ESP300

Overview of Radiative Heat Transfer

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
for the United States Department of Energy under contract DE-AC04-94AL85000.

ESP = Engineering Sciences Program

ESP100 is a course on computational solid mechanics

ESP200 is a course on digital signal processing with MATLAB

ESP300 is a course on heat transfer analysis using the finite element method

There are plans to offer additional courses in the future.

All of these courses are intended to provide a continuing education opportunity – in the spirit of the INTEC courses some years ago

Introductory Info

Evacuation Procedures:

- **Exits are located…**
- **Restrooms out back**

Classification:

- **Absolutely no classified discussions**
- **If you have a concern, let us know**
- **Some material may be OUO, it will be marked as such**

Please remind me to show this slide each time we have class !

Questions for "Overview of Radiative Heat Transfer"

What do we mean by:

- Emissive power, irradiation, radiosity, intensity?
- Spectral, hemispherical, total properties?
- "Blackbody," Planck distribution, Stefan-Boltzmann, blackbody fraction?
- Semitransparent, opaque?
- Emissivity, absorptivity, reflectivity (specular and diffuse)?

We really didn't dig deeply into the broad topic of radiative heat transfer. It can easily take a semester to cover that topic, with participating media radiation, etc. My objective here was to provide a really high-level overview, with some detail on the formulation of the enclosure radiation equations that we use with the FEM for heat transfer analyses.

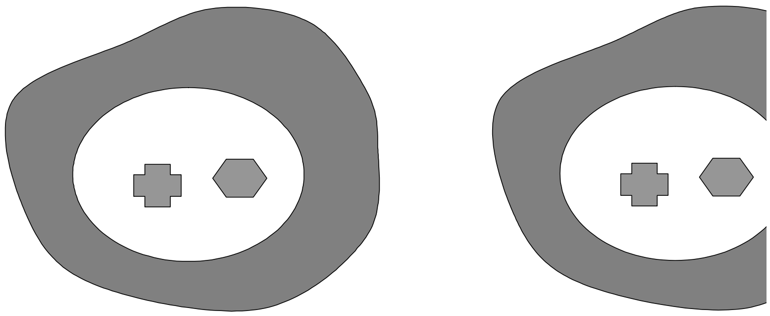
More Questions for "Overview of Radiative Heat Transfer"

How do we couple heat conduction and radiative transfer in enclosures?

- What are view factors?
  - calculating them,
  - reciprocity relationship,
  - sum to unity relationship
- "Radiosity" verses "Net-radiation" formulation?
- Considerations for coupling with heat conduction?

How do we compute the radiative heat transfer within an enclosure?

We need to couple heat conduction in the solids with the radiative energy transferred within the enclosure.



"Complete" Enclosure   "Partial" Enclosure

We will consider an "overview" (I mean really an OVERVIEW) of some of the concepts needed for computing radiative heat transfer in general, and for enclosures, in particular.

So, what's an enclosure?

What's a complete enclosure? What's a partial enclosure?

Thermal Radiation Concepts – Objectives for this Class

Overview of thermal radiative concepts to provide sufficient understanding to

- understand assumptions in enclosure radiation
- assist in making simplifications in analyses
- conduct simple enclosure radiation analyses

For this class, we will focus on basic radiative concepts and their application to enclosure radiation problems

References:

Introduction to Heat Transfer, Frank P. Incropera and David P. DeWitt, 2nd ed, 1990

Thermal Radiation Heat Transfer, Robert Siegel and John R. Howell, 4th ed, 2002

Most of the basic concepts for today come from Incropera's book.

I prefer the discussion in Howell's book for the enclosure radiation discussion.

We will cover the basic terms that are needed to understand the radiative heat transfer in enclosures.

We will NOT cover the specifics of participating media radiative transfer…. Far beyond the intended scope of this class.

Thermal Radiation Concepts

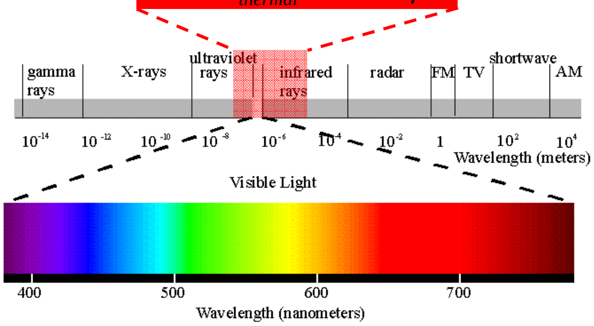
Thermal radiation is a unique heat transfer mechanism:

- energy transfer by virtue of a temperature difference
- energy transported without matter (transport across a void)
- thermal radiation emission occurs everywhere
  - Volumetric phenomenon – gases, glasses … semi-transparent media
  - Surface phenomenon – solids and some liquids
- different approaches to describe radiation
  - Propagation of particles – photons or quanta
  - Propagation of electromagnetic waves at the speed of light (frequency & wavelength)

Thermal radiation is different….

Where's Thermal Radiation in the Electromagnetic Spectrum?

Thermal radiation $0.1 \le \lambda_{thermal} \le 100\ \mu m$



- frequency-wavelength relationship

$$\lambda = \frac{c}{\nu}$$ where c is the speed of light in the medium

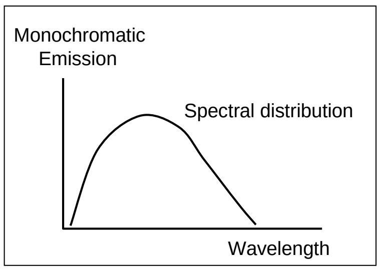
Image reference: http://www.yorku.ca/eye/spectru.htm

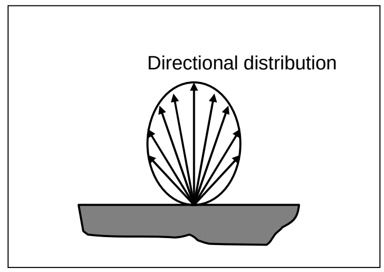
Thermal radiation occupies a small portion of the spectrum. The visible spectrum is a small portion of the thermal region.

Frequency and wavelength are related through the speed of light.

