Initialization of the thermo-mechanical state of an ice sheet model

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Mauro Perego
(Center for Computing Research, Sandia National Laboratories)

SNL team:
Luca Bertagna, John D. Jakeman, Mauro Perego,
Irina K. Tezaur, Jerry Watkins, Andrew Salinger (emeritus)

LANL Collaborators:
Matthew Hoffman, Stephen Price, Tong Zhang

NYU collaborator: Georg Stadler

Emory collaborator: Alessandro Barone

University of Montana, October 14, Missoula, MT, 2019

Center for Computing Research

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Funded by (ProSPect)

SciDAC Scientific Discovery through Advanced Computing

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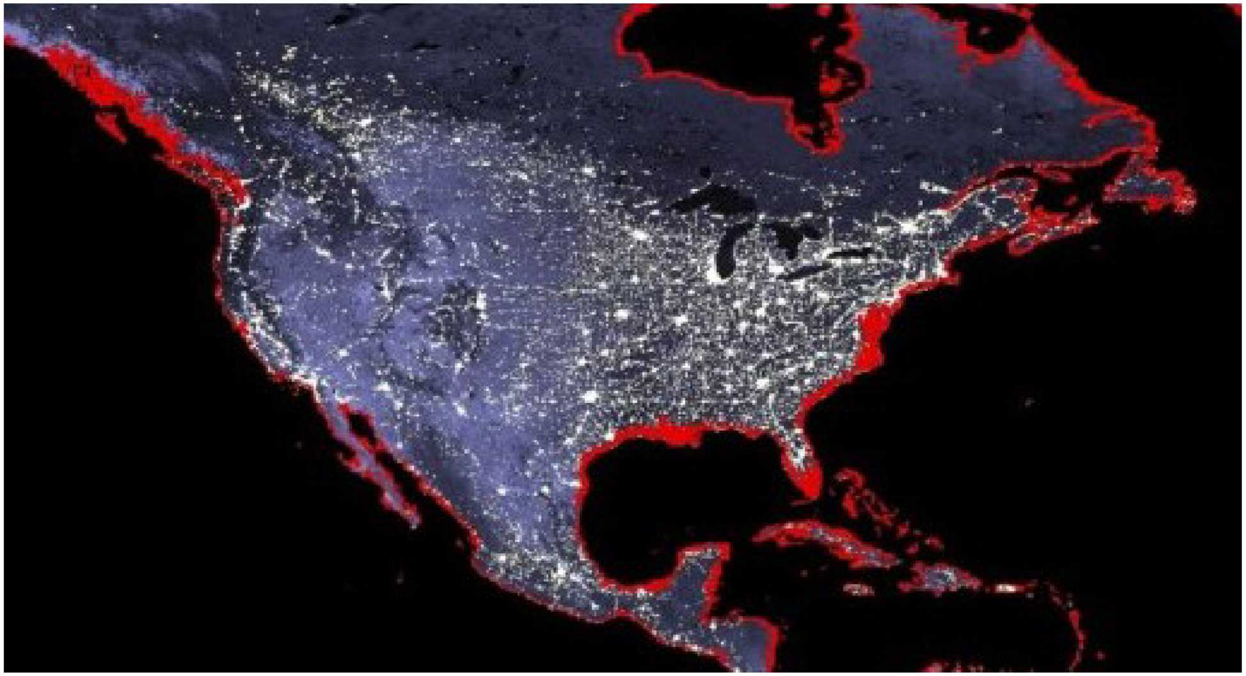
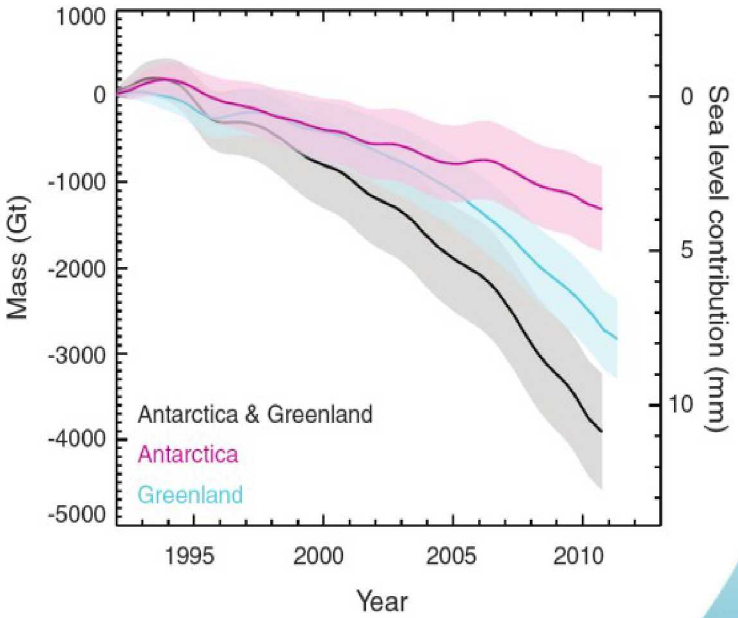
Brief introduction and motivation

- Greenland and Antarctica ice sheets store most of the fresh water on hearth.
- Modeling ice sheets (Greenland and Antarctica) dynamics is essential to provide estimates for sea level rise* and and fresh water circulation.
- Global mean sea-level is rising at the rate of 3.2 mm/yr and the rate is increasing.
- Latest studies suggest possible increase of 0.3 – 2.5m by 2100.



Map with 6 meters sea-level rise in red (NASA).



total mass loss of ice sheets in 1992-2011 (sheperd et al. 2012)

*DOE SciDAC project **ProSPect** (**Pro**babilistic **S**ea Level **P**roj**ect**ion from Ice Sheet and Earth System Models), Institutes: LANL, LBNL, SNL, ONL, NYU, Univ. of Michigan

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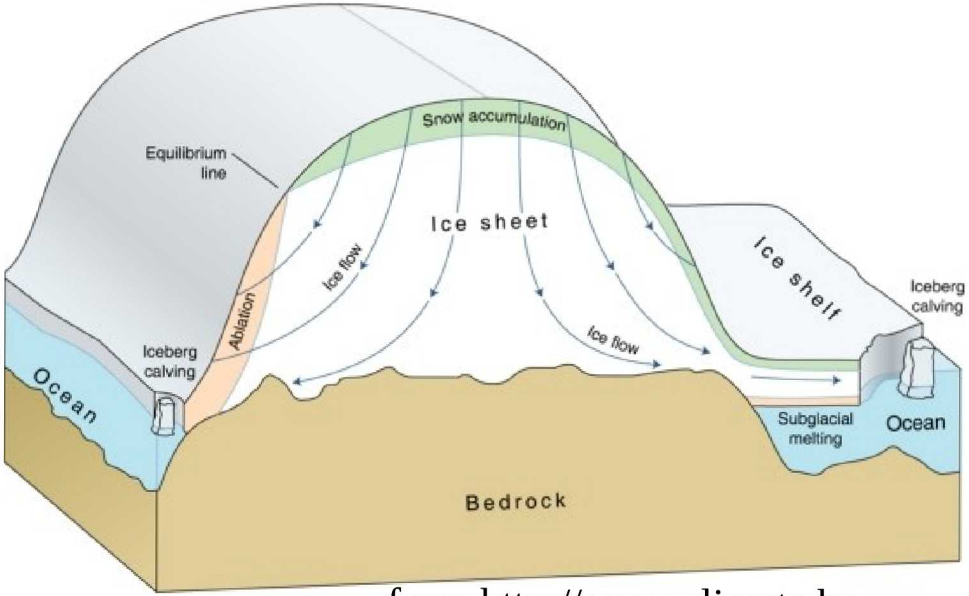
Brief introduction and motivation

- Ice behaves like a very viscous shear-thinning fluid (similar to lava flow) driven by gravity. Source: snow packing/water freezing. Sink: ice melting / calving in ocean.
- Greenland and Antarctica have a shallow geometry (thickness up to 4 km, horizontal extensions of thousands of km).

Perito Moreno glacier



from http://www.climate.be

Outline

- Ice sheet flow model
- PDE-constrained optimization approach for initializing ice sheet flow model
- Introduction of implicit temperature model
- Improved optimization approach that accounts for temeprature and thickness tendencies

Ice Sheet Modeling

Ice momentum equations

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} \rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) - \nabla\cdot\sigma = \rho\mathbf{g} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

Ice Sheet Modeling

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Ice Sheet Modeling

Ice momentum equations

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

with:

$$\sigma = 2\mu \mathbf{D} - pI, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Nonlinear viscosity:

$$\mu = \frac{1}{2}\alpha(T)\,|\mathbf{D}(\mathbf{u})|^{\frac{1}{n}-1}, \quad n \geq 1, \quad (\text{tipically } n \simeq 3)$$

Viscosity is singular when ice is not deforming:

What about iterative algorithms?

- Picard scheme not affected by singularity, rate of convergence: $\left(1 - \frac{1}{n}\right)$
- Newton does not converge if viscosity is close to singularity → regularization:

$$\mu = \frac{1}{2}\alpha(T)\ \left(|\mathbf{D}(\mathbf{u})|^2 + \delta^2\right)^{\frac{1}{2n}-\frac{1}{2}}, \quad \delta > 0$$

Ice Sheet Modeling

Ice momentum equations

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$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

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Viscosity is singular when ice is not deforming

Stiffening/Damage factor

$$\mu^*(x,y,z) = \phi(x,y)\,\mu(x,y,z)$$

ϕ : stiffening factor that accounts for modeling errors in rheology

Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

- **Model for the ice sheet evolution**
  (thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot \int_z \mathbf{u}\, dz$$

- **Temperature equation (cold ice)**

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \rho c \mathbf{u} \cdot \nabla T + 2\dot{\varepsilon}\sigma$$

- **Coupling with other climate components (e.g. ocean, atmosphere)**

Stokes approximations in different regimes

$\text{Stokes}(\mathbf{u}, p)$

$$\begin{cases} -\nabla \cdot (2\mu \mathbf{D}(\mathbf{u}) - p\mathbf{I}) = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

$\text{FO}(u, v)$

First Order* or
Blatter-Pattyn model

$$-\nabla \cdot \left(2\mu \tilde{\mathbf{D}} - \rho g (s - z)\mathbf{I}\right) = \mathbf{0}$$

*Dukowicz, Price and Lipscomb, 2010. J.

Stokes approximations in different regimes

$$\text{Stokes}(\mathbf{u}, p)$$

$$\begin{cases} -\nabla \cdot (2\mu \mathbf{D}(\mathbf{u}) - p\mathbf{I}) = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Drop terms using **scaling argument** based on the fact that ice sheets are shallow

$$\mathbf{D}(\mathbf{u}) = \begin{bmatrix} u_x & \frac{1}{2}(u_y + v_x) & \frac{1}{2}(u_z + \cancel{w_x}) \\ \frac{1}{2}(u_y + v_x) & v_y & \frac{1}{2}(v_z + \cancel{w_y}) \\ \frac{1}{2}(u_z + \cancel{w_x}) & \frac{1}{2}(v_z + \cancel{w_y}) & w_z \end{bmatrix} \qquad \mathrm{u} := \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\mu = \mu(|\mathbf{D}(\mathbf{u})|)$$

$$\text{FO}(u, v)$$

First Order* or Blatter-Pattyn model

*Dukowicz, Price and Lipscomb, 2010. J.

