

Statistical analysis of Z stellar opacity data

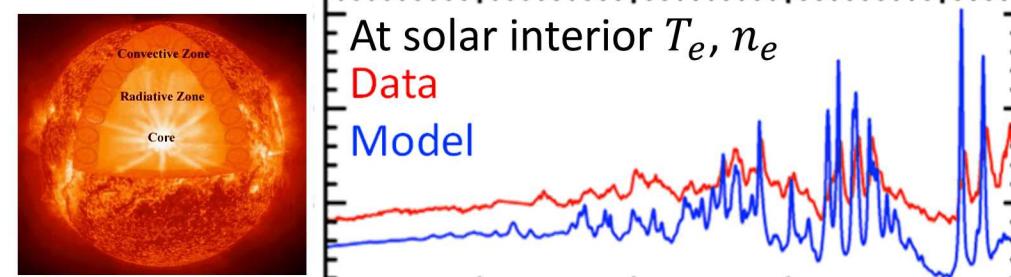
Taisuke Nagayama, Jim Bailey, Guillaume Loisel, Stephanie Hansen,
Greg Dunham, and Greg Rochau

Sandia National Laboratories, New Mexico

New analysis method confirmed experiment reproducibility and enabled accurate systematic study of Cr, Fe, and Ni opacity

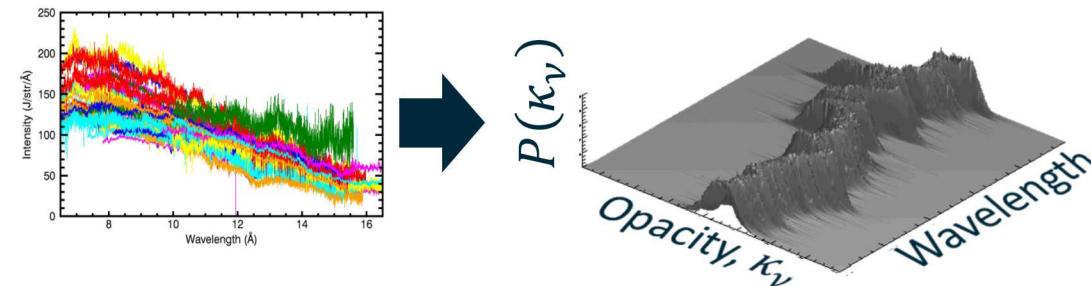
Is iron opacity inaccurate?

- Fe opacity is measured at solar interior condition
- **Severe disagreement with modeled opacity → Why?**



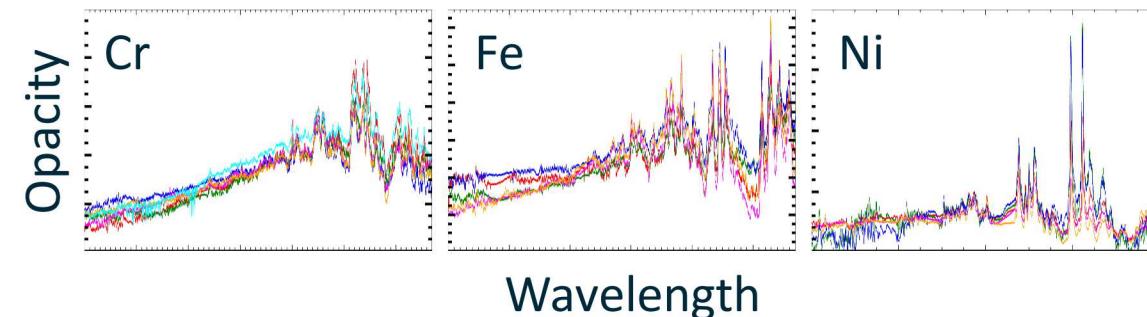
Data analysis method is refined

- Large volume of calibration-shot statistics
- Error propagation with Monte Carlo
- Method tested



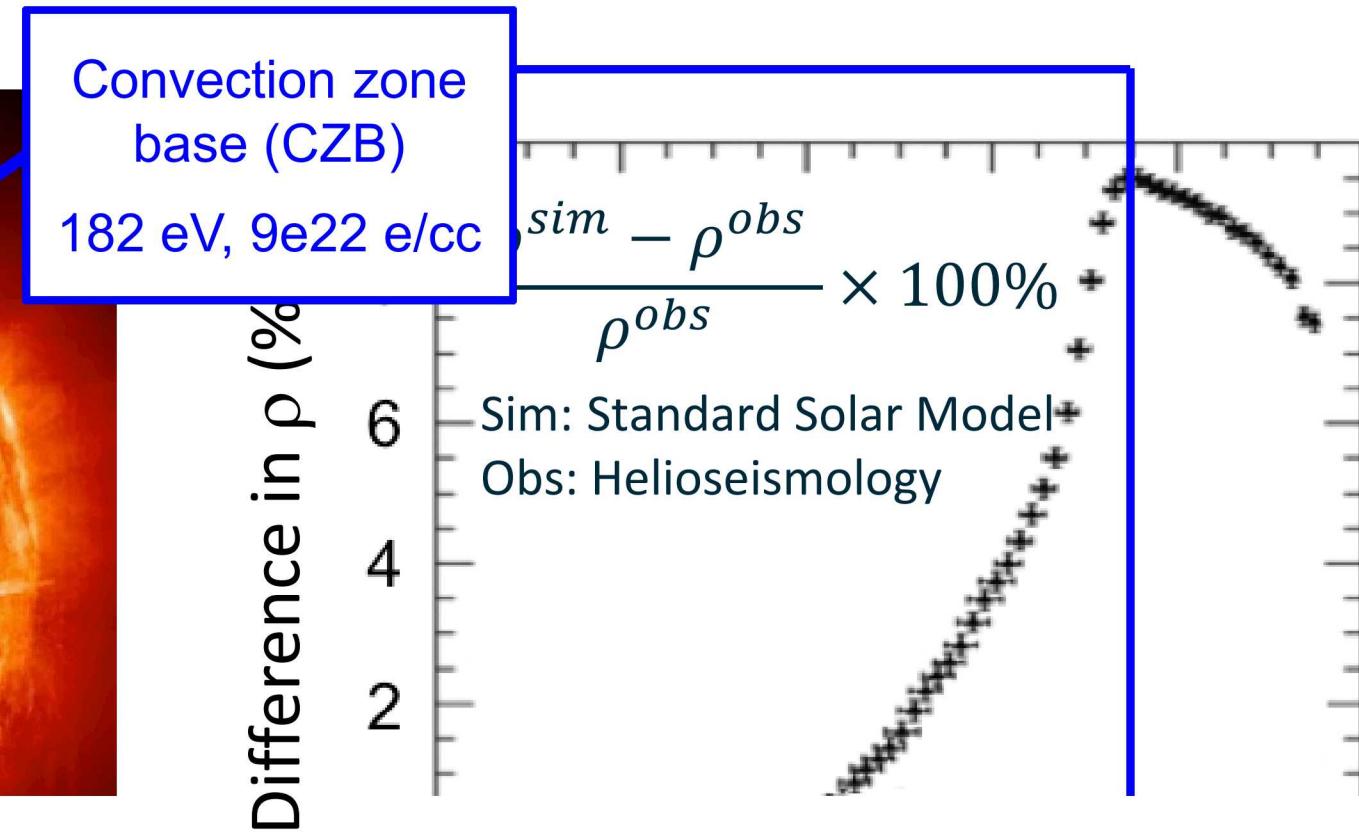
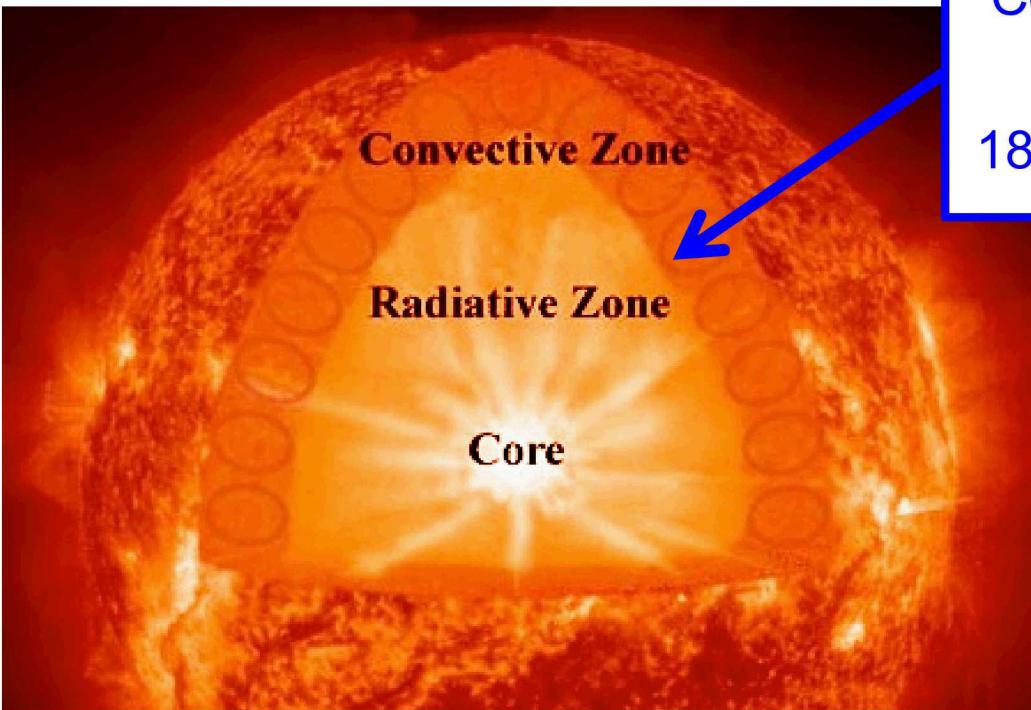
New analysis improved reproducibility, providing insight into the problem

- Improved reproducibility: 10-20%
- First systematic study published by PRL



T. Nagayama et al, PRL 122, 235001 (2019)

Is the decade-old solar problem caused by inaccuracy of opacity models?

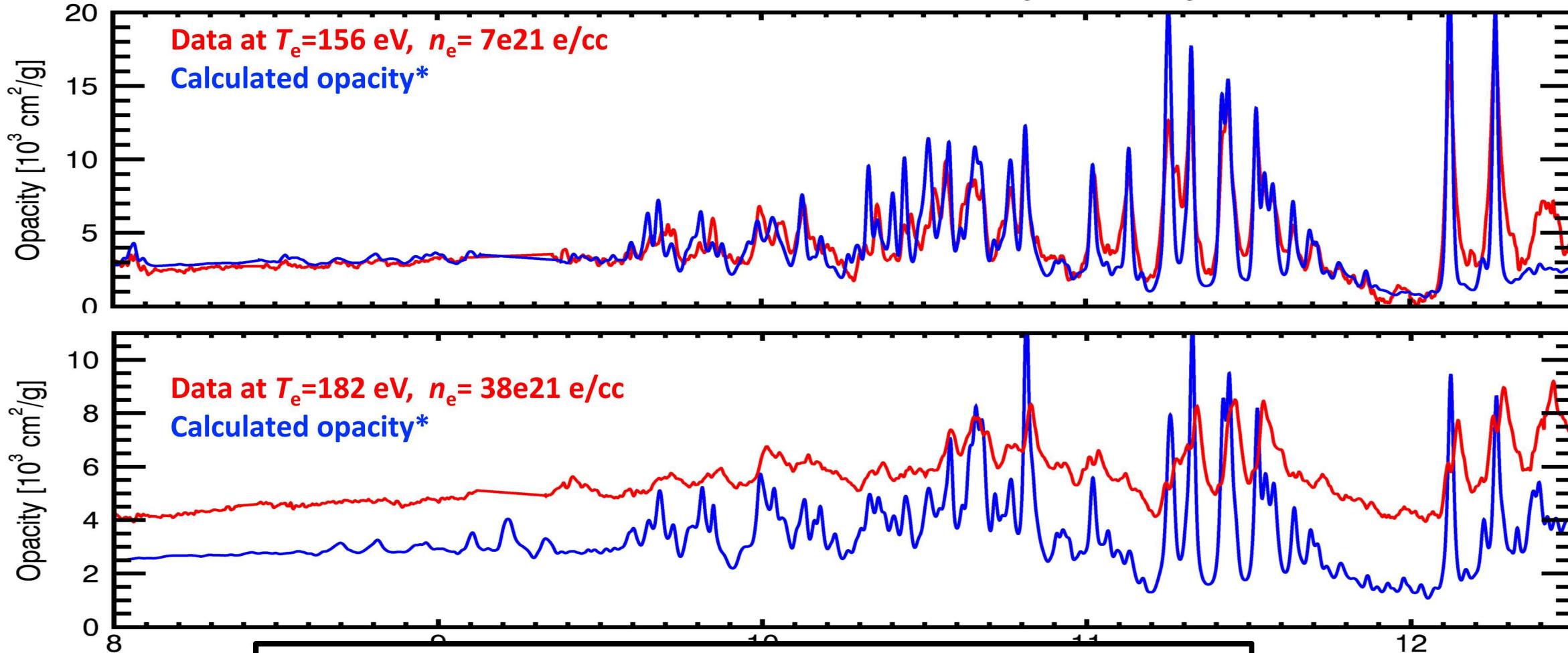


- Solar physicists: solar models need 10-30% higher mean opacity at CZB [1]
- Hypothesis: Iron opacity calculated at CZB is underestimated

Let's measure and check Fe opacity at CZB conditions

Severe opacity model-data disagreement was found as condition approaches solar interior conditions

Convection Zone Base: $T_e=185$ eV, $n_e = 90e21$ e/cc



What's causing the discrepancy? Theory? Experiment?

There are long lists of hypotheses for the discrepancy both in opacity experiment and theory



Experiments:

- Plasma T_e and n_e errors
- Sample areal density errors
- Analysis errors
- Spatial non-uniformities
- Temporal non-uniformities
- Departures from LTE
- Fe self emission
- Tamper self emission
- Extraneous background
- Sample contamination
- Tamper transmission difference

Theory:

- Atomic data accuracy:
 - Cross-section
 - Oscillator strengths
 - Transition energy
- Population
 - Completeness in excited states
- Density effects
 - Spectral line shape
 - Continuum lowering
- Missing physics
 - Multi-photon processes
 - Transient space localization

No systematic error has been found that explains the model-data discrepancies



Potential systematic errors :

→ Evaluated with experiments and simulations

Experimental evidence

- Plasma T_e and n_e errors → $\pm 4\%$ and $\pm 25\%$, respectively [1]
- Sample areal density errors → RBS measurements agree with Mg spectroscopy
- Analysis errors → Transmission analysis on null shot shows $\pm 5\%$
- Spatial non-uniformities → Al and Mg spectroscopy
- Temporal non-uniformities → Backlight radiation lasts 3ns
- Departures from LTE
- Fe self emission → Measurement do not show Fe self-emission
- Tamper self emission →
- Extraneous background → Quantified amount do not explain the discrepancy
- Sample contamination → RBS measurements show no contamination
- Tamper transmission difference →

No systematic error has been found that explains the model-data discrepancies



Potential systematic errors :

→ Evaluated with experiments and simulations

- Plasma T_e and n_e errors —————→ Suggested n_e error did not explain the discrepancy
Nagayama et al, *High Energ Dens Phys* (2016)
Iglesias et al, *High Energ Dens Phys* (2016)
- Sample areal density errors
- Analysis errors
- Spatial non-uniformities
- Temporal non-uniformities
- Departures from LTE
- Fe self emission —————→ Simulation found they were negligible
Nagayama et al, *Phys Rev E* **93**, 023202 (2016)
- Tamper self emission
- Extraneous background
- Sample contamination
- Tamper transmission difference

Numerical evidence

No systematic error has been found that explains the model-data discrepancies

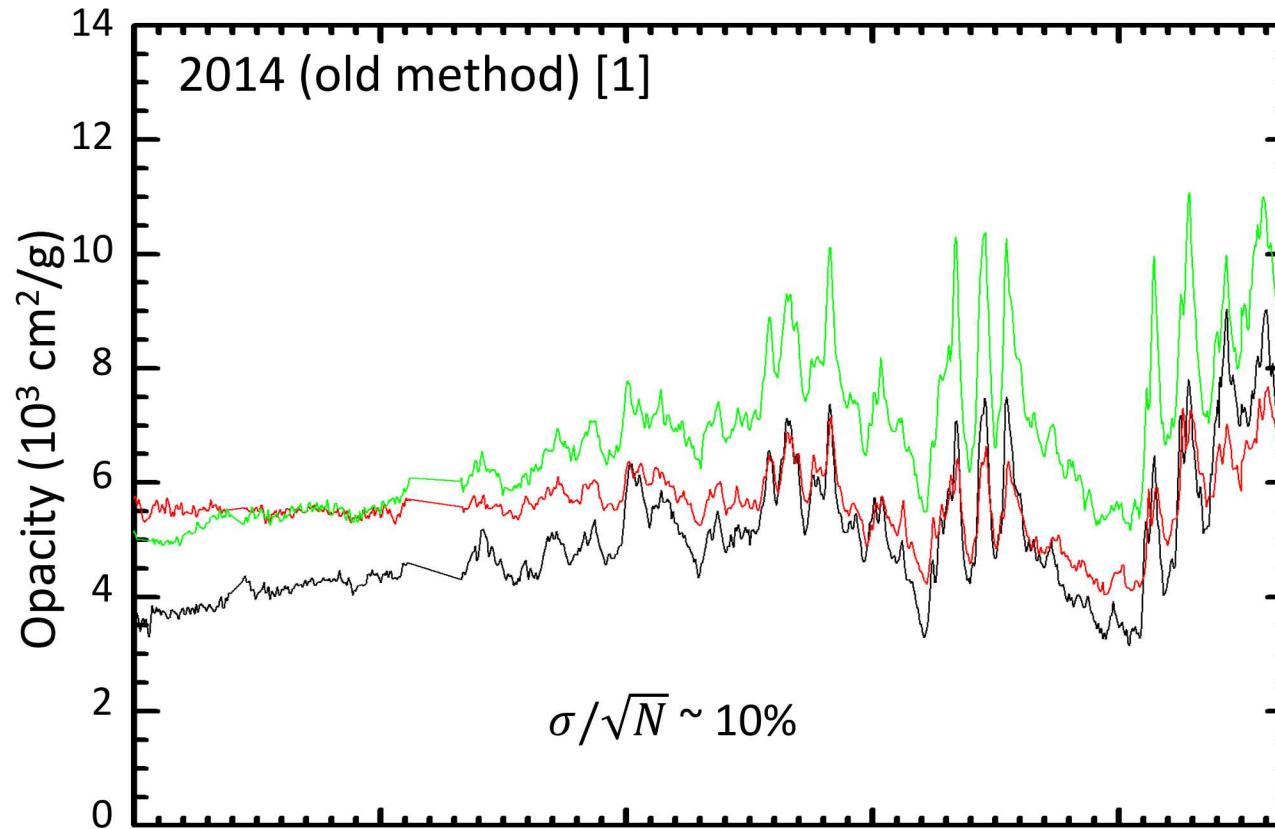


Potential systematic errors :

→ Evaluated with experiments and simulations

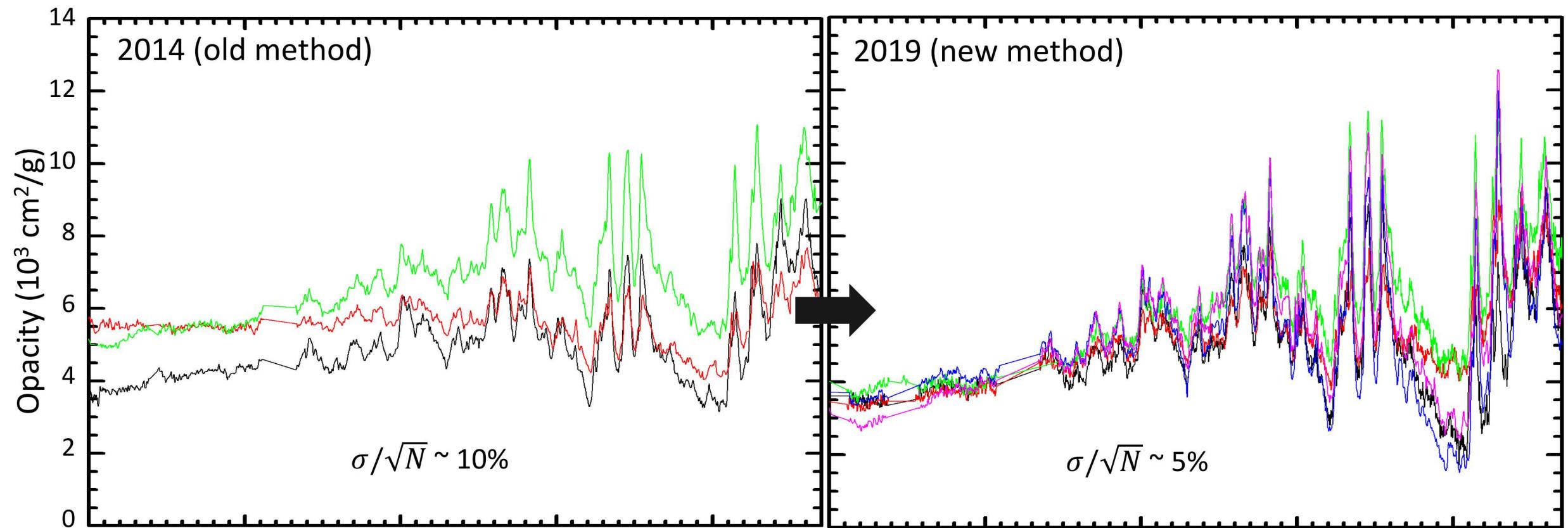
- Plasma T_e and n_e errors
- Sample areal density errors
- **Analysis errors** ← Rescrutinized recently
- Spatial non-uniformities
- Temporal non-uniformities
- Departures from LTE
- Fe self emission
- Tamper self emission
- Extraneous background
- Sample contamination
- Tamper transmission difference

Old analysis method revealed experiments are reproducible within $\pm 20\%$



- Fundamental property of HED plasma measured within $\pm 20\%$ is decent
- But, is it possible that some of this variation is caused by insufficient analysis accuracy?

