

## Sandia National Laboratories Final Report to ATA

for

Contract: Navy STTR N18B-T029  
Optimization of Fatigue Test Signal Compression Using the Wavelet  
Transform

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## 1 Introduction

This report summarizes Sandia National Laboratories (SNL) contribution to ATA Engineering, Inc's (ATA) project for the Naval Air Systems Command (NAVAIR), entitled "Optimization of Fatigue Test Signal Compression Using the Wavelet Transform." Sandia National Laboratories were a subcontractor to ATA. We were involved because this was a Small Business Technology Transfer (STTR) project that required ATA to partner with a national laboratory or academic institution. ATA selected SNL, and specifically the Environments Engineering Department (1557) because ATA has a long-standing working relationship with this department and the department staff have experience in environment definition, signal processing, fatigue testing protocols and traditional methods of generating inputs for accelerated fatigue testing. Reference [1] contains ATA's summary of the overall project. Reference [2] contains a summary of ATA's main results.

Mr. Tyler Van Fossen was the ATA Principal Investigator. Dr. Vit Babuska was the SNL principal investigator. Mr. Jerome Cap served as technical advisor on environment definition and signal processing methods. Mr. Daniel Clarke was the SNL Contract Representative.

ATA's period of performance on the STTR Prime Contract was from August 2018 to March 2019. Sandia's period of performance was January 2019 through July 2019. Contracting and administrative complexities prevented an earlier start date. As a result, our contribution to ATA's Phase 1 final deliverable was not significant.

Sandia National Laboratories' effort on this contract was based on a Statement of Work (SOW), which was formally defined in Appendix A of Ref. [3]. Section 2 describes the results of SNL's work on this project. Section 3 concludes the report.

## 2 Tasks and Results

The SOW negotiated between SNL and ATA consisted of the follow tasks:

- a. Using unclassified, unlimited release (UUR) non-stationary mechanical environments, generate representative time histories for the Sponsor (ATA) to use in the development, evaluation, and verification of the wavelet transform fatigue equivalent time compression algorithms.
- b. Assist the Sponsor in the development of the wavelet transform fatigue equivalent time compression algorithms to realize maximum signal compression while maintaining damage equivalence through modeling and simulation and independent verification and validation of candidate approaches.
- c. Use Sandia's unique expertise in single input, multi output (SIMO) and multi-input, multi-output (MIMO) long duration random vibration environments definition to ensure that the Sponsor's algorithms can be extended to SIMO and MIMO problems.

### 2.1 Representative Non-Stationary Random Vibration Time Histories

SOW Task: Using unclassified, unlimited release (UUR) non-stationary mechanical environments, generate representative time histories for the Sponsor to use in the development, evaluation, and verification of the wavelet transform fatigue equivalent time compression algorithms.

SNL's primary contribution was the development of non-stationary data sets that the sponsor could use to test out the wavelet transform time compression algorithms. To that end, SNL developed MATLAB codes that generate non-stationary random vibration signals. Table 1 summarizes the MATLAB scripts and functions.

*Table 1 MATLAB Scripts and Functions for Creating Non-stationary Random Vibration Datasets*

Script/Function Name	Description
NS_RandomVibe_Driver.m	Driver script that demonstrates the use, input and outputs for Make_RandomVibe_Sets.m and Concat_RandomVibe_Sets.m
Make_RandomVibe_Sets.m	Function to make a set of random vibration time histories with ASDs based on a $2^{N-1}$ design matrix
Concat_RandomVibe_Sets.m	Function to concatenate random vibration time histories into a non-stationary random vibration data set.

Function Make\_RandomVibe\_Sets generates  $2^{N-1}$  random vibration time histories, where  $N$  is the number of frequency bands in an acceleration power spectral density (ASD). The number of frequency bands can be between 1 and 5. Each time history is a stationary data segment. Each time history could represent a different flight regime for an aircraft and the concatenated data set could represent all or part of an aircraft flight.

## UNCLASSIFIED/UNLIMITED RELEASE

Each frequency band is parameterized by a minimum ASD level, a maximum ASD level, and lower and upper frequency bounds. The  $2^{N-1}$  permutations are generated by the ASD level in the frequency band – whether the band's ASD level is high or low. Table 2 - Table 5 show the permutations for 2 – 5 frequency bands. A 0 entry means the ASD level is low in that frequency band; a 1 means that the level is high in the band. The output of Make\_RandomVibe\_Sets is  $2^{N-1}$  random vibration time histories and ASDs. Figure 1 illustrates the ASDs for the 3 band case.

