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NUCLEAR CRITICALITY SAFETY PROGRAM

Module 10
Reactor Physics

LA-UR-21
Unclassified

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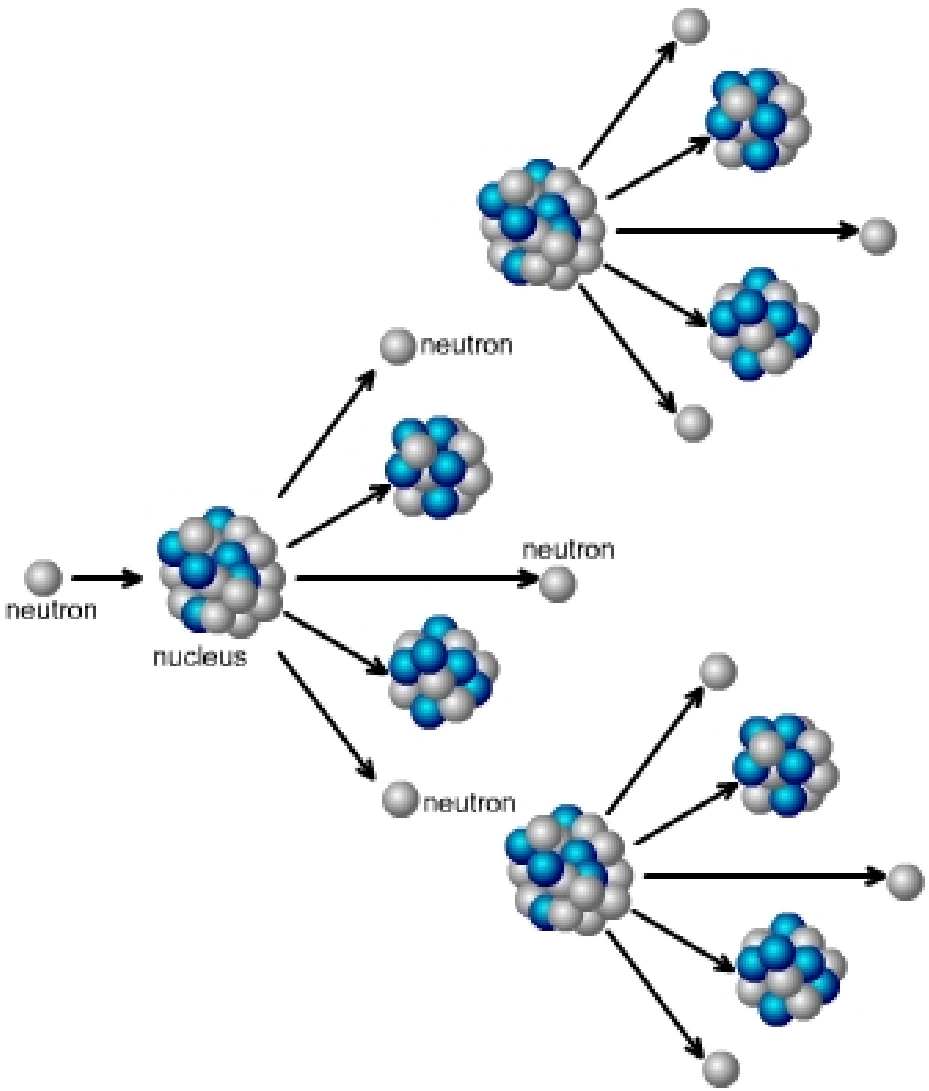
Goals

- Students will gain a understanding of some basic reactor physics concepts, which will prepare them for learning more detailed concepts about the design and operations of the experiments performed using the NCERC critical assemblies.

Fission Chain Process

- What is a fission chain?
- What is the multiplication factor?
- What is neutron multiplication?
- What is the difference between delayed critical and prompt critical?

Fission Chain



Fission Chain

- Main idea for a reactor is to use one neutron from a fission to induce another fission, etc
  - Statistical process due to very large number of neutrons and fission chains at one time
  - Other neutrons are absorbed (and don't cause a fission) or leak out of the reactor
- A reactor is critical when it maintains a steady-state neutron chain - the total number of neutrons (power) in the reactor is constant
- If a reactor is subcritical, the total number of neutrons (power) decreases
- If a reactor is supercritical, the total number of neutrons (power) increases

Multiplication Factor

- k_{eff} is the multiplication factor
- k_{eff} is a measure of the number of fission neutrons in one generation compared to the previous generation

$$k = \frac{fission\ neutrons\ in\ a\ generation}{fission\ neutrons\ in\ preceeding\ generation}$$

$$k = \frac{fissions\ in\ a\ generation\ caused\ by\ fission\ neutrons}{fissions\ in\ preceeding\ generation\ caused\ by\ fission\ neutrons}$$

