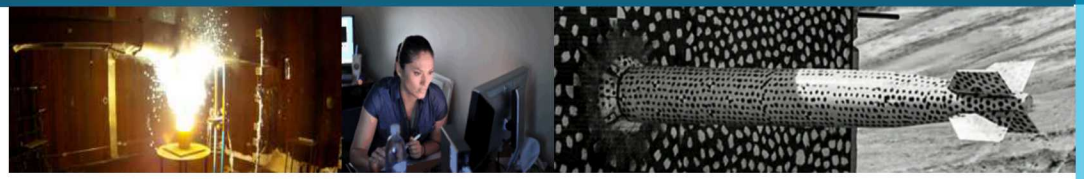
SAND2020-0631PE

Introduction to Belief and Plausibility



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Topics

Uncertainty quantification basics

Why use belief/plausibility?

Overall process

Conceptual description of belief/plausibility

Assignment of evidence

Interpretation of belief/plausibility

Example

What are we doing? Characterizing Uncertainty

Goal: Characterize the state-of-knowledge and uncertainty in that knowledge, i.e. what do we know and how well do we know it?

Methods:

- Statistical inference (frequentist statistics)
  - Useful where there is plentiful experimental data or simulation data from well-validated models
- Bayesian inference
  - Useful where there is some data that is supplemented by expert knowledge
- Belief and Plausibility/Evidence Theory/Dempster Schafer
  - Useful when there is little to no data but plenty of expertise

Which method should you use?

- Choose the method based on the quantity, quality, and completeness of the available data

Why use belief/plausibility?

There is substantial uncertainty

The data does not cover the entire space of possibilities

- How common are manufacturing defects?
- What contaminants are present and what are the effects?
- What is the current state of every component that could affect resistance?
- What is the valid range of resistance when in RESET?

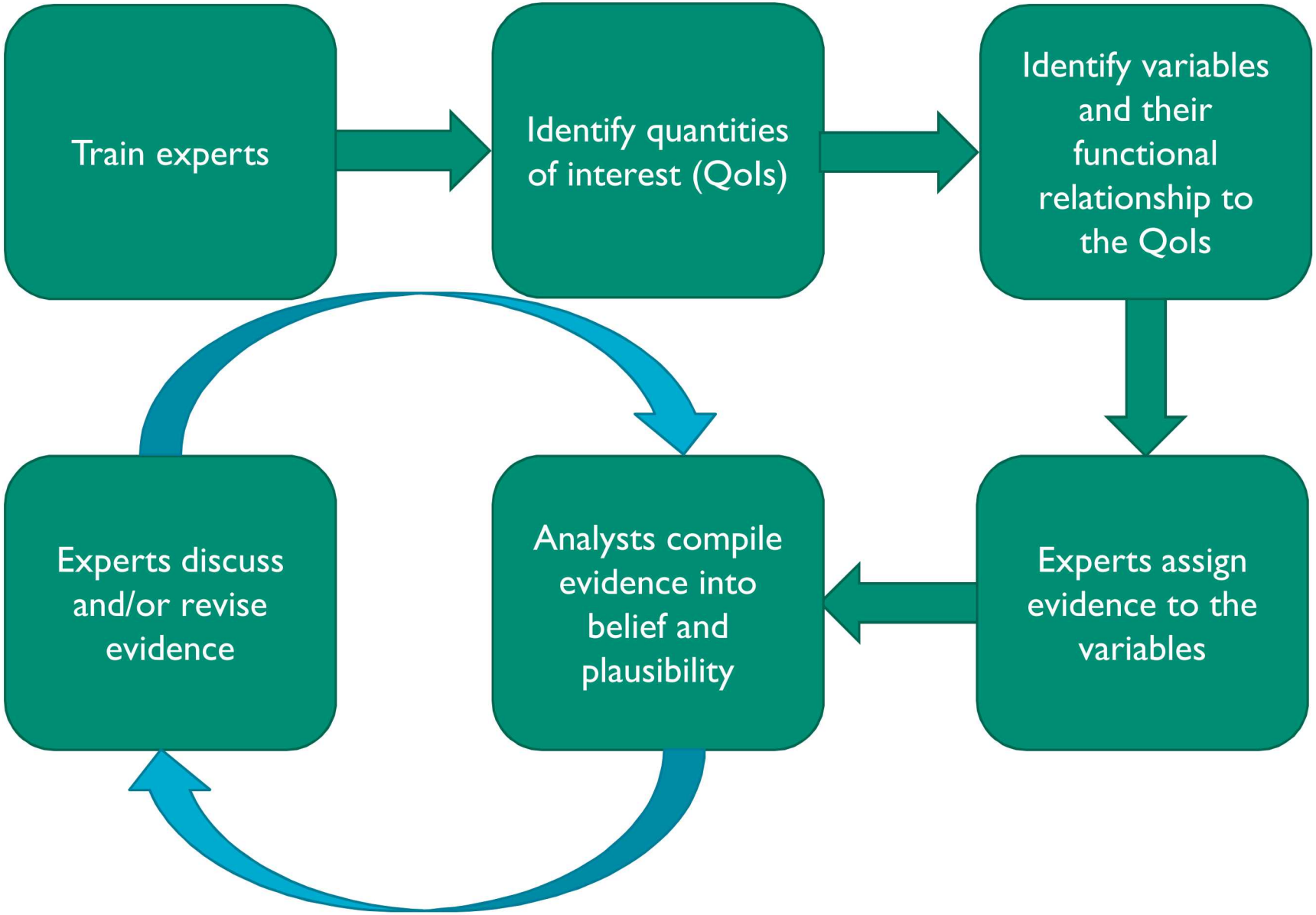
Belief/Plausibility does not require expert consensus

- Classical statistical inference would require experts to agree on the underlying assumptions, which can be unjustified
- Bayesian inference requires expert consensus on the prior distribution, which has a strong effect on results when data is sparse

Belief/Plausibility results are more explicit about the state-of-knowledge

- Results are characterized by bounds only with no "mean" or "best estimate"

How do we use belief/plausibility?



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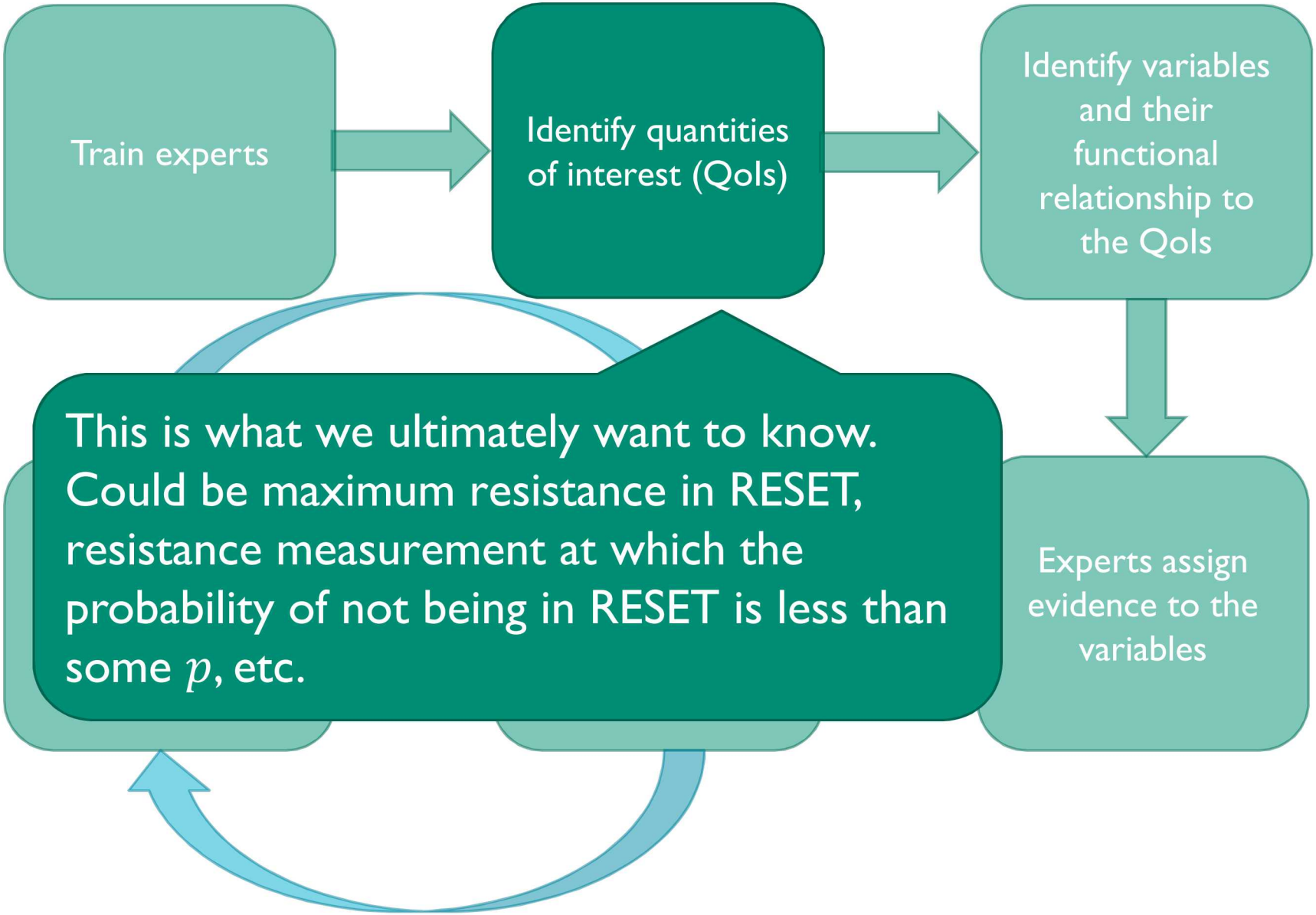

