

# Overview of Fracture Simulation Methods



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Sandia National Laboratories

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for the United States Department of Energy's National Nuclear Security Administration  
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# Outline



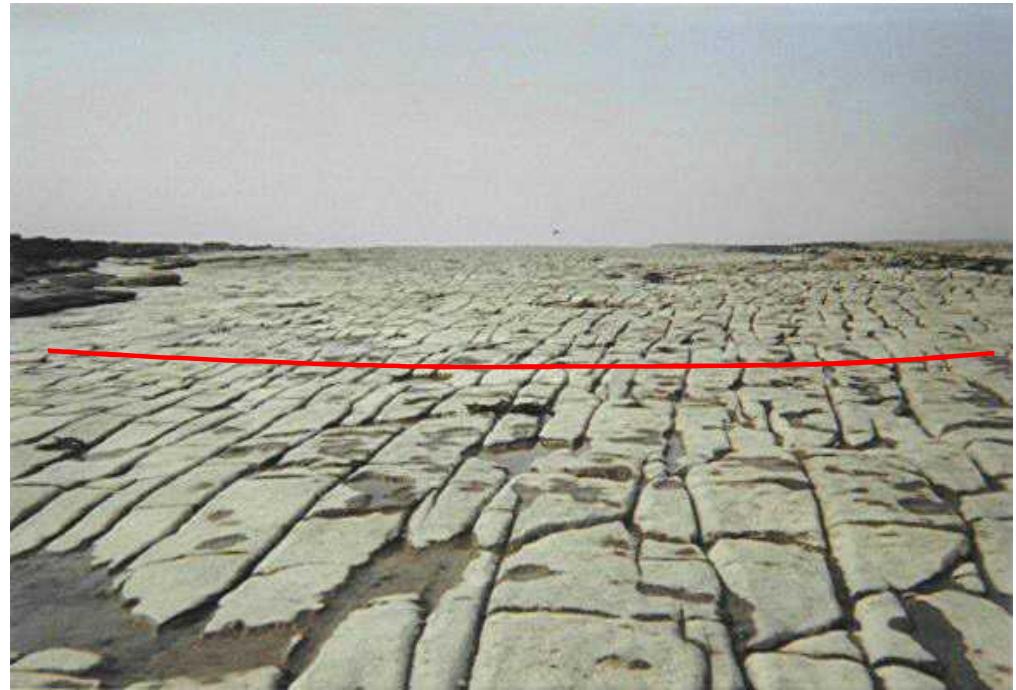
- Basics (Defining Fractures)
  - Density
  - Length
  - Orientation
  - Aperture & Transmissivity
- Discrete Fracture Models
  - Different approaches for fracture locations
- Pixel-Based
  - Fracnet
  - FCM

# How Do Rocks Break?

- Multiple processes and stress fields lead to the final fracture pattern that we can observe

Scanline: measurement of fracture locations, or distances between locations, on a 1-D line perpendicular to the orientation of the fractures

Examine a few end-member fracturing mechanisms to understand spacing distributions



# Fracture Measurements

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- Intensity,  $\lambda$ , fractures per length (1/L in 1-D)

$$\lambda = \frac{N}{L}$$

- In 2-D, length/area and in 3-D, area/volume
- Spacing,  $S$ , length between fractures (L in 1-D)

$$\bar{S} = \frac{1}{N} \sum_{i=1}^N S_i = \frac{L}{N} = \frac{1}{\lambda}$$

# Random Breakage

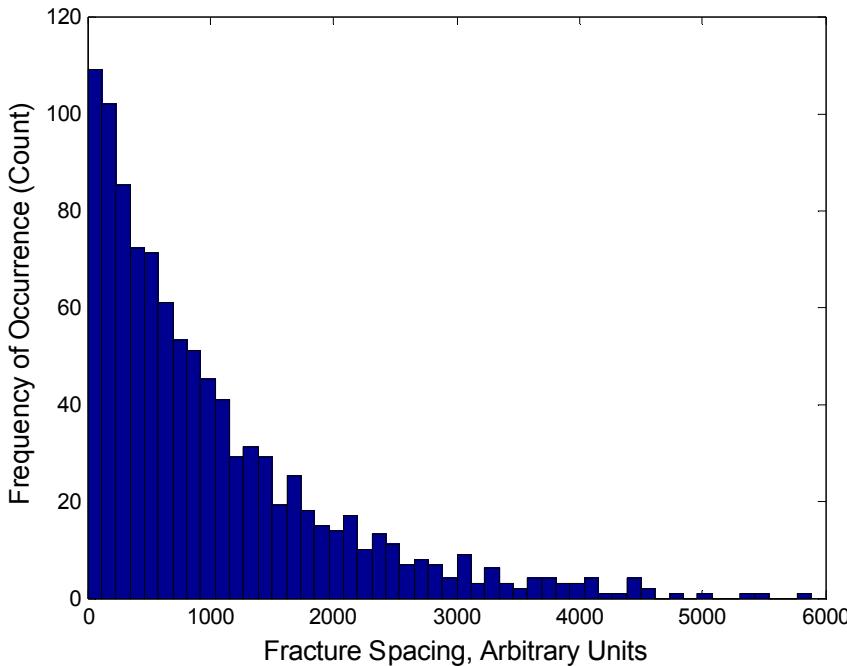


- Fracture locations are random over a distance of rock
- Could occur due to uniform stress applied to a rock with randomly located pre-existing flaws
- Fractures are the result of a Poisson process
  - What does this say about fracture spacing?

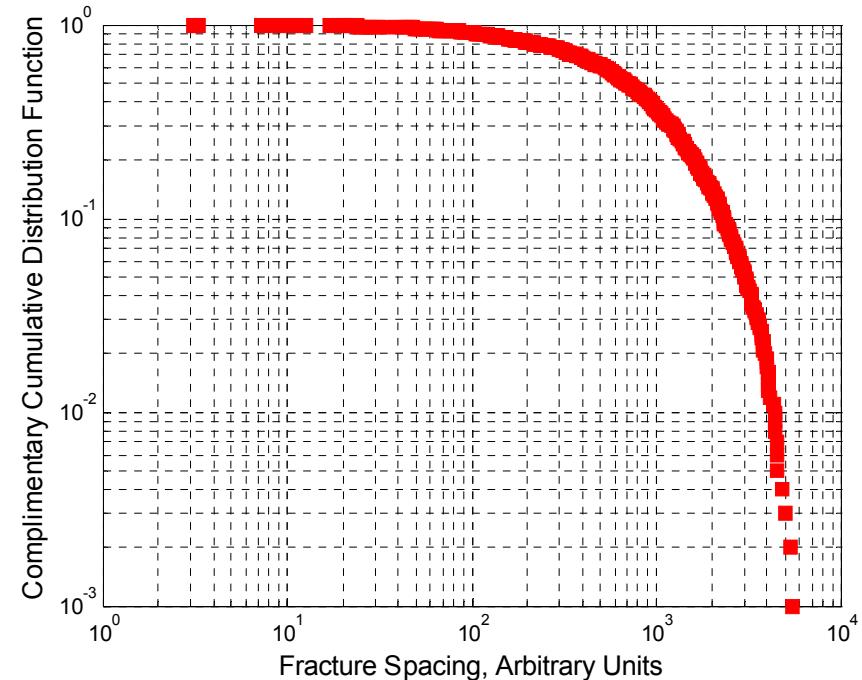
# Random Breakage

Random locations of breaks (Poisson process) leads to exponential distribution of spacing between fractures

Histogram

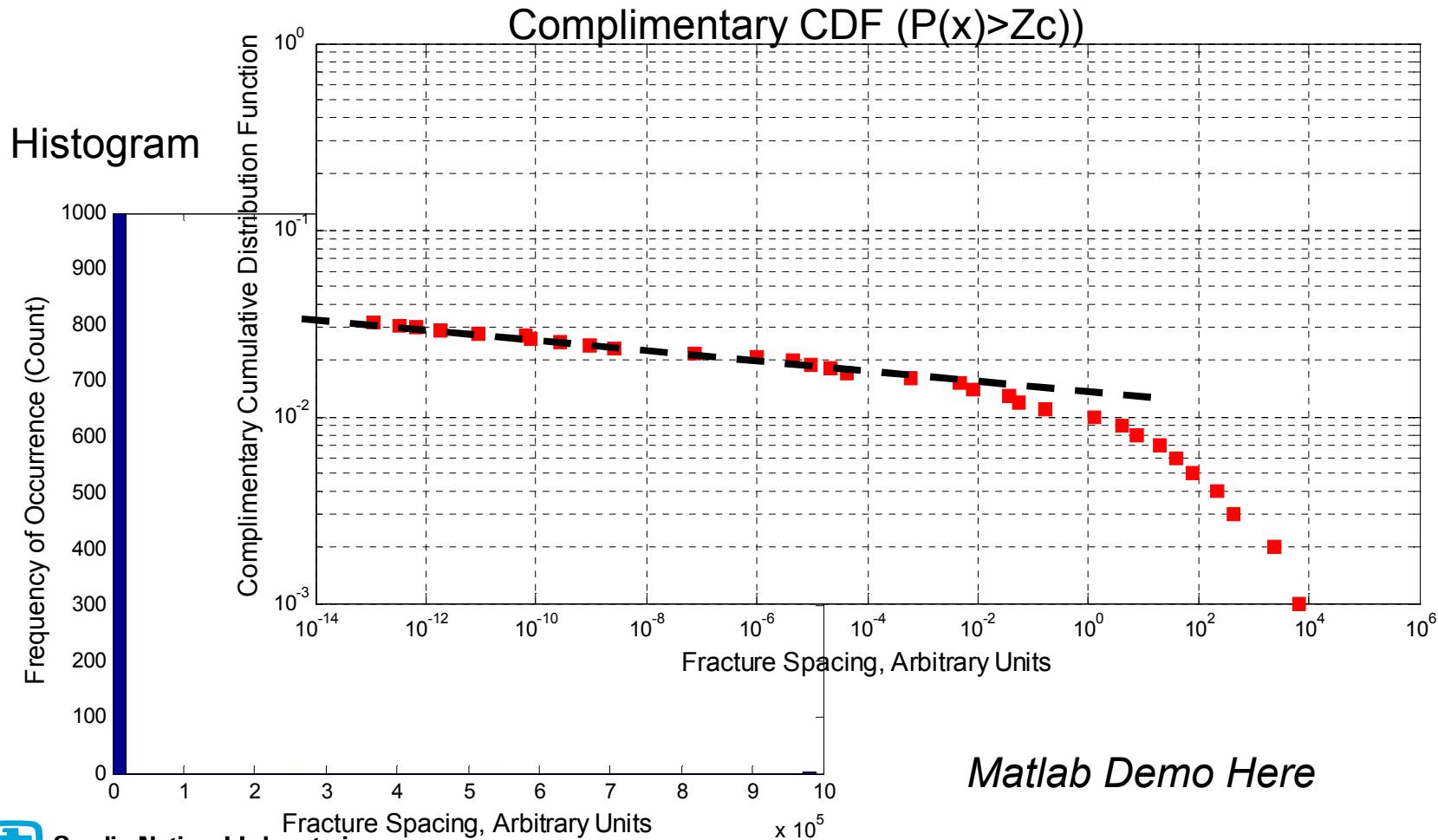


Complimentary CDF ( $P(x) > Z_c$ )



# Non-Random Breakage

Preferential breaking of smallest piece leads to a power-law distribution of fracture spacing. Power-law distribution has a straight line in log-log space



# Power-Law Relationship



$$Y = \alpha X^\beta$$

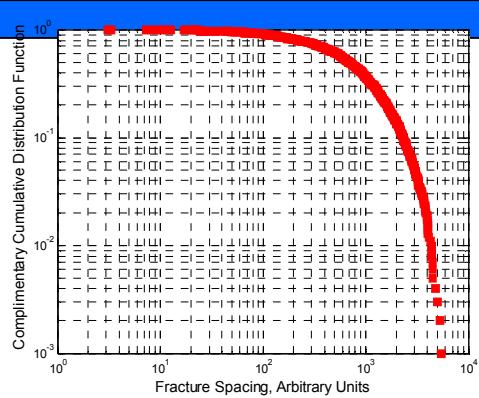
$$\log 10 Y = \beta_0 + \beta_1 \log 10 X$$

$$\alpha = 10^{\beta_0} \quad \beta = \beta_1$$

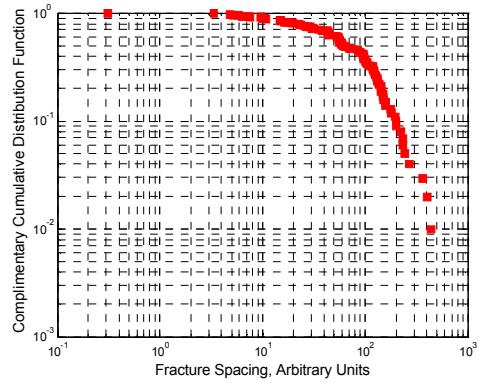
Slope, in log-log space, is the fractal dimension ( $\beta = D$ )



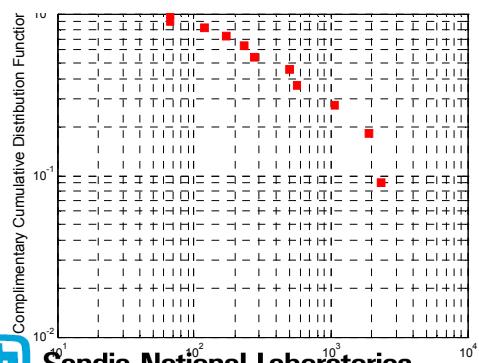
# Limited Sampling: Spacing



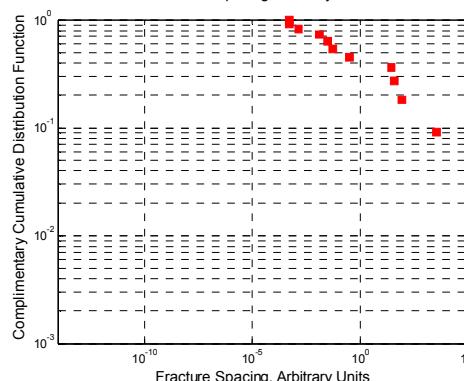
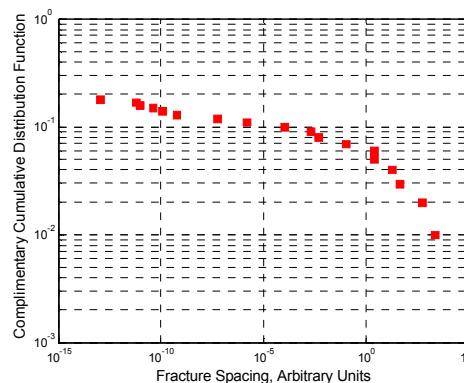
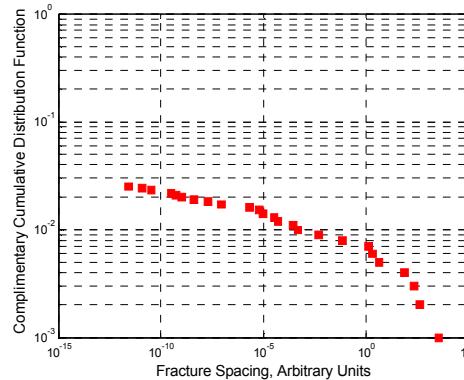
$N = 1000$



$N = 100$



$N = 10$



# Fracture Spacing



- We just covered several things
  - Measured fracture spacing (scanline, borehole, etc) will be exponential if fractures are randomly located
  - Spacing will have a power-law distribution if fractures preferentially break the smallest intact piece
  - Inferring statistical distributions from limited data is a risky approach