Fundamental Concepts of Thermal Radiation

- For any frequency, both the magnitude of the radiation and the spectral distribution are functions of temperature
- Radiation from a surface must account for both spectral and directional effects

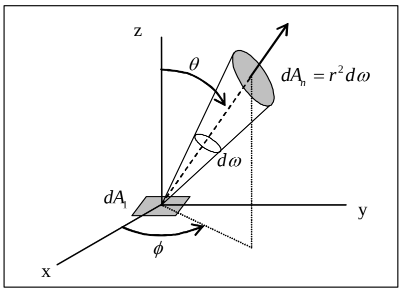




Fundamental Concepts of Thermal Radiation (2)

Radiative "intensity" is the basis for the concepts of

- "Emissive power" – energy emitted from a surface
- "Irradiation" – energy incident on a surface
- "Radiosity" – net outgoing energy from a surface



Intensity is defined at the rate at which radiant energy is emitted

- at the wavelength, λ ,
- in the (θ,ϕ) direction,
- per unit area of emitting surface normal to this direction, dA_n ,
- per unit solid angle, $d\omega$, about this direction,
- per unit wavelength interval $d\lambda$, about wavelength, λ

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Intensity can be associated with energy leaving the surface (that is, emitted or radiosity)

Intensity can be associated with energy incident on the surface (irradiation)

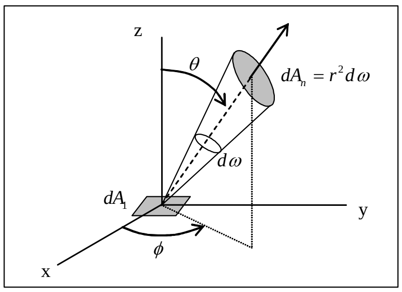
Fundamental Concepts of Thermal Radiation (3)

Mathematically, radiative "intensity" is

$$I_{\lambda,e}(\lambda,\theta,\phi) \equiv \frac{dQ}{dA_1 \cos\theta \, d\omega \, d\lambda} \qquad \left(W/m^2 \ sr \ \mu m\right)$$

or

$$dQ_\lambda \equiv \frac{dQ}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\, dA_1 \cos\theta \, d\omega \qquad \left(W/\mu m\right)$$



Intensity is defined at the rate at which radiant energy is emitted

- at the wavelength, λ ,
- in the (θ,ϕ) direction,
- per unit area of emitting surface normal to this direction, dA_n ,
- per unit solid angle, $d\omega$, about this direction,
- per unit wavelength interval $d\lambda$, about wavelength, λ

dq_lambda - rate at which energy leaves dA1 and passes through dAn

Fundamental Concepts of Thermal Radiation (4)

Radiative flux as a function of radiative “intensity”

$$dQ_\lambda \equiv \frac{dQ}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\, dA_1 \cos\theta\, d\omega \qquad (W/\mu m)$$

$$d\omega = \sin\theta\; d\theta\; d\phi$$

$$dq_\lambda = I_{\lambda,e}(\lambda,\theta,\phi)\, \cos\theta\, \sin\theta\; d\theta\; d\phi$$

$$q_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\, \cos\theta\, \sin\theta\; d\theta\; d\phi \qquad (W/m^2\,\mu m)$$

$$q = \int_0^\infty q_\lambda(\lambda)\, d\lambda \qquad (W/m^2)$$

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To develop and explain this appropriate detail, we would need to consider a spherical coordinate system, write the flux in those terms, and work through the mathematics.

Most textbooks go through this in all the gory detail.

For our purposes today, it is sufficient to recognize that we can obtain the radiative flux by integrating the intensity over the appropriate hemisphere.

After integrating the directional variation, the radiative flux may still be a function of wavelength, which can also be integrated to obtain the total radiative flux.

Fundamental Concepts of Thermal Radiation (5)

"Emissive power," "Irradiation," and "Radiosity"

- are functions of radiative "intensity" $I\left(\lambda,\theta,\phi\right)$
- are functions of wavelength (spectral) and direction

Three simplifications are commonly used:

- "hemispherical" properties → integrated over all directions
- "total" properties → integrated over all wavelengths
- "total, hemispherical" properties → integrated over all wavelengths and directions

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Here are some of the more common simplifications to radiative flux quantities.

The intent here is to simply state the adjectives typically used to describe the different ways to simplify the spectral/directional radiative quantities.

We will often consider radiative quantities/properties for which we have integrated either the spectral or directional dependence (or both).

Fundamental Concepts for Emissive Power

"Emissive power" – amount of radiation ***emitted*** per unit of surface area per unit wavelength $\left(W/m^2\mu m\right)$

- Spectral, hemispherical emissive power

$$E_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\,\sin\theta\,d\theta\,d\phi$$

- Total, hemispherical emissive power

$$E = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\,\sin\theta\,d\theta\,d\phi\,d\lambda$$

- For a diffuse emitter (independent of direction)

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda) \quad \text{and} \quad E = \pi I_e$$

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Emissive power is determined by integrating the intensity of the radiative energy emitted from the surface.

Spectral, hemispherical – integrate the directional dependence

Total, hemispherical – also integrate over the wavelength

Fundamental Concepts for Irradiation

"Irradiation" – amount of radiation ***incident*** per unit of surface area per unit wavelength $\left(W/m^2 \mu m\right)$

- Spectral, hemispherical irradiation

$$G_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta \sin\theta \, d\theta \, d\phi$$

- Total, hemispherical irradiation

$$G = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

- For a diffuse radiation (independent of direction)

$G_\lambda(\lambda) = \pi I_{\lambda,i}(\lambda)$ and $G = \pi I_i$

(in some textbooks, irradiation is denoted as q_i)

Irradiation is determined by integrating the intensity of the radiative energy incident on the surface.

Fundamental Concepts for Radiosity

"Radiosity" – all radiation ***leaving a surface*** per unit of surface area per unit wavelength $\left(W/m^2\mu m\right)$

- Spectral, hemispherical irradiation

$$J_\lambda\left(\lambda\right)=\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}\left(\lambda,\theta,\phi\right)\cos\theta\,\sin\theta\;d\theta\;d\phi$$

- Total, hemispherical irradiation

$$J=\int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}\left(\lambda,\theta,\phi\right)\cos\theta\,\sin\theta\;d\theta\;d\phi\;d\lambda$$

- For a diffuse radiation (independent of direction)

$J_\lambda\left(\lambda\right)=\pi I_{\lambda,e+r}\left(\lambda\right)$ and $J=\pi I_{e+r}$

(in some textbooks, radiosity is denoted as q_o)

Radiosity includes both energy emitted by the surface and incident energy that is reflected from the surface, hence (e+r) subscript on intensity

How is Radiative Intensity Determined?