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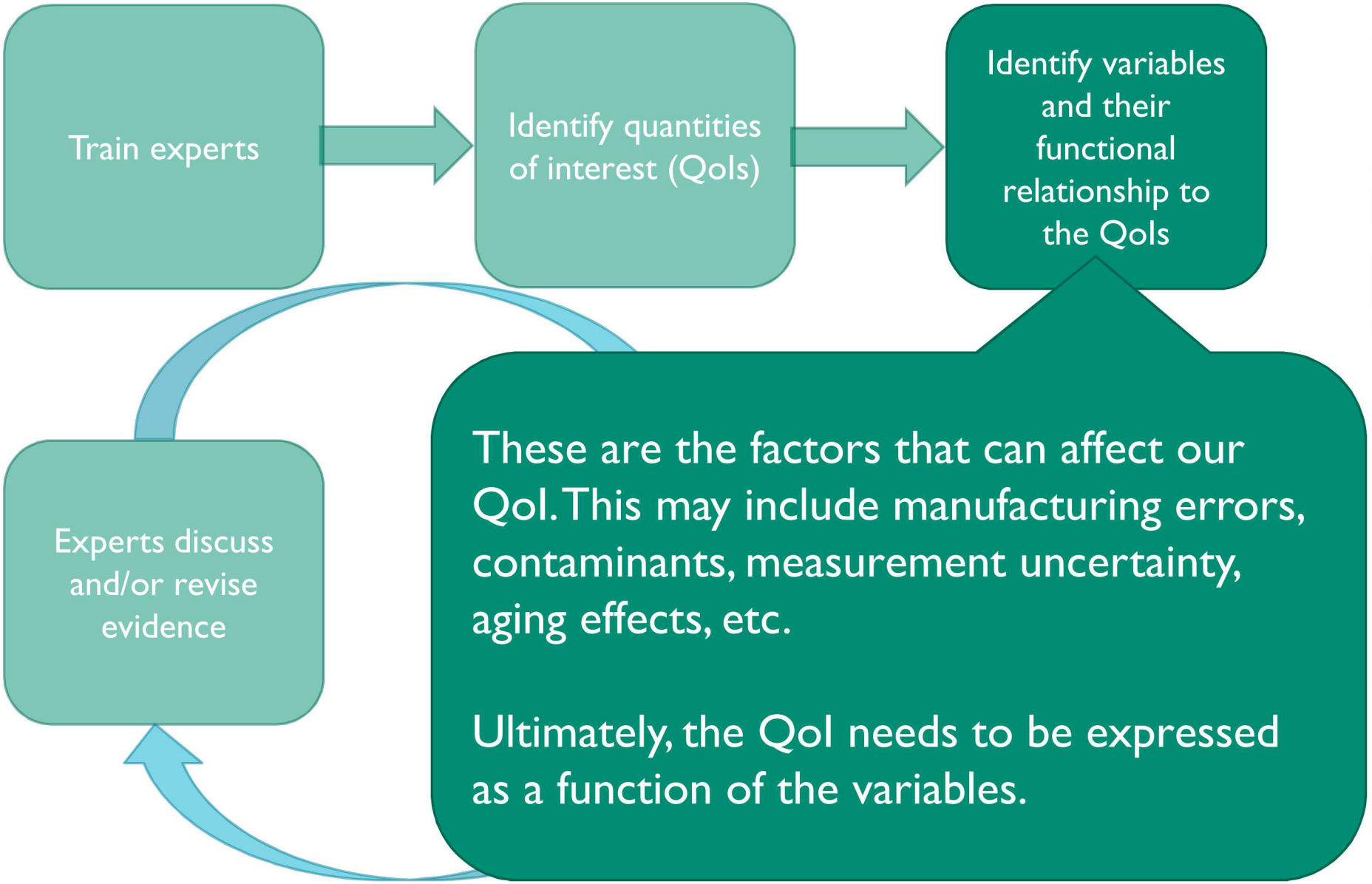
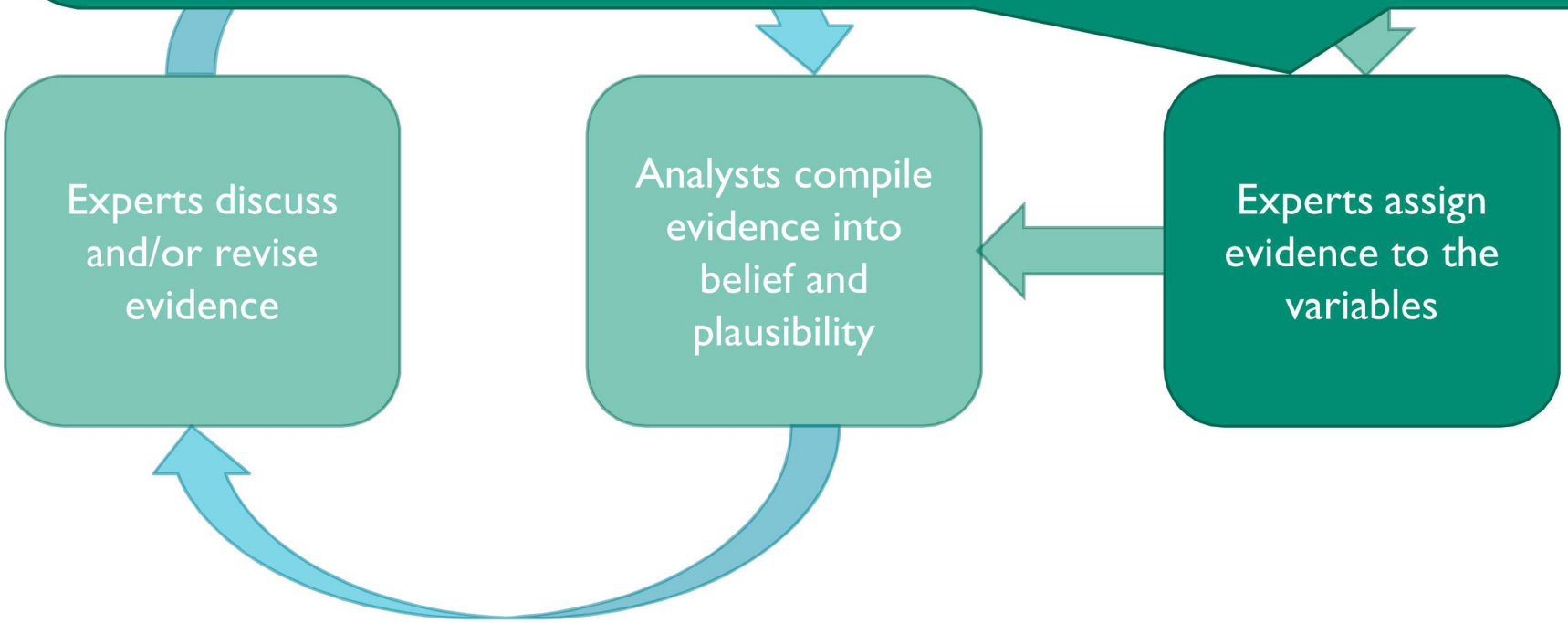


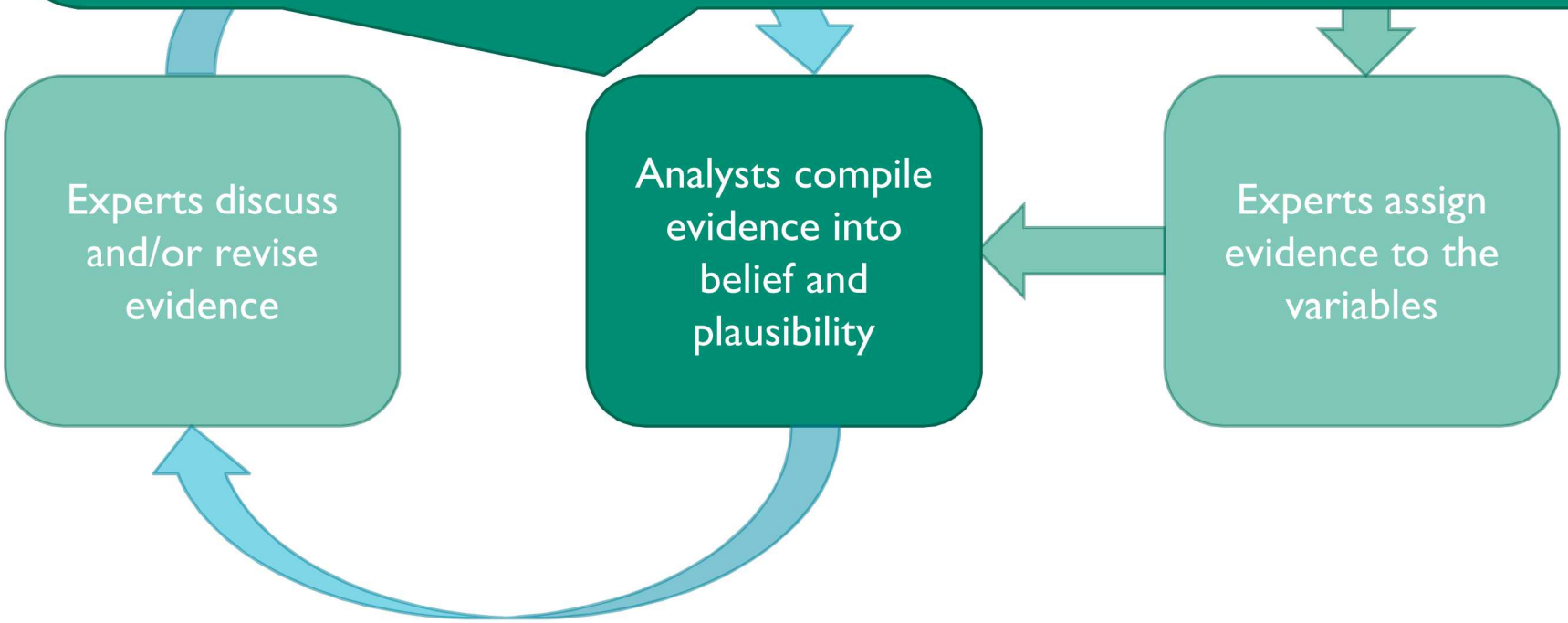
How do we use belief/plausibility?

Evidence will characterize expert belief about each of the variables that contribute to the QoI. Experts assign evidence independently and document their justifications for each assignment of evidence. Justifications can include properties of physics, simulation results, experimental data, operational experience, statistical analyses, etc.



How do we use belief/plausibility?

We do this with the BeliefConvoluion code (developed by John Darby). The input for this code is the functional relationship between the variables and the QoI and the evidence provided by each expert; experts are weighted evenly.