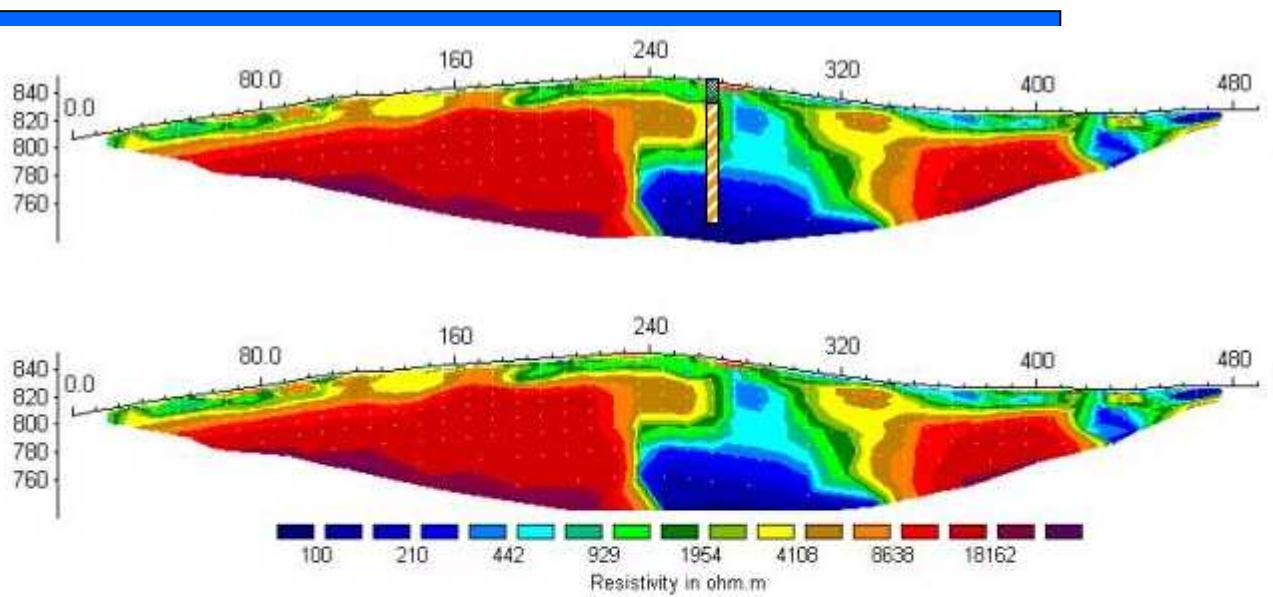
# Intensity in 2-D



- Homogeneous Poisson Process (HPP) and Non-Homogeneous Poisson Process (NHPP) as models for fracture locations



# Intensity Data: Resistivity Profiling

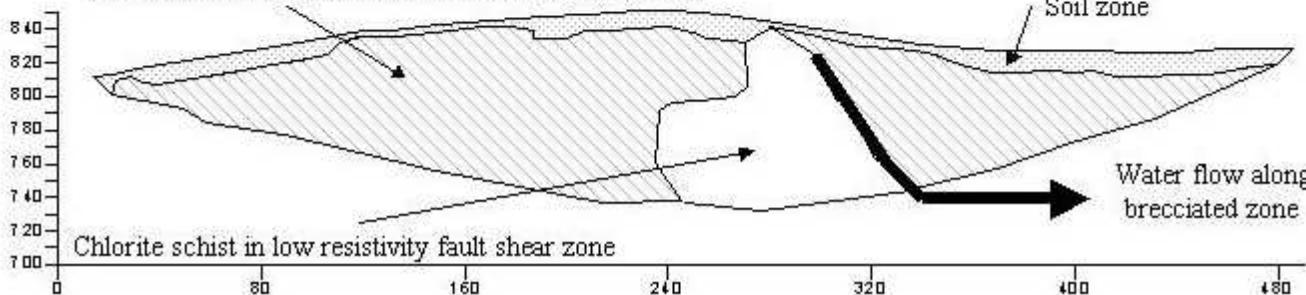


From: Thomas  
Burbey, Virginia Tech  
University

Granite gneiss in high resistivity massive bedrock zone

Soil zone

Water flow along  
brecciated zone



Two-dimensional surface resistivity profiles collected using a variety of array techniques combined with borehole geophysical logs revealed anomalous low resistivity areas in crystalline bedrock associated with fault zones.



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# Length



- Measuring fracture length
  - Have to derive from outcrop data
  - Has anyone here ever seen both ends of a significant fracture?



# Length Distributions

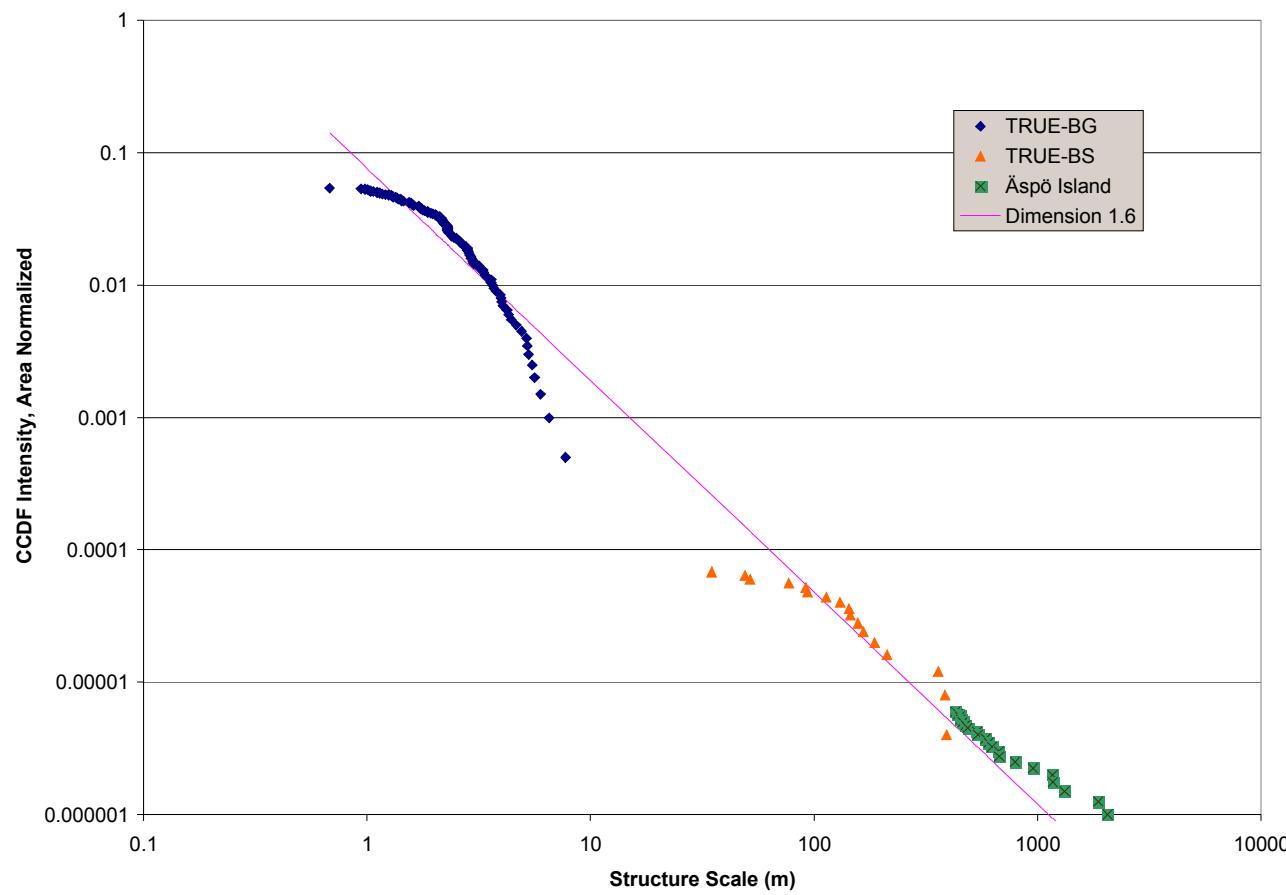


- Exponential
  - Uniformly random growth of all fractures
- Power-Law
  - Preferential growth of long fractures (growth is proportional to current length)
- Log-Normal
  - Products of uniform random numbers produce log-normal distributions



# Length

- Simulated Aspo feature length distribution



Aspo site characterization:

Borehole-Cannister Scale  
(0.1-5m)

Block Scale (200m)

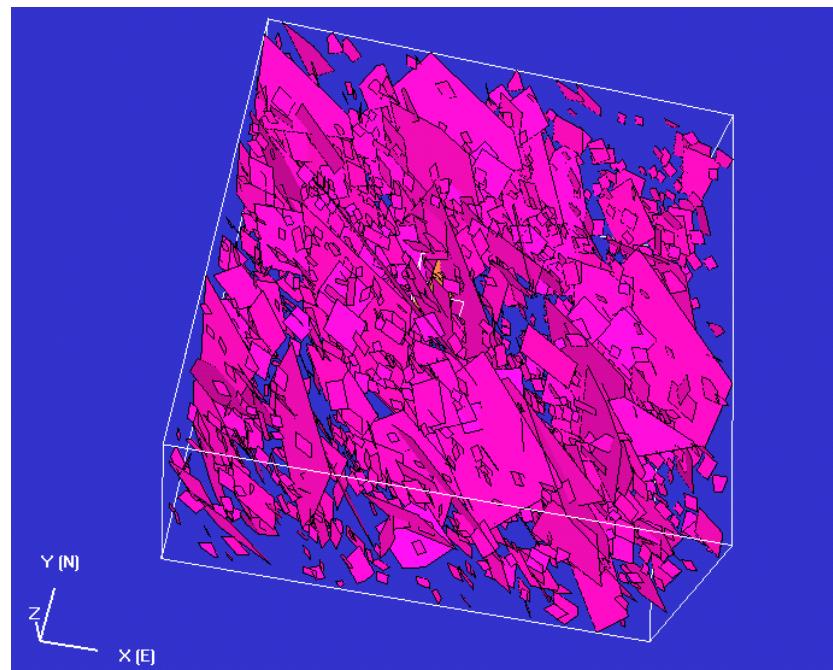
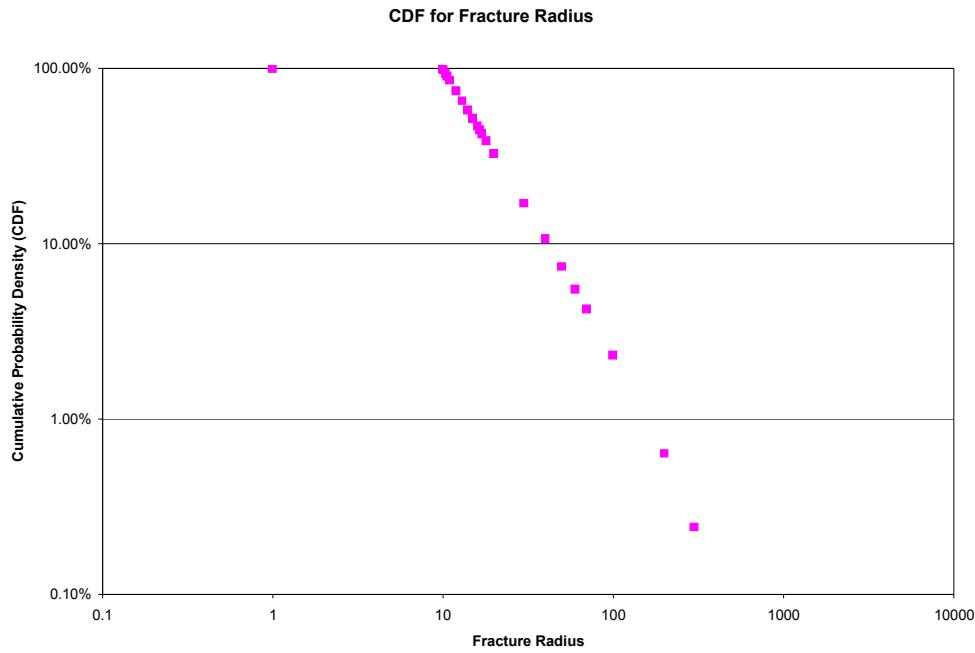
Site Scale (2000m)

Power-Law?

Significant effect of  
censored measurements

# Length/Size

Discrete feature simulation of rectangular objects with a power-law size distribution

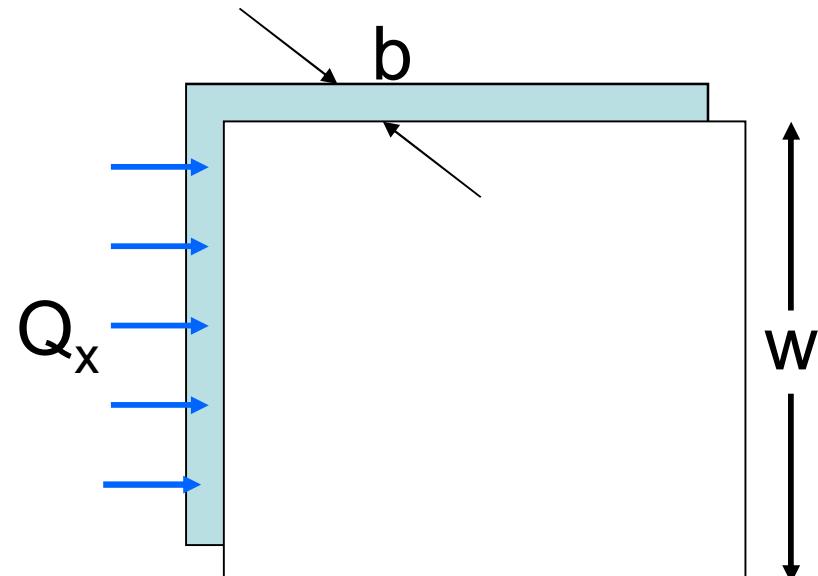


# Aperture and Transmissivity

- Cubic Law
  - Flow  $\propto$  aperture<sup>3</sup>

$$Q = \frac{\rho_w g b^2}{12\mu} (bw) \frac{\partial h}{\partial l}$$

$$K = \frac{\rho_w g b^2}{12\mu}$$



- Snow's Equation (multiple fractures)

$$K = \frac{\rho_w g N b^3}{12\mu} \quad k = \frac{N b^3}{12}$$

# Fracture Measurements



- *“However, in most subsurface cases, there will be insufficient fractures having the size of interest (i.e., large conductive fractures) to derive a statistically significant estimate of spacing”.*

Ortega et al., 2006, AAPG Bulletin



# Stochastic Simulation



- Observational limits force us to use a stochastic approach to fracture modeling to capture significant uncertainty



