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Toward Scalable Solvers for Stochastic Multi-Stage Long-Term Generation and Transmission Capacity Expansion

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Federal Energy Regulatory Commission

Washington, DC

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Combinatorial Optimization R&D at Sandia

- Efforts are centered on two primary research thrusts
  - Risk Management
    - Multi-stage, general mixed-integer
    - Efficient risk versus cost tradeoff analysis
    - Scalable Conditional Value-at-Risk (CVaR) computation
  - Multi-Stage Stochastic Optimization
    - Multi-stage, general mixed-integer
    - Massively parallel environments
- Application drivers
  - Contamination sensor network design (INFORMS Edelman Finalist)
  - Network interdiction for critical infrastructure
  - Biofuel network design
  - Electrical grid generation and transmission capacity expansion
  - Scalable unit commitment with large renewables penetration
- Funding sources
  - DOE Office of Science, US EPA, Sandia LDRD

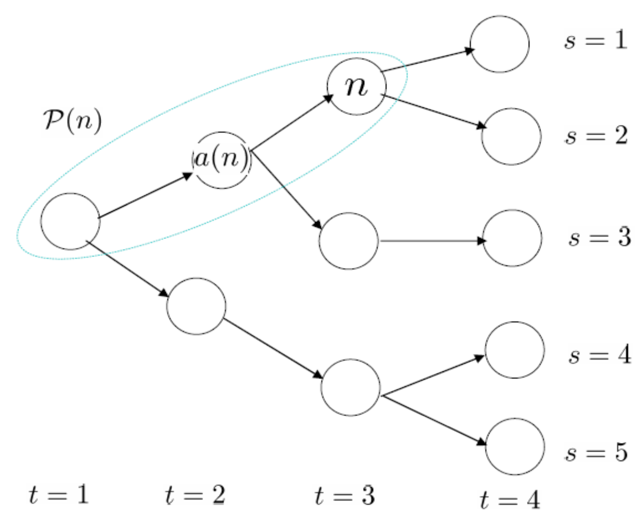
Resource Allocation: Integer and Stochastic Programming

- Deterministic Mixed-Integer Programming (MIP)
  - The PDE of Operations Research

$$\begin{aligned} \min \quad & \boldsymbol{c}'\boldsymbol{x} + \boldsymbol{h}'\boldsymbol{y} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in Z_+^n (\boldsymbol{x} \geq 0, \boldsymbol{x} \text{ integer}) \\ & \boldsymbol{y} \in R_+^n (\boldsymbol{y} \geq 0) \end{aligned}$$

  - Approximable for most real-world problems (NP-Hard)
- Stochastic Mixed-Integer Programming (SMIP)
  - SMIP = MIP + *uncertainty* + *recourse*

$$\begin{aligned} \min \quad & f(\mathbf{x}) = \mathbf{c}^T\mathbf{x} + \mathbb{E}[Q(\mathbf{x},\omega)] \\ \text{s.t.} \quad & A\mathbf{x} \geq \mathbf{b}, \quad \mathbf{x} \in \mathbb{R}_+^{n_1-p_1} \times \mathbb{Z}_+^{p_1} \\ Q(\mathbf{x},\omega) = \quad & \min \quad \mathbf{q}(\omega)^T\mathbf{y} \\ & \text{s.t.} \quad W\mathbf{y} \geq \mathbf{h}(\omega) - T(\omega)\mathbf{x} \\ & \qquad \mathbf{y} \in \mathbb{R}_+^{n_2-p_2} \times \mathbb{Z}_+^{p_2} \end{aligned}$$



  - Still NP-Hard, but far more difficult than MIP in practice

Capacity Expansion as Stochastic Mixed-Integer Programming

- Many historical planning models are either deterministic or linear (or both)
  - Driven by combinations of data availability and solver maturity

- With advances in IT and solver technology, multi-stage stochastic mixed-integer formulations are becoming more prevalent in the literature
  - Singh et al. (2009), Wang and Ryan (2010), Huang and Ahmed (2009)
  - General paradigm captures key aspects of capacity expansion problems

- Key technological challenges to deploying multi-stage stochastic MIP models
  - No canonical generation and transmission capacity expansion model
  - Multi-stage stochastic MIP solvers are not yet general-purpose
  - The difficulty of multi-stage stochastic MIPs *likely* requires parallelism

- Key requirement to solve the deployment barrier
  - Modeling and solver framework to facilitate rapid prototyping of alternative solution strategies, supporting built-in parallelism