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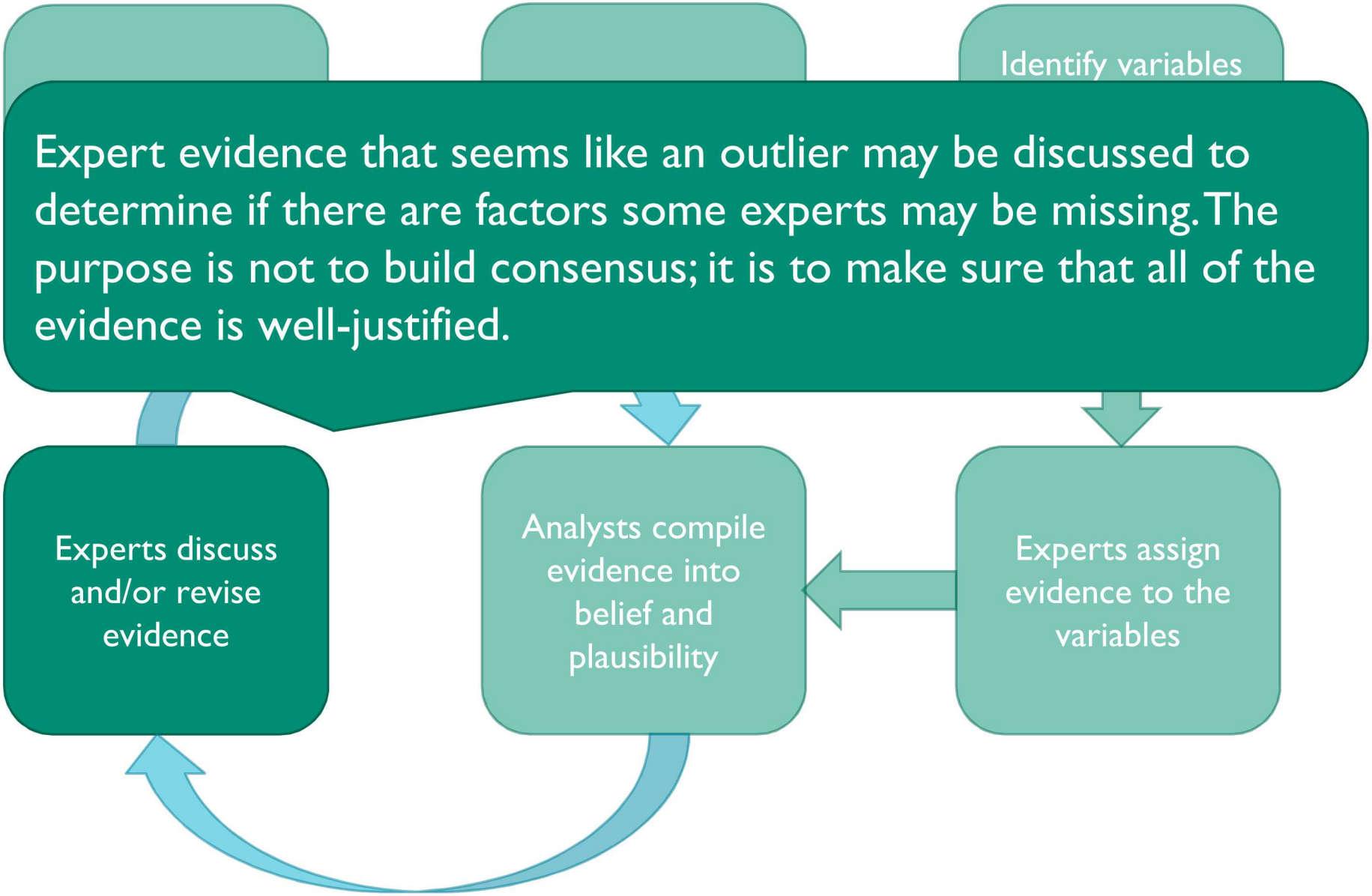

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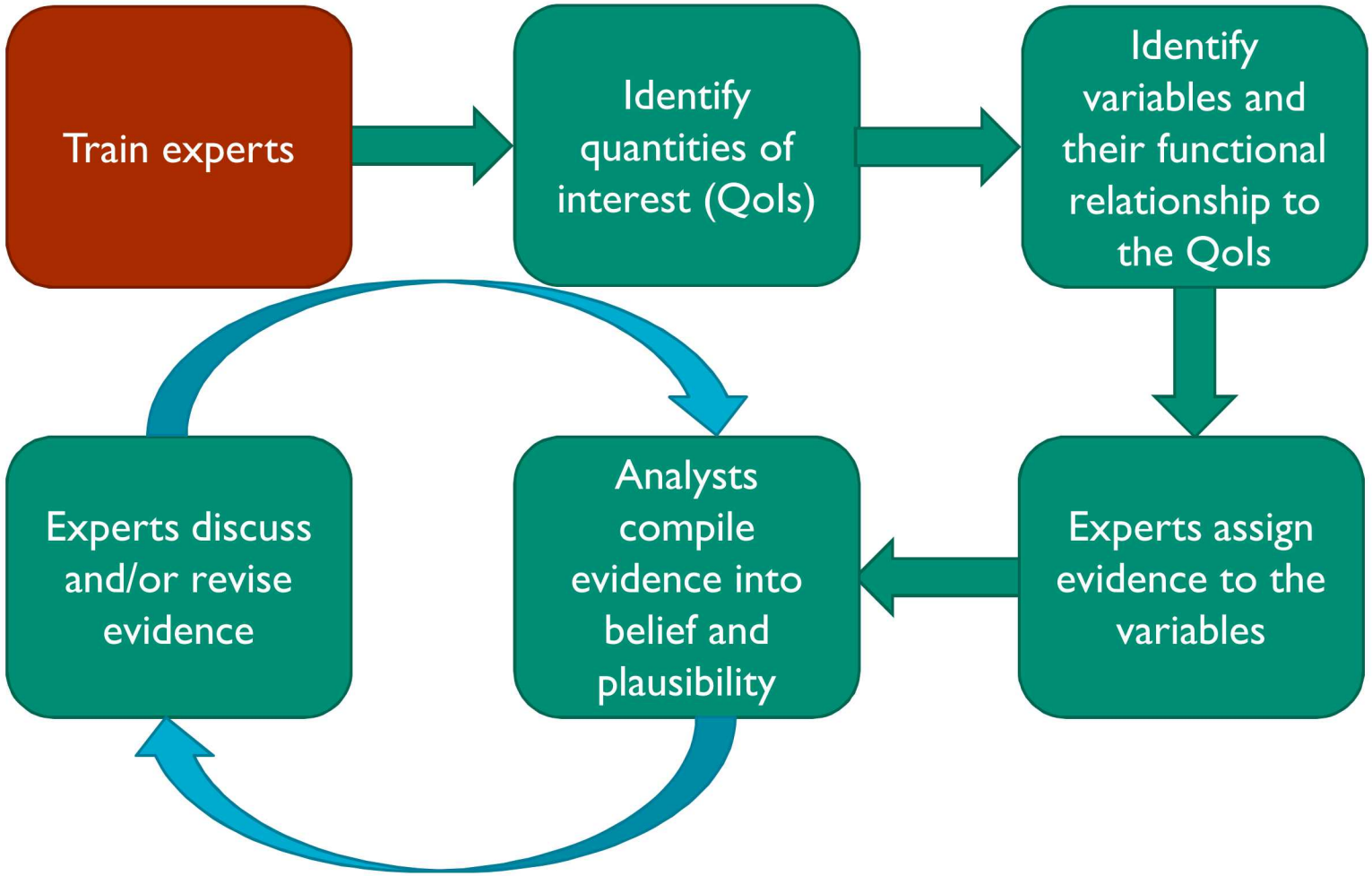
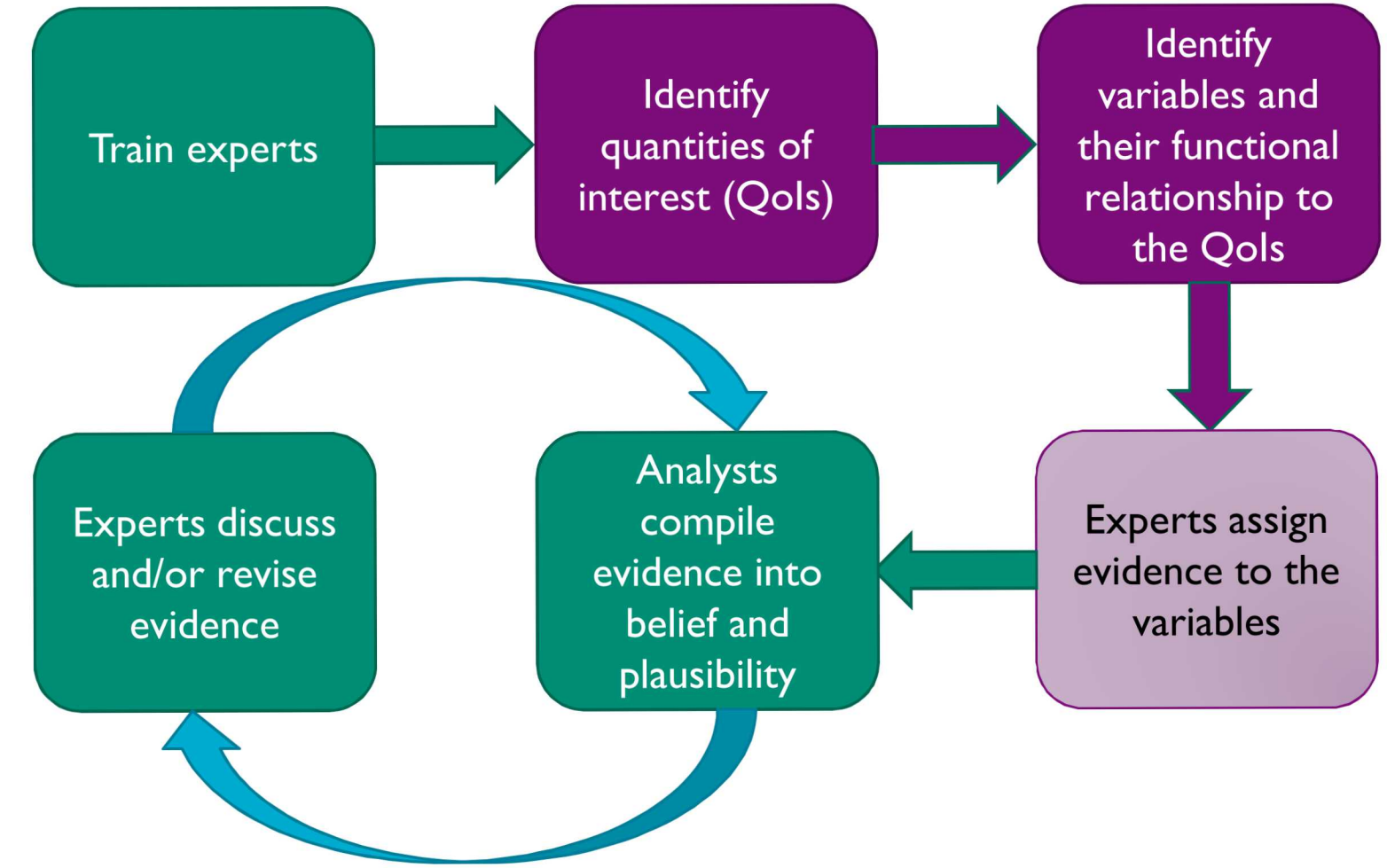