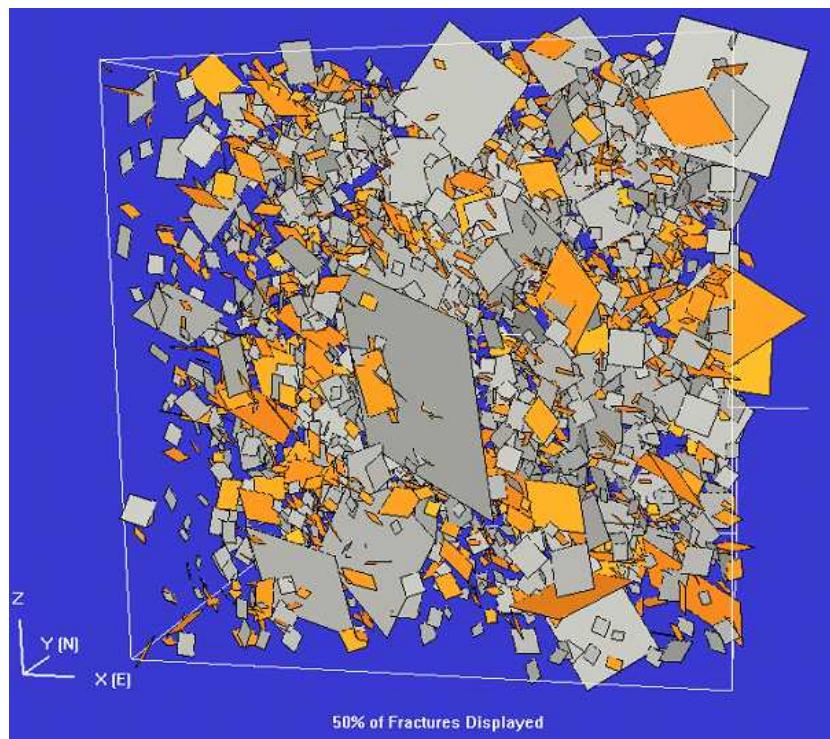
# Discrete Fracture Models



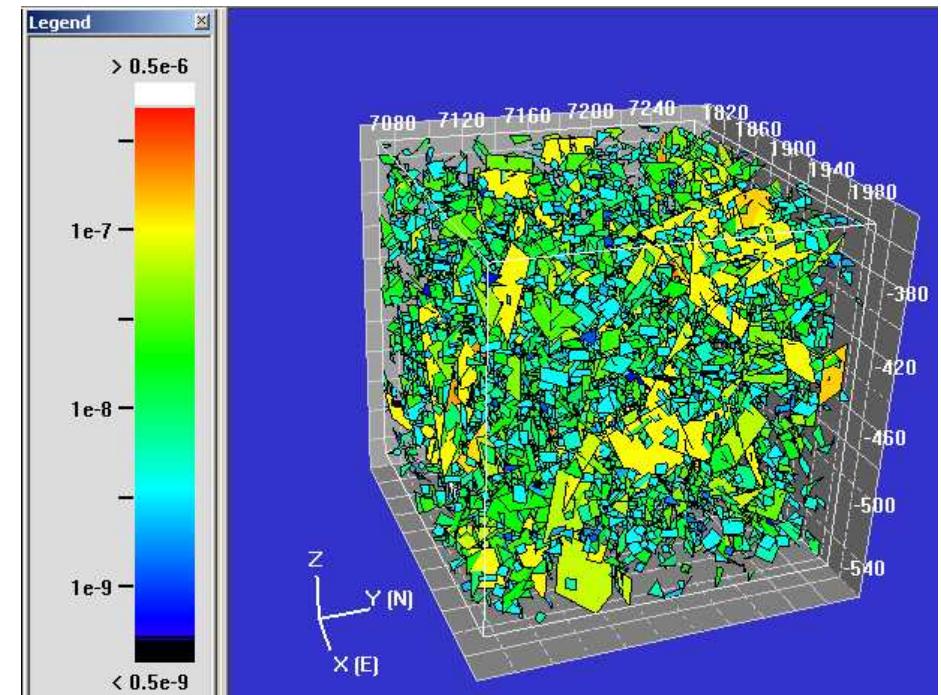
- Statistically-based approach to simulating objects (fractures) that represent observational data base
- Typically objects have a very large (length to aperture) aspect ratio



# DFM Examples: Aspo



Transmissivity Distribution



Dershowitz et al., 2002



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Background fractures coloured by transmissivity (log scale) in 200m cube

# DFM: Drawbacks



- DFM's are completely observationally-based (Statistical models)
  - Do not account for genesis of fractures.
  - DFM can work well if observational database is complete
    - Impossible to determine degree of completeness
    - Observations of length and shape are censored
  - My Experience: Difficult to use in stochastic flow simulation

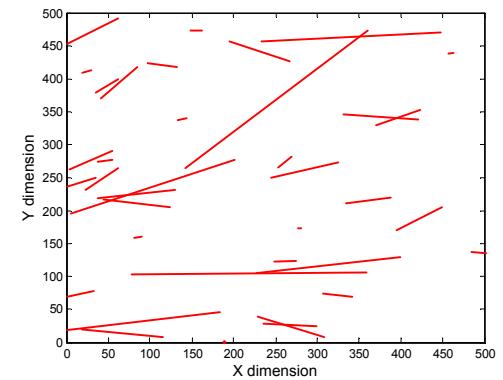
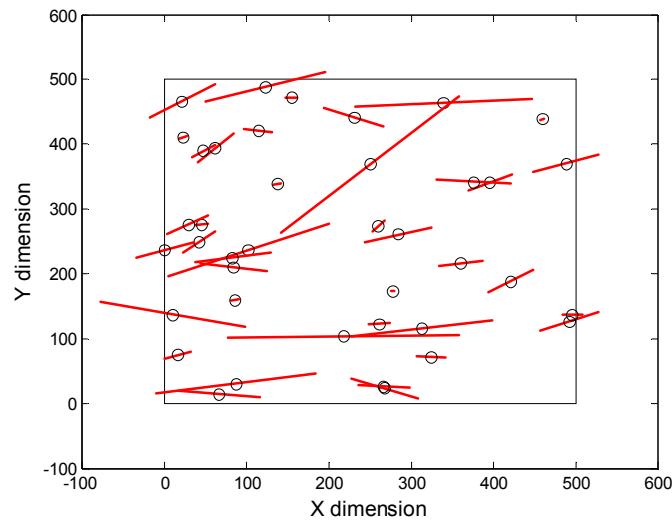
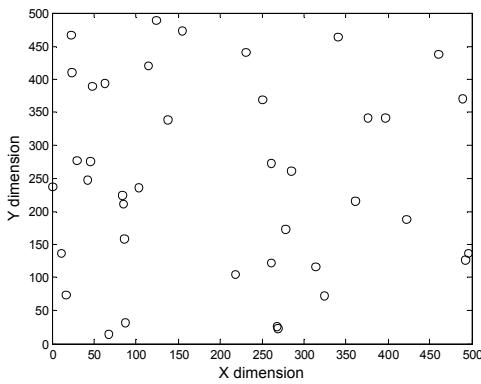
# Cell (Pixel)-Based



- Similar to DFN's in that stochastic simulation is used to place objects (fractures)
- Main difference is that fractures are placed on a grid
  - Fracture simulation will not be mesh independent (minimum support is mesh size)
  - May be easier to incorporate rule-based growth and termination (FracNet)
  - Amenable to dual-permeability and stochastic simulation approaches

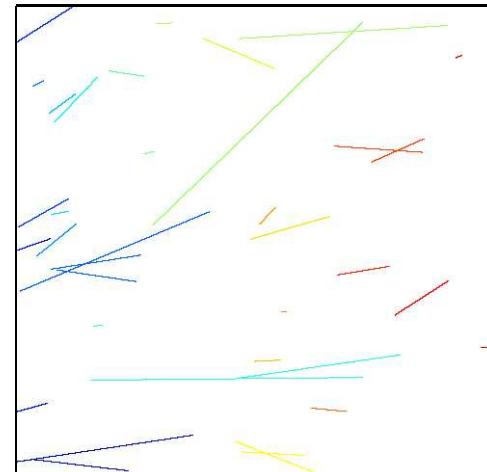
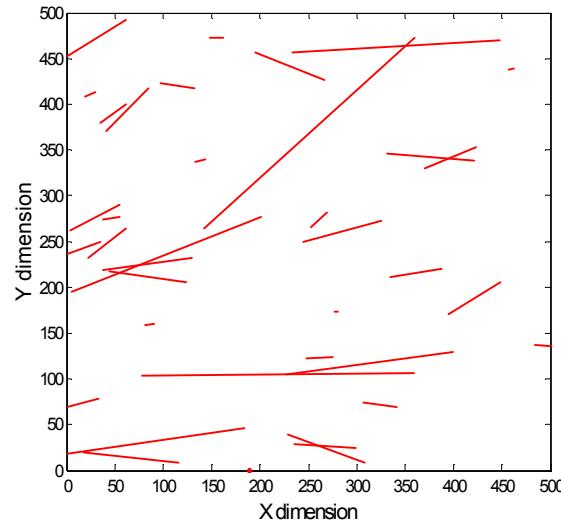
# Cell Based: Simulation Example

40 Fracture centers located randomly in domain (left). Fracture lengths and orientations drawn from exponential (80) and normal (10,20) distributions (center). Fractures then trimmed to domain boundaries (right)



# Fracture Network

Connectedness of fracture network dictates the amount of flow across the domain. Image on left shows locations of 40 individual fractures. Image on right shows 35 distinct clusters of connected fractures.

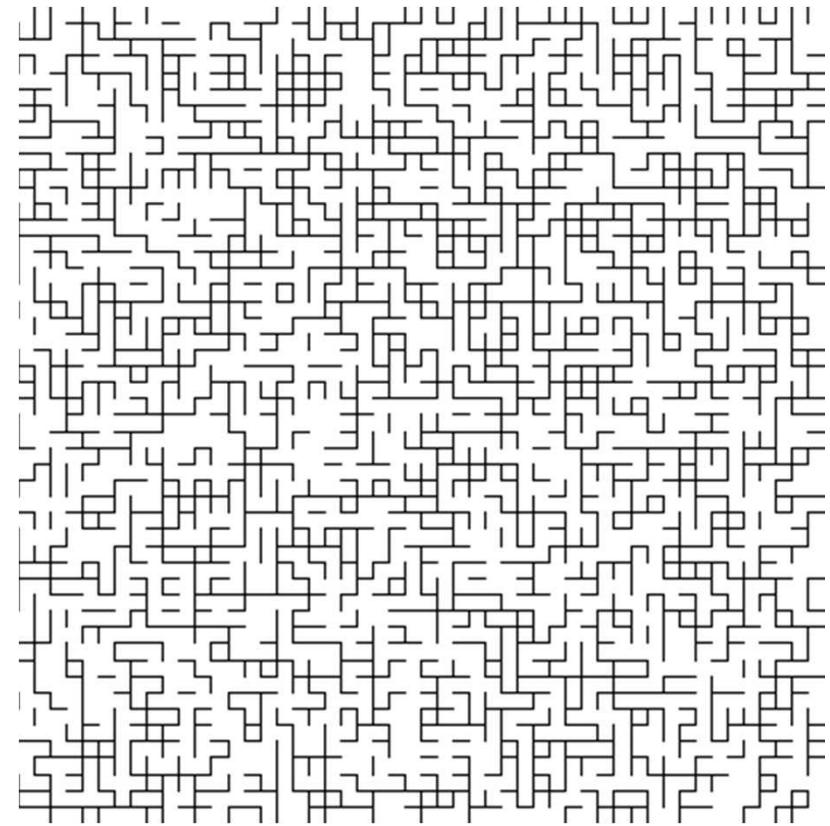


# Percolation: Background

Development in statistical physics

Bond percolation on a square lattice where  $P(\text{connect})$  is 0.51 for any given edge

Percolation threshold is 0.50 for 2-D and approximately 0.249 For 3-D (square lattice with  $z = 4$  and 6, respectively)



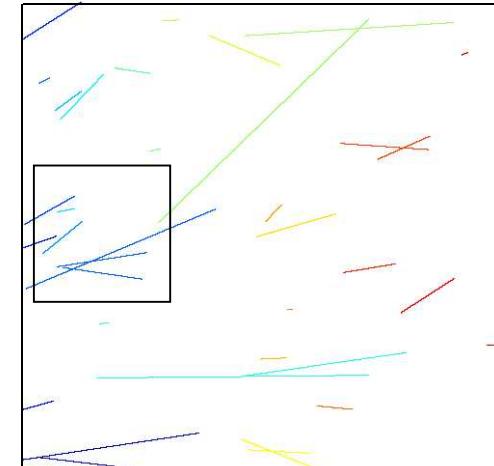
See: Sahimi, M., 1995, *Flow and Transport in Porous Media and Fractured Rock*, VCH, New York, 482 pp.



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# Percolation

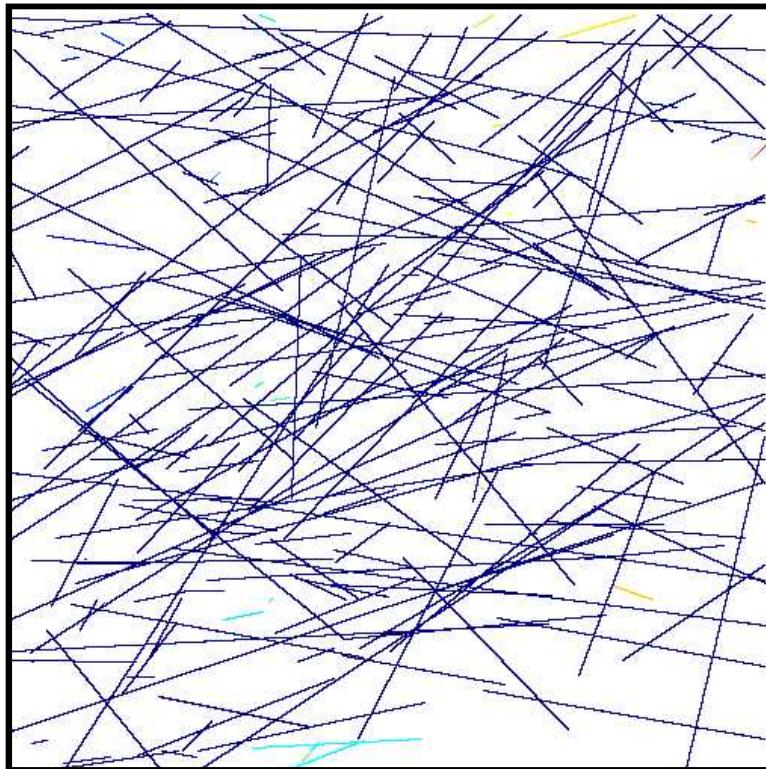
- Critical point at which the fracture network goes from impermeable to permeable
  - Network becomes connected across a volume
- Percolation only has definition within a *specified area/volume*