Stochastic Mixed-Integer Programming: The Algorithm Landscape

- The Extensive Form or Deterministic Equivalent
  - Write down the full variable and constraint set for all scenarios
  - Write down, either implicitly or explicitly, non-anticipativity constraints
  - *Attempt* to solve with a commercial MIP solver
    - Great if it works, but often doesn't due to memory or time limits
- Time-stage or "vertical" decomposition
  - Benders / L-shaped methods (including nested extensions)
  - Pros: Well-known, exact, easy for (some) 2-stage, parallelizable
  - Cons: Master problem bloating, multi-stage difficulties
- Scenario-based or "horizontal" decomposition
  - Progressive hedging / Dual decomposition
  - Pros: Inherently multi-stage, parallelizable, leverages specialized MIP solvers
  - Cons: Heuristic (depending on algorithm), parameter tuning
- Important: *Development of general multi-stage SMIP solvers is an open research area*

Progressive Hedging: A Review and/or Introduction

1. $k := 0$
2. For all $s \in \mathcal{S}$, $x_s^{(k)} := \text{argmin}_x(c \cdot x + f_s \cdot y_s) : (x, y_s) \in \mathcal{Q}_s$
3. $\bar{x}^k := (\sum_{s\in\mathcal{S}} p_s\, d_s x_s^{(k)}) / \sum_{s\in\mathcal{S}} p_s\, d_s$
4. For all $s \in \mathcal{S}$, $w_s^{(k)} := \rho(x_s^{(k)} - \bar{x}^{(k)})$
5. $k := k + 1$
6. For all $s \in \mathcal{S}$, $x_s^{(k)} := \text{argmin}_x(c \cdot x + w_s^{(k-1)}x + \rho/2 \left\|x - \bar{x}^{(k-1)}\right\|^2 + f_s \cdot y_s)$ $: (x, y_s) \in \mathcal{Q}_s$
7. $\bar{x}^{(k)} := (\sum_{s\in\mathcal{S}} p_s\, d_s x_s^{(k)}) / \sum_{s\in\mathcal{S}} p_s\, d_s$
8. For all $s \in \mathcal{S}$, $w_s^{(k)} := w_s^{(k-1)} + \rho \left(x_s^{(k)} - \bar{x}^{(k)}\right)$
9. $g^{(k)} := \frac{(1-\alpha)|\mathcal{S}|}{\sum_{s\in\mathcal{S}} p_s\, d_s} \sum_{s\in\mathcal{S}} \left\|x^{(k)} - \bar{x}^{(k)}\right\|$
10. If $g^{(k)} < \epsilon$, then go to step 5. Otherwise, terminate.

Rockafellar and Wets (1991)

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Progressive Hedging as a Stochastic Mixed-Integer Heuristic

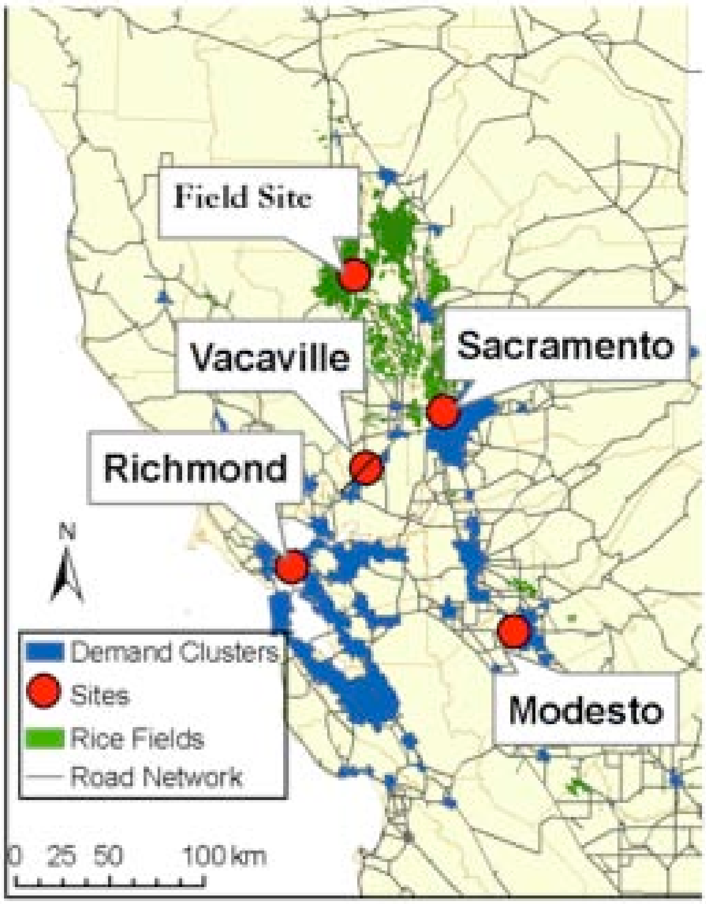
- Progressive Hedging does provably converge in the *convex* case, in linear time
  - NOTE: As practitioners know well, linear time can take a *long* time