Table 2 2 Frequency Band Matrix

Case	Band 1	Band 2
1	0	1
2	1	0

Table 3 3 Frequency Band Matrix

Case	Band 1	Band 2	Band 3
1	0	0	1
2	0	1	0
3	1	0	1
4	1	1	1

Table 4 4 Frequency Band Matrix

Case	Band 1	Band 2	Band 3	Band 4
1	0	0	0	1
2	0	0	1	0
3	0	1	0	0
4	0	1	1	1
5	1	0	0	0
6	1	0	1	1
7	1	1	0	1
8	1	1	1	0

Table 5 5 Frequency Band Matrix

Case	Band 1	Band 2	Band 3	Band 4	Band 5
1	0	0	0	0	1
2	0	0	0	1	0
3	0	0	1	0	0
4	0	0	1	1	1
5	0	1	0	0	0
6	0	1	0	1	1
7	0	1	1	0	1
8	0	1	1	1	0
9	1	0	0	0	0

10	1	0	0	1	1
11	1	0	1	0	1
12	1	0	1	1	0
13	1	1	0	0	1
14	1	1	0	1	0
15	1	1	1	0	0
16	1	1	1	1	1

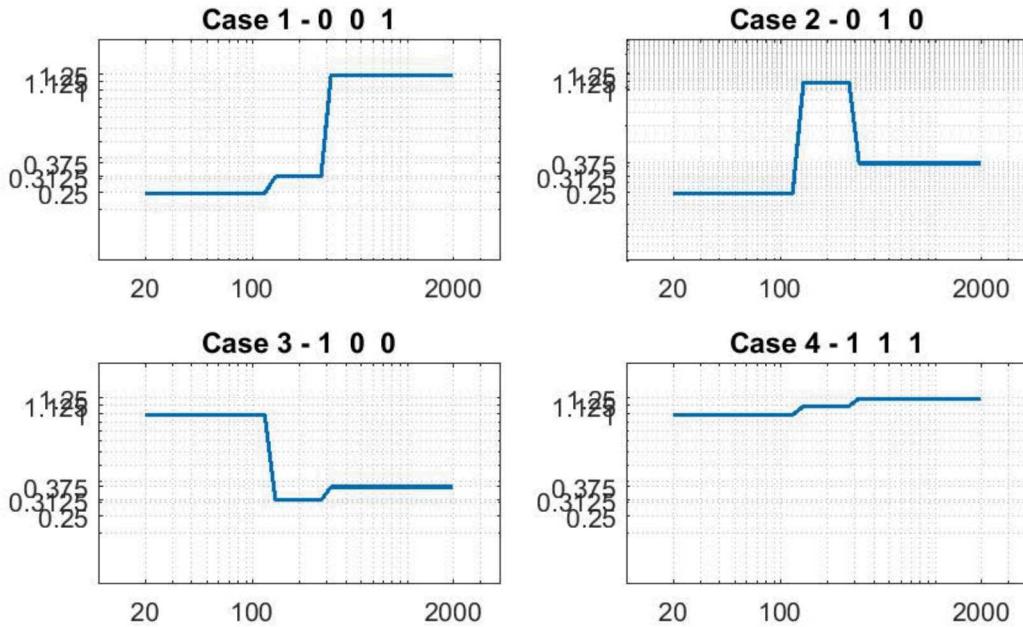


Figure 1 ASDs from the 3 Frequency Band Matrix

Currently, the duration of each segment is fixed based on the lowest frequency of the ASD. The criterion is that the segment contains at least 100 cycles at the lowest ASD frequency. For example, if the lowest frequency is 20 Hz, the minimum record length is 5 sec. The sample rate is also a function of the ASD. It is set at 10X the highest frequency in the ASD. For example, if the highest frequency is 2000 Hz, the sample rate is 20000 Hz. These parameters are hard coded into the function but could be made user selectable parameters in future versions.

A non-stationary data set is generated with `Concat_RandomVibe_Sets`. Up to  $2^{N-1}$  segments generated with function `Make_RandomVibe_Sets` can be concatenated together to form non-stationary data that can be fed into a time compression algorithm. Figure 2 and Figure 3 show two data sets created from the 3-band design matrix. In Figure 2, cases 1 and 3 were concatenated and in Figure 3, all four cases, ordered 2,1,3,4 were used.