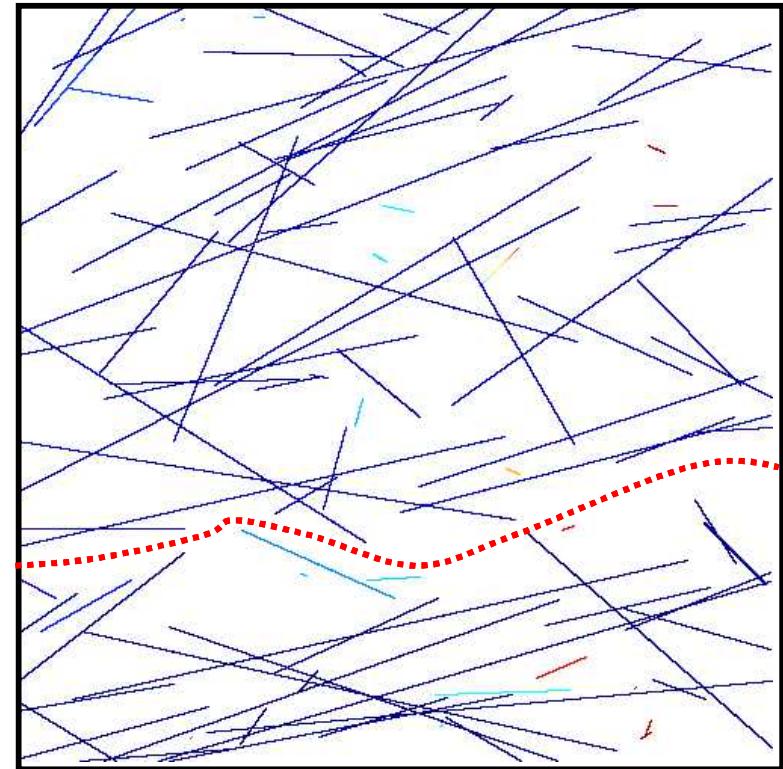
# Percolation and Fractures

Fractures drawn from same length and orientation distributions

240 Fractures



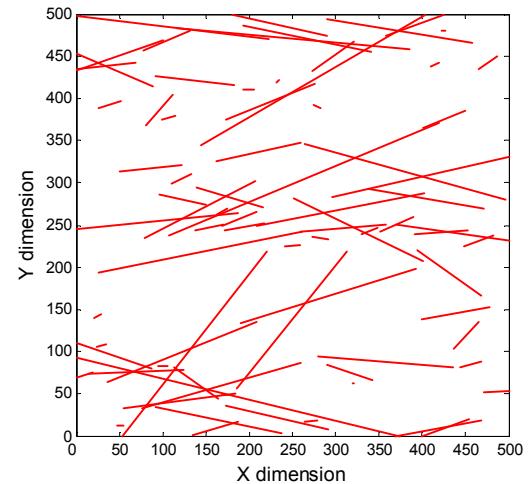
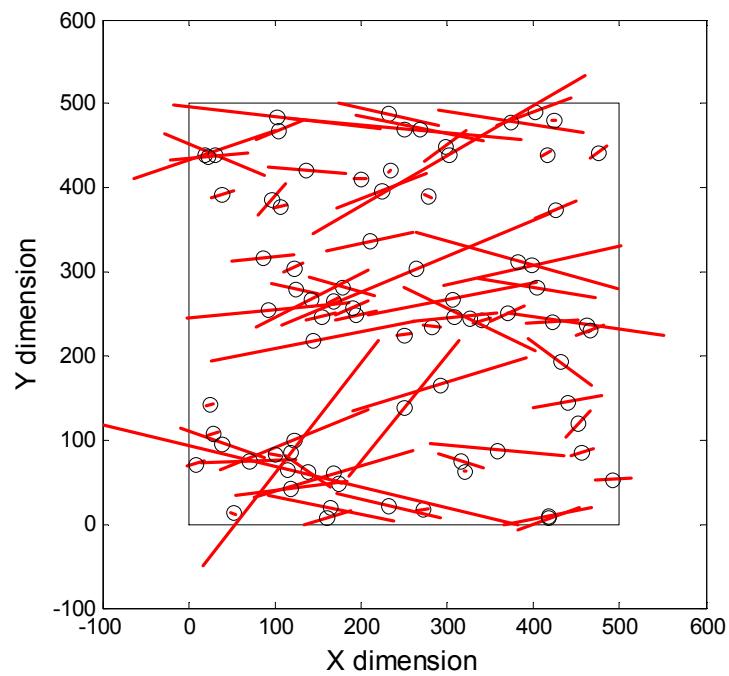
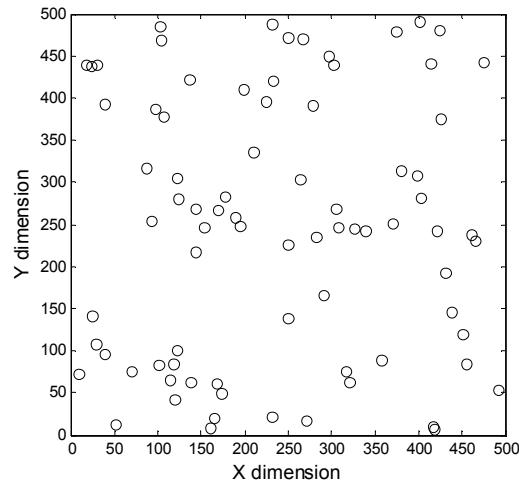
100 Fractures



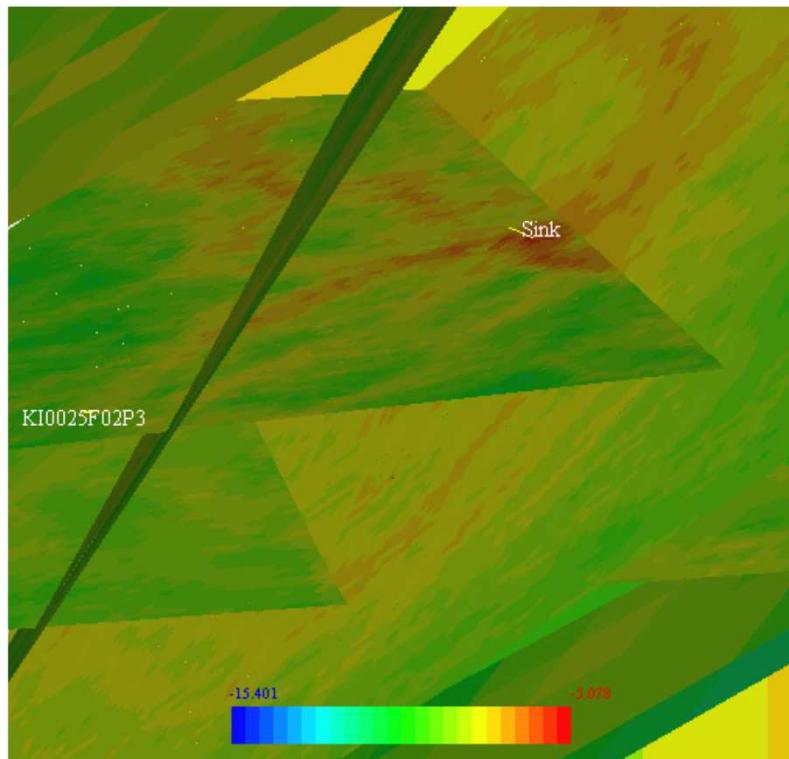
*Pressure testing at Kamaishi mine*

# Cell Based: Simulation Example

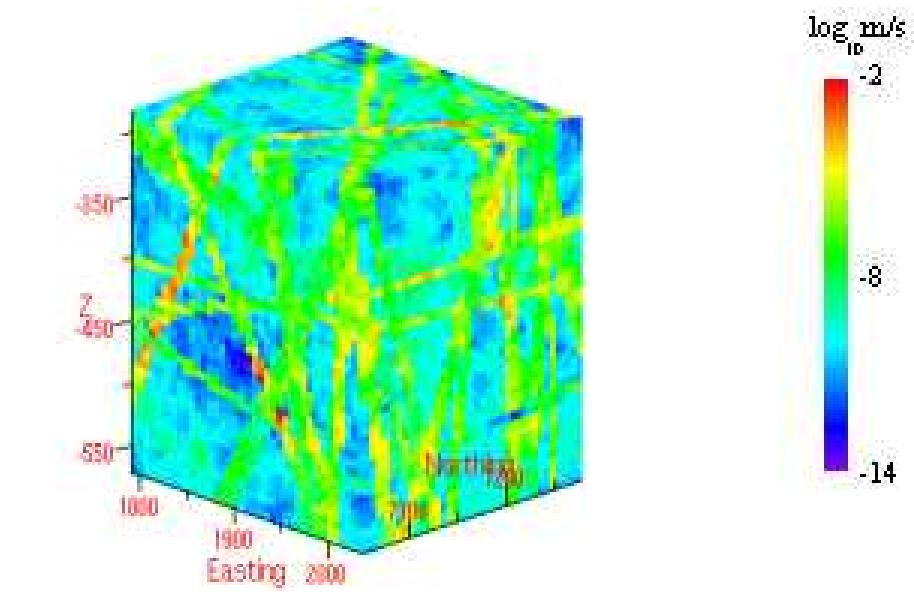
80 Fracture centers located randomly in domain (left). Fracture lengths and orientations drawn from exponential (120) and normal (10,20) distributions (center). Fractures then trimmed to domain boundaries (right)



# Cell-Based Model Examples



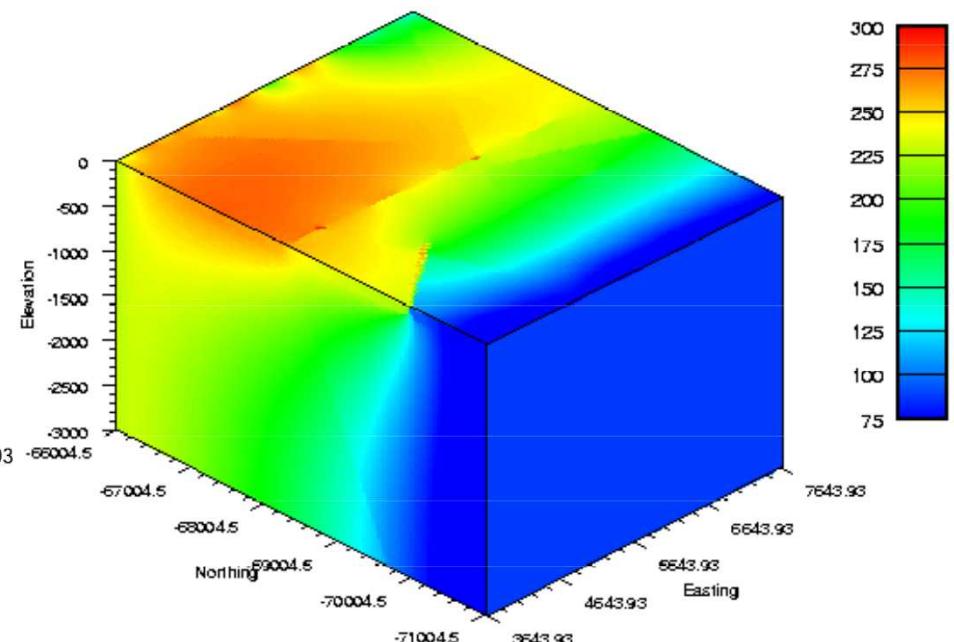
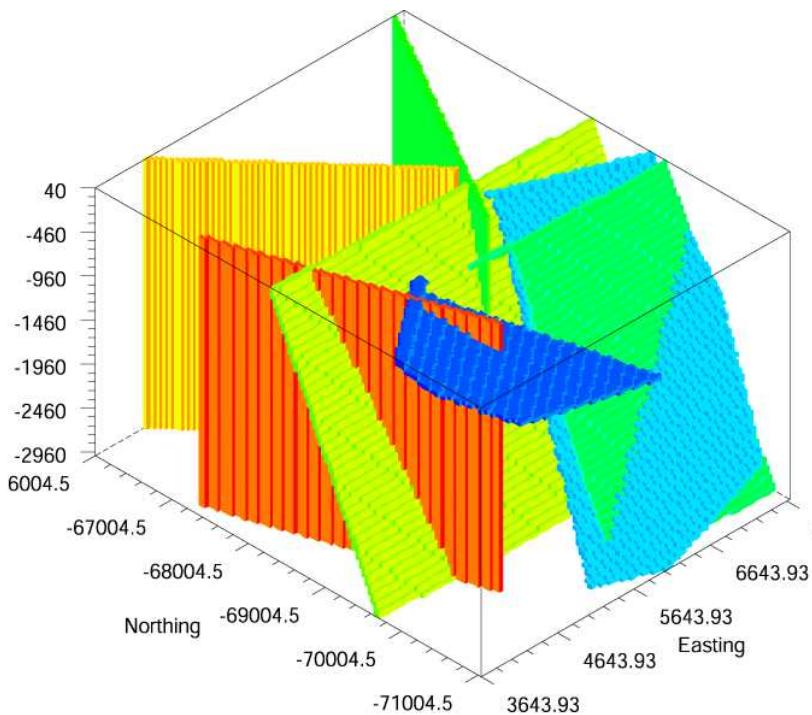
Stochastic transmissivity field on  
Structure 20 (Holton, 2001)



*Stochastic field of hydraulic conductivity  
on structures (Gómez-Hernández et al., in  
prep.)*

# Cell Based: JNC Shobasama Site

Model of seven existing (confirmed faults)



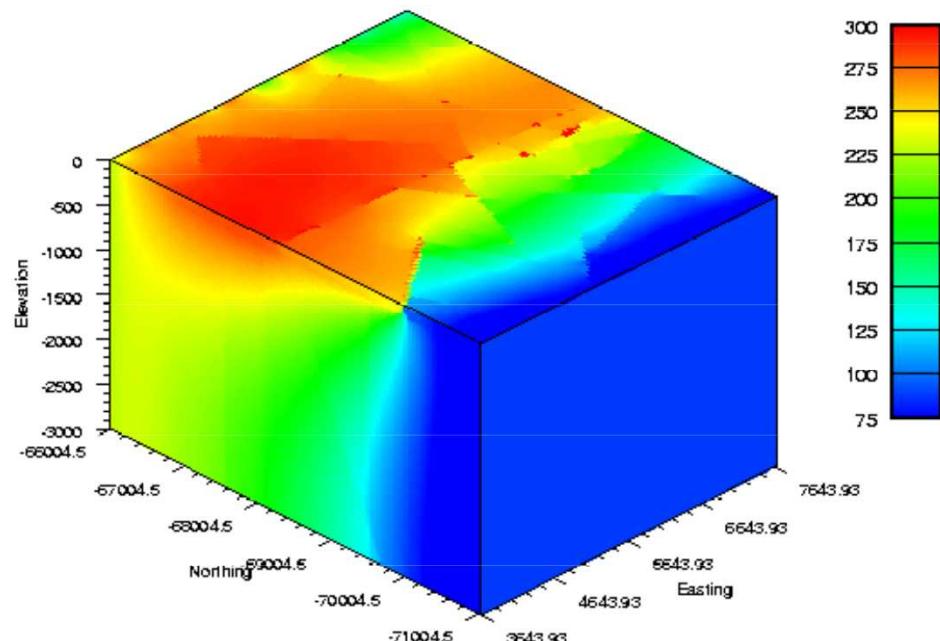
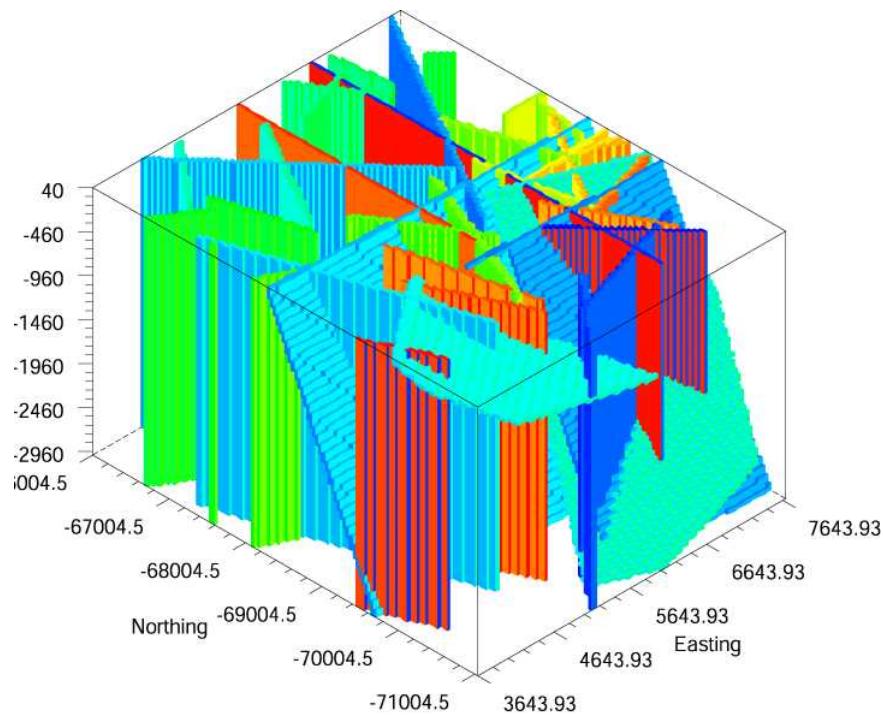
937,500 cells, 40m cube gridblocks



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# Cell Based: JNC Shobosama Site

Model of existing (confirmed faults) + unconfirmed faults from lineament analysis (23 faults total)



McKenna, S.A., M. Eliassi, K. Inaba and H. Saegusa, 2001, Steady-state groundwater flow modeling of the MIU site area, Groundwater Flow in Discrete Fractures Symposium, Japanese Geotechnical Society, Tokyo, September 10-11, 14 pp.