Refined analysis revealed actual experiment reproducibility is much better than originally believed

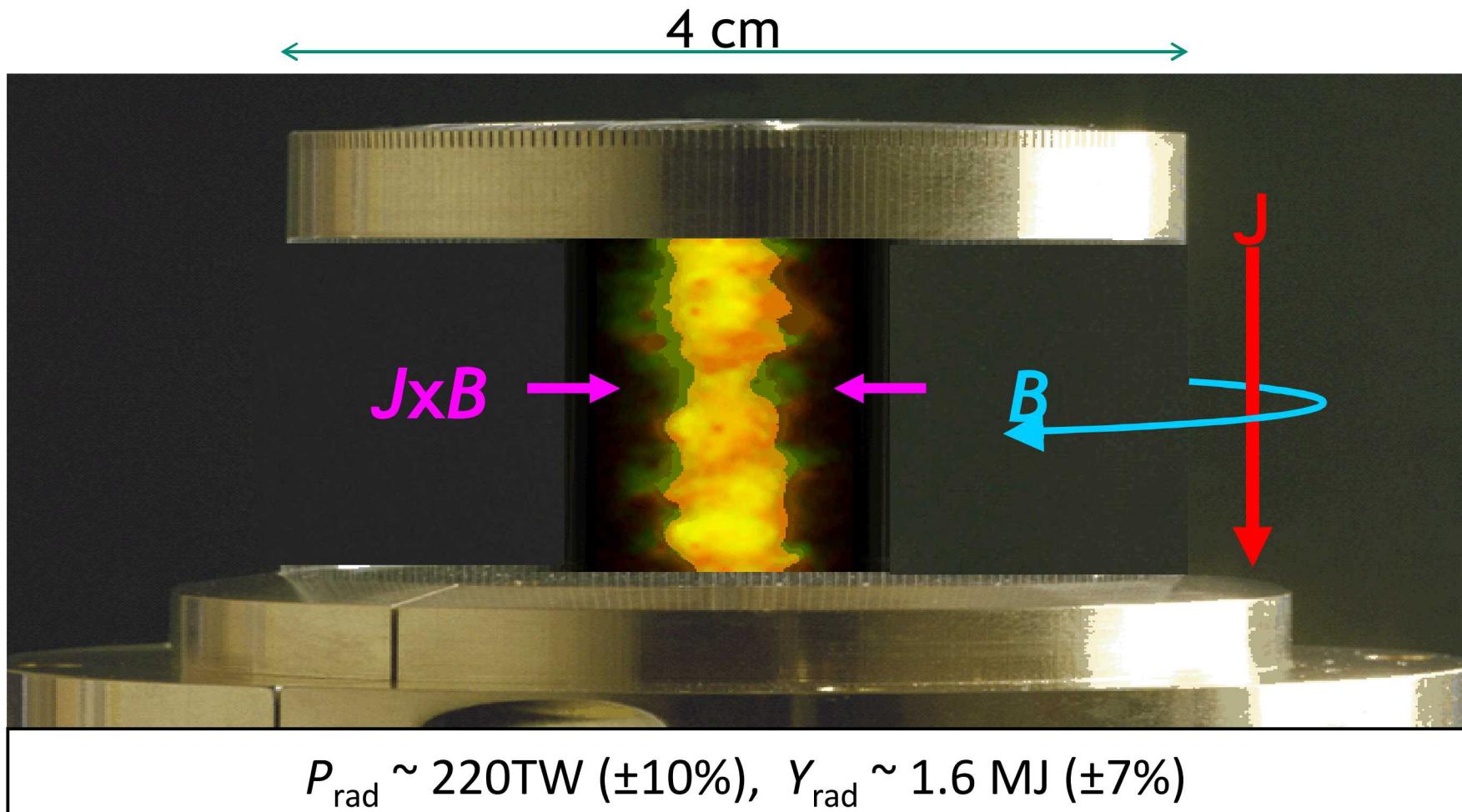


Focus of the talk:

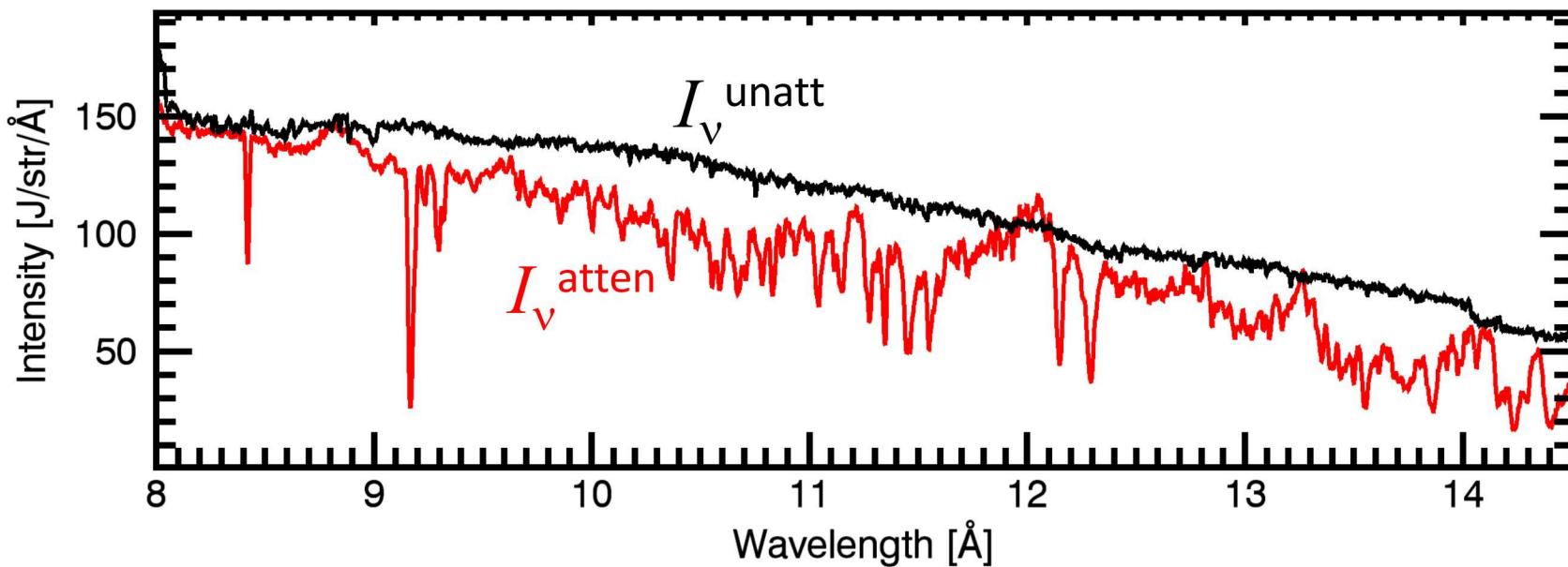
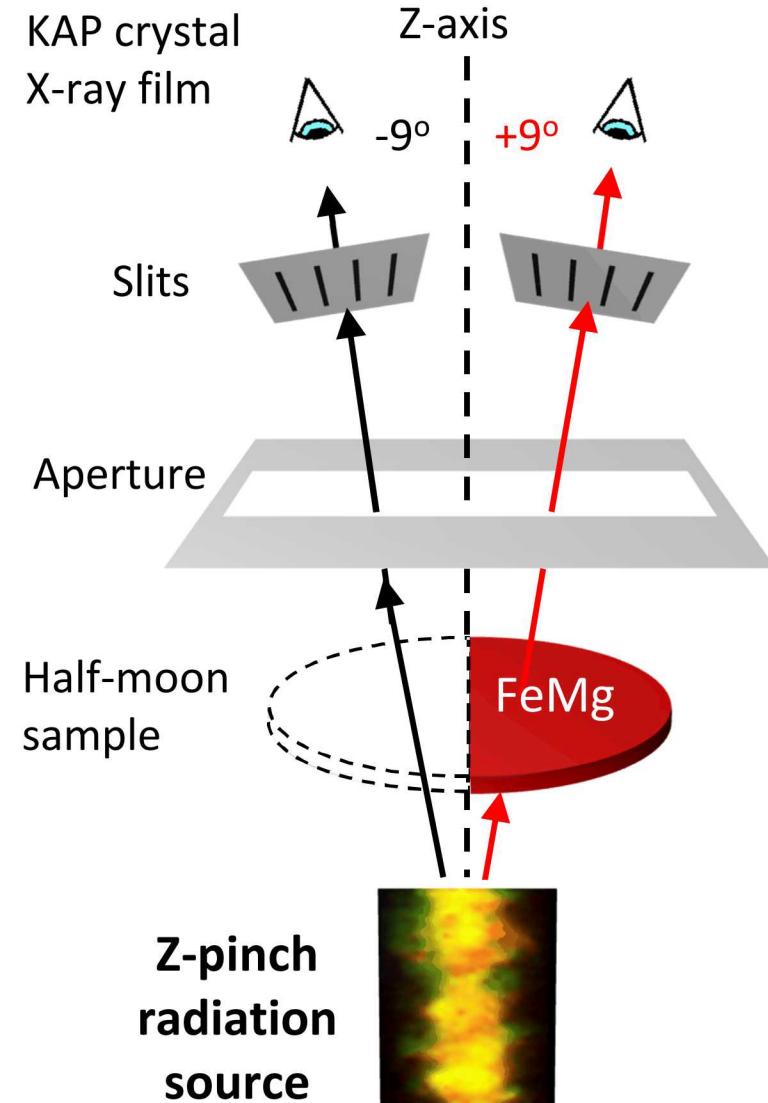
- i) Challenges associated with opacity data analysis
- ii) How we analyze the data statistically

How does experiment work?

Iron opacity at solar interior conditions is measured using bright radiation generated by Z-pinch



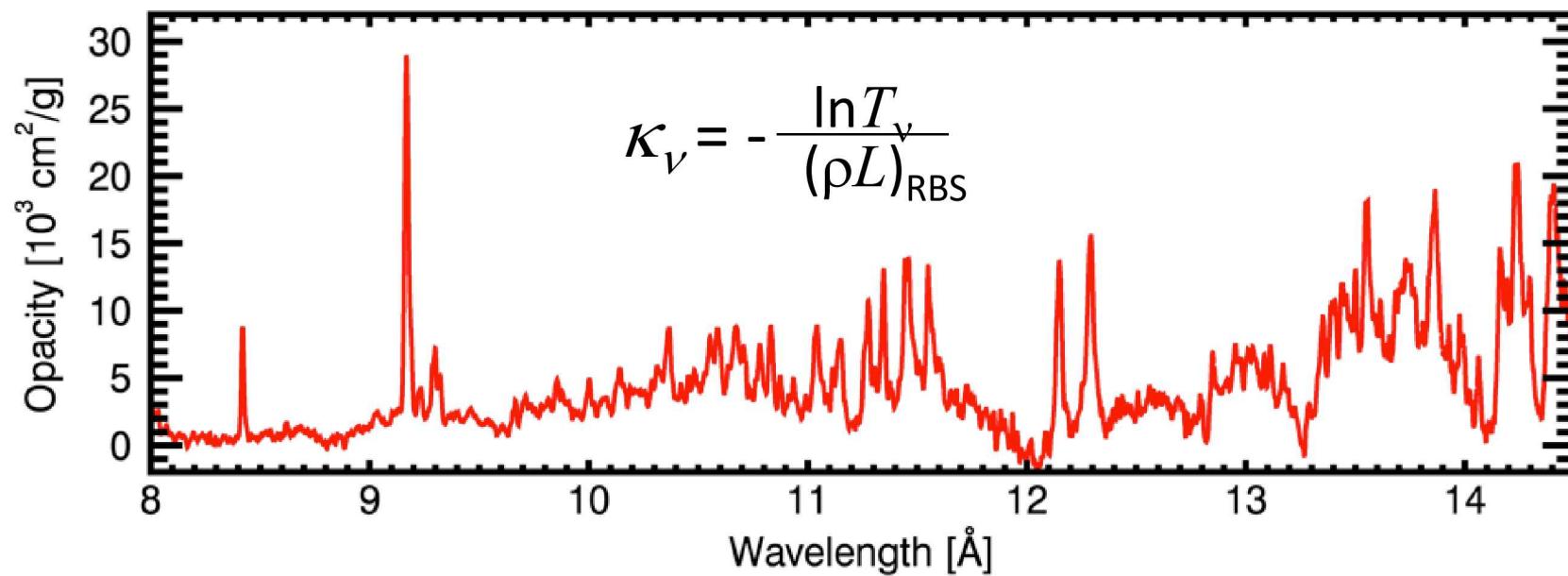
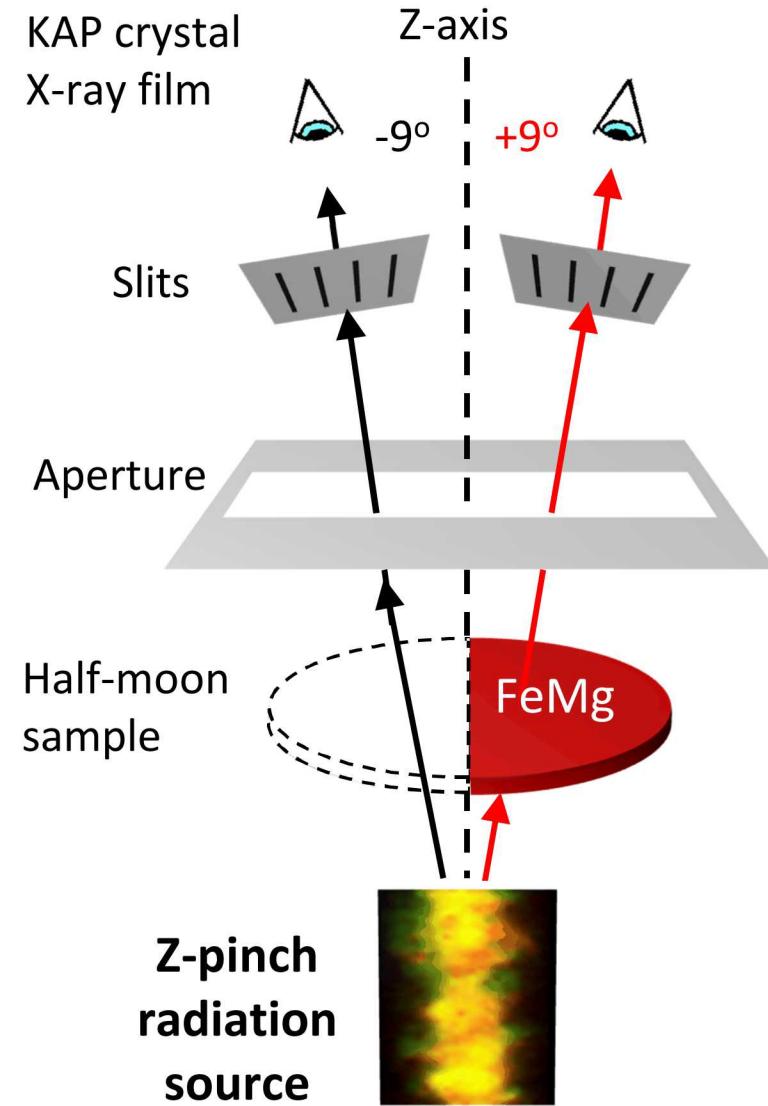
Iron opacity at solar interior conditions is measured using bright radiation generated by Z-pinch



Z experiment satisfies challenging requirements:

- Uniform heating
- Mitigating self emission
- Condition measurements
- Checking reproducibility

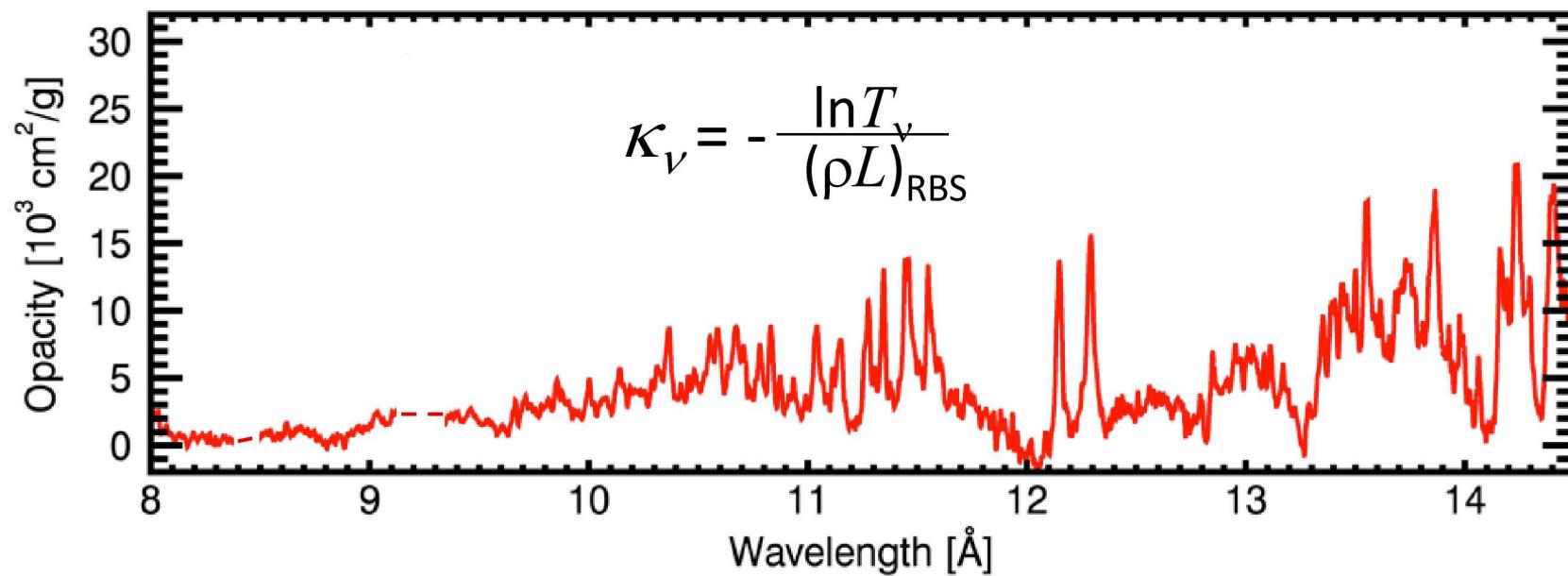
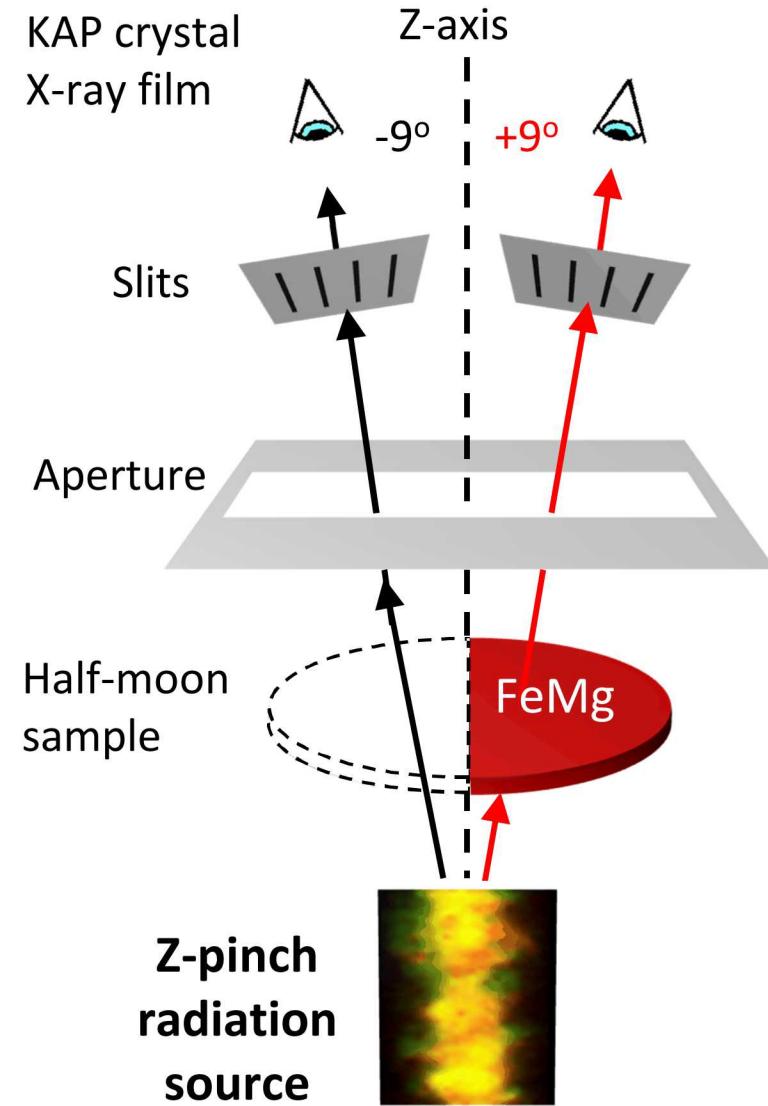
Iron opacity at solar interior conditions is measured using bright radiation generated by Z-pinch



Z experiment satisfies challenging requirements:

- Uniform heating
- Mitigating self emission
- Condition measurements
- Checking reproducibility

Iron opacity at solar interior conditions is measured using bright radiation generated by Z-pinch

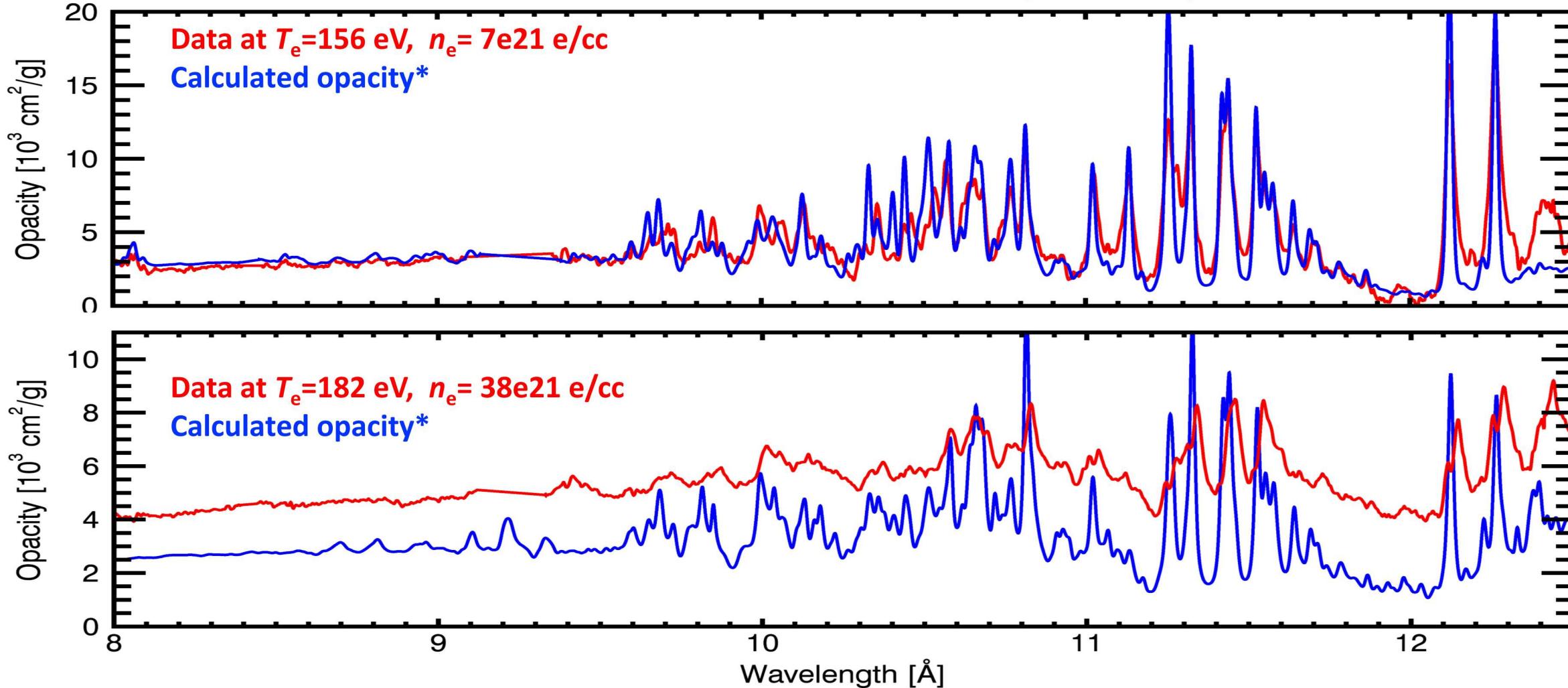


Z experiment satisfies challenging requirements:

- Uniform heating
- Mitigating self emission
- Condition measurements
- Checking reproducibility