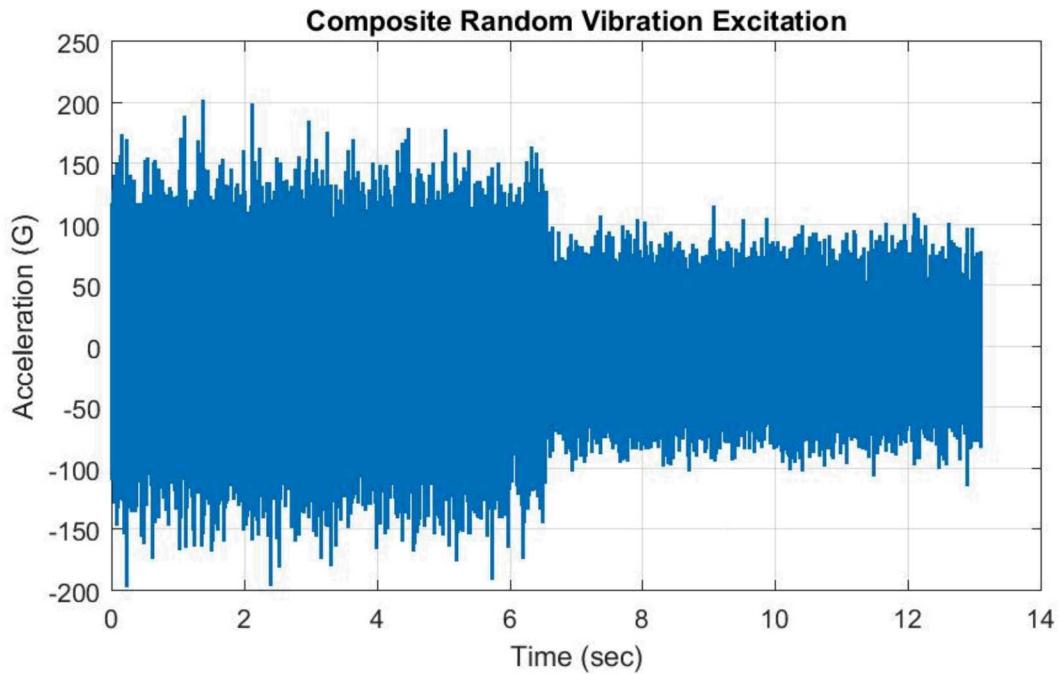


Figure 2 Nonstationary Random Vibration Signal made from Cases 1 and 3 of the 3 Band Design Matrix

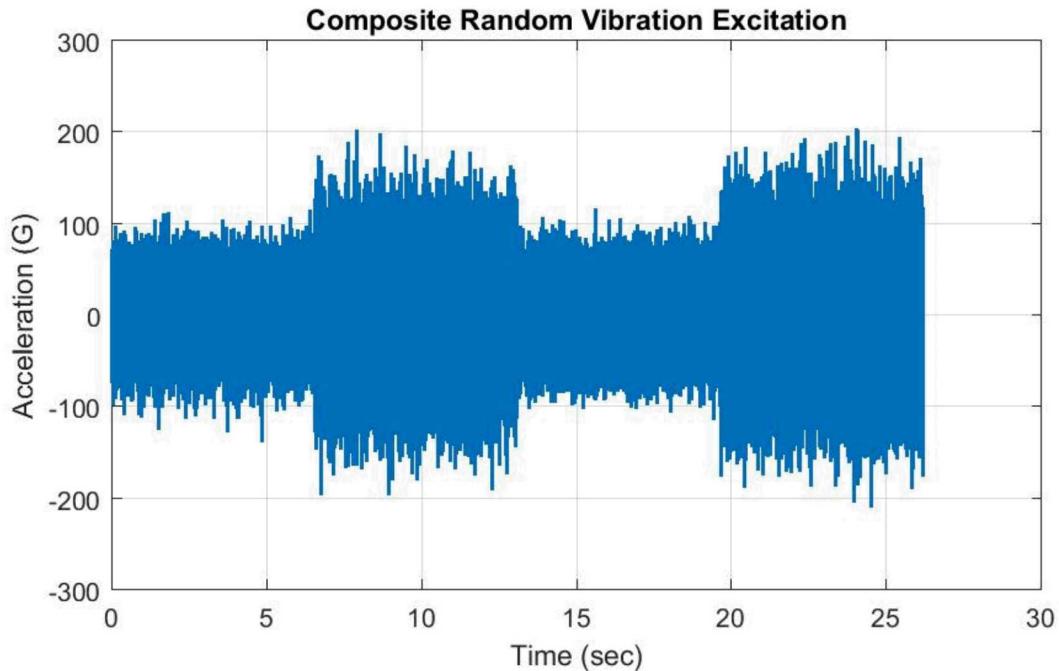


Figure 3 Nonstationary Random Vibration Signal made from Cases 2,1,3,4 of the 3 Band Design Matrix

The time histories can be overlapped. A  $\frac{1}{2}$  haversine mollifier can be applied to the overlap portion of the signals as illustrated in Figure 4. The size of the overlapped segment is an input to the function. Figure 5 shows a 0.82 sec trailing edge mollifier.

