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Implementing the Restoring Force Surface Method to Fit Experimentally Measured Modal Coupling Effects

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ABSTRACT
In complex structures, particularly those with jointed interfaces, the dynamic response of individual modes can behave nonlinearly. To simplify analyzing and modeling this response, it is typically assumed that modes are uncoupled, in that each responds independently of the excitation level of other modes. This assumption is derived from the belief that, while modal coupling generally exists in physical structures, its effects are relatively small and negligible. This practice is reinforced by the fact that the actual causes of modal coupling are poorly understood and difficult to model. To that end, this work attempts to isolate and fit a model to the effects of modal coupling in experimental data from a nonlinear structure. After performing a low-level test to determine the linear natural frequencies and damping ratios of several modes, sine beat testing is used to individually excite each mode and record its nonlinear dynamic response. The Restoring Force Surface (RFS) method is then implemented to fit a nonlinear model to each isolated modal response. Sine beats are then done on multiple modes simultaneously, in which the response is assumed to be a combination of the nonlinear models of each isolated mode and some coupling term between them. As the terms modeling the individual modes are known, the only unknown is the coupling term. This procedure is performed on several mode pairs and excitation levels to evaluate the effectiveness of all proposed coupling models and gauge the significance of modal coupling in the structure.

Keywords: Modal Coupling, Restoring Force Surface, Nonlinear System Identification, Hilbert Transform, Sine Beat

INTRODUCTION

Creating models to accurately predict the dynamic response of complex structures becomes increasingly difficult as sources of nonlinearity become more significant. This is especially true in designs that contain many bolted joints, as even seemingly minute motion through the interface can cause the opposing faces to slip relative to one another. This motion, called microslip, can be a considerable source of damping in the structure due to the frictional forces that develop in the joint. A side effect of this is that the dynamic modes of the structure can exhibit some degree of coupling, where the response of one is altered by the excitation of another. These phenomena are poorly understood and have proven quite difficult to accurately measure and model. Consequently, it is usually assumed that the modes are only weakly coupled and any effects due to this are insignificant and may be neglected. While this approach has thus far yielded acceptable results, future designs may not allow for such simplifications and require an accurate description of modal coupling. This work explores the applicability of implementing the Restoring Force Surface (RFS) method of nonlinear system identification to detect and model modal coupling in experimental data from a physical structure. This is done by first characterizing the linear modal properties of the structure, the mass-normalized modeshapes, natural frequencies, and damping ratios of each mode. A modal shaker is then used to excite individual modes to high amplitude, such that a RFS model can be fit to the nonlinear response of each mode. The shaker is then used to excite two modes simultaneously and, using the linear parameters and the nonlinear model of each isolated mode as knowns, RFS is used to fit a model to any residual modal coupling effects.

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THEORY

As proposed by Masri and Caughey [1], RFS is a method of nonlinear system identification that leverages the fundamental equation of motion (EOM) of a system to isolate its nonlinearity. A single degree-of-freedom modal EOM with general nonlinearity is given in Eq. (1), where $\ddot{q}$ is acceleration, ζ is the linear damping ratio, ω_n is the linear natural frequency, $\dot{q}$ is velocity, q is displacement, $f_\mathrm{NL}(q,\dot{q})$ is some internal nonlinear force that is a function of the displacement and velocity, and f_ext is an external force acting upon the system.

$$\ddot{q} + 2\zeta\omega_\mathrm{n}\dot{q} + \omega_\mathrm{n}^2 q + f_\mathrm{NL}(q,\dot{q}) = f_\mathrm{ext} \tag{1}$$

By transferring the linear terms to the RHS, as in Eq. (2), one obtains an expression for the cumulative nonlinearity in the system, or the restoring force, in terms of easily measured quantities.

$$f_\mathrm{NL}(q,\dot{q}) = f_\mathrm{ext} - \ddot{q} - 2\zeta\omega_\mathrm{n}\dot{q} - \omega_\mathrm{n}^2 q \tag{2}$$

A model for the restoring force can be formed as in Eq. (3), where it is assumed that the nonlinearity can be characterized by polynomial expressions that are an even and odd function of the displacement and velocity raised to some power.