- Three possible values for k_{eff}:
  - $k_{eff}<1$, system is **subcritical**, neutron population **drops** from generation to generation
  - $k_{eff}=1$, system is **critical**, neutron population is **constant**
  - $k_{eff}>1$, system is **supercritical**, neutron population **grows** with each generation

Neutron Multiplication

- Neutron Multiplication (M) is not the same as the multiplication factor (k_{eff})!
- Neutron Multiplication is the total number of neutrons that would be generated through fission from a single starter neutron
  - How is the original starter neutron "amplified" through the fission process?
- Neutron Multiplication is only meaningful for **subcritical** reactors (ie, $k_{eff} < 1$)
  - $1 \leq M < \infty$
  - If the system is critical or supercritical, the number of neutrons generated for a single starter neutron will be infinity, so M has no meaning in that case
- Simple formula relates Multiplication to multiplication factor:

$$M = \frac{1}{1 - k_{eff}}$$

Neutron Multiplication

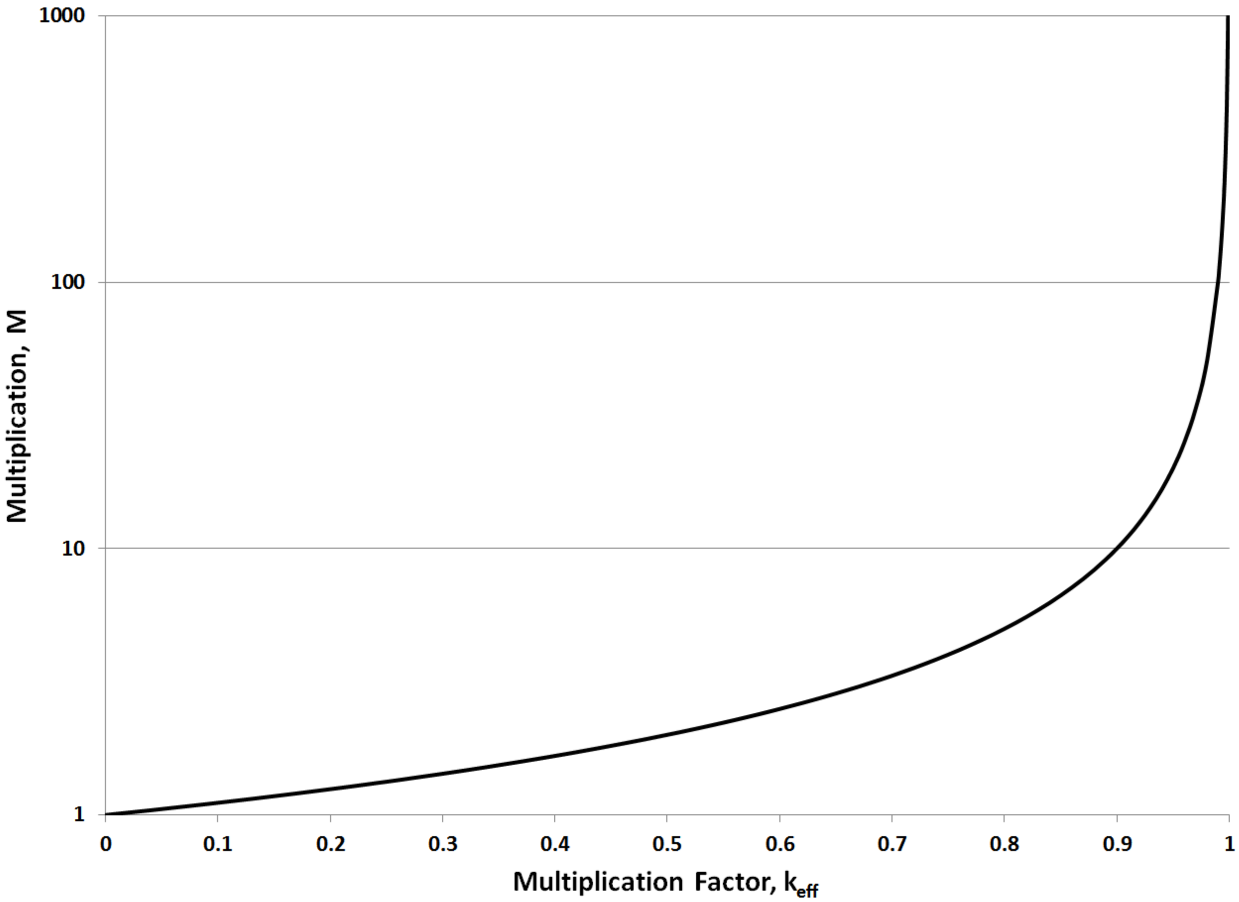
- Example: System with k_{eff}=0.5, adding in 1000 neutrons per generation
  - After each generation, the number of neutrons are cut by half due to k_{eff}