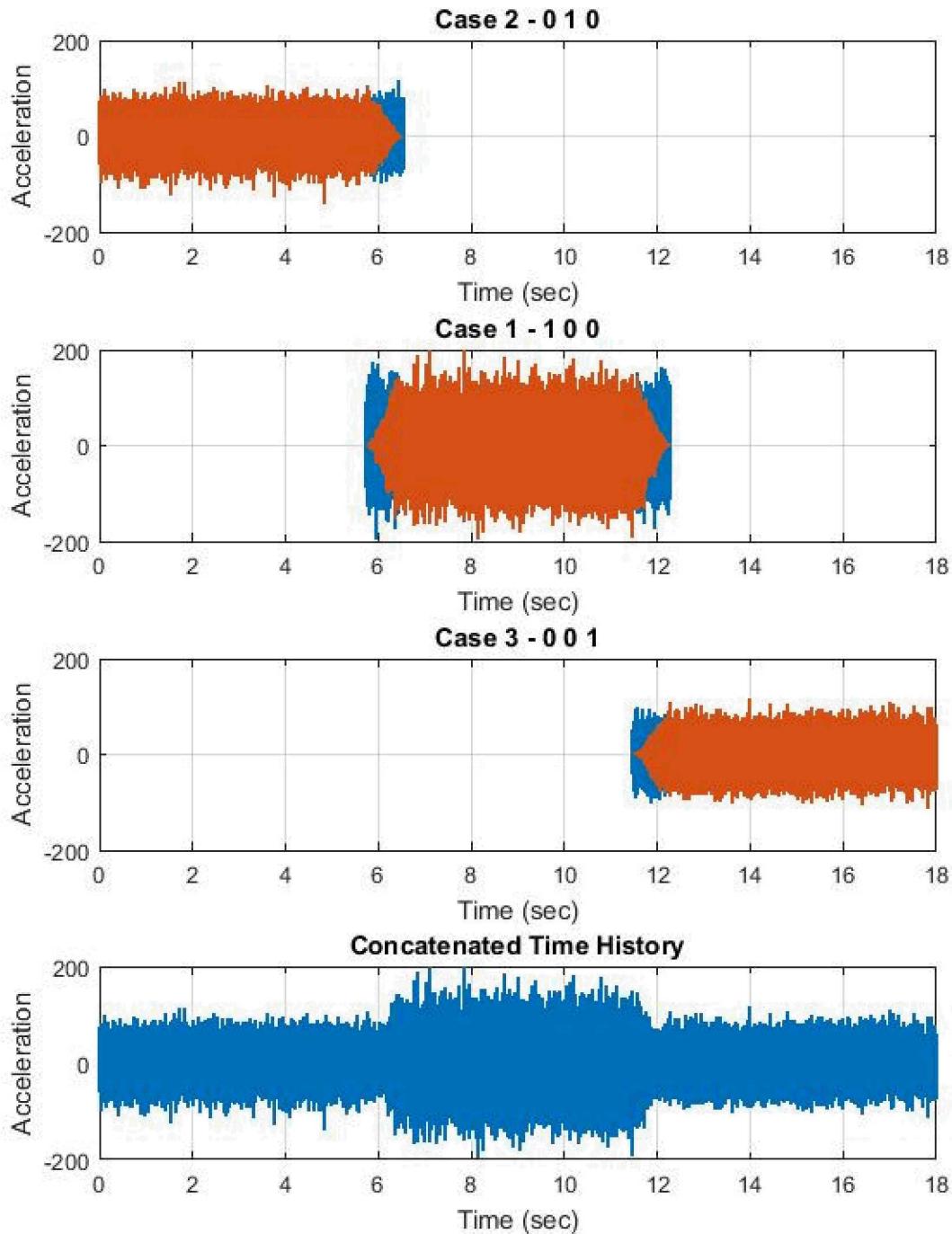


Figure 4 Three Concatenated Overlapping Mollified Random Vibration Segments

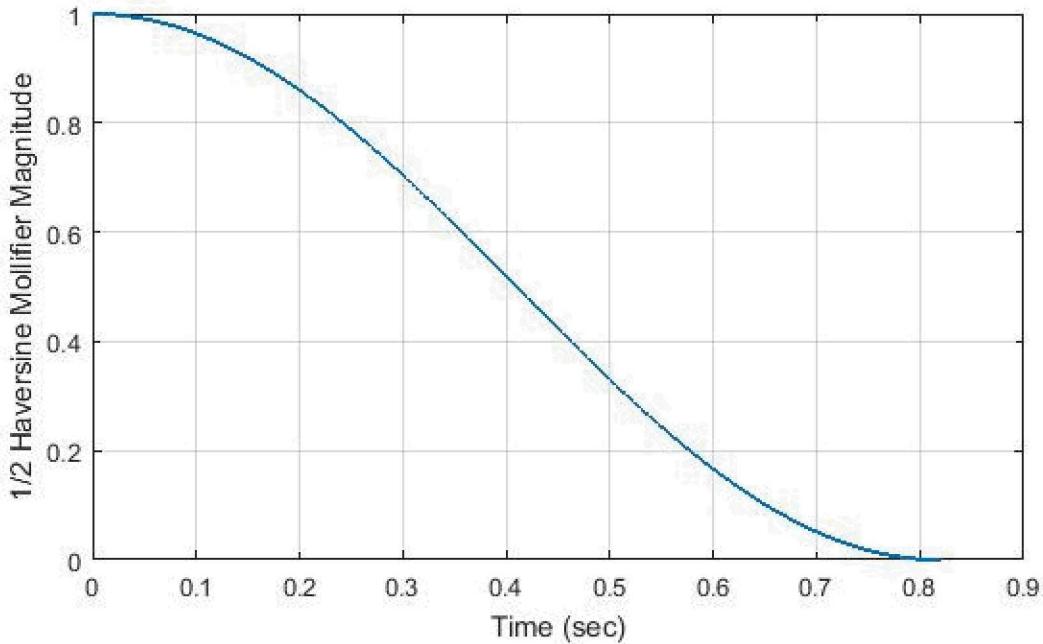


Figure 5 0.82 sec 1/2 Haversine Mollifier

## 2.2 Wavelet Transform Fatigue Equivalent Time Compression Algorithms

SOW Task: Assist the Sponsor in the development of the wavelet transform fatigue equivalent time compression algorithms to realize maximum signal compression while maintaining damage equivalence through modeling and simulation and independent verification and validation of candidate approaches.

Collaboration with ATA on the development of the wavelet transform compression algorithms was limited. SNL did not have much opportunity to interact with the ATA PI. Sandia's main contributions under this task were in the independent verification and validation of candidate approaches.

