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subject: Failure Formulations in Modular Plasticity Models

1 Introduction

Computational prediction of ductile failure remains a challenging and important problem as demonstrated by the recent Sandia Fracture Challenges [1, 2]. In addition to emphasizing the complexity of such problems, the variety of solution strategies also highlighted the number of possible approaches to this problem. A common engineering approach for such efforts is to use a failure model in conjunction with element deletion. In the second Sandia Fracture Challenge [2], for instance, nine of the fourteen teams used some form of element deletion. For such schemes, a critical decision pertains to the selection of the appropriate failure model; of which many may be found in the literature (see the review of Corona and Reedlunn [3]). The variety may also be observed in the aforementioned second Sandia Fracture Challenge in which at least eight different failure criteria are listed for the nine element deletion based approaches.

The selection of the appropriate failure model is a difficult challenge depending on the material being considered and such criteria can variously depend on stress state (*i.e.* triaxiality, Lode angle) and loading conditions (*i.e.* strain rate, temperature). Separate implementations of each criteria with different plasticity models can be a repetitive and cumbersome process which may limit available models for an engineering analyst. To mitigate this issue, an effort was pursued to flexibly implement failure models in which different failure models could be specified and utilized within the same elastic-plastic constitutive routine by simply changing the input syntax. Similarly, the same models are implemented across a suite of elastic-plastic formulations enabling consistent definitions. As will be discussed later, a specific "modular failure" model is also implemented which allows for the selection or specification of different dependencies depending on the current need. At this stage, this effort is limited to defining failure models; progression/damage evolution in the constitutive model is not treated and left to future efforts.

To present these efforts, Section 2 gives the functional forms of the implemented models while Section 3 describes the corresponding syntax. Sample verification exercises are discussed in Section 4. Finally, some concluding thoughts are found in Section 5.

2 Theory

For use with different plasticity models, various failure models were implemented in a consistent framework. These models include the "tearing parameter" approach of Wellman [4], the Johnson-Cook model [5], the model of Wilkins *et al.* [6], and a "modular failure" model allowing for different combinations of terms. These models will be briefly introduced here although details are left to the cited works. In general, all of these models seek to evaluate a damage parameter, d, in terms of stress state and loading history. Failure is predicted when they reach a critical value, $d = d_{\mathrm{crit}}$. For convenience, however, in what follows the expressions are normalized such that $d = 1$ corresponds to the critical damage value.

2.1 Tearing Parameter

The "Tearing Parameter" formulation implemented here is the modified form of Wellman [4] that is given as,

$$d = \frac{1}{d_{\mathrm{crit}}} \int_0^{\bar{\varepsilon}^{\mathrm{p}}} \left\langle \frac{2\sigma_{\mathrm{max}}}{3\left(\sigma_{\mathrm{max}} - p\right)} \right\rangle^m d\hat{\bar{\varepsilon}}^{\mathrm{p}}. \tag{1}$$

If σ_{ij} is the Cauchy stress tensor, then $p = (1/3)\sigma_{kk}$ is the pressure, σ_{max} is the maximum principal stress, and $\bar{\varepsilon}^{\mathrm{p}}$ is the equivalent plastic strain. The two parameters of the model are m, a fit exponent, and the critical failure (tearing) parameter, d_{crit}. Additionally, $\langle \cdot \rangle$ are Macauley brackets such that,

$$\langle x \rangle = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases} . \tag{2}$$

Note, unlike some of the later models, as written in Eqn. 1 the tearing parameter formulation implicitly, rather than explicitly, depends on stress state characteristics such as the pressure, Lode angle, and triaxiality. It is also independent of loading conditions like temperature and strain rate.

2.2 Johnson-Cook

The Johnson-Cook model [5] was formulated by experimental observation of failure in a variety of metals. This investigation resulted in a proposed model of the form,

$$d = \int_0^{\bar{\varepsilon}^{\mathrm{p}}} \frac{1}{\left(D_1 + D_2 \exp\left(D_3 \eta\right)\right) \left(1 + D_4 \ln \frac{\dot{\bar{\varepsilon}}^{\mathrm{p}}}{\dot{\varepsilon}_0}\right) \left(1 + D_5 T^*\right)} d\hat{\bar{\varepsilon}}^{\mathrm{p}}, \tag{3}$$

in which D_1, D_2, D_3, D_4 and D_5 are fitting constants and $\dot{\varepsilon}_0$ is a reference strain rate. Note, if the plastic strain rate, $\dot{\bar{\varepsilon}}^{\mathrm{p}}$ is less than the reference rate, their ratio is taken to be one. Furthermore, η is the triaxiality defined as,

$$\eta = \frac{p}{\sigma_{vM}}, \tag{4}$$

with σ_{vM} being the von Mises stress such that,

$$\sigma_{vM} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}}, \tag{5}$$

and σ'_{ij} is the deviatoric stress,

$$\sigma'_{ij} = \sigma_{ij} - p\delta_{ij}. \tag{6}$$

Defined in this fashion, the triaxiality is a non-dimensional number giving the ratio of the hydrostatic to deviatoric stress and is a common measure of the stress state. The homologous temperature, T^*, is given by,

$$T^* = \frac{T - T_{\mathrm{ref}}}{T_{\mathrm{melt}} - T_{\mathrm{ref}}}, \tag{7}$$

where T_{ref} and T_{melt} are a reference temperature (room temperature in the original work of Johnson and Cook [5]) and the melting temperature.