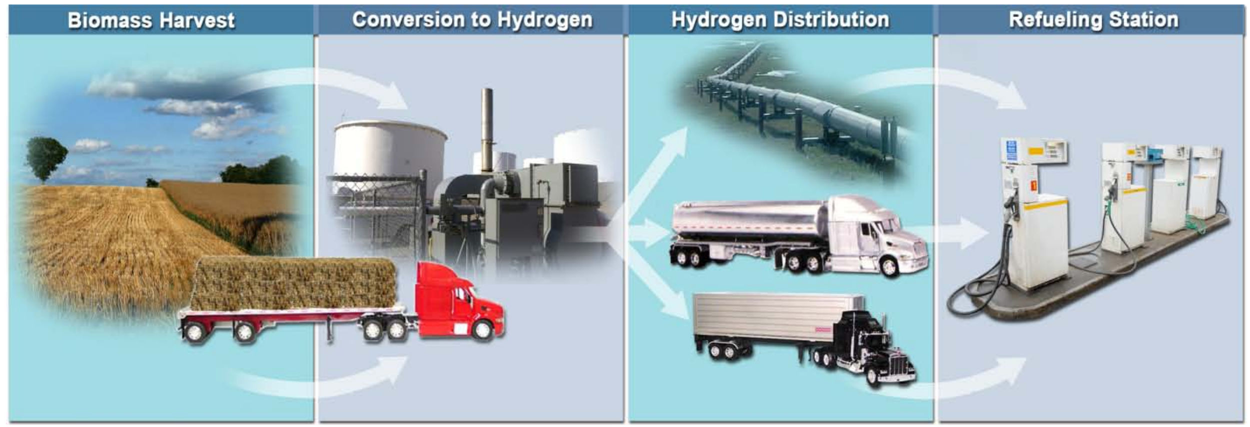
- Progressive Hedging (PH) has been successfully used as a heuristic for multi-stage mixed-integer stochastic programming
  - Løkketangen and Woodruff (1996)
  - Numerous others (Birge, Gendreau, Crainc, Rei)

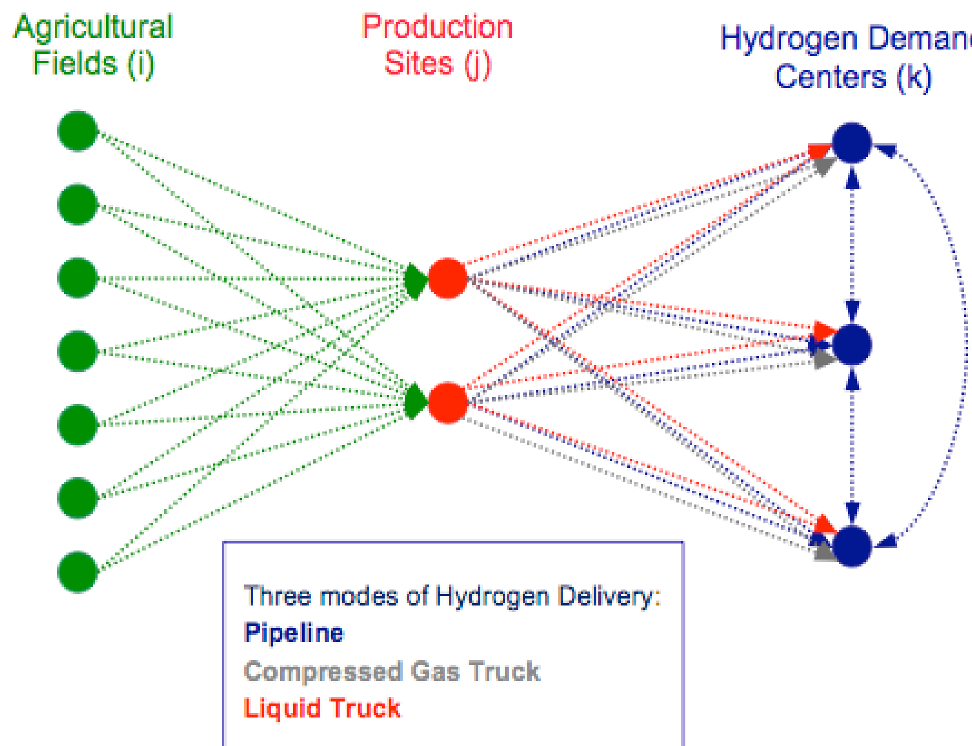
- Practical and critical issues of note
  - How to pick ρ?
  - Cycle detection
  - Convergence acceleration
    - Variable fixing
    - Slamming