### 2.2.1 Wavelet Transform Fatigue Equivalent Time Compression Algorithms

The SNL PI reviewed ATA's algorithm and the wavelet functions on which it is based. Wavelet transforms are not naturally applicable to time compression. They are primarily used for lossy compression of signals and feature identification. Lossy compression means creating a lower dimensional representation (i.e., a reduced order) that retains the dominant features of the signal. Lossy compression does not change the duration or extent of the signal. In the feature extraction identification problem, the time-frequency nature of the wavelet is used to locate specific elements, such as transients and sharp transitions in a signal, akin to running a matched-filter across the data.

One difficulty with any wavelet-based fatigue equivalent time compression algorithm is that during the compression in wavelet space, the phase component is lost. The phase must be

estimated/reconstructed to perform the inverse transform back into temporal space. This is not a simple problem and the quality of the compressed time domain signal depends on the estimated phase.

The wavelet phase reconstruction problem is similar the phase reconstruction problem in Fourier space. If the magnitude of the Fourier transform of a signal is all one has, then the original data cannot be reconstructed with the inverse Fourier transform. There are several Fourier phase techniques that can be used for general data, and the wavelet phase reconstruction methods are similar. The SNL PI is familiar with alternate projection algorithms such as Gerchberg-Saxon and Fienup for Fourier phase reconstruction. These algorithms are relatively simple to implement but they are not universally robust and their applicability to time varying signals and wavelets is unknown to the SNL PI. ATA surveyed wavelet phase reconstruction algorithms and selected an approach. The mathematics are rather complex, at least for the SNL PI, and SNL did not experiment with different algorithms to evaluate them.

It seems that different wavelets will perform differently for the time compression problem. We recommend that ATA try to exploit specific characteristics of the data to potentially simplify the phase reconstruction problem. First, the time domain data are causal. This suggests that the wavelet transform should be analytic. For Fourier transforms of real, causal sequences, the Fourier magnitude and phase are related by the Hilbert transform (with some constraints). That means that the Fourier phase can be estimated from the Fourier magnitude. This same relationship applies to the wavelet transform if the wavelets are analytic -i.e., generated with an analytic wavelet transform. This is an example of characteristics of the time domain signal and wavelet transform that could be exploited to make the phase reconstruction problem easier.

SNL tried the Hilbert transform approach on ATA's "I'm sorry" data set by modifying ATA's scripts, but the results were unsatisfactory. Different wavelets than the ones in ATA's scripts maybe required.

### 2.2.2 Independent Verification and Validation

Time compression techniques have been used for decades, e.g., Ref [4] was published in 1972. Sandia National Laboratories has analysis tools for compressing random vibration data. These tools have been developed to enable laboratory tests to represent field environments. A structure may experience multiple, long duration, random vibration environments in the field. It is not practical or possible to reproduce the environments in the laboratory to qualify a structure to withstand the field environments, so time compression of field environments, also called accelerated testing, is needed.

Sandia National Laboratories uses the damage-based equivalence approach, which is consistent with the ATA wavelet-based time compression project. Therefore, Sandia's methods could serve as an independent reference against which to compare the efficacy of the wavelet-based algorithms.

When a non-stationary random vibration environment is comprised of an ensemble of vibration environments, time compression can be realized into two ways:

- 1) Compress each segment in the ensemble separately and concatenate the compressed segments;
- 2) Create a ASD envelope that produces the same fatigue damage as the ensemble of ASDs and then scale the amplitude to achieve additional duration reductions.

The approaches are often combined, particularly if the ensemble is comprised of a mixture of low amplitude long duration segments and high amplitude short duration segments. The Sandia developed Graflab function `genv_fatigue` [5] performs the time compression. The function computes the fatigue damage associated with an ensemble of spectra. The function can be used to determine a test time and final spectra associated with the accumulated damage for the ensemble. To do this, the function scales each member of the ensemble with a simple fatigue damage model (Miner's rule) and envelopes the result. The function creates the composite spectrum with the lowest overall magnitude as a function of frequency. Therefore, the algorithm compresses the lower magnitude spectra until they are equal in magnitude to the highest spectra in the ensemble. If this produces a shorter effective duration than the user requested then no further compression is required, and the effective compressed time is reported as a function of frequency. If the effective duration is shorter than the user defined duration, then the analyst could consider reducing the envelop using the same fatigue damage model.

### 2.2.2.1 Time Compression Example

A random vibration signal comprised of five segments was created with the approach presented in Section 2.1. Figure 6 shows the composite acceleration time history, and Figure 7 shows the ASD of each segment. The signals were compressed by a factor of 2 in time using the function `genv_fatigue`. The function generated a composite signal whose ASD is a minimum amplitude ASD envelope that produces at least as much fatigue damage as the underlying spectra. Figure 8 shows the time compressed signal and it's ASD.

One of the outputs of `genv_fatigue` is the per frequency band final scaled duration. Any frequency band for which the scaled final time is less than the specified compressed duration will be overtested. In this example, the compressed duration for the second frequency band is 6.9 sec, which is less than the 14.75 sec duration selected.

Both the original signal and time-compressed signal were run through a rainflow cycle counting function. Miner's Damage indices were computed for each time history assuming a simple power law SN curve.

Figure 9 shows the histogram of cycle peaks of the original signal (Figure 6) and Figure 10 shows the histogram of the cycle peaks of the compressed signal (Figure 8). Note that there are compression only cycles in each data set. There are 5153 compression only cycles in the original data set and 2497 compression only cycles in the compressed data set. These are not counted when computing Miner' Damage index.

