

Study of Greenland subglacial hydrology at medium-high resolutions



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Abstract

The interaction between ice sheets and the underlying bedrock is one of the most important factors driving the dynamics of ice. Quantifying the extent of a subglacial hydrology system can help to obtain more realistic sliding laws, and therefore help to improve the reliability of ice sheet simulations. Recently, several modeling choices have been proposed for subglacial hydrology, covering both a distributed drainage system as well as concentrated channels networks.

In this work, we consider the case of a distributed drainage system, and we explore some of the models already proposed in literature (Schoof et al. 2012, Hewitt 2013, Bueler 2015). Such models are usually formulated in terms of two unknowns, namely the water pressure and the thickness of the water layer, and typically involve on the order of 10 scalar parameters. Here we are interested in studying the sensitivity of the model to some of these parameters, with the goal to identify which parameters most impact the ice sliding law, and therefore need to be accurately estimated from available measurements (via data assimilation techniques).

Mathematical model

We consider a steady distributed system for subglacial hydrology of the form

$$F(\Phi, h) : \begin{cases} \frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \omega + \frac{m}{\rho_w} & \text{mass conservation} \\ \frac{\partial h}{\partial t} = \frac{h_r - h}{l_r} |u| - c_c A h N^3 & \text{cavities evolution} \end{cases} \quad (1)$$

with melting $m = (G + \beta |u|^2)/L$, water discharge $\mathbf{q} = -kh^\alpha |\nabla \Phi|^\beta \nabla \Phi$, ice thickness H , transmissivity k , water source ω , geothermal flux G , ice sliding velocity u , bed bumps height/length h_r, l_r are given, effective pressure $N = \rho_w g H - \Phi$, and sliding friction coefficient $\beta = \beta(|u|, N)$, with functional form

$$\beta = \mu N \left(\frac{|u|}{|u| + \lambda A N^3} \right)^q \frac{1}{|u|}.$$

Goal: estimate μ, λ, q in the functional form of β , as well as k, c_c in the hydrology model.

Method: minimize mismatch between computed β and target $\bar{\beta}$, where $\bar{\beta}$ has been estimated solving another inverse problem for the ice only, assimilating surface velocity measures.

$$[\mu, \lambda, q, k, c_c] = \arg \min \|\beta - \bar{\beta}\| \quad s.t. \quad F(\Phi, h) = 0.$$

Implementation

We used **Albany**, a C++ parallel finite element library (in particular the subpackage **LandIce**) which provides an implementation of the FO ice model, the hydrology model and analysis/sensitivities capabilities, for the estimation of scalar and distributed parameters. It is part of E3SM, as a dycore for the land ice component of MPAS.

Albany relies on several packages within the **Trilinos** library. In particular, **Belos** for the solution of linear systems, **ML**, **Ifpack**, **MueLu** and **Ifpack2** for the preconditioners, **NOX** and **ROL** for nonlinear solvers and optimization, **Interpid2** and **Phalanx** for finite element assembly.

Numerical experiments

- Greenland ice sheet mesh: 25k triangular elements (8km resolution).
- Pre-processing: compute $\bar{\beta}$ solving a First-Order ice problem, assimilating surface velocity measures.
- Optimization: BFGS with backtrack line search.

