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Sandia National Laboratories

Engineered thermalization in quantum many-body systems

Mohan Sarovar
Sandia National Laboratories, Livermore

Physics of Quantum Electronics, January 2019

U.S. DEPARTMENT OF ENERGY

National Nuclear Security Administration

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Analog quantum simulation

- Emulate one quantum system by engineering another to mimic it
- Feynman's original idea for *quantum supremacy*
  Feynman, Int. J. Theor. Phys. **21**, 467 (1982)
- Already an experimental reality:

nature physics | ARTICLES

PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NPHYS2232

Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas

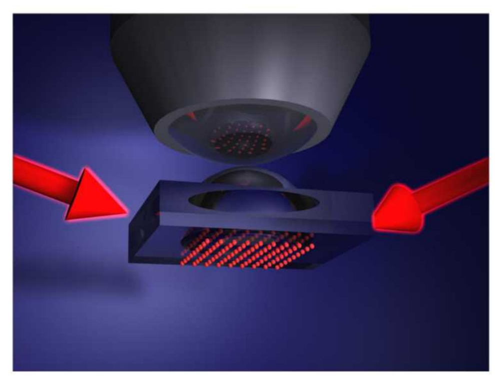
S. Trotzky[1,2,3]★, Y-A. Chen[1,2,3], A. Flesch[4]★, I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7,8] and I. Bloch[1,2,3]

nature | Vol 465 | 3 June 2010 | doi:10.1038/nature09071

LETTERS

Quantum simulation of frustrated Ising spins with trapped ions

K. Kim[1], M.-S. Chang[1], S. Korenblit[1], R. Islam[1], E. E. Edwards[1], J. K. Freericks[2], G.-D. Lin[3], L.-M. Duan[3] & C. Monroe[1]



Greiner Lab, Harvard

ARTICLE

doi:10.1038/nature09994

Quantum simulation of antiferromagnetic spin chains in an optical lattice

Jonathan Simon[1], Waseem S. Bakr[1], Ruichao Ma[1], M. Eric Tai[1], Philipp M. Preiss[1] & Markus Greiner[1]

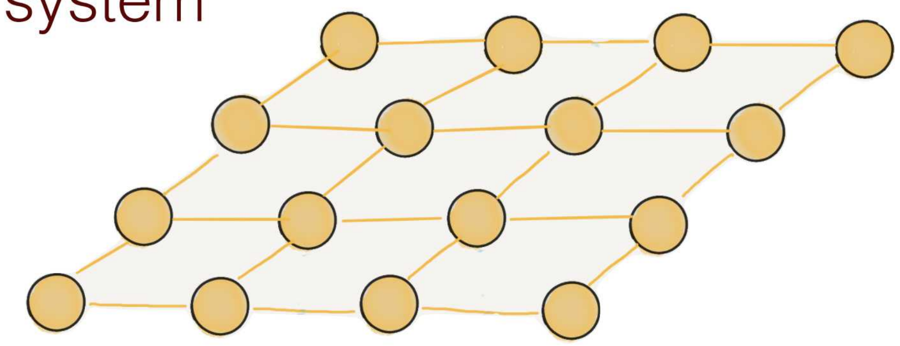
New Journal of Physics
The open-access journal for physics

Quantum simulation of the transverse Ising model with trapped ions

K Kim[1,5,6], S Korenblit[1], R Islam[1], E E Edwards[1], M-S Chang[1,7], C Noh[2,8], H Carmichael[2], G-D Lin[3,9], L-M Duan[3], C C Joseph Wang[4], J K Freericks[4] and C Monroe[1]

Simulation of many-body systems

Many-body lattice system



Engineered Hamiltonian

$$H$$

How do we prepare a thermal state at temperature T? $\rho(\beta) = \frac{e^{-\beta H}}{\mathcal{Z}}$

The apparatus has a physical temperature, but this is not always relevant.

The errors/noise on the logical degrees of freedom may be non-equilibrating.

e.g. Spin lattice, and independent depolarizing channel on each spin

$$\rho(t) \to \frac{\mathbf{1}}{n}$$

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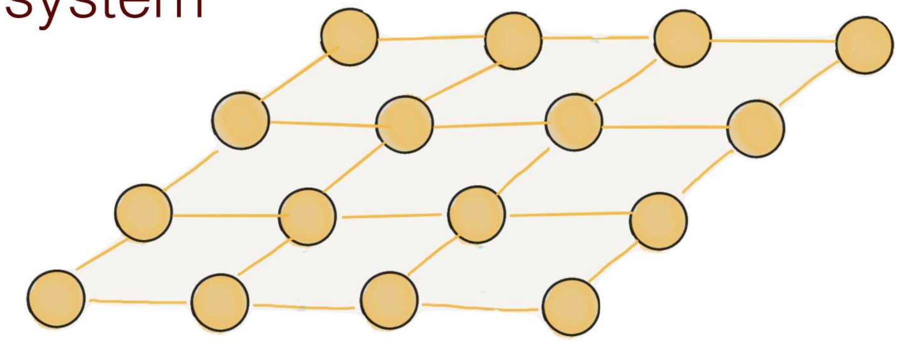

