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# Adjoint Discrete Scattering Models for Electron Dose Calculations

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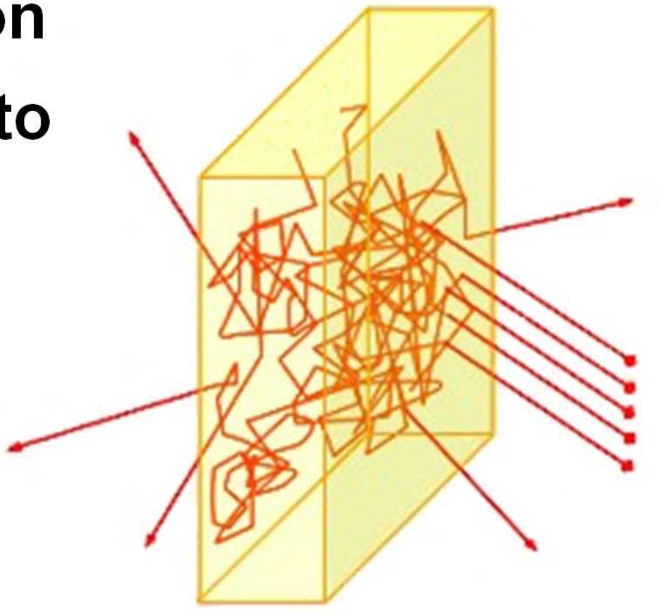
# Overview

- Radiation Transport
- GBFP Method
  - Discrete Model
- Forward Transport
- Adjoint Transport
- Weight Correction
- Results