Like the yield stress model separately put forth by Johnson and Cook, the denominator of the integrand in Eqn. 3 is decomposed into contributions related to triaxiality, strain rate, and temperature and may be considered to be the current "failure strain". As such, failure occurs when the integrated value of d equals one. Furthermore, such a decomposition and decoupling helps motivate a modular failure formulation.

2.3 Wilkins

As with the previous Johnson-Cook model, the failure model of Wilkins *et al.* [6] assumes a multiplicative decomposition such that,

$$d = \frac{1}{d_{\text{crit}}} \int_0^{\bar{\varepsilon}^{\text{p}}} w_1 w_2 d\hat{\bar{\varepsilon}}^{\text{p}}, \tag{8}$$

with w_1 being the pressure dependent term defined as,

$$w_1 = \left(\frac{1}{1 - \frac{p}{B}} \right)^{\alpha}, \tag{9}$$

where α and B are fitting parameters and w_2 is a Lode angle related term,

$$w_2 = (2 - A)^{\beta}, \tag{10}$$

in which β is a fitting parameter and,

$$A = \max\left(\frac{\sigma_2'}{\sigma_3'}, \frac{\sigma_2'}{\sigma_1'} \right), \tag{11}$$

with $\sigma_{1,2,3}'$ being the deviatoric principal stresses so that $\sigma_1' \geq \sigma_2' \geq \sigma_3'$. In the original work of Wilkins *et al.* [6], these ratios are described as being related to "stress field symmetry" so that $0 \leq A \leq 1$ with the limits corresponding to tension and torsion. As such, while not explicitly depending on the Lode angle the meaning and value of w_2 is clearly related to that parameter.

2.4 Modular Failure

In considering the Johnson-Cook and Wilkins models, it is interesting to note that while they share a similar multiplicative decomposition in terms of dependencies, the underlying variables are not the same. Specifically, while the Johnson-Cook form depends on the stress-state via the triaxiality and loading conditions such as strain rate and temperature, the Wilkins model depends only on the stress-state but via the pressure and a Lode angle related term instead of triaxiality. An interesting and intriguing possibility to consider is if these terms may be used in different combinations to enable enhanced descriptions of failure models to meet specific needs of a given material. The "modular failure" model allows for such a description.

The "modular failure" models uses a multiplicative decomposition of any (or all) of the five aforementioned variables so that,

$$d = \frac{1}{d_{\text{crit}}} \int_0^{\bar{\varepsilon}^{\text{p}}} w_1(p)\, w_2(\theta)\, w_3(\eta)\, w_4\left(\dot{\bar{\varepsilon}}^{\text{p}}\right) w_5(T)\, d\hat{\bar{\varepsilon}}^{\text{p}}. \tag{12}$$

In Eqn. 12, each w_i is a "multiplier" giving the contribution of the corresponding dependence and θ is the Lode angle which is defined via the relation [7],

$$\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}} \tag{13}$$

with J_2 and J_3 the second and third invariant, respectively, of the deviatoric stress tensor. Currently, each term may be independently specified with available possibilities being,

$$\begin{aligned}
w_1(p) &= 1,\ w_1^{\mathrm{w}},\ w_1^{\mathrm{ud}} && (14)\\
w_2(\theta) &= 1, w_2^{\mathrm{w}} && (15)\\
w_3(\eta) &= 1,\ w_3^{\mathrm{jc}},\ w_3^{\mathrm{ud}} && (16)\\
w_4(\dot{\bar{\varepsilon}}^{\mathrm{p}}) &= 1,\ w_4^{\mathrm{jc}},\ w_4^{\mathrm{ud}} && (17)\\
w_5(T) &= 1,\ w_5^{\mathrm{jc}},\ w_5^{\mathrm{ud}}, && (18)
\end{aligned}$$

in which 1 denotes independence of the modular failure model with respect to that variable, a superscript "w" is used for a Wilkins model related term (w_1^{w} is given by Eqn. 9 and w_2^{w} via Eqn. 10), "jc" refers to a Johnson-Cook expression, and "ud" is user-defined. For the Johnson-Cook terms the expressions are given as,

$$\begin{aligned}
w_3^{\mathrm{jc}}(\eta) &= \frac{1}{D_1 + D_2\exp(D_3\eta)}, && (19)\\
w_4^{\mathrm{jc}}(\dot{\bar{\varepsilon}}^{\mathrm{p}}) &= \frac{1}{1 + D_4 \ln\frac{\dot{\bar{\varepsilon}}^{\mathrm{p}}}{\dot{\varepsilon}_0}}, && (20)\\
w_5^{\mathrm{jc}}(T) &= \frac{1}{1 + D_5 T^*}. && (21)
\end{aligned}$$