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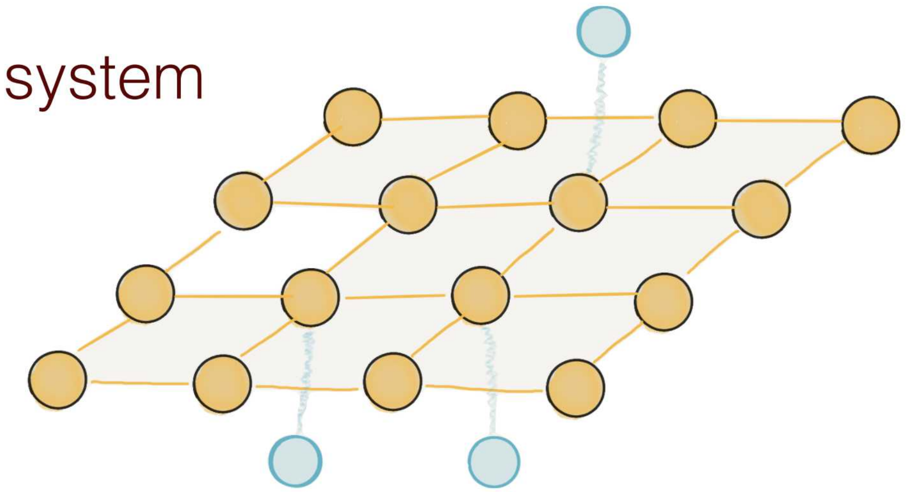
Need to *engineer* thermalization also

How to thermalize?

Many-body lattice system



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Engineered dissipative evolution

$$\dot{\rho(t)} = \mathcal{L}\rho(t)$$

How to thermalize?

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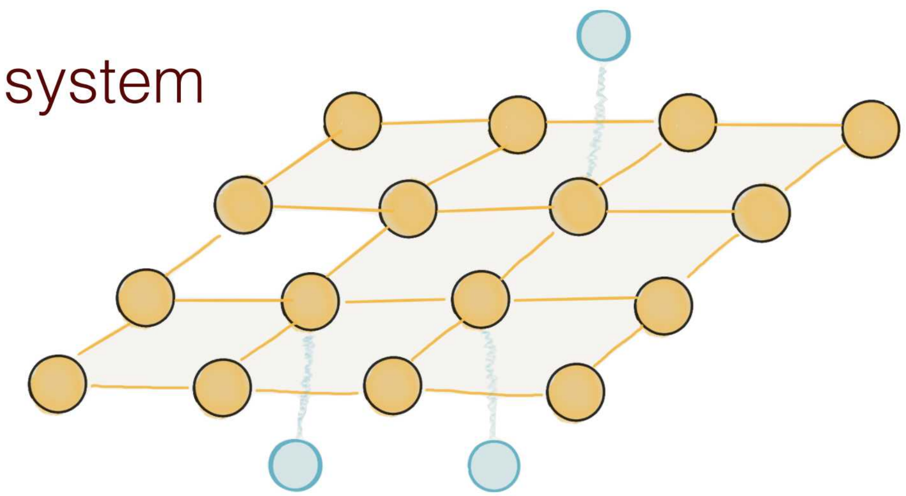


Engineered Hamiltonian

$$H$$

Engineered dissipative evolution

$$\dot{\rho}(t) = \mathcal{L}\rho(t)$$

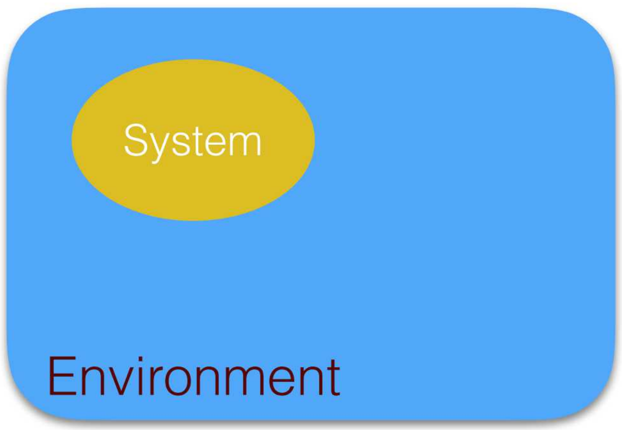
But what kind of evolution will result in a thermal state?

Only general characterization known [*e.g.* Breuer & Petruccione, The theory of open quantum systems]:

Born-Markov master equation + ergodicity + KMS conditions => thermal steady state

How to thermalize?

Born-Markov master equation + ergodicity + KMS conditions => thermal steady state

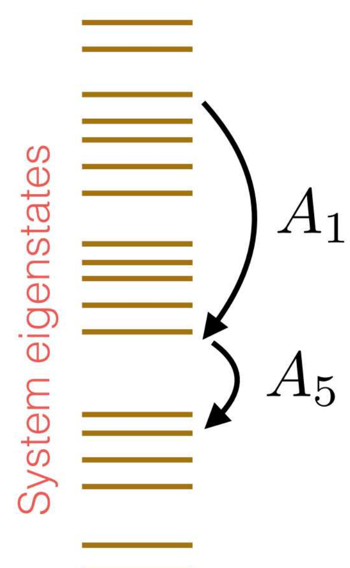


- Weak coupling to many degrees of freedom
- Fast relaxing environment

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{\omega,\alpha,\beta} \gamma_{\alpha\beta}(\omega)\left(A_\beta(\omega)\rho A_\alpha^\dagger(\omega) - \frac{1}{2}\{A_\alpha^\dagger(\omega)A_\beta(\omega)\}\right)$$

$$[A_\alpha, X] = [A_\alpha^\dagger, X] = 0$$

$$\Rightarrow X \propto 1$$



$$\frac{\gamma_{\alpha\beta}(-\omega)}{\gamma_{\beta\alpha}(\omega)} = e^{-\beta\omega}$$