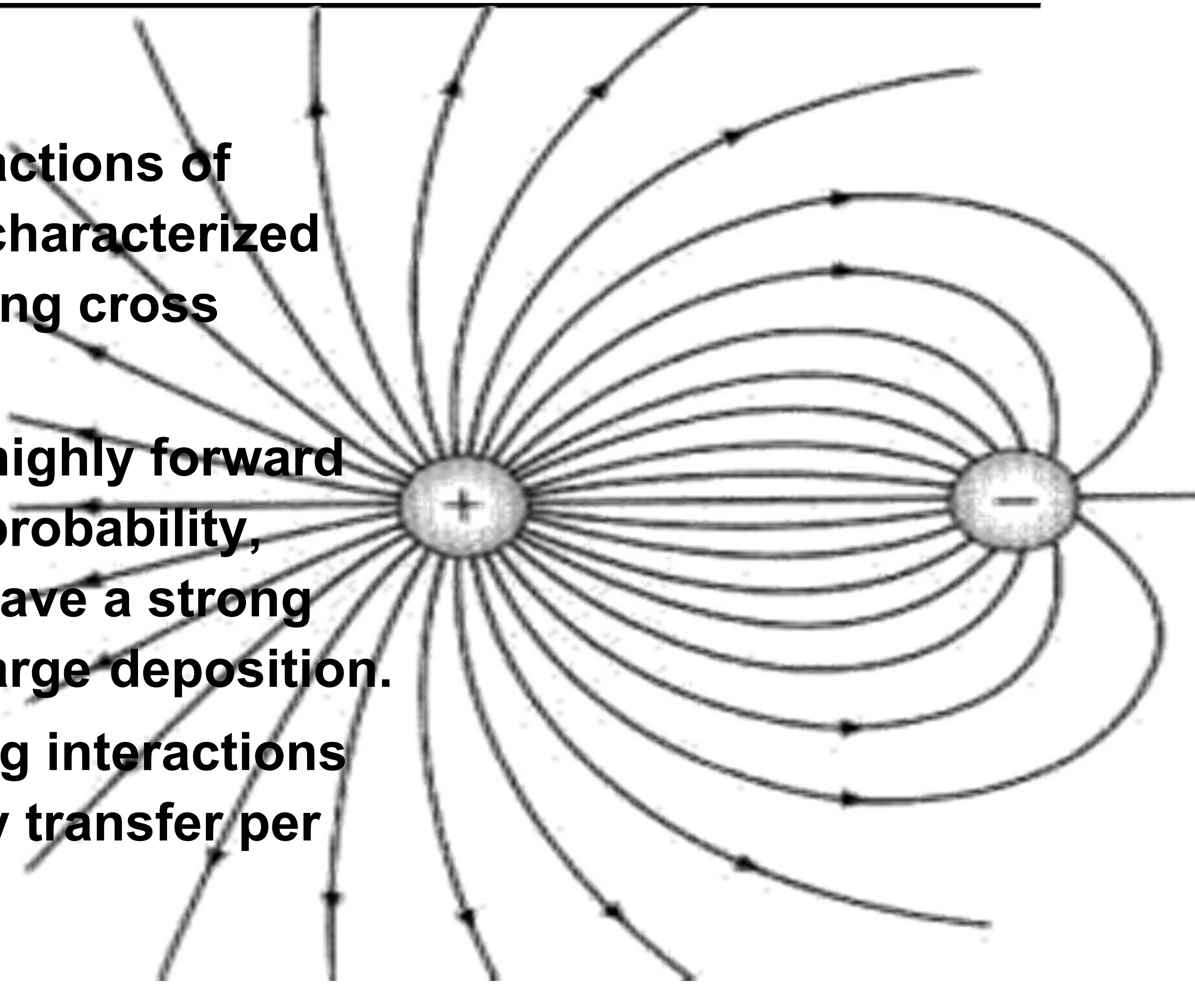
# Monte Carlo Method

For either adjoint or forward calculations:

1. Sample a distance to interaction
2. Sample an isotope with which to interact
3. Sample type of interaction
4. Sample result of interaction
5. Adjust Particle/Tally Statistics
6. Repeat upon survival



# Electron Transport



- The non-binary interactions of charged particles is characterized by large total scattering cross sections.
- Elastic scattering is highly forward peaked, but the low-probability, high-angle scatters have a strong impact on energy/charge deposition.
- Small-angle scattering interactions result in small energy transfer per interaction.

# Forward Radiation Transport

**Forward Linear Boltzmann Transport Equation:**

$$\vec{\Omega}\cdot\nabla\psi\left(\vec{r},\vec{\Omega},E\right)+\left[\sigma_{s,el}\left(\vec{r},E\right)+\sigma_{s,in}\left(\vec{r},E\right)\right]\psi\left(\vec{r},\vec{\Omega},E\right)=$$

$$\int_{4\pi}\sigma_{s,el}\left(\vec{r},\vec{\Omega}\cdot\vec{\Omega}',E\right)\psi\left(\vec{r},\vec{\Omega}',E\right)d\vec{\Omega}'+\int_{0}^{\infty}\sigma_{s,in}\left(\vec{r},\vec{\Omega},E'\rightarrow E\right)\psi\left(\vec{r},\vec{\Omega},E'\right)dE'$$

