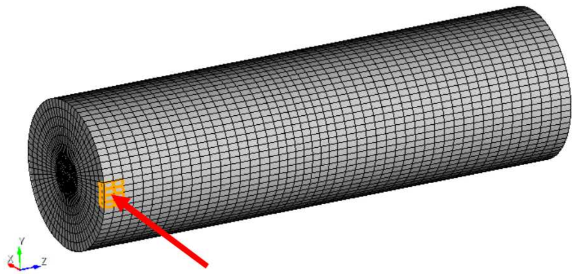
Acoustic Mode Shape Expansion with SEREP

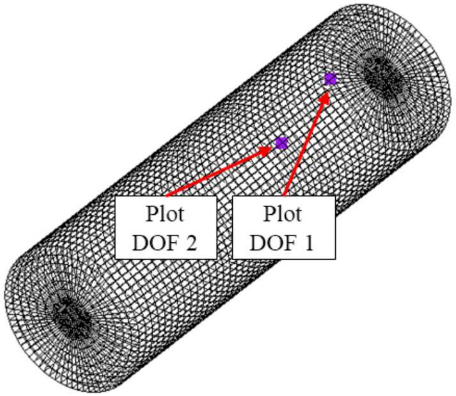
- "Measured" acoustic pressure mode shapes at the a-DOF
  - All with simulation data
  - Added noise to the a-DOF shapes
- Expanded to the n-DOF (all the acoustic DOF) to compare the actual and expanded mode shapes



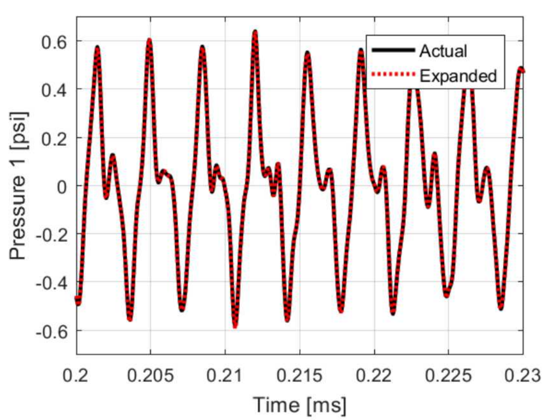
Time Response and PSD Expansion with SEREP

- Expand from a-DOF to two n-DOF selected in the cavity
- Applied a 1 ms haversine pulse to the corner of the cylinder