$$f_\mathrm{NL}(q,\dot{q}) = \sum k_\mathrm{q} q^i + \sum k_{|\mathrm{q}|}|q|^j + \sum c_{\dot{\mathrm{q}}}\dot{q}^k + \sum c_{|\dot{\mathrm{q}}|}|\dot{q}|^l \tag{3}$$

After choosing the number and order of these terms, their coefficients are found via a least-squares solution as in Eq. (4), where the restoring force is premultiplied by the pseudoinverse of the nonlinear terms. A more detailed explanation of these computations is given in [2].

$$\left[\{k_\mathrm{q}\} \quad \{k_{|\mathrm{q}|}\} \quad \{c_{\dot{\mathrm{q}}}\} \quad \{c_{|\dot{\mathrm{q}}|}\}\right]^T = [\mathbf{q}^i \quad |\mathbf{q}|^j \quad \dot{\mathbf{q}}^k \quad |\dot{\mathbf{q}}|^l]^\dagger\,(f_\mathrm{ext} - \ddot{q} - 2\zeta\omega_\mathrm{n}\dot{q} - \omega_\mathrm{n}^2 q) \tag{4}$$

Coupling between two modes can be modeled as in Eq. (5), where the restoring force is now taken to be a function of responses from both modes and nonlinear terms are formed from combinations of those responses; exponents are neglected here for brevity.

$$f_\mathrm{NL,C}(q_1,q_2,\dot{q}_1,\dot{q}_2) = \sum \sigma_{\mathrm{q_1q_2}} q_1 q_2 + \sum \sigma_{\mathrm{q_1}\dot{\mathrm{q}}_2} q_1\dot{q}_2 + \sum \sigma_{\dot{\mathrm{q}}_1\mathrm{q_2}}\dot{q}_1 q_2 + \sum \sigma_{\dot{\mathrm{q}}_1\dot{\mathrm{q}}_2}\dot{q}_1\dot{q}_2 \tag{5}$$

In general, the order of any of these nonlinear terms is not necessarily an integer number; fractional powers may be considered. While this allows for very flexible selection of parameters when choosing the model form, there is consequently an infinite number of models that could be fit to the restoring force, with widely varying levels of success.

TEST CASE

The structure analyzed in this work is an aluminum cylinder-plate-beam (CPB) assembly that has been extensively studied in previous works, such as [2, 3]. The test setup, shown in Figure 1, was used to collect linear data from low-level burst random input from the shaker, and high-level nonlinear responses from sine beat shaker excitations; [3] explores these testing methods in great detail. These processes are implemented to characterize the first three elastic modes of the CPB, which are the first cantilever modes of the beam in the soft and stiff directions and the first drum mode of the plate. Coupling between these modes is investigated by exciting the possible pairings between them; 1-2, 1-3, and 2-3.

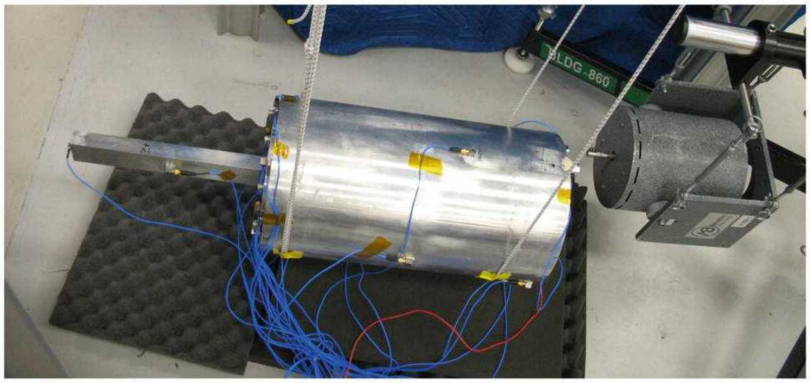

Figure 1: Experimental Setup - aluminum cylinder-plate-beam excited by a shaker through connecting stinger

To form the restoring force, the system velocity and displacement are found by integrating the measured acceleration. This was done in the frequency domain by dividing the acceleration by $i\omega$ and $-\omega^2$ for velocity and displacement respectively, and then applying a high pass filter to eliminate low frequency drift. The model form fit to this nonlinear restoring force was determined through a simple combination of trial and error and Monte Carlo simulations, where a specified number of terms were given a random exponent from a pool of possible values. With coefficients from the least squares solution between the restoring force and the nonlinear terms, the response of the resultant model is computed by numerically integrating its EOM. The accuracy of the model is determined by comparing the instantaneous natural frequency and damping ratio of the integrated response to those of the measured experimental response. For the first soft cantilever beam mode of the CPB, four of each of the nonlinear displacement and velocity terms were given random powers between 1.1 and 4, at 0.1 increments, until a sufficiently accurate result was found; the final set of exponents are given in Table 1.

