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Multi-Stage Stabilized Continuation for Indirect Optimal Control of Hypersonic Trajectories

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Overview

Hypersonic Trajectory Generation

- Hypersonic optimal trajectory generation is generally difficult due to its nonlinearity.
- Direct optimization is the usual approach because it is easier to implement.
- Indirect optimization provides smooth and continuous solutions and can distill the trajectory into a compact representation.

Indirect Optimal Control

- Indirect optimal control transforms the optimization problem into a boundary value problem.
- This transformation is through the maximum principle or Euler-Lagrange conditions of optimality.
- A representation of the control inputs in terms of states and co-states is computed.

Planar Hypersonic Equations of Motion and Cost Function

- Planar hypersonic trajectory generation problem is governed by the following dynamics:

$$\dot{h} = v \sin\gamma$$
$$\dot{\theta} = \frac{v\cos\gamma}{r}$$
$$\dot{v} = \frac{-D}{m} - \frac{\mu \sin\gamma}{r^2}$$
$$\dot{\gamma} = \frac{L}{mv} + \left(\frac{v}{r} - \frac{\mu}{vr^2}\right)\cos\gamma$$

- With cost function:

$$J = -v(t_f)^2$$

Stabilized Continuation

Stabilized continuation is a powerful root solver which provides 3 unique benefits over other root solvers:

- Adaptive step-size selection
- Guaranteed error attenuation over the continuation interval in a numerically stable fashion
- Guaranteed convergence in one continuation interval

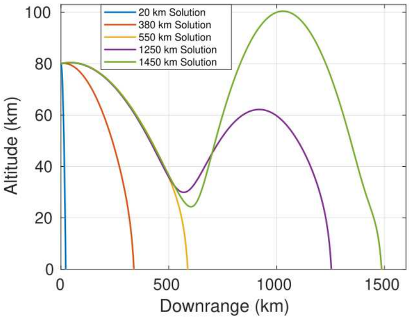
$$0 = \frac{d}{ds}F(z,s) = \frac{\partial F}{\partial s} + \frac{\partial F}{\partial z}\frac{dz}{ds} \doteq A_m F(z,s) + \nu_s \tag{1}$$

$$\frac{\mathrm{d}z}{\mathrm{d}s} = \left[\frac{\partial F}{\partial z}\right]^{-1}\left[A_m F(z,s) + \nu_s - \frac{\partial F}{\partial s}\right] \tag{2}$$

Continuing on Terminal Downrange

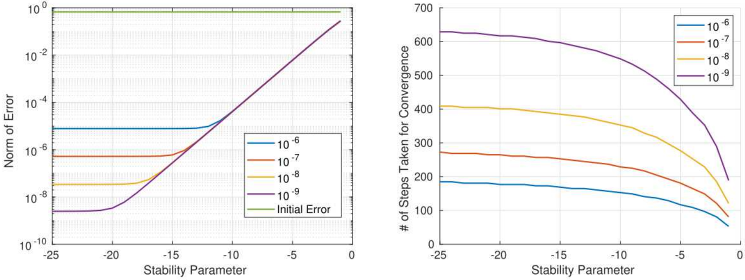
Continuation can be implicit or explicit. An example of explicit continuation on terminal downrange is:

$$\theta_f(s) = s\theta_{\text{desired}} + (1 - s)\theta_{\text{initial}}$$

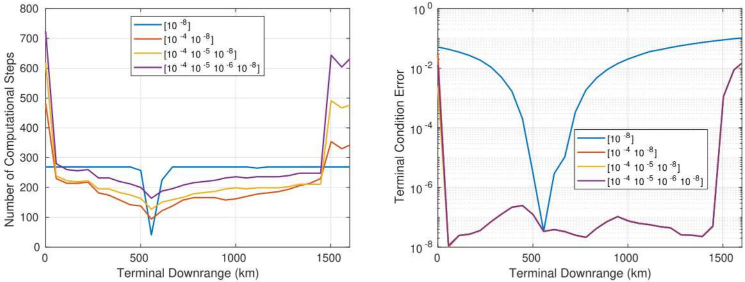


Parameter Tuning

When the stability parameter is small, stabilized continuation is invariant to integration tolerance. However, the algorithm takes more steps for a stricter tolerance.

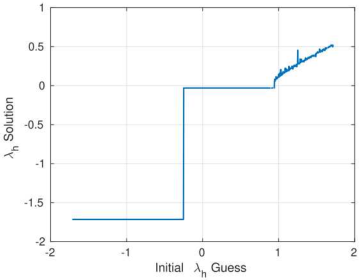


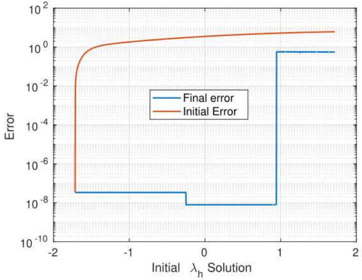
Multi-staged stabilized continuation reduces the computational effort of the algorithm, while increasing the radius of convergence.



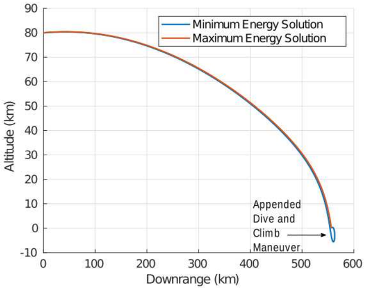
Initial Guess Robustness Study

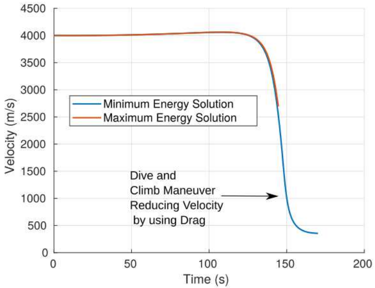
Solutions switch when giving a bad initial guess to the stabilized continuation solution with a “loose” integration tolerance.





Even where multi-stage stabilized continuation encounters numerical stiffness, it is still robust enough to find a high-quality trajectory.





Discussion

- Stabilized Continuation can transform high quality trajectories into very different equal quality trajectories over one continuation interval.
- Multi-Staging the Stabilized Continuation algorithm reduces the number of floating point operations required while expanding the robustness of the algorithm.
- Multi-Staged Stabilized Continuation converges even when given poor numerical accuracy.

Future Work

- Testing the Multi-Staged approach on more difficult Hypersonic applications (Including path constraints).
- Developing an auto-tuner for the number of stages and tolerance per stage of the Multi-Staged algorithm