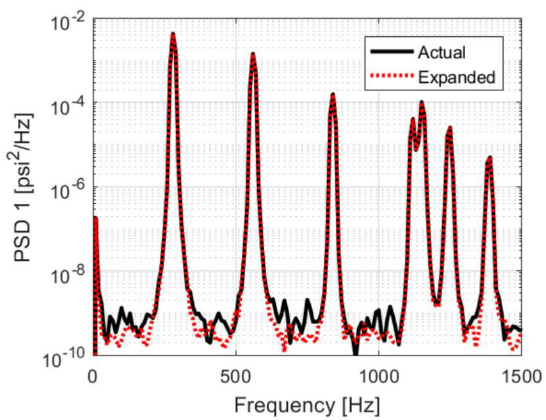




n-DOF 1



Pressure Time Response



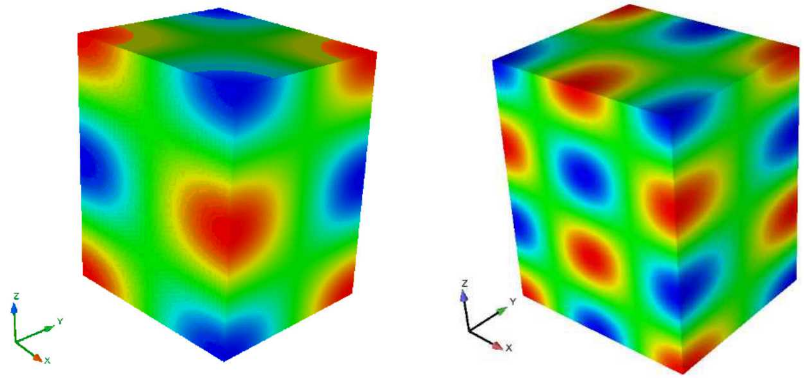
Pressure PSD

Acoustic Expansion Demonstration: Large Domain

So Many Modes…

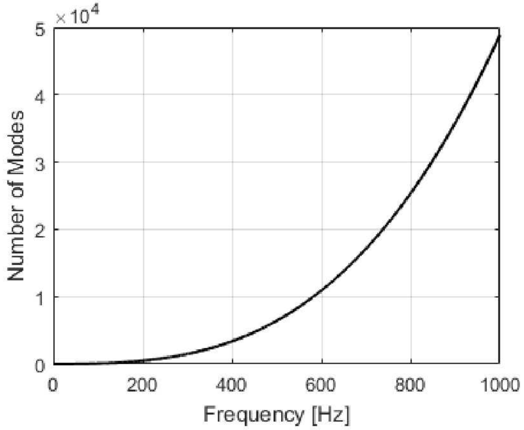
Example Large Domain System

- Large Domain = Many Modes
- Acoustic test chamber
  - Reverberation chamber
  - 21 x 25 x 30 ft



Example Chamber Modes

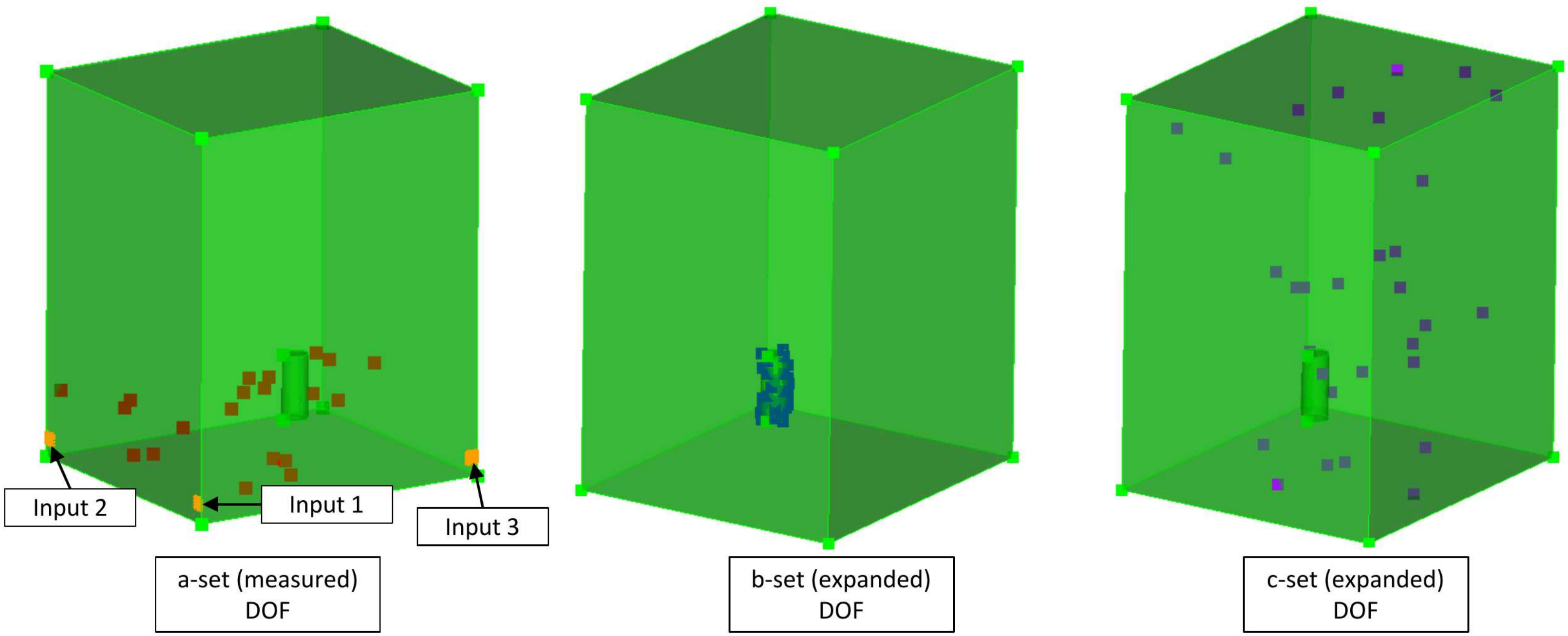
- Can we use SEREP for expansion?
  - No!
  - Requires *thousands* of a-DOF



40,000 modes below 1kHz!