"User-defined" simply means that the user may specify a function in the input deck. Functions may be defined in any fashion allowed by Sierra/SolidMechanics. Possible forms of functions (*e.g.* piecewise linear or analytical) and there syntax may be found in the Sierra/SolidMechanics User Guide [8] and should be specified in terms of the relevant variable and returning the multiplier value. As an example, for w_1^{ud} the pressure, p, would be the variable and the function should be specified such that $w_1^{\mathrm{ud}} = 1$ corresponds to pressure-independence with $w_1^{\mathrm{ud}} > 1$ increasing and $w_1^{\mathrm{ud}} < 1$ lowering damage accumulation.

3 Failure Model Specification

Section 2 laid out a variety of the failure model options for use within a consistent, flexible framework. While the flexibility can be desirable in terms of enabling different formulations, it does also increase complexity with respect to specification in an input file. Such an issue is exacerbated by the "modular failure" model and its variable dependencies. As such, the usage of these capabilities is briefly discussed here to try and address that complexity.

The failure capabilities of interest have been added into existing modular plasticity models. Specifically, the currently described features have been added to and tested with both solid (*e.g.* J_2, Hosford, Hill, and Barlat) and plane stress (*e.g.* Modular Plane Stress Plasticity Fail) implementations. Details of these formulations and their model syntax may be found elsewhere (*e.g.* [9]). To use the failure capabilities, the optional `FAILURE MODEL` command must be specified with a given model. If this term is not provided, the damage parameter is not updated or calculated. Additionally, a `CRITICAL FAILURE PARAMETER` must be specified for every model except `JOHNSON_COOK_FAILURE`. The below syntax gives an example for the specifying the failure model using the J_2 formulation as a representative plasticity model. Note the same failure syntax may used with any other hardening model or aforementioned plasticity formulations.

```
BEGIN PARAMETERS FOR MODEL J2_PLASTICITY

    YOUNGS MODULUS        = E
    POISSONS RATIO        = ν
    YIELD_STRESS          = σ_y^0

    HARDENING MODEL       = LINEAR
    HARDENING MODULUS     = H'

    FAILURE MODEL         = TEARING_PARAMETER | JOHNSON_COOK_FAILURE
                             | WILKINS | MODULAR_FAILURE

    CRITICAL FAILURE PARAMETER = d_crit
                        •
                        •
          failure model specification
                        •
                        •
END PARAMETERS FOR MODEL J2_PLASTICITY
```

3.1 Models from the Literature

For any of the three existing, models from the literature (tearing parameter, Johnson-Cook, and Wilkins) the needed parameters may be seen in Eqns. 1, 3, 9, and 10. The relevant syntax for these models are given below.

```
FAILURE MODEL = TEARING_PARAMETER
   TEARING PARAMETER EXPONENT = m
```

```
FAILURE MODEL = JOHNSON_COOK_FAILURE
    JOHNSON COOK D1        = D1
    JOHNSON COOK D2        = D2
    JOHNSON COOK D3        = D3
    JOHNSON COOK D4        = D4
    JOHNSON COOK D5        = D5
    REFERENCE RATE         = ε̇0
    REFERENCE TEMPERATURE  = Tref
    MELTING TEMPERATURE    = Tmelt

FAILURE MODEL = WILKINS
    WILKINS ALPHA    = α
    WILKINS BETA     = β
    WILKINS PRESSURE = B
```

With respect to the `JOHNSON_COOK_FAILURE` model, some terms (`REFERENCE RATE`, `REFERENCE TEMPERATURE`, and `MELTING TEMPERATURE`) are common to both the isotropic hardening [10] and failure forms of the model. These parameters, however, are assumed to be common across both model features and can only be specified once.