Now that we have emissive power, irradiation, and radiosity in terms of radiative intensity, how do we compute radiative intensity?

Consider an ideal surface known as a "blackbody"

- absorbs all incident radiation (all wavelengths and directions)
- surface emits maximum energy for a given wavelength and temperature
- surface emits energy diffusely (independent of direction)

A "blackbody cavity" is conceptually a large cavity in which the radiative environment is diffuse. In this environment, energy is emitted and reflected multiple times so that the emission is equal to that of a blackbody at the cavity temperature.

We will show that "blackbody" emission is the maximum emission that a surface can produce. It is an ideal case.

The Planck Distribution – Spectral Distribution of Blackbody Emission

Planck distribution – intensity (wavelength, temperature)

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_o^2}{\lambda^5\left[\exp\left(hc_o / k\lambda T\right) - 1\right]}$$

$h = 6.6256 \times 10^{-34}\ J \bullet s$	Planck constant
$k = 1.3805 \times 10^{-23}\ J/K$	Boltzmann constant
$c_o = 2.998 \times 10^{8}\ m/s$	speed of light (vacuum)

Emissive Power is a function of both wavelength and temperature.

We will examine the functional relationship in more detail in the following discussion

The Planck Distribution –
Spectral Distribution of Blackbody Emission (2)

Integrating over direction, the emissive power is then

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}$$

in terms of the “first” and “second” radiation constants

$$C_1 = 2\pi h c_o^2 = 3.742\times10^8 \quad W\bullet\mu m^4/m^2$$

$$C_2 = {}^{hc_o}\!/_{k} = 1.439\times10^4 \quad \mu m\bullet K$$

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Emissive Power is obtained by integrating Planck’s distribution and the result is a function of both wavelength and temperature.

We will use this equation in a number of ways for both blackbodies and real surfaces.

The Planck Distribution –
Spectral Distribution of Blackbody Emission (3)

So, where is the maximum intensity?

Differentiating wrt wavelength.….

$$\frac{\partial}{\partial \lambda}\left(E_{\lambda,b}\left(\lambda,T\right)\right) = \frac{\partial}{\partial \lambda}\left(\frac{C_1}{\lambda^5\left[\exp\left(C_2/\lambda T\right)-1\right]}\right) = 0$$

yields a relationship of

$\lambda_{\max} T = C_3$ Wien's Displacement Law

where

$C_3 = 2897.8\ \mu m \bullet K$ is the "third" radiation constant

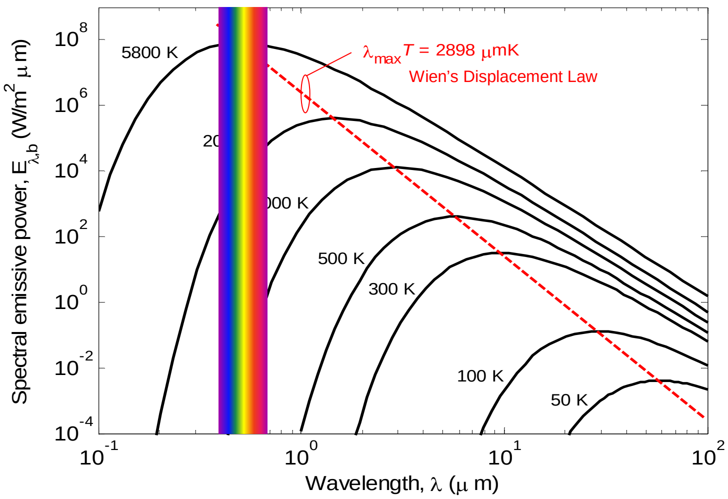
Wien's displacement law gives the relationship between the temperature of an emitting surface and the wavelength of maximum emission. Temperature and maximum wavelength are inversely proportional. C_3 is a function of C_1 and C_2

We will take a look at a graphical representation of Planck's distribution and Wien's displacement law

The Planck Distribution –
Spectral Distribution of Blackbody Emission (4)

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T)$$

Planck Distribution



Emissive Power is a function of both wavelength and temperature is shown by Planck’s law.

As temperature increases, the smaller wavelengths make up a greater fraction of the emissive power.

5800K is representative of solar radiation.

Clearly, lot’s of thermal energy is outside the visible spectrum.

Fundamental Concepts - the “Stefan-Boltzmann” Law

Integrating over all wavelengths

$$E_b(T) = \int_0^\infty E_{\lambda,b}(\lambda,T)\,d\lambda = \int_0^\infty \frac{C_1}{\lambda^5\left[\exp\left(C_2/\lambda T\right)-1\right]}\,d\lambda$$

yields the blackbody emission

$$E_b(T) = \sigma T^4$$

where

$$\sigma = 5.670\times10^{8} \;\; W/m^2 \bullet K^4$$

is the Stefan-Boltzmann constant

The “Stefan-Boltzmann” equation. We will use this result extensively.

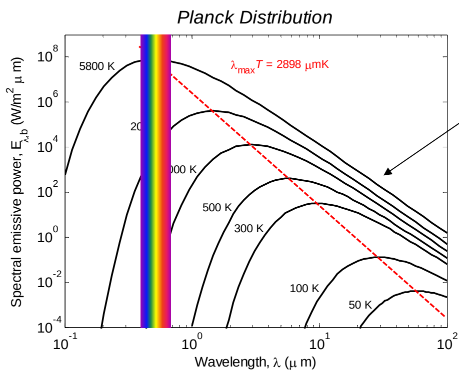
For a blackbody, emission is in all directions, over all wavelengths and is a function of temperature only.

Fundamental Concepts - the “Stefan-Boltzmann” Law (2)

Integrating over all wavelengths

$$E_b(T) = \int_0^\infty E_{\lambda,b}(\lambda, T)\, d\lambda$$

$$E_b(T) = \sigma T^4$$



amounts to determining the “area under the emissive power curve”

For a blackbody, emission is in all directions, over all wavelengths and is a function of temperature only.