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$$\mu = \mu(|\mathbf{D}(\mathbf{u})|)$$

Quasi-hydrostatic approximation

3rd momentum equation

$$-\cancel{\partial_x(\mu u_z)} - \cancel{\partial_y(\mu v_z)} - \partial_z(2\mu w_z - p) = -\rho g,$$

continuity equation

$$w_z = -(u_x + v_y)$$

$$\Longrightarrow p = \rho g(s - z) - 2\mu(u_x + v_y)$$

$$\text{FO}(u, v)$$

First Order* or Blatter-Pattyn model

*Dukowicz, Price and Lipscomb, 2010. J.

Stokes approximations in different regimes

$\text{Stokes}(\mathbf{u}, p)$

$$\begin{cases} -\nabla \cdot (2\mu \mathbf{D}(\mathbf{u}) - p\mathbf{I}) = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Drop terms using **scaling argument** based on the fact that ice sheets are shallow

$$\mathbf{D}(u,v) = \begin{bmatrix} u_x & \frac{1}{2}(u_y + v_x) & \frac{1}{2}(u_z + \cancel{w_x}) \\ \frac{1}{2}(u_y + v_x) & v_y & \frac{1}{2}(v_z + \cancel{w_y}) \\ \frac{1}{2}(u_z + \cancel{w_x}) & \frac{1}{2}(v_z + \cancel{w_y}) & -(u_x + v_y) \end{bmatrix} \qquad \mathrm{u} := \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\mu = \mu(|\mathbf{D}(u,v)|)$$

Quasi-hydrostatic approximation

3rd momentum equation

$$-\cancel{\partial_x(\mu\, u_z)} - \cancel{\partial_y(\mu\, v_z)} - \partial_z(2\mu\, w_z - p) = -\rho g,$$

continuity equation

$$w_z = -(u_x + v_y)$$

$$\Longrightarrow \; p = \rho g(s - z) - 2\mu(u_x + v_y)$$

$\text{FO}(u, v)$

First Order* or Blatter-Pattyn model

$$-\nabla \cdot \left(2\mu \tilde{\mathbf{D}} - \rho g(s-z)\mathbf{I}\right) = \mathbf{0}$$

with $\tilde{\mathbf{D}}(u,v) = \begin{bmatrix} 2u_x + v_y & \frac{1}{2}(u_y + v_x) & \frac{1}{2}u_z \\ \frac{1}{2}(u_y + v_x) & u_x + 2v_y & \frac{1}{2}v_z \end{bmatrix}$

Dukowicz, Price and Lipscomb, 2010. J. Glaciol

MPAS-Albany Landice model (MALI)

Algorithm and Software

ALGORITHM	SOFTWARE TOOLS
Linear Finite Elements on test/hexas	Albany
Quasi-Newton optimization (L-BFGS)	ROL
Nonlinear solver (Newton method)	NOX
Krylov linear solvers/Prec	AztecOO/ML, Belos/MueLu
Automatic differentiation	Sacado

***MPAS:** Model for Prediction Across Scales, fortran finite volume library:*

- *works on Voronoi Tessellations*
- *conservative Lagrangian schemes for advecting tracers*
- *evolution of ice thickness*

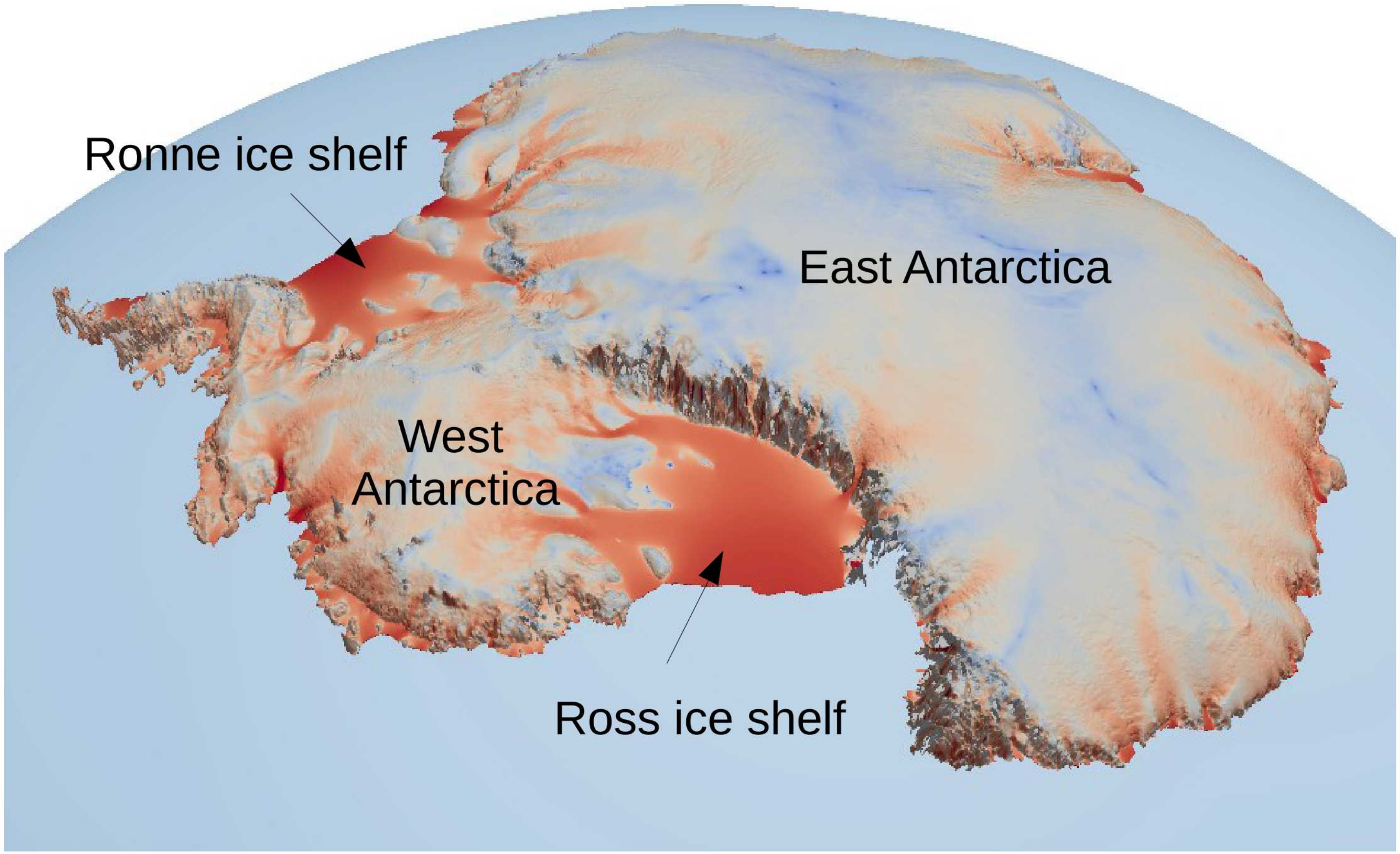
Albany: C++ finite element library built on Trilinos to enable multiple capabilities:

- Jacobian/adjoints assembled using automatic differentiation (Sacado).
- nonlinear and parameter continuation solvers (NOX/LOCA)
- large scale PDE constrained optimization (Piro/ROL)
- linear solver and preconditioners (Belos/AztecOO, ML/MeuLu/Ifpack)

Hoffman, et al. GMD, 2018
Tuminaro, Perego, Tezaur, Salinger, Price, SISC, 2016.
Tezaur, Perego, Salinger, Tuminaro, Price, Hoffman, GMD, 2015
Perego, Price, Stadler, JGR, 2014

MPAS-Albany Landice model (MALI)

Antarctic Ice Sheet velocity



Colored by ice sheet velocity surface velocity
(blue = slow, red = fast)

Deterministic Inversion

GOAL

Find ice sheet initial state that

- matches observations:
  - State variables (e.g. surface velocity, temperature, etc.)
  - Tendencies (thickness tendencies)
- is in compliance with model physics (Stokes, temperature, subglacial hydrology..)

(Early) Bibliography

- *Arthern, Gudmundsson*, J. Glaciology, 2010
- *Price, Payne, Howat and Smith*, PNAS, 2011
- *Petra, Zhu, Stadler, Hughes, Ghattas*, J. Glaciology, 2012
- *Pollard DeConto*, TCD, 2012
- W. J. J.Van Pelt et al., The Cryosphere, 2013
- *Morlighem et al.* Geophysical Research Letters, 2013
- *Goldberg and Heimbach*, The Cryosphere, 2013
- Brinkerhoff and Johnson, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014
- *Perego, Price, Stadler,* Journal of Geophysical Research, 2014
- *Goldberg et al.*, The Cryosphere Discussions, 2015

Deterministic Inversion

PDE-constrained optimization problem: cost functional

Problem: find initial conditions such that the ice matches available observations.

Optimization problem:

find β and H that minimize the functional* $\mathcal{J}$

$$\mathcal{J}(\beta, \phi) = \int_\Omega \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds \quad \text{surface velocity mismatch}$$

$$+ \int_\Omega \frac{1}{\sigma_\phi^2} |\phi - 1|^2 \, ds \quad \text{stiffening factor mismatch}$$

$$+ \mathcal{R}(\beta, \phi) \quad \text{regularization terms.}$$

subject to ice sheet model equations (FO or Stokes)

$\mathbf{u}$: computed depth averaged velocity
ϕ: stiffening factor
β: basal sliding friction coefficient
$\mathcal{R}(\beta, \phi)$ regularization term