G1	G2	G3	G4	G5	G6
1000	500	250	125	63	32
	1000	500	250	125	63
		1000	500	250	125
			1000	500	250
				1000	500
					1000

- If you continue to do this, each row will eventually add up to 2000
  - Originally started with 1000 neutrons and generated 2000 neutrons
  - M = 2
- Or use formula to calculate M:

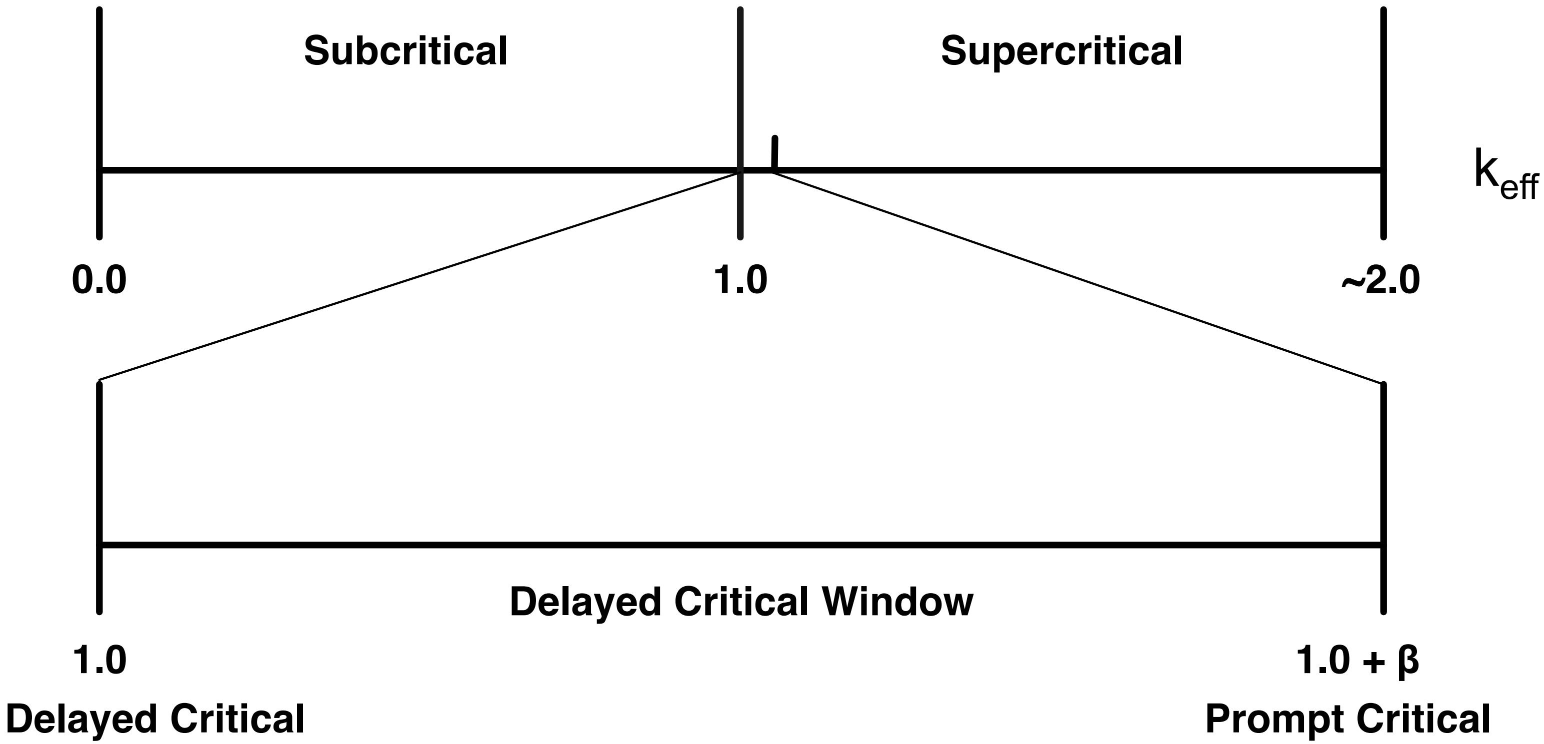
$$M = \frac{1}{1-k_{eff}} = \frac{1}{1-0.5} = \frac{1}{0.5} = 2$$

Neutron Multiplication



k_{eff}	M
0	1
0.5	2
0.9	10
0.95	20
0.99	100
0.999	1000

Regions of Criticality



- Recall β and the Delayed Critical Window
  - β is the delayed neutron fraction (β < 1%)
  - Delayed Critical Window is where delayed neutrons are required to maintain a critical (or slightly supercritical) system
    - $1.0 \leq k_{eff} < 1.0 + \beta$