3.2 Modular Failure

The modular failure model builds off of the capabilities of the existing literature models and adds flexibility to allow for failure model specification depending on the individual needs of the user. As a first step for using this model, the various dependencies and functional forms must be defined in terms of the form of the different multipliers w_1 through w_5. The options and syntax for these different functions are given below. Note, while the `INDEPENDENT` options may be directly input, all of the multipliers are set to default to the `INDEPENDENT` choice ($w_i = 1$) if no option is given. Thus, each of the multipliers is an optional input and the user need only specify the terms of interest.

```
 FAILURE MODEL = MODULAR_FAILURE
    PRESSURE MULTIPLIER    = PRESSURE_INDEPENDENT | WILKINS |
                             USER_DEFINED (PRESSURE_INDEPENDENT)
    LODE ANGLE MULTIPLIER  = LODE_ANGLE_INDEPENDENT |
                             WILKINS (LODE_ANGLE_INDEPENDENT)
    TRIAXIALITY MULTIPLIER = TRIAXIALITY_INDEPENDENT | JOHNSON_COOK |
                             USER_DEFINED (TRIAXIALITY_INDEPENDENT)
    RATE FAIL MULTIPLIER   = RATE_INDEPENDENT | JOHNSON_COOK |
                             USER_DEFINED (RATE_INDEPENDENT)
    TEMPERATURE FAIL MULTIPLIER = TEMPERATURE_INDEPENDENT |
                                  JOHNSON_COOK | USER_DEFINED
                                  (TEMPERATURE_INDEPENDENT)
```

After identifying the dependencies, the additional parameters must be specified for the corresponding multiplier. If an `INDEPENDENT` option is requested, no additional input or parameters need to be identified. If an existing multiplier model or user defined function is desired, further input is needed. Largely this corresponds to defining the corresponding terms of the previously described literature models. Nonetheless, for completeness the needed inputs for the various multipliers are given here. Below is the syntax for the various pressure multiplier models. `FUNCTION_NAME` corresponds to the name of a Sierra function defined outside the material model.

```
PRESSURE MULTIPLIER = WILKINS
   WILKINS ALPHA = α
   WILKINS PRESSURE = B

PRESSURE MULTIPLIER = USER_DEFINED
   PRESSURE MULTIPLIER FUNCTION = FUNCTION_NAME (w_1^ud (p))
```

For the Lode angle multiplier, only the Wilkins model (which is implicitly, not explicitly, dependent on the Lode angle) may be specified. Only one parameter is needed for that term.

```
LODE ANGLE MULTIPLIER = WILKINS
   WILKINS BETA = β
```

For the triaxiality multiplier, w_3, either the Johnson-Cook expression or a user-defined function may be defined. The corresponding syntax is given below.

```
TRIAXIALITY MULTIPLIER = JOHNSON_COOK
   JOHNSON COOK D1 = D_1
   JOHNSON COOK D2 = D_2
   JOHNSON COOK D3 = D_3

TRIAXIALITY MULTIPLIER = USER_DEFINED
   TRIAXIALITY MULTIPLIER FUNCTION = FUNCTION_NAME (w_3^ud (η))
```

Similar to the triaxiality multiplier, strain-rate dependence may take the form of either the Johnson-Cook model or a user defined function. Note, as with the `JOHNSON_COOK_FAILURE` model, the `REFERENCE RATE` may only be defined once between the hardening and failure definitions.

```
RATE FAIL MULTIPLIER = JOHNSON_COOK
   JOHNSON COOK D4 = D_4
   REFERENCE RATE  = ε̇_0
```

```
RATE FAIL MULTIPLIER = USER_DEFINED
    RATE FAIL MULTIPLIER FUNCTION = FUNCTION_NAME (w_4^ud (ε̇^p))
```

Temperature failure dependence may also be given via a JOHNSON_COOK or user defined dependence. As with the previous JOHNSON_COOK_FAILURE model, the REFERENCE TEMPERATURE and MELTING TEMPERATURE should only be defined once in the hardening and failure definition.

```
 TEMPERATURE FAIL MULTIPLIER = JOHNSON_COOK
    JOHNSON COOK D5         = D_5
    REFERENCE TEMPERATURE = T_ref
    MELTING TEMPERATURE     = T_melt

TEMPERATURE FAIL MULTIPLIER = USER_DEFINED
    TEMPERATURE FAIL MULTIPLIER FUNCTION = FUNCTION_NAME (w_5^ud (T))
```

4 Verification

Like other elements of a constitutive model, the dependence of the failure models on the current stress-state can lead to some complexity in terms of verification. However, by carefully selecting loading paths, specific stress states may be induced and maintained allowing for simplifications and reductions in the various terms. Such efforts produce analytic expressions that can be used for verification of the underlying failure model.

For the verification tests, previous efforts deriving analytic solutions for various boundary value problems are leveraged. Details may be found in the LAMÉ manual [9] and these approaches have been used extensively in the verification of modular plasticity models. As such, details are omitted here and focus is instead on terms related to the failure models. In what follows, the J_2 model (J2_PLASTICITY) is used with Voce type isotropic hardening and power-law breakdown [11] rate dependence. Needed elastic-plastic model parameters are given in Table 1. Other combinations of yield surfaces, hardening laws, and rate-dependence have also been successfully tested. For purposes of brevity and clarity, these results are not presented here.