Perego, Price, Stadler, Journal of Geophysical Research, 2014

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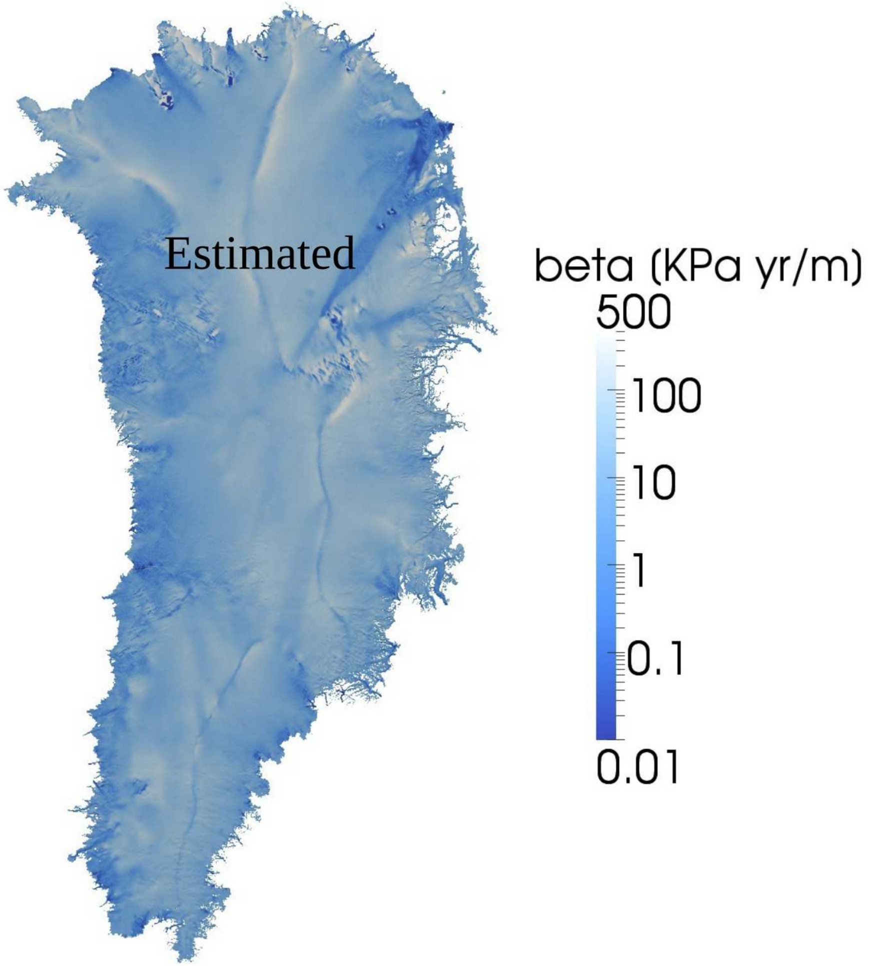
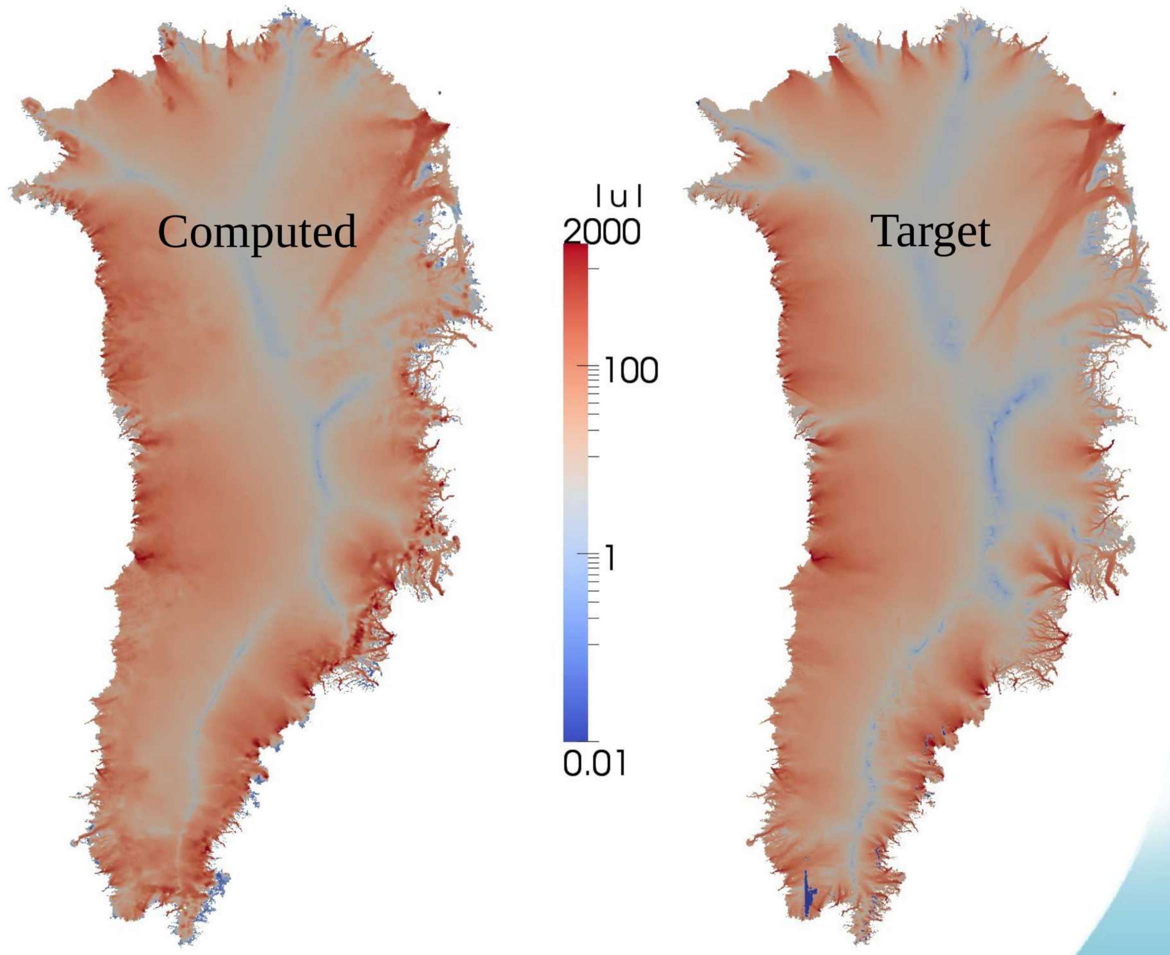
Greenland Inversion

velocity mismatch only, tuning basal friction

Inversion with 1.6M parameters





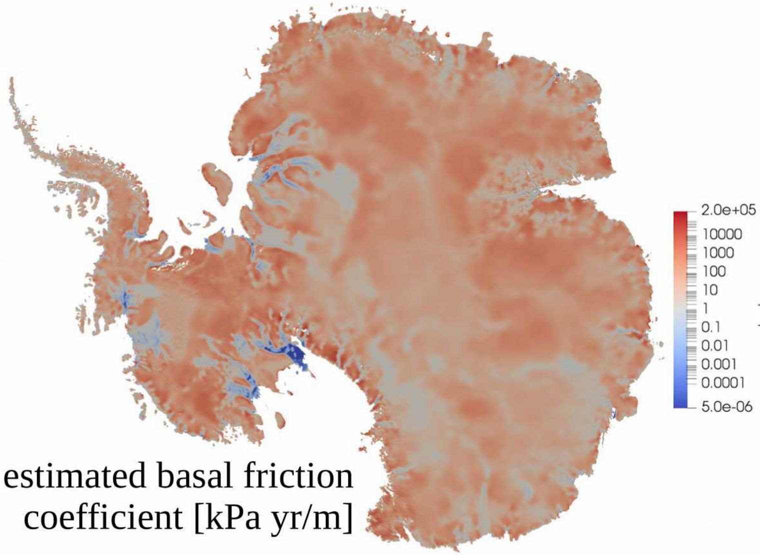
Basal friction coefficient (kPa yr/m)

surface velocity magnitude (m/yr)

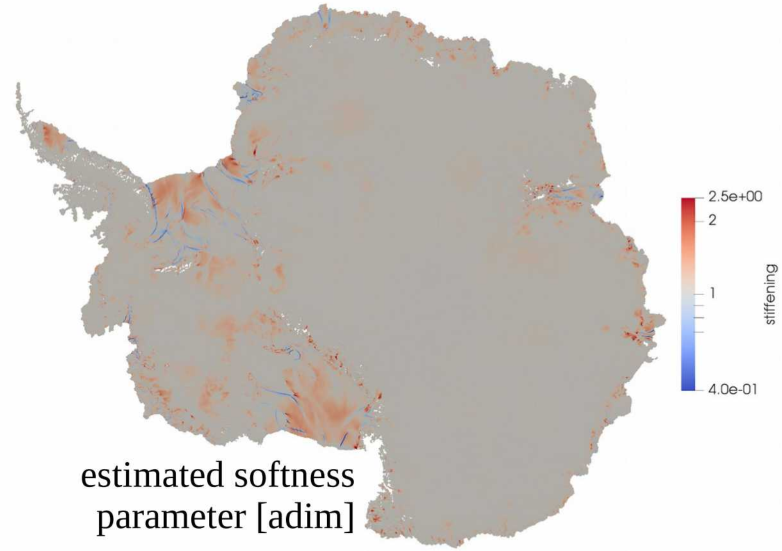
Geometry (Morlighem et al., Nature Geo., 2014)

Antarctica Inversion

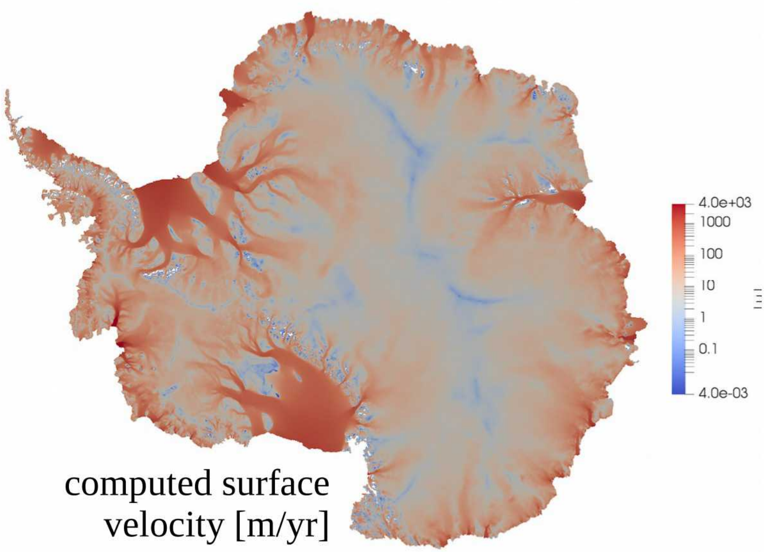
velocity and stiffening mismatches, tuning basal friction and stiffening



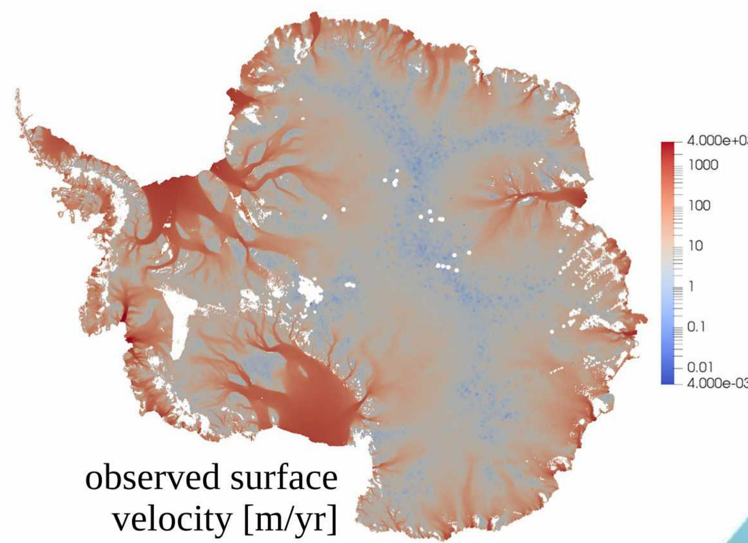
estimated basal friction coefficient [kPa yr/m]



estimated softness parameter [adim]



computed surface velocity [m/yr]



observed surface velocity [m/yr]

simulation details

#parameters: 2.5M	#cores: 8640
#unknowns: 30M	#nodes:180
machine: Edison (NERSC)	#hours:18