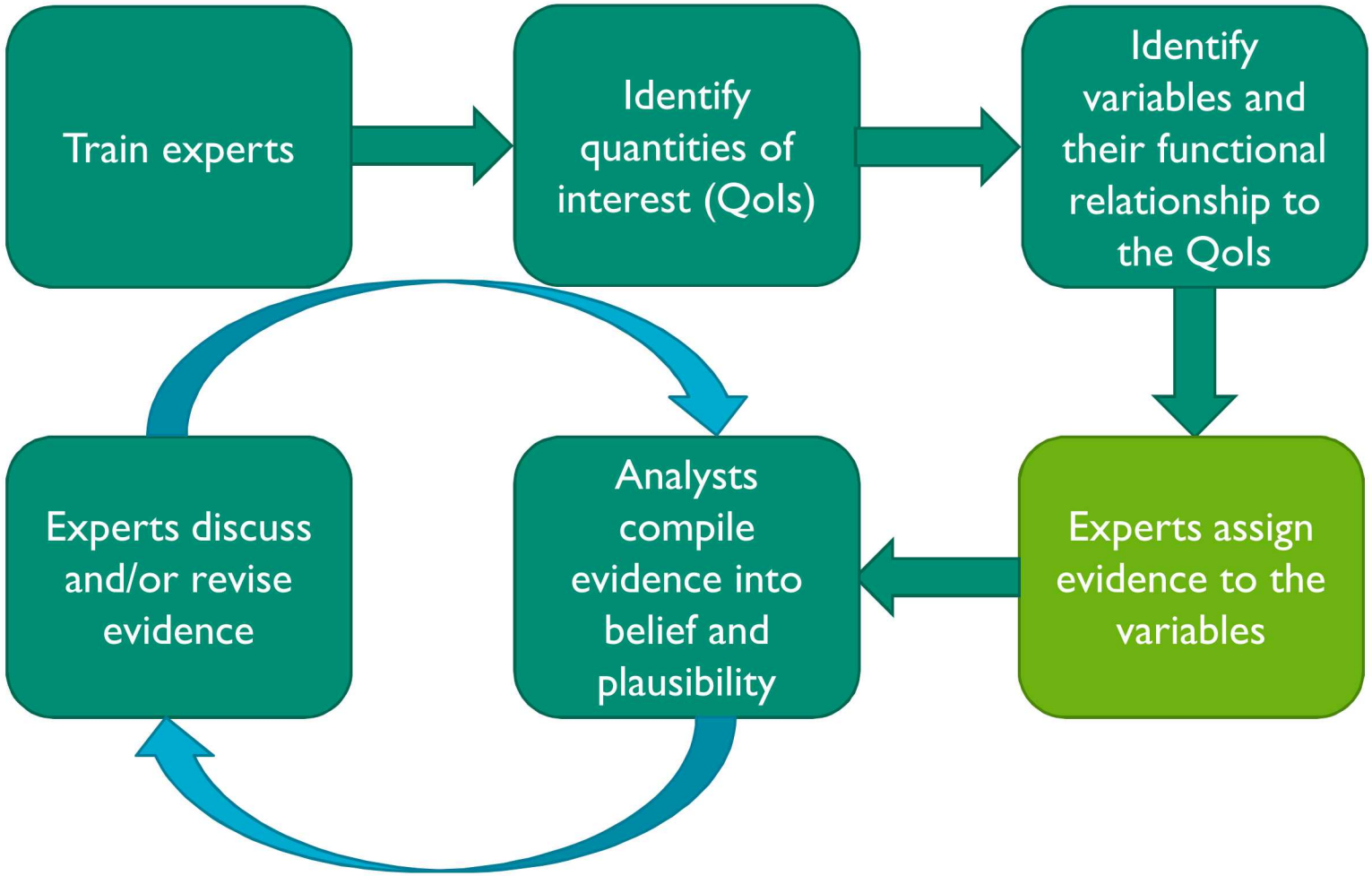
Monday

How do we use belief/plausibility?

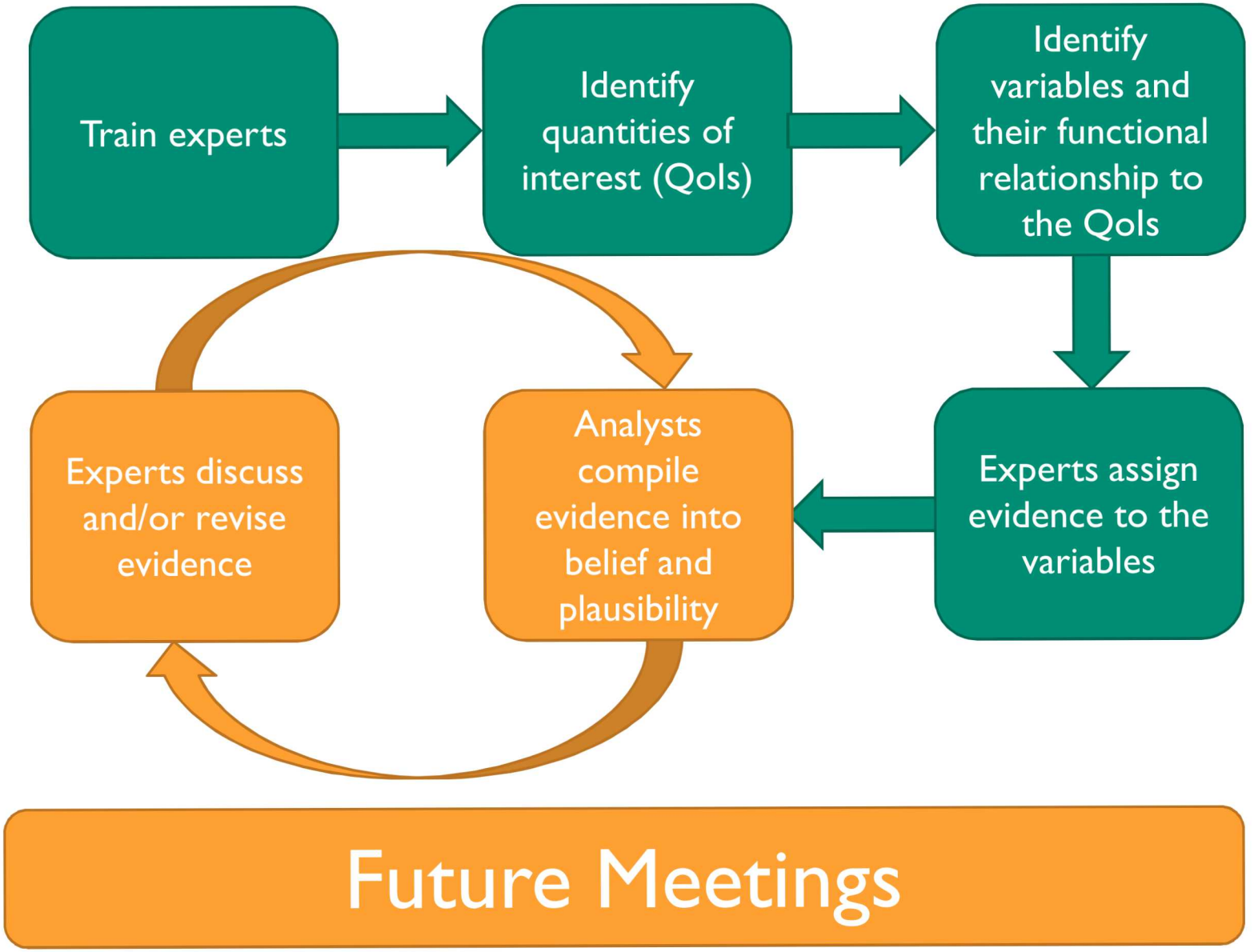


Tuesday

How do we use belief/plausibility?



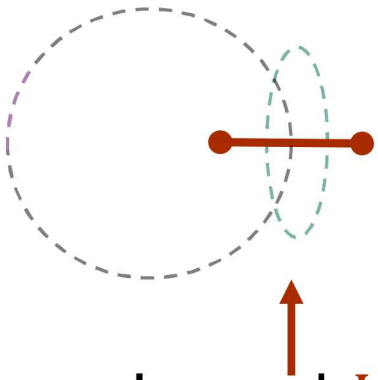
How do we use belief/plausibility?



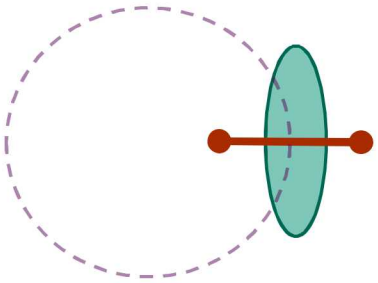
Belief/Plausibility Concept: Basics

Belief and plausibility are lower and upper bounds on probability obtained by collecting evidence that characterizes expert knowledge

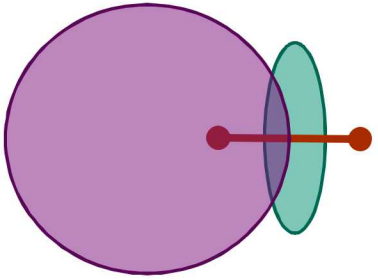
- Estimated based on assignment of **evidence**
- **Focal elements** are intervals that are assigned **evidence**
- The **belief** value for an interval is the sum of evidence over all intervals contained within it (strict condition)
- The **plausibility** value for an interval is the sum of evidence over all intersecting intervals (less strict condition)



Interval *I* with evidence



Evidence contributing to belief of *I*



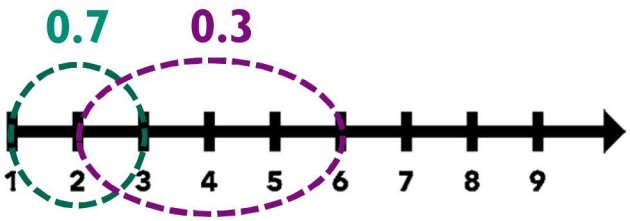
Evidence contributing to plausibility of *I*

Assigning Evidence

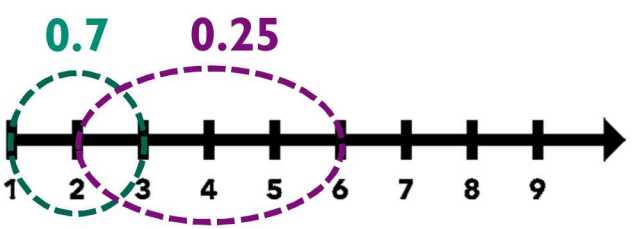
Rules for assigning evidence

- The total evidence you provide for a variable must equal 1

Correct



Incorrect

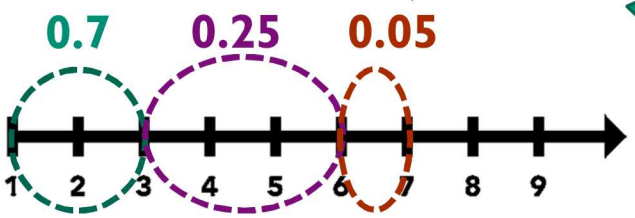


- Expert interpretation: "There is a 70% chance this variable is between 1 and 3. There is a 30% chance this variable is between 2 and 6. There is no chance it is greater than 6."