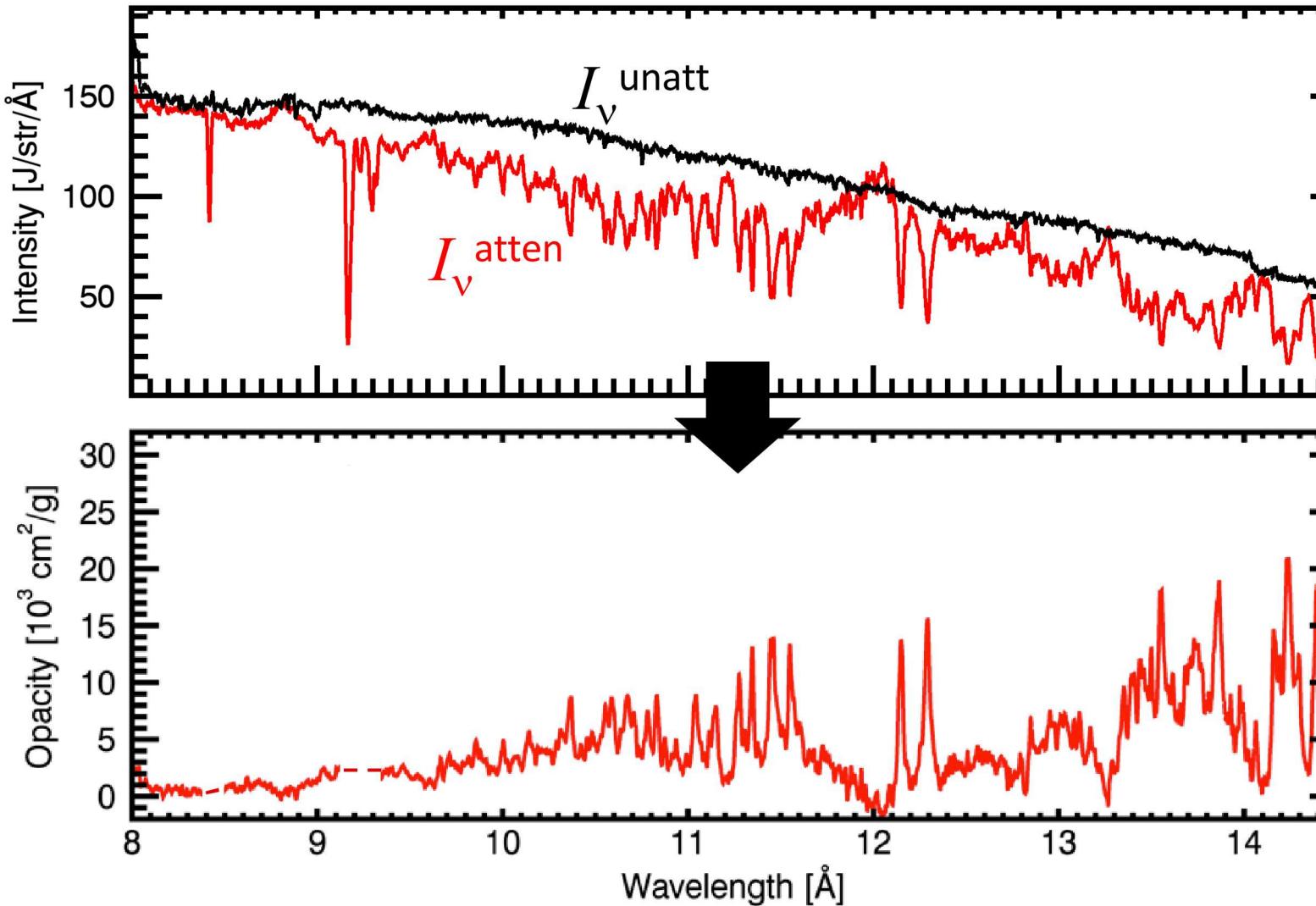
Severe opacity model-data disagreement was found as condition approaches solar interior conditions

Convection Zone Base: $T_e=185$ eV, $n_e = 90e21$ e/cc



What are the main sources of measurement uncertainty?

Iron opacity at solar interior conditions is measured using bright radiation generated by Z-pinch



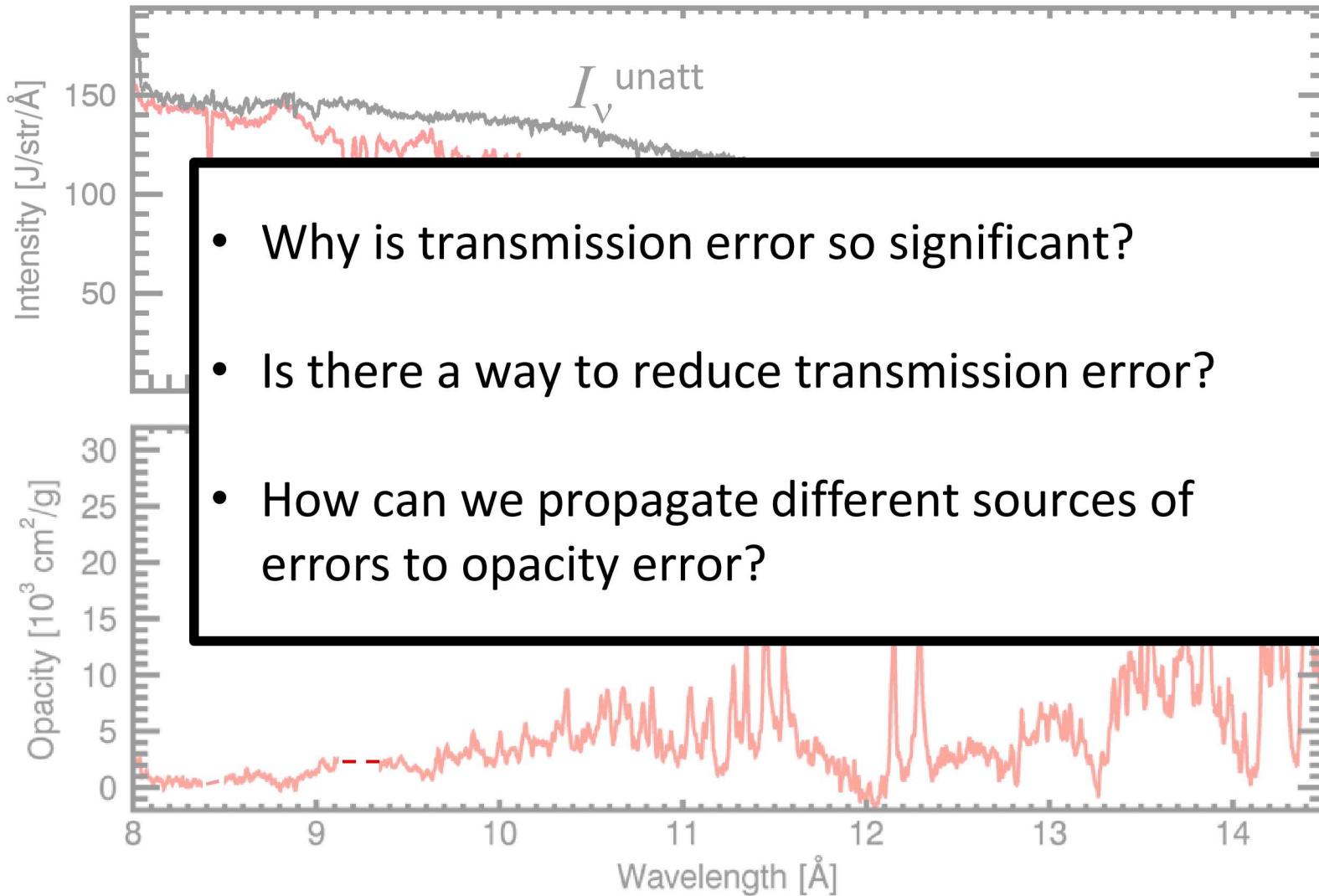
$$T_{\nu} = I_{\nu}^{\text{atten}} / I_{\nu}^{\text{unatt}}$$

$$\kappa_{\nu} = - \frac{\ln T_{\nu}}{(\rho L)_{\text{RBS}}}$$

Source of κ_{ν} uncertainty:

- Transmission error, $\Delta T'_{\nu}$ (20%)
- Areal density error, $\Delta \rho L$ (4%)
- Background subtraction error, ΔB_{ν} , (3%)

Iron opacity at solar interior conditions is measured using bright radiation generated by Z-pinch



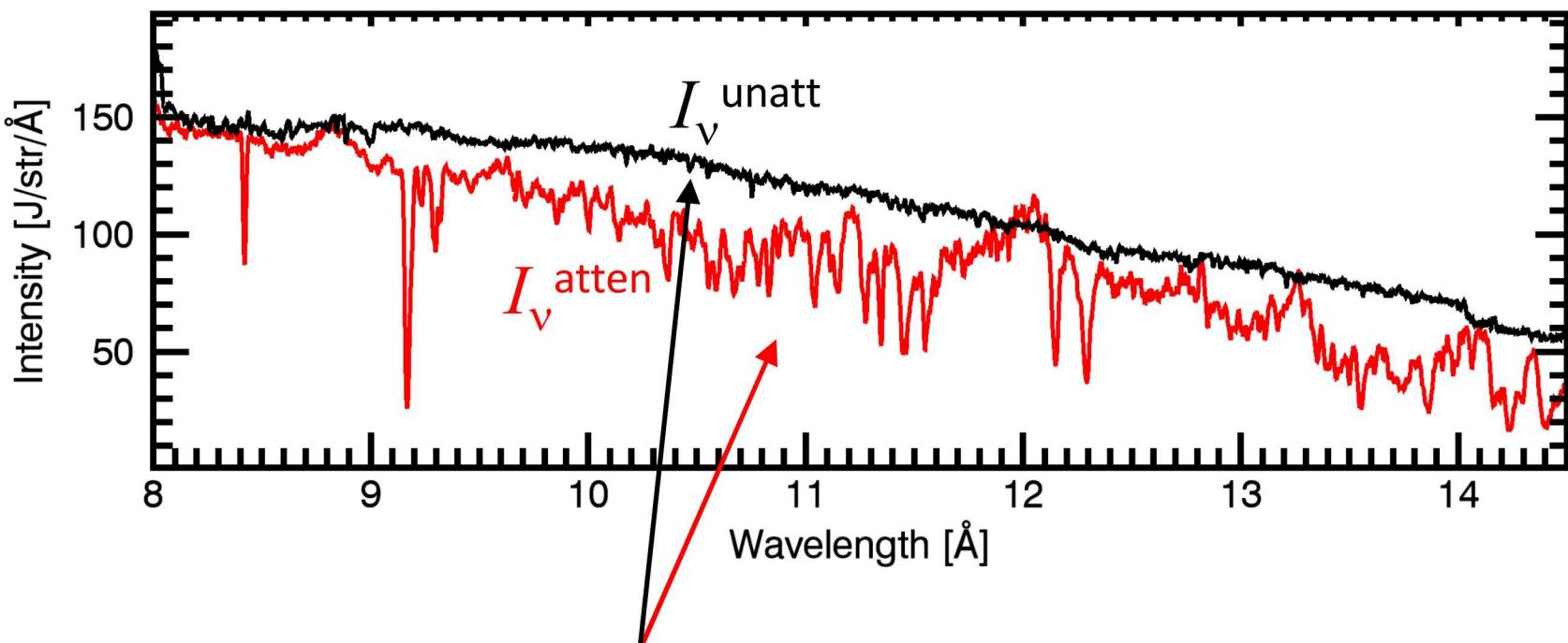
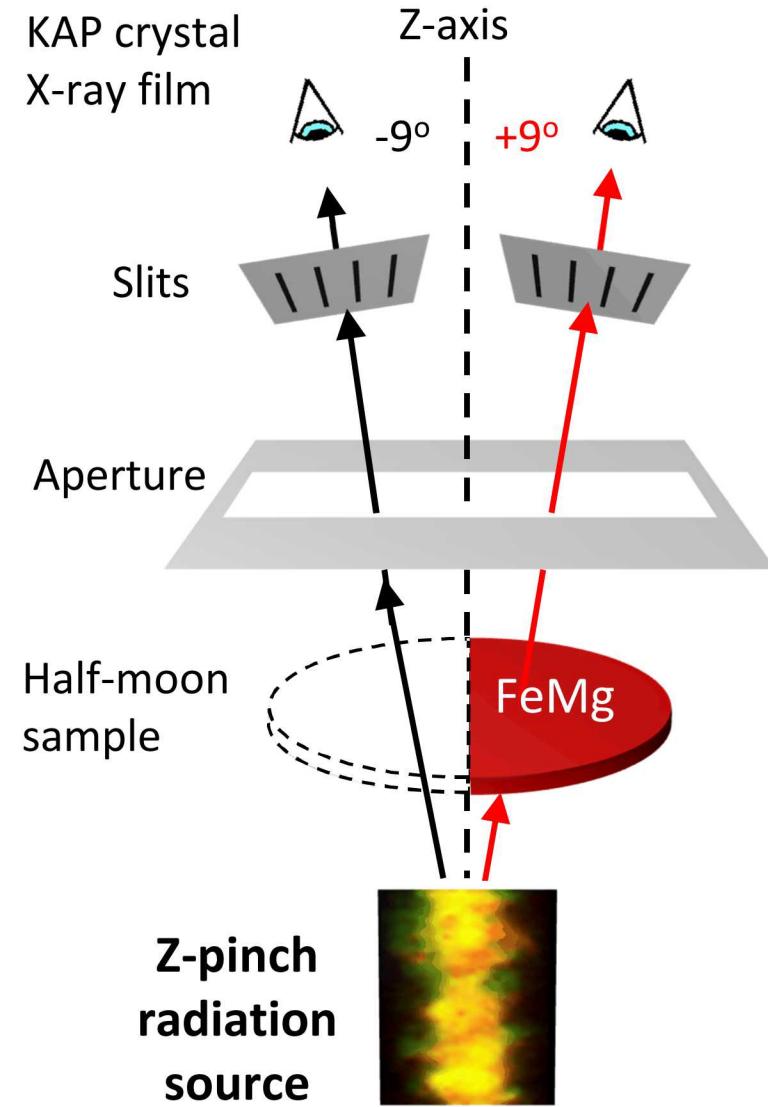
$$T_\nu = I_\nu^{\text{atten}} / I_\nu^{\text{unatt}}$$

$$\kappa_\nu = - \frac{\ln T_\nu}{(\rho L)_{\text{RBS}}}$$

Source of κ_ν uncertainty:

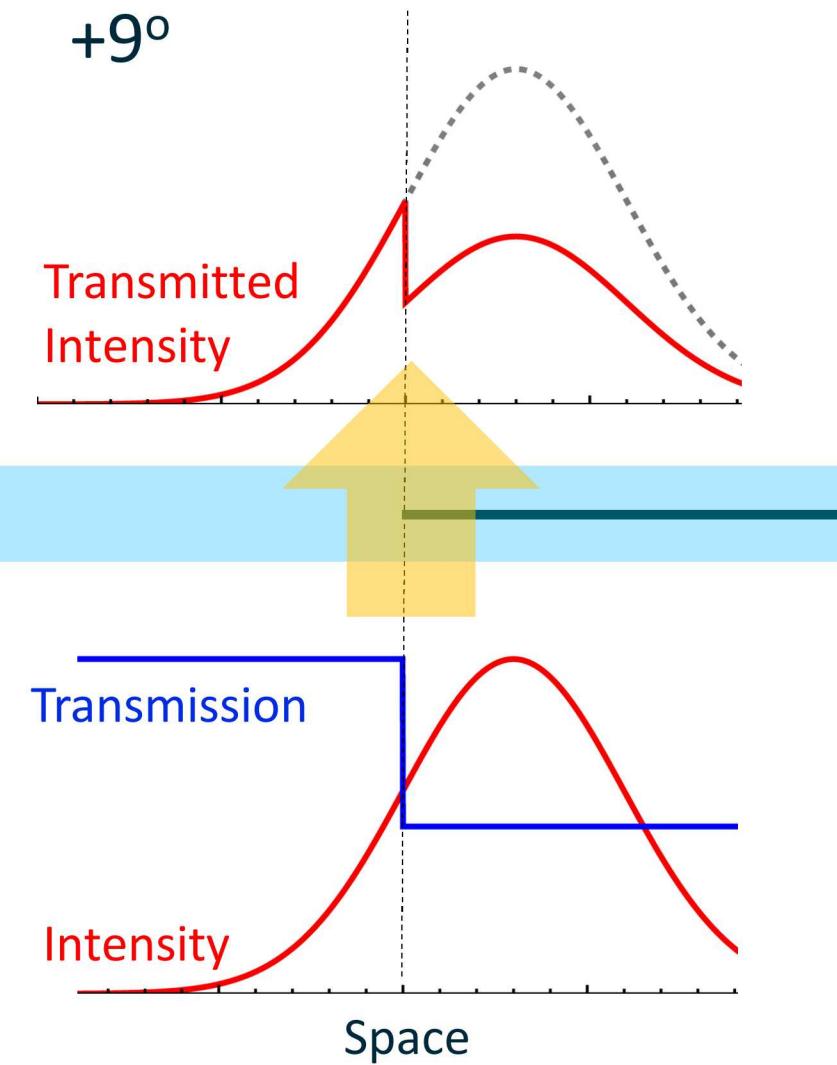
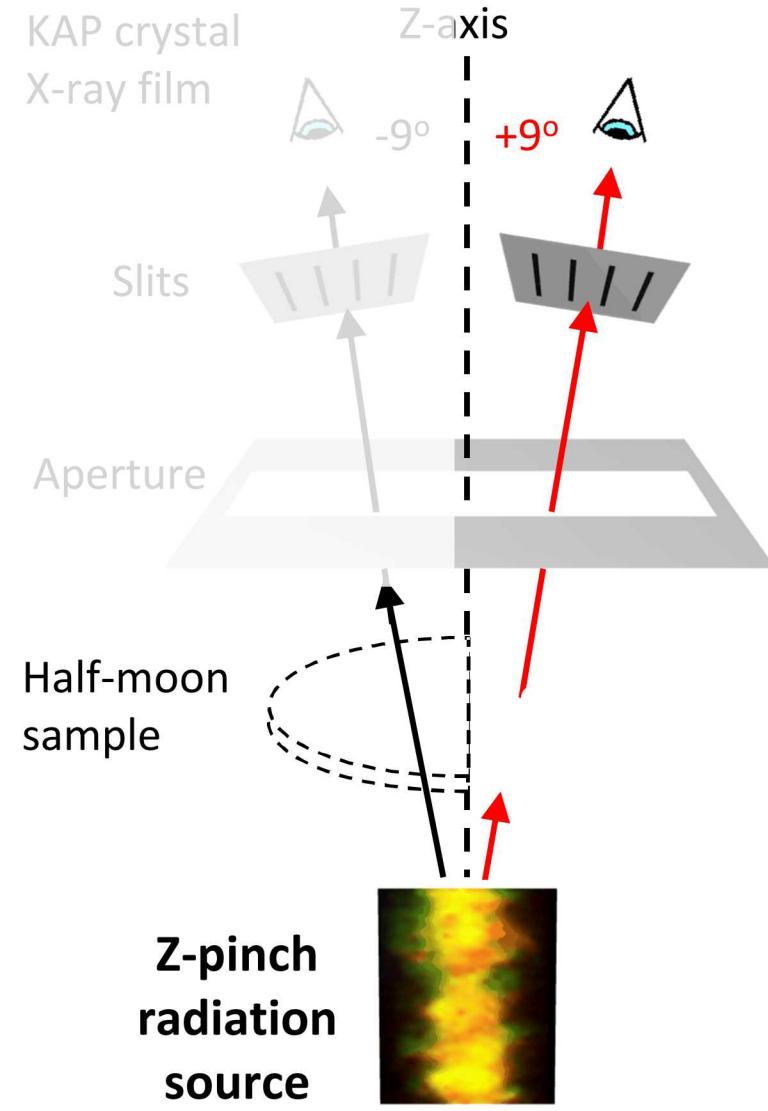
- Transmission error, $\Delta T'_\nu$ (20%)
- Areal density error, $\Delta \rho L$ (4%)
- Background subtraction error, ΔB_ν , (3%)

Sample transmission is inferred by dividing sample-attenuated radiation by unattenuated radiation

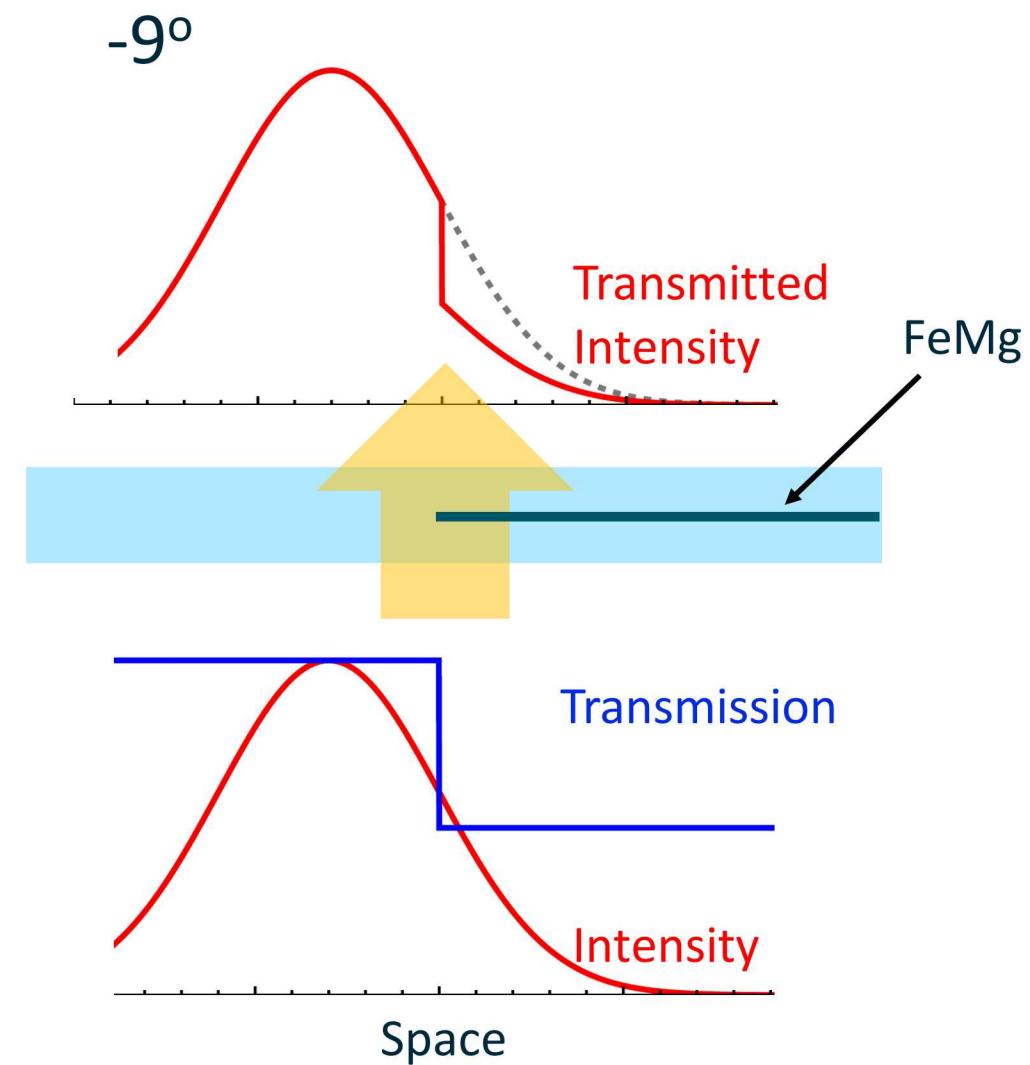
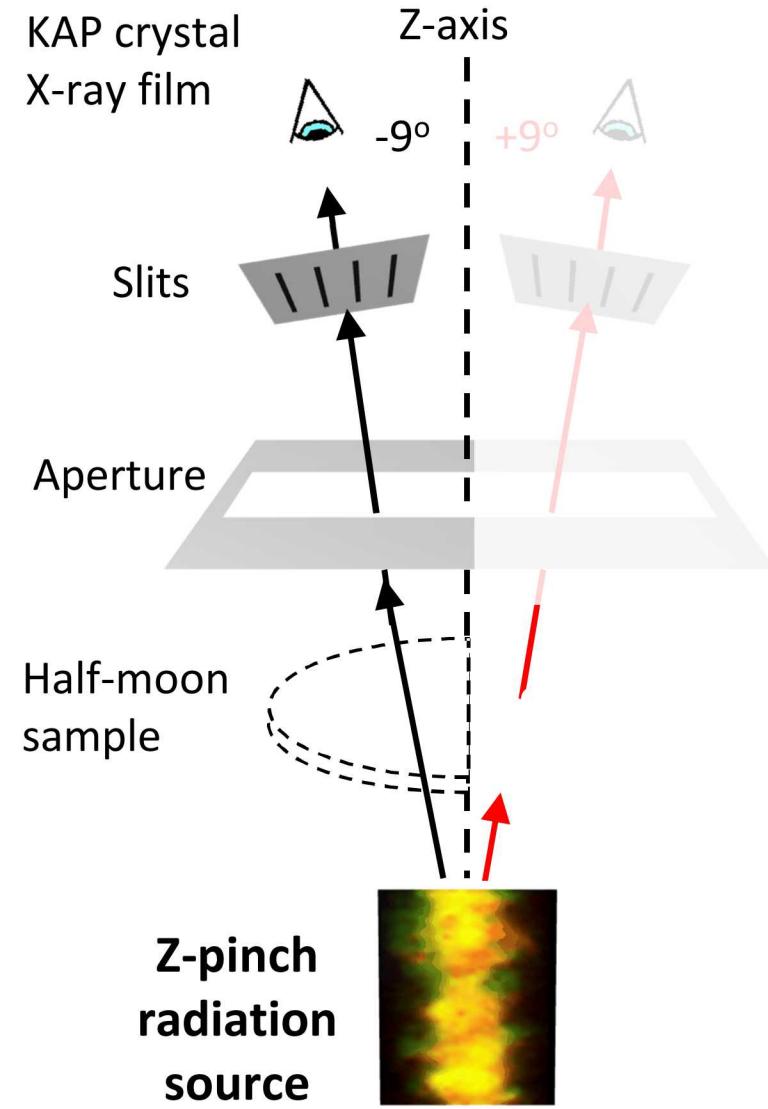