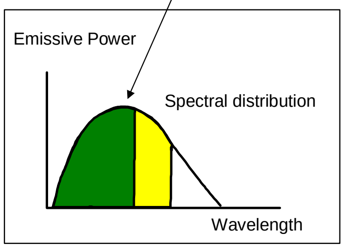
For a given temperature, integrating yields the area under the emissive power curve and is the total hemispherical emission

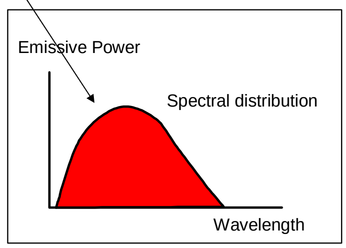
Fundamental Concepts - the “Blackbody Fraction” Concept

How much energy is in a wavelength interval?

We use a “ Blackbody Fraction” concept, defined as

$$F_{(0-\lambda)} \equiv \frac{\int_0^{\lambda} E_{\lambda,b}(\lambda,T)\,d\lambda}{\int_0^{\infty} E_{\lambda,b}(\lambda,T)\,d\lambda} = \frac{\int_0^{\lambda} E_{\lambda,b}(\lambda,T)\,d\lambda}{\sigma T^4}$$





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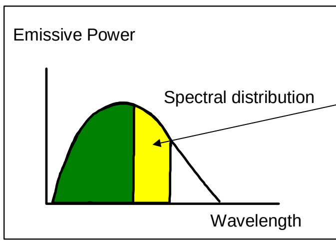
The “blackbody fraction” is useful for determining the emission in all directions over a wavelength interval.   Again, it is a function of temperature only for a blackbody.

For a given temperature, integrating over all wavelengths yields the area under the emissive power curve and is the total hemispherical emission.

Fundamental Concepts - the “Blackbody Fraction” Concept

“Blackbody Fractions” are often tabulated or approximated using functions

$$F_{(\lambda_1-\lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda,b}(\lambda,T)\,d\lambda - \int_0^{\lambda_1} E_{\lambda,b}(\lambda,T)\,d\lambda}{\sigma T^4}$$



$$F_{(\lambda_1-\lambda_2)} = F_{(0-\lambda_2)} - F_{(0-\lambda_1)}$$

$$E_{(\lambda_1-\lambda_2),b} = \left(F_{(0-\lambda_2)} - F_{(0-\lambda_1)}\right)\sigma T^4$$

ESP300: Radiative Heat Transfer

Blackbody fractions are often approximated using an equation or presented as tables. For these tables/equations, the dependence is often presented in terms of the product of wavelength and temperature.

Howell’s book provides an equation that is useful for numerical computations.

Emissivity of Real Surfaces

- Real surfaces behave differently than blackbodies

$$\varepsilon_\lambda(\lambda,T) = \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)}$$

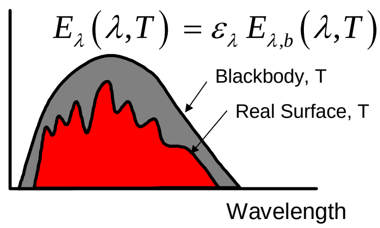
spectral, hemispherical emissivity

$$\varepsilon(T) = \frac{E(T)}{E_b(T)}$$

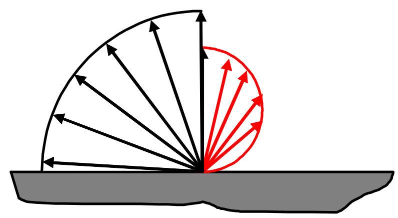
total, hemispherical emissivity

Spectral Emission

$$E_\lambda(\lambda,T) = \varepsilon_\lambda E_{\lambda,b}(\lambda,T)$$



$$I_\lambda(\lambda,\theta,T) = \varepsilon_{\lambda,\theta} I_{\lambda,b}(\lambda,T)$$



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Emissivity is the ratio of actual emission to ideal (blackbody) emission.

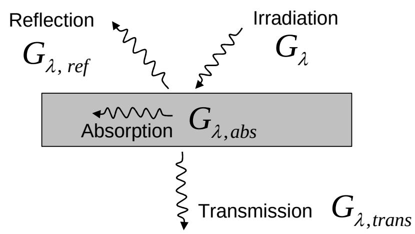
These figures graphically demonstrate the differences between real and ideal surfaces.

Radiative Energy Incident on a Semi-transparent Medium

Radiative processes for a semi-transparent medium:

- reflection (surface)
- absorption (volumetric)
- transmission

$$G_\lambda = G_{\lambda,abs} + G_{\lambda,ref} + G_{\lambda,trans}$$



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Semi-transparent media includes materials like glass, gasses, etc. … materials in which radiative energy is transmitted through the medium and exits the "other" side.

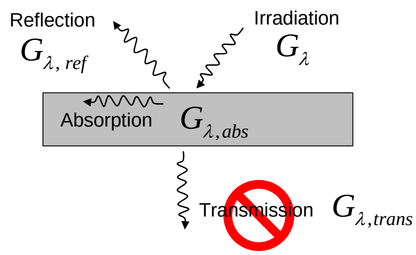
We will not consider semi-transparent materials.
For the radiative transfer that we will consider….. We will be considering the radiative transfer with enclosures with a non-participating (or transparent) medium.

Radiative Energy Incident on an Opaque Medium

Radiative processes for an opaque medium:

- reflection (surface)
- absorption (surface)
- transmission (zero)

$$G_\lambda = G_{\lambda,abs} + G_{\lambda,ref} + \cancelto{0}{G_{\lambda,trans}}$$



For our analyses, we will consider opaque surfaces for which the radiation is essentially absorbed at the surface of the material.

No radiative energy is assumed to penetrate into the volume of the medium.

Radiative Properties for an Opaque Medium

Radiative properties for an opaque medium:

- absorptivity α_λ
- reflectivity ρ_λ
- ~~transmissivity~~ τ_λ → 0

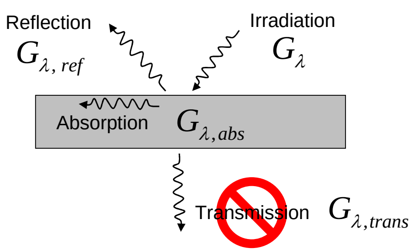
$$G_\lambda = G_{\lambda,abs} + G_{\lambda,ref} + \cancel{G_{\lambda,trans}} \rightarrow 0$$

$$\frac{G_\lambda}{G_\lambda} = \frac{G_{\lambda,abs}}{G_\lambda} + \frac{G_{\lambda,ref}}{G_\lambda} + \frac{\cancel{G_{\lambda,trans}}}{G_\lambda} \rightarrow 0$$

$$1 = \alpha_\lambda + \rho_\lambda + \cancel{\tau_\lambda} \rightarrow 0$$

$$1 = \alpha_\lambda + \rho_\lambda$$

$$1 = \alpha + \rho$$

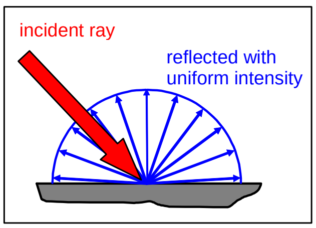


For opaque surfaces, all of the radiative energy is either absorbed or reflected. Next we will look the concept of reflection in more detail.