Assigning Evidence

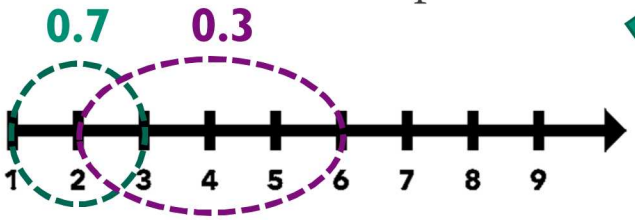
Rules for assigning evidence: there are multiple representations and experts should use whichever representation makes the most sense to them

- Evidence can be disjoint



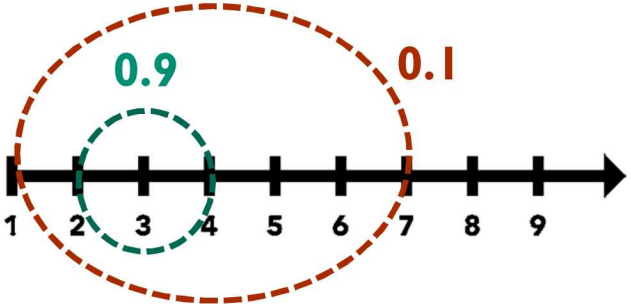
This is easiest for me because I am used to thinking "probabilistically"

- Evidence can overlap



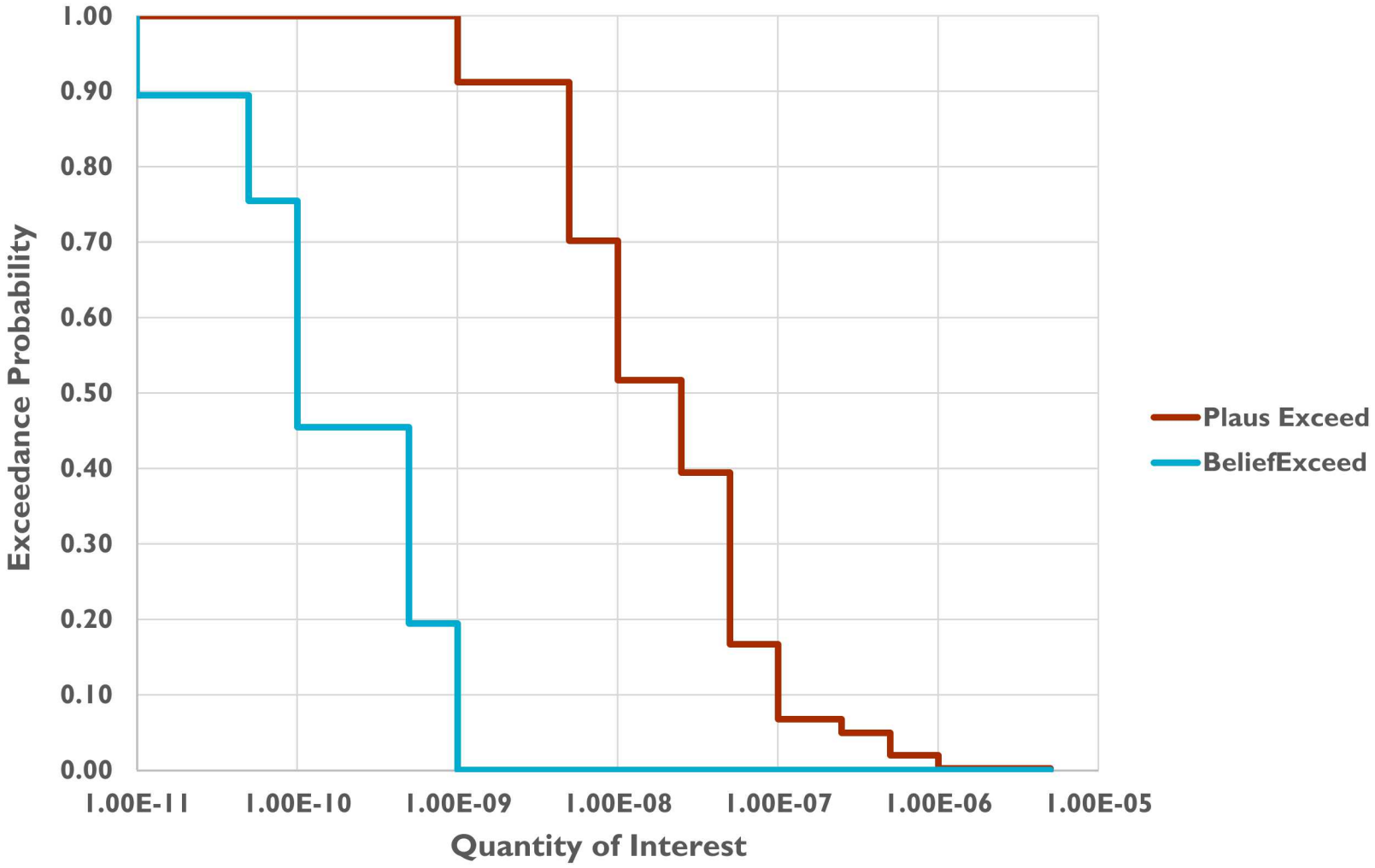
This could be based on two experiments, one of which is a much higher quality (0.7 vs 0.3) but there is some overlap in results so evidence from both contributes to the overlapping results

- Evidence can be nested

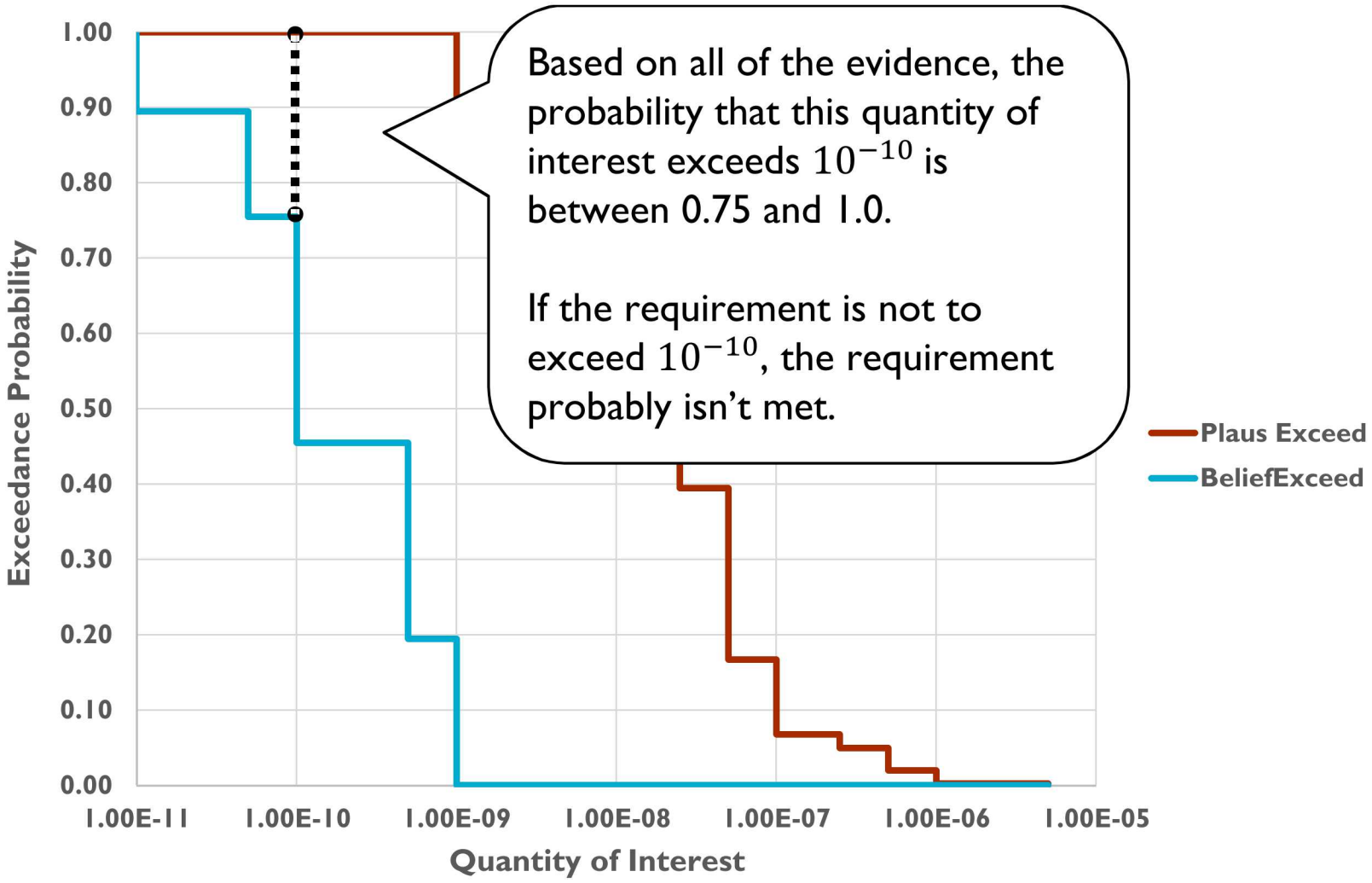


This could be a way to express that there is good evidence the value is between 2 and 4, but there isn't enough evidence to completely exclude values (Darby calls this "hedging")