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# Fractured Continuum Model



- Problem Statement:
  - How to honor observations made on various discrete fractures in continuum models of fracture permeability?
- What we care about:
  - Flow characteristics of the fracture network
- What we measure:
  - Characteristics of individual fractures

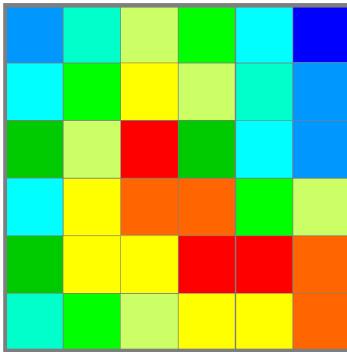
McKenna, S.A. and P.C. Reeves, 2006, Chapter 14: Fractured Continuum Approach to Stochastic Permeability Modeling, in: Coburn, Yarus, and Chambers, eds., *Stochastic Modeling and Geostatistics: Principles, Methods, and Case Studies, Volume II: AAPG Computer Applications in Geology 5*, p. 173–186.



# MODELING APPROACHES

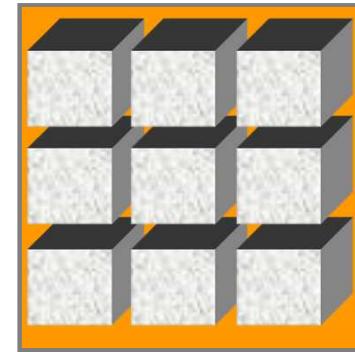
## *Dual Porosity / Permeability Systems*

Equivalent Continuum Model

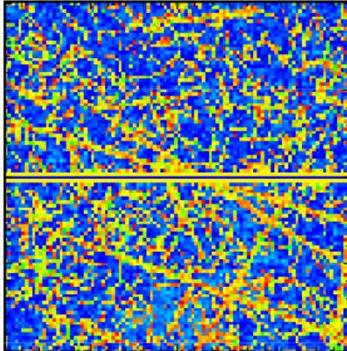


*No Fractures*

Dual Permeability (K) Model

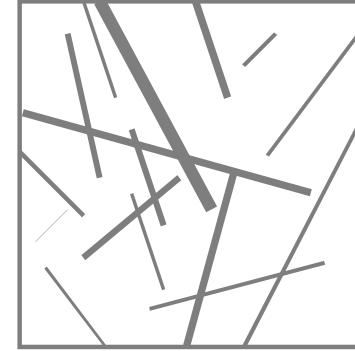


Fractured Continuum Model



*No Matrix*

Discrete Fracture Network



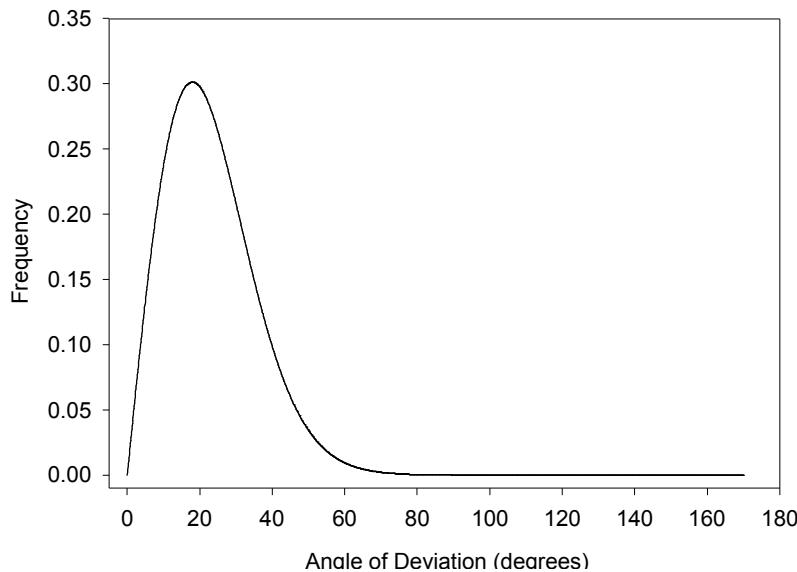
# Honoring Discrete Fracture Observations

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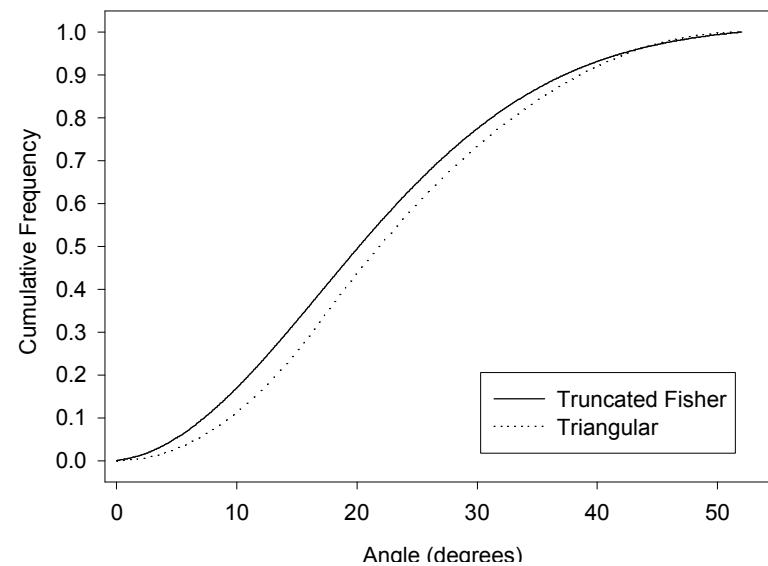
- At the gridblock scale, FCM is an effective permeability value derived from knowledge of discrete fracture network
- For this study, observations were made on discrete fractures to characterize:
  - Radius: Truncated Power-Law
  - Frequency: Poisson
  - Orientation: Fisher (approximated by Triangular)
  - Transmissivity: Log-normal
  - Aperture: Deterministic relation with Transmissivity

# Feature Orientation

Fisher distribution of fracture orientation deviations about the mean orientation

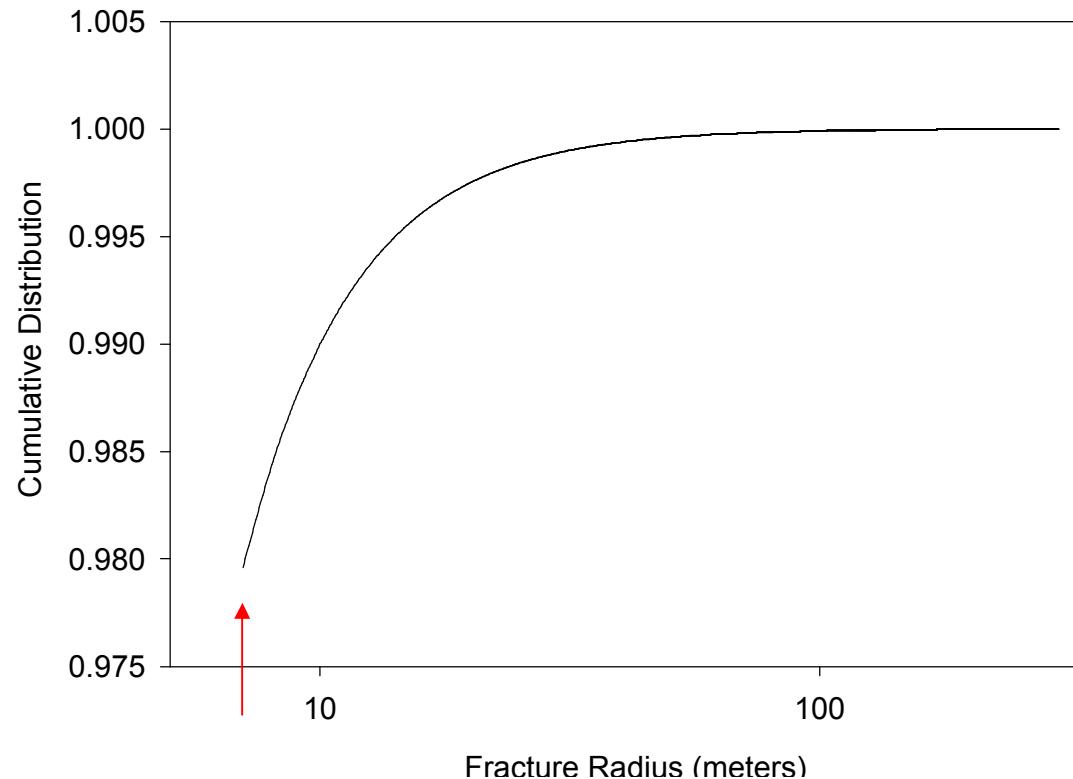


Fisher distribution approximated by triangular distribution



# Fracture Radius

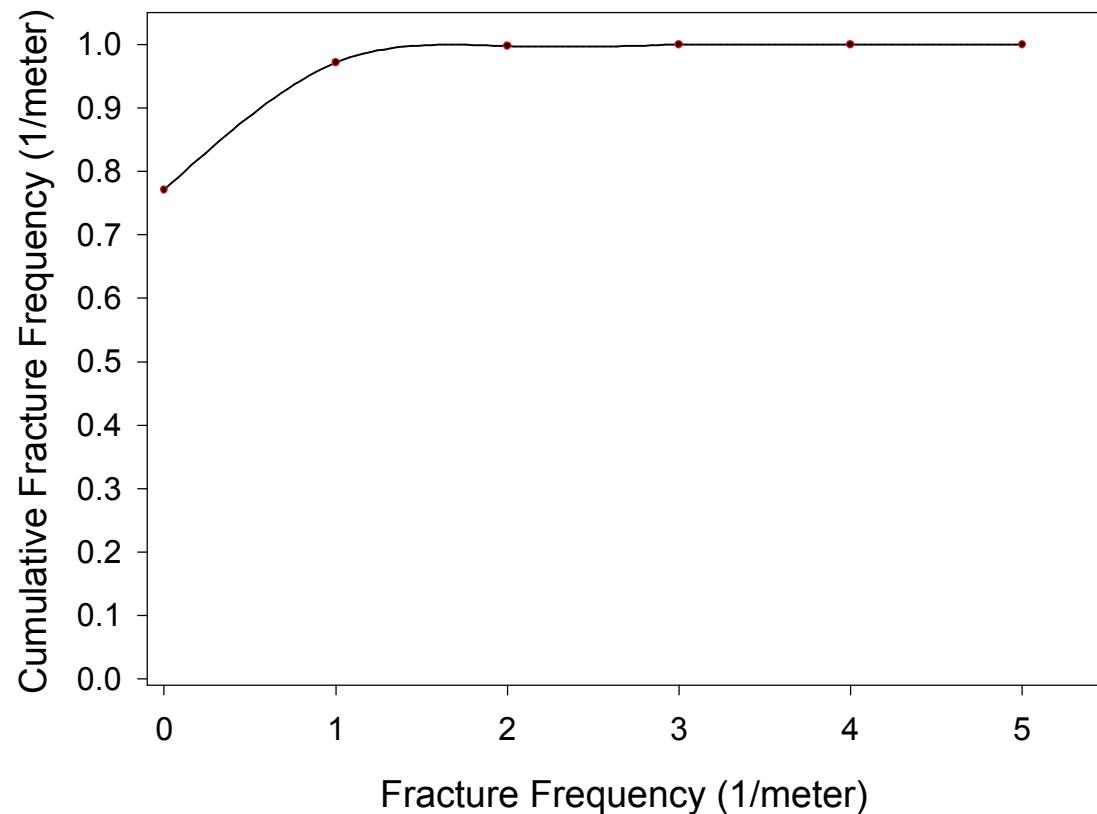
Fracture radius specified with a truncated power-law  
(minimum length = 7.0 meters)



# Fracture Frequency

Poisson distribution of fracture frequency (1/m)

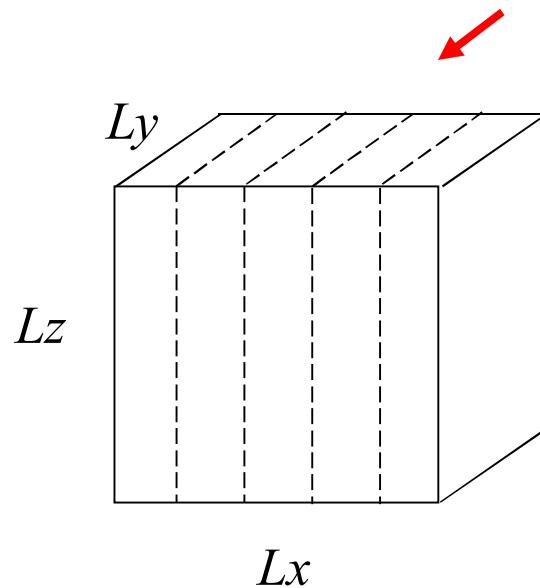
Poisson distribution modified to define frequency per grid-block. Note that Poisson distribution is only defined for integer values.