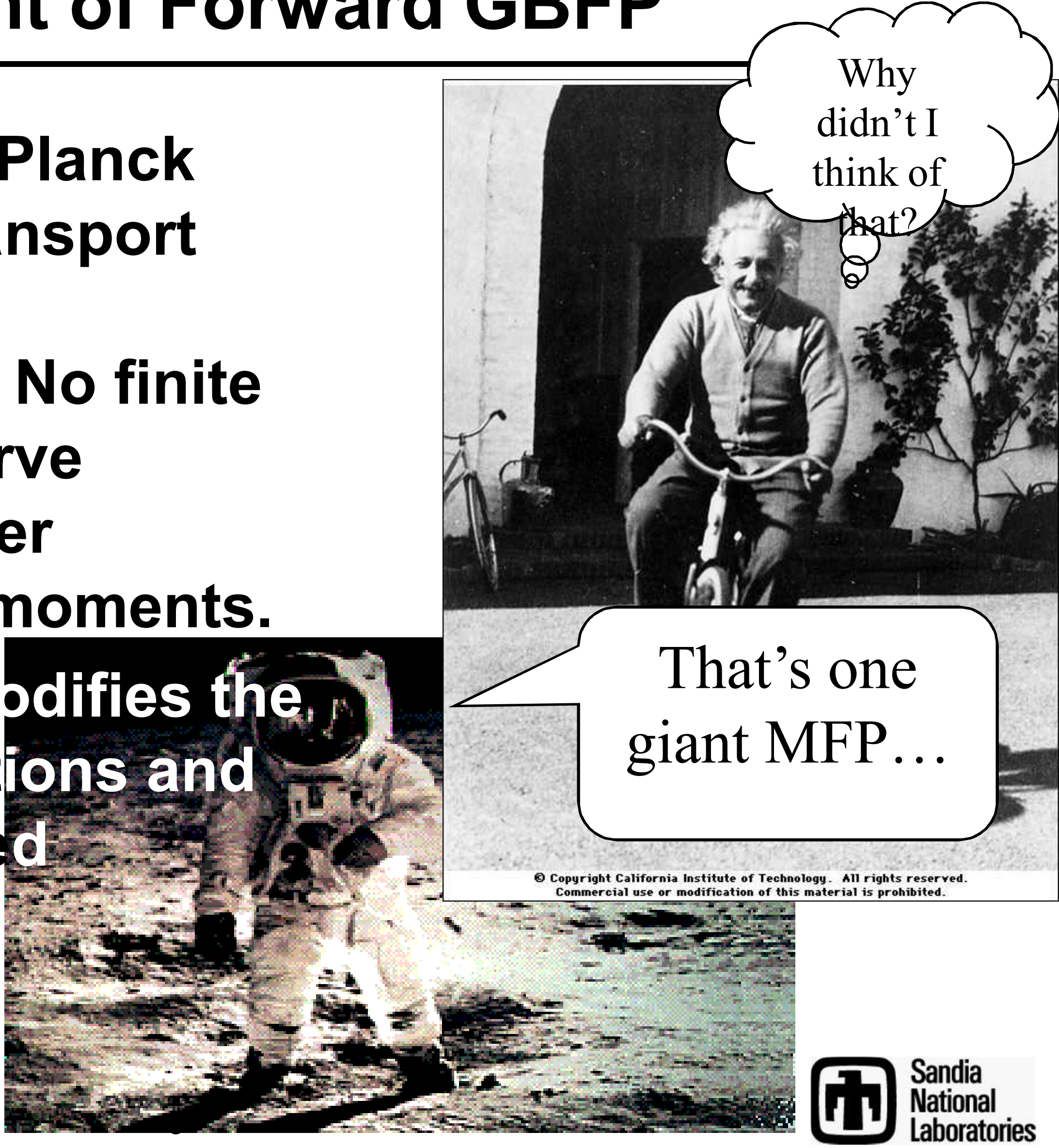
**Lewis theory suggests a direct correlation between preserving moments of the differential cross section (DCS) and the accuracy of the model as measured by space-angle moments of the infinite medium solution**

**Momentum-Transfer Moments:**

$$\sigma_n = 2\pi\int_{-1}^{1} d\mu_0\left(1-\mu\right)^n\sigma_s\left(\vec{r},\mu_0,E\right)$$

# Development of Forward GBFP

- Higher order Fokker-Planck expansions of the transport equation are asymptotic/unstable. No finite truncation can preserve arbitrarily higher-order momentum transfer moments.
- The GBFP method modifies the scattering cross sections and then solves a modified transport equation.



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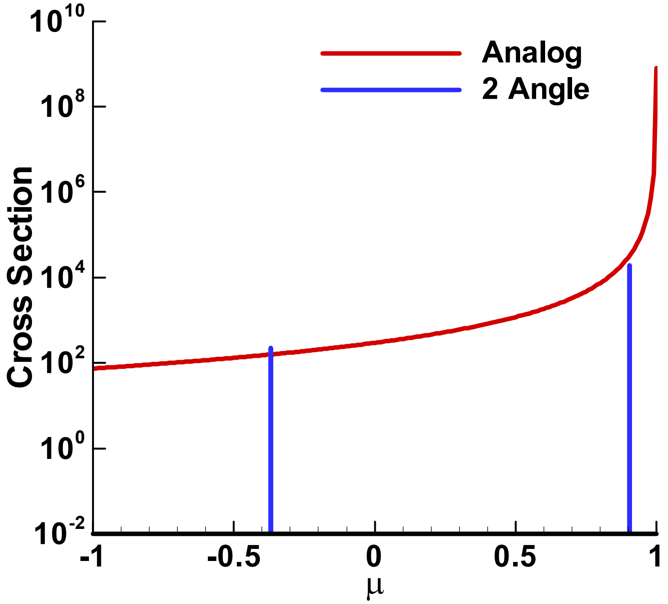
# Discrete Model for Electron Scattering

