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Interpolation of V-Gamma Map Failure Data for Use with LAPS

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This document outlines the approach used for interpolation of a dataset provided by the Jet Propulsion Laboratory (JPL) which contains failure predictions of the Mars 2020 entry vehicle (EV). JPL used Navier-Stokes simulations to predict the failure altitude of the EV, along with a corresponding velocity and flight path angle at failure. These simulations were executed for several initial trajectory points in the entry envelope (the V-Gamma map).

The Loop Analysis Program Software (LAPS) is a driver code which provides solutions for trajectory, aeroheating, and material thermal response of a spacecraft over an entire entry envelope. LAPS has the capability to run complete trajectories of a vehicle which undergoes sequential breakup of its components. It is desired that LAPS complete trajectories by beginning with the EV failure data provided by JPL. In order to do this, the JPL data must be interpolated onto the approximately 1500 trajectory points used by LAPS to designate the complete V-Gamma map.

JPL provides trajectory points which contain the initial velocity and flight path angle, along with the predicted EV failure altitude, velocity, and flight path angle for each trajectory point. That is to say that failure data is provided for several points on the V-Gamma map, and this failure data must be interpolated onto the complete V-Gamma map. The interpolation strategy must account for several factors:

- The JPL data is scattered. Simple bilinear interpolation would not be appropriate.
- If the data were to be triangulated, it would extend beyond the boundaries of the LAPS V-Gamma map in some regions. It would fail to contain the entire V-Gamma map within its boundaries, i.e., the data must also be extrapolated in some regions.

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- The interpolation dimensions have different units, and are of disparate magnitude, which is important to account for if an interpolation scheme were to use a norm-based weighting of points.

To meet these requirements, the interpolation was completed using a radial basis function (RBF) method, which when applied correctly can meet all of the above criteria. For a set of n distinct points for which the solution $f(\vec{x})$ is known, an interpolant can be given by

$$s(\vec{x}) = \sum_{j=1}^{n} \lambda_j \phi(\|\vec{x} - \vec{x_j}\|), \tag{1}$$

where λ_j are a set of coefficients which must be computed ϕ is a radial basis function, and $\|\cdot\|$ is the Euclidean norm. The coefficients λ_j can be found by solving the linear system

$$A_{ij}\lambda_j = f_i, \tag{2}$$

where the elements of A_{ij} are given by $\phi(\|\vec{x_i} - \vec{x_j}\|)$. The radial basis function used for this work is a thin plate spline, defined as

$$\phi(r) = r^2 \log r, \tag{3}$$

with the special case of $\phi(0) = 0$. The thin plate spline has the advantages of producing a smooth interpolation which runs through all the data points and does not require any input parameters.

The above method was used to interpolate all three failure quantities onto the entire V-Gamma map. Since the interpolation dimensions do not share the same units, the Euclidean norm becomes unsuitable. Therefore, the V-Gamma map is nondimensionalized such that both axes span the range from 0 to 1, and the interpolation is then based on fractional changes in flight path angle and velocity. In this context the Euclidean norm is appropriate, and after interpolation the data is transformed back into dimensional units.

This interpolation has no protection against producing aphysical quantities, e.g. negative altitudes. In order to produce a more physical result, additional control points were added to the JPL dataset around some of the boundaries to force the spline to asymptote to physical values. Specifically, control points were added on the lower boundary to prevent negative altitudes, and on the right boundary to prevent flight path angles and velocities higher than initial values. Any remaining aphysical values in the interpolated map were truncated to their extrema: negative altitudes were set to zero, flight path angles higher than initial were set to initial, and velocities higher than initial were set to initial.

It should be noted that since the JPL data is sparse, the interpolation could easily be missing physical features (i.e., peaks, valleys, and gradients) between the data, and aside from one's intuition, the only way to mitigate this problem is to increase the size of the dataset to be interpolated.