# Effective Gridblock Permeability

## Local Model

Define gridblock permeability in terms of geometric information on fracture network, gridblock dimensions and effective fracture conductance



$$C_i = \frac{gb^3 \rho_w}{12 \mu L}$$

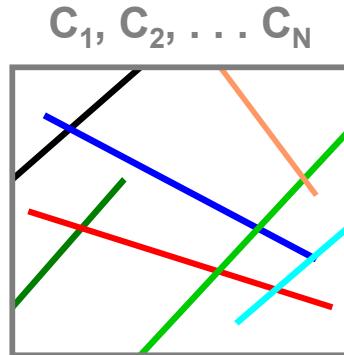
$$Q_y = N_x C^* \Delta H$$

$$Q_y = K_y L_z L_x \Delta H / L_y$$

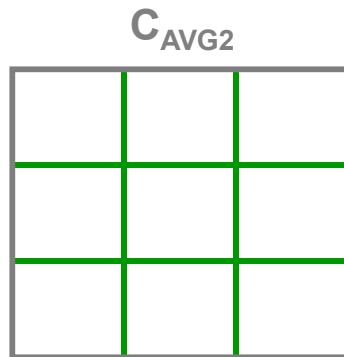
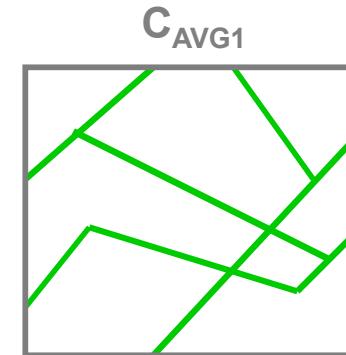
$$K_y = \frac{L_y N_x C^*}{L_x L_z}$$

# Effective Medium Approximation

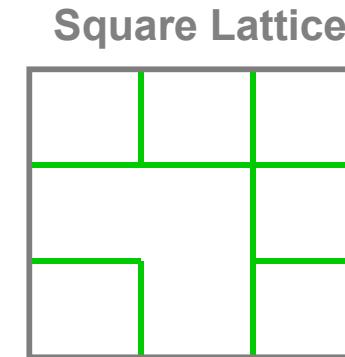
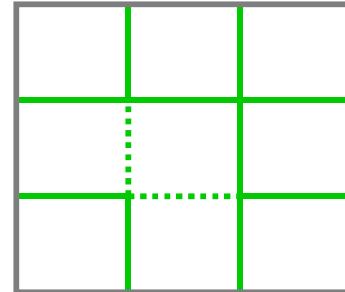
## Local Model



$$\sum_{i=1}^N \left[ \frac{z}{2} - 1 \right] C_{AVG} + C_i = 0$$



Non-Conducting  
Bonds



# Spatial Simulation

## Domain Model

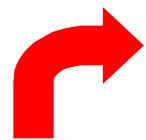
- FCM uses combination of geostatistical and object-based simulation to populate flow model domain
- Components of Effective Medium Approximation (frequency and coordination number) are considered to be realizations of a spatial random function and are modeled with geostatistical simulation.
- Proportion of “conductive/non-conductive” cells, based on Poisson distribution, modeled with object-based simulation
- Three simulations combined to produce final permeability model



# GEOSTATISTICAL MODEL

## “Fractured Continuum”

*Fractured Continuum Model*

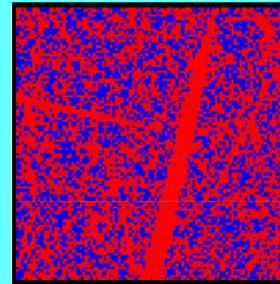


### Fracture Statistics

- *Shape*
- *Radius*
- *Orientation*
- *Transmissivity*
- *Aperture*
- *Spatial Frequency*

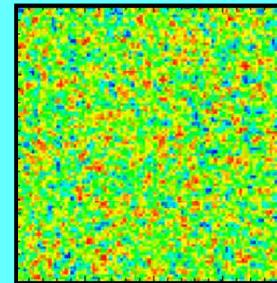
“Conductive Domains”

*Boolean*

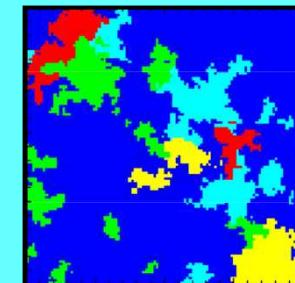


*Coordination Number*

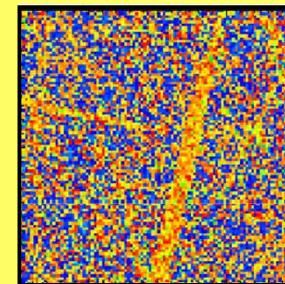
*Multigaussian*



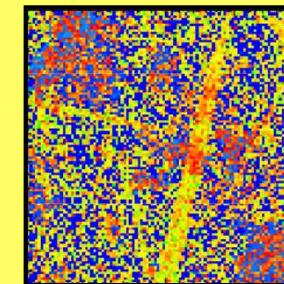
*Number of Fractures*



*Permeability*



*Porosity*



# FRACTURED CONTINUUM MODEL

*Dual Porosity / Permeability Systems*

## Advantages

- Matrix is Not Ignored → *Unlike DFNs*
- Fractures are Not Abstracted to a Network of 1-D Pipes → *Unlike DFNs*
- Spatial Correlation Reflects Underlying Spatial Structure of Fractures → *Unlike ECMs / DFNs*
- Statistics Underlying Fracture Geometry Are Directly Utilized → *Unlike ECMs / DFNs*
- Influence of Fracture Geometry on Flow and Transport Can be Explicitly Studied → *Unlike ECMs / DFNs*



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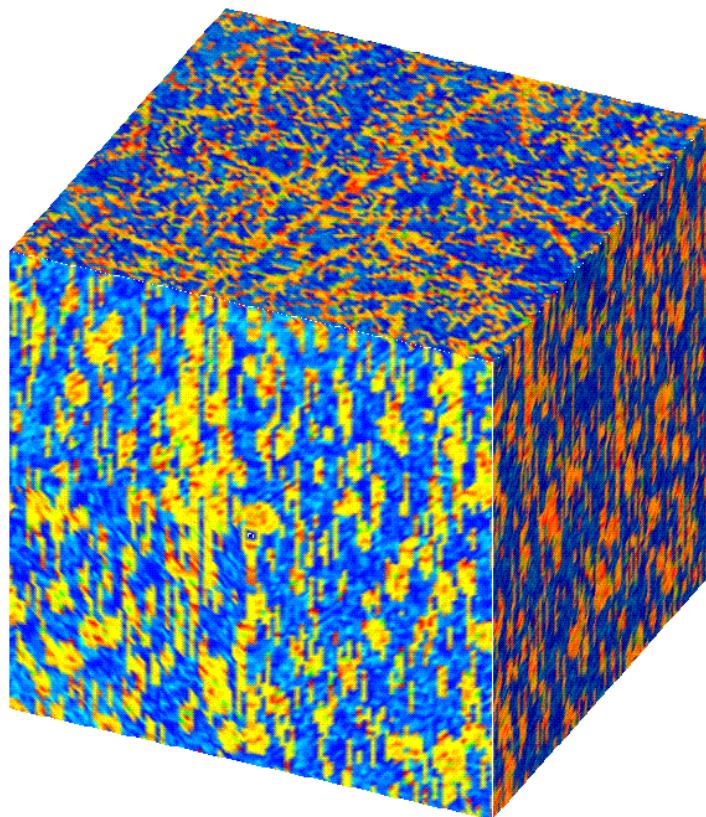
# EXAMPLE REALIZATION

Stage 1 R44/50

POR-SALSA: H12 FLOW COMPARISON

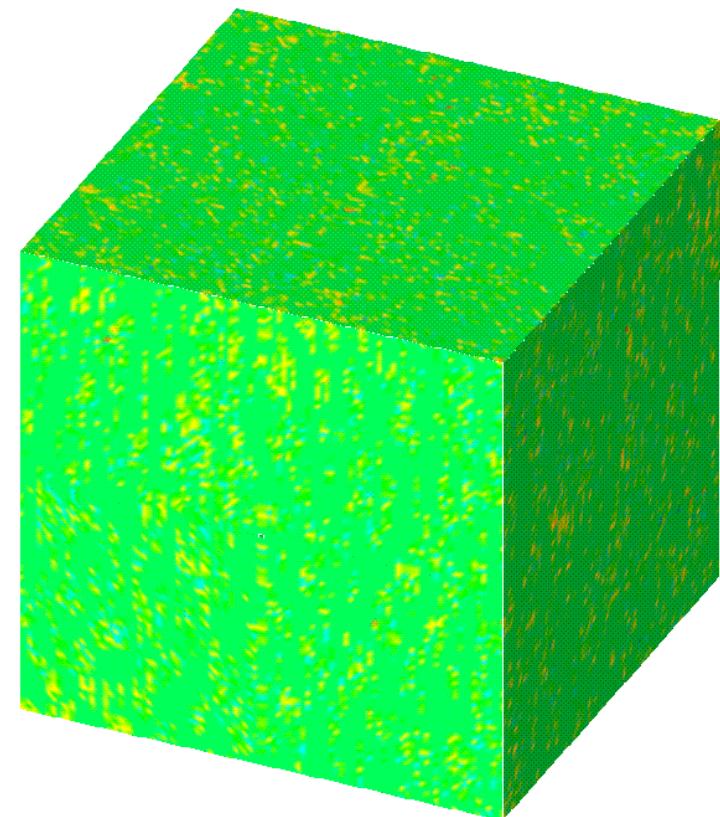
Intrinsic Permeability

Porosity



$\text{Log}_{10} (k)$  [meters<sup>2</sup>]

-18 -17 -16 -15

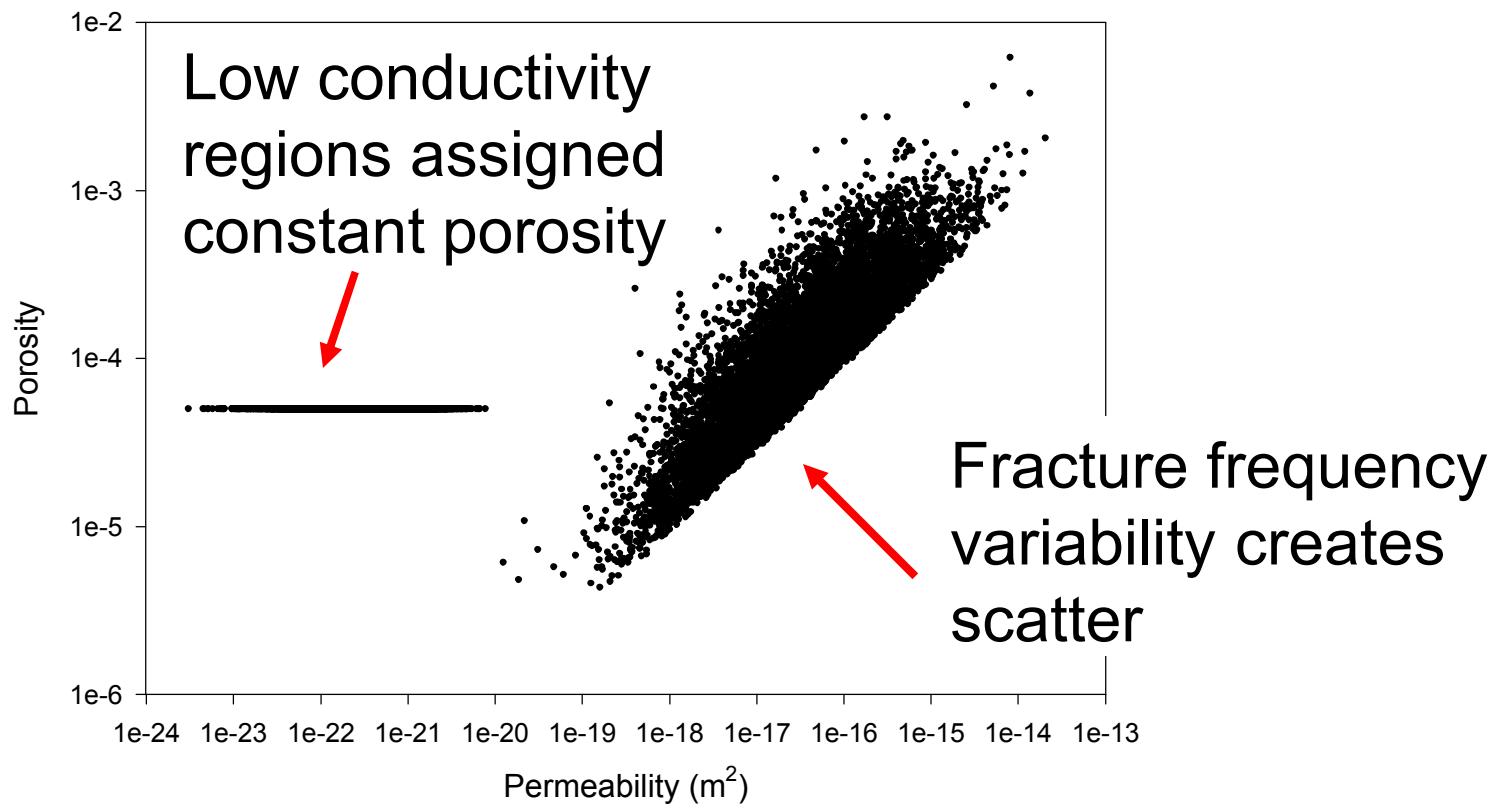


$\text{Log}_{10} (\phi)$

-6.0 -5.4 -4.9 -4.3 -3.7 -3.1 -2.6 -2.0

# Permeability-Porosity Relationship

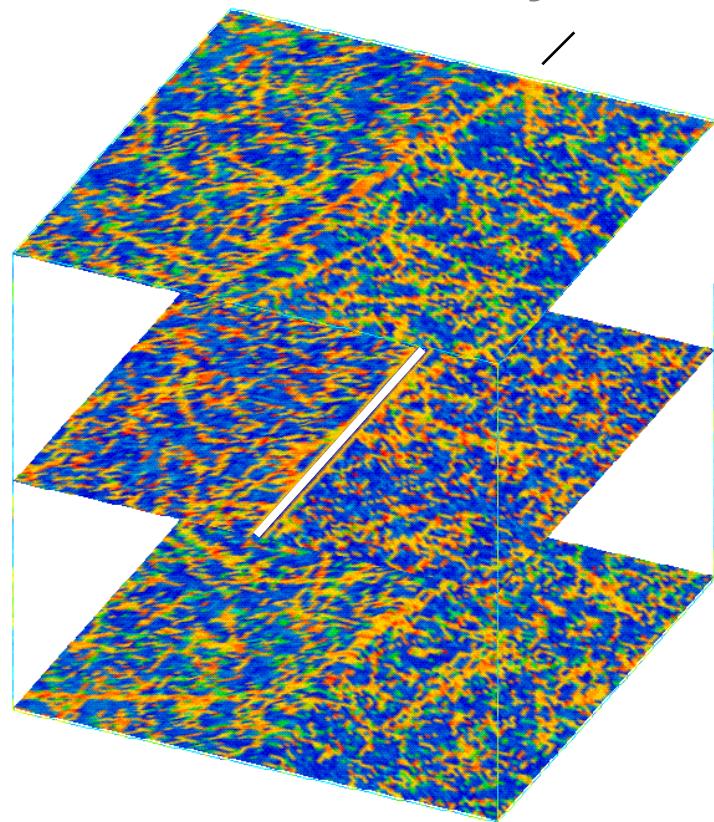
Stage 1 result, single realization



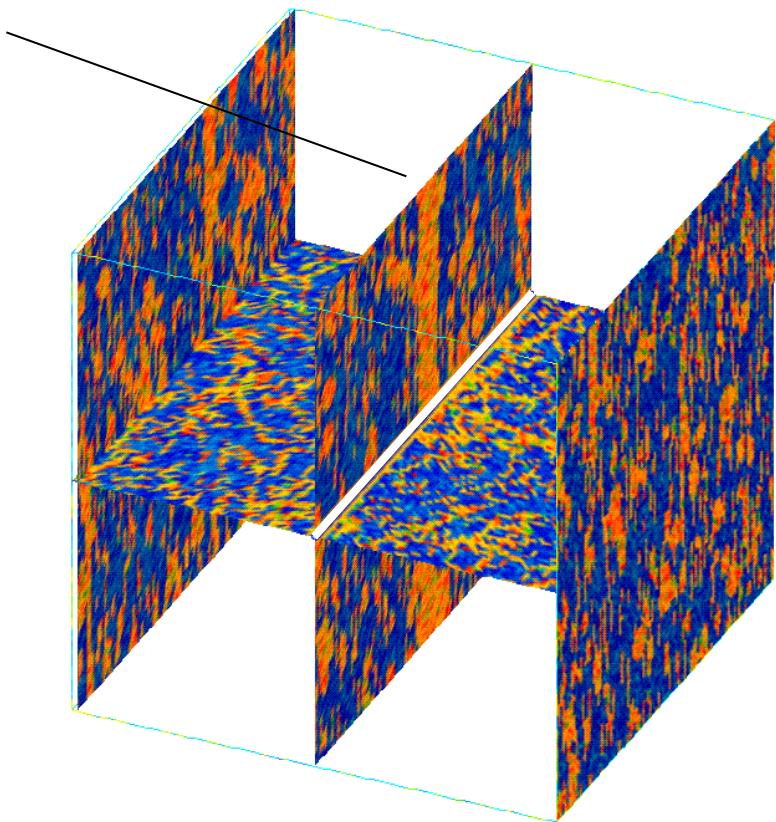
# EXAMPLE REALIZATION

Stage 1 R44/50

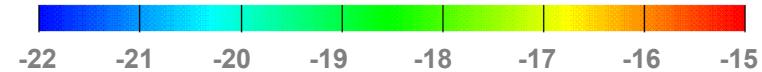
POR-SALSA: *H12 FLOW COMPARISON*  
Intrinsic Permeability Transects