E	70 GPa	A_{Voce}	200 MPa	g	0.21 s^{-1}
ν	0.33 (-)	n_{Voce}	20 (-)	m_{plb}	16.4 (-)
σ_y^0	200 MPa				

Table 1: Elastic-plastic model parameters used for verification testing. Note, m_{plb} is the exponent for the power-law breakdown rate multiplier. In [9] this parameter is denoted m although here it is renamed to avoid confusion with the tearing parameter exponent.

With respect to the failure models, model parameters are given in Table 2. For the tearing parameter, the exponent of $m = 4$ is recommended by Wellman in his report [4] while

Johnson-Cook [5] and Wilkins [6] parameters are taken from those specific references. For convenience in what follows, $d_{\text{crit}} = 1$ and $\dot{\varepsilon}_0 = 1 \times 10^{-4}$ s^{-1}.

m	4	D_1	0.54 (-)	α	1.8
		D_2	4.89 (-)	β	0.75
		D_3	-3.03 (-)	B	750 MPa
		D_4	0.014 (-)		
		D_5	1.12 (-)		
		T_{ref}	293 K		
		T_{melt}	1356 K		

Table 2: Failure model parameters used for verification testing.

For the modular failure model, the variability in specification of the dependencies leads to a large number of possible model combinations. While a large set of such possibilities have been tested and verified, here for brevity only one combination is presented. That form is given by,

$$d_{\text{mf}} = \int_0^{\bar{\varepsilon}^{\text{p}}} w_1^{\text{ud}} w_2^{\text{w}} w_3^{\text{ud}} w_4^{\text{ud}} w_5^{\text{ud}} d\hat{\bar{\varepsilon}}^{\text{p}}, \tag{22}$$

in which w_1^{ud} is set to match the corresponding Wilkins term and w_3^{ud}, w_4^{ud}, and w_5^{ud} are defined to be equivalent to the corresponding Johnson-Cook expressions. Using the definition in this way, the user-defined capabilities are also tested.

4.1 Uniaxial Stress

In the case of uniaxial stress (under tension), the stress state may simply be given as $\sigma_{ij} = \sigma \delta_{i1} \delta_{j1}$. For the tearing parameter model, the expression simplifies to $d_{\text{tp}} = \bar{\varepsilon}^{\text{p}}$. If the plastic strain rate and temperature are constant (as is the case here), w_4^{jc} and w_5^{jc} are also constant. Additionally, with a uniaxial stress state, $\eta = 1/3$ and w_3^{jc} is fixed through loading. Therefore,

$$d_{\text{jc}} = \frac{\bar{\varepsilon}^{\text{p}}}{\left(D_1 + D_2 \exp\left(D_3/3\right)\right) \left(1 + D_4 \ln \frac{\dot{\bar{\varepsilon}}^{\text{p}}}{\dot{\varepsilon}_0}\right) \left(1 + D_5 T^*\right)}. \tag{23}$$

The impact of the pressure dependence on the Wilkins criteria may be observed in identifying an analytic verification expression. Unfortunately, as p will increase with time in the case of uniaxial stress, arriving at an analytically integrable expression is not possible. As such, for the uniaxial stress case only $B \approx \infty$ to remove the pressure dependence and make $w_1^{\text{w}} = 1$. Furthermore, in such a loading $A = 1$ resulting in $w_2^{\text{w}} = 1$ leading to $d_{\text{w}} = d_{\text{tp}} = \bar{\varepsilon}^{\text{p}}$. For the modular failure model, as $w_1^{\text{w}} = w_2^{\text{w}} = 1$, $d_{\text{mf}} = d_{\text{jc}}$. Results for the different cases are presented in Fig. 1 for the case of $T = T_{\text{ref}}$ and $\dot{\bar{\varepsilon}}^{\text{p}} = \dot{\varepsilon}_0$ for both numerical (denoted by the

(N)) and analytical cases ((A)). Note, given that some of the analytic expressions reduce to other forms, the only analytic results presented are those of the Johnson-Cook and tearing parameter models.