Belief/Plausibility: Result Interpretation



Belief/Plausibility: Result Interpretation



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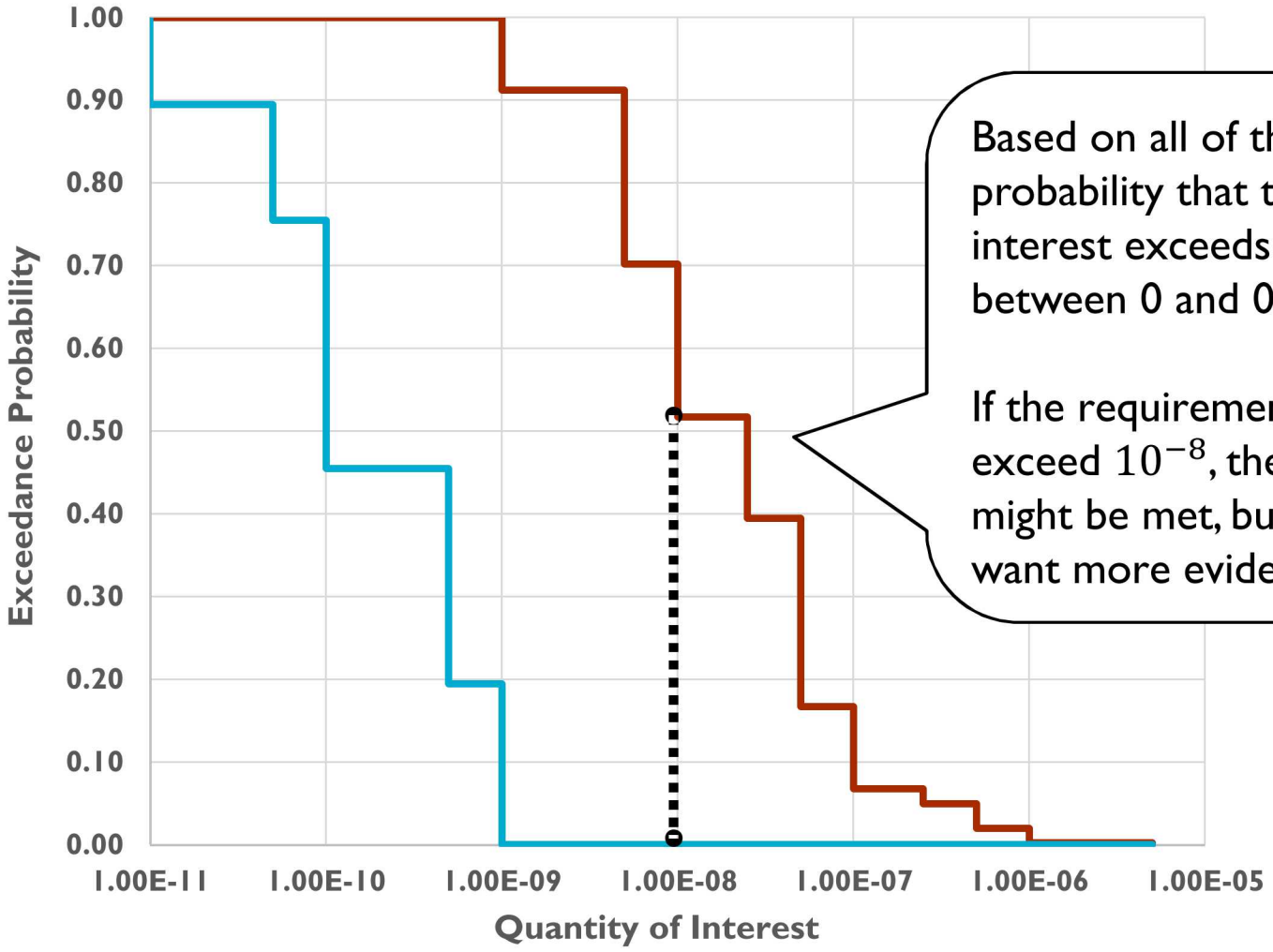
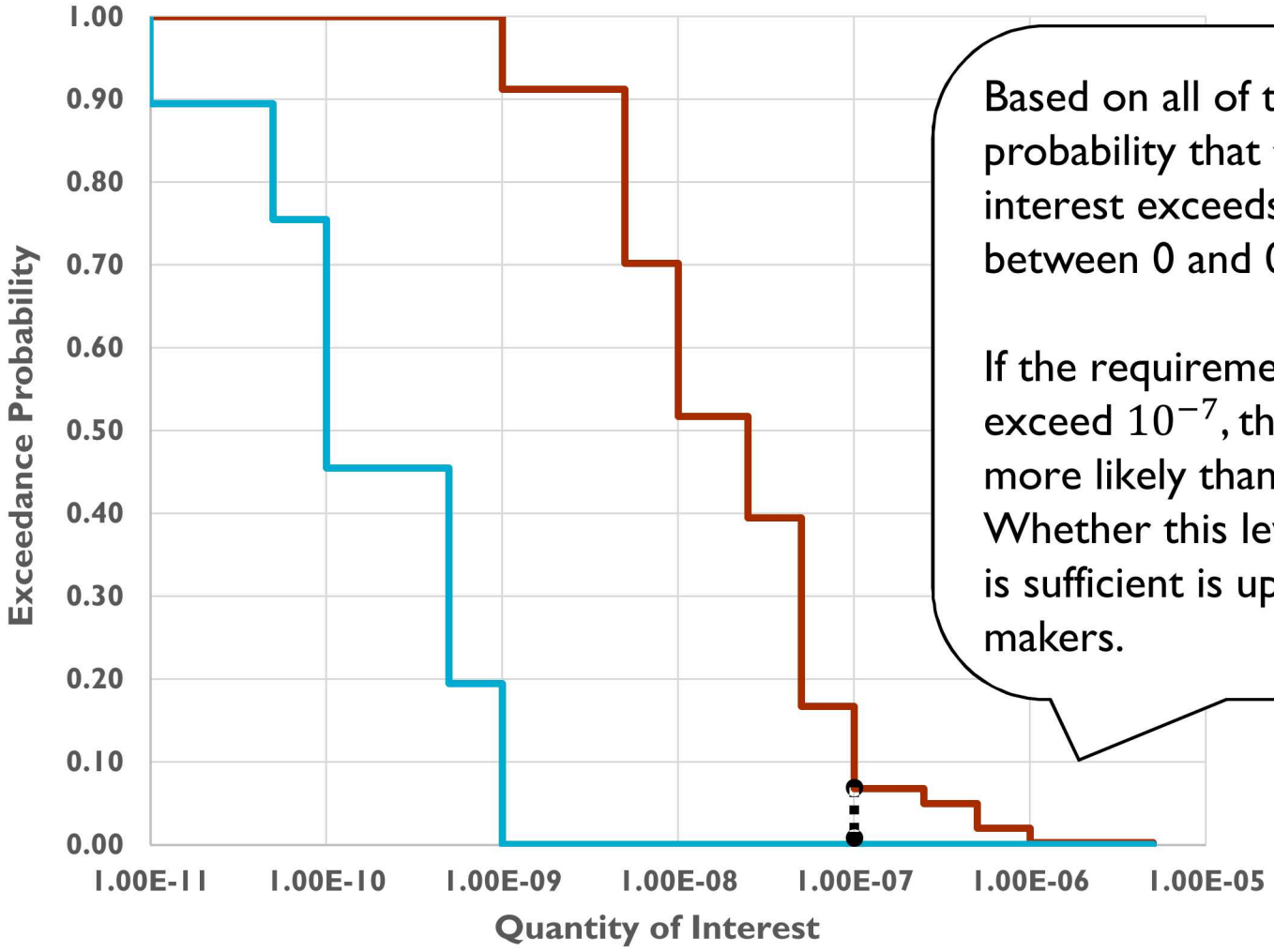


Based on all of the evidence, the probability that this quantity of interest exceeds 10^{-8} is between 0 and 0.52.

If the requirement is not to exceed 10^{-8}, the requirement might be met, but you probably want more evidence.

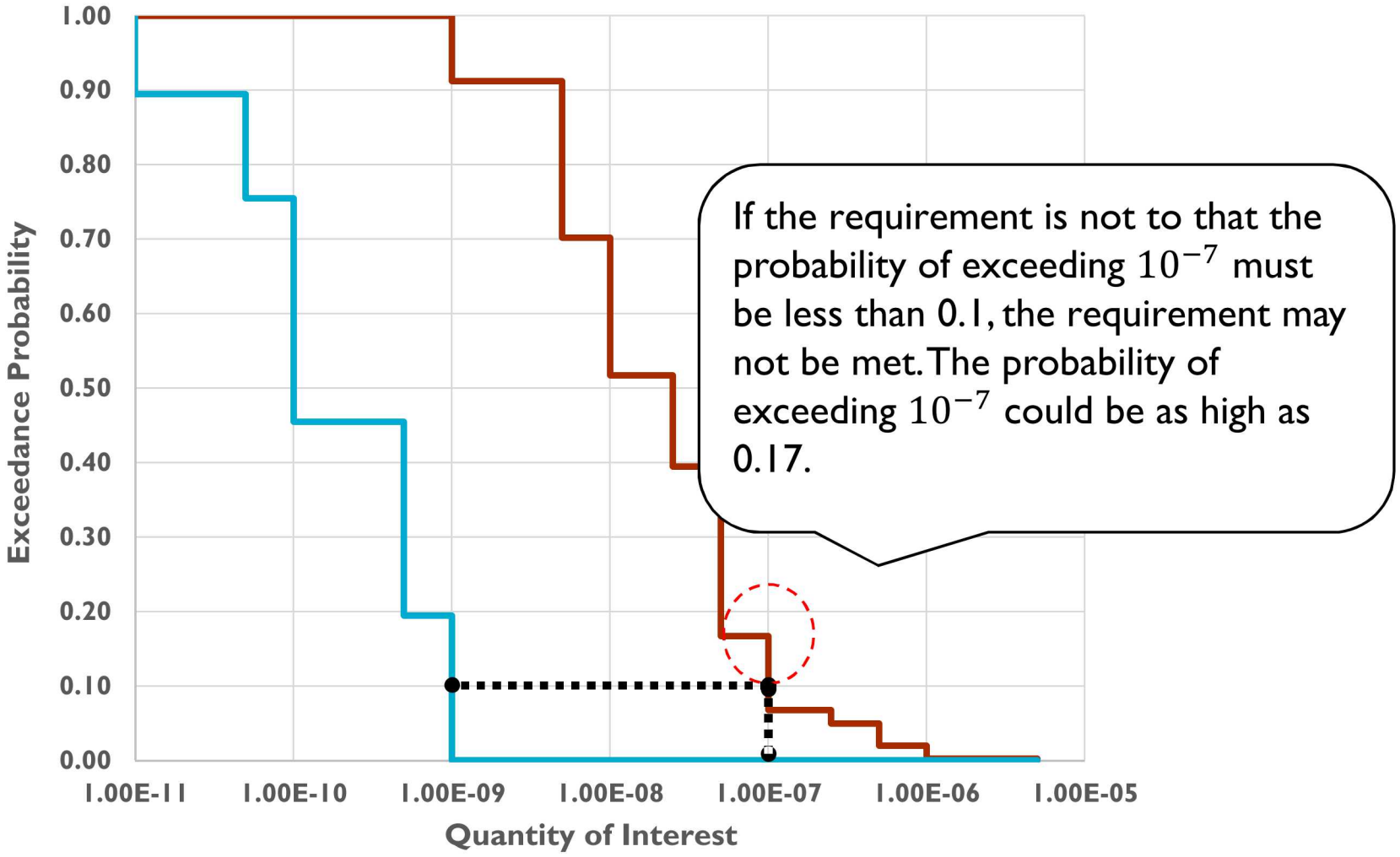
Belief/Plausibility: Result Interpretation