Progressive Innovations for a Class of Stochastic Mixed-Integer Resource Allocation Problems
(Watson/Woodruff, Sandia Technical Report, Journal Article Under Revision)

The Impact of Decomposition: Biofuel Infrastructure and Logistics Planning







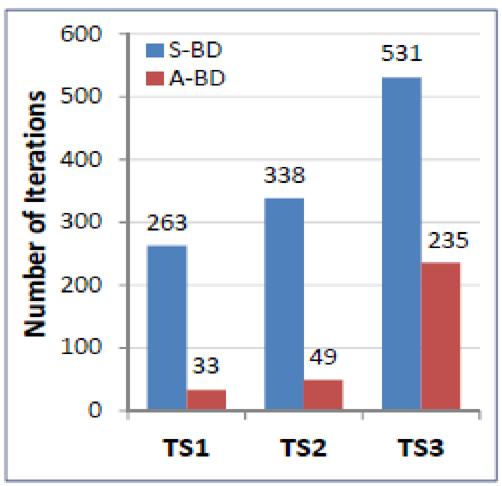
<u>Example of PH Impact:</u>

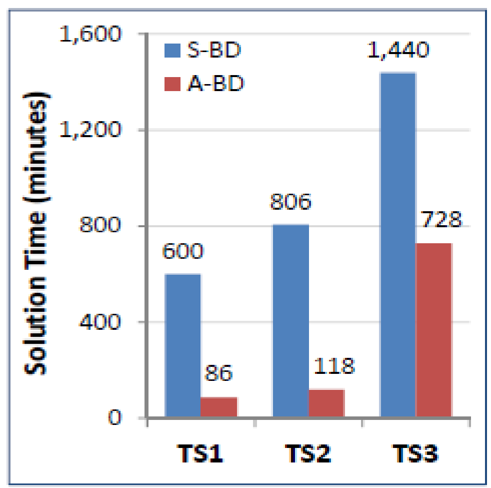
- Extensive form solve time: >20K seconds
- PH solve time: 2K seconds

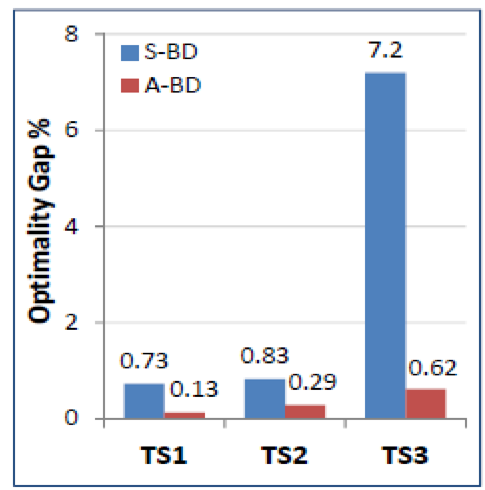
Slide courtesy of Professor YueYue Fan (UC Davis)

The Impact of Decomposition: Wind Farm Network Design

- Where to site new wind farms and transmission lines in a geographically distributed region to satisfy projected demands at minimal cost?
- Formulated as a two-stage stochastic mixed-integer program
  - First stage decisions: Siting, generator/line counts
  - Second stage "decisions": Flow balance, line loss, generator levels
- 8760 scenarios representing coincident hourly wind speed, demand
- Solve with Benders: Standard and Accelerated







- Summary: A non-trivial Benders variant is *required* for tractable solution

Slide courtesy of Dr. Richard Chen (Sandia California)