The Miner's Damage index ratio of compressed to uncompressed signals is 1.29. Since the ratio is greater than one, the compressed signal produces more fatigue damage than the original signal. This means that the ASDs were scaled up more than necessary for the specified duration, or that the signal could have been compressed to 11.43 sec (i.e.,  $14.75/1.29$ ) rather than 14.75 sec. The difference is because of genv-fatigue treats each frequency line in the ASD independently. For this example, the Miner's Damage index ratio is independent of compressed time selected in genv\_fatigue.

These results show the sensitivity of the results to the time compression algorithm as well as the metric(s) by which the two signals are compared, and the assumptions underlying them (e.g., choice of SN curve parameters).

The functions created for this example are listed in Table 6

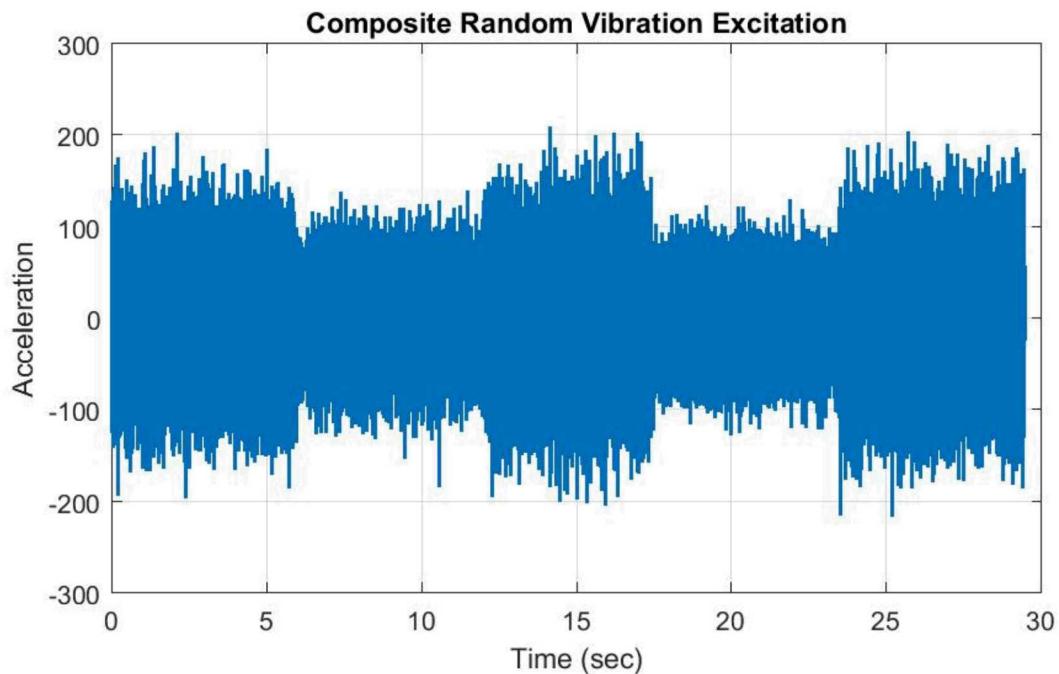