Delayed Critical and Prompt Critical

- Delayed Critical: Reactor needs both prompt and delayed neutrons to be critical
  - increase in neutron population (power) dominated by time for delayed neutrons to appear
    - long time for power increase allows for control mechanisms
- Prompt Critical: Reactor only needs prompt neutrons to be critical
  - increase in neutron population (power) dominated by time for prompt neutrons to appear
    - short time for power increase does not allow for control mechanisms

Reactor Power and Critical

- If a reactor is critical, it does not mean the reactor is at high power!
- If a reactor is subcritical, it does not mean the reactor is at low power!
- If a reactor is supercritical, it does not mean the reactor is at high power!
- Whether a system is subcritical, critical, or supercritical depends upon the **change** in neutron population (ie, power), not the **magnitude**
- Possible to have low power systems that are critical (or supercritical)
- Possible to have high power systems that are subcritical

Multiplication Factor

- k_{eff} is useful, but is not always used to determine how close to critical an assembly is
- For example:
  - k_{eff}=1.000
  - k_{eff}=1.007
  - 7/1000 of a difference between the two numbers, so very small
  - But, in reality the difference is quite large (second is prompt critical for U-235 assembly)
- Reactivity is another description of how close to criticality a system is

Reactivity

- How is reactivity defined?
- What are the units of reactivity?
- How is reactivity “measured”?

Reactivity

- Reactivity is another description of how far or close from critical an assembly is, represented by ρ
- Related to the multiplication factor:
  - $\rho = \frac{k_{eff}-1}{k_{eff}}$
- Possible values for ρ:
  - $\rho < 0$, which means $k_{eff} < 1$, so the reactor is subcritical
  - $\rho = 0$, which means $k_{eff} = 1$, so the reactor is critical
  - $\rho > 0$, which means $k_{eff} > 1$, so the reactor is supercritical
  - $\rho = \beta$, the reactor is prompt critical
    - Delayed Critical Window: $0 \leq \rho < \beta$

Units of Reactivity

- Reactivity itself is unitless
- However, it can be quoted in units of:
  - Dollars and cents
  - pcm (per cent millirho)
- Take the Delayed Critical Window ($0 \leq \rho < \beta$) and split that interval into 100 units (called cents)
  - One dollar is 100 cents
- 1 cent of reactivity: ρ = \$0.01 = β / 100
- 1 dollar of reactivity: ρ = \$1.00 = β
- By definition, \$1.00 of reactivity is prompt critical

Multiplication Factor for Prompt Critical

- Reactor is prompt critical at ρ = β
- From definition of reactivity

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad k_{eff} = \frac{1}{1-\rho} \qquad k_{pc} = \frac{1}{1-\beta}$$

- Oftentimes quoted as (from Taylor expansion):