Shortcomings of current optimization approach

velocity and stiffening mismatches, tuning basal friction and stiffening

Main issues with the proposed optimization approach:

1. initial state **is not consistent** with **temperature**

2. intial state **does not match** observed **thickness tendencies**

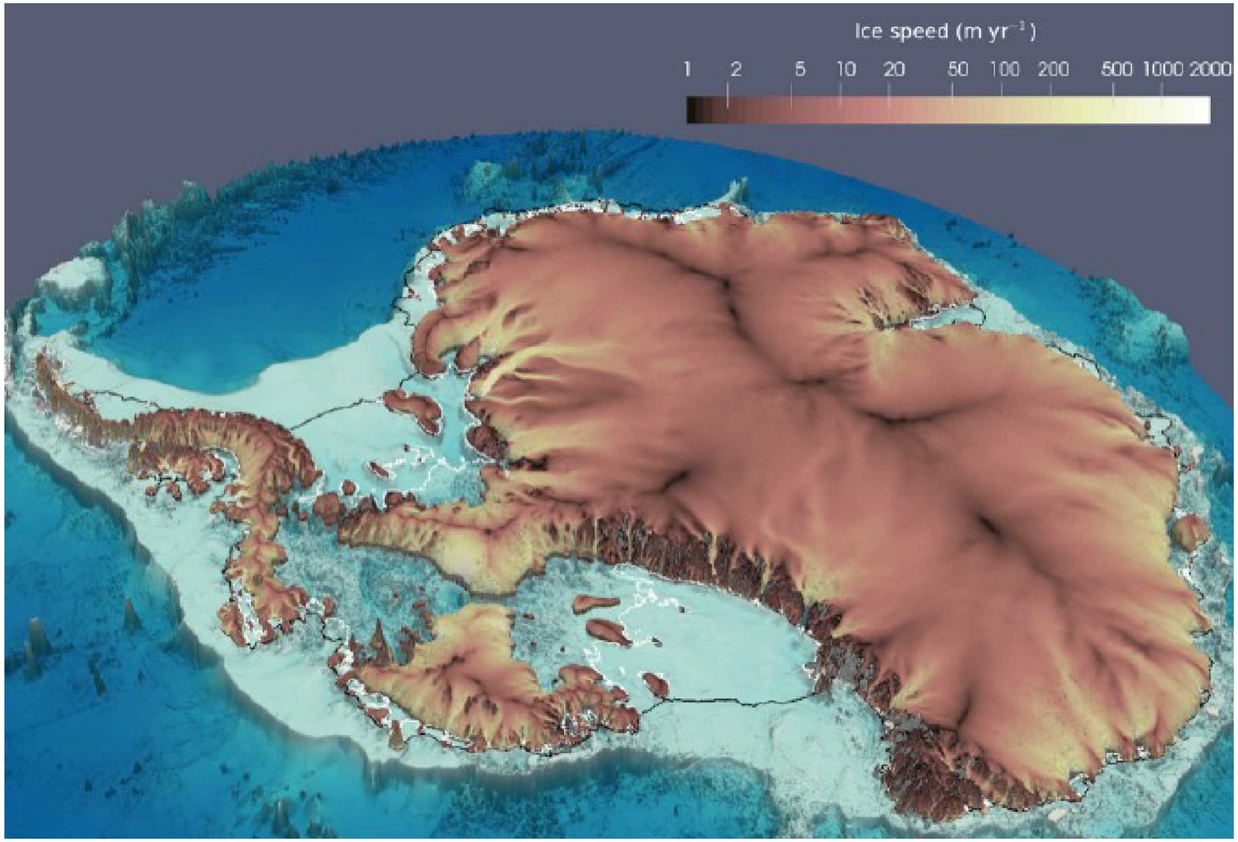
3. basal friction field **is not steady** in time

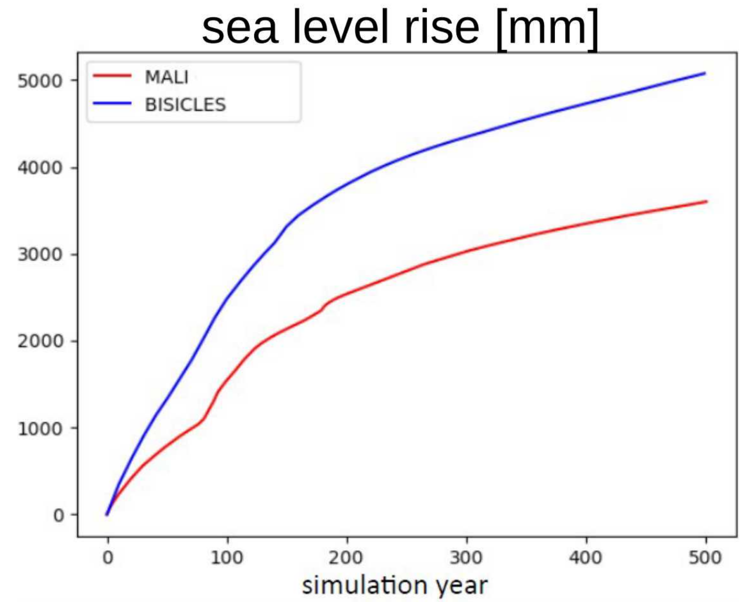
As soon as we start evolving the ice sheet in time, we experience fast unphysical transients (mainly because of 2.) and the modeled dynamics won't be accurate, especially in the medium term (50-100 years).

Typically one "spins up" the model for O(100-1000) years, but this can lead to an initial state that is far form the present day one.

Today we focus on 1. and 2. To address 3. a subglacial hydrology model is needed.

Ice sheet response under extreme (unrealistic) forcing





ABUMIP targets the response of the ice sheet model to instantaneous removal of all ice shelves, to understand the sensitivity of ice sheet to extreme climate forcing

Significance of Temperature solver

- Temperature strongly affects ice rheology and sliding conditions at the bed

- Time response of ice to temperature changes is of the order of several thousands of years.
  In order to obtain a self consistent initial state, temperature model is typically spun up for $O(10^4)$ years.

- With an <u>implicit steady-state</u> temperature model, coupled with the flow model, it is in principle possible to obtain a self-consistent state in one shot. Moreover it allows for larger time step when marching in time.

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Significance of Temperature solver

- Our goal is to create a robust and efficient implicit temperature solver for large scale problems that is adequately accurate.
- We chose to implement the enthalpy formulation in *Aschwaden et al., 2012*, with the gravity driven drainage term presented in *Hewitt and Schoof, 2016.*
- This formulation accounts for temperate ice (mixture of ice and water) and melting/refreezing a the bed.

Relevant references:

- A. Barone and M. Perego, Implementation of Enthalpy model for polythermal Glaciers, CCR Summer Proceedings, 2016

- I. Hewitt, and C. Schoof: Models for polythermal ice sheets and glaciers, The Cryosphere, 2016

- C. Schoof and I. J. Hewitt, A model for polythermal ice incorporating gravity-driven moisture transport, Journal of Fluid Mechanics, 2016

- H. Zhu, N. Petra, G. Stadler, T. Isaac, T. J. R. Hughes and O. Ghattas, Inversion of geothermal heat flux in a thermomechanically coupled nonlinear Stokes ice sheet model, The Cryosphere, 2016

- T. Kleiner, M. Rückamp, J. H. Bondzio, and A. Humbert: Enthalpy benchmark experiments for numerical ice sheet models, The Cryosphere, 2015

- W. Leng, L. Ju, M. Gunzburger, S. Price, A Parallel Computational Model for Three-Dimensional, Thermo-Mechanical Stokes Flow Simulations of Glaciers and Ice Sheets, CCP, 2014

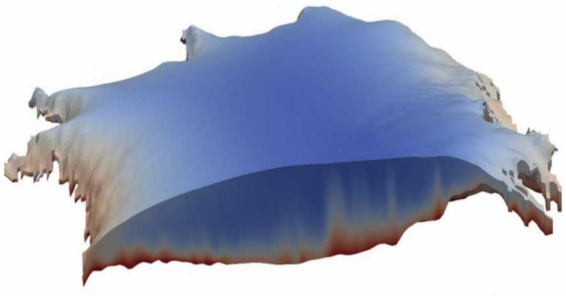
- A. Aschwaden et al., An Enthalpy formulation for glaciers and ice sheets, Journal of Glaciology, 2012

Temperature Model

Cold ice temperature equation:

$$\rho c \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) + \rho c \mathbf{u} \cdot \nabla T = \tau : \dot{\epsilon}$$

ice heat capacity — ice velocity — dissipation heat



Because of the internal dissipation heat, there can be internal melting and the ice become temperate (mixture of ice and water)

It is convenient to model the temperature and porosity (water content) equations in terms of the enthalpy defined as

$$\mathrm{h} := \rho c (T - T_0) + \rho_w L \phi$$

enthalpy — temperature — latent heat — porosity (water content)

	cold ice $h < h_m$	temperate ice $h \geq h_m$
T	$T = T_0 + \frac{1}{\rho c} h$	$T = T_m$
ϕ	0	$\frac{1}{\rho_w L}(h - h_m)$

$$\mathrm{h}_m := \rho c (T_m - T_0)$$

- *A. Aschwaden et al., An Enthalpy formulation for glaciers and ice sheets, Journal of Glaciology, 2012*

Temperature Model

Steady-state Enthalpy equation reads:

$$\nabla \cdot \mathbf{q}(h) + \mathbf{u} \cdot \nabla h = \tau : \dot{\epsilon}$$

$$\mathbf{q} = \begin{cases} -\frac{k}{\rho c}\nabla h & \text{cold}(h < h_m) \\ -\frac{k}{\rho c}\nabla h_m + \rho_w L\,\mathbf{j}(h) & \text{temperate} \end{cases}$$

total enthalpy flux

$$\mathbf{j}(h) = \frac{1}{\eta_w} k_0 \left(\frac{h - h_m}{\rho c}\right)^{\gamma} (\rho_w - \rho)\mathbf{g}$$

gravity driven water flux

Stefan's condition at the bed

$$\mathrm{m} = \mathrm{G} + \tau_b \cdot \mathbf{u} - k\nabla T \cdot \mathbf{n}$$

melting rate geothermal heat flux frictional heating

At surface elevation:

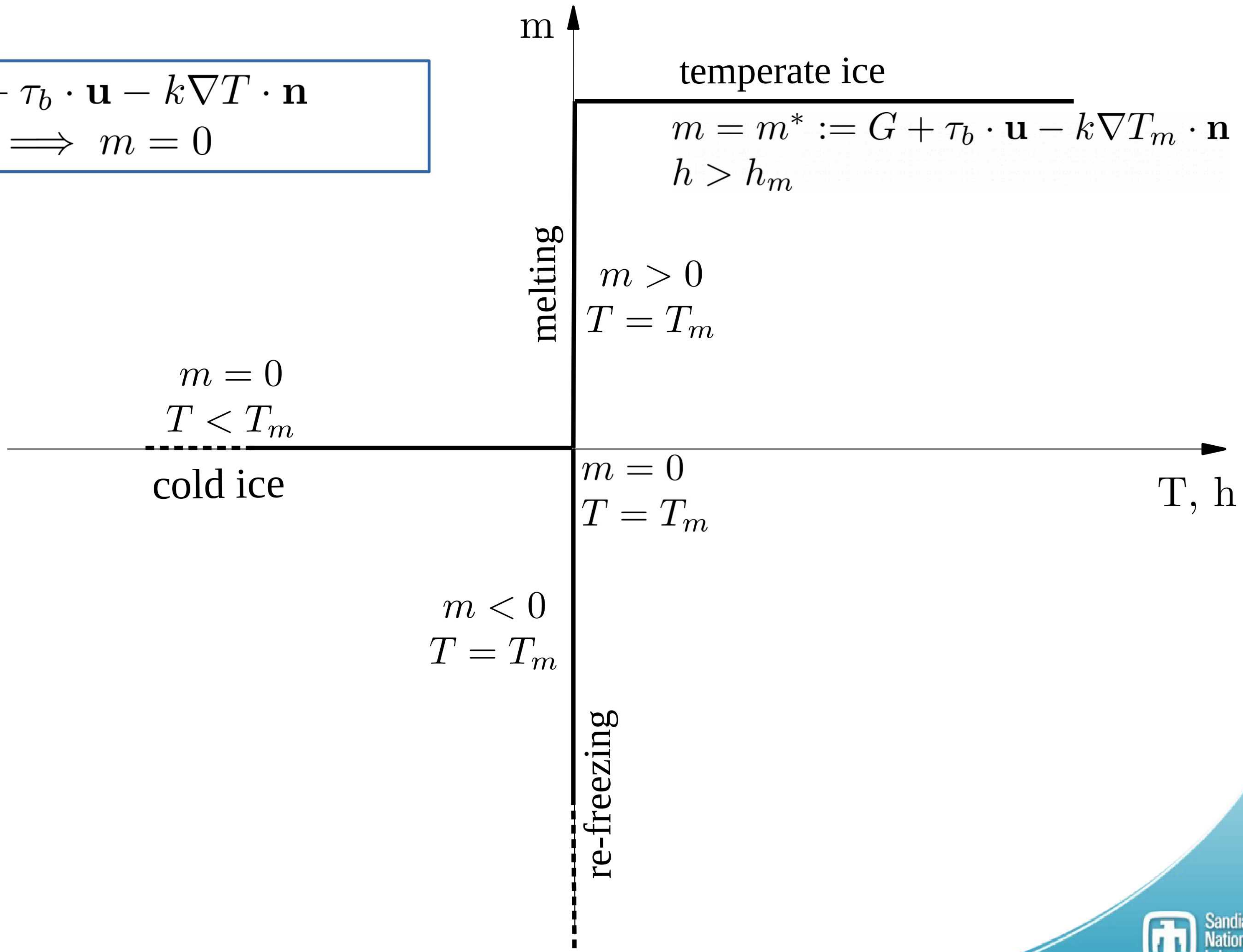
$$\mathrm{T} = \mathrm{T}_{\mathrm{air}}$$

- *I. Hewitt, and C. Schoof: Models for polythermal ice sheets and glaciers, The Cryosphere, 2016*

- *C. Schoof and I. J. Hewitt, A model for polythermal ice incorporating gravity-driven moisture transport, Journal of Fluid Mechanics, 2016*

Melting/Enthalpy graph at the bed interface

$$m = G + \tau_b \cdot \mathbf{u} - k\nabla T \cdot \mathbf{n}$$
$$T < T_m \implies m = 0$$



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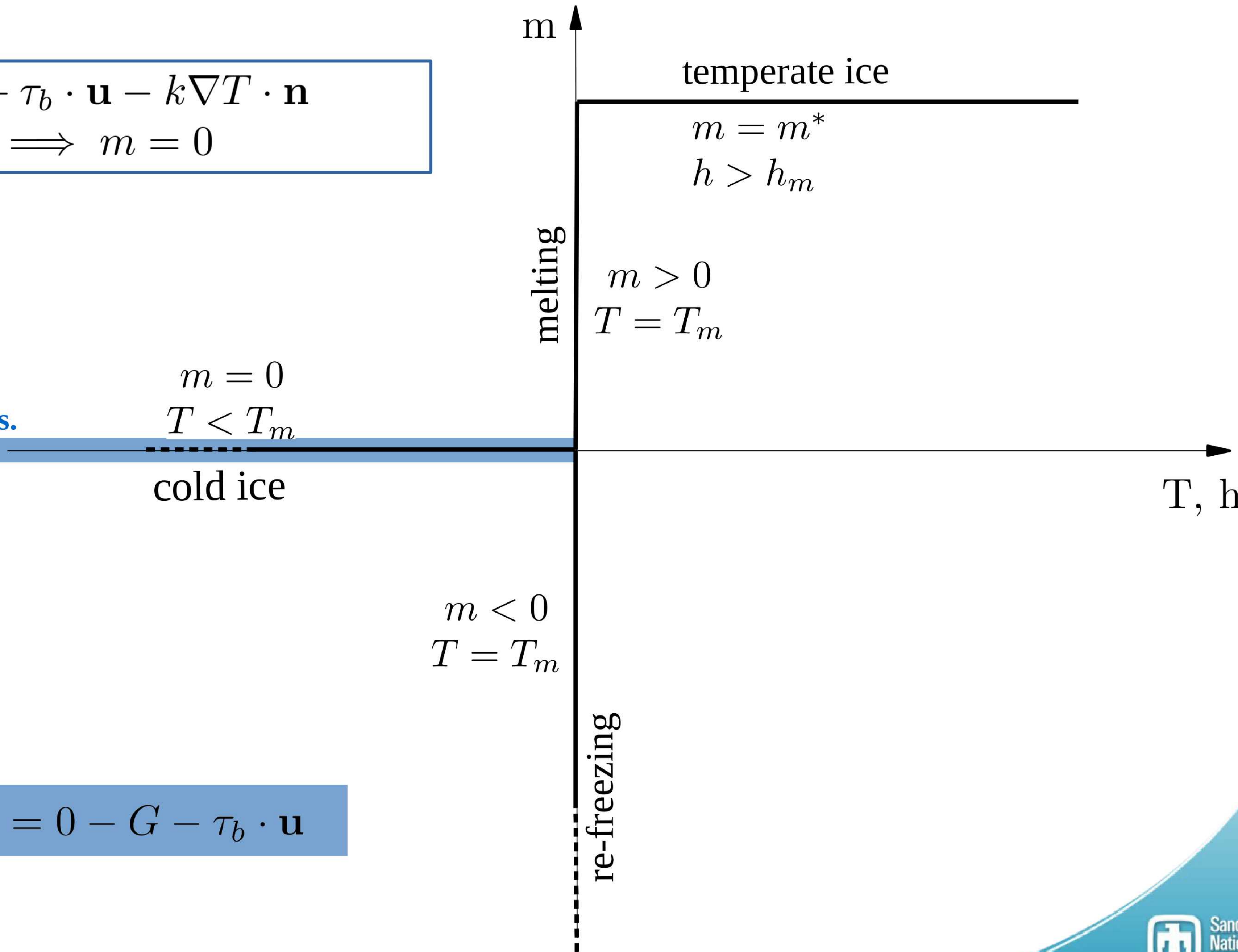

Neumann BCs.

$$-\frac{k}{\rho c}\nabla h \cdot \mathbf{n} = 0 - G - \tau_b \cdot \mathbf{u}$$

Melting/Enthalpy graph at the bed interface

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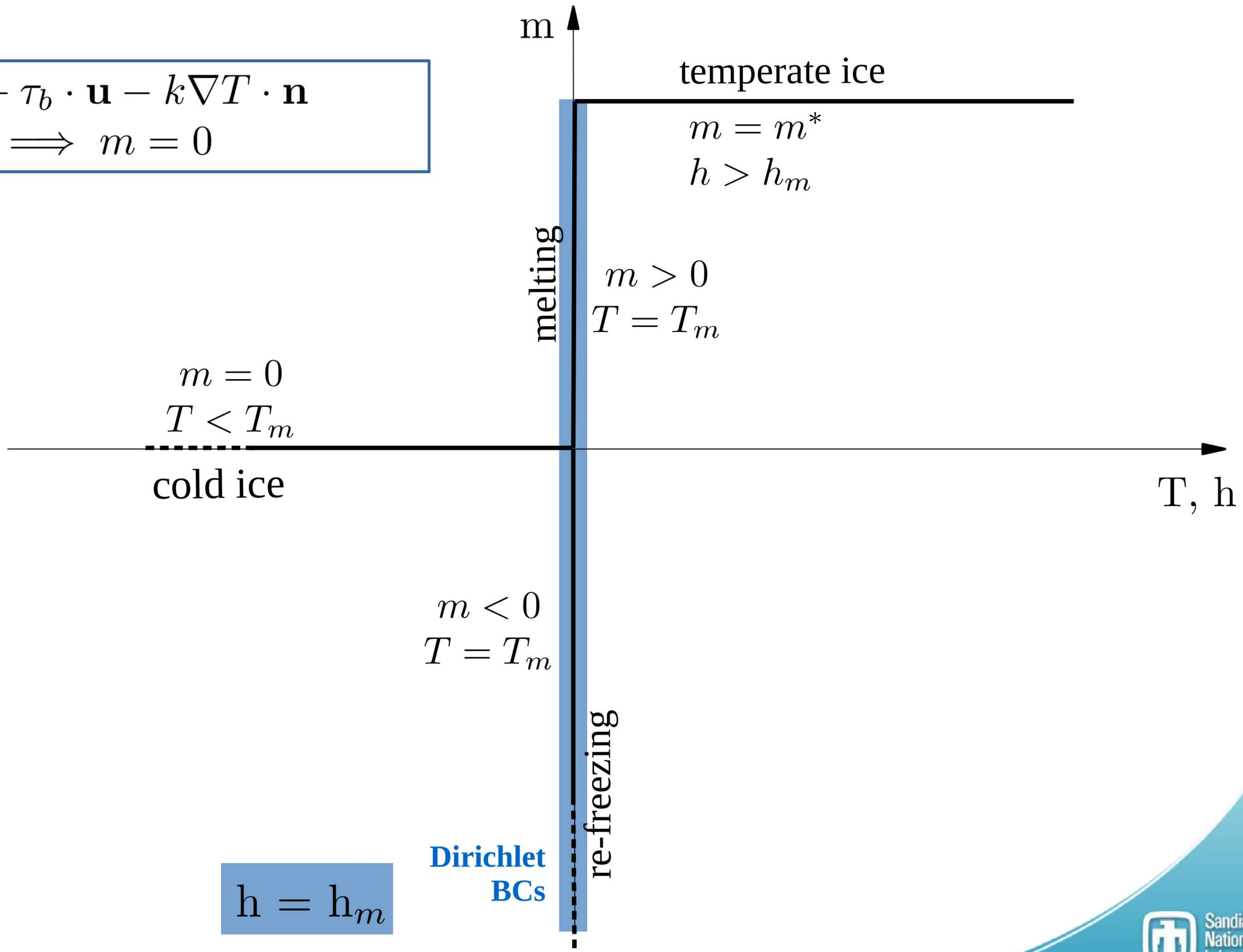