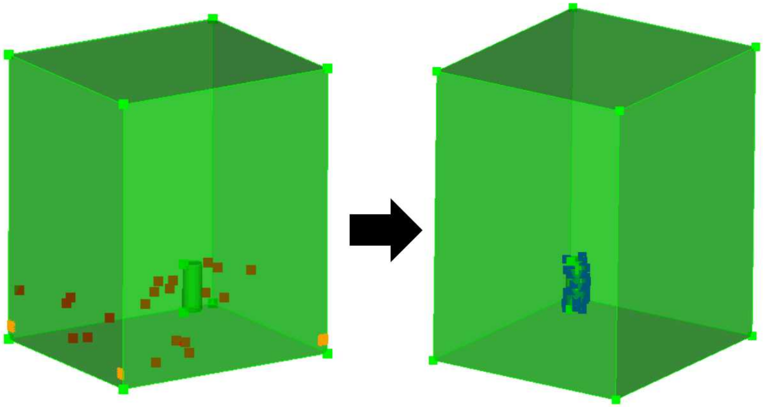
Example Large Domain System

- Cannot use SEREP because too many modes in the bandwidth
- Use Transmissibility expansion instead
- Setup:
  - 3 inputs (speakers in the corners)
  - 20 a-DOF scattered throughout the bottom portion (mic locations)
  - Expansion sets b and c (test article and throughout the volume)
  - Transient simulation. Obtain $p(t)$ then compute CPSDs and FRFs

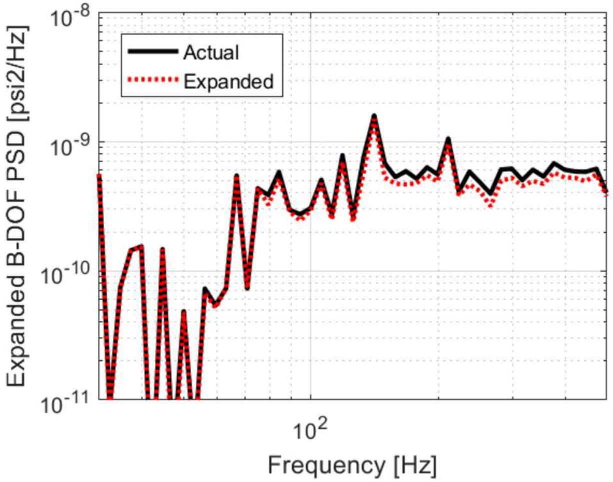


Transmissibility Expansion

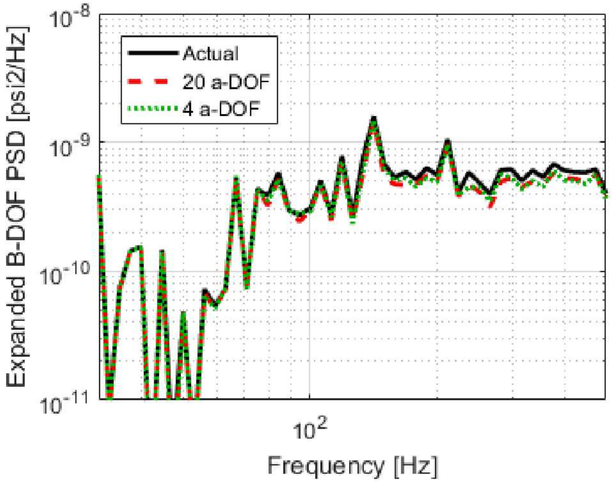
- a-DOF to b-DOF (test article surface pressure)
  - Model validation case
- 20 a-DOF ($N_a > N_b$)
- 4 a-DOF ($N_a < N_b$)