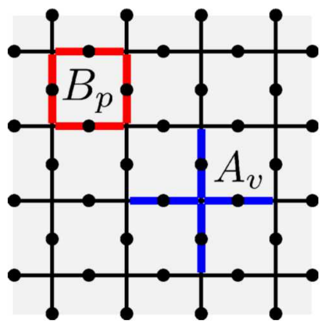
Bath induced fluctuations and dissipation satisfy detailed balance

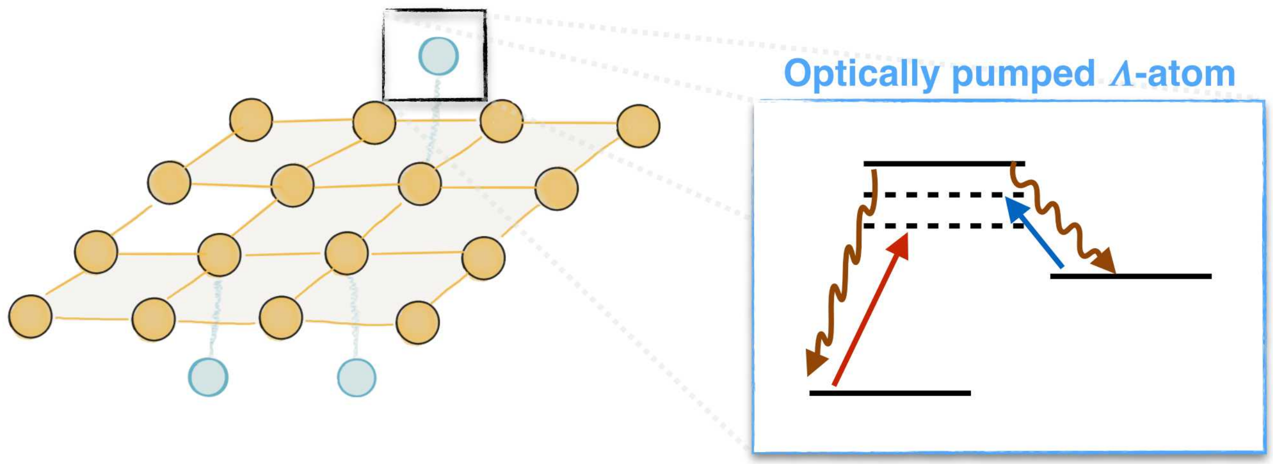
A constructive approach to thermalization for stabilizer models

K. Young, M.S. *et al.* J. Phys. B, **45,** 154012 (2012)

e.g. toric code

$$H_{TC} = -\lambda_e \sum_v A_v - \lambda_m \sum_p B_p$$



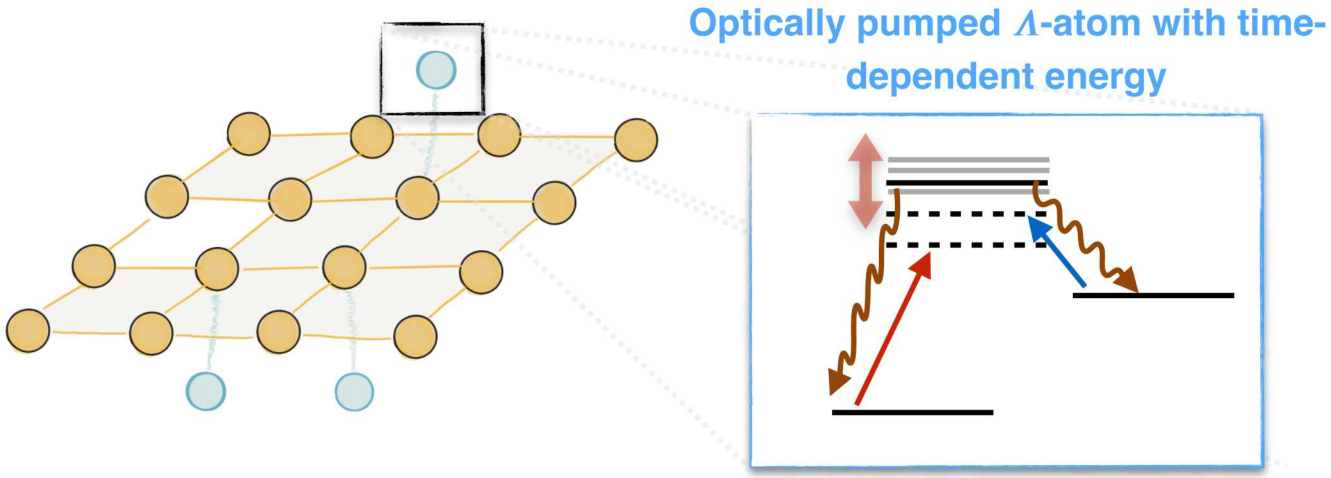


A number of special properties of stabilizer Hamiltonians make this work:

1. Easily predicted <u>equidistant</u> spectra => we can choose the energies of <u>few</u> ancilla to achieve resonant energy exchange