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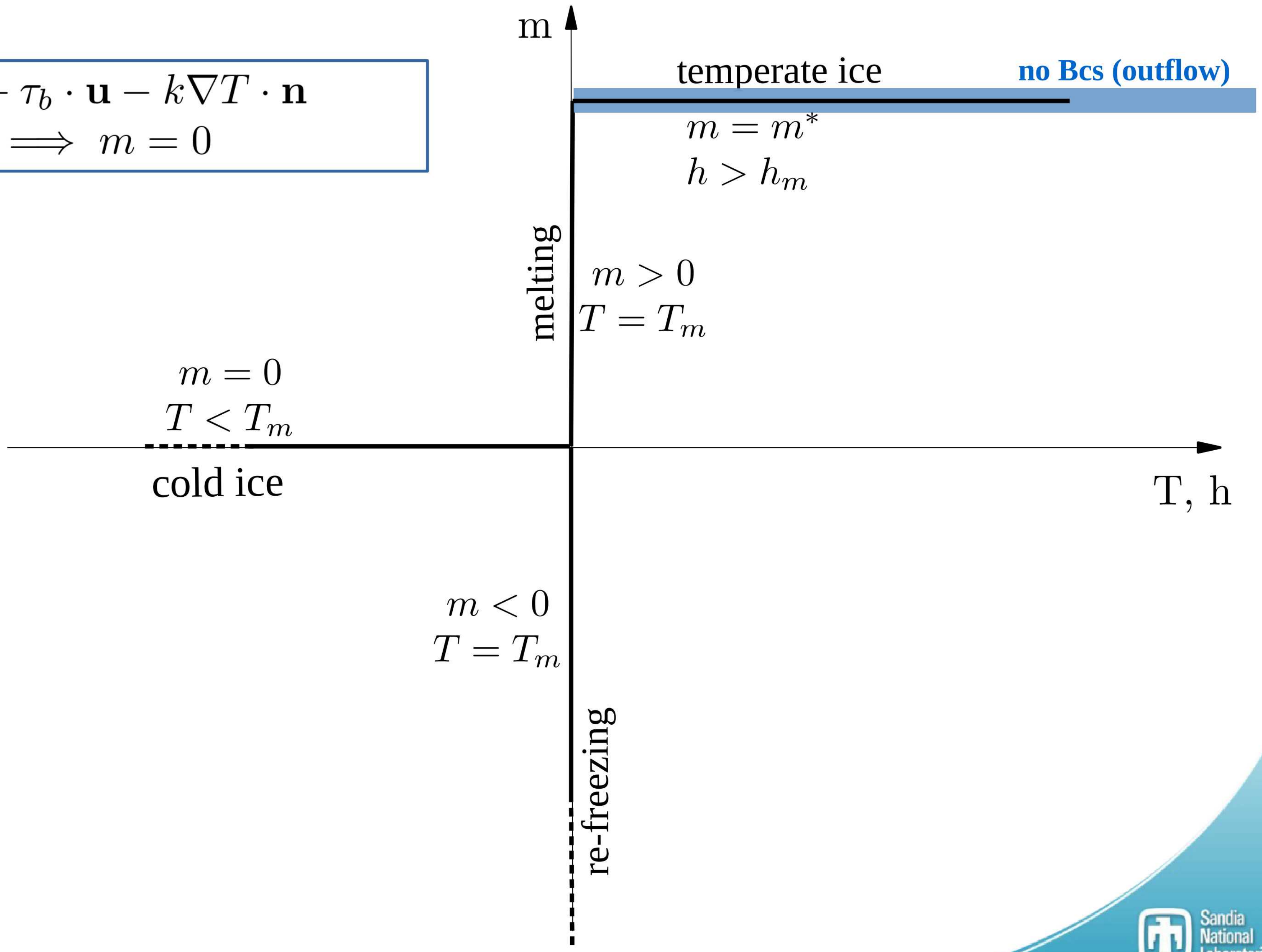
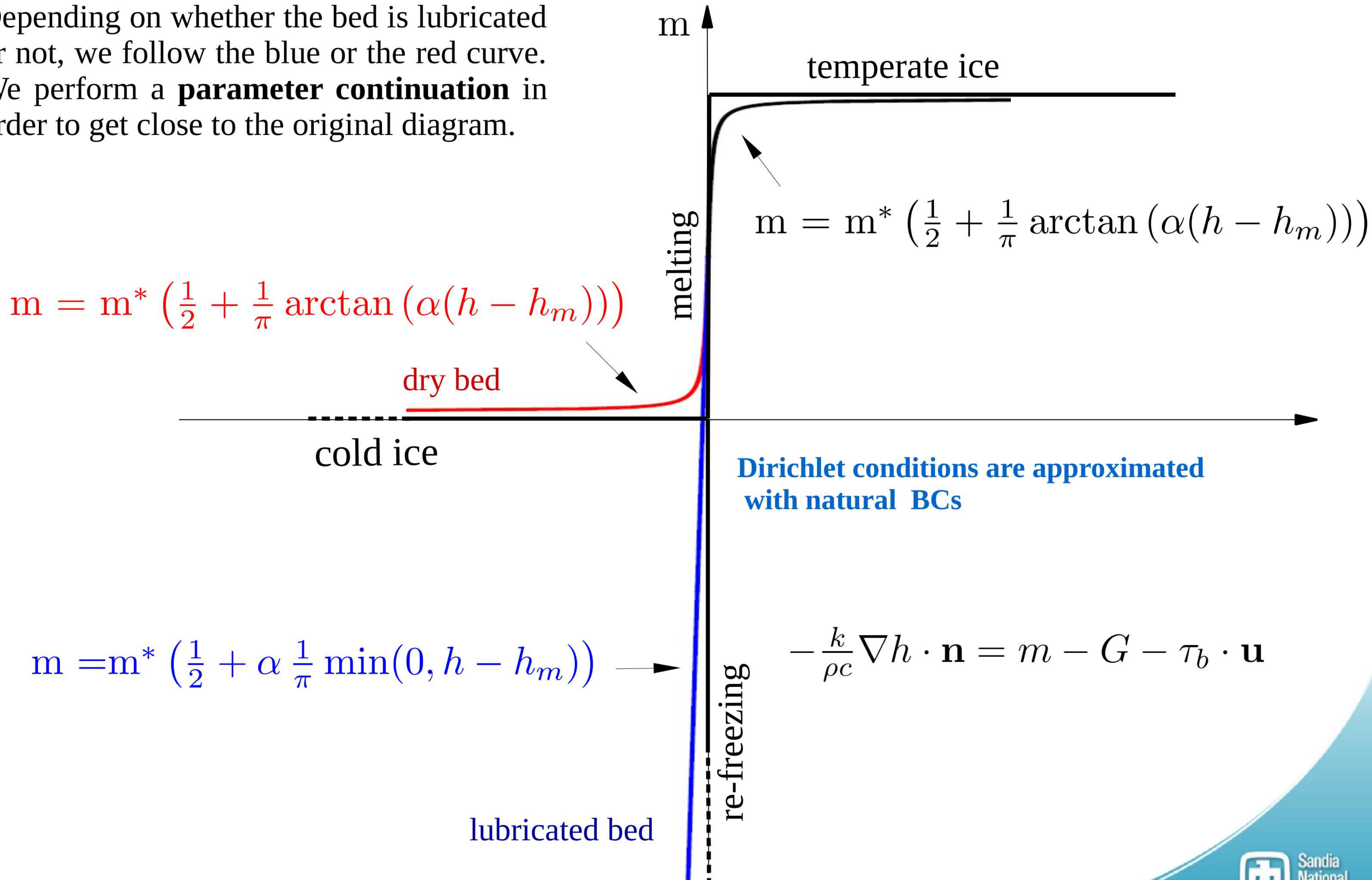


Approximation/smoothing of the enthalpy/melting graph

Depending on whether the bed is lubricated or not, we follow the blue or the red curve. We perform a **parameter continuation** in order to get close to the original diagram.



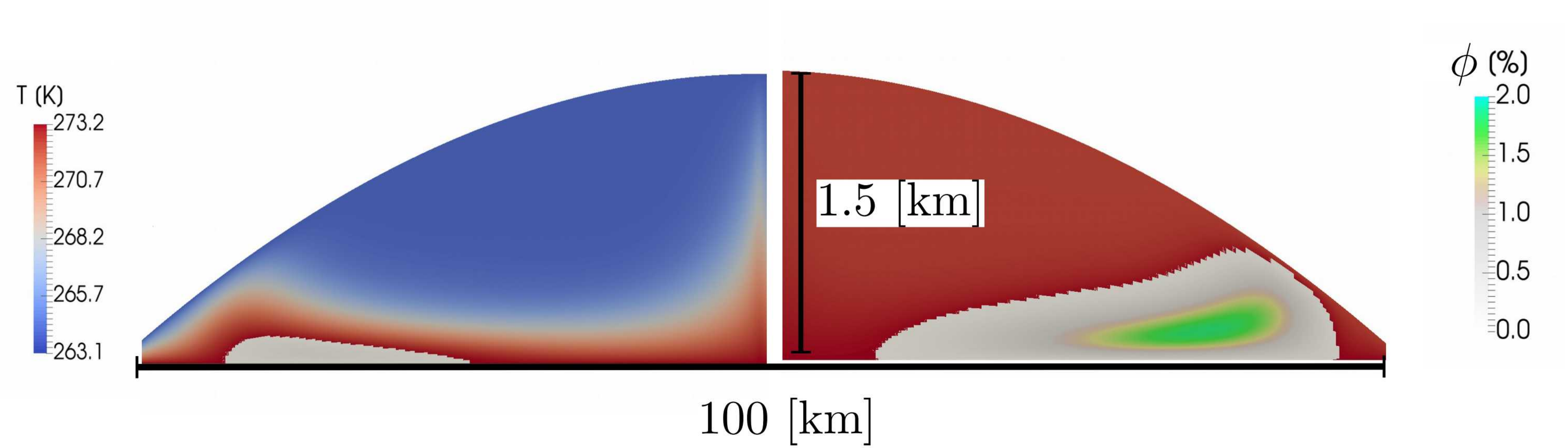
α: continuation parameter (e.g. α goes form 1e-3 to 1e3)

Preliminary Results

Dome problem: (based on Hewitt and Schoof, The Cryosphere, 2016)
We explore different scenarios and report, in each picture, the temperature (for cold ice) and porosity (for temperate ice).