20 a-DOF



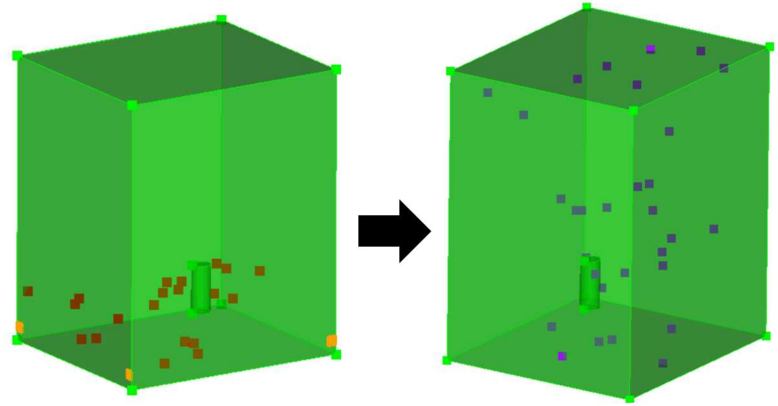
4 a-DOF



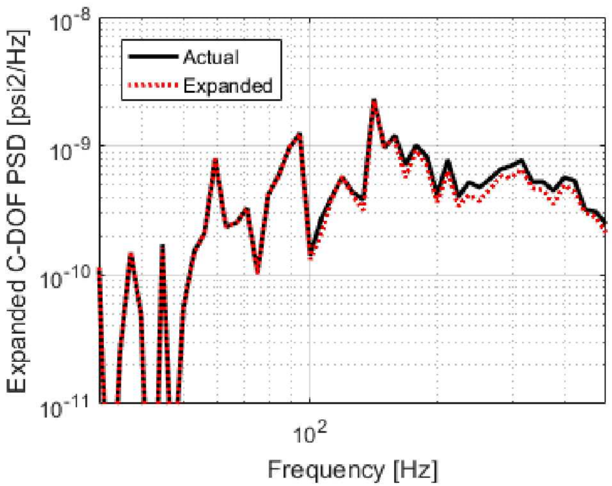
Only need $N_a > N_{inputs}$
(remain overdetermined)

Transmissibility Expansion

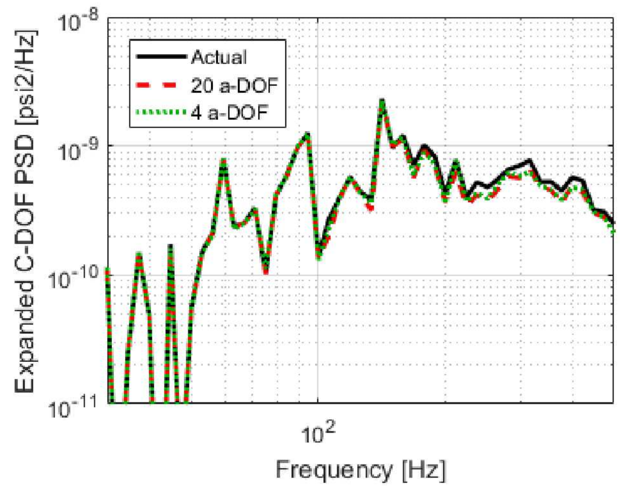
- a-DOF to c-DOF (pressure anywhere)
  - Additional instrumentation
- 20 a-DOF ($N_a > N_b$)
- 4 a-DOF ($N_a < N_b$)



20 a-DOF



4 a-DOF



Conclusions

Conclusions

- Expansion is useful for making the most of limited measurements
- Mode-based expansion works well when there are a relatively small number of modes
- Transmissibility or FRF expansion can work when the number of modes is large
- Two examples were provided to demonstrate expansion applied to acoustic problems:
  - Small Domain
  - Large Domain
- **Typical expansion methods work just as well for acoustic problems as structural problems, and many of the same rules apply**

- Future Work:
  - Demonstrate this with actual measurements
  - Obtain model pressure loads from expanded test pressures for a V&V effort