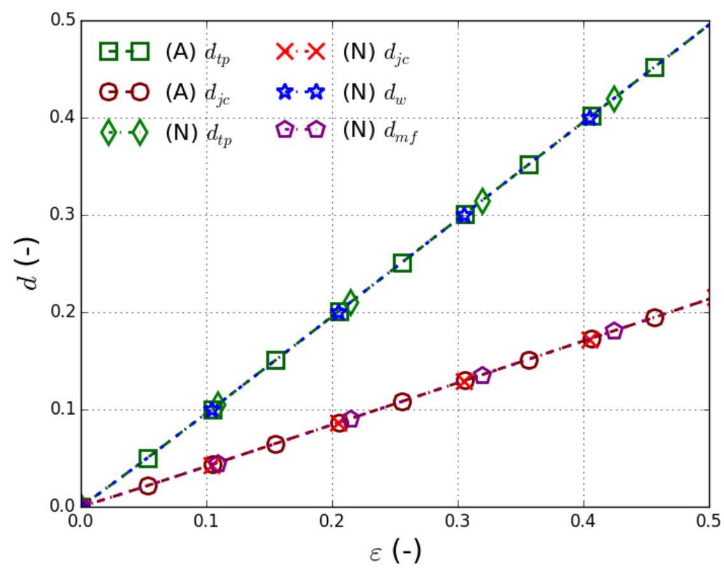


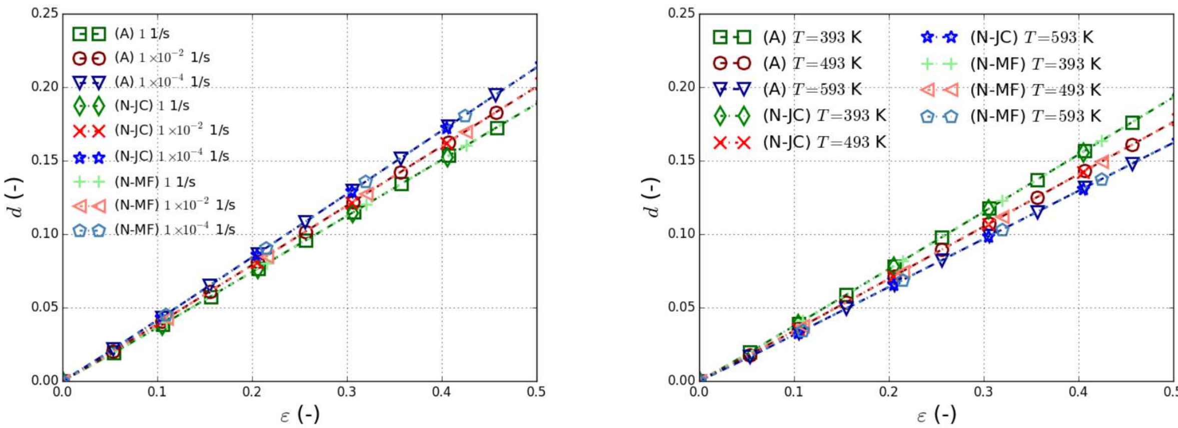
Figure 1: Uniaxial verification test results with $T = T_{\mathrm{ref}}$ and $\dot{\bar{\varepsilon}}^{\mathrm{p}} = \dot{\varepsilon}_0$. Analytically determined expressions are denoted by (A) while numerical results from finite element computations are indicted by an (N).

Clear agreement is observed between the numerical, finite element results and the expected analytical expressions. As further verification, Fig. 2 presents results under different loading conditions for the Johnson-Cook (JC) and modular failure (MF) models. In both cases the analytical results correspond to the expression in Eqn. 23. Specifically, Fig. 2a gives the rate-dependence of the failure model in which the temperature is fixed at $T = T_{\mathrm{ref}}$ while Fig. 2b presents the influence of temperature with the rate fixed such that $\dot{\bar{\varepsilon}}^{\mathrm{p}} = \dot{\varepsilon}_0$.

Importantly, Fig. 2 points to agreement between the analytical and numerical results giving further verification. These results also highlight that increasing the rate and/or temperature will lower damage accumulation thereby delaying the onset of failure. Furthermore, this case demonstrates that the modular failure model may be specified to reproduce the Johnson-Cook model.

4.2 Pure Shear

A second verification case is that of pure shear in which $\sigma_{ij} = \tau\,(\delta_{i1}\delta_{j2} + \delta_{i2}\delta_{j1})$. Under such a loading more variability is observed in terms of analytic damage expressions. To that end, with pure shear $p = 0$ and,



(a) $T = T_{\mathrm{ref}}$ (b) $\dot{\bar{\varepsilon}}^{\mathrm{p}} = \dot{\varepsilon}_0$

Figure 2: Uniaxial damage evolution determined analytically (denoted (A)) and numerically ((N)) under different loading conditions: (a) rate-dependence with applied equivalent plastic strain rates of $\dot{\bar{\varepsilon}}^{\mathrm{p}} = 1 \times 10^{-4},\ 1 \times 10^{-2}$, and $1\mathrm{s}^{-1}$ and (b) temperature dependence with $T = 393,\ 493$, and 593K.

$$d_{\mathrm{tp}} = \left(\frac{2}{3}\right)^m \bar{\varepsilon}^{\mathrm{p}}, \tag{24}$$

$$d_{\mathrm{jc}} = \frac{\bar{\varepsilon}^{\mathrm{p}}}{(D_1 + D_2)\left(1 + D_4 \ln \frac{\dot{\bar{\varepsilon}}^{\mathrm{p}}}{\dot{\varepsilon}_0}\right)(1 + D_5 T^*)}, \tag{25}$$

$$d_{\mathrm{w}} = 2^\beta \bar{\varepsilon}^{\mathrm{p}}, \tag{26}$$

$$d_{\mathrm{mf}} = \frac{2^\beta \bar{\varepsilon}^{\mathrm{p}}}{(D_1 + D_2)\left(1 + D_4 \ln \frac{\dot{\bar{\varepsilon}}^{\mathrm{p}}}{\dot{\varepsilon}_0}\right)(1 + D_5 T^*)}. \tag{27}$$

Results are presented in Fig. 3 for the above analytic expressions (denoted (A) in Fig. 3) and determined through corresponding numerical, finite element simulations ((N)). Excellent agreement is noted for all four models.