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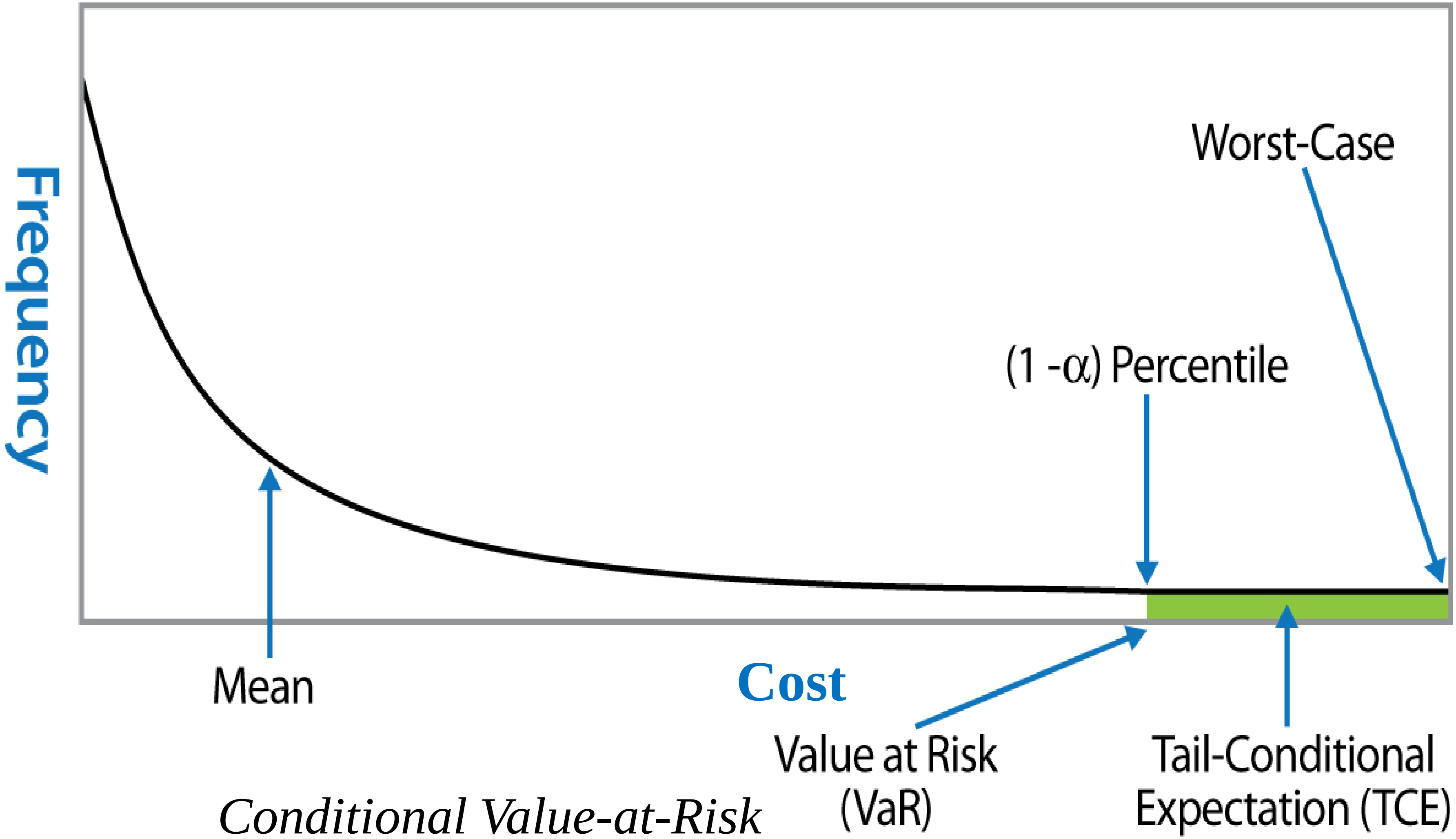
Mean versus Risk? Some Terminology



Conditional Value-at-Risk (CVaR) is a linear approximation of TCE

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Progressive Hedging and Conditional Value-at-Risk

- Scenario-based decomposition of Conditional Value-at-Risk models is conceptually straightforward (Schultz and Tiedemann 2006)

Proposition 5.1. *Assume that μ is discrete with finitely many scenarios $h_1, \ldots, h_J$ and corresponding probabilities $\pi_1, \ldots, \pi_J$. Let $\alpha \in (0, 1)$. Then the stochastic program*

$$\min\{Q_{CVaR_\alpha}(x) \ : \ x \in X\} \tag{11}$$

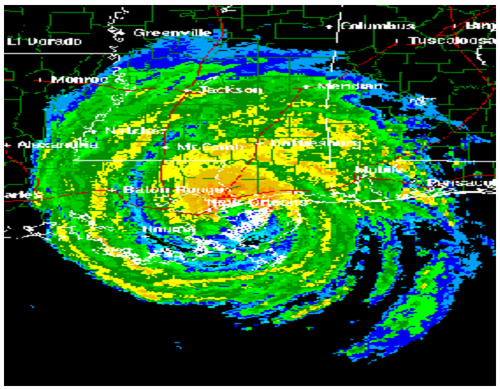
can be equivalently restated as

$$\begin{aligned} \min_{x,y,y',v,\eta} \Bigg\{ \eta + \frac{1}{1-\alpha} \sum_{j=1}^{J} \pi_j v_j \ : \ & W y_j + W' y'_j \ = \ h_j - T x, \\ & v_j \geq c^\top x + q^\top y_j + q'^\top y'_j - \eta, \\ & x \in X, \quad \eta \in \mathbb{R}, \quad y_j \in \mathbb{Z}_+^{\bar{m}}, \\ & y'_j \in \mathbb{R}_+^{m'}, \ v_j \in \mathbb{R}_+, \quad j = 1, \ldots, J \Bigg\}. \end{aligned} \tag{12}$$