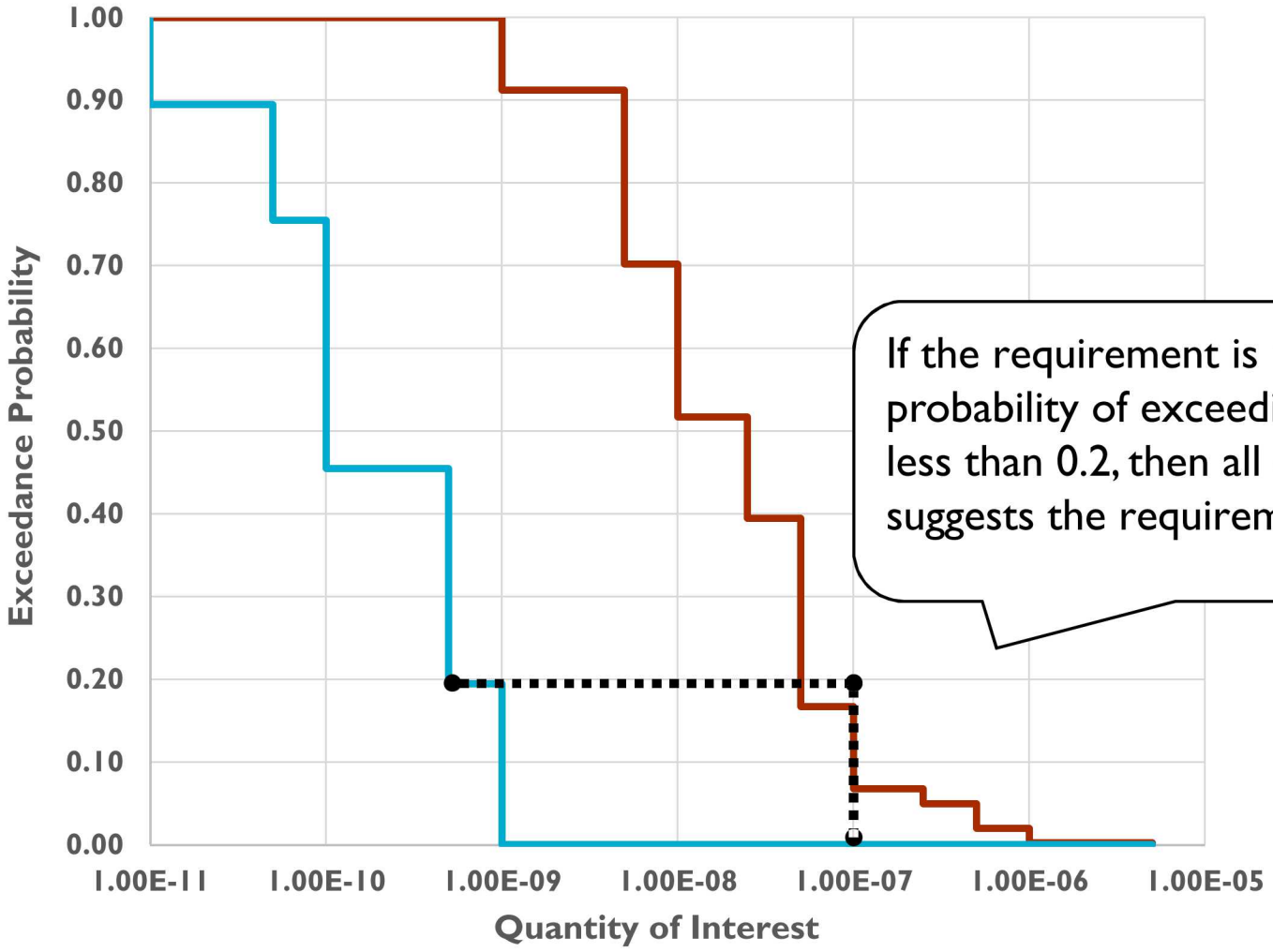
Based on all of the evidence, the probability that this quantity of interest exceeds 10^{-7} is between 0 and 0.07.

If the requirement is not to exceed 10^{-7}, the requirement is more likely than not to be met. Whether this level of confidence is sufficient is up to decision-makers.

Belief/Plausibility: Result Interpretation



Belief/Plausibility: Result Interpretation



If the requirement is not to that the probability of exceeding 10^{-7} must be less than 0.2, then all evidence suggests the requirement is met.

Belief and Plausibility

Subjective means of organizing subject matter expert state of knowledge

A means of communicating "state of knowledge uncertainty".

Belief is a lower bound

- Value is believed to be greater than this amount.
- Evidence must fully support this.
- *Alternative view*: Evidence that supports.

Plausibility is a upper bound

- It is not perceived likely that the value could be larger than this.
- Evidence at least partially supports this.
- *Alternative view*: Evidence that does not contradict.

Evidence is a form of information elicited from an expert.

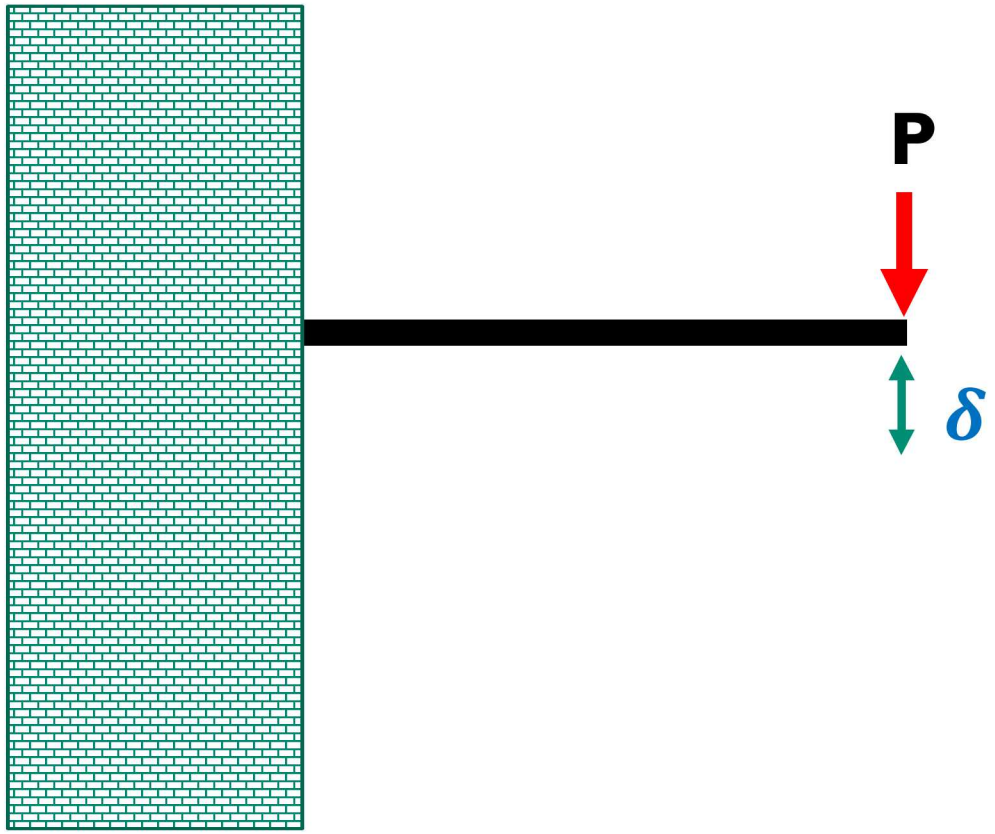
- Weighted information
- Subjective
- Flexible

Example Question of Interest

What is the maximum amount of deflection (δ) that should be expected for a beam attached on one side to a wall, with a point load P applied to the free end?

Specificity Required:

- Beam dimensions
  - L, W, H
- Material
  - Type A
- Loading
  - No twisting
- Wall
  - Fixed point



Evidence Elicited from a Single Expert

Could elicit evidence for variables such as L, W, H, E, P and convolute evidence (analytically) to deflection space, but will use simplified example here.

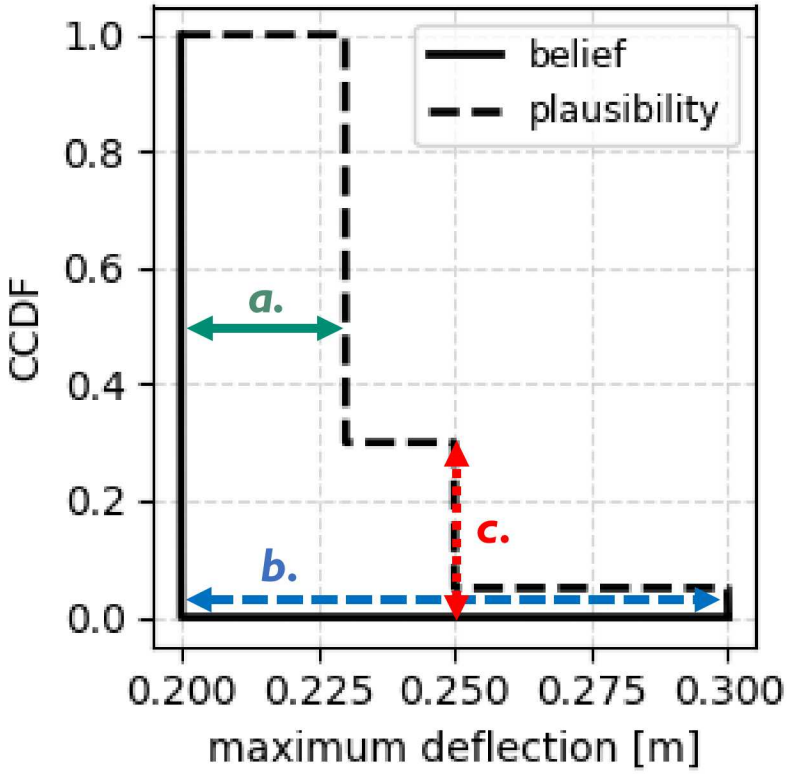
- Current limitation: convolution can be performed for +, -, x

Expert believes it is highly likely that the maximum beam deflection is between 0.2 and 0.23 meters.

Expert concedes lesser possibility that imperfections in beam manufacturing process could lead to deflections between 0.2 and 0.25 meters.

An additional safety hedge for unforeseen aging effects on the beam, deemed highly unlikely, is consider to extend the possible deflection span between 0.2 and 0.3 meters.

Visualizing Evidence

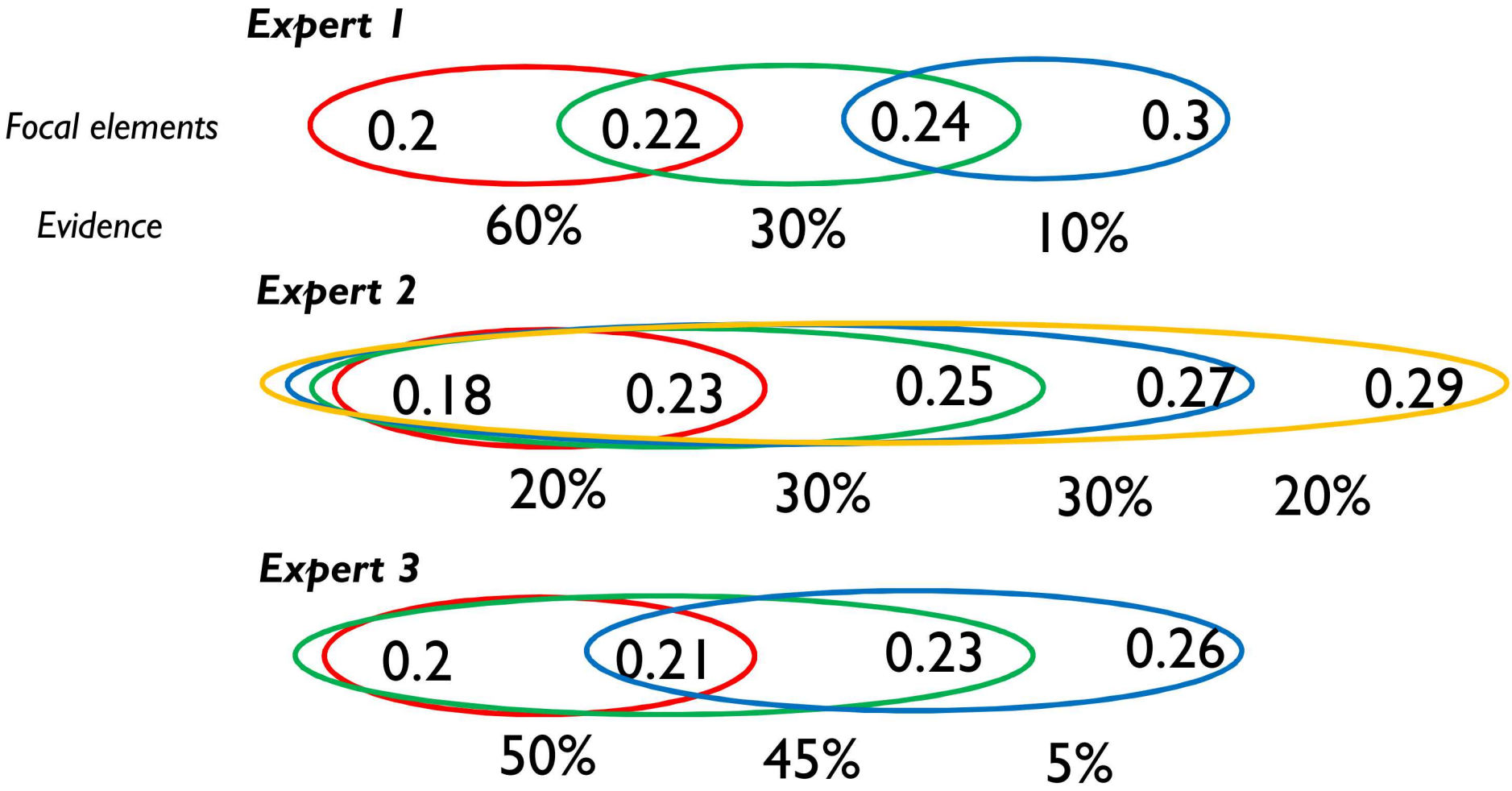


a. Median estimate is that the maximum deflection will be less than between 0.2 and 0.23 meters.