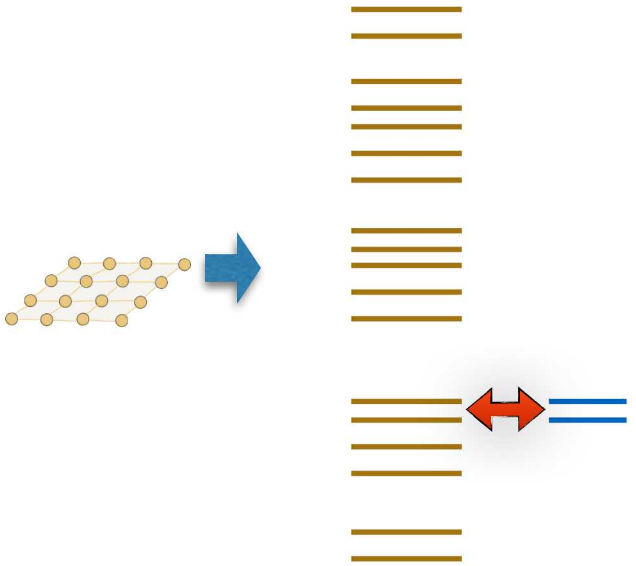
2. Local perturbations create energy excitations => so local couplings to ancilla move one up and down between eigenstates (ergodicity easy)

A generalization to arbitrary models



We assume ergodicity is satisfied by the system-ancilla couplings

This mimics a macroscopic bath over some longer timescale



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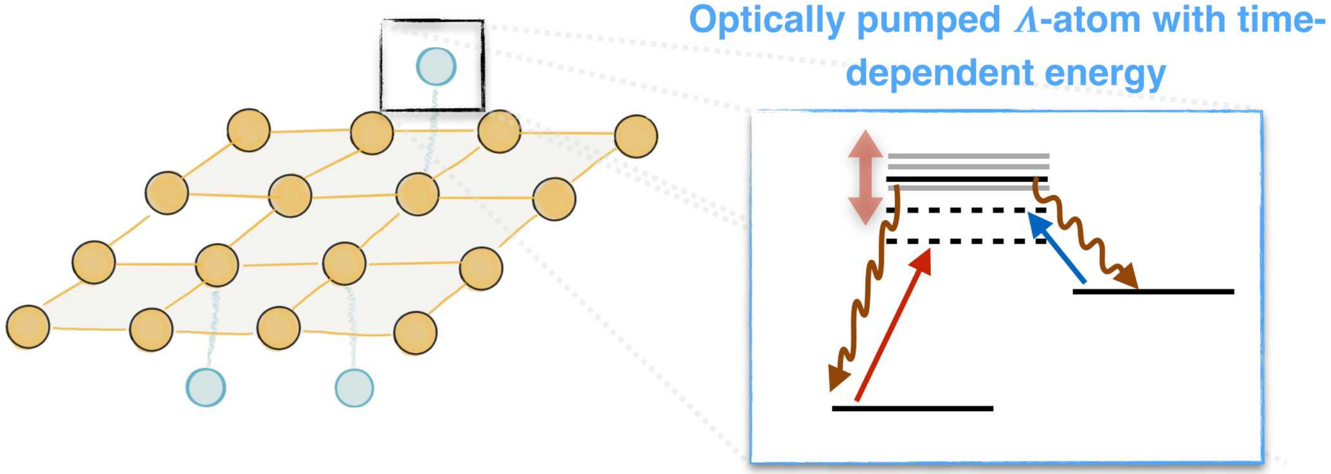

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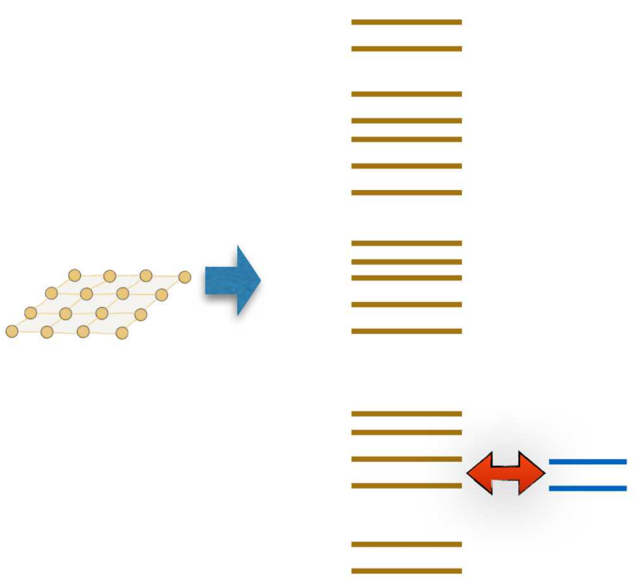