$\text{Log}_{10} (k) [\text{meters}^2]$

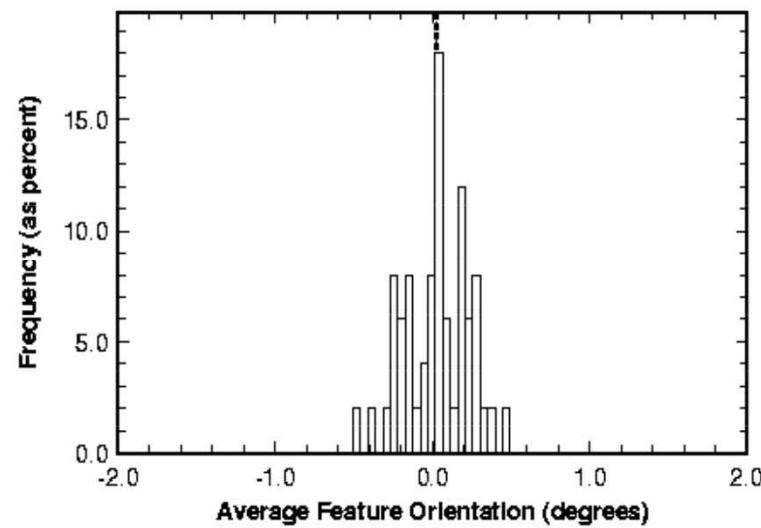
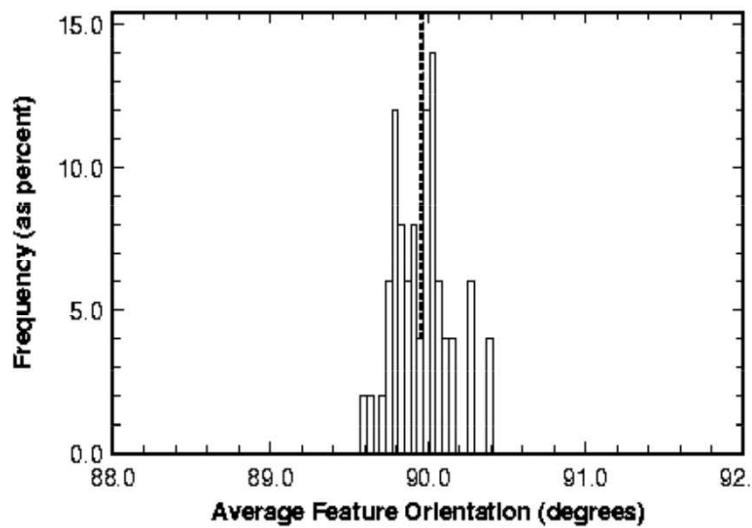


$\text{Log}_{10} (k) [\text{meters}^2]$



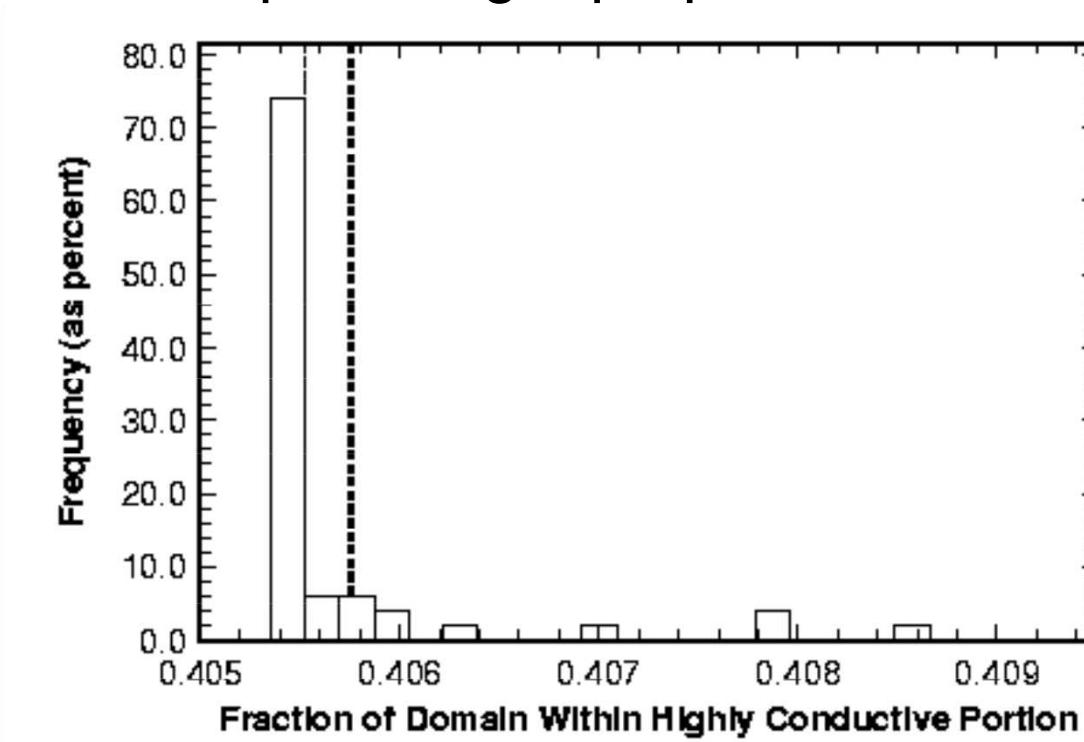
# Model Validation (orientation)

Mean orientation of both feature sets calculated for 50 realizations and compared to target values of 0.0 and 90.0 degrees



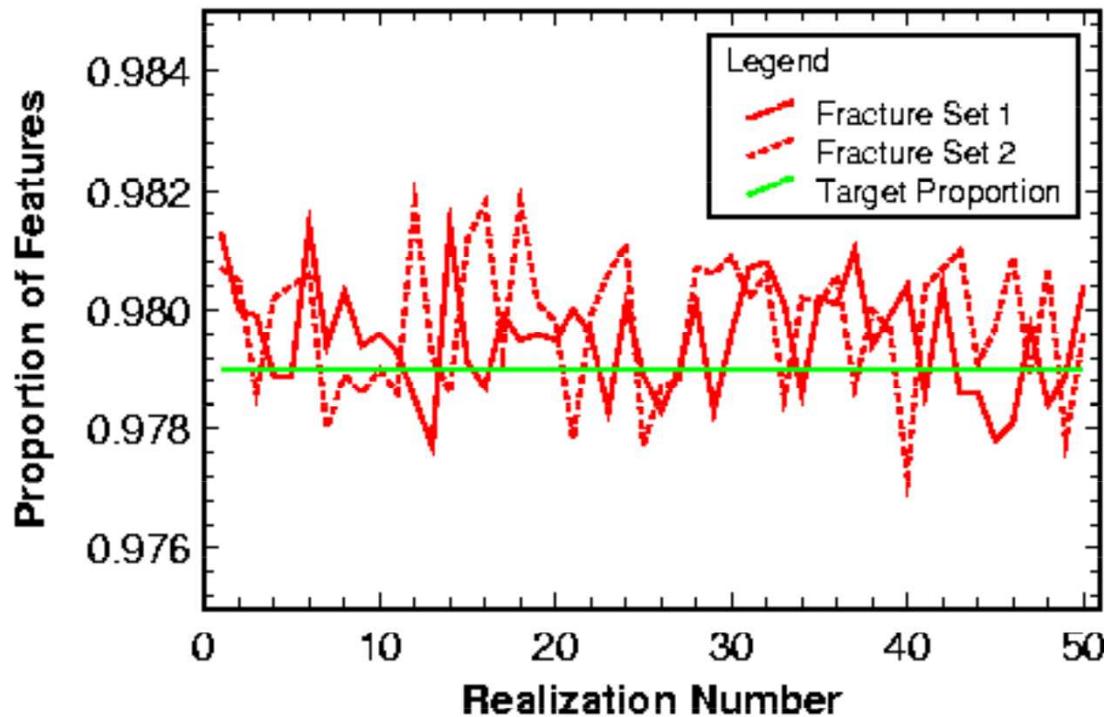
# Model Validation (Frequency Minimum)

Median proportion of conductive gridblocks across 50 realizations equals target proportion of 0.4055



# Model Validation (Minimum Fracture Radius)

97.9 percent of all features have radius greater than 7 meters

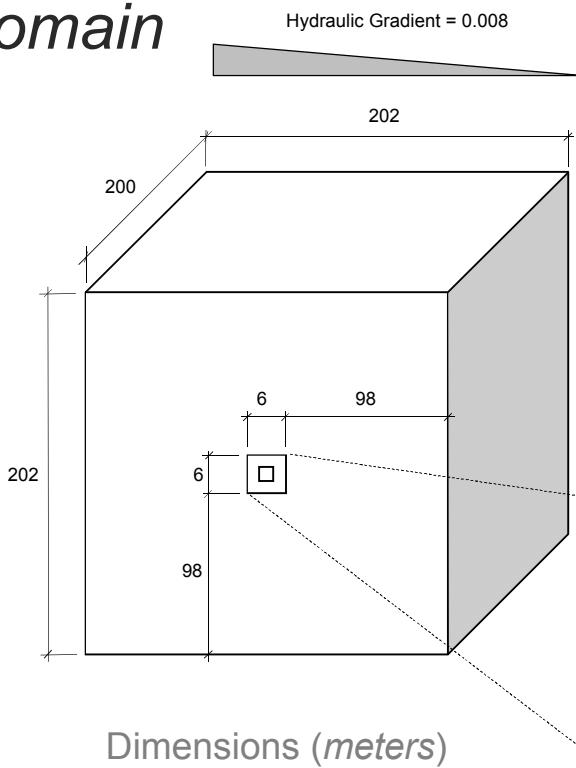


Proportion of features with radii greater than 7 meters is shown against the target proportion for both feature sets

# Flow Modeling (MESH DESCRIPTION)

POR-SALSA: H12 FLOW COMPARISON

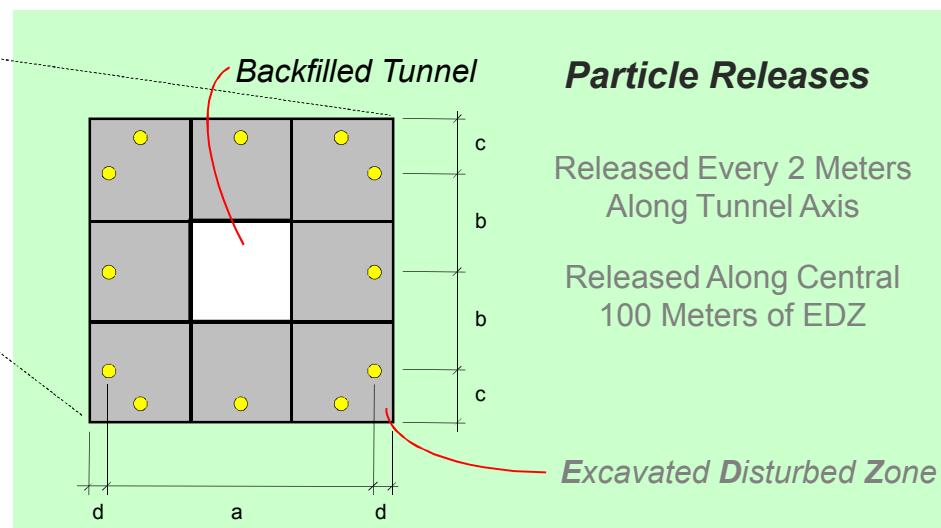
Domain



100 x 101 x 101 Elements  
Regular Hexahedral Elements  
(2 x 2 x 2 Meter / 8 Nodes )

**1,020,000 Elements**  
**1,050,804 Nodes**

1 Primary Unknown ( $\rho_w$ )  
4 Secondary Unknowns ( $H, v_x, v_y, v_z$ )



# Flow and Transport Model PERFORMANCE

POR-SALSA: *H12 FLOW COMPARISON*

## Flow Solution

- *1,050,804 Primary Unknowns*
- *4,203,216 Secondary Unknowns*

**Per Realization**

*15-25 Minutes*

## Particle Tracking

- *600 Particles*
- *Advection Travel Times / Distances*
- *Cumulative F-Quotients*

*~10 Seconds*

**50 Realizations Overnight**



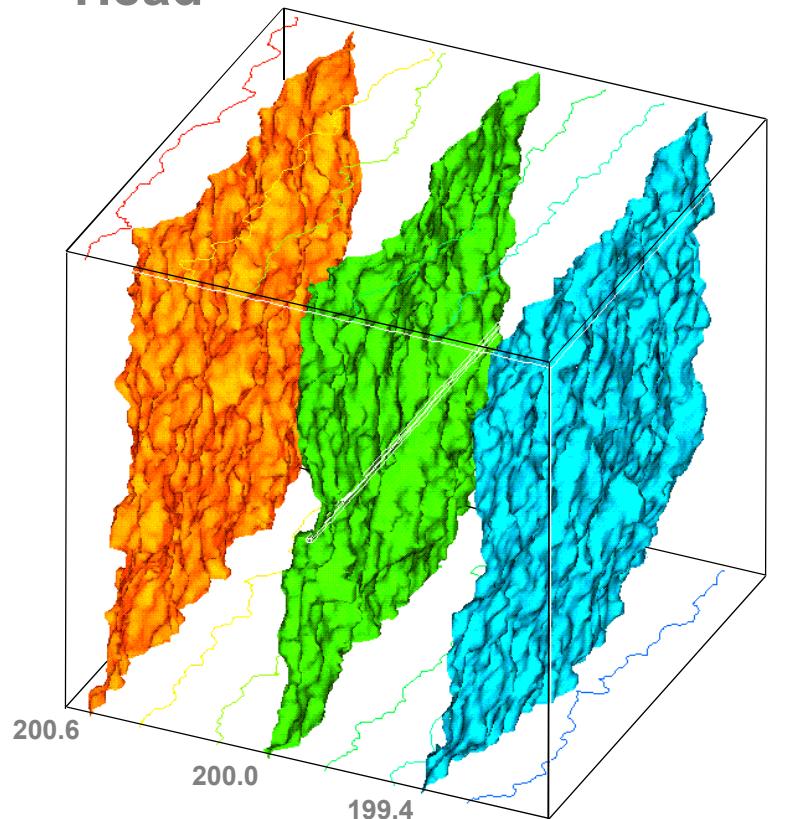
Sandia National Laboratories

# FLOW CALCULATIONS

## Stage 1 R44/50

POR-SALSA: *H12 FLOW COMPARISON*

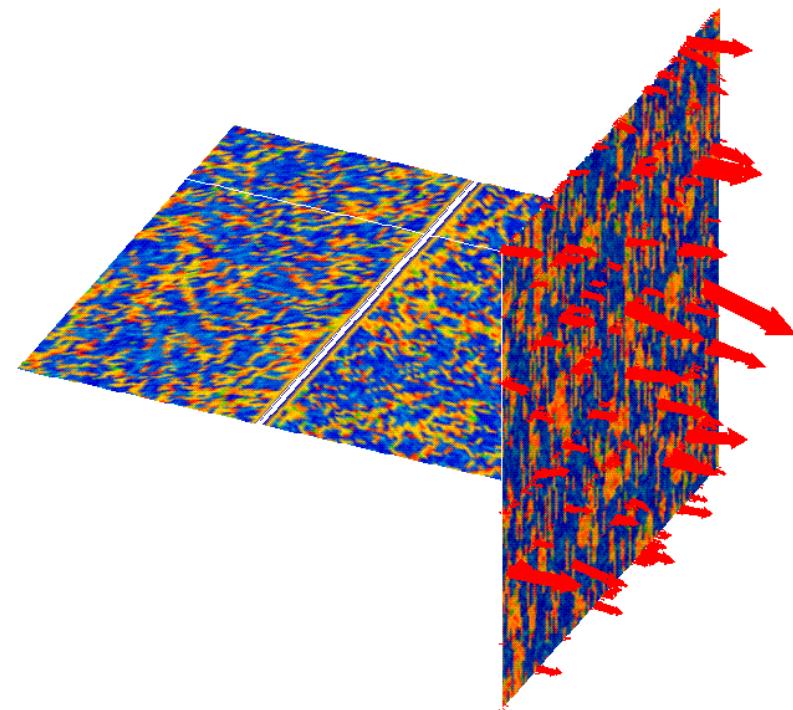
Head



Head (meters)