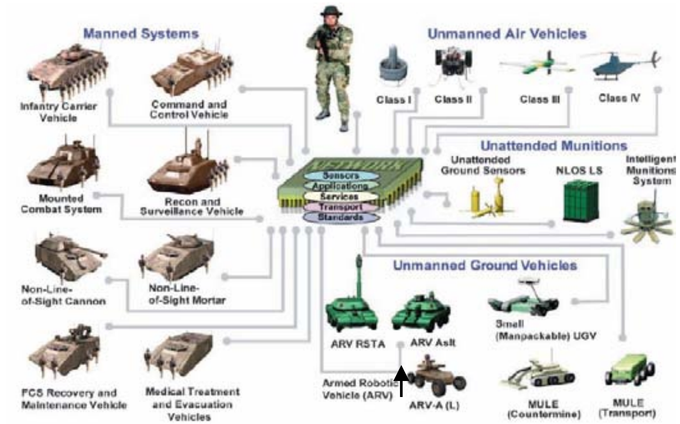
- But
  - Computational issues are largely unexplored

Selecting Scenarios to Ignore in Stochastic Optimization: Advances in Probabilistic Integer Programming Solvers

Ignoring the 100-year Flood
(Infrastructure Planning)



Capacitated Storage
(US Army Future Combat Systems)



Force-on-Force "Anomalies"
(Mission Planning)

Central Theme: The Need to Ignore a Small Fraction α of Scenarios During Optimization

$$\begin{aligned} \text{minimize} \quad & c \cdot x + \textstyle\sum_{s \in \mathcal{S}} p_s (f_s \cdot y_s) \qquad \text{(E)} \\ \text{subject to:} \quad & (x, y_s) \in \mathcal{Q}_s, \quad \forall s \in \{\mathcal{S} : d_s = 1\} \\ & \textstyle\sum_{s \in \mathcal{S}} p_s \, d_s \geq (1 - \alpha) \\ & d_s \in \{0, 1\}, \quad \forall s \in \mathcal{S} \end{aligned}$$

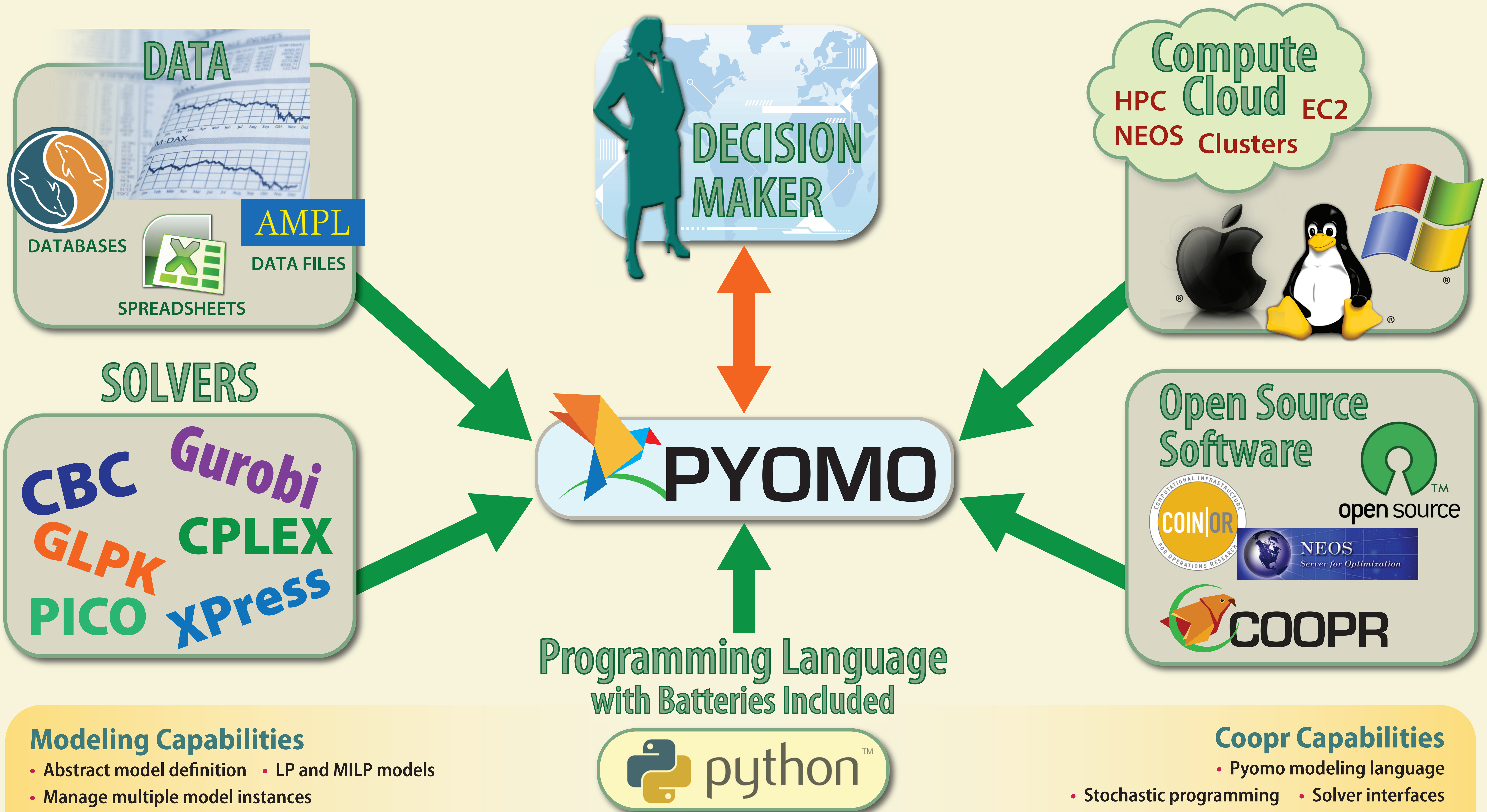
Results for network design:

- 2-8% better solutions than CPLEX, 1440m versus ~10m

Impact: - Best available heuristic for solving probabilistic integer programs
- First demonstration on large-scale, real-world problems

PYOMO

An Open-Source Optimization Modeling Tool



Modeling Capabilities

- Abstract model definition
- LP and MILP models
- Manage multiple model instances
- Stochastic modeling extensions

Key Features

- Parallel solver execution
- Extensible framework
- Interface to many data sources
- Portability
- Embedded in modern programming language
- Freely available
- Unrestricted open source license

Coopr Capabilities

- Pyomo modeling language
- Stochastic programming
- Solver interfaces
- Modeling extensions
- GUI front-end

Coopr Resources

- Coopr installer script
- Wiki documentation
- Examples
- Trouble tickets
- Mailing lists

TO LEARN MORE VISIT >>

https://software.sandia.gov/pyomo

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Ideas into Action

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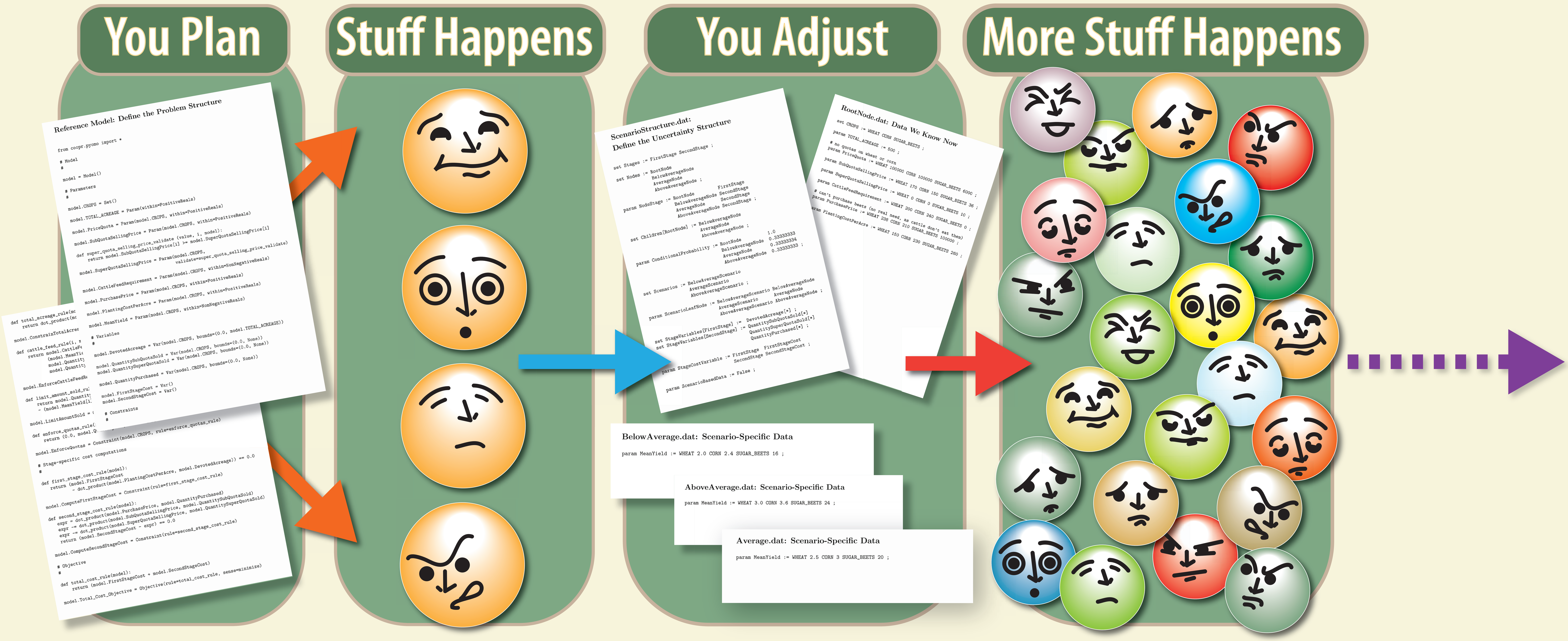
Hedging Against Uncertainty:
A Modeling Language and Solver Library