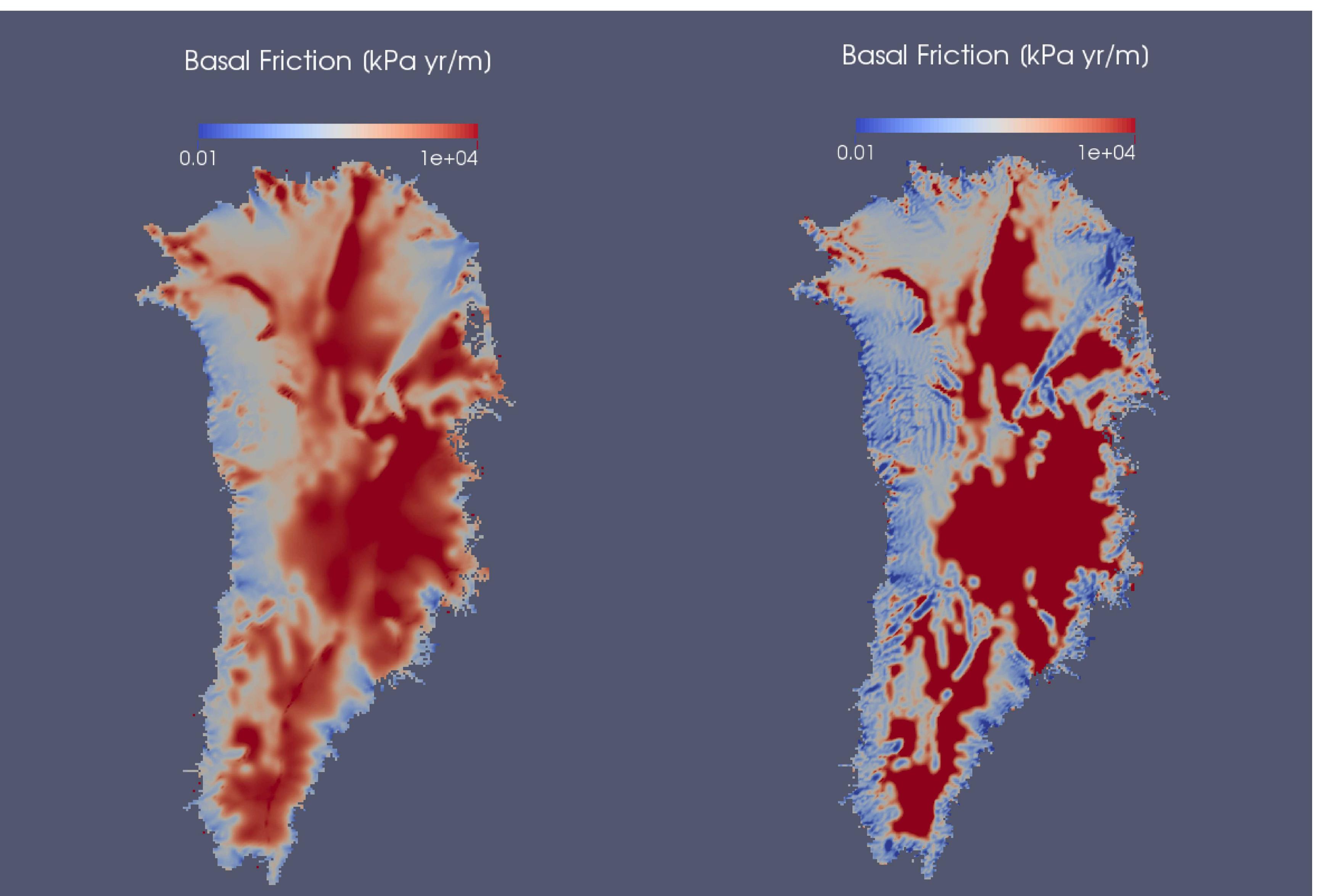


Figure 1: Computed (left) and target (right) basal friction coefficient β . Optimal parameters: $\mu \simeq 0.02, q \simeq 0.049, \lambda \simeq 0.022, k \simeq 0.2, c_c \simeq 0.13$.