Outflow Fluxes



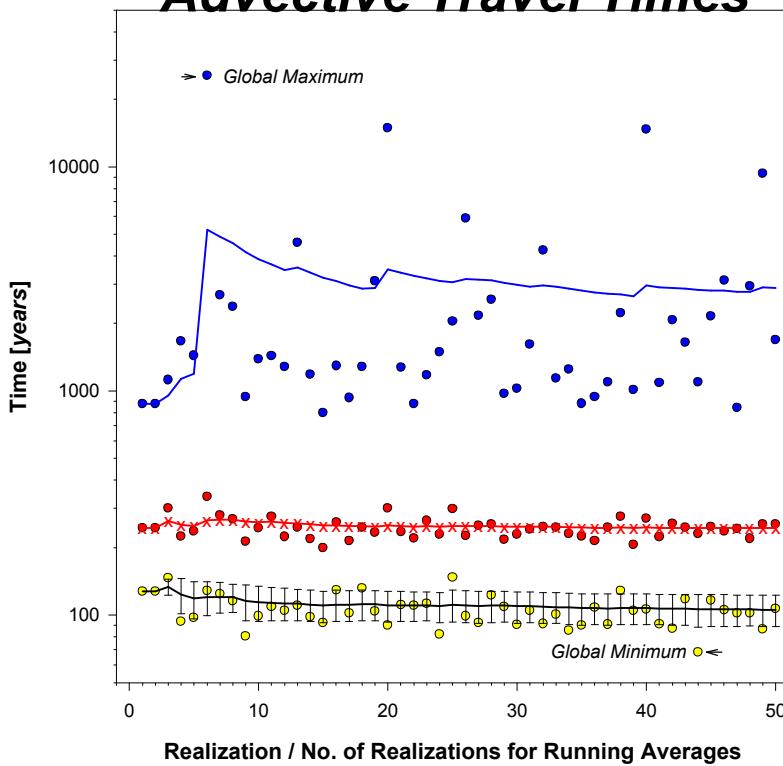
$\log_{10}(k)$  [meters $^2$ ]



# PARTICLE TRACKING

## Stage 1: 50 Realizations

### POR-SALSA: H12 FLOW COMPARISON *Advection Travel Times*

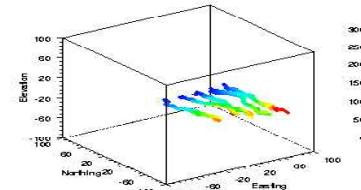


- Realization Mean
- Realization Minimum
- Realization Maximum
- Ensemble Mean
- Ensemble Mean Minimum and Standard Deviation
- Ensemble Mean Maximum

Model: TRACKWAY  
(Alex H. Treadway, Dept. 6849)

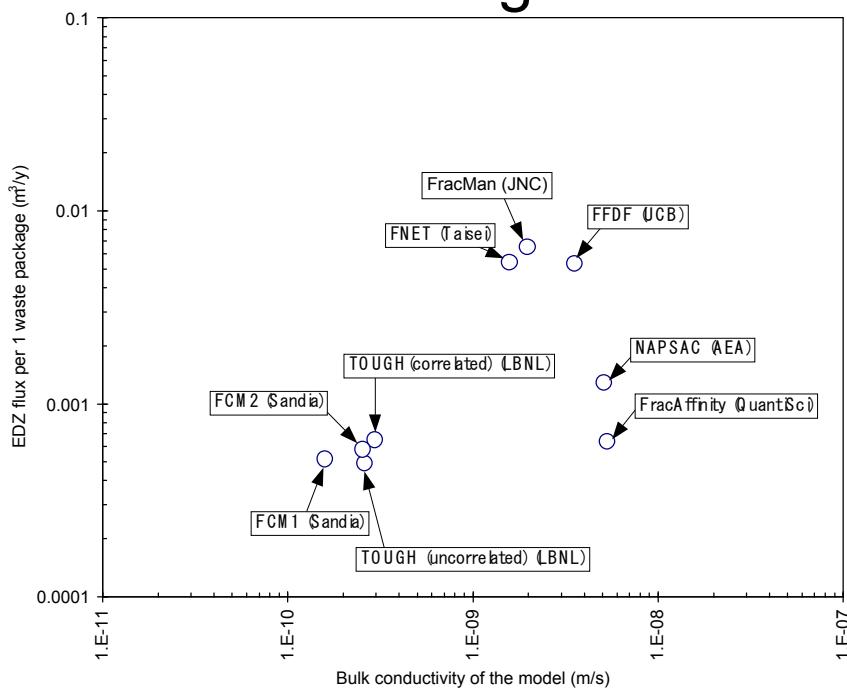
600 Particles Per Realization  
< 10 Seconds

Ensemble Mean Minimum  
 $105.6 \pm 15.6$  Years

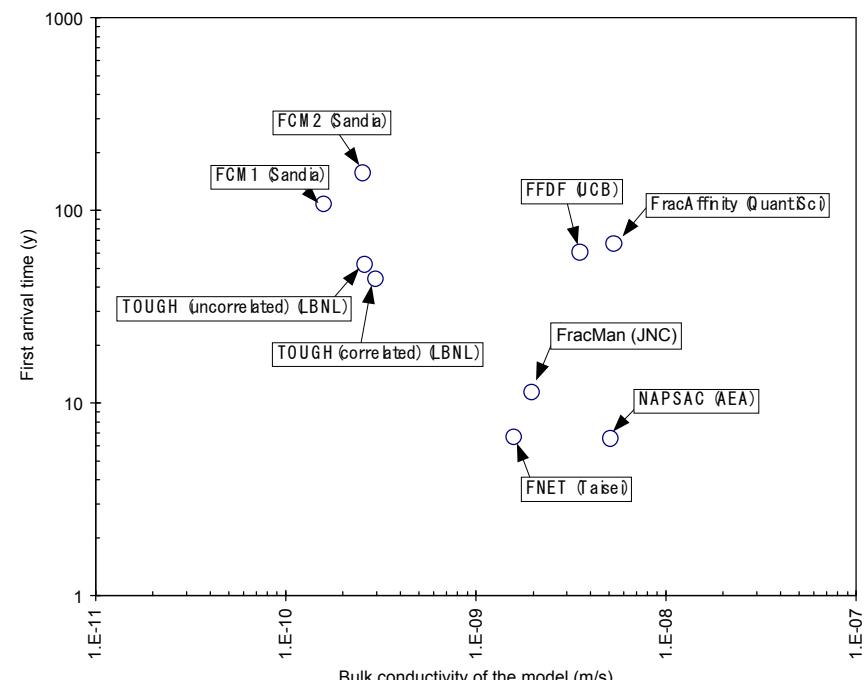


# Comparing Conceptual Models

## Bulk Conductivity vs. Flux through EDZ



## Bulk Conductivity vs. First Arrival Time

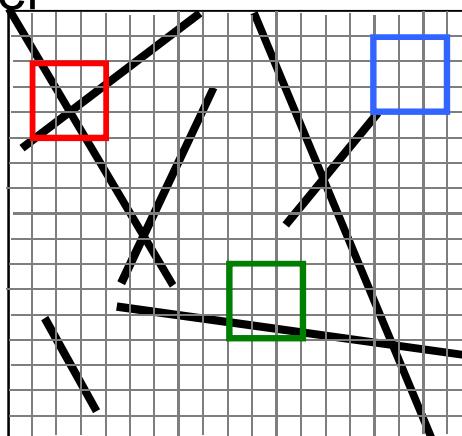


After Sawada, DRAFT, 1999

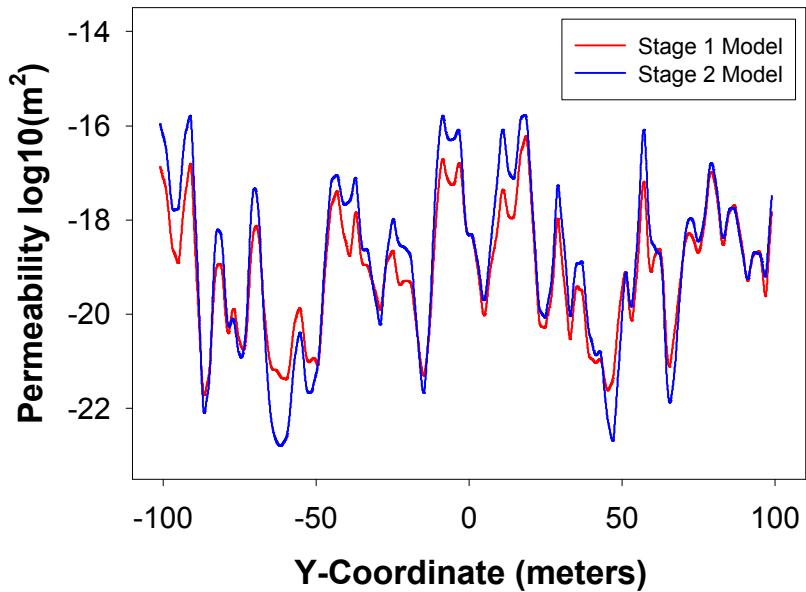
# Variation on Conceptual Model

In Stage 1 frequency & coordination number values were drawn independently from location of conductive features

In Stage 2 proportion of conductive features in moving template is used to draw from frequency and coordination number

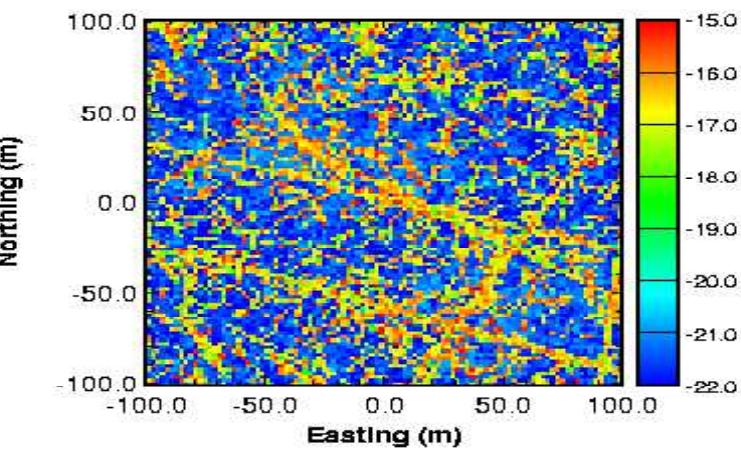


**Results:** greater difference between conductive and non-conductive zones

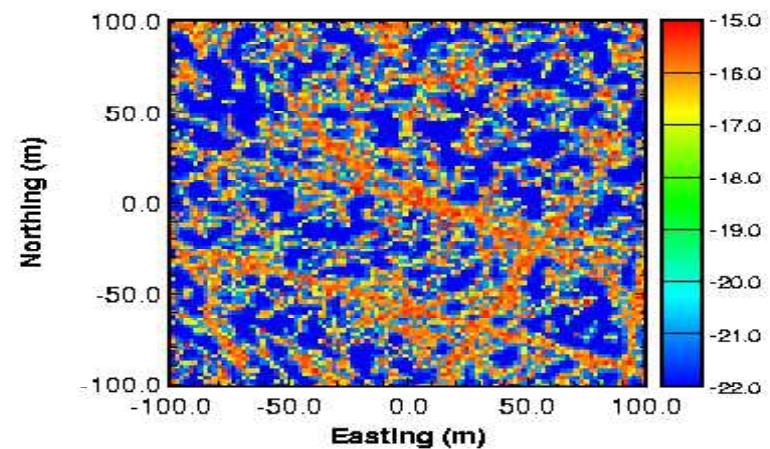


# Comparing Permeability Models

Stage 1



Stage 2



Permeability in m<sup>2</sup>

# PROJECT OVERVIEW

## POR-SALSA: H12 FLOW COMPARISON

### Flow Simulator: **POR-SALSA**

#### **Mesh**

100 x 101 x 101 Elements  
Regular Hexahedral Elements  
(2 x 2 x 2 Meter / 8 Nodes )

**1,020,000 Elements**  
**1,050,804 Nodes**

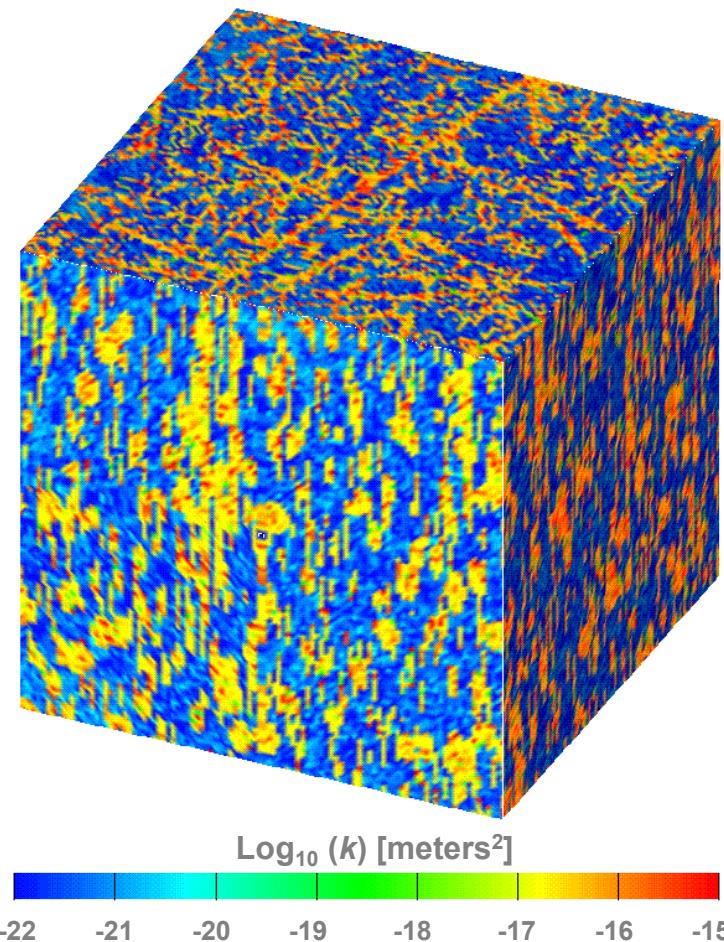
#### **Flow Problem**

Steady-State, Saturated  
1 Primary Unknown ( $\rho_w$ )  
4 Secondary Unknowns ( $H, v_x, v_y, v_z$ )

#### **Performance**

**50 Realizations**  
**20 Processors**  
**17 Hours Total Time**

### Geostatistical Representation: **Fractured Continuum Model**



# Summary



- FCM represents new approach to modeling fracture permeability (EMT + Spatial Simulation)
  - Bridge between DFN and ECM models
  - Exploits capability of MPP flow and transport capabilities
- Compared to other approaches, results indicate:
  - FCM Bulk hydraulic conductivity similar to ECM's (about 1 order of magnitude lower than DFN models)
  - FCM produces longest times to first arrival (100+) years (DFN model first arrivals are roughly 5-15 years)