But, interpretation of measured intensities are complicated by spatial distribution and limited reproducibility

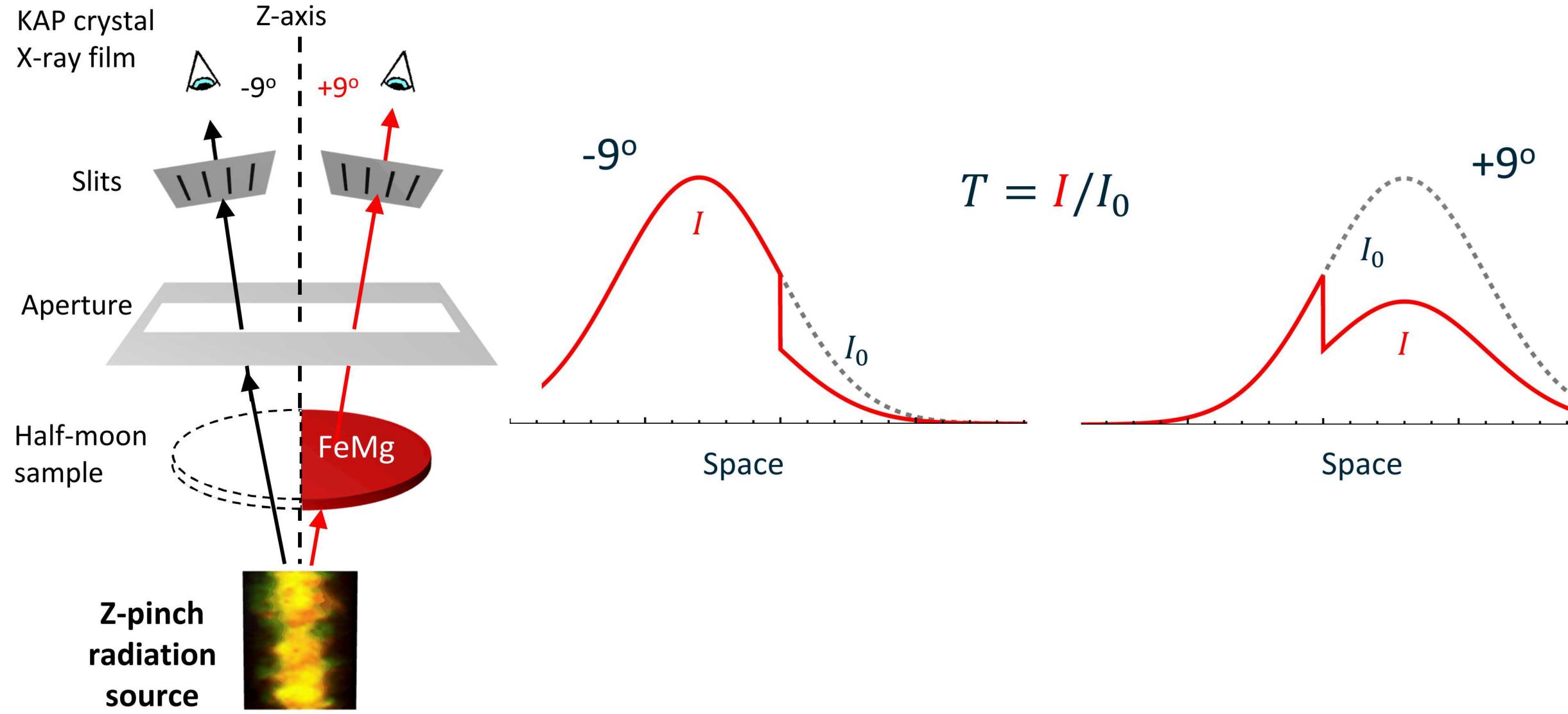
Observing finite-area backlighter through half-moon sample at $\pm 9^\circ$ produces complicated spatial shape



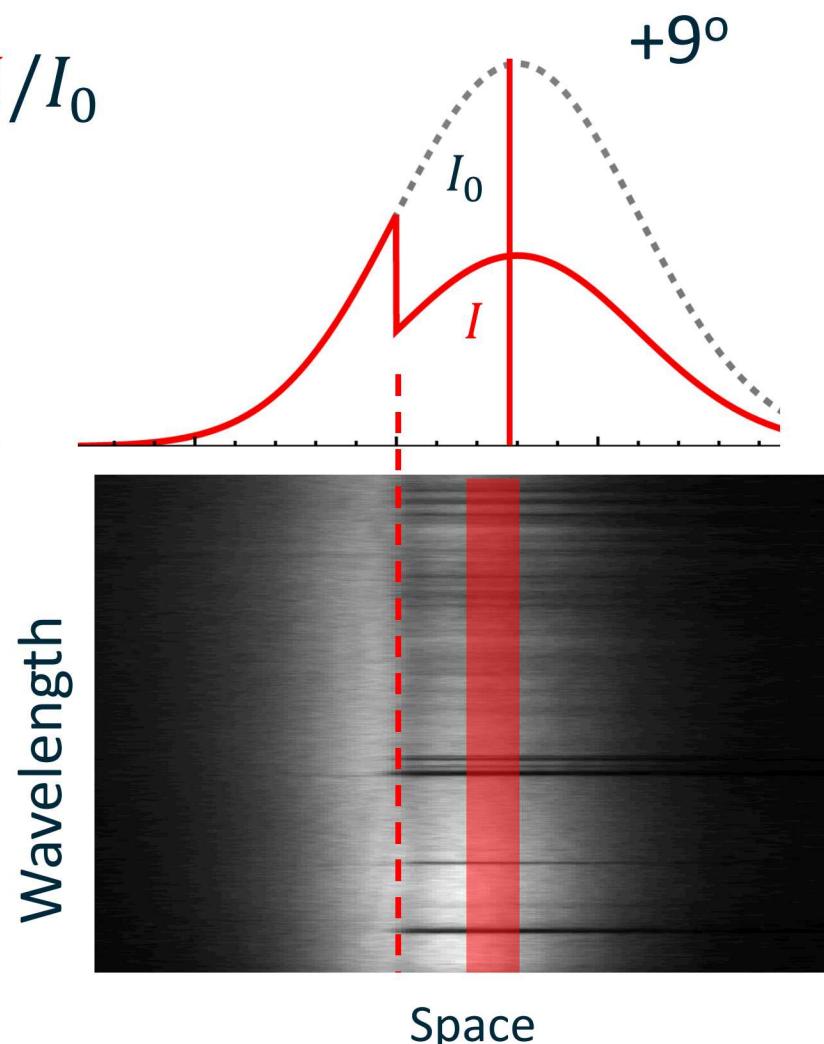
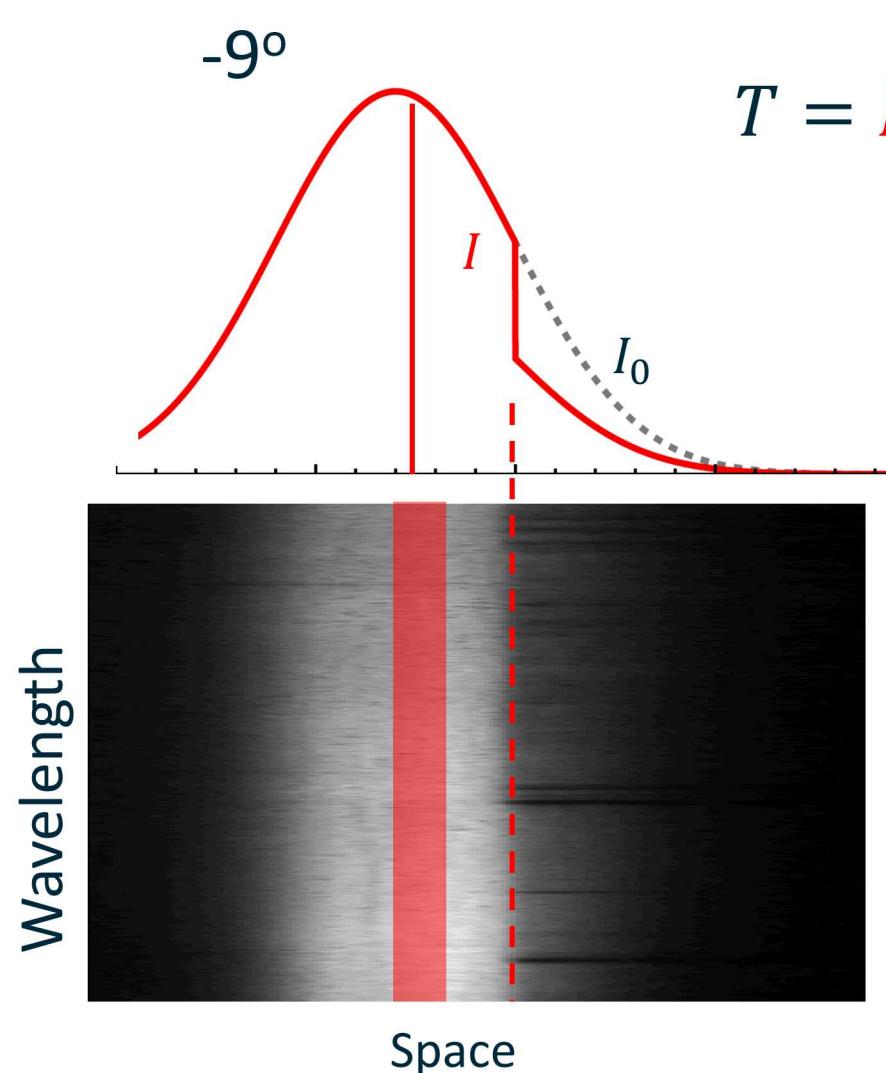
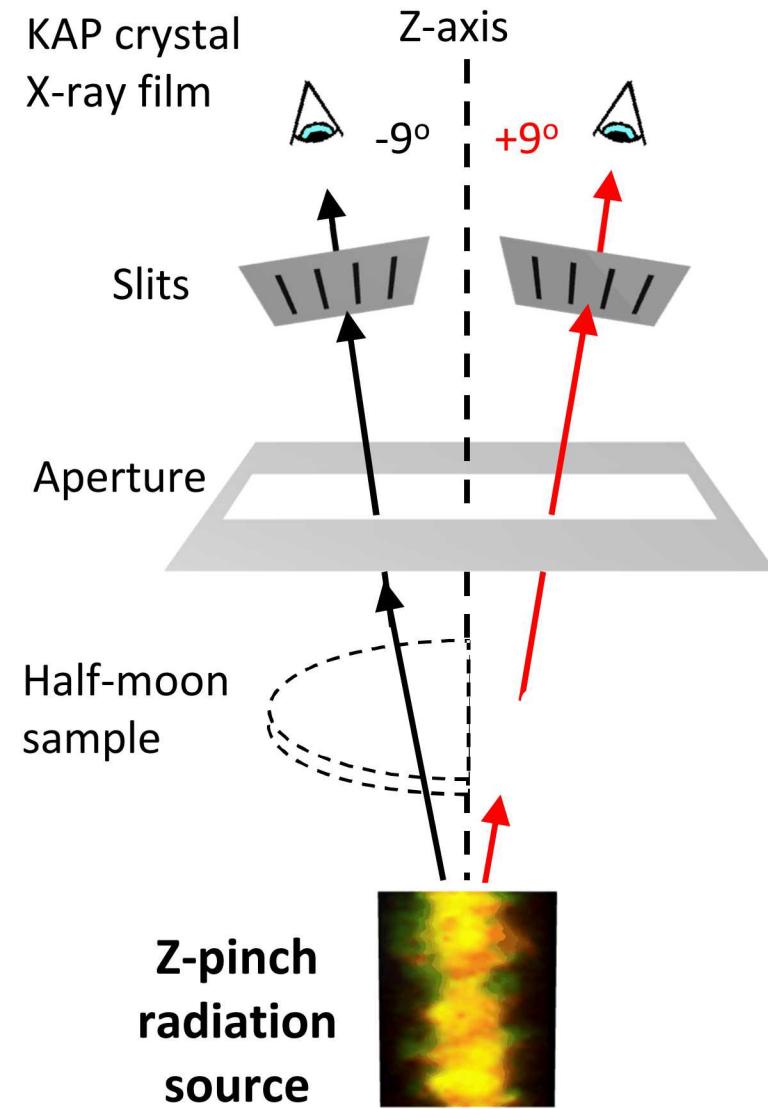
Observing finite-area backlighter through half-moon sample at $\pm 9^\circ$ produces complicated spatial shape



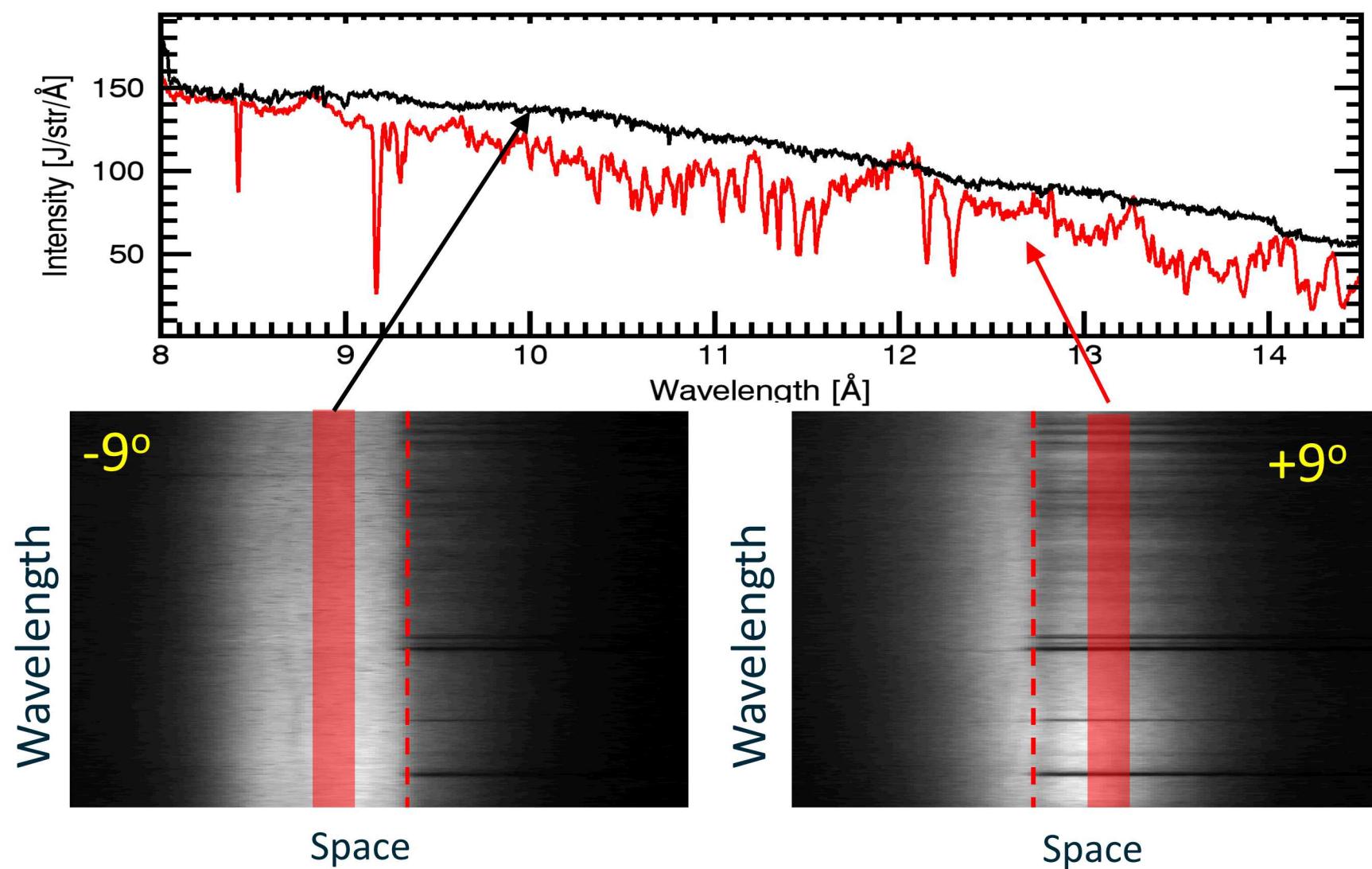
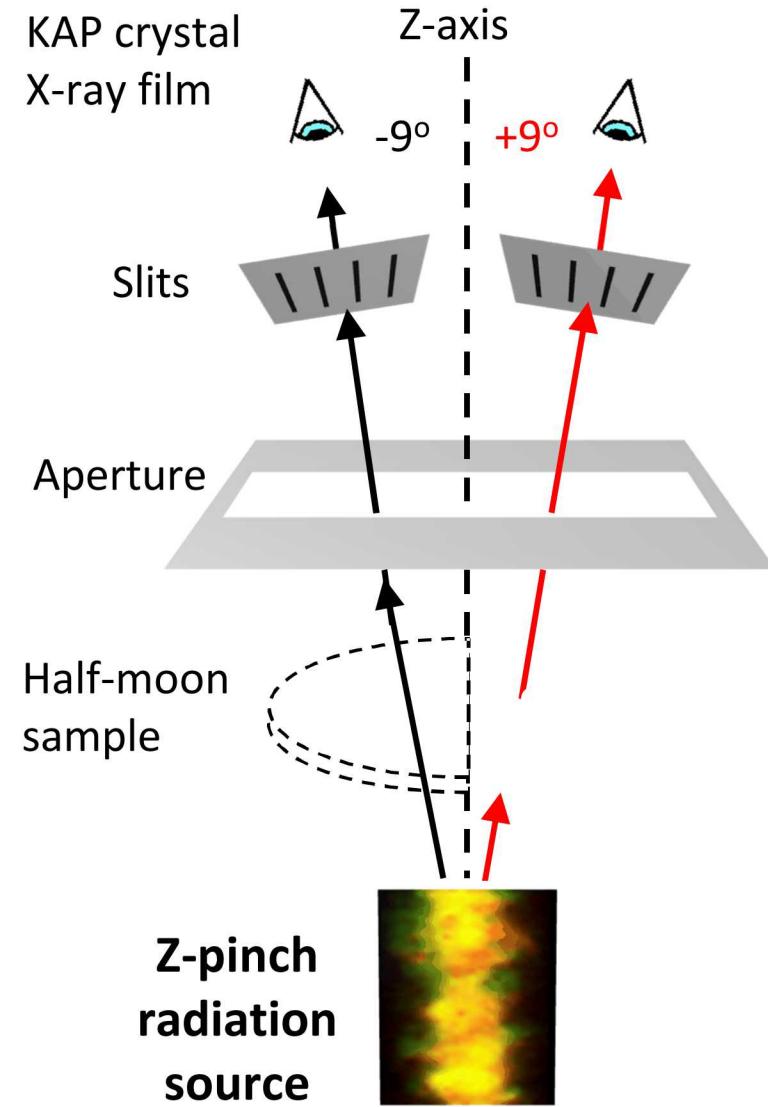
Observing finite-area backlighter through half-moon sample at $\pm 9^\circ$ produces complicated spatial shape



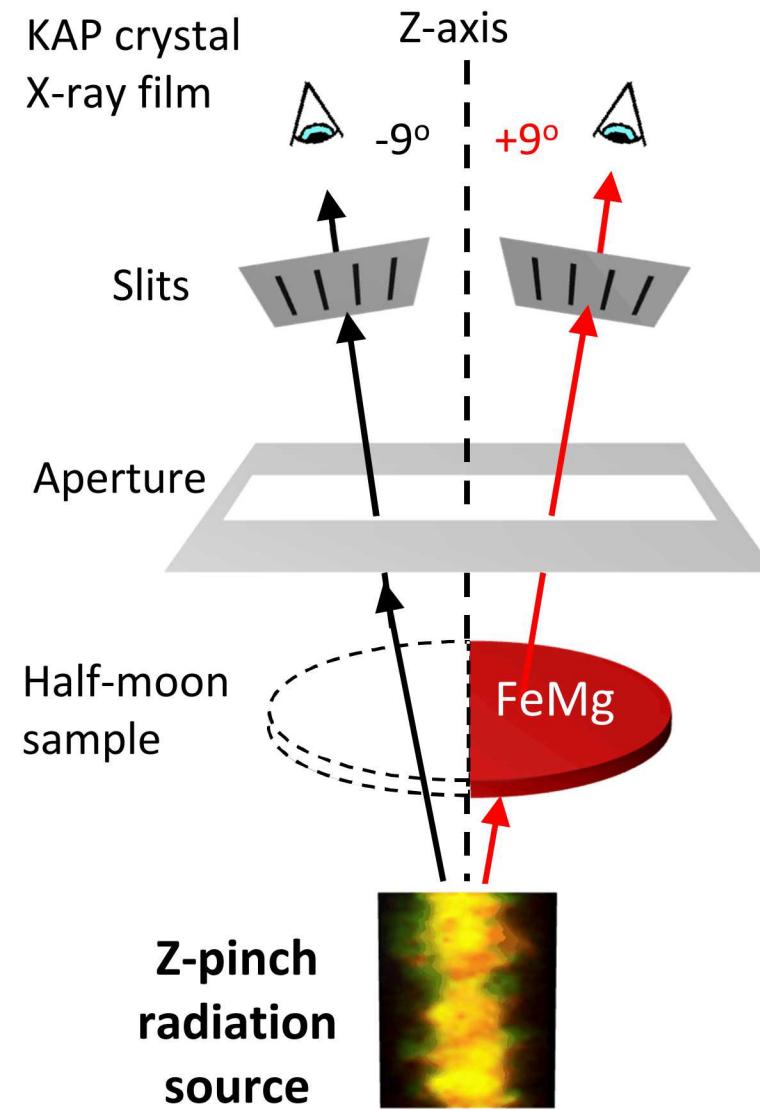
Z opacity platform measures transmitted intensity as a function of space and wavelength