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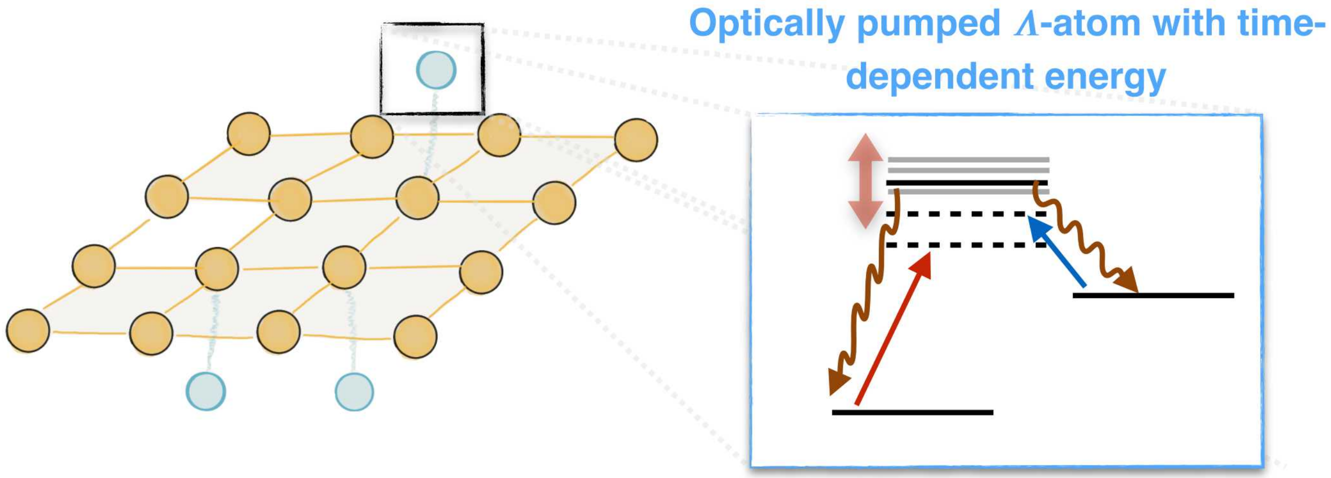

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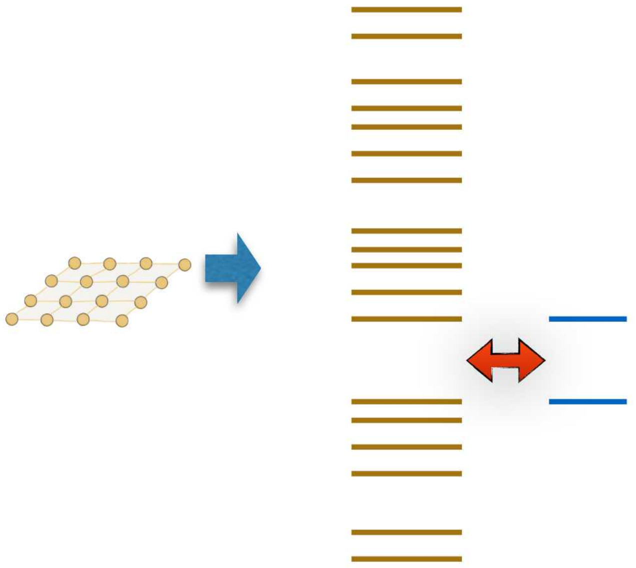
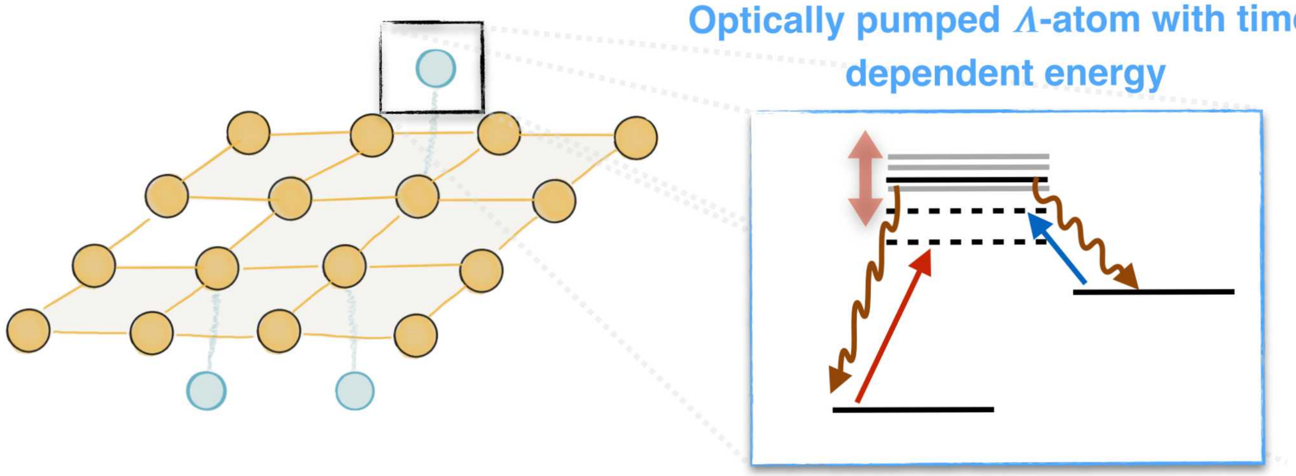


Ancillas will be resonant with different energy transitions in the system at different times, so as long as we maintain Boltzmann populations in the ancilla at all time, we will induce transitions that thermalize the system.

Spin lattice example



$$\frac{dr^m(t)}{dt} = \mathcal{L}_m(t)[r^m(t)] = \gamma_+^m(t)\mathcal{D}[\tau_+^m]r^m(t) + \gamma_-^m(t)\mathcal{D}[\tau_-^m]r^m(t),$$

$$H_T(t) = H_{\rm sys} - \sum_{m=1}^{M} \frac{\Omega_m(t)}{2}\tau_z^m + \sum_{m=1}^{M} g_m\left(\sigma_x^{k_m}\tau_x^m\right),$$

