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Baselined Conditions to Observe Modal Coupling in a Nonlinear Bolted Structure

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1 Abstract

Traditional experimental modal analysis assumes that structures under test exhibit strictly linear behavior. However, many structural assemblies display nonlinear response caused by frictional contact at bolted joints. In previous works, the authors rigorously investigated this type of nonlinear response in a modal framework using traditional modal excitation techniques and modeling the nonlinearity on a mode-by-mode basis. A cursory investigation was completed using force appropriation techniques to extract nonlinear modal information from a jointed structure. This work continues that investigation. Significant modal coupling was observed between the 9th harmonic of the target resonance and a higher frequency mode of the system. The structure under test consists of a cylinder, plate, and beam connected by bolted joints This work describes the conditions present when modal interactions were observed in a bolted structure based on further studies using force appropriation testing. Three primary conditions are discussed: harmonic frequency crossings, high-energy state, and mode shape compatibility.

Keywords – nonlinear normal modes, force appropriation, modal interactions, nonlinear modal damping

2 Approach

The objective of this work is to develop a set of conditions necessary to observe modal coupling in an experimental bolted structure. The structure under test is referred to as the Cylinder Plate Beam (CPB) which has been studied previously in [1, 2, 3]. The experiments performed for this work were completed using force appropriation testing described in [4, 5] and first implemented on the CPB by the authors in [6]. The method follows the assumption that, for a sinusoidal excitation of a resonance, the excitation force will be 90 degrees out of phase with the response acceleration. An experimental execution of this method involves many steps. First, the CPB is excited with a single tone using a modal shaker. The shaker driven frequency is determined using a closed-loop control system, which updates the excitation frequency until the quadrature condition is stably satisfied. Once this occurs, a frame of data is recorded, and the amplitude is increased allowing the control system to find a new frequency when quadrature is met for this increased amplitude level. This process continues as the excitation force sweeps from small to large amplitude. These measurements are used to understand the frequency and damping of the nonlinear system as a function of response amplitude.

In a previous study [6], the authors observed significant modal coupling using the CPB structure. A force appropriation test targeting the first bending mode of the system was completed. A large 9:1 modal interaction was observed between the 1st and 2nd bending mode of the structure. The magnitude of the harmonic response was greater than that of the prescribed fundamental frequency leading to large vibration amplitude of the structure where force input was not prescribed. This interaction occurred when the frequency of 9th harmonic aligned with that of the 2nd bending mode. As the vibration amplitude of the target mode increases, the resonance frequency of the target mode softens. Thus, a crossing point exists when the harmonic response of the fundamental frequency lines up with a higher frequency mode. This **harmonic frequency crossing** is an essential condition to observe modal interactions in a bolted structure as it provides input energy at a compatible frequency as shown previously in [7].

To begin, the authors attempted to replicate these previous results using the harmonic frequency crossing condition as the primary criterion. A force appropriation test on the reassembled structure from [6] was performed and

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negligible modal coupling was observed at the 9th harmonic crossing. This conundrum led the authors to investigate the difference between the system dynamics from this study and those of [6]. A frequency energy plot of the two measurements provided valuable insight, the nonlinear dynamic response in [6] was significantly weaker than the reassembled structure (the reassembled structure had steeper frequency shift). Thus, the interaction with the 9th harmonic occurred at a much lower vibration amplitude than that of [6]. Next, the reassembled structure was driven to even higher energy and 10th and 11th harmonic frequency crossings were observed near the energy level of previous coupling. These higher-energy frequency crossings led to significant coupling similar to the previous study. Thus, a harmonic frequency crossing alone is insufficient to observe a modal interaction, but **high levels of energy** are required to activate the nonlinearity which allows coupling to occur.

In this work and the previous study, other high frequency modes of the structure have harmonic frequency crossings when in a high-energy state, but not every instance of these pairs led to observable modal coupling. This led the authors to investigate a third condition: **mode shape compatibility**. A finite element model (FEM) was used to investigate this condition. A FEM of a CPB structure was used along with the Modal Assurance Criterion (MAC). The MAC of the modes of interest results in linearly independent modes. The nonlinear harmonic response of the bolted structure is caused due to frictional contact at the bolted joints. The MAC was recalculated, using only the degrees of freedom (DOFs) in contact near the nonlinear interface, to provide more insightful information. The MAC of the interface DOFs shows high compatibility between mode shapes that display coupling (the off diagonal between the first and second bending mode), and low compatibility between shapes that do not display coupling. Thus, the motion at the nonlinear joint is similar between mode pairs that exhibit modal interactions and this motion is important to understand when a modal interaction will be observed under test.

The repeatability of this nonlinear coupling phenomena was assessed to best understand these three conditions: harmonic frequency crossing, high-energy state, and mode shape compatibility. Most instances with these conditions satisfied resulted in coupling, however; a few measurements failed to exhibit significant coupled response. The condition that is the most difficult to evaluate is the high-energy state, as no consistent energy threshold was found. The authors investigated other techniques to characterize system energy including a cursory investigation of the nonlinear dissipation present during force appropriation testing.

To evaluate the dissipation in the structure, two main techniques were evaluated. Both techniques rely on the assumption that the force input into the system is equal to the force dissipated. This can be further broken down to assume the applied load is equal to the summation of a linear damping force due to the modes of the structure and the nonlinear dissipation forces caused by the nonlinearity in the system. The principles are discussed in detail for linear dynamics in [8].

$$F_{applied} = F_{dissipation} = F_{d,nl} + F_{d,lin} \tag{1}$$

$$F_{d,nl} = F_{applied} - F_{d,lin} \tag{2}$$

In a force appropriation test, a single resonance is appropriated. As such, these forces can be converted to modal forces, where the linear damping force can be calculated using the linear modal parameters: natural frequency (ω_n), damping ratio (ζ), and mode shape (Φ). These nonlinear forces were compared for measurements with and without coupled nonlinear dynamics and rungs exhibiting coupling had significantly larger nonlinear modal damping forces.

The second nonlinear dissipation method was based solely on work done and dissipation energy. The work done within a cycle of oscillation can be written as:

$$W_d = \oint F_a dq = \oint F_a \dot{q} dt \tag{3}$$

As with the first method, the energy into the system during a cycle of oscillation must exactly overcome the energy dissipated. Thus, the work done is equal to the sum of energy dissipated due to linear and nonlinear structural response.

$$W_d = E_{dis} = E_{dis,lin} + E_{dis,nl} = \oint 2\zeta\omega_n \dot{q} dq + E_{dis,nl} \tag{4}$$

$$E_{dis,nl} = W_d - \oint 2\zeta\omega_n \dot{q}\, dq \tag{5}$$

The resulting nonlinear energy dissipated can be energy dissipated due to nonlinear dynamics was compared for multiple test runs. Significant modal interactions occurred in measurements with large nonlinear energy dissipation.

3 Concluding Remarks

This abstract summarized research completed to experimentally identify the conditions present when significant modal interactions occurred in the response of a bolted assembly. Three such conditions were identified through experimental testing. First, a **harmonic frequency crossing** should be identified. This consists of finding a vibration amplitude at which the frequency of a higher harmonic aligns with a higher frequency mode. Second, the mode which aligns with this harmonic should be tested for **compatible mode shape** motion with the target mode. Finally, some means of a **high energy level** should be achieved. Future work will further investigate whether this high-energy state is required in the target mode or the dissipation forces within the system. This study contains two means by which dissipation forces and energy can be computed and compared to understand differences in force appropriation measurements.

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