Problem 1: Settings
- top surface b.c.: $T = \text{-}10$ C
- bottom surface b.c.: $h = h_m$
- prescribed SIA velocity profile

Problem 2: Settings
- top surface b.c.: $T = \text{-}1$ C
- bottom surface b.c.: $h = h_m$
- prescribed SIA velocity profile



Ice Sheet velocity equations

First order approximation of Stokes equations for horizontal velocities:

$$-\nabla \cdot \left(2\mu\tilde{\mathbf{D}}(u,v) - \rho g(s-z)\mathbf{I}\right) = \mathbf{0}$$

with $\tilde{\mathbf{D}}(u,v) = \begin{bmatrix} 2u_x + v_y & \frac{1}{2}(u_y + v_x) & \frac{1}{2}u_z \\ \frac{1}{2}(u_y + v_x) & u_x + 2v_y & \frac{1}{2}v_z \end{bmatrix}$

Nonlinear viscosity:

$$\mu = \alpha \exp(-\gamma/T)\,|\mathbf{D}(u,v)|^{-\frac{2}{3}}$$

Boundary conditions:

$$\begin{cases} 2\,\mu\tilde{\mathbf{D}}\mathbf{n} + \beta \begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{0} & \text{on bed} \\ 2\mu\tilde{\mathbf{D}}\mathbf{n} = \mathbf{0} & \text{elsewhere.} \end{cases}$$

Need to reconstruct vertical velocity:

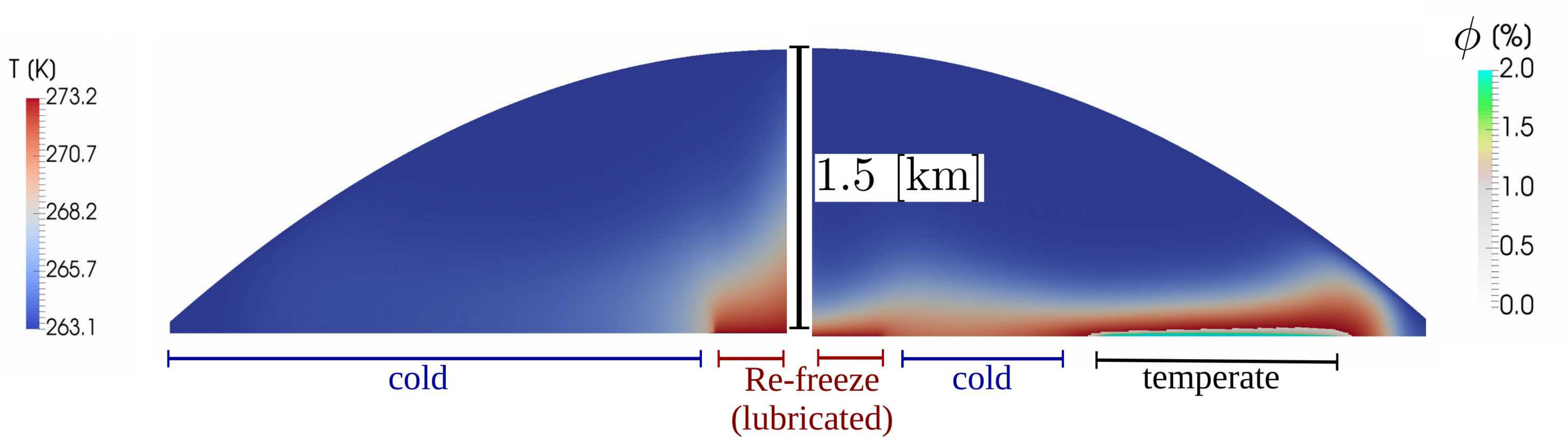
$$w = w_b + \int_z -\partial_x u - \partial_y v \,\mathrm{d}z, \quad w_b = -\frac{m}{L\rho_w(1-\phi)} - \mathbf{j}\cdot\mathbf{n}$$

Preliminary Results:

Dome problem: based on Hewitt and Schoof (in preparation)
We explore different scenarios and report, in each picture, the temperature (for cold ice) and porosity (for temperate ice)

Problem 3: Settings
- top surface b.c.: T = -10 C
- no dissipation inside the dome
- bed lubricated near the center of the dome
- basal heat flux = 0.0 [W m^{-2}]
- coupled with FO velocity solver

Problem 4: Settings
- top surface b.c.: T = -10 C
- basal heat flux = 0.01 [W m^{-2}]
- bed lubricated near the center of the dome
- coupled with FO velocity solver



Deterministic Inversion

PDE-constrained optimization problem: cost functional

Problem: find initial conditions such that the ice is close to thermo-mechanical equilibrium, given the geometry and the SMB, and matches available observations.

Optimization problem:

find β and H that minimize the functional* $\mathcal{J}$

$$\mathcal{J}(\beta, H) = \int_\Omega \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds \qquad \text{surface velocity mismatch}$$

$$+ \int_\Omega \frac{1}{\sigma_\tau^2} \left| \operatorname{div}(\bar{\boldsymbol{u}} H) - \tau_{\text{smb}} + \left\{ \frac{\partial H}{\partial t} \right\}^{\text{obs}} \right|^2 ds \qquad \text{SMB mismatch}$$

$$+ \int_\Omega \frac{1}{\sigma_H^2} |H - H^{obs}|^2 \, ds \qquad \text{thickness mismatch}$$

$$+ \mathcal{R}(\beta, H) \qquad \text{regularization terms.}$$

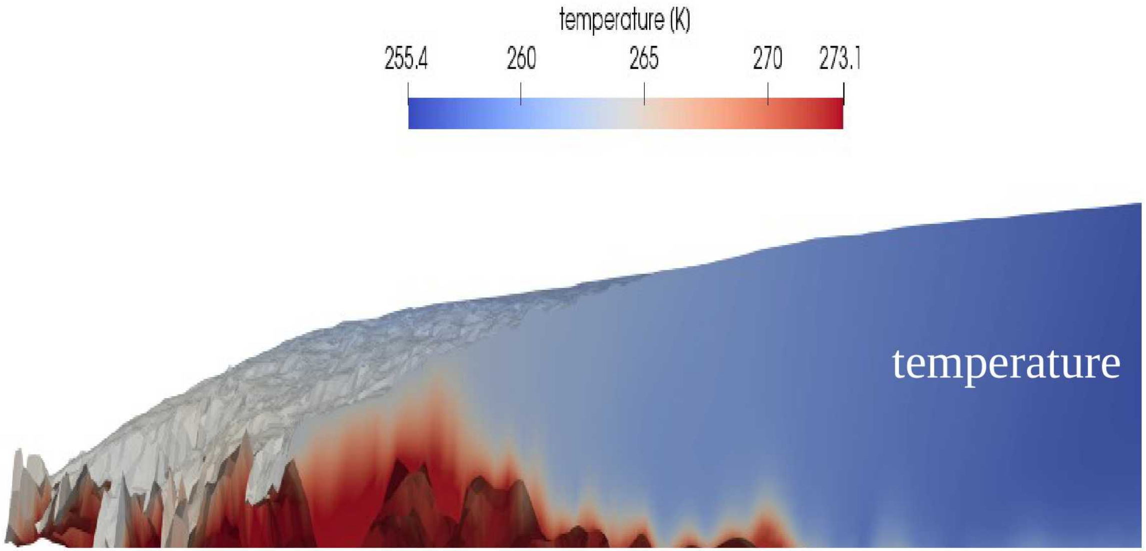
subject to ice sheet model equations (FO + enthalpy)

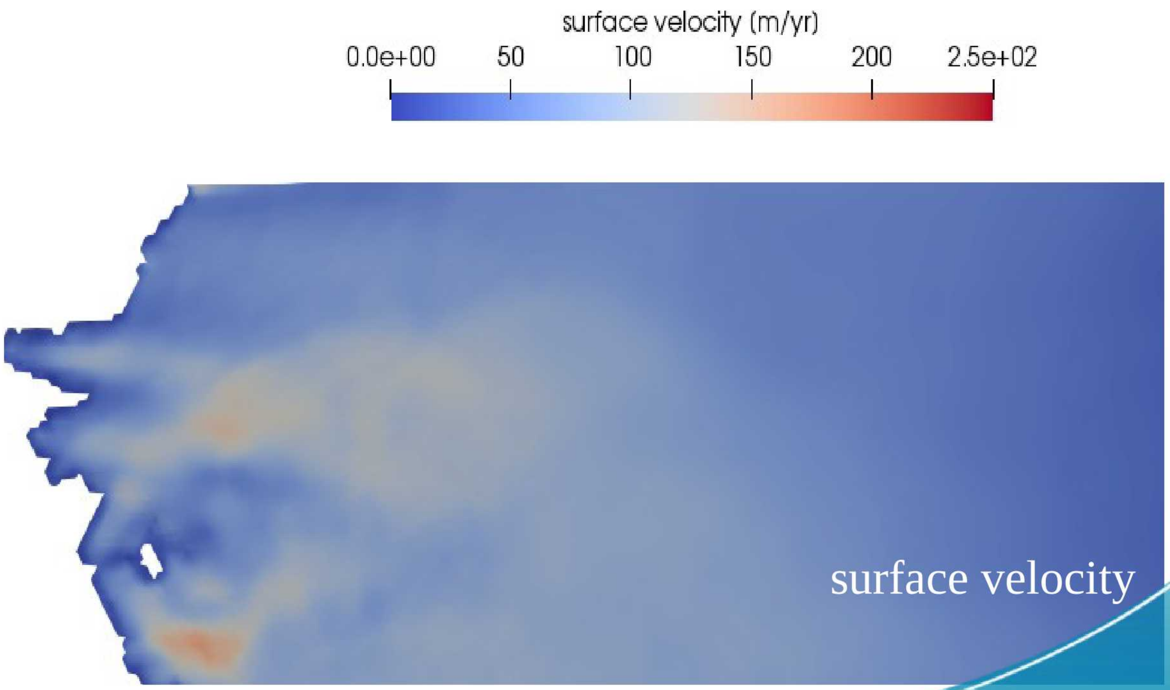
$\bar{\boldsymbol{u}}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

Perego, Price, Stadler, Journal of Geophysical Research, 2014

Realistic Geometry:
Isunnguata Sermia glacier from Western Greenland

Solution obtained after calibrating the basal friction and the bed topography with a PDE-constrained optimization approach where the constraint is the coupled enthalpy/velocity system and the cost functional is the mismatch with observed surface velocity, thickness, and tendencies