We require the parameter regime

$$|\frac{d\Omega_m(t)}{dt}| \ll g_m \sim \Gamma^m \ll ||H_{\rm sys}||, \quad \forall m, t$$

$$\Gamma^m \equiv \gamma_+^m + \gamma_-^m$$

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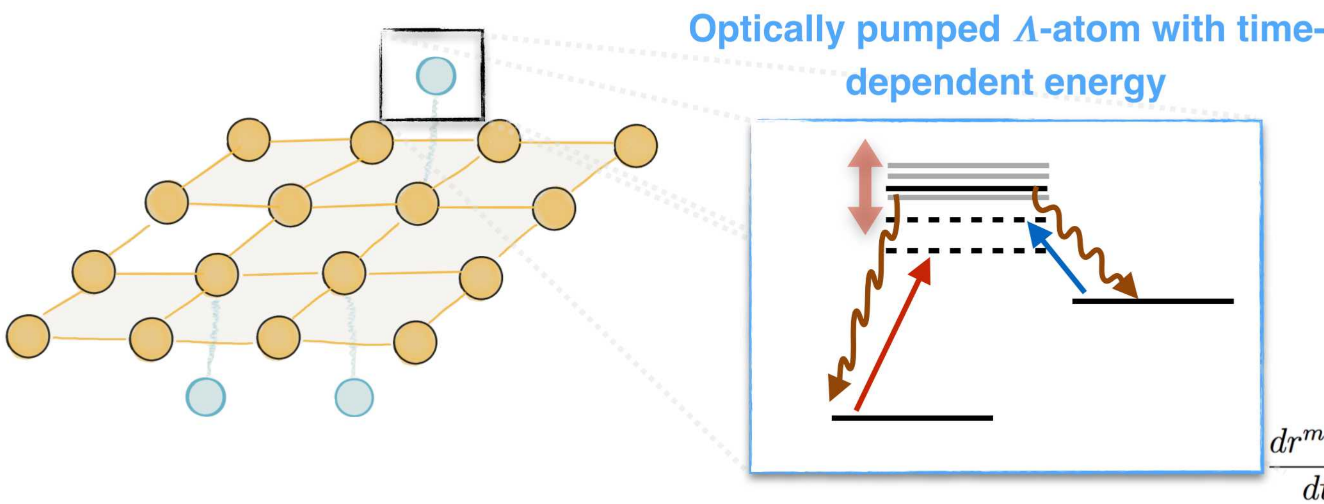

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In this regime we can derive a reduced master equation for the system alone that describes its evolution when the ancilla dynamics is averaged over.

Removing the ancilla DOF

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$$\left|\frac{d\Omega_m(t)}{dt}\right| \ll g_m \sim \Gamma^m \ll ||H_{\text{sys}}||, \quad \forall m, t$$

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In this regime we can derive a reduced master equation for the system alone that describes its evolution when the ancilla dynamics is averaged over.

A reduced master equation describes system evolution

when the ancilla dynamics is averaged over

Operator on System

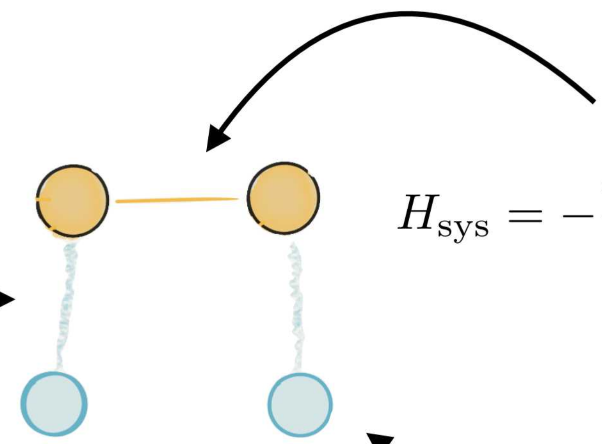
$$\dot{\rho} = g^2 \sum_{\omega} \gamma(t,\omega) \left(X(\omega)\rho X^\dagger(\omega) - \frac{1}{2}\{X^\dagger(\omega)X(\omega), \rho\} \right) \quad X(\omega) = \sum_{\epsilon'-\epsilon=\omega} |\epsilon\rangle \langle\epsilon| \sigma_x^k |\epsilon'\rangle \langle\epsilon'|$$

Ancilla Correlation Functions

$$\gamma(t,\omega) = \frac{P_0(t)\Gamma}{\left(\frac{\Gamma}{2}\right)^2 - (\omega - \Omega(t))^2} + \frac{P_1(t)\Gamma}{\left(\frac{\Gamma}{2}\right)^2 - (\omega + \Omega(t))^2}$$

A simple example

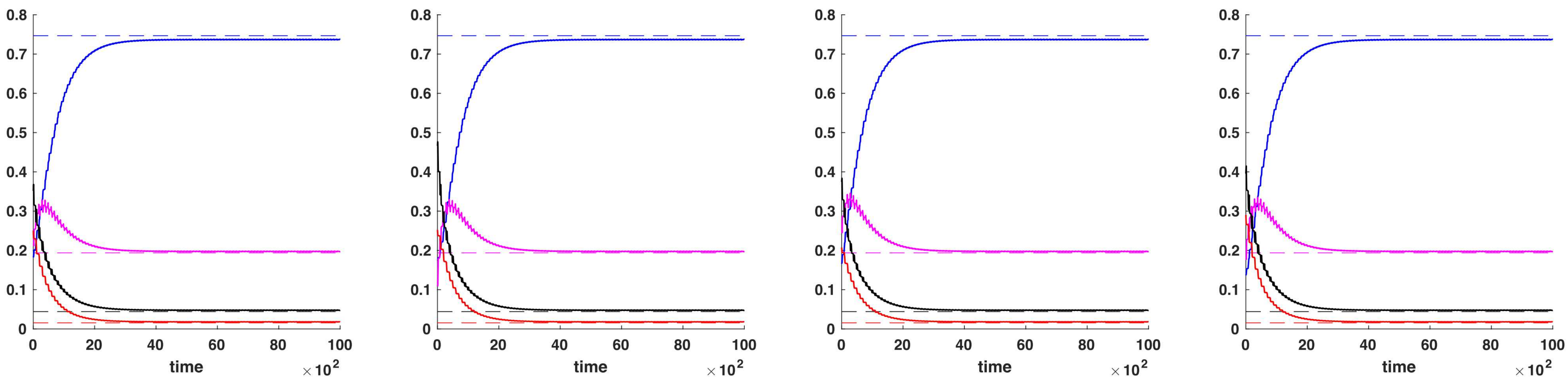
$$H_{\text{s-a}} = \sum_{i=1}^{2} g(\sigma_x^i \tau_x^1)$$