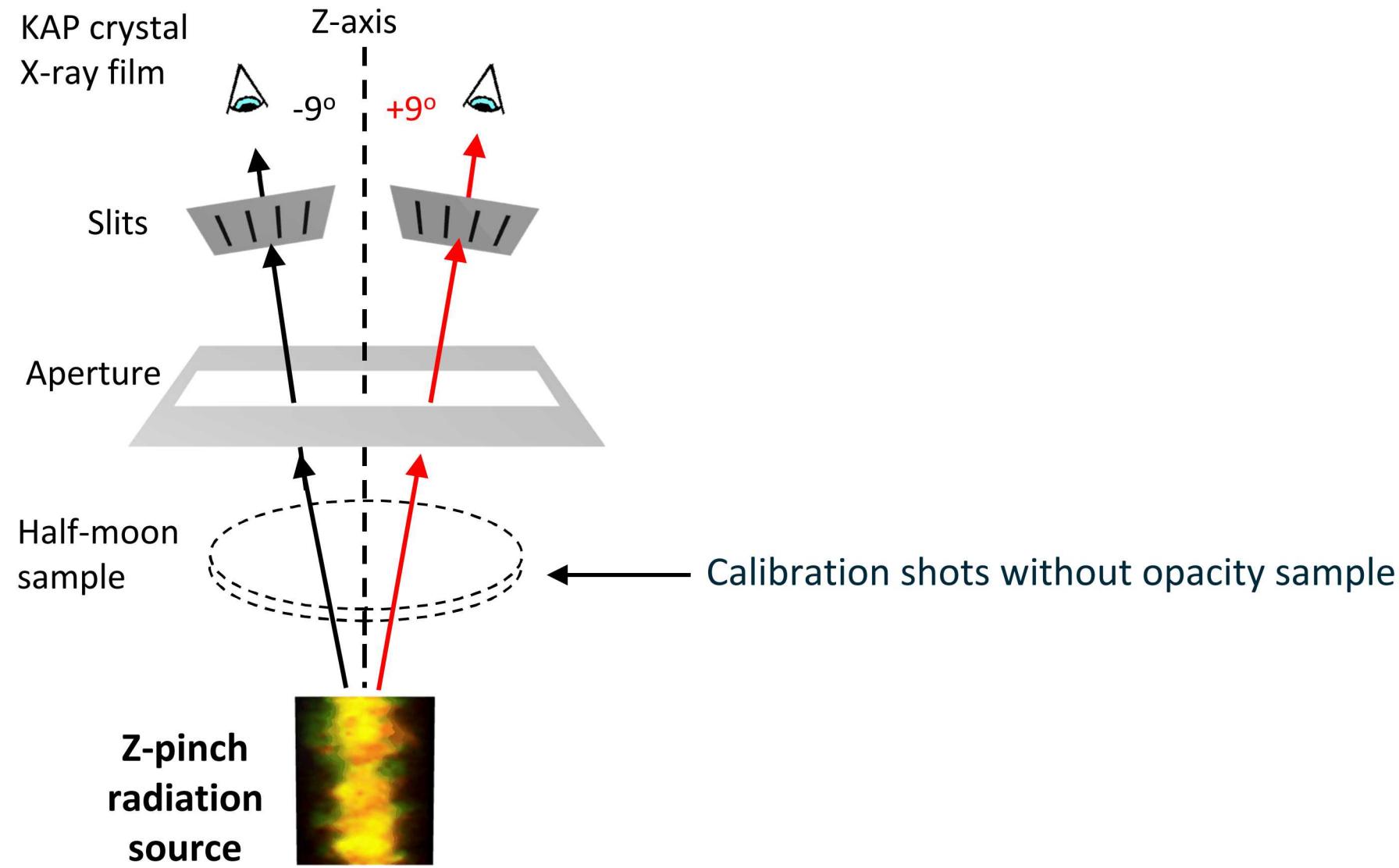
If backlighter is perfectly reproducible at two sides, analysis is as simple as described earlier



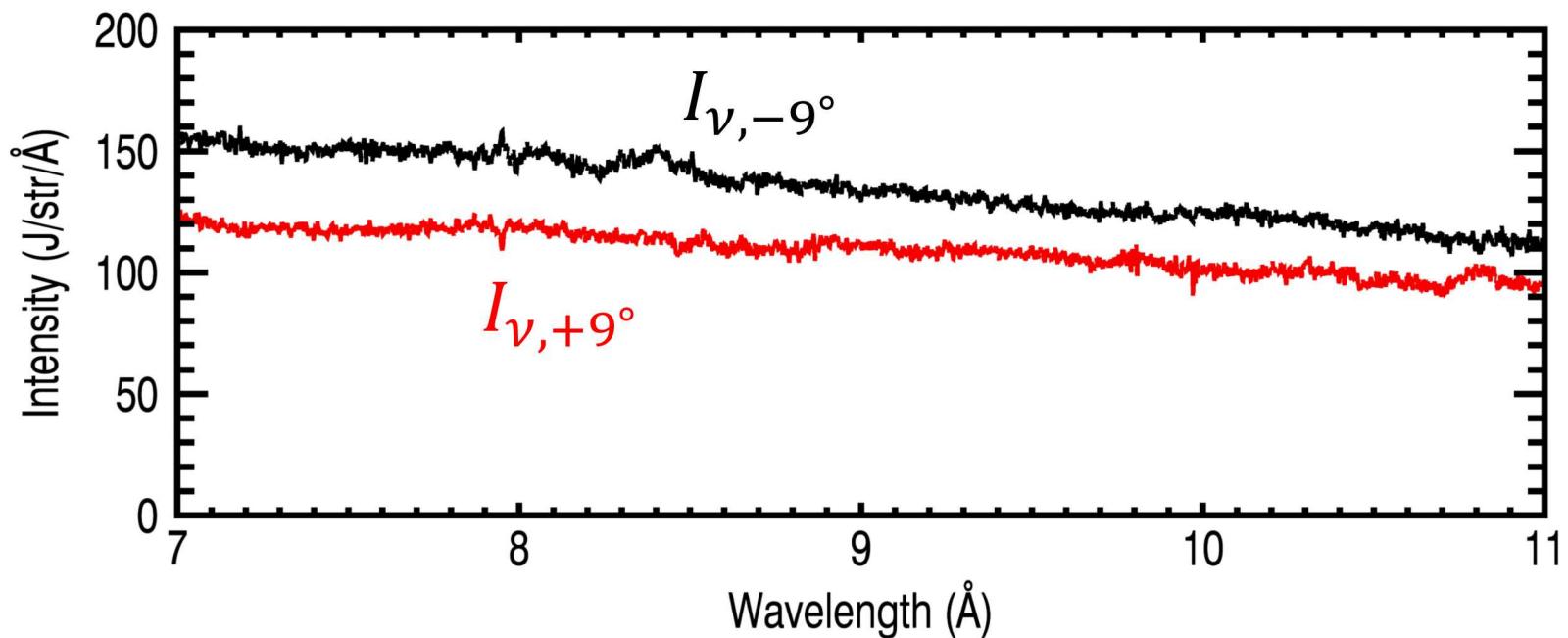
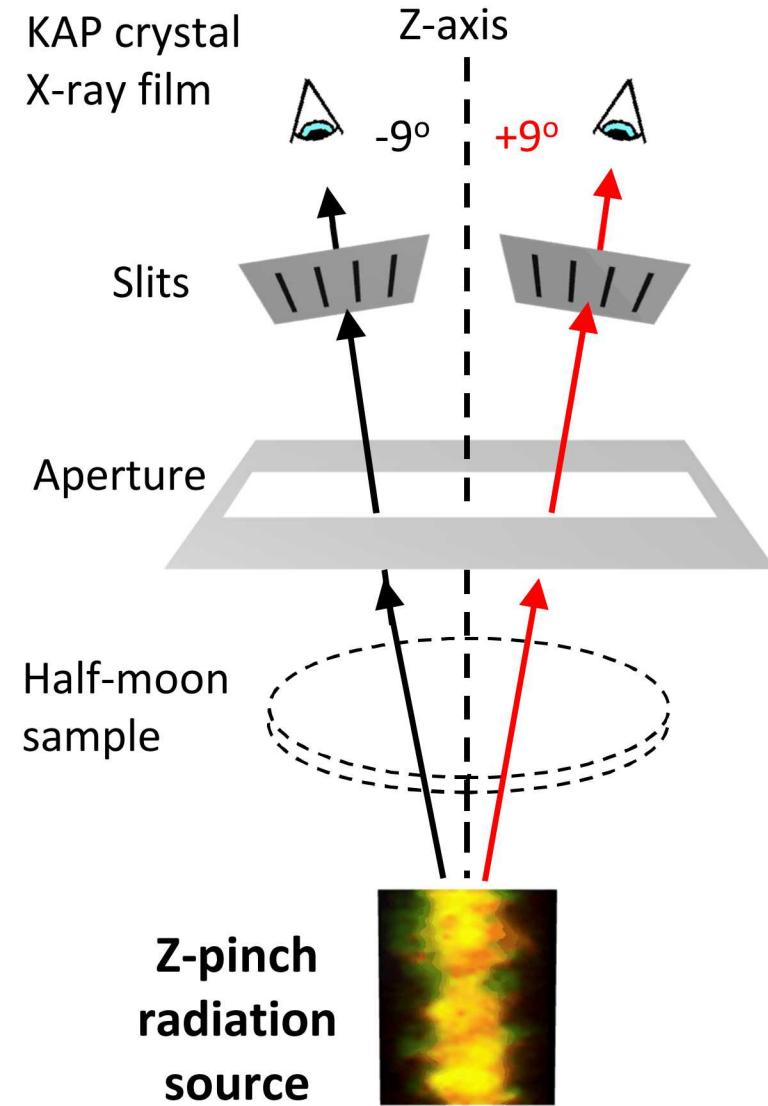
However, calibration shot shows backlight reproducibility at two sides are limited to +/-20%



However, calibration shot shows backlight reproducibility at two sides are limited to +/-20%



However, calibration shot shows backlight brightness reproducibility at two sides are limited to +/-20%

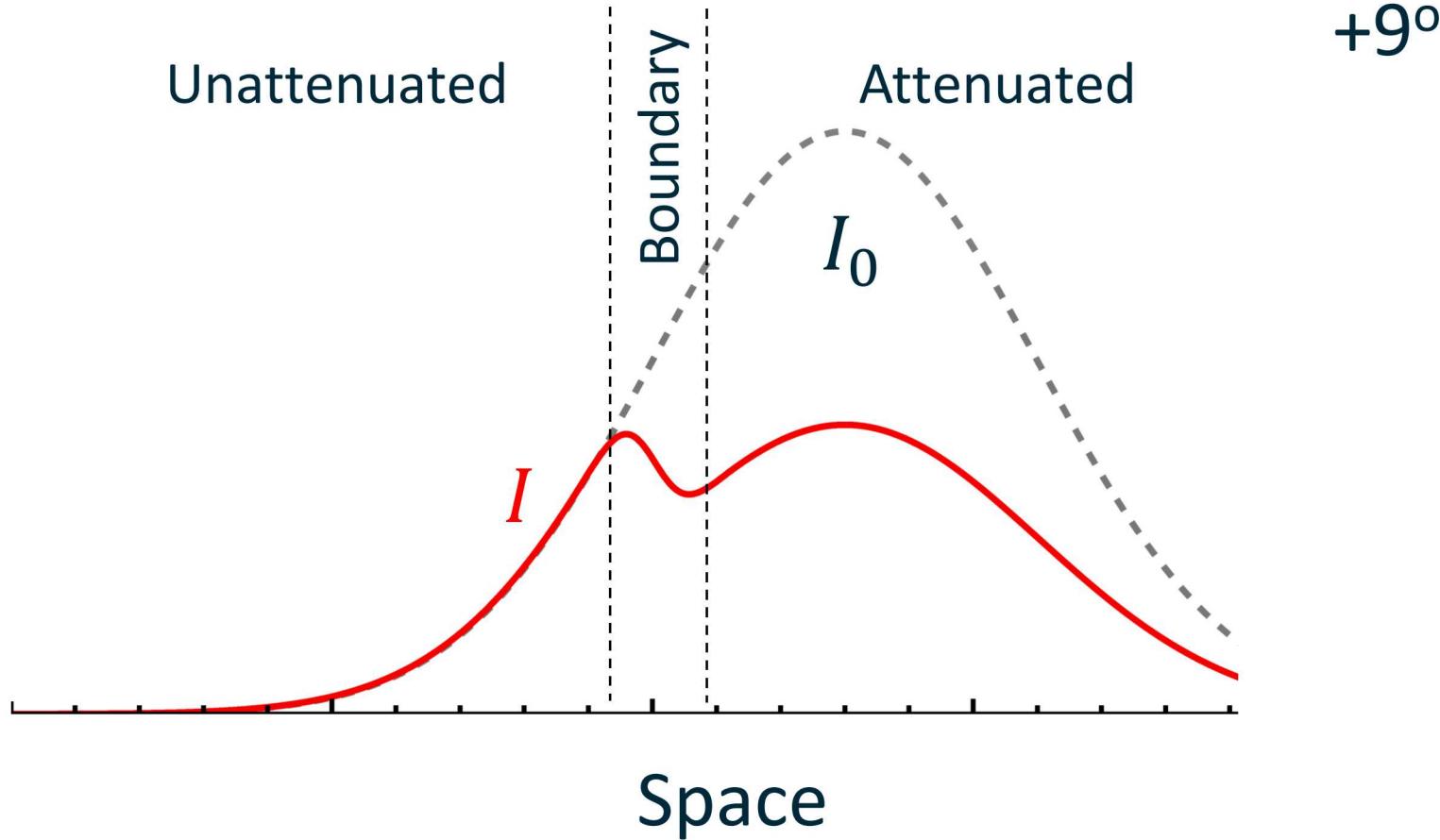


- Reproducibility at two sides \approx Shot-to-shot reproducibility
- This error is not systematic and hard to correct
- Hypothesis: Misalignment of aperture?

Brightness reproducibility only will not be sufficiently accurate

We should analyze spatial shape too!

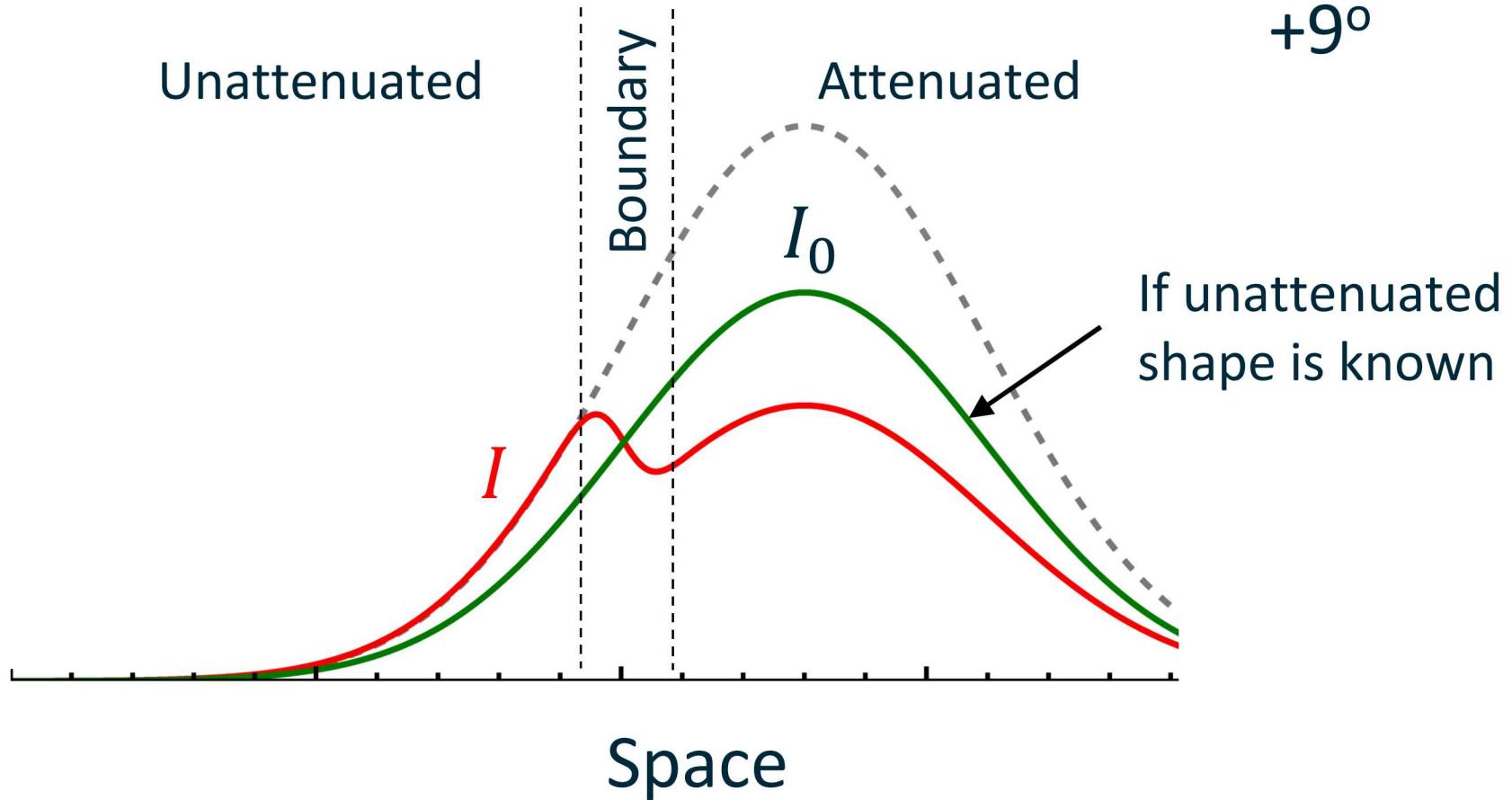
Half-moon spatial profile has both attenuated and unattenuated intensities, enabling accurate analysis



If the unattenuated shape is known, we can determine FeMg transmission accurately

We should analyze spatial shape too!

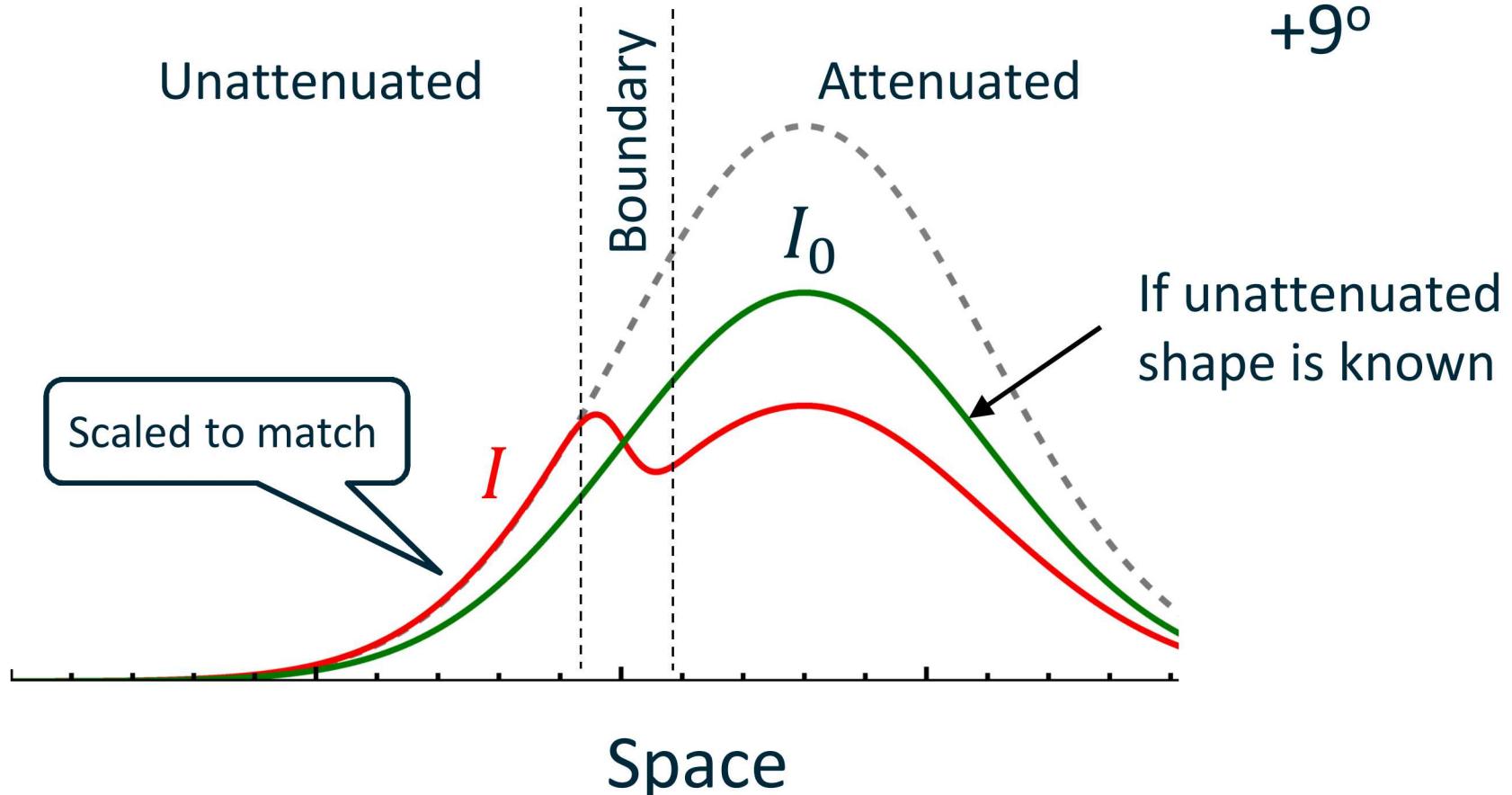
Half-moon spatial profile has both attenuated and unattenuated intensities, enabling accurate analysis



If the unattenuated shape is known, we can determine FeMg transmission accurately

We should analyze spatial shape too!