Figure 6 Five Segment Non-stationary Random Vibration Signal

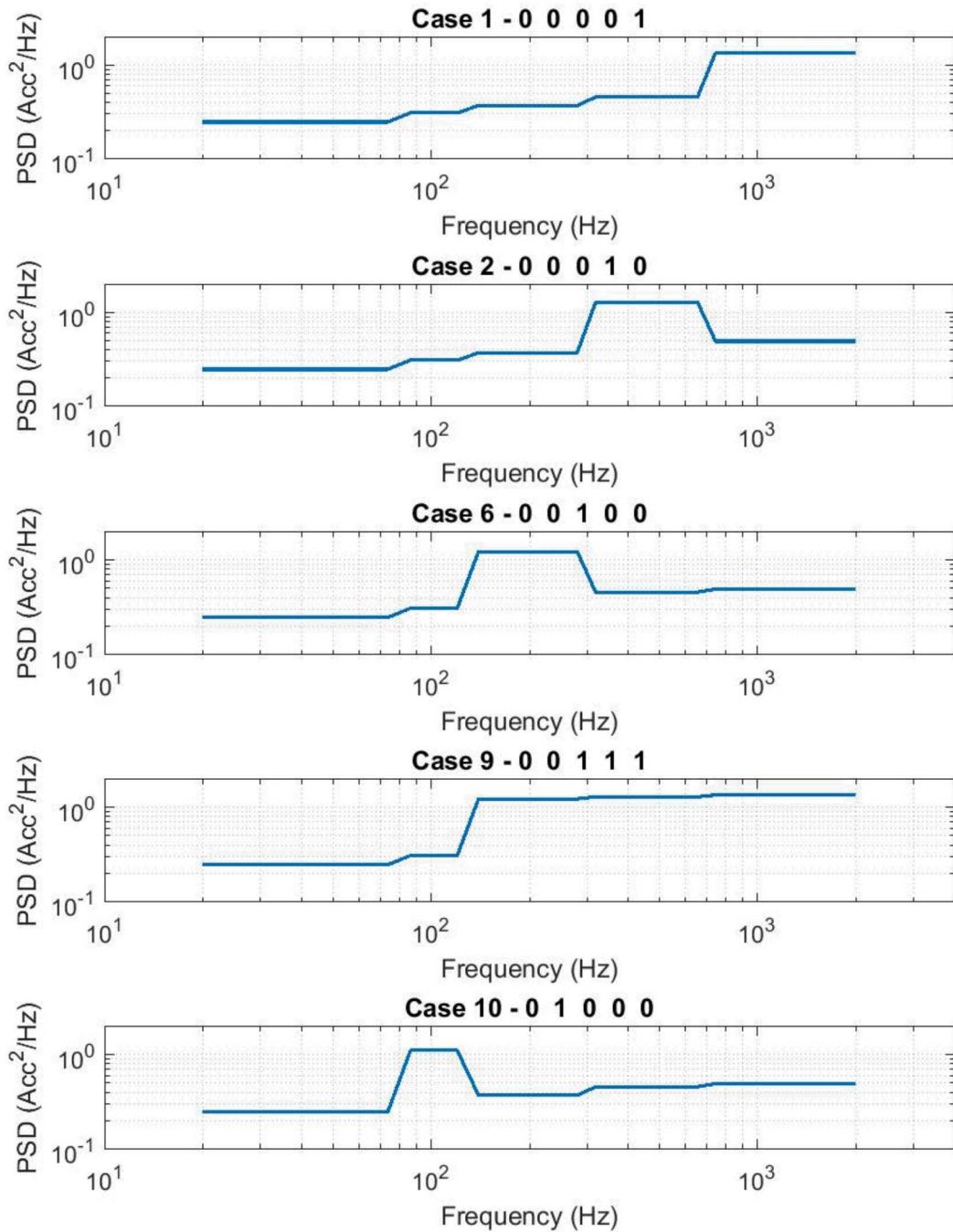


Figure 7 ASDs of the Five Concatenated Segments

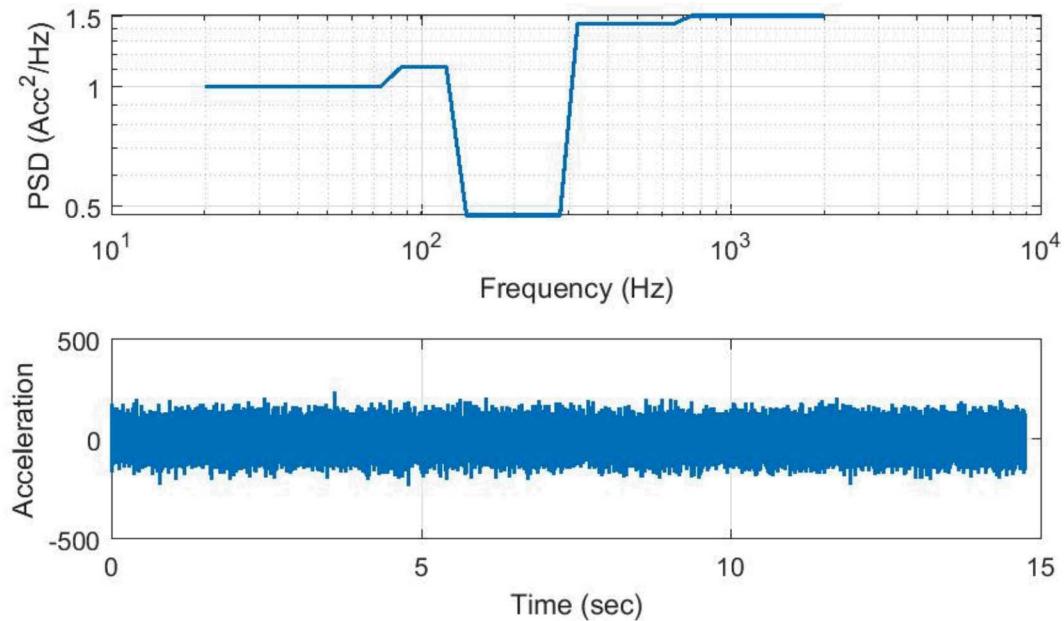


Figure 8 Compressed Random Vibration Signal and its ASD

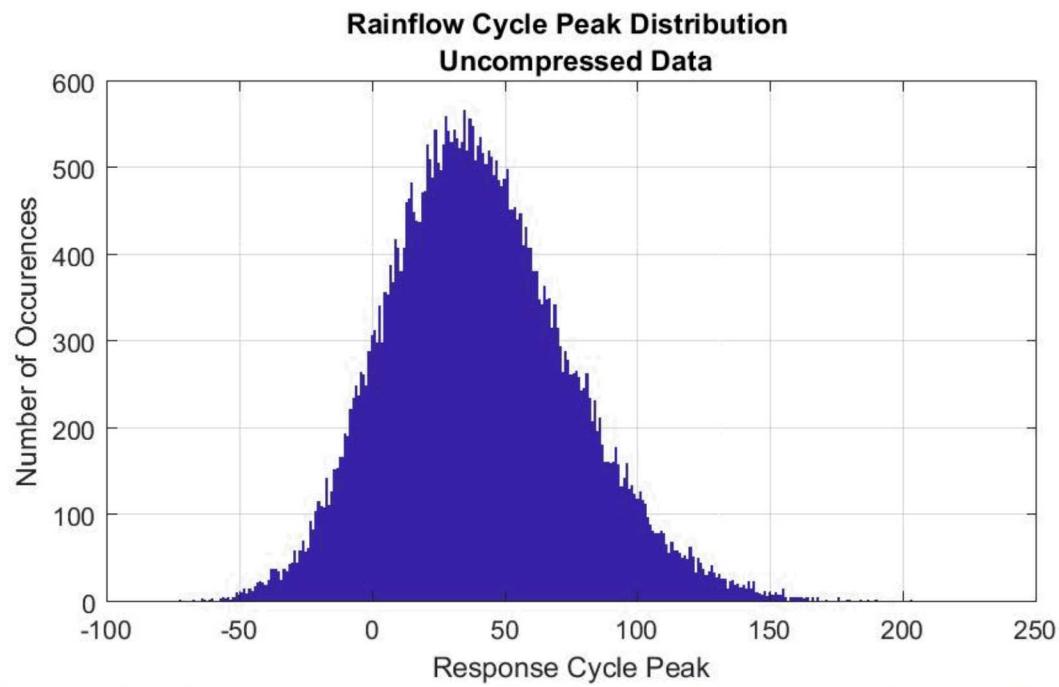


Figure 9 Rainflow Cycle Histogram of Five Segment Non-stationary Random Vibration Signal

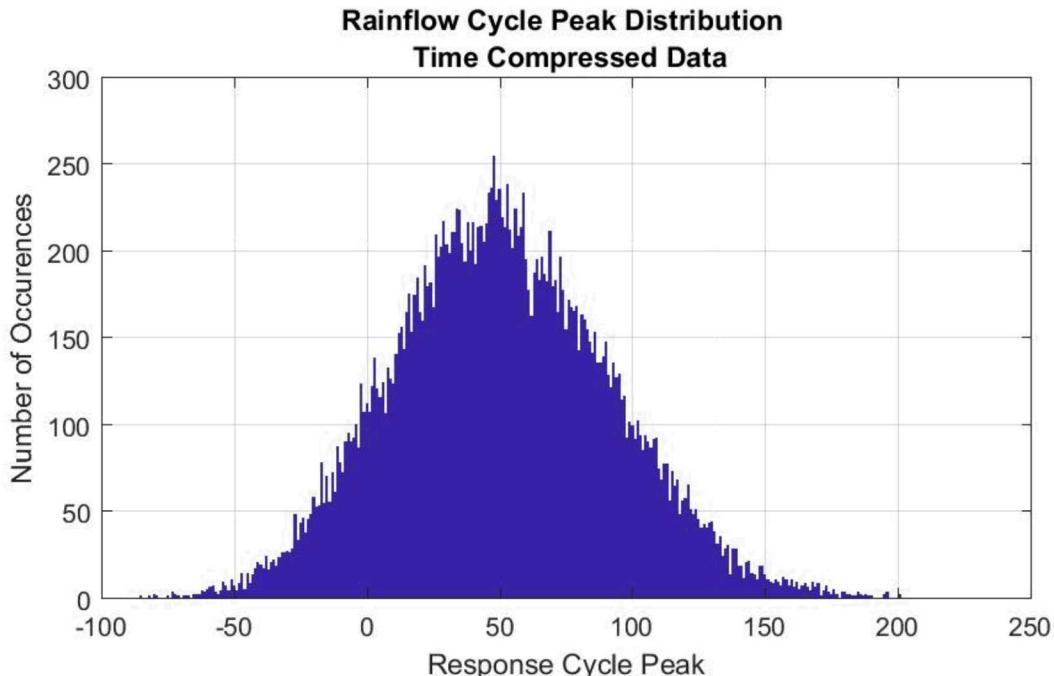


Figure 10 Rainflow Cycle Histogram Compressed Random Vibration Signal

Table 6 MATLAB Scripts and Functions for Compressing Time Histories with Graflab  
Function `genv_fatigue`

Script/Function Name	Description
Compress_RandomVibe_Script.m	Driver script that demonstrates time compression with <code>genv_fatigue.m</code>
Rainflow_v13.m	Function to compute rainflow cycles
MinersDamageIndex_V2.m	Function to compute Miner's Damage Index

### 2.3 Extension of ATA Algorithms to SIMO and MIMO Problems

ATA did not develop SIMO and MIMO algorithms, so work on this task was not performed.

### 3 Conclusions

Sandia National Laboratories was a partner with ATA on a U.S. Navy STTR Contract entitled Optimization of Fatigue Test Signal Compression Using the Wavelet Transform. Contracting and administrative complexities led to a late start date. As a result, our contribution to ATA's Phase 1 final deliverable was not significant.

SNL's primary contributions to ATA were:

- 1) the development of non-stationary data sets that ATA could use to test out the wavelet transform time compression algorithms.
- 2) Functions that ATA could use to generate time compressed data that could serve as an independent reference against which to compare the efficacy of the wavelet-based algorithms.

## References

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## Acknowledgements

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## Disclaimer

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