$$H_{\text{sys}} = -\frac{\omega_1}{2}\sigma_z^1 - \frac{\omega_2}{2}\sigma_z^2 + J_x\sigma_x^1\sigma_x^2 + J_y\sigma_y^1\sigma_y^2 + J_z\sigma_z^1\sigma_z^2$$

$$\Omega_i(t) = \Omega_{\text{min}} + \Omega_{\text{max}}\sin(\omega t)$$

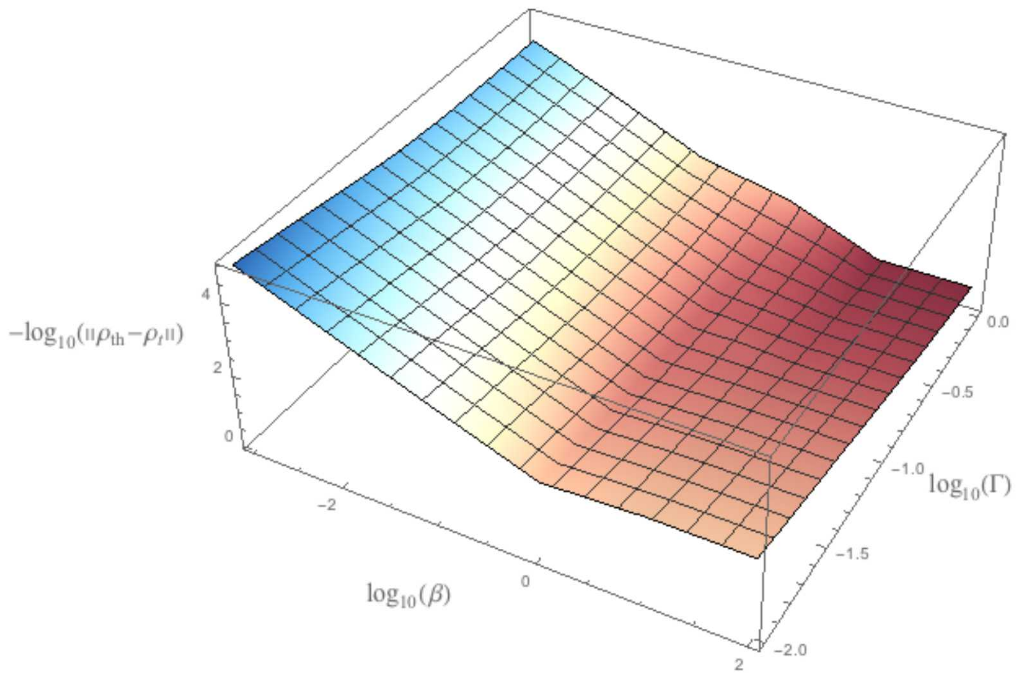
Thermalization from four random initial states