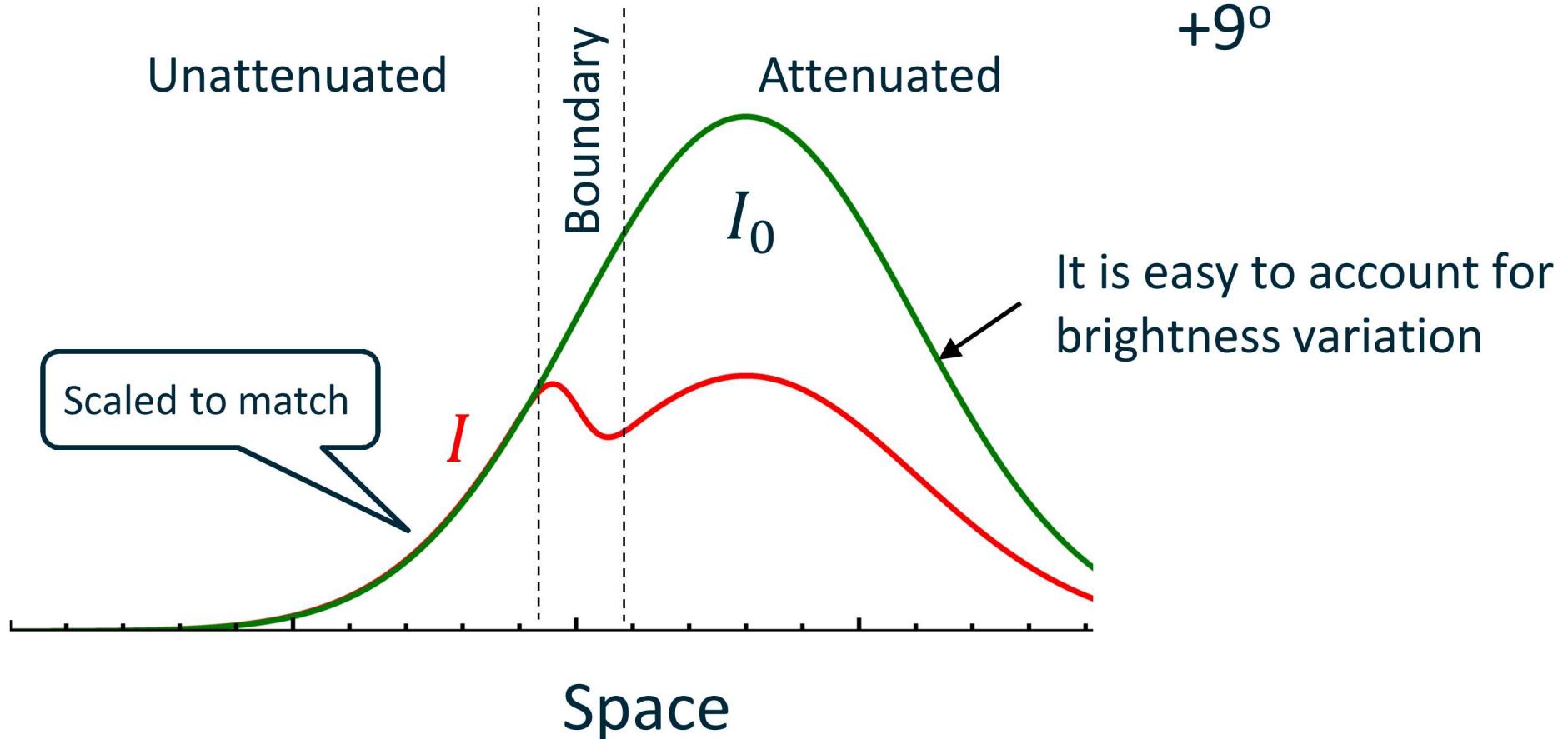
Half-moon spatial profile has both attenuated and unattenuated intensities, enabling accurate analysis



If the unattenuated shape is known, we can determine FeMg transmission accurately

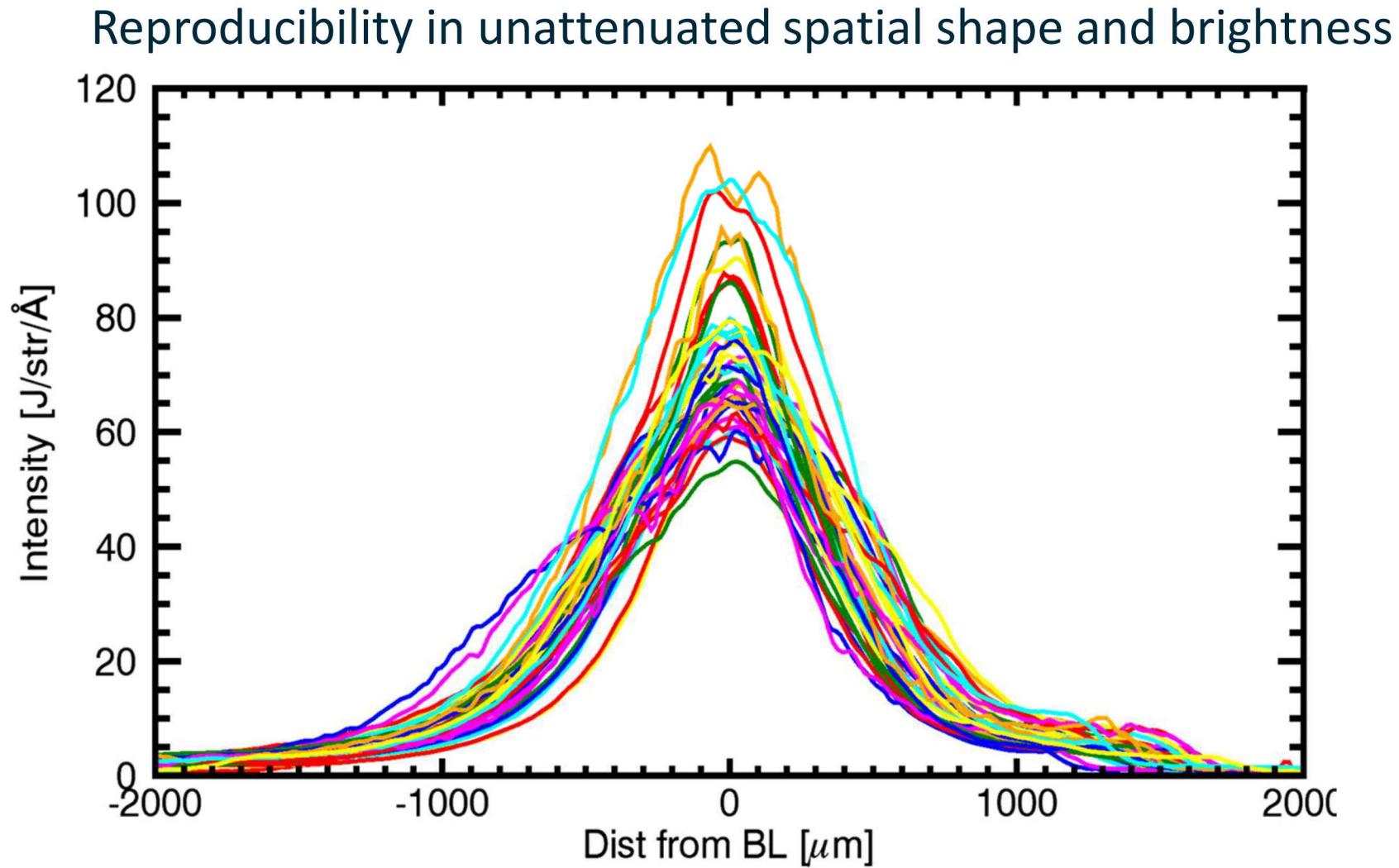
We should analyze spatial shape too!

Half-moon spatial profile has both attenuated and unattenuated intensities, enabling accurate analysis



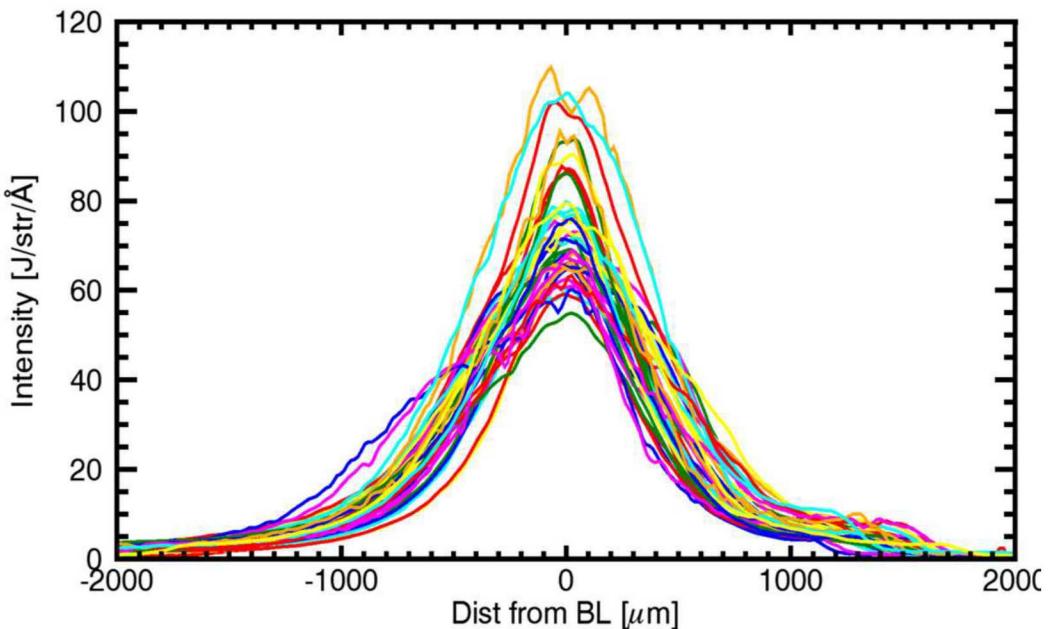
If the unattenuated shape is known, we can determine FeMg transmission accurately

Challenge comes from the fact that both shape and brightness are known to limited accuracy

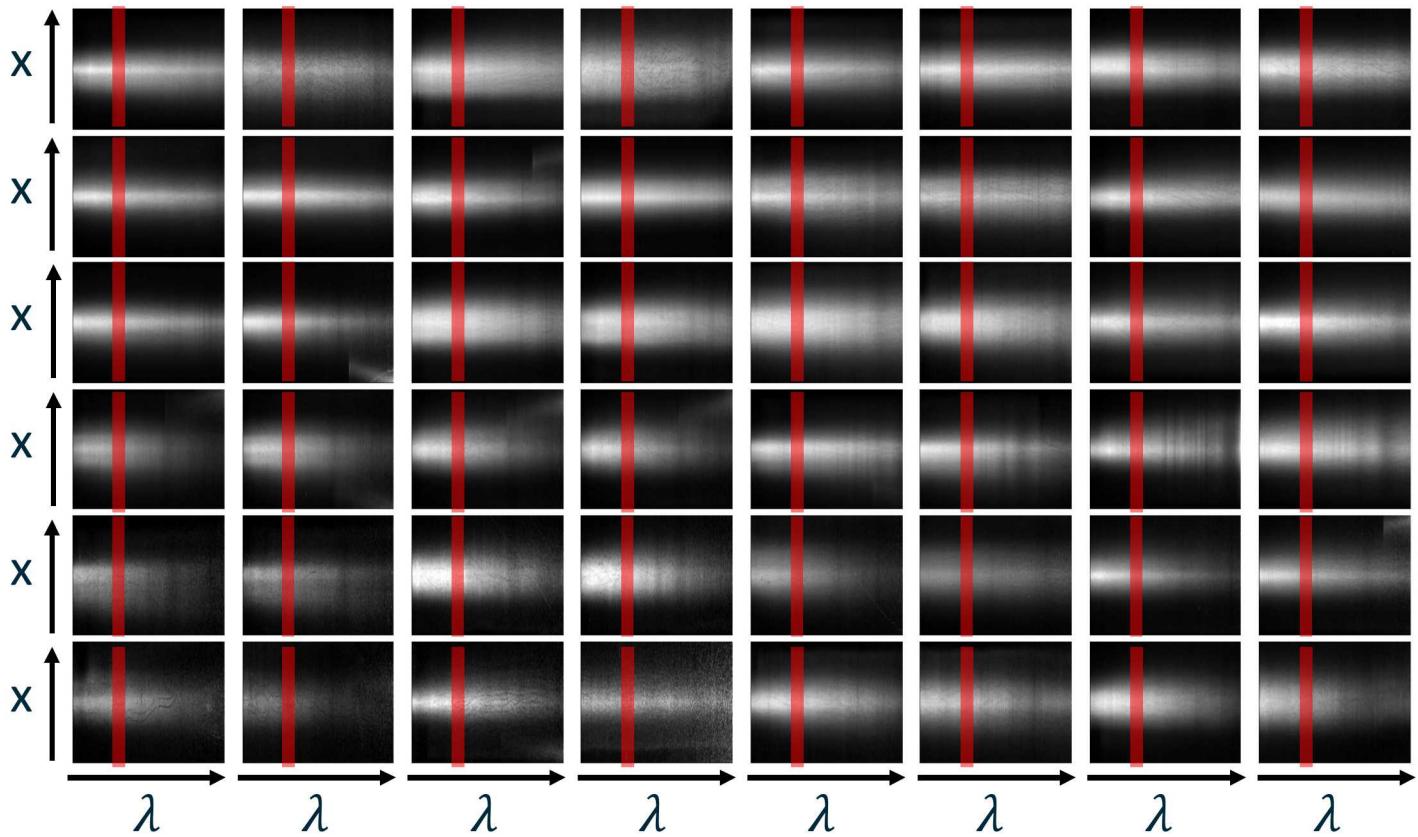


Challenge comes from the fact that both shape and brightness are known to limited accuracy

Reproducibility in unattenuated spatial shape and brightness

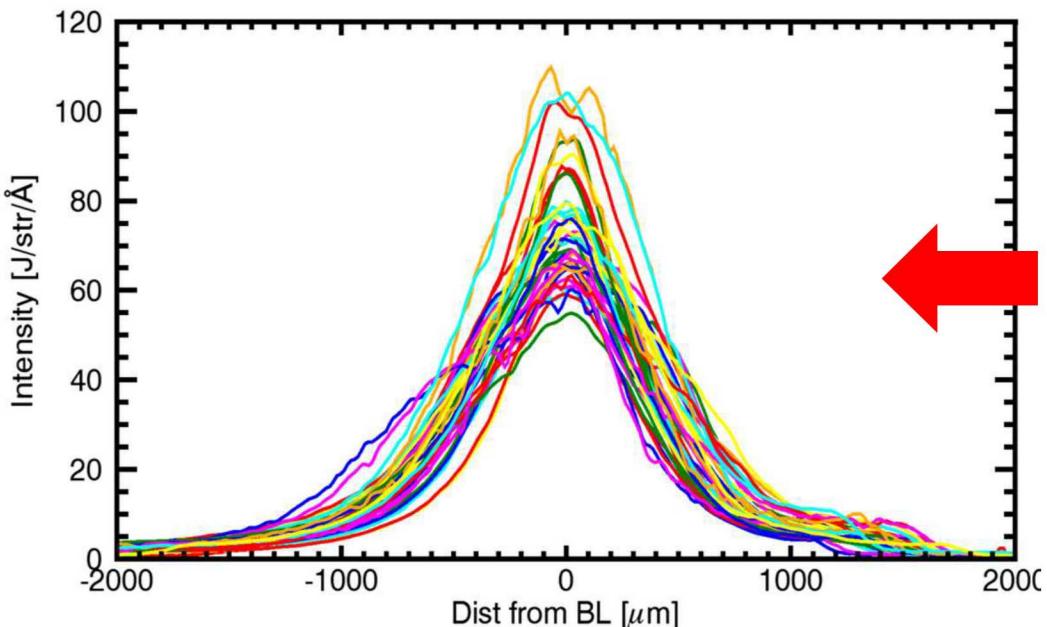


48 spectral images from 12 calibration shots collected over a decade



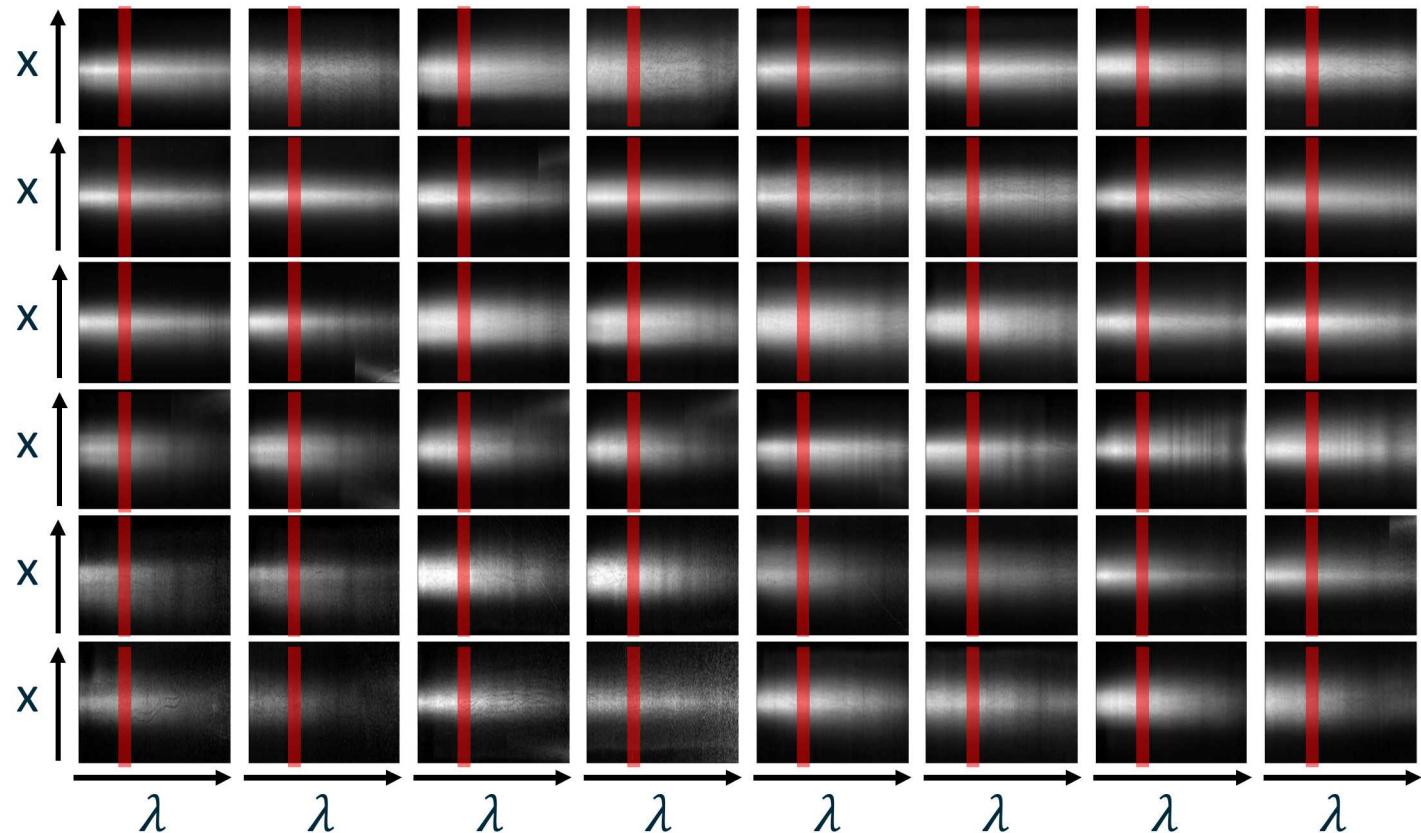
Challenge comes from the fact that both shape and brightness are known to limited accuracy

Reproducibility in unattenuated spatial shape and brightness



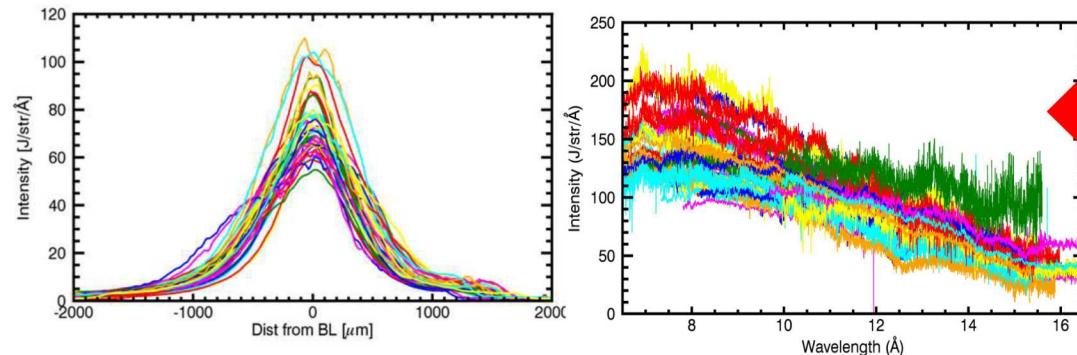
Good news: We have accumulated large volume of backlight radiation statistics

48 spectral images from 12 calibration shots collected over a decade



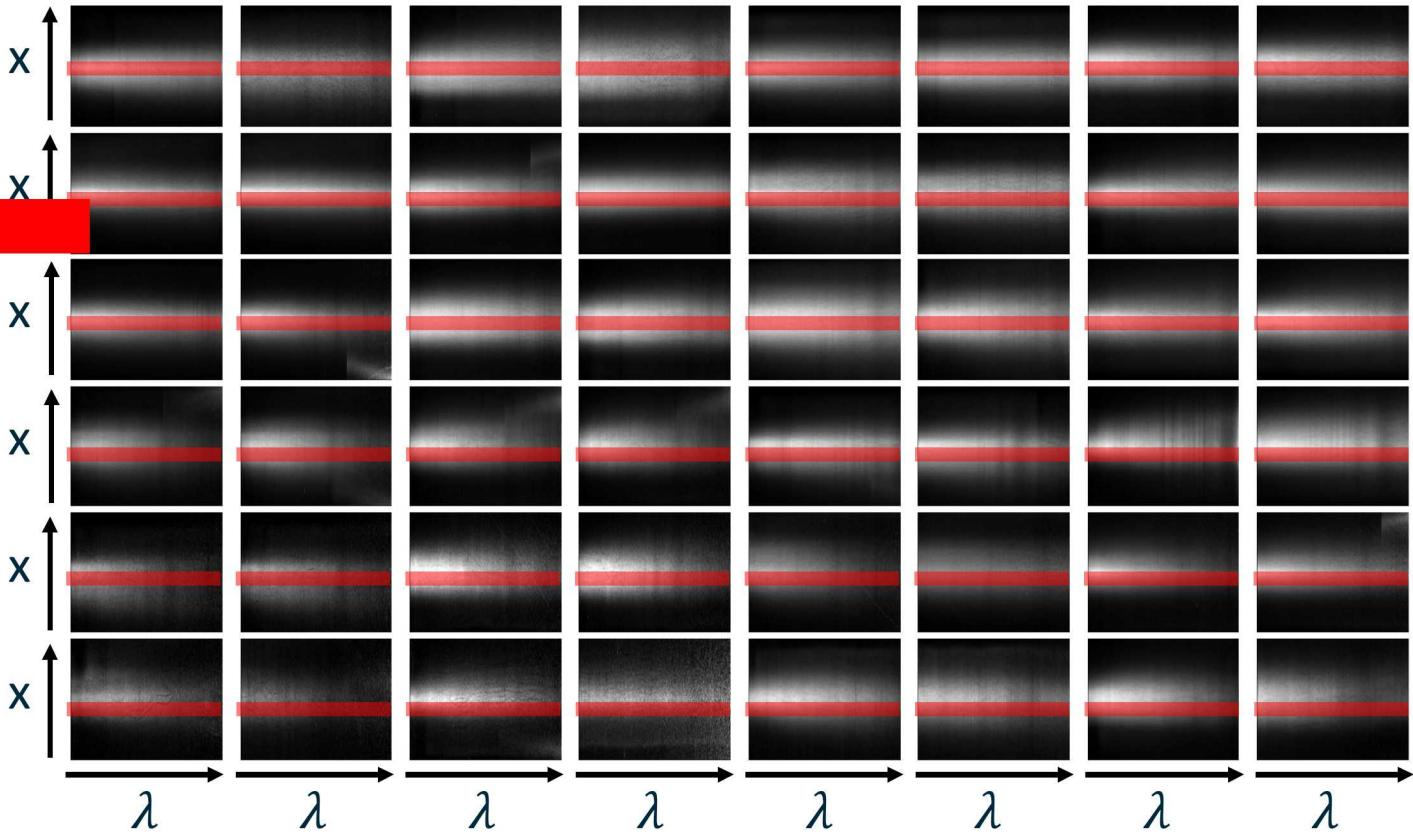
Challenge comes from the fact that both shape and brightness are known to limited accuracy