Table 1: RFS Model Terms and Coefficients for CPB Elastic Mode 1

Term	q^i	$\lvert q\rvert^j$	$\dot{q}^k$	$\lvert\dot{q}\rvert^l$
Exponent	2.3, 2.6, 2.7, 2.8	1.5, 1.7, 2.4, 4	1.7, 2, 2.5, 3.2	1.7, 2.6, 2.9, 3.2

The restoring force and the resultant least squares fit of the nonlinear terms is shown in the top left plot of Figure 2; there is good agreement throughout the frequency range. From numerically integrating the model, where the measured force was interpolated and used as the applied force in the EOM, the response is shown with the measured experimental acceleration in the top right of Figure 2, and the instantaneous natural frequency and damping ratio are given in the bottom left and right plots, respectively. While the frequency of the response is very close to the truth data, the damping ratio is less accurate. As this is the best model that could be found from any number of terms and range of exponents, it would seem that damping is difficult to model regardless of how well the restoring force is fit. It was also observed that, even if the restoring force is accurately modeled, the resultant simulation can deviate greatly from the measured response. The best fit models of the other modes, and the coupling between them, will be explored in the accompanying presentation.

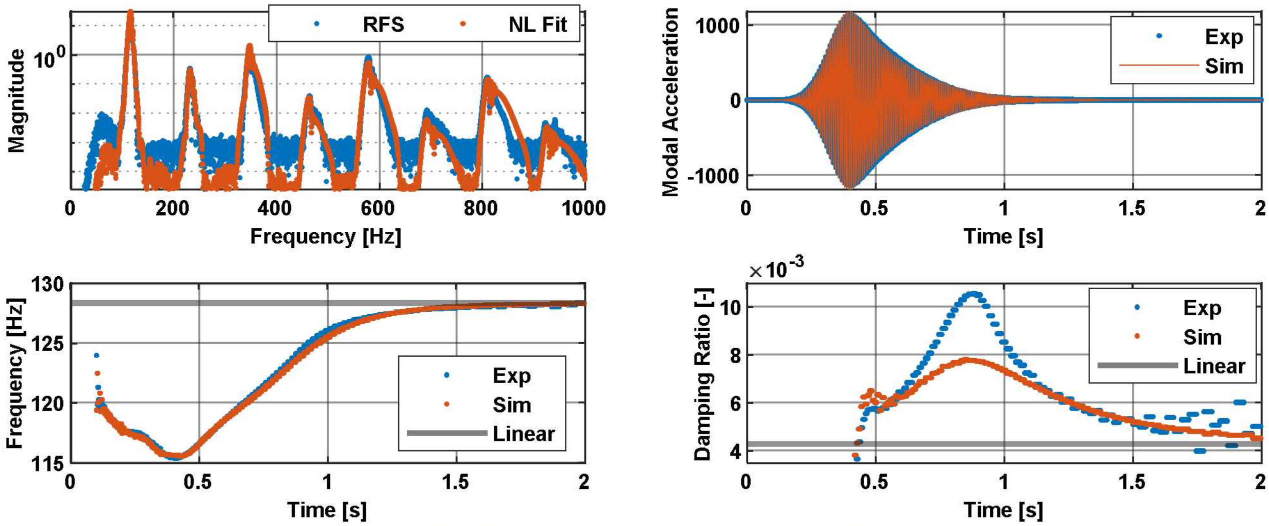

Figure 2: Top left - Experimental restoring force for mode 1 and the least squares fit of the nonlinear terms ; Top right - Experimental vs Simulation modal acceleration; Bottom left: Instantaneous Natural Frequency; Bottom right: Instantaneous Damping Ratio

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