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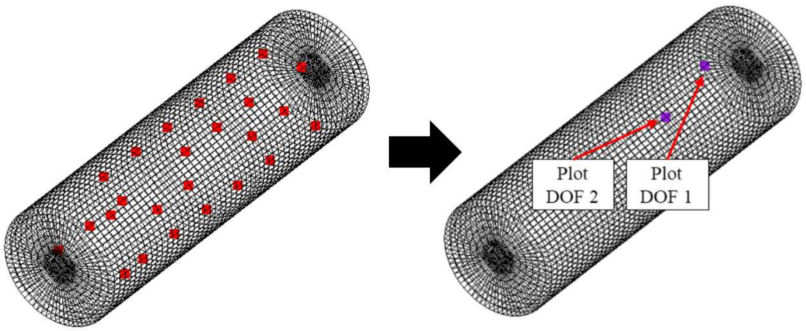
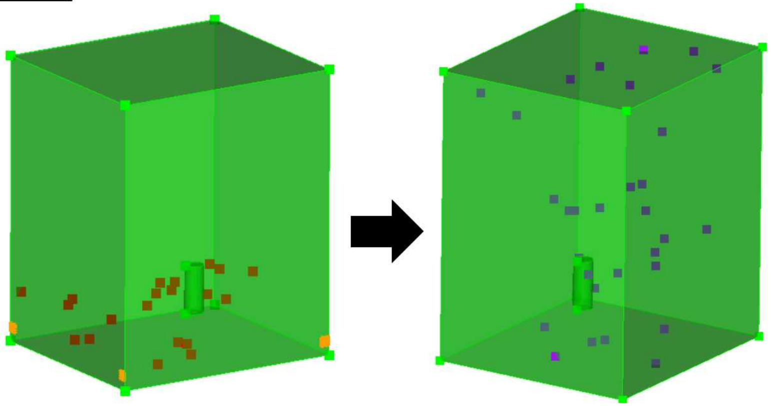
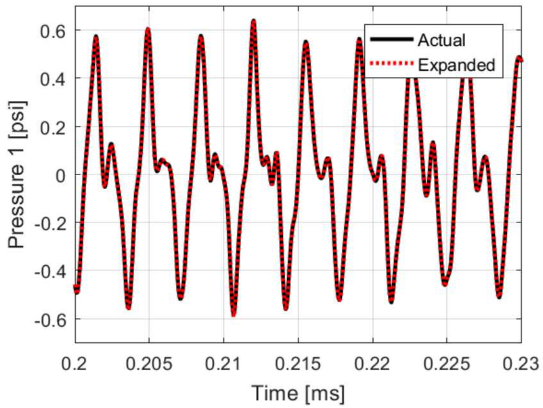
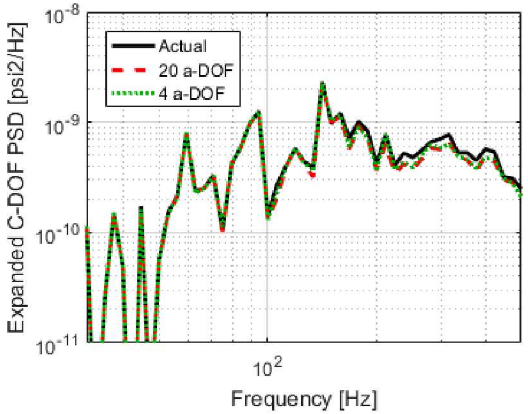


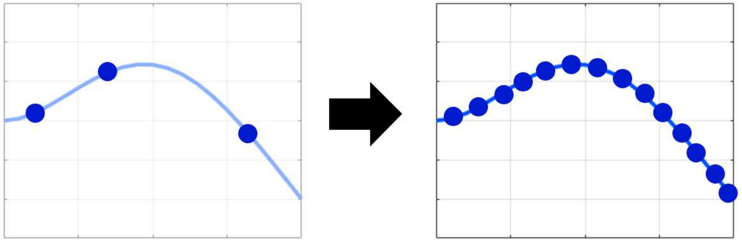


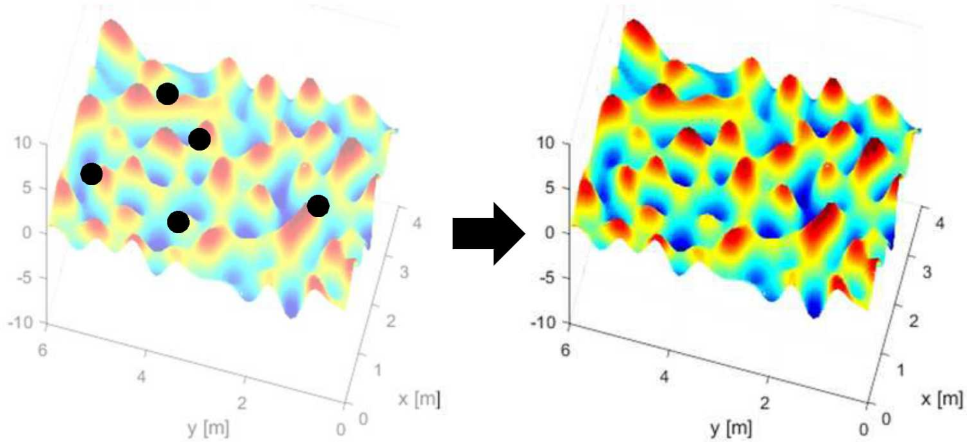
$$[T_{na}] = [U_{n,k}][U_{a,k}]^+$$

$$[T_{na}] = [H_{ni}][H_{ai}]^+$$





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