Reproducibility in unattenuated spatial shape and brightness



Good news: We have accumulated large volume of backlight radiation statistics

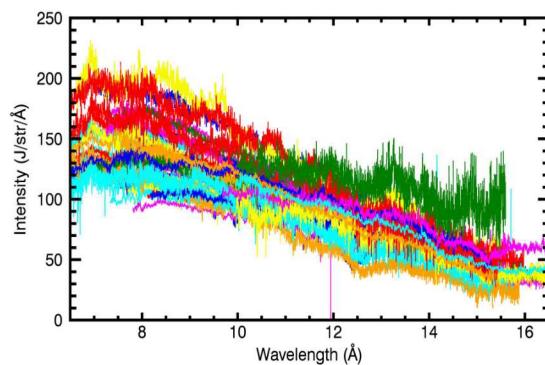
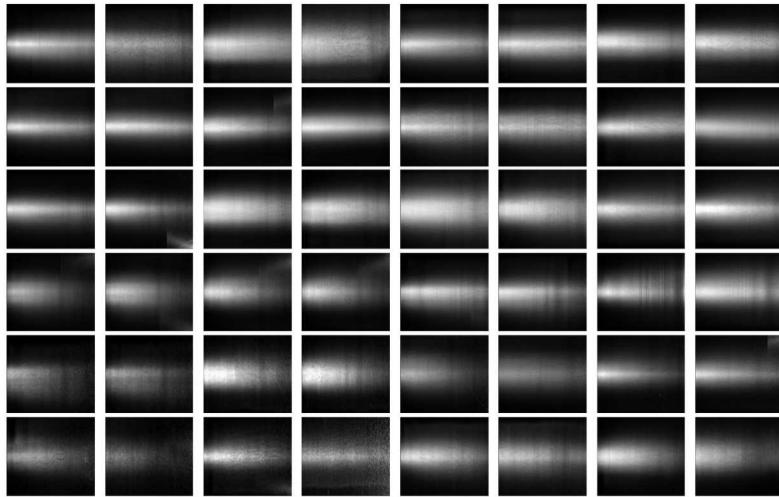
48 spectral images from 12 calibration shots collected over a decade



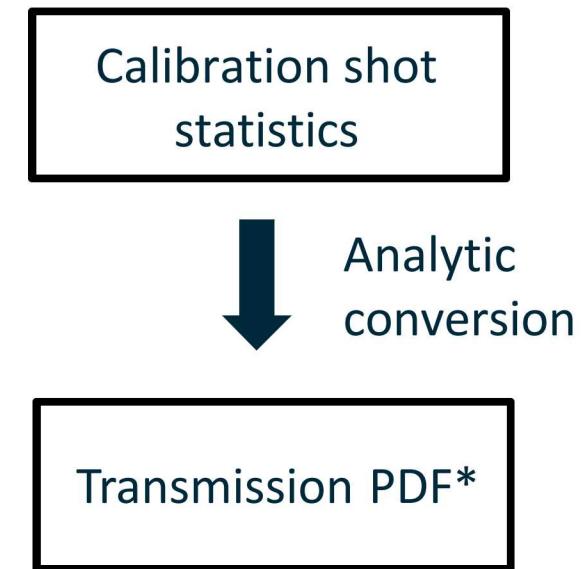
The analysis method can be improved by performing rigorous propagation of this statistics

New analysis propagate calibration-shot statistics and propagate three sources of uncertainties properly

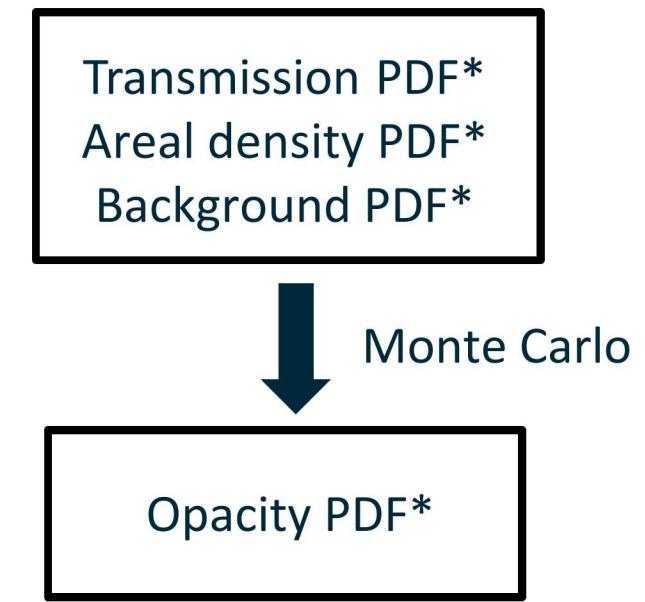
Step 1: Calibration-shot statistics



Step 2: Statistics conversion



Step 3: Error propagation

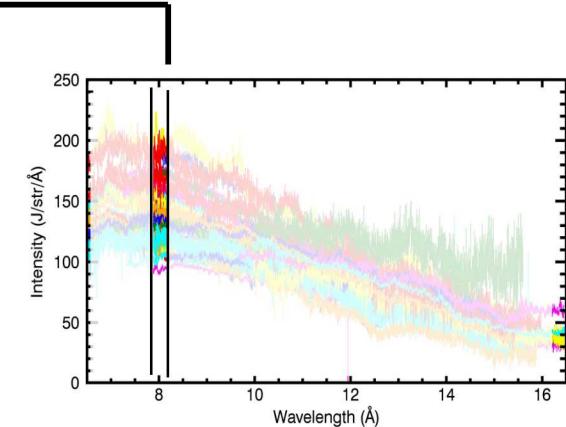
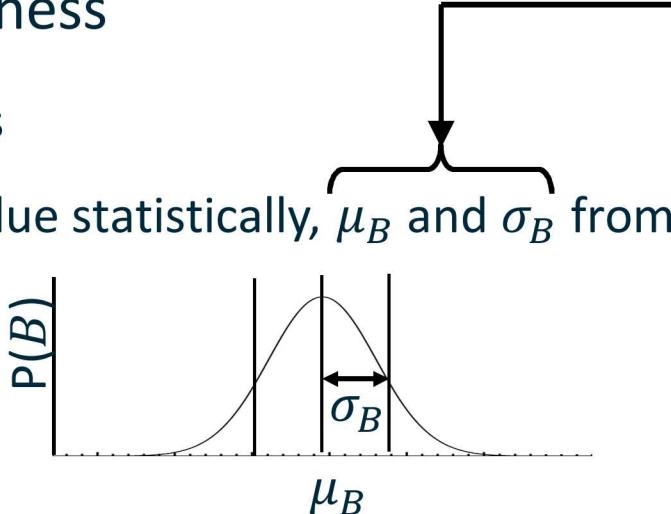


It reduces transmission error by combining multiple uncorrelated statistics, and propagate all uncertainties to the opacity uncertainty

ii) Calibration-shot statistics can be analytically converted to transmission PDF (\equiv Probability Distribution Function)

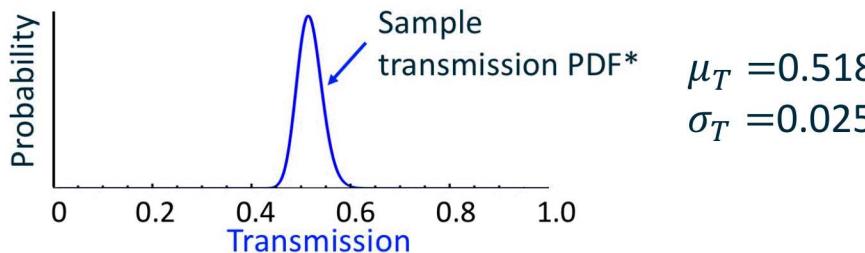
Example: Transmission at 8Å from brightness

$$T_{8\text{\AA}} = \frac{I_{8\text{\AA}}}{B_{8\text{\AA}}} \quad \begin{array}{l} \text{We measure this} \\ \text{We know this value statistically, } \mu_B \text{ and } \sigma_B \text{ from} \end{array}$$



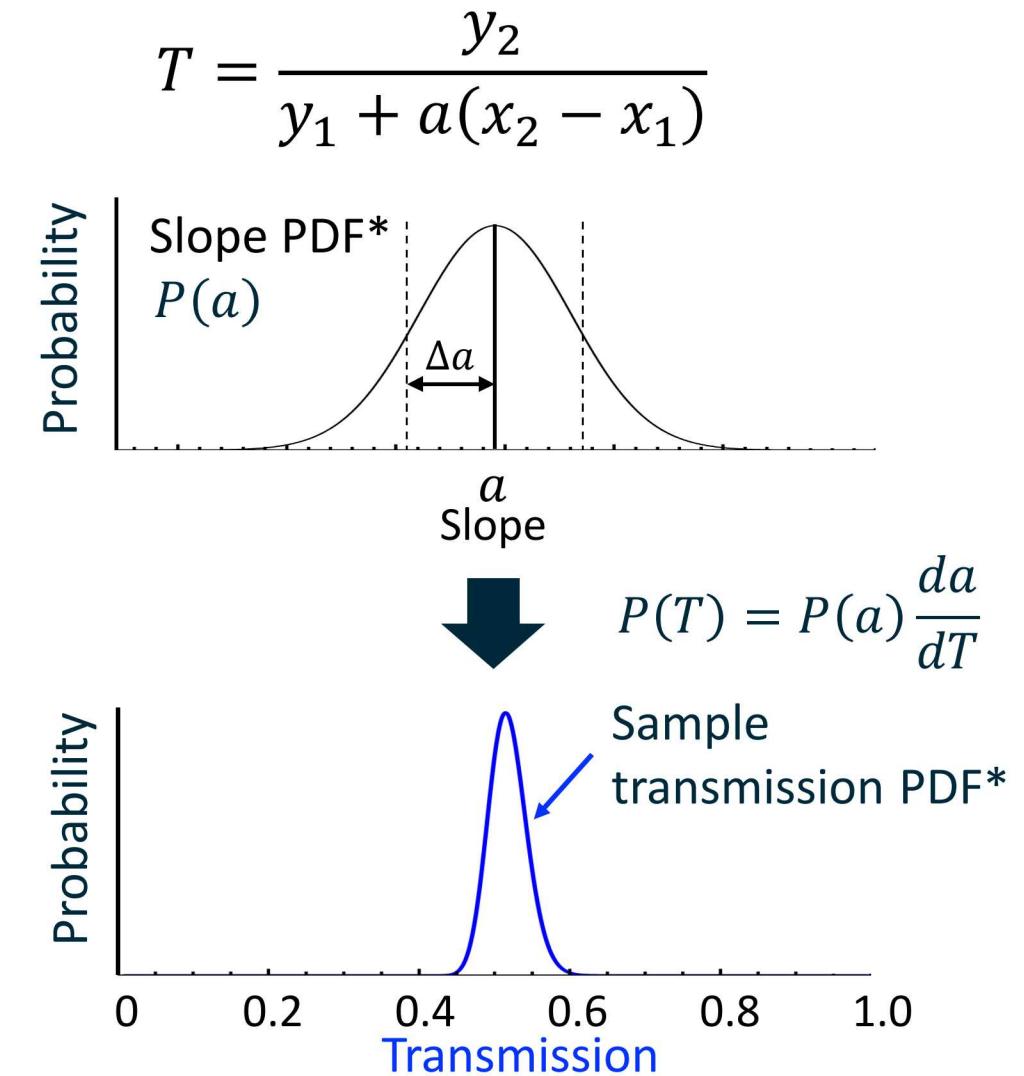
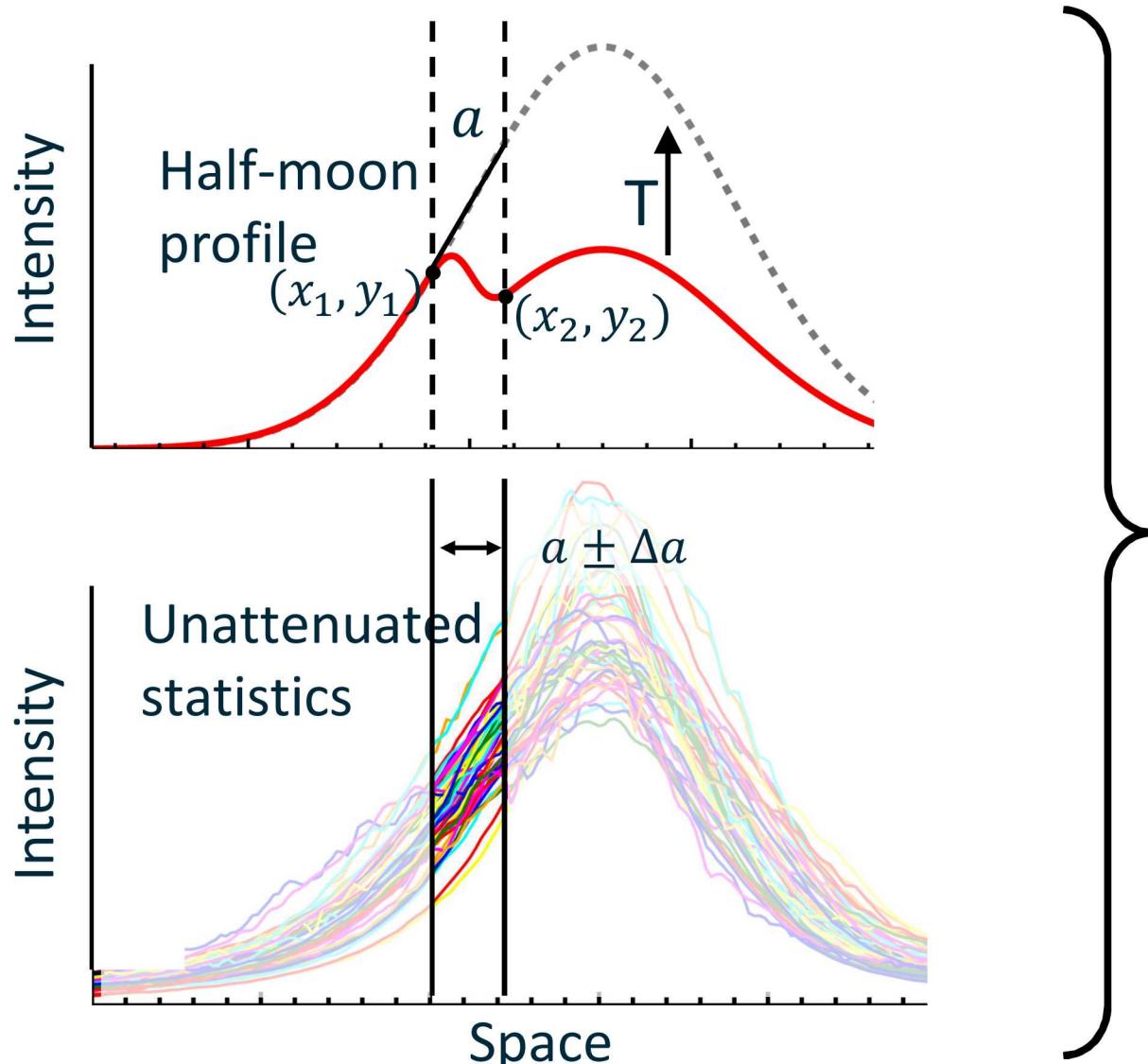
Key idea: If we know $P(B)$, we can analytically derive $P(T)$

$$P(T) = P(B) \frac{dB_{8\text{\AA}}}{dT_{8\text{\AA}}} \quad \rightarrow$$



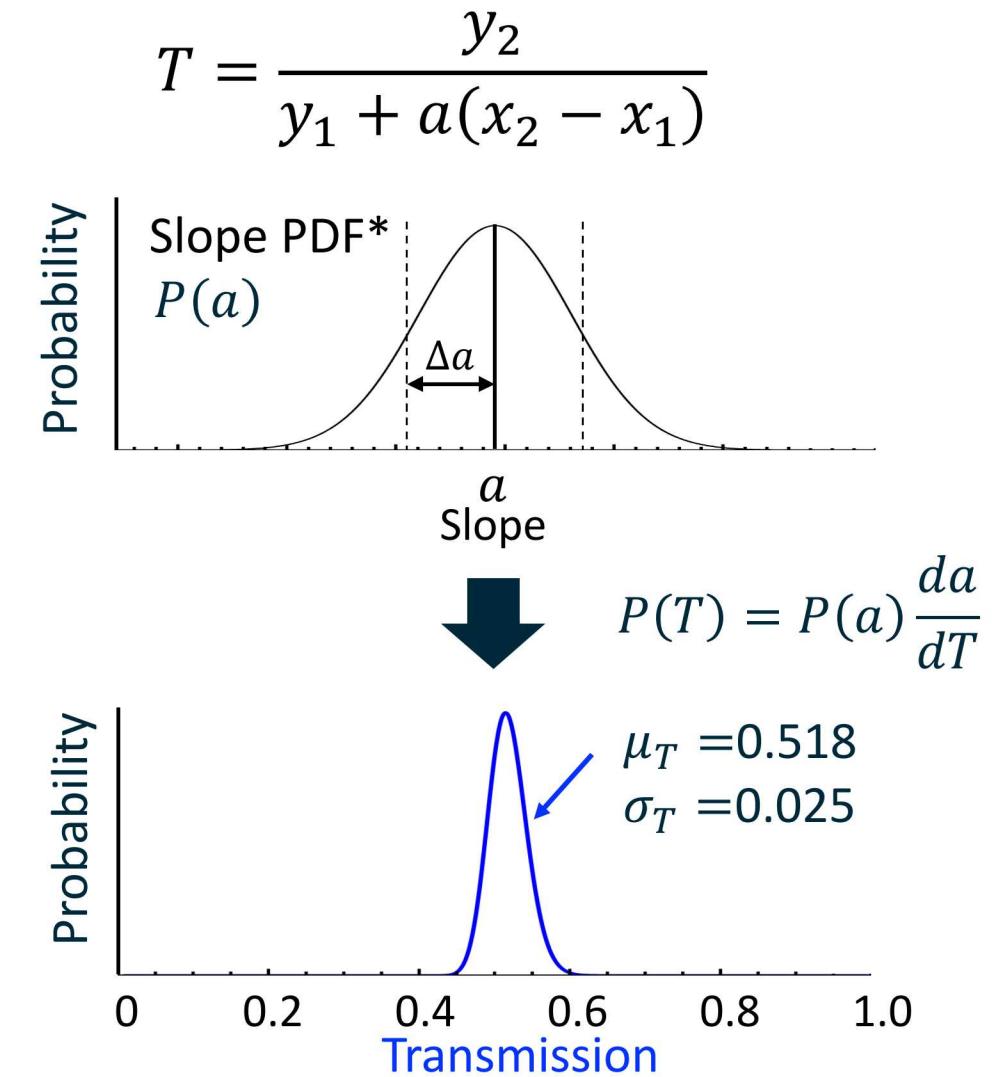
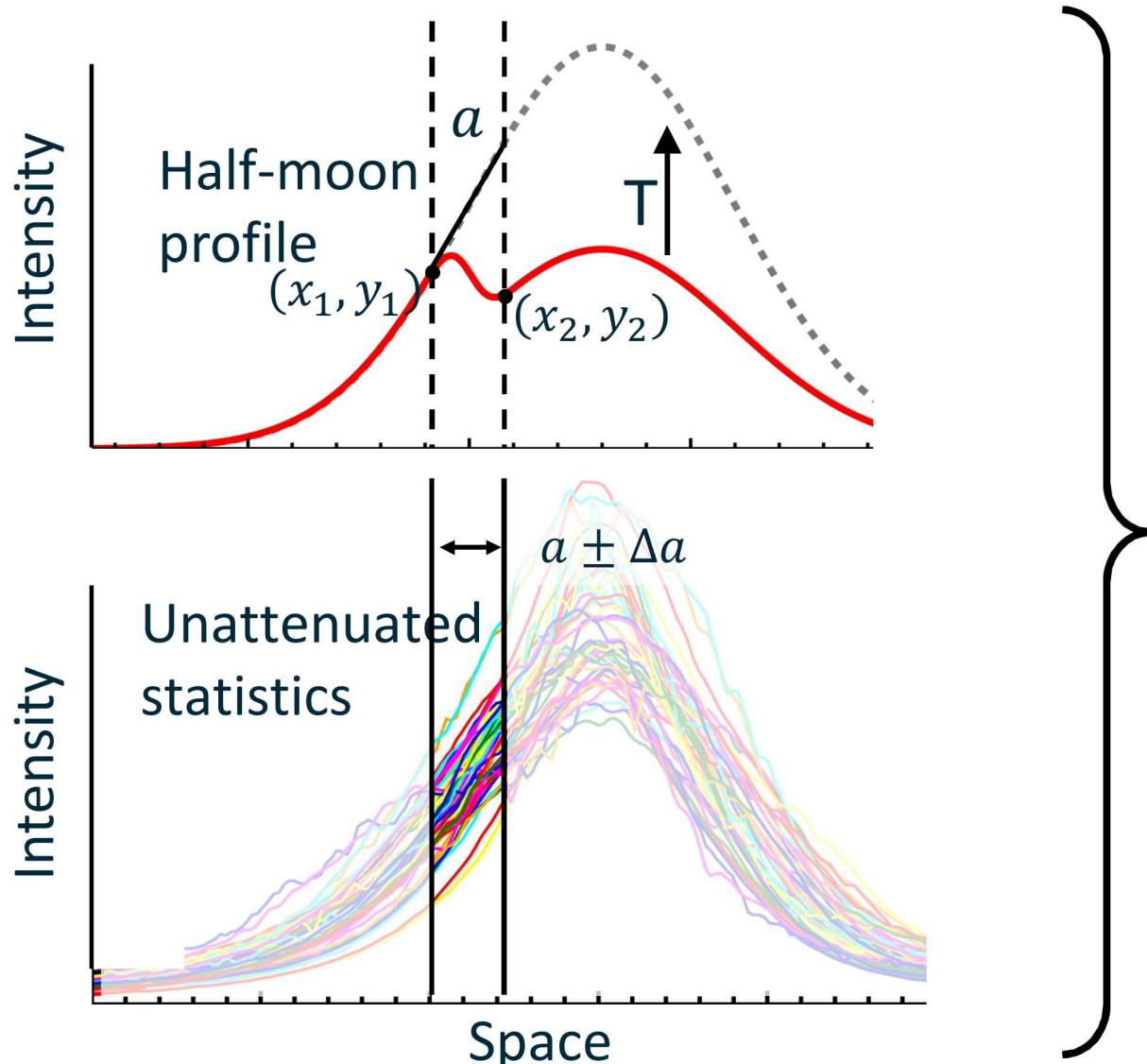
Calibration shot gives statistics on absolute, spectra, and spatial shapes \rightarrow Multiple ways to get PDF