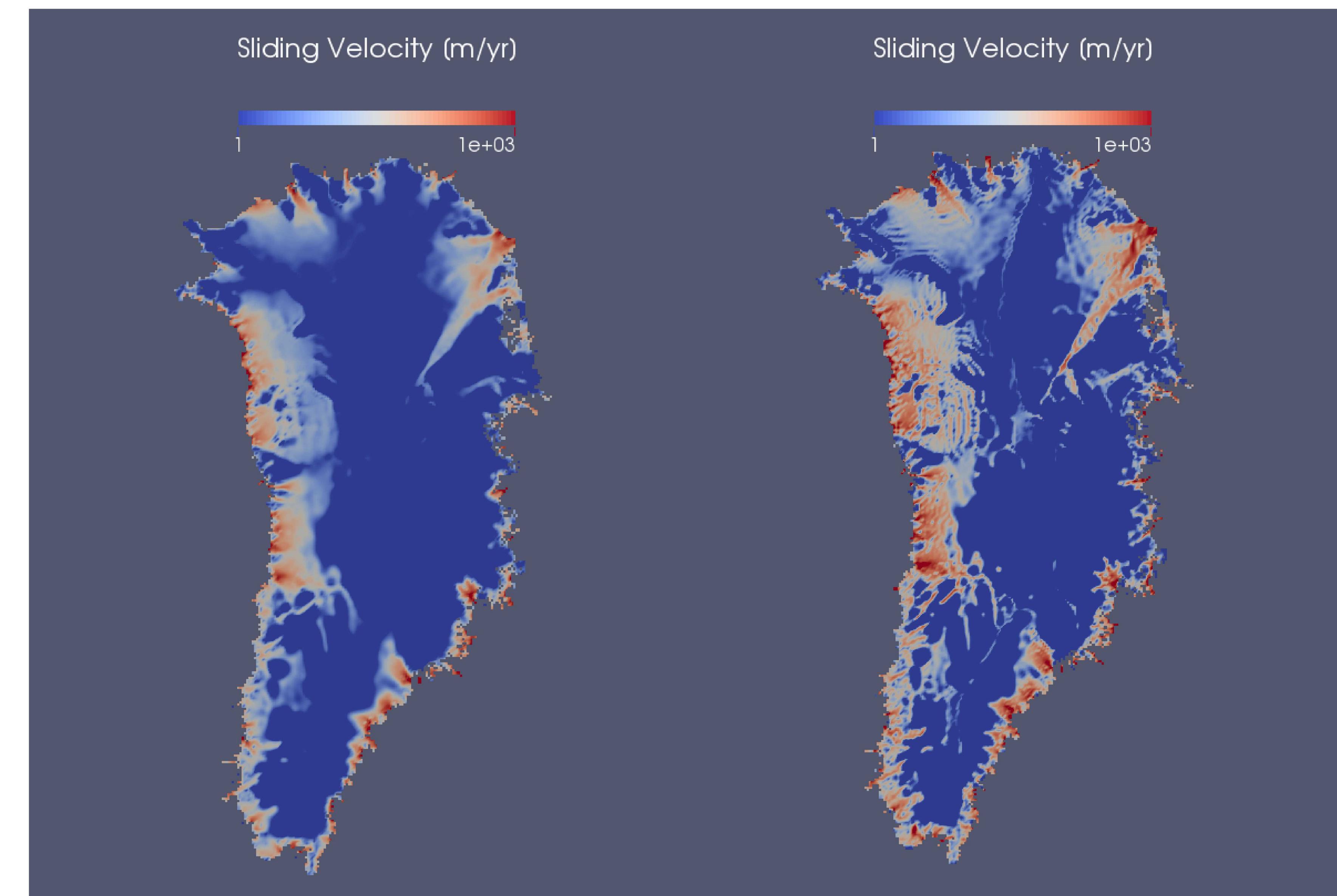


Figure 2: Computed (left) and target (right) basal sliding velocity.

Considerations and future directions

- Sliding law yields much more *regular* fields.
- Loss of details compensated by simplicity and potential reusability of parameters.
- Optimal parameters seem off compared to literature 'suggestions'.
- Question: are optimal parameters mesh dependent?
- Todo: run simulations at high resolution (2km). Try unsteady.
- Todo: improve convergence properties of inverse problem.
- Todo: verification & validation.

References:

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- C. Schoof, *Coulomb friction and other sliding laws in a higher-order glacier flow model*, Math. Models Methods Appl. Sci., 2010

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