PYOMO

PySP: Stochastic Programming in Python

COOPR

Multi-Stage Planning for Uncertain Environments

- Explicitly capture recourse
- Uncertainty modeling framework
- Integrated solver strategies

What We Do:

- Mixed decision variables
  - Continuous
  - Integer/Binary
- General multi-stage
- Stochastic programming
  - Expected value
  - Conditional Value-at-Risk
  - Scenario selection
- Cost confidence intervals

How We Do It:

- Deterministic equivalent
- Scenario-based decomposition
  - Progressive Hedging
  - Customizable accelerators
- Algebraic modeling via Pyomo
- SMP and cluster parallelism
- Integrated high-level language support
- Multi-platform, unrestrictive license
- Open source, actively supported by Sandia
- Co-Managed by Sandia and COIN-OR

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TO LEARN MORE VISIT > https://software.sandia.gov/trac/coopr/wiki/PySP

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Stochastic Programming and High-Performance Computing

- Decomposition algorithms for solving multi-stage stochastic mixed-integer programs are "naturally" parallelizable
  - L-shaped and Progressive Hedging are particularly amenable
- Practical issues arise as the number of scenarios grows
  - Even the most modest branching processes in multi-stage decision environments lead to thousands to millions of scenarios
  - MIP solve times are heterogeneous, leading to poor parallel efficiency
- Current capabilities in PySP:
  - Scalability to order-thousand scenarios and processors
- In-progress efforts
  - Asynchronous decomposition algorithms
  - IBM Research Blue Gene deployment
  - EC2 / Gurobi deployment
- Major deployment issue: MIP solver licensing to thousands of processors
  - Mitigated in part by Gurobi EC2 deployment

Scenario Sampling: How Many is Enough?

- Discretization of the scenario tree is "standard" in stochastic programming
  - Often, no mention of solution or objective stability
  - Let alone rigorous statistical hypothesis-testing of stability
  - *Don't trust anyone who doesn't show you a confidence interval*

- Two general approaches in the literature
  - Has the solution converged? (Sample Average Approximation)
  - Has the objective converged? (Multiple Replication Procedure)

- Formal question we are concerned with
  - What is the probability that $\widehat{x}$'s objective function value is suboptimal by more than α%?

- Initial implementation available in PySP
  - Preliminary results for various network expansion and design problems indicates that we are using *far* too few samples

Conclusions

- Multi-stage stochastic mixed-integer programs are a natural modeling paradigm for solving generation/transmission capacity expansion problems
- Solver technologies capable of solving realistic instances are emerging
  - But many challenges remain, both in terms of research and deployment
- Sandia is developing software to address what we view as the challenges
  - Frameworks to support rapid modeling and solver prototyping
  - Scalable parallelization of decomposition strategies
  - Rigorous quantification of uncertainty bounds on solution costs
  - Open-source solutions
    - Sandia is mandated to collaborate with and aid industry – not compete
- For more information:
  - https://software.sandia.gov/trac/coopr/wiki/PySP -or- jwatson@sandia.gov