$$k_{pc} = 1 + \beta$$

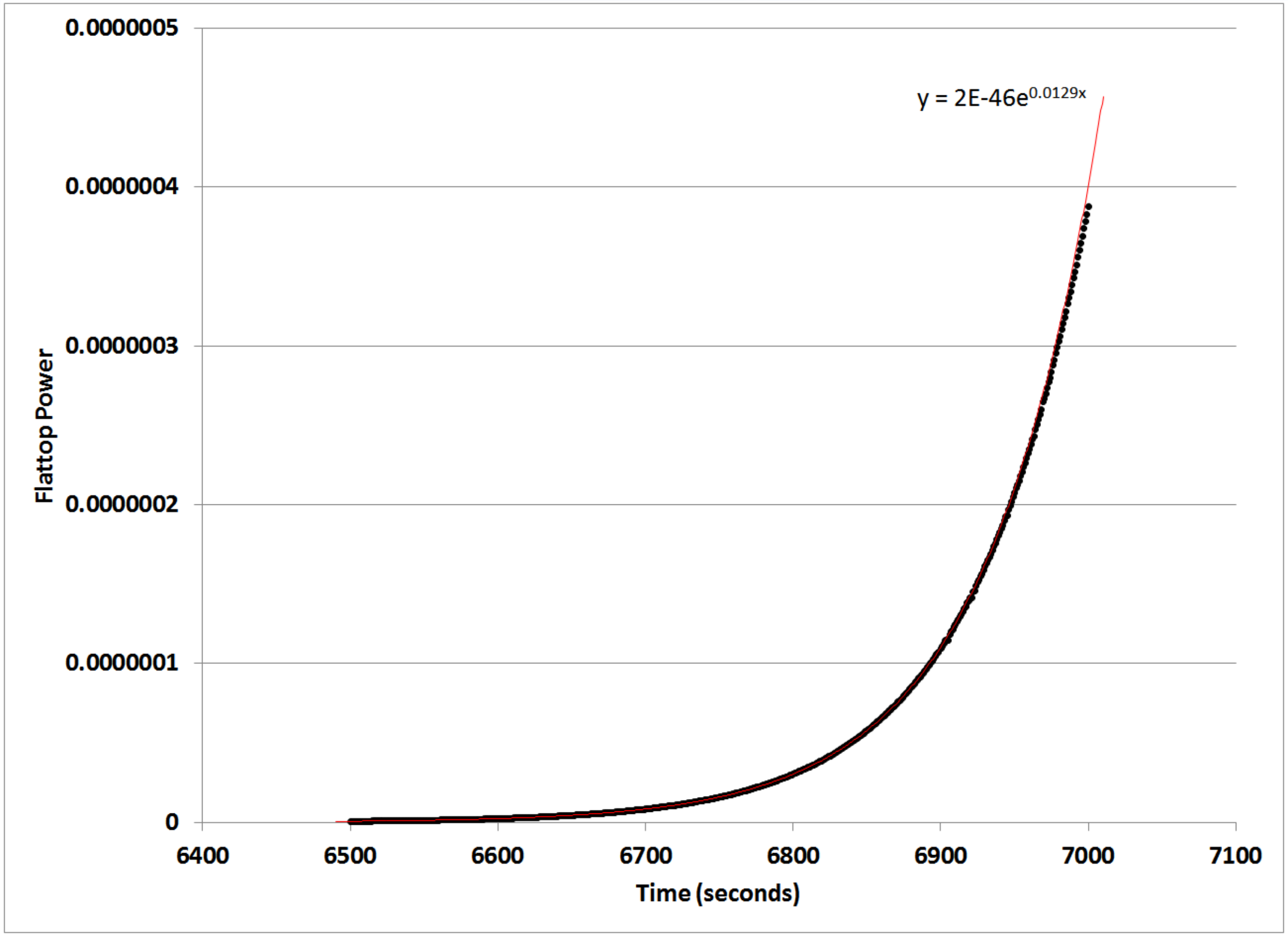
"Measuring" Reactivity

- Question: How can we measure reactivity or k_{eff}?
- Reactivity (and multiplication factor) cannot be directly measured
- Reactivity can be inferred from other measurements
- We assume that neutron leakage out of the reactor is proportional to the number of neutrons in the reactor
  - Larger the neutron leakage, the larger the neutron population
  - Smaller the neutron leakage, the smaller the neutron population
  - Neutron detectors can be used to determine the neutron leakage, giving an idea of what the neutron population is
  - We expect that as power increases, the neutron population in the system increases, which lead to greater leakage
- Measurements of how power (ie, leakage) increases or decreases over time can give an indication of reactivity of the system

"Measuring" Reactivity

- Measuring the change in leakage essentially is measuring the change in reactor power, which can give the reactor period, *T*
- The reactor period is defined as the time it takes to increase (or decrease) the power by a factor of e (e is Euler's Number, e=2.71828...)
- Period can be measured by examining neutron detector response as a function of time as the reactor power increases (ie, as the leakage increases and detector response increases)
  - Fit an exponential curve, $P = P_0 e^{t/T}$
- In February, 2012 a freerun operation was performed on the Flattop machine (ie, reactivity was inserted into the machine and the power was allowed to rise exponentially)
- The following plot shows the power rise and an associated calculation of the reactor period

"Measuring" Reactivity



- The fit gives an $\omega=0.0129\ s^{-1}$, and the reactor period is then:
- $\tau = \frac{1}{\omega} = 77.5\ s$

"Measuring" Reactivity

- Once the reactor period is known, how do we convert between the period and the reactivity of the system?
- Once the period of the reactor has been measured, the reactivity of the system can be inferred using the Inhour Equation:

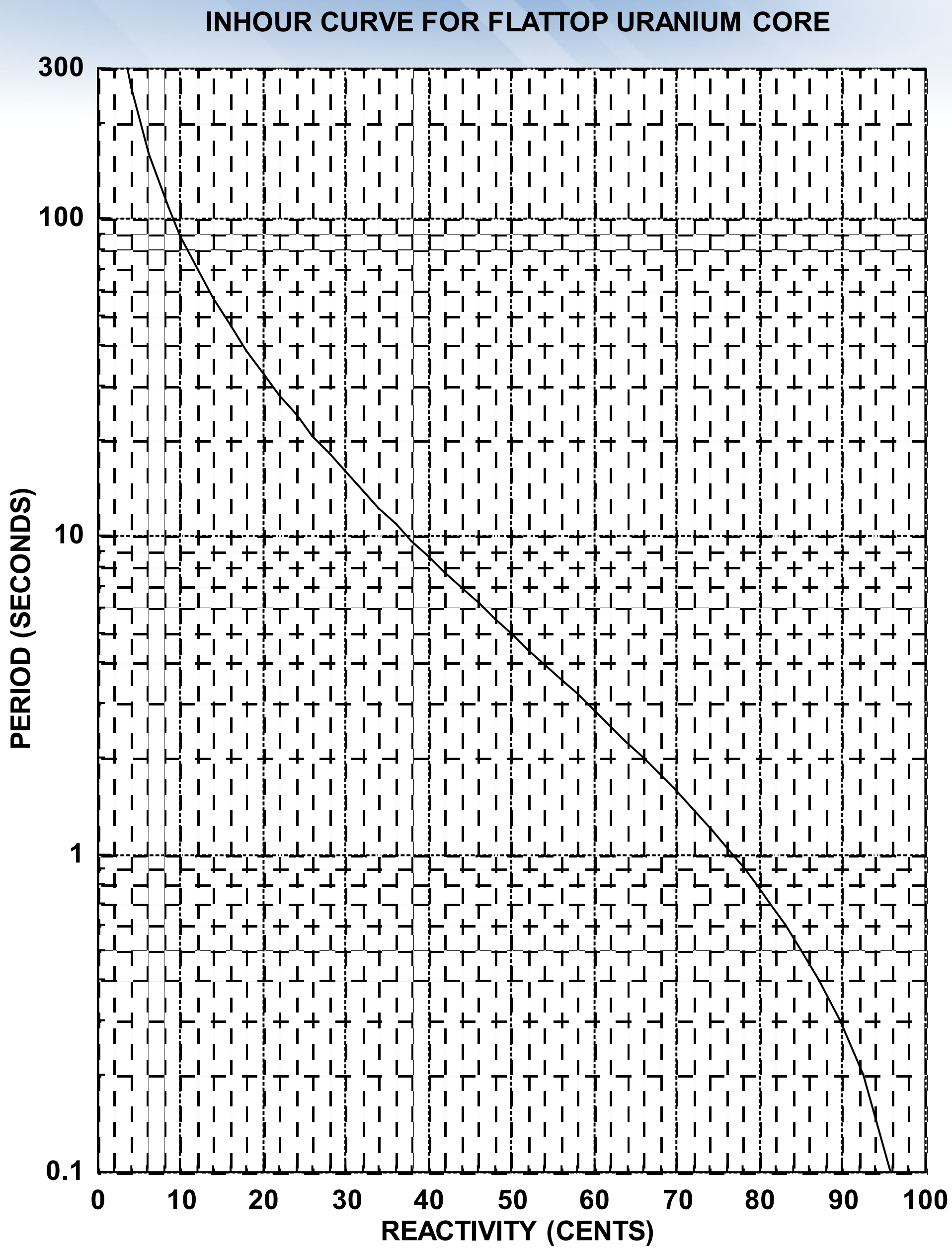
$$\rho(\$) = \frac{\Lambda}{\beta_{eff} T} + \sum_{i=1}^{6} \frac{\beta i / \beta_{eff}}{1 + \lambda_i T}$$

- The Inhour Equation can be derived from the point-reactor kinetics equations, which relates reactor power to prompt and delayed neutron generation in the system
- The coefficients β_i and λ_i are the precursor yields and the decay constants related to each delayed neutron precursor group, which can be material and system dependentT
- For the February 2012 Flattop freerun the initial reactivity of the system was 11 cents, which corresponds to a reactor period of approximately 75 seconds

Period for Delayed Critical and Prompt Critical

- Reactor period changes as reactivity increases from delayed critical to prompt critical
- At critical (steady state, k_{eff}=1), power does not increase, so reactor period is technically infinite
- As reactivity increases, reactor period decreases
- In the delayed critical region, reactor period can range from minutes to seconds to fractions of a second
  - In the delayed critical region, reactor period dominated by the time for delayed neutrons to appear, which can be seconds or greater
  - This large amount of time for power increase allows for control mechanisms to be used
- However, period strongly decreases once reactor is prompt critical, from milliseconds to microseconds
  - In the prompt critical region, reactor period is dominated by the time for prompt neutrons to appear (ie, neutron generation time)
  - This small amount of time for power increase does not allow for control mechanisms to be used

Period


INHOUR CURVE FOR FLATTOP URANIUM CORE
300
100
10
1
0.1
PERIOD (SECONDS)
0
10
20
30
40
50
60
70
80
90
100
REACTIVITY (CENTS)

Functional Approximation for k_{eff}

- Is there a way we can derive a formula to calculate k_{eff}?
  - For most systems, no. However, there are approximations that we can use that might be useful.
- One approximation that we use at NCERC is:
  - $k_{eff} \approx \left(\frac{m}{m_c}\right)^{0.3}$
  - m is the mass on the machine
  - m_c is the mass needed to go critical
- This relation applies to compact systems, similar to systems we see at NCERC
  - Has been found from fitting experimental and calculational data
- Question: Why do we need to use this?
- Answer: NCERC has specific limits on the maximum amount of reactivity we are allowed on our machines. This formula can be used to get an idea of the maximum amount of reactivity on the machine (and allows us to make adjustments if necessary).