Another example: transmission from boundary-slope statistics



*PDF = Probability distribution function

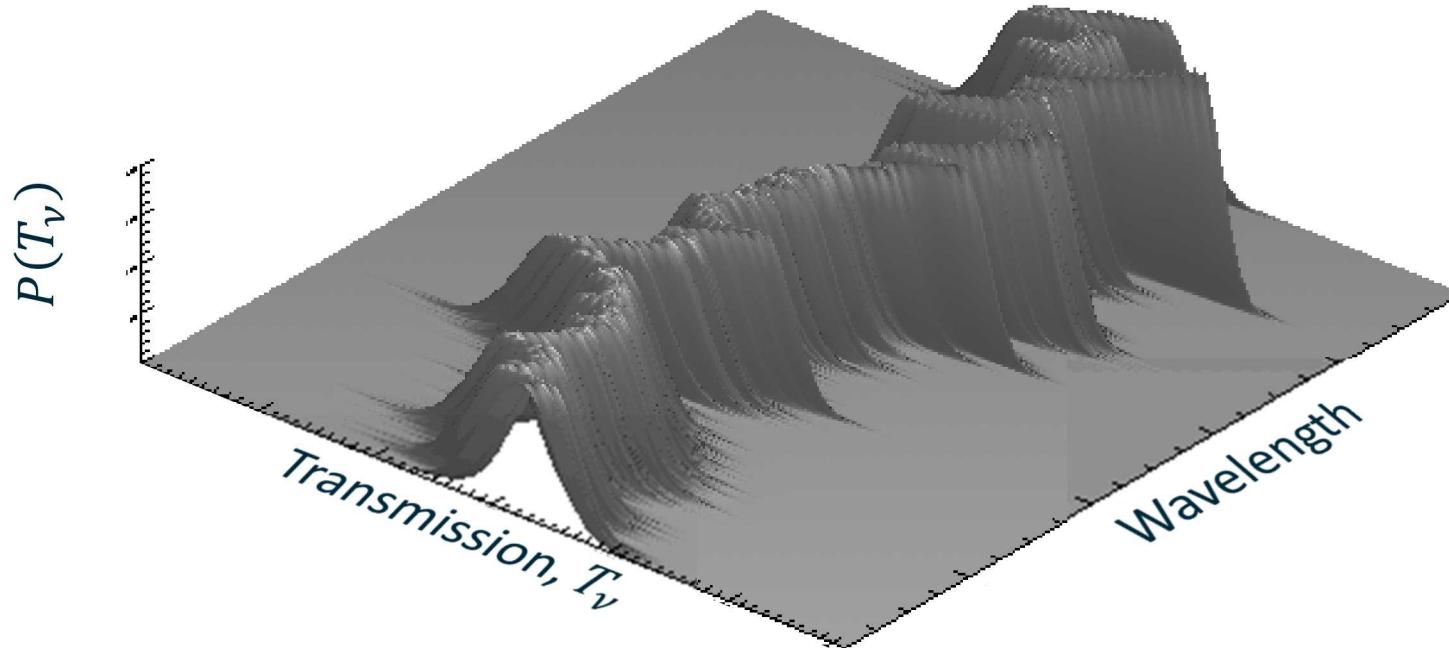
Another example: transmission from boundary-slope statistics



*PDF = Probability distribution function

ii) Calibration-shot statistics can be analytically converted to transmission PDF (\equiv Probability Distribution Function)

Repeating this analysis at every wavelength gives you:

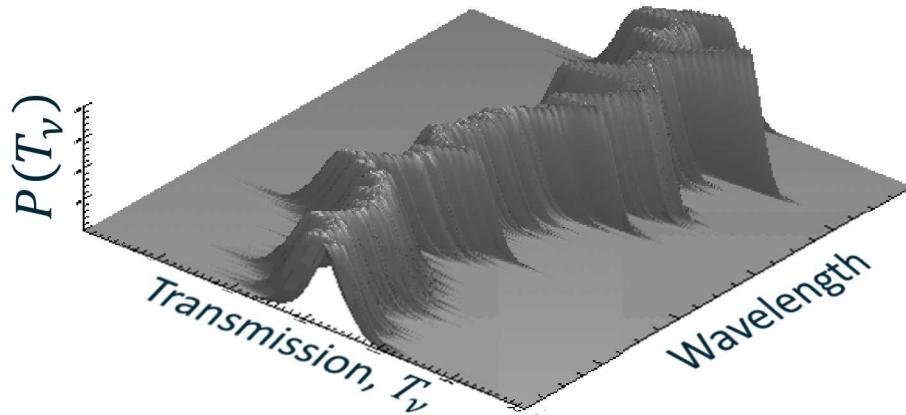


- We follow detailed transmission PDF
- Multiple methods and data are easily combined through joint probability

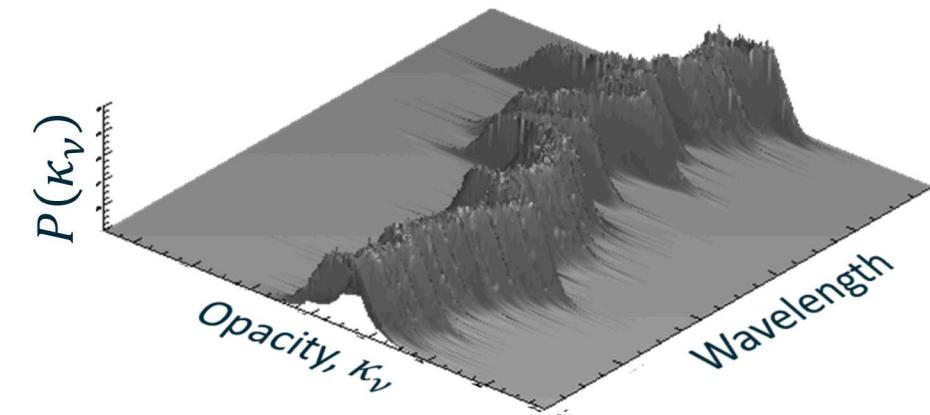
Analysis-method accuracy is confirmed through synthetic-data analysis

iii) Transmission PDF is converted to opacity PDF using Monte-Carlo technique, propagating various uncertainties

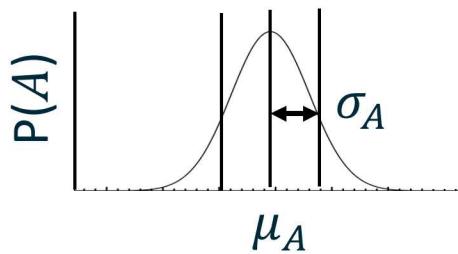
Transmission PDF, $P_\lambda(T)$



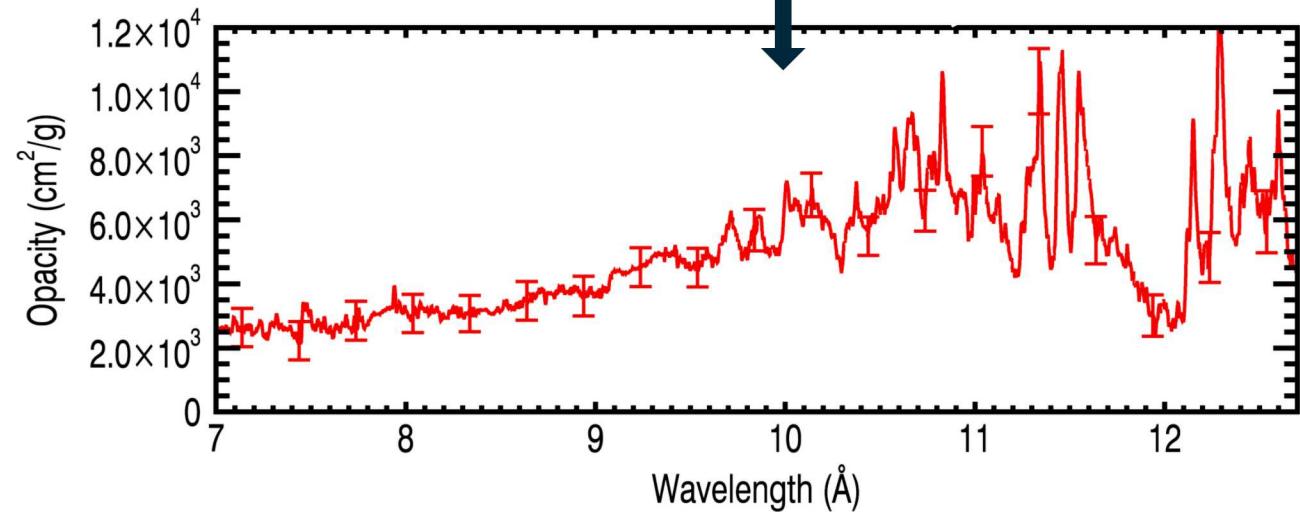
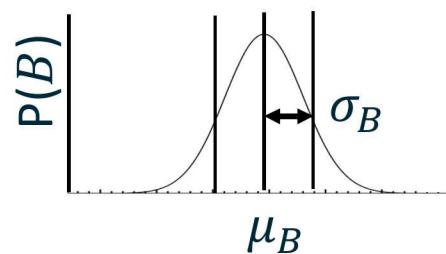
Monte Carlo



Areal-density PDF

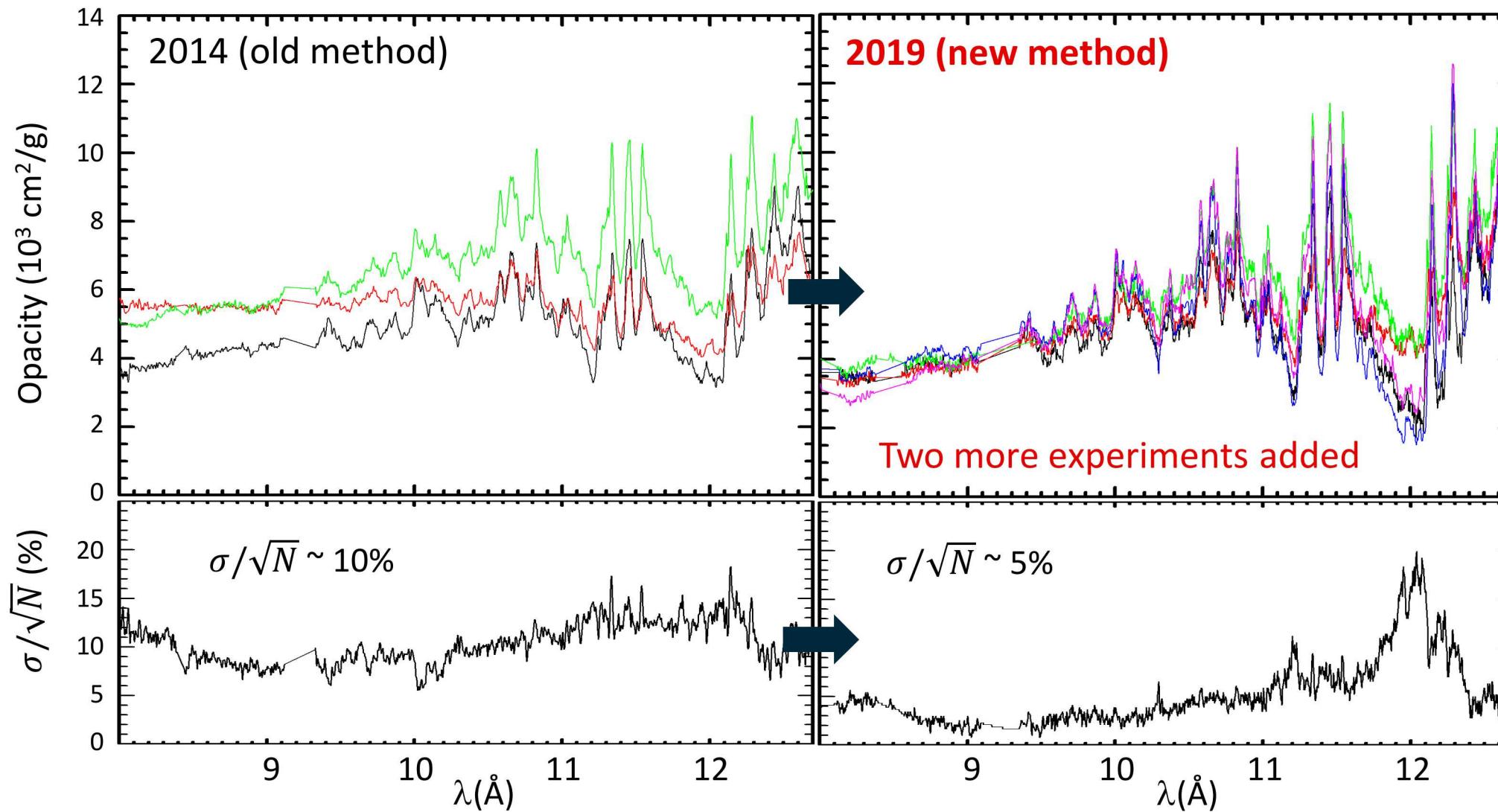


Background PDF

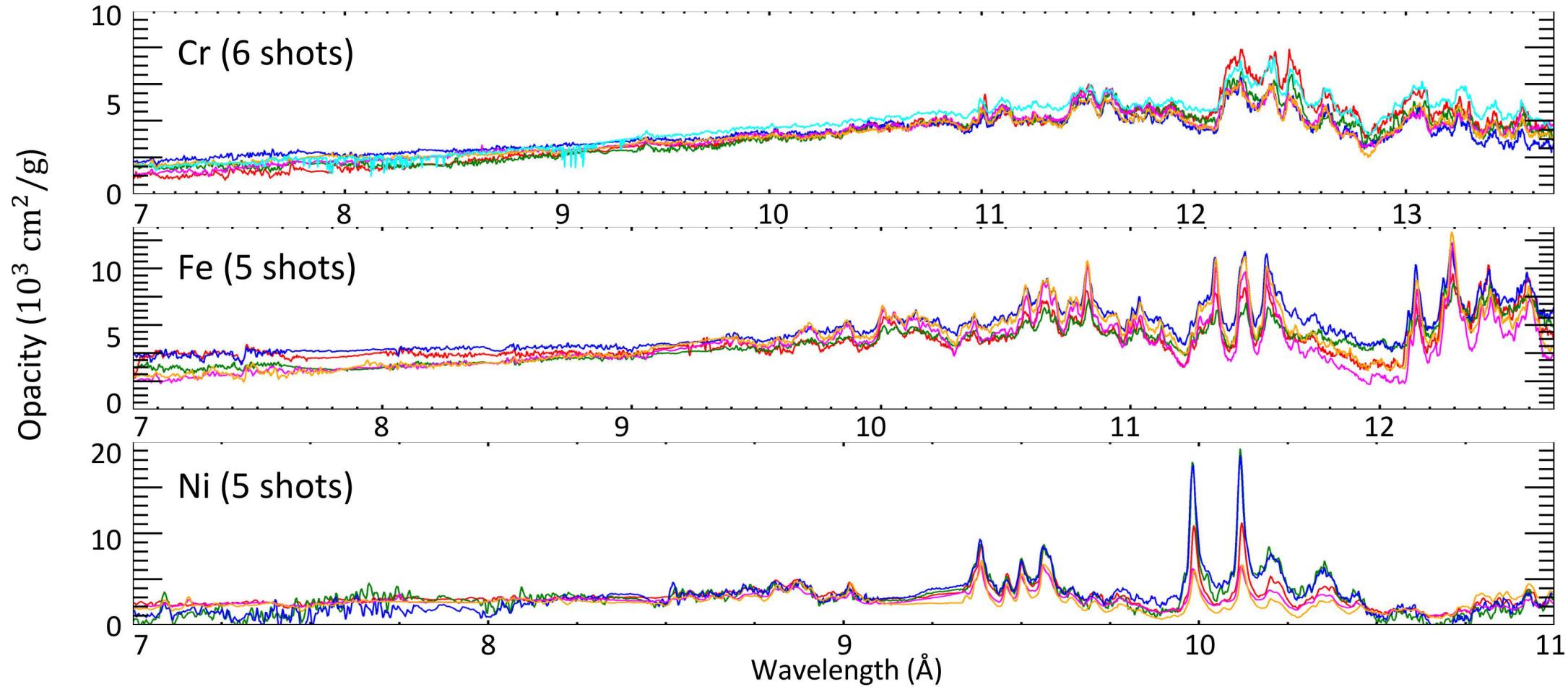


Analysis returns asymmetric non-Gaussian opacity PDF as a function of wavelengths

New analysis was applied to old data; Experiment reproducibility is better than we believed

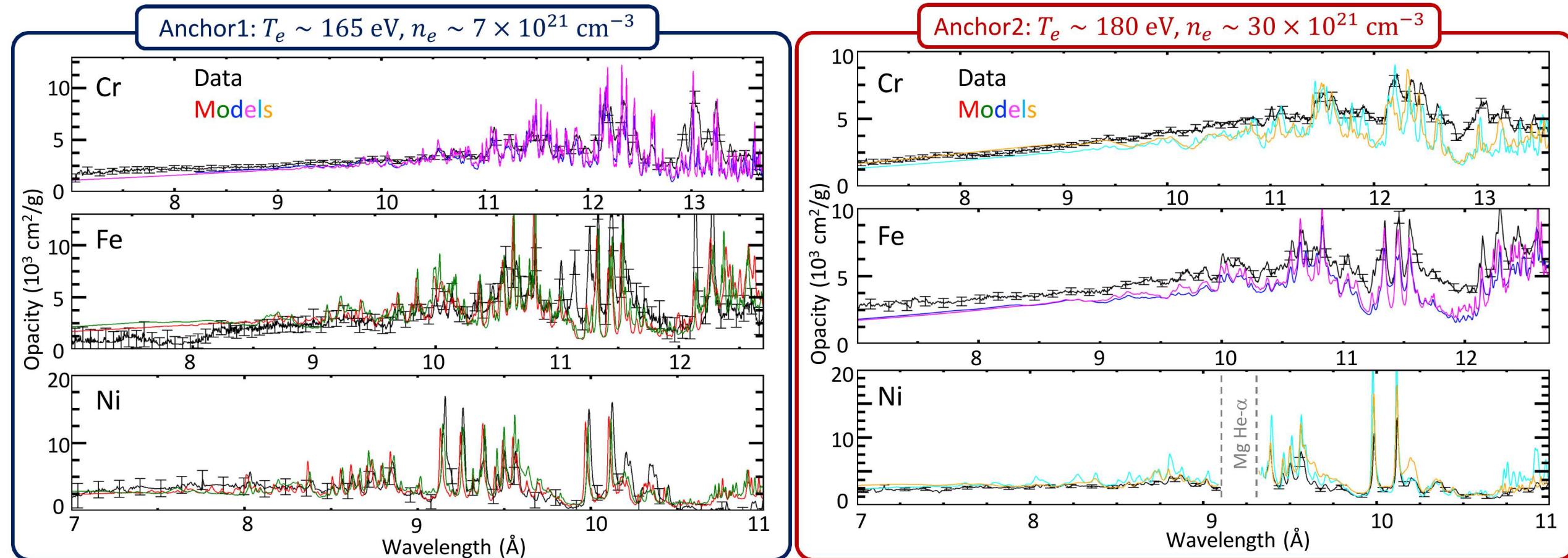


Excellent reproducibility is confirmed from all three elements, demonstrating experiment/analysis reliability



Model-data discrepancy as a function of atomic number helped narrow down sources of discrepancies [1]

First systematic study of high-temperature L-shell opacities were performed for Cr, Fe, and Ni at two conditions



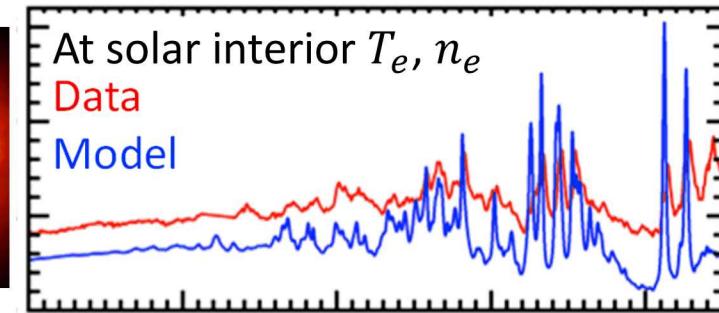
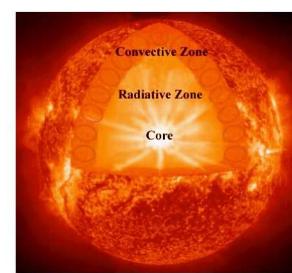
- Opacities are measured at $T_e > 150$ eV
- T_e and n_e are diagnosed independently
- Reproducibility is confirmed

Systematically performed for Cr, Fe, Ni at two conditions

New analysis method confirmed experiment reproducibility and enabled accurate systematic study of Cr, Fe, and Ni opacity

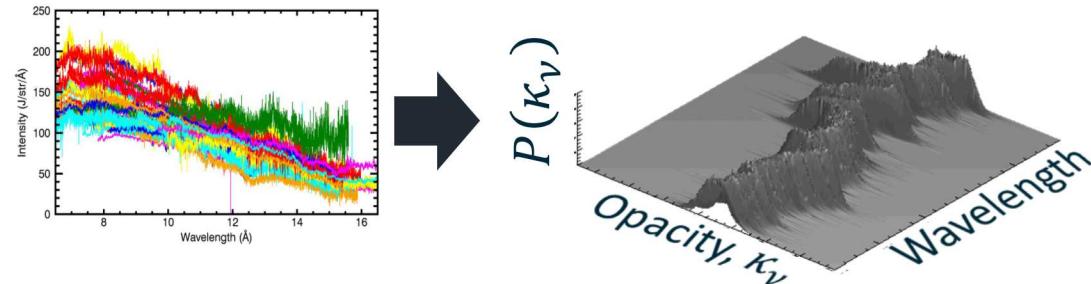
Is iron opacity inaccurate?

- Fe opacity is measured at solar interior condition
- **Severe disagreement with modeled opacity → Why?**



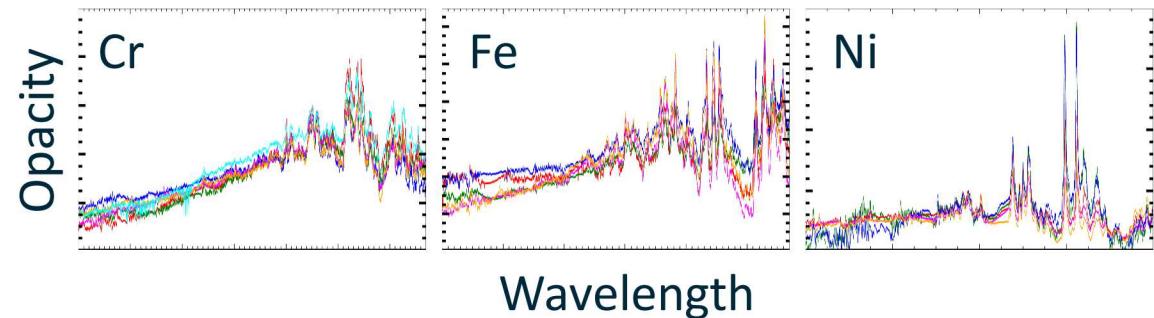
Data analysis method is refined

- Large volume of calibration-shot statistics
- Error propagation with Monte Carlo
- Method tested



New analysis improved reproducibility, providing insight into the problem

- Improved reproducibility: 10-20%
- First systematic study published by PRL



T. Nagayama et al, PRL 122, 235001 (2019)