Examining the fixed point of dynamics

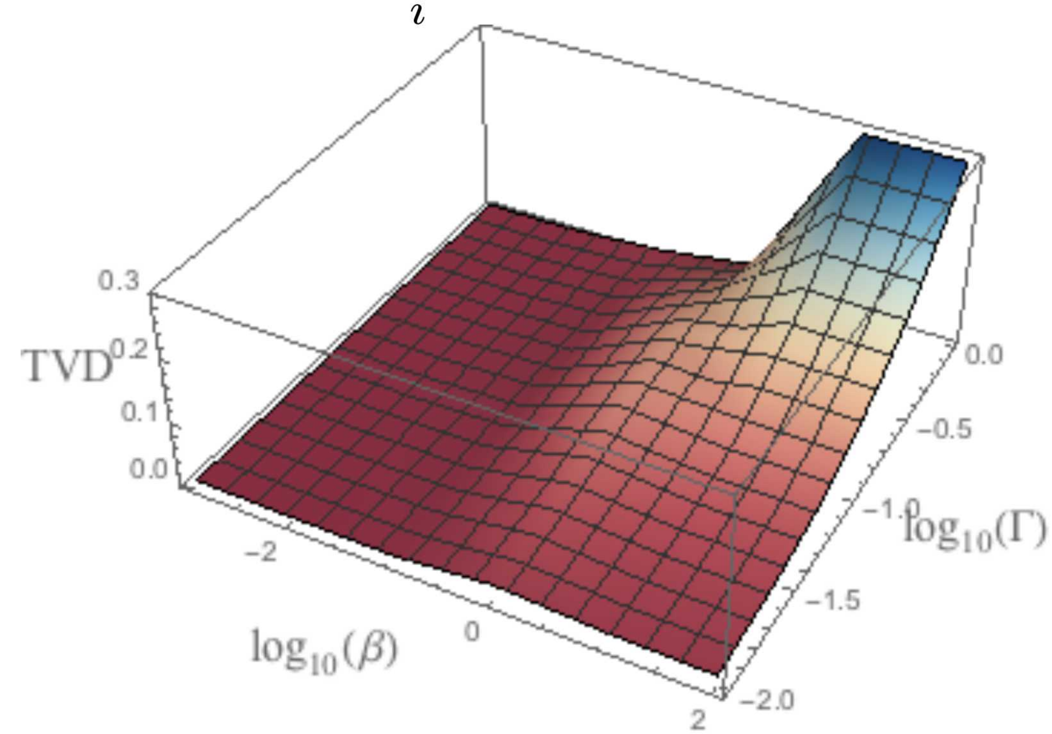
Frobenius Norm

$$||\rho^{th} - \rho^{*}||$$



Total Variation Distance in system's eigenbasis

$$\sum_i \left| p_i^{th} - p_i^{*} \right|$$



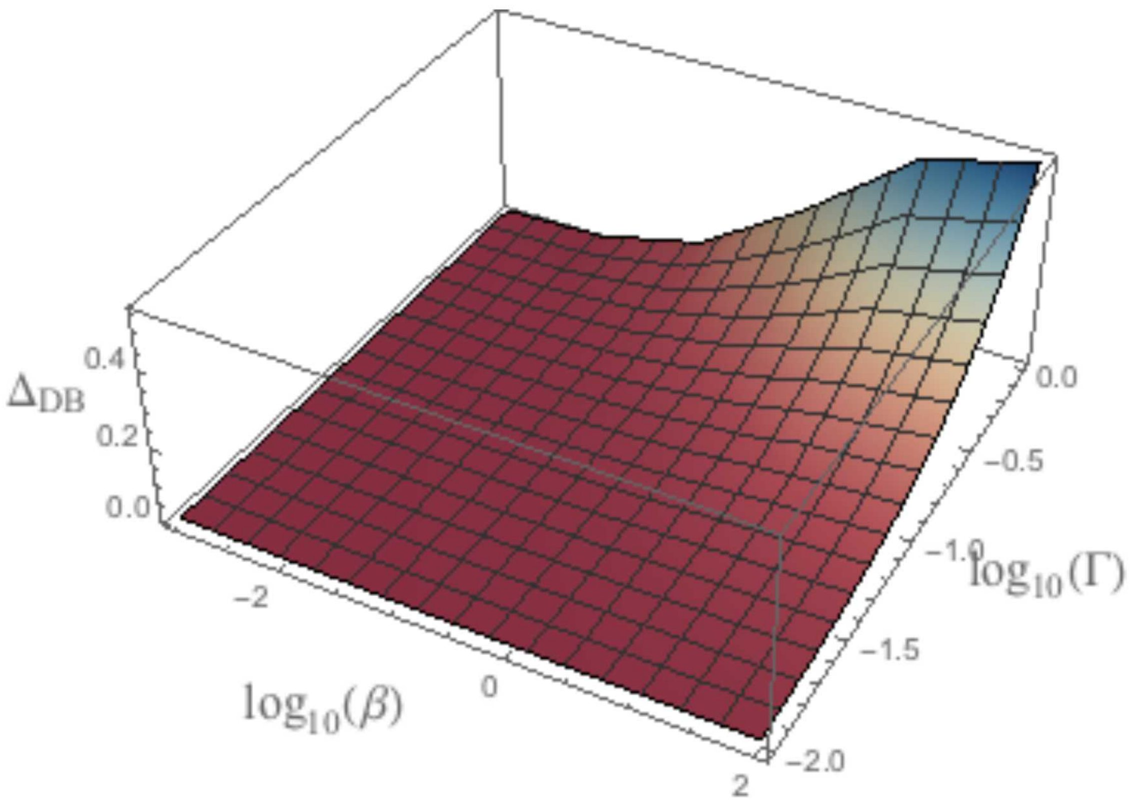
Detailed balance

The regimes of poor thermalization are marked by a breakdown of detailed balance in the engineered reservoir

$$\frac{\gamma(-\omega,t)}{\gamma(\omega,t)} = e^{-\beta\omega}$$

$$\gamma(t,\omega) = \frac{P_0(t)\Gamma}{\left(\frac{\Gamma}{2}\right)^2 - (\omega - \Omega(t))^2} + \frac{P_1(t)\Gamma}{\left(\frac{\Gamma}{2}\right)^2 - (\omega + \Omega(t))^2}$$

$$\Delta = \int_0^{\omega_m} d\omega \int_0^T dt \left(\frac{\gamma(-\omega,t)}{\gamma(\omega,t)} - e^{-\beta\omega} \right)$$



Ongoing work

1. Bounds on thermalization time of this protocol

2. Discrete-time (gate-based version of the protocol
    1. Can derive strong bounds on performance in this case
    2. Useful for comparison to other discrete-time (gate-based) thermalization protocols (e.g. quantum Metropolis)

Thanks!

Collaborators

Mekena Metcalf
Sandia National Labs

Jonathan Moussa
Molecular Sciences Software Institute

Funding

U.S. DEPARTMENT OF ENERGY | Office of Science

ASCR programs:
Quantum Computing Application Teams
Quantum Algorithm Teams

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