Functional Approximation for k_{eff}

- Example: The Class Foils Experiment on the Planet Machine stacks HEU foils (each foils is 70 grams of HEU) to achieve a critical state.
  - Each foil is a discrete mass (ie, we cannot add 1/10th of a foil)
  - It is unlikely that we will be exactly critical when we add the final foil – we will have a small amount of excess reactivity on the system.
  - However, our TSRs state that our maximum excess reactivity on Planet is $0.80. We can use this approximation for k_{eff} to get an idea of how much excess reactivity we will have when we add that final foil.
- Example Setup: We predict that we will go critical with 24.8 foils. If we add the 25th foil to the machine, how much excess reactivity do we predict we will have on the machine? Will we have gone over our TSR limit? Assume that β_{eff}=0.0068 for this experiment.
  - Step 1: Calculate our approximate k_{eff} value, using the above equation.
  - Step 2: Using this calculated k_{eff} and β_{eff}, calculate our excess reactivity on the machine.

Functional Approximation for k_{eff}

- Step 1: Calculate our approximate k_{eff}
  - m_c=24.8 foils = 1736 grams
  - m=25 foils = 1750 grams
  - $k_{eff} \approx \left(\frac{m}{m_c}\right)^{0.3} = \left(\frac{1750}{1736}\right)^{0.3} = (1.008065)^{0.3} = 1.0024$
- Step 2: Calculate our excess reactivity
  - $\rho = \frac{k_{eff}-1}{k_{eff}} = \frac{1.0024-1}{1.0024} = 0.002394$
  - $\rho(\$) = \frac{\rho}{\beta} = \frac{0.002394}{0.0068} = \$0.35$
- Step 3: Compare to our limit of $0.80
  - $0.35 < $0.80
  - We estimate that adding the 25th foil will **not** bring us above our maximum excess reactivity limit

Period Calculation

- Using these curves, reactor period can be found (if system reactivity is known) or reactivity can be found (if reactor period is known)
- Example 1: If Flattop is being operated at a reactivity of $0.10, what would the corresponding reactor period that we would expect to see?
  - ~90 seconds
- Example 2: If we measure a reactor period of 22 seconds on Flattop, what would the corresponding reactivity be?
  - ~$0.24

Criticality Information

- Two main factors that determine criticality:
  - Size (geometry): leakage
    - Surface area to volume ratio, reflectors
  - Composition: absorption
    - Enrichment, fuel to moderator ratio
- Both size and composition can be used to control assembly
- Composition: Addition of absorbers or poisons
- Size: Modification of geometry to change leakage
- NCERC uses size modification for control

Reactivity Feedback

- What is reactivity feedback?
- What are the main reactivity feedback mechanisms for fast and thermal systems?
- What is a self-limiting excursion?

Reactivity Feedback

- As energy is added to a system (ie, fission energy), the properties of the system can change
  - Temperature rise -> Volume expansion -> Density decrease
- As the properties change, the reactivity of the system can change
- Energy deposition leads to reactivity feedback

Reactivity Feedback

- Reactivity feedback is oftentimes negative
  - Increases in energy (or temperature) lead to decreases in reactivity
  - Fast systems: Volume expansion introduces more leakage
  - Thermal systems: Extra leakage, more resonance absorption, and decreasing fission cross section
- Reactivity feedback modeled with single coefficient,

