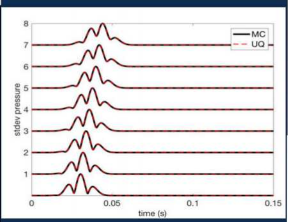
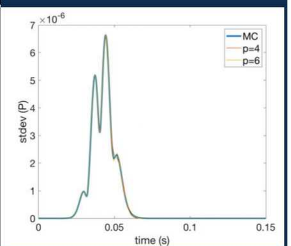
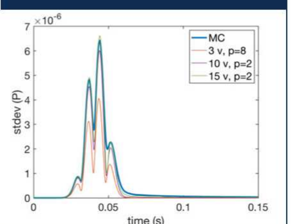
SAND2020-0731D

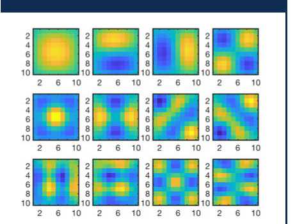
Uncertainty Quantification of Geophysical Inversion Using Stochastic Partial Differential Equations

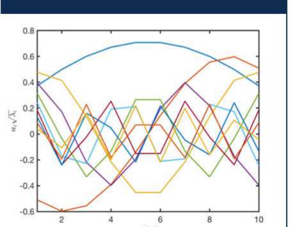
PI: Leiph Preston, Team Members: Christian Poppeliers and Eric Phipps, PM: Kyle Jones











Problem

- Parameters inferred from inversion of geoscience data rarely have uncertainties attached, or simplifying assumptions, such as Gaussian uncertainties, are made.
- This is generally due to the computational complexity of many geomodels.

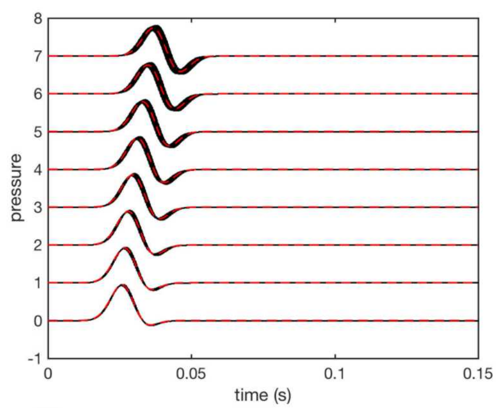
Objective

- Use stochastic methods to more efficiently propagate uncertainty through geo-inversion models.

Research Plan

- **Year 1**: Apply the method to simple linear problem, i.e., full waveform seismic moment tensor inversion and compare results with baseline methods.
- **Year 2**: Using what we learned in Year 1, apply the method to the full waveform structural inversion problem (nonlinear).

Uncertainty quantification of pressure waves through uncertain acoustic model



Results

- **Linear moment tensor inversion**: $d = GS$, where d is observed data, G is the matrix of green's functions, and S is the source term, all in the frequency domain. Model uncertainty is computationally expensive.
- **Classical inversion** (assumes Gaussian noise): $S = (G^T C^{-1} G)^{-1} G^T C^{-1} d$, where C is the data and/or model covariance matrix.
- **Bayesian inversion**: $P(S|d) \propto P(d|S)P(S)$, where the likelihood, $P(d|S)$ is a function of $d - GS$
- **Pure Chaos Polynomials with only data uncertainty**: Chaos polynomials compactly describe probability distributions. Decompose d and S into chaos polynomials, e.g., $d = \sum d_i \psi_i(\theta)$, where $\psi_i(\theta)$ is a chaos polynomial basis function of the random variable θ and d_i is the coefficient of that basis. One can solve for the coefficients, which solves the complete stochastic problem, obtaining: $d_i = GS_i$, i.e., each coefficient is independent of the others.
- **Pure Chaos Polynomials with model uncertainty**: Decompose d, G, and S into chaos polynomials. One can then solve the stochastic problem via: $d_k = \sum_i \sum_j \epsilon_{ijk} G_i S_j$ where $\epsilon_{ijk} = \langle \psi_i \psi_j \psi_k \rangle$, i.e, a large linear system must be solved for each coefficient.
- **Improved Chaos Polynomial expansion**: Build computationally efficient surrogate model (e.g., collocation, regression) and only solve for subset of coefficients.

Exceptional service in the national interest

Sandia National Laboratories

Built on LDRD Laboratory Directed Research and Development

U.S. DEPARTMENT OF ENERGY

NNSA