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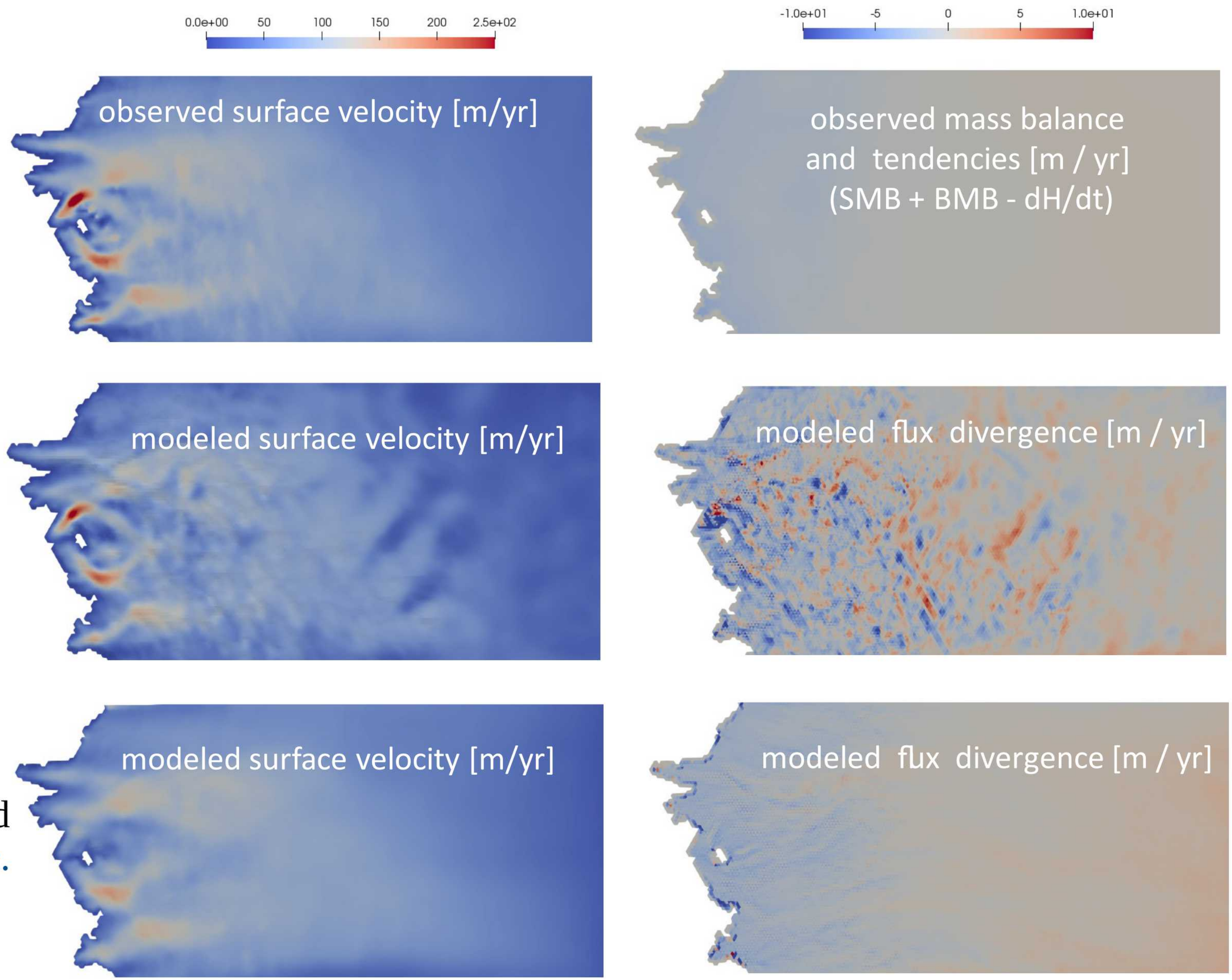
Initialization:
Isunnguata Sermia glacier from Western Greenland



Observations
(target)

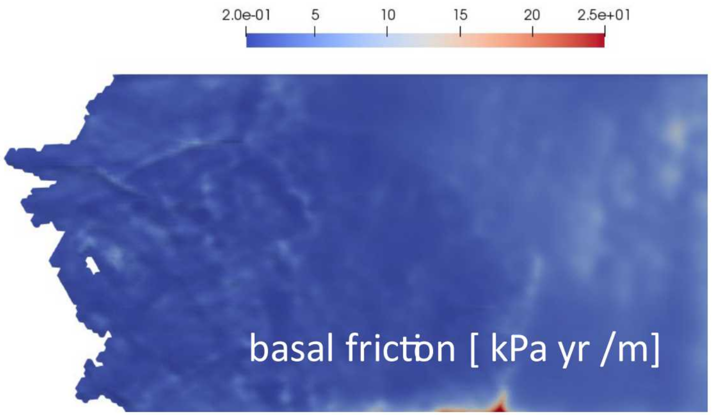
Basic optimization
(calibrate basal friction to match obs. sfc. velocity)

Improved optimization
(calibrate basal friction and thickness to match obs. sfc. velocity and tendencies)

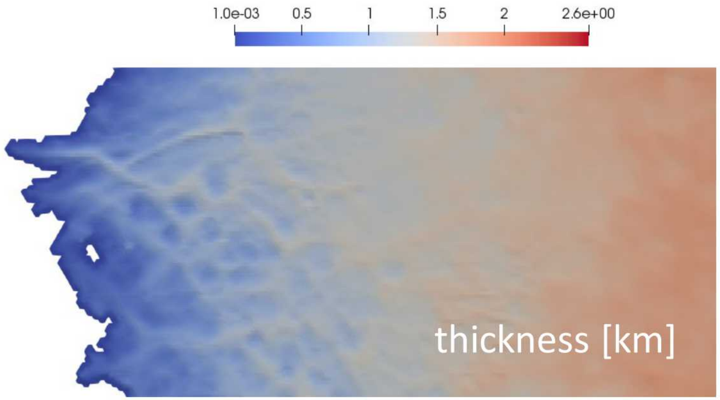
Initialization:
Isunnguata Sermia glacier from Western Greenland

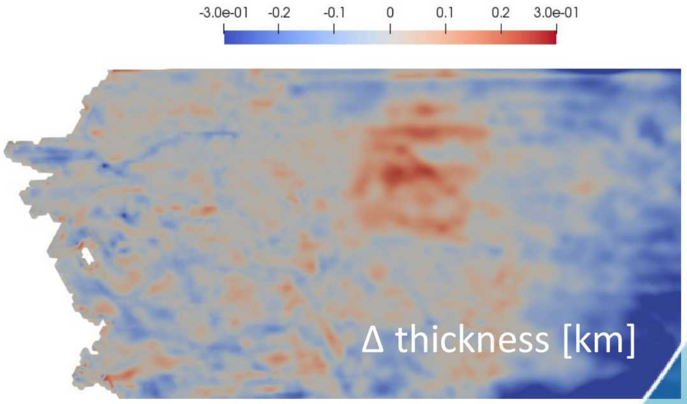
Fields estimated with improved optimization:

- basal friction:



- thickness (bed topography):



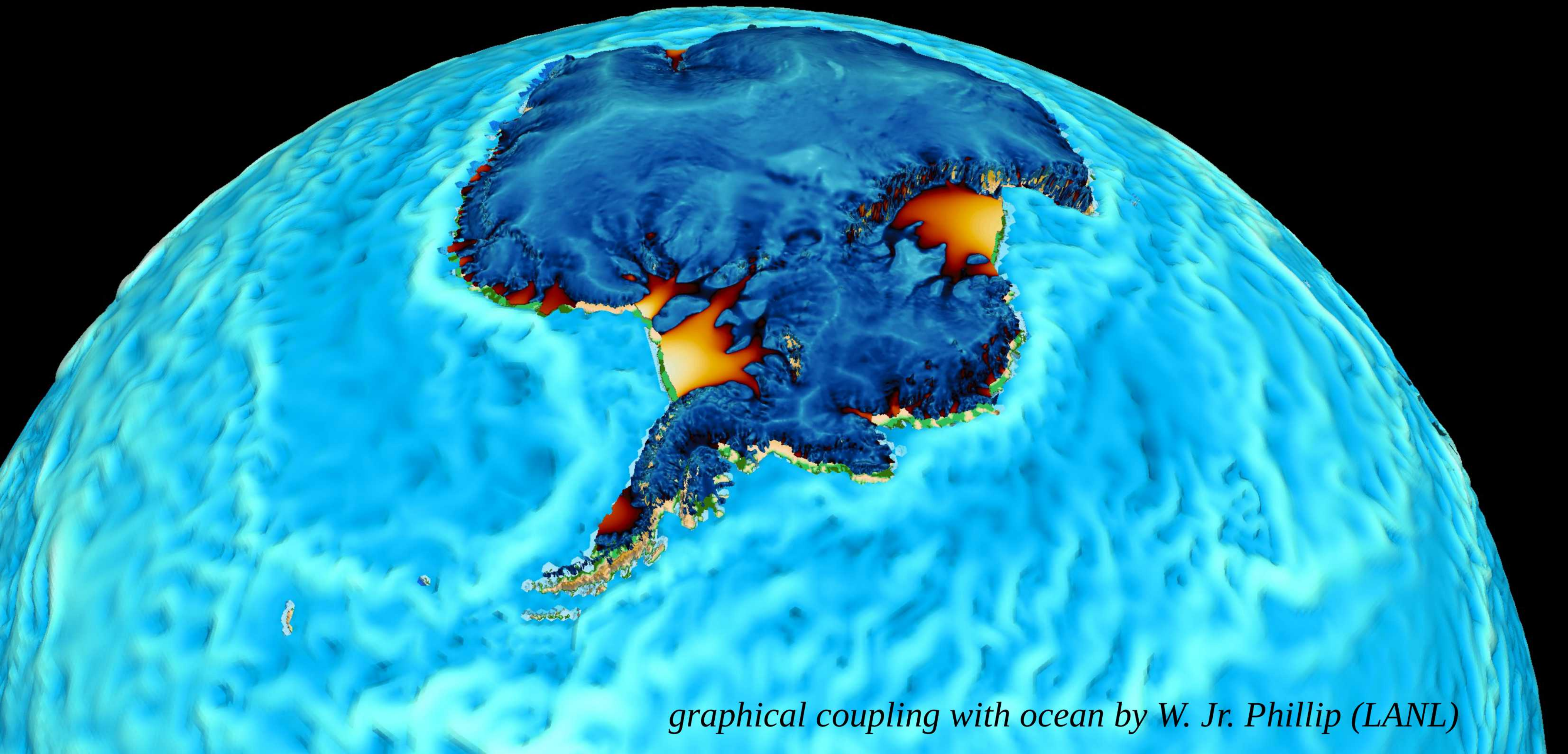


Conclusion

Current challenges and future developments:

- Include basal hydrology model and invert for model parameter rather than for basal friction
- Implement physics based block preconditioner for temperature/hydrology
- Address fully coupled large scale problems

Thank you for your attention



graphical coupling with ocean by W. Jr. Phillip (LANL)