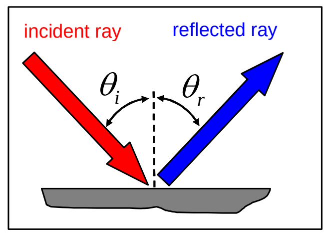
Additional Aspects of Reflectivity – Diffuse and Specular Surfaces

Reflective surface properties depend on both:

- direction of incident energy
- direction of reflected energy



Diffuse reflection



Specular reflection $\theta_i = \theta_r$

Reflection from opaque surfaces is different in that BOTH the direction of incident AND the direction of the reflected energy are important.

Specular reflections are common with polished surfaces, like mirrors.

For all of the analyses we will consider, we will assume surfaces are diffuse reflectors.

Kirchhoff's Law

Void forms a "blackbody cavity" $G = E_b(T_s)$

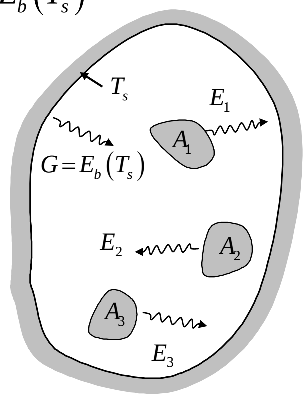
For steady-state $T_1 = T_2 = \cdots = T_s$

Energy balance for "A_1"

$$\alpha_1 G A_1 - E_1(T_s) A_1 = 0$$

or

$$\frac{E_1(T_s)}{\alpha_1} = E_b(T_s)$$



For all the others

$$\frac{E_1(T_s)}{\alpha_1} = \frac{E_2(T_s)}{\alpha_2} = \cdots = E_b(T_s)$$

In a blackbody cavity, the emitted radiative energy is given as a blackbody at the cavity surface temperature.   For this case, the irradiation is given by the blackbody emissive power.

For thermal equilibrium, the temperatures of the small masses are equal to the temperature of the cavity surfaces.

So, what's important here?

Kirchhoff's Law (2)

$$\frac{E_1(T_s)}{\alpha_1} = \frac{E_2(T_s)}{\alpha_2} = \cdots = E_b(T_s)$$

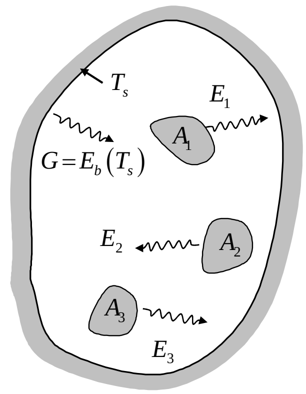
since $0 \le \alpha \le 1$ $\quad E(T_s) \le E_b(T_s)$

Using $\varepsilon_1 = \dfrac{E_1(T_s)}{E_b(T_s)}$

then $\dfrac{\varepsilon_1}{\alpha_1} = \dfrac{\varepsilon_2}{\alpha_2} = \cdots = 1$

So, for any surface in the enclosure

$$\varepsilon = \alpha$$



Two important observations:

1. No real surface will have an emissive power exceeding that of a blackbody at the same temperature (blackbody is ideal and maximum)
2. For surfaces in the enclosure, total hemispherical emissivity is equal to the total hemispherical absorptivity

The "Diffuse-Gray" Surface

For our purposes, a surface is termed a "diffuse-gray" surface if the

- Directional emissivity and absorptivity are independent of direction (diffuse)
- Spectral emissivity and absorptivity are independent of wavelength (gray)

"Diffuse-gray" surfaces are an important assumption used in developing radiative equations for enclosures

There are some more detailed definitions of gray surfaces, but for our purposes, we will use the above.

Whew!! What a mess!
Why would we do this intentionally?

Some was intended to just be an overview (really high-level) or review of radiative heat transfer concepts

Some was needed because these assumptions are used for enclosure radiation

In any case, on we go!

Now, on to formulating the equations for radiative exchange in an enclosure

ESP300: Radiative Heat Transfer

So, that’s an overview of the fundamental concepts for radiative transfer that set us up to consider energy transfer in enclosures.

Radiative Heat Transfer Between Surfaces (within Enclosures)

Radiative HT between surfaces depends on:

- radiative properties of surfaces
- geometric relationship between surfaces
- the medium between surfaces (nonparticipating in our case)

We will discuss:

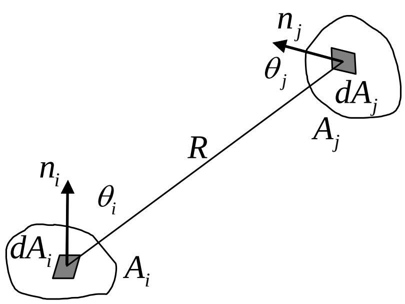
- radiation “view factors”
- conservation of radiative energy for enclosures
- equations describing radiative transfer in “enclosures”
- coupling with equations for conduction heat transfer

Some general comments on enclosure radiation before we get going and a little peek at what we will cover.

Radiative Heat Transfer Between Surfaces Depends on Geometry

Compute energy leaving surface *"i"* that arrives at surface *"j"*

$$dQ_{i \to j} = I_i \, \frac{\cos\theta_i \, \cos\theta_j}{R^2} \, dA_i \, dA_j$$



For a diffusely emitting and reflecting surface *"i"*

$$dQ_{i \to j} = J_i \, \frac{\cos\theta_i \, \cos\theta_j}{\pi R^2} \, dA_i \, dA_j$$

$$Q_{i \to j} = J_i \int_{A_i} \int_{A_j} \frac{\cos\theta_i \, \cos\theta_j}{\pi R^2} \, dA_j \, dA_i$$

function of geometry only!