To consider the impact of different loading conditions, Fig. 4 presents damage evolution under pure shear loading at different temperatures and equivalent plastic strain rates. Results are given for both the Johnson-Cook (JC) and modular failure (MF) models determined both analytically ((A)) and numerically ((N)) through finite element calculation. As with previous results, the analytical and numerical results are in agreement. Similarly, increasing temperature and/or plastic strain rate decreases damage accumulation through loading.

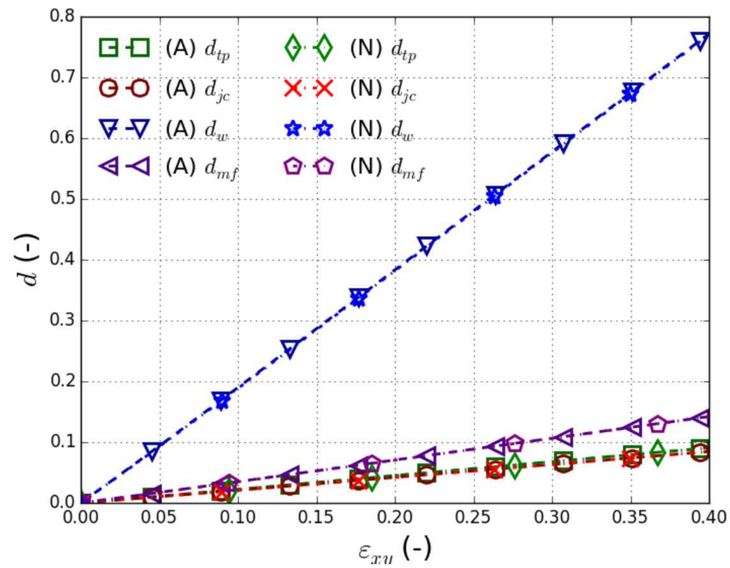
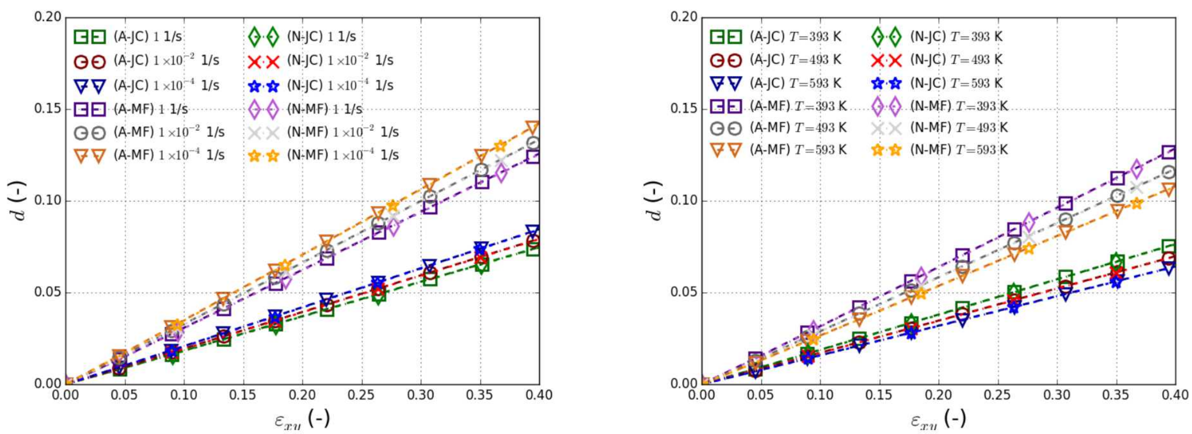


Figure 3: Pure shear verification test results with $T = T_{\text{ref}}$ and $\dot{\bar{\varepsilon}}^{\text{p}} = \dot{\varepsilon}_0$. Analytically determined expressions are denoted by (A) while numerical results from finite element computations are indicated by an (N).