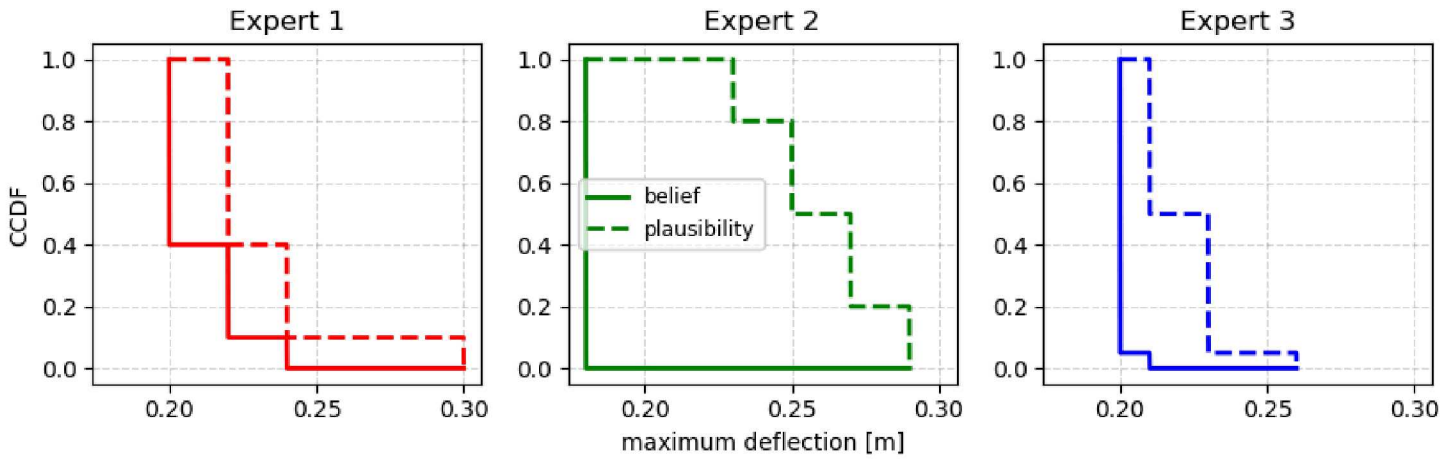
b. 95% probability that the maximum deflection will be less than 0.3 meters.

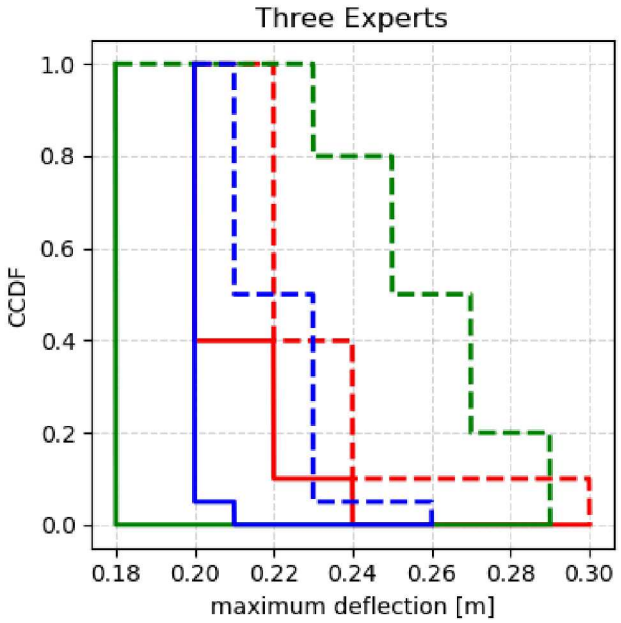
c. There is between 0 and 30% chance that the maximum deflection could be as high as 0.25 meters.

Elicited Evidence from Multiple Experts



Visualizing Evidence

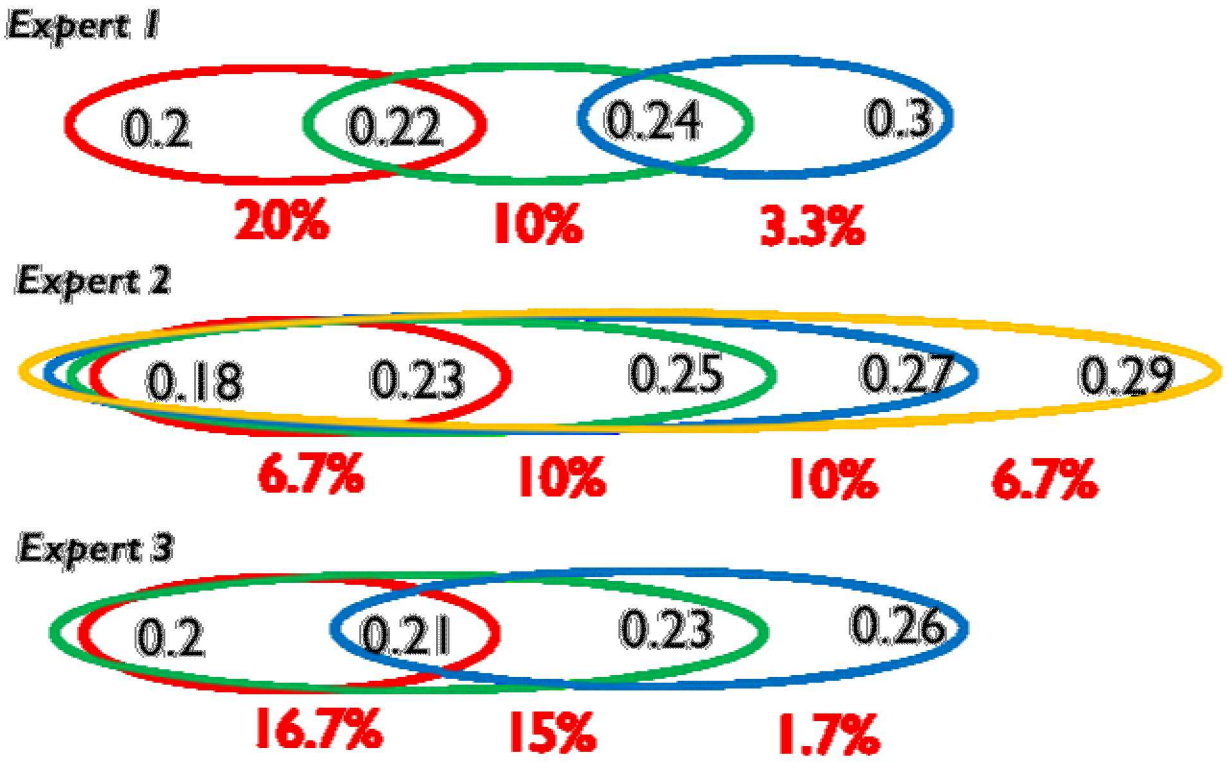




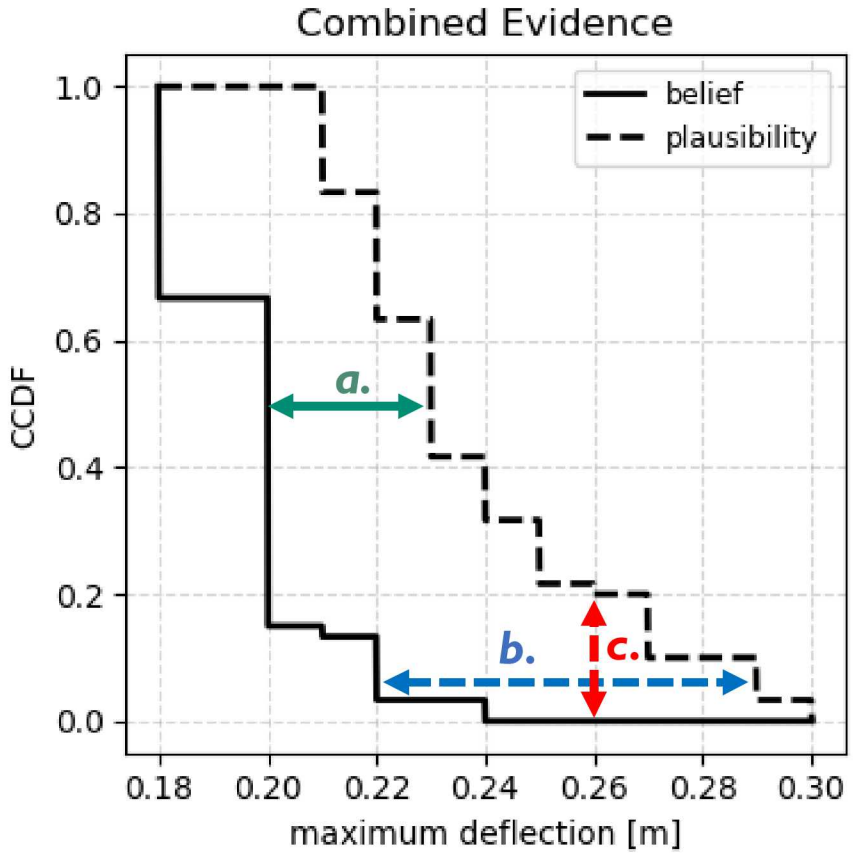
Combining Evidence from Multiple Experts

Combing evidence here using equal weight

- Single expert evidence x 1/3
- Could place different weights based on expertise or basis of evidence



Visualizing Combined Evidence



a. 50% probability that maximum deflection is as high as 0.23 meters.

b. 95% probability that maximum deflection is as high as 0.29 meters.

c. Probability that maximum deflection exceeds 0.26 meters is between 0 – 20%.