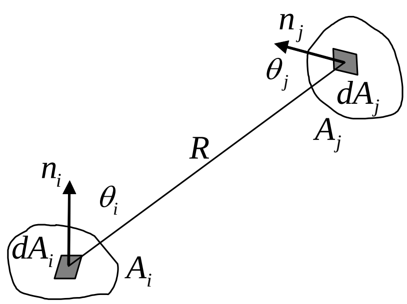
ESP300: Radiative Heat Transfer

Note that the concept of a view factor implies a diffusely emitting and reflecting surface!

Radiative Heat Transfer Between Surfaces Depends on Geometry (2)

"View factors, configuration factors, or shape factors"

$F_{ij} \equiv$ fraction of the radiation leaving surface i which is intercepted by surface j



$$Q_{i \to j} = J_i \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_j \, dA_i$$

$$F_{ij} = \frac{Q_{i \to j}}{A_i J_i}$$

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_j \, dA_i$$

View factors are known by several names…

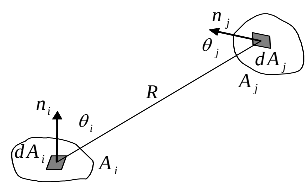
These factors are functions of geometry only.

Methods for Computing View Factors

"View factors, configuration factors, or shape factors" can be computed / obtained from a variety of sources:

- Collections of analytical formulas for some geometries (in form of equations or presented as figures)
- Numerical approximations
  - contour integrations
  - "hemi-cube" approaches
  - graphical approaches (crossed-string)
  - combinations using view factor algebra principles

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2}\, dA_j\, dA_i$$



ESP300: Radiative Heat Transfer

Computation/evaluation of view factors can be done in a number of ways. View factors for particular geometries can be found in many textbooks. Computer codes are also available using a variety of methods to compute view factors for complex geometries, with blocking surfaces, etc. These may be particularly useful for numerical implementation.

One important concept that we will skip is the concept of "view factor algebra." It is not needed for the development being considered here. This concept is useful in determining view factors for "composite" surfaces in terms of the view factors for the simpler, constituent surfaces using algebraic manipulation. Many textbooks will cover this concept in detail. The discussion of this topic is left to the interested person.

View Factor Relationships (1)

Reciprocity relationship

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \, \cos\theta_j}{\pi R^2} \; dA_j \, dA_i$$

$$F_{ji} = \frac{1}{A_j} \int_{A_j} \int_{A_i} \frac{\cos\theta_i \, \cos\theta_j}{\pi R^2} \; dA_i \, dA_j$$

$$A_i F_{ij} = A_j F_{ji}$$

- Reciprocity is important in developing radiative exchange equations for enclosures
- Numerically, it's not always satisfied exactly! So what will we do in those cases?

Useful for computing view factors given one view factor and the areas of the two surfaces.

Will have an important part in the development of radiative transfer equations for enclosures.

When computing view factors numerically, this is one of the "objectives" for smoothing numerical errors in the view factor calculations.

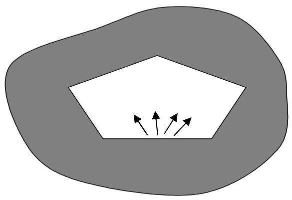
The concept of view factor smoothing will be covered in more detail when we cover the numerical implementation.

View Factor Relationships (2)

Summation relationship

$$Q_{1-1} + Q_{1-2} + Q_{1-3} + Q_{1-4} + Q_{1-5} = Q_{total}$$

$$A_1 J_1 F_{1-1} + A_1 J_1 F_{1-2} + A_1 J_1 F_{1-3} + A_1 J_1 F_{1-4} + A_1 J_1 F_{1-5} = A_1 J_1$$



$$F_{1-1} + F_{1-2} + F_{1-3} + F_{1-4} + F_{1-5} = 1$$

$$\sum_{j=1}^{N} F_{ij} = 1$$

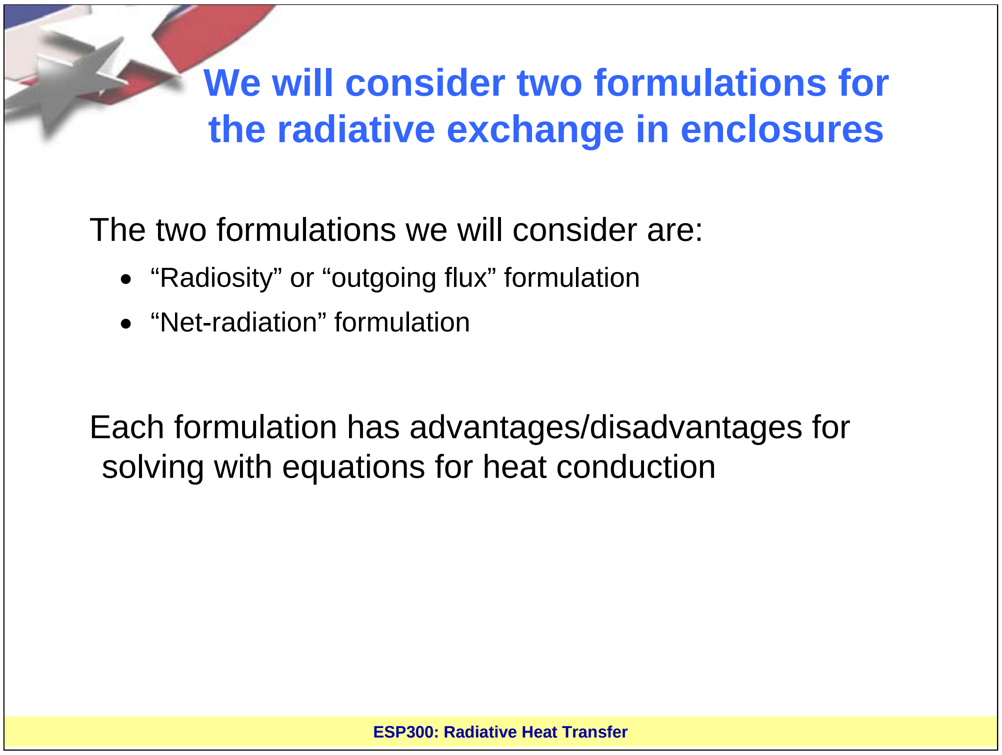
- Equivalent to a statement of "conservation of energy!" As such, it's important!
- Numerically, it's not always satisfied exactly! So what will we do in those cases?

Reciprocity is an important concept because it is a statement of "conservation of radiative energy" for each surface. Collectively, the surfaces make up the enclosure.