(a) $T = T_{\text{ref}}$ (b) $\dot{\bar{\varepsilon}}^{\text{p}} = \dot{\varepsilon}_0$

Figure 4: Pure shear damage evolution determined analytically (denoted (A)) and numerically ((N)) under different loading conditions: (a) rate-dependence with applied equivalent plastic strain rates of $\dot{\bar{\varepsilon}}^{\text{p}} = 1 \times 10^{-4},\ 1 \times 10^{-2}$, and 1s^{-1} and (b) temperature dependence with $T = 393,\ 493$, and 593K. "JC" refers to results pertaining to the Johnson-Cook model while "MF" signifies modular-failure.

4.3 Pure Shear with Superposed Pressure

While the uniaxial and pure shear test cases help verify a variety of the different terms, the pressure contribution, w_1, of the Wilkins model has only been tested in the most trivial of cases. As a final test to interrogate this particular feature, the pure shear case is revisited but with a constant, hydrostatic pressure superposed. In this fashion, the pressure is constant "activating" the w_1 term while still allowing for the determination of an analytic expression. For this test, only the Wilkins and modular failure models are considered as the changing stress states leads to expression for the other damage variables that are not easily integrated. Additionally, for the modular failure model, $w_3 = 1$ is used as the triaxiality changes over the course of such loadings. Similarly, $T = T_{\text{ref}}$ and $\dot{\bar{\varepsilon}}^{\text{p}} = \dot{\varepsilon}_0$ so that $w_4 = w_5 = 1$ and $d_{\text{mf}} = d_{\text{w}}$. In such instances,

$$d_{\text{w}} = \left(\frac{1}{1-\frac{p}{B}}\right)^{\alpha} 2^{\beta}\bar{\varepsilon}^{\text{p}}. \tag{28}$$

Figure 5 presents results determined analytically (denoted with an (A)) and numerically through finite element calculations ((N)) at three different superposed pressures. Agreement is again noted between the analytical and the two numerical sets of results. Additionally, these results also demonstrate that the modular failure successfully reduces to the Wilkins criteria and that increasing the hydrostatic pressure leads to higher rates of damage accumulation.

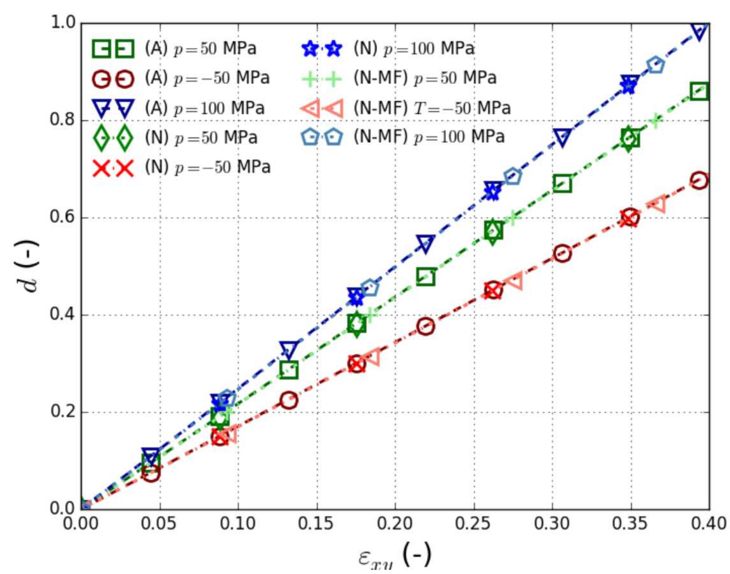


Figure 5: Verification test results with a hydrostatic pressure superposed on a pure shear test with $T = T_{\text{ref}}$ and $\dot{\bar{\varepsilon}}^{\text{p}} = \dot{\varepsilon}_0$. Analytically determined expressions are denoted by (A) while numerical results from finite element computations are indicated by an (N). Modular failure results are denoted with an "MF" while the rest of the cases correspond to the Wilkins model.

5 Summary and Conclusion

In this work, a framework for flexibly evaluating different failure models has been discussed and implemented in a variety of existing plasticity formulations. Such capabilities give greater flexibility in selecting a criteria to meet the needs of a specific material and/or loading condition. Expanding on this concept, a "modular failure" model combining different dependencies on stress-state and loading conditions was also implemented. Enabling the specification of different dependencies and forms (including user-defined functions) further broadens the pool of possible models giving further flexibility in selecting an appropriate criteria. Examples of usage and syntax were given along with simple verification problems to demonstrate and give confidence in current capabilities. While only a sampling of such exercises were presented, these same tests were also successfully performed with a suite of different plasticity and hardening functions with the results omitted for brevity. These problems, however, are quite simple and additional examples of responses on different structural problems are needed to fully compare and contrast the impact of the different models. Additionally, while the current models enable use with element deletion type approaches, additional extensions to include damage progression need to be incorporated to provide further capabilities. Finally, while substantial flexibility in terms of specification of models may result from the modular failure models, experimental studies on the calibration and characterization of these different dependencies are needed to fully utilize this formulation.

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