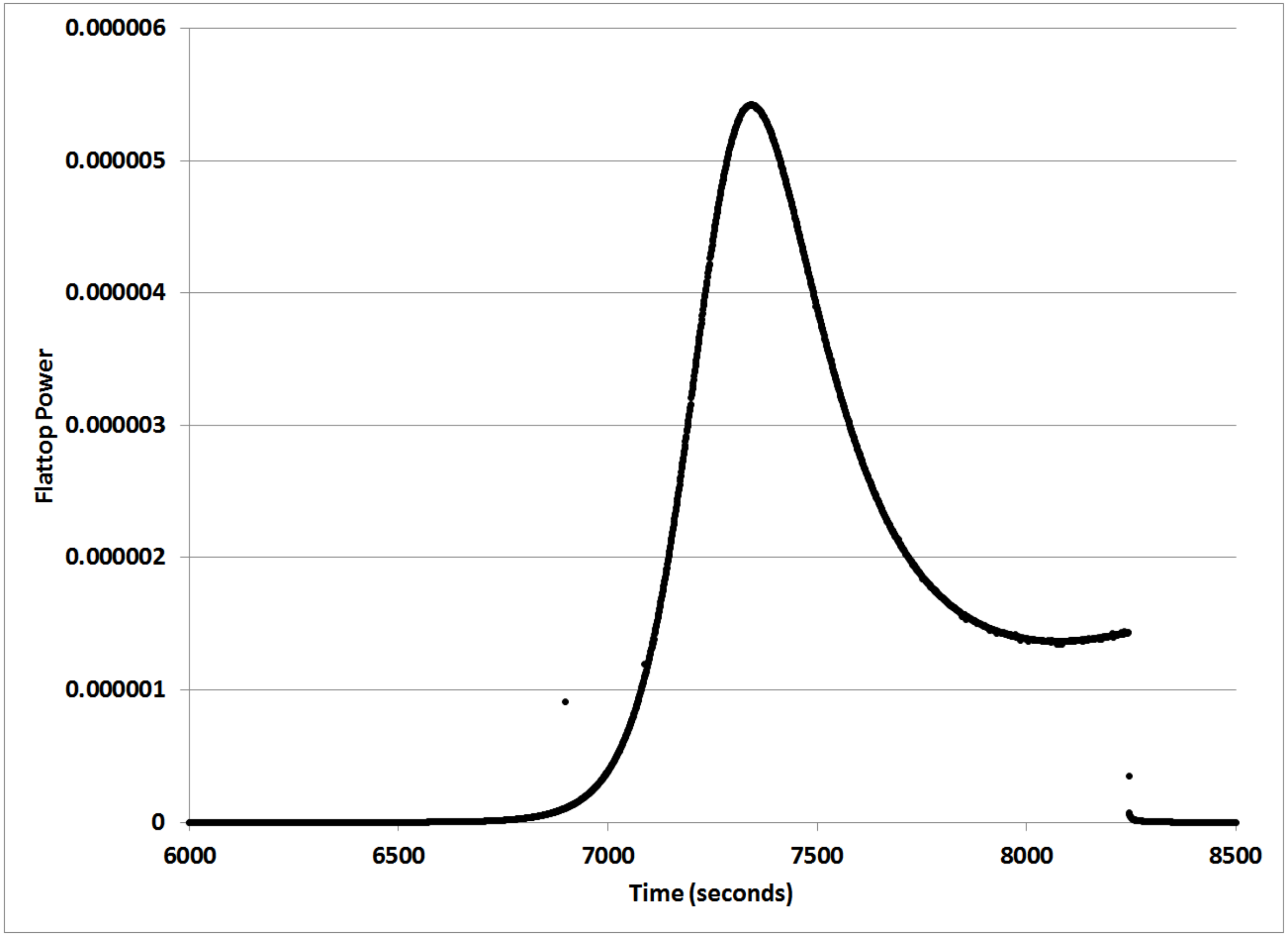
$$\rho = \rho_0 + \alpha T$$

- Some example values:

Assembly	Coefficient (cents/Kelvin)
Godiva IV, Flattop (U)	-0.3
Godiva IV (dynamic)	-0.2
Flattop (Pu)	-0.2
SHEBA	-4.0 -10.0

Self-Limiting Excursion

- Increase in reactivity -> power increase -> temperature increase -> reactivity feedback -> decrease in reactivity -> power decrease



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Reactivity Feedback Model Example

- Using this simple formula for reactivity feedback, the effects of temperature on system reactivity can be calculated
- Example: A Flattop freerun has an initial reactivity of \$0.10. Assuming a negative temperature coefficient of reactivity of α=-0.3 cents/Kelvin, what is the approximate temperature increase in the Flattop machine need to bring Flattop to a critical state (ie, ρ=\$0.00)?
  - $\rho = \rho_0 + \alpha T$
  - ρ_0 = \$0.10
  - α = -0.3 cents/Kelvin = -\$0.003 K^{-1}
  - ρ = \$0.00
  - T = unknown
  - $T = \frac{\rho - \rho_0}{\alpha} = \frac{\$0.00 - \$0.10}{-\$0.003\ K^{-1}} = 33\ K$
  - So a temperature change of 33 K would be needed to reduce the excess reactivity of Flattop by 10 cents.

Power and NCERC Machines

- NCERC machines operate at a **very low power** (ie, low energy generation)
  - There is no need for water cooling, cooling towers, etc.
  - Rough power of milliwatts to kilowatts
    - Compared to power reactors (~1000 megawatts)
  - Very low fission product inventory

Review

- Questions?

- Review:
  - What is the fission chain process?
  - What is reactivity and how is reactivity related to the ideas of delayed and prompt critical?
  - What are the simple reactivity feedback mechanisms?