Reciprocity will have an important part in the development of radiative transfer equations for enclosures.

When computing view factors numerically, this is one of the "objectives" for smoothing numerical errors in the view factor calculations.

The concept of view factor smoothing will be covered in more detail when we cover the numerical implementation.



Now, on to formulations for the radiative energy transfer in enclosures.

We will defer the detailed discussion of advantages and disadvantages until we cover the numerical implementation of coupled problems.

Radiative Transfer for Enclosures Using the "Radiosity" Formulation

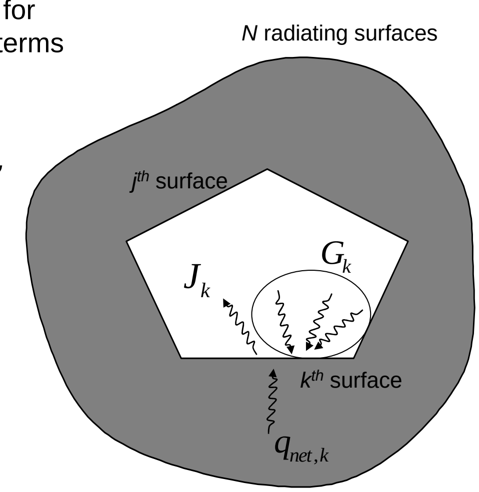
Objective is to develop equations for the radiative energy transfer in terms of radiosity

Begin with the radiosity and net conduction heat flux for the "k^{th}" surface

$$J_k = \varepsilon_k E_{b,k} + \rho_k G_k$$

$$q_{net,k} = J_k - G_k$$

Assumptions:
- Uniform surface temperatures
- Diffuse-gray surfaces
- Uniform surface heat fluxes

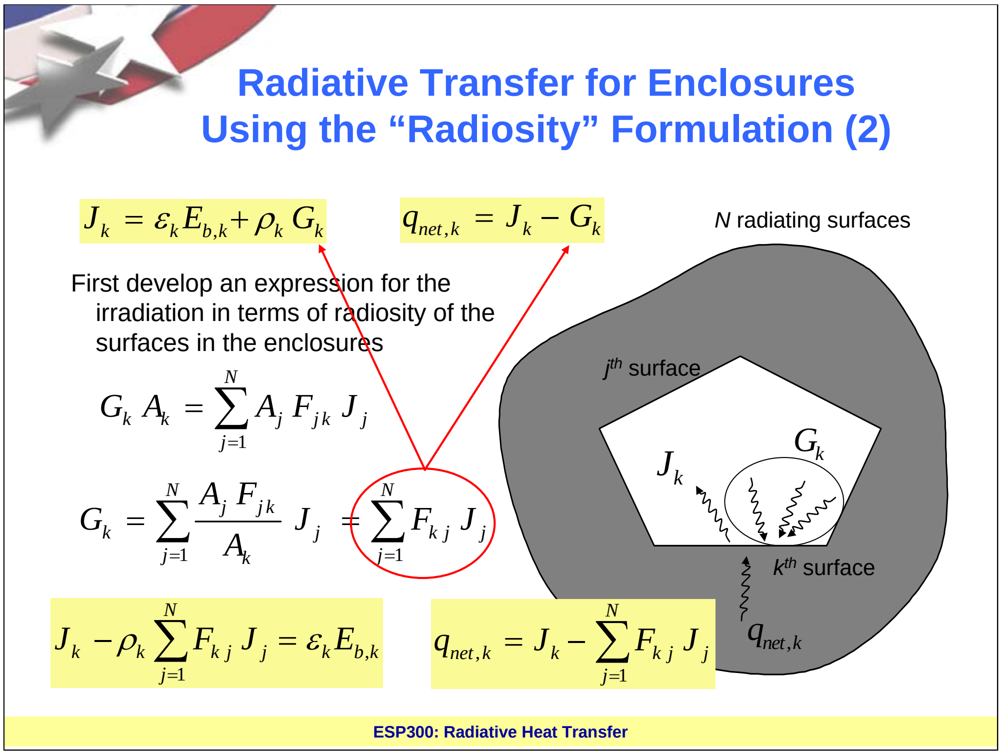


We begin with an energy balance on a typical surface. With those concepts for a single surface, we can apply them to the entire enclosure, simply a collection of all the individual surfaces.

Assumptions:

- Temperatures are uniform over each surface.
- Radiative properties are independent of wavelength and direction.
- All energy is emitted and reflected diffusely.
- Incident and reflected energy flux is uniform over each surface.

Radiosity is the sum of the emitted energy and the reflected energy. The irradiation is the sum of the energy incident from all the surfaces. The net heat conducted to the surface into the solid is the difference between the reflected



Simplifying the expressions using the irradiation and view factor reciprocity.

Radiative Transfer for Enclosures Using the "Radiosity" Formulation (3)

$$J_k - \rho_k \sum_{j=1}^{N} F_{kj} J_j = \varepsilon_k E_{b,k} \qquad q_{net,k} = J_k - \sum_{j=1}^{N} F_{kj} J_j$$

Now, replace the absorptivity with an expression using the surface emissivity

$$\rho_k = (1-\alpha_k) \qquad \text{for opaque surfaces}$$

$$\alpha_k = \varepsilon_k \qquad \text{for diffuse, gray surfaces}$$

then $\rho_k = (1-\varepsilon_k)$

substituting and reordering, we now have

$$\sum_{j=1}^{N} \left(\delta_{kj} - (1-\varepsilon_k) F_{kj} \right) J_j = \varepsilon_k E_{b,k} \qquad \sum_{j=1}^{N} \left(\delta_{kj} - F_{kj} \right) J_j = q_{net,k}$$

Continuing to simplify the equation set in terms of emissivity and radiosity.

Radiative Transfer for Enclosures Using the "Radiosity" Formulation (4)

$$\sum_{j=1}^{N}\left(\delta_{kj}-(1-\varepsilon_k)F_{kj}\right)J_j=\varepsilon_k E_{b,k} \qquad \sum_{j=1}^{N}\left(\delta_{kj}-F_{kj}\right)J_j=q_{net,k}$$

Expressing the emissive power using the Stefan-Boltzmann equation,

we get $E_{b,k}(T)=\sigma T_k^4$

Then, the two equations we will use to compute radiosities are

$$\sum_{j=1}^{N}\left(\delta_{kj}-(1-\varepsilon_k)F_{kj}\right)J_j=\varepsilon_k\sigma T_k^4$$ for surfaces with known temperatures

$$\sum_{j=1}^{N}\left(\delta_{kj}-F_{kj}\right)J_j=q_{net,k}$$ for surfaces with known heat fluxes

We now have two equations for each radiating surface. However, there are three unknowns (radiosities, surface temperatures, and net heat fluxes). We will need to specify one of the three unknowns in order to have a solvable set of equations.

One of these two equations will be used to solve for the radiosities, depending on whether the surface temperature or the net heat flux is specified for that surface. The other will be used to compute the remaining unknown.

Radiative Transfer for Enclosures Using the "Radiosity" Formulation (5)

For an enclosure with *'N'* surfaces,

- *'m'* of which have specified temperatures and
- the remaining *'N-m'* of which have specified net heat fluxes,

we have a set of *'N'* equations given by

for surfaces with known temperatures, we will use

$$\sum_{j=1}^{N}\left(\delta_{kj}-\left(1-\varepsilon_k\right)F_{kj}\right)J_j=\varepsilon_k\sigma T_k^{4} \qquad 1\le k\le m$$

for surfaces with known heat fluxes, we will use

$$\sum_{j=1}^{N}\left(\delta_{kj}-F_{kj}\right)J_j=q_{net,k} \qquad m+1\le k\le N$$

Solving for the radiosities, the remaining unknowns can be computed

We now have two equations for each radiating surface, with three unknowns (radiosities, surface temperatures, and net heat fluxes)

We really have a set of 2N equations, with a total of 3N unknowns (radiosities, surface temperatures, and net heat fluxes)

If we specify a total of N variables (temperatures or net heat flux for each surface), then we have 2N surfaces with a total of 2N unknowns

Of the 2N unknowns, we can form a set of N equations, with only radiosities as unknowns…

Solving this set for the surface radiosities, then the remaining N unknown surface temperatures and net heat fluxes can be determined.

Radiative Transfer for Enclosures Using the "Net-Radiation" Formulation

The equations describing the radiative transfer in an enclosure can be written in terms of the "net heat flux" conducted away from the surface and the surface temperature.

In the "net-radiation" formulation, the set of equations becomes

$$\sum_{j=1}^{N}\left[\frac{\delta_{kj}}{\varepsilon_j} - F_{kj}\left(\frac{1-\varepsilon_j}{\varepsilon_j}\right)\right]\frac{Q_{net,j}}{A_j} = \sum_{j=1}^{N}\left(\delta_{kj} - F_{kj}\right)\sigma T_j^4$$

This form will turn out to be useful when solved in a fully-coupled manner with the conduction equations for the enclosure surfaces

In this case, we now have a set of N equations, with 2N unknowns (surface temperatures and net heat fluxes).

Knowing N of these, we can solve the remaining N unknowns.

For example, given surface temperatures for all the surfaces, then the net heat flux can be computed.

Consider an example with three surfaces defining the enclosure

$$\sum_{j=1}^{N}\left(\delta_{kj}-(1-\varepsilon_k)F_{kj}\right)J_j=\varepsilon_k\sigma T_k^4 \qquad \sum_{j=1}^{N}\left(\delta_{kj}-F_{kj}\right)J_j=q_{net,k}$$

$$\left(1-(1-\varepsilon_1)F_{1-1}\right)J_1-(1-\varepsilon_1)F_{1-2}J_2-(1-\varepsilon_1)F_{1-3}J_3=\varepsilon_1\sigma T_1^4$$

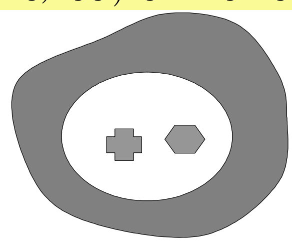
$$-(1-\varepsilon_2)F_{2-1}J_1+\left(1-(1-\varepsilon_2)F_{2-2}\right)J_2-(1-\varepsilon_2)F_{2-3}J_3=\varepsilon_2\sigma T_2^4$$

$$-(1-\varepsilon_3)F_{3-1}J_1-(1-\varepsilon_3)F_{3-2}J_2+\left(1-(1-\varepsilon_3)F_{3-3}\right)J_3=\varepsilon_3\sigma T_3^4$$

$$\left(1-F_{1-1}\right)J_1-F_{1-2}J_2-F_{1-3}J_3=q_{net,1}$$

$$-F_{2-1}J_1+\left(1-F_{2-2}\right)J_2-F_{2-3}J_3=q_{net,2}$$

$$-F_{3-1}J_1-F_{3-2}J_2+\left(1-F_{3-3}\right)J_3=q_{net,3}$$



In this three-surface enclosure, we have six equations, with nine unknowns (3 each radiosity, temperature, and net heat flux). By specifying either the surface temperature or net heat flux for each surface, you reduce the set to six equations and six unknowns. You can compute the radiosities from the equations in which you specified a temperature or heat flux. Finally, you can compute the remaining unknowns using the previously computed radiosities. Whew!

Coupling with Heat Conduction Equations via FEM

To couple with the FEM equations for conduction, we will have to consider (in the future), the following

- which radiative transfer formulation to use - the details of the implementation
- implementing surface fluxes with the conduction solution
- converting/transforming between “surface” temperatures and “nodal” temperatures

For now, we’ll leave it at that!

We will discuss the details of coupling with the heat conduction equations for a later class.

We've "covered" a bunch of radiative concepts – anything ring a bell?

Thermal radiation concepts:

- Emissive power, irradiation, radiosity, radiative intensity
- Spectral, hemispherical, total properties
- "Blackbody," Planck distribution, Stefan-Boltzmann, blackbody fraction
- Real surfaces – semitransparent, opaque, emissivity, absorptivity, reflectivity (specular and diffuse)

We really didn't dig deeply into the broad topic of radiative heat transfer. It can easily take a semester to cover that topic, with participating media radiation, etc. My objective here was to provide a really high-level overview, with some detail on the formulation of the enclosure radiation equations that we use with the FEM for heat transfer analyses.

We've "covered" a bunch of radiative concepts – anything ring a bell?

For radiative transfer in enclosures:

- View factors
  - calculating them,
  - reciprocity relationship,
  - sum to unity relationship
- "Radiosity" formulation
- "Net-radiation" formulation
- Discussed a simple problem
- Considerations for coupling with heat conduction