The discrete scattering kernel is:

$$\tilde{\sigma}_s(\vec{r},\mu_0,E)=\sum_{n=1}^{N}\frac{\alpha_n(\vec{r},E)}{2\pi}\delta[\mu_0-\xi_n(\vec{r},E)]$$

$\alpha_n$ are amplitudes

$\xi_n$ are scattering cosines



Requiring that $\alpha_n$ and $\xi_n$ preserve *2N* momentum transfer moments ($\sigma_n$, *n=1,…,2N*) produces a nonlinear algebraic system, which is solved using Sloan's MORSE algorithm.

Sloan, D.P., "A New Multigroup Monte Carlo Scattering Algorithm for Neutral and Charged-Particle Boltzmann and Fokker-Planck Calculations," Technical Report SAND83-7094, Sandia National Laboratories (1983).

# Discrete Model for Electron Scattering

**ADVANTAGES**

- Very fast, direct sampling
- Only requires moments (should be independent of DCS )
- Accuracy and speed depend on moment preservation, easily adjusted
- In practice, only 4-8 discrete angles provided excellent accuracy

**DISADVANTAGES**

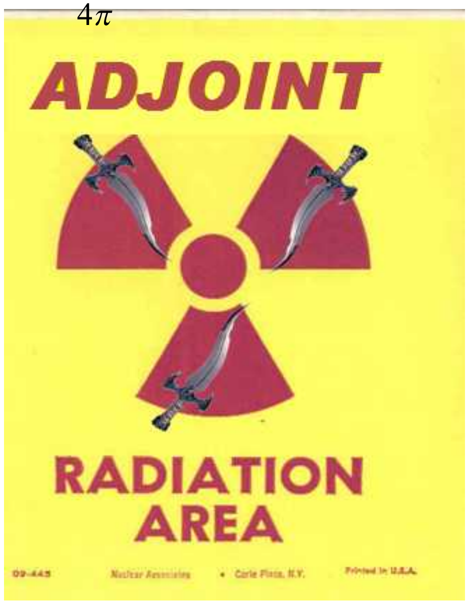
- Ray effects may skew angular tallies/integrated quantities of interest

# Adjoint Radiation Transport

**Adjoint Linear Boltzmann Transport Equation:**

$$-\vec{\Omega}\cdot\nabla\psi^{\dagger}\left(\vec{r},\vec{\Omega},E\right)+\left[\sigma_{s,el}\left(\vec{r},E\right)+\sigma_{s,in}\left(\vec{r},E\right)\right]\psi^{\dagger}\left(\vec{r},\vec{\Omega},E\right)=$$

$$\int_{4\pi}\sigma_{s,el}\left(\vec{r},\vec{\Omega}\cdot\vec{\Omega}',E\right)\psi^{\dagger}\left(\vec{r},\vec{\Omega}',E\right)d\vec{\Omega}'+\int_{0}^{\infty}\sigma_{s,in}\left(\vec{r},\vec{\Omega},E\rightarrow E'\right)\psi^{\dagger}\left(\vec{r},\vec{\Omega},E'\right)dE'$$



Anti-symmetry in the first operator makes the problem “backwards”

Notice the daggers, and the energetic changes

Angular DCS is unchanged by symmetry

Don’t worry about boundary conditions…

# Transforming the Adjoint Equation

- Differences stem in the total and DCS:

$$\sigma_{e,tot}\left(E\right)\neq\int_{0}^{\infty}\sigma_{e}\left(E\rightarrow E'\right)\psi^{\dagger}\left(E'\right)dE'$$

- Can the problem be made to look forward?

YES

- Adjusting the DCS is akin to classical variance reduction techniques

$$\int_{0}^{\infty}\underset{\uparrow}{\sigma_{e}\left(E'\rightarrow E\right)}\underset{\uparrow}{\frac{\sigma_{e}\left(E\rightarrow E'\right)}{\sigma_{e}\left(E'\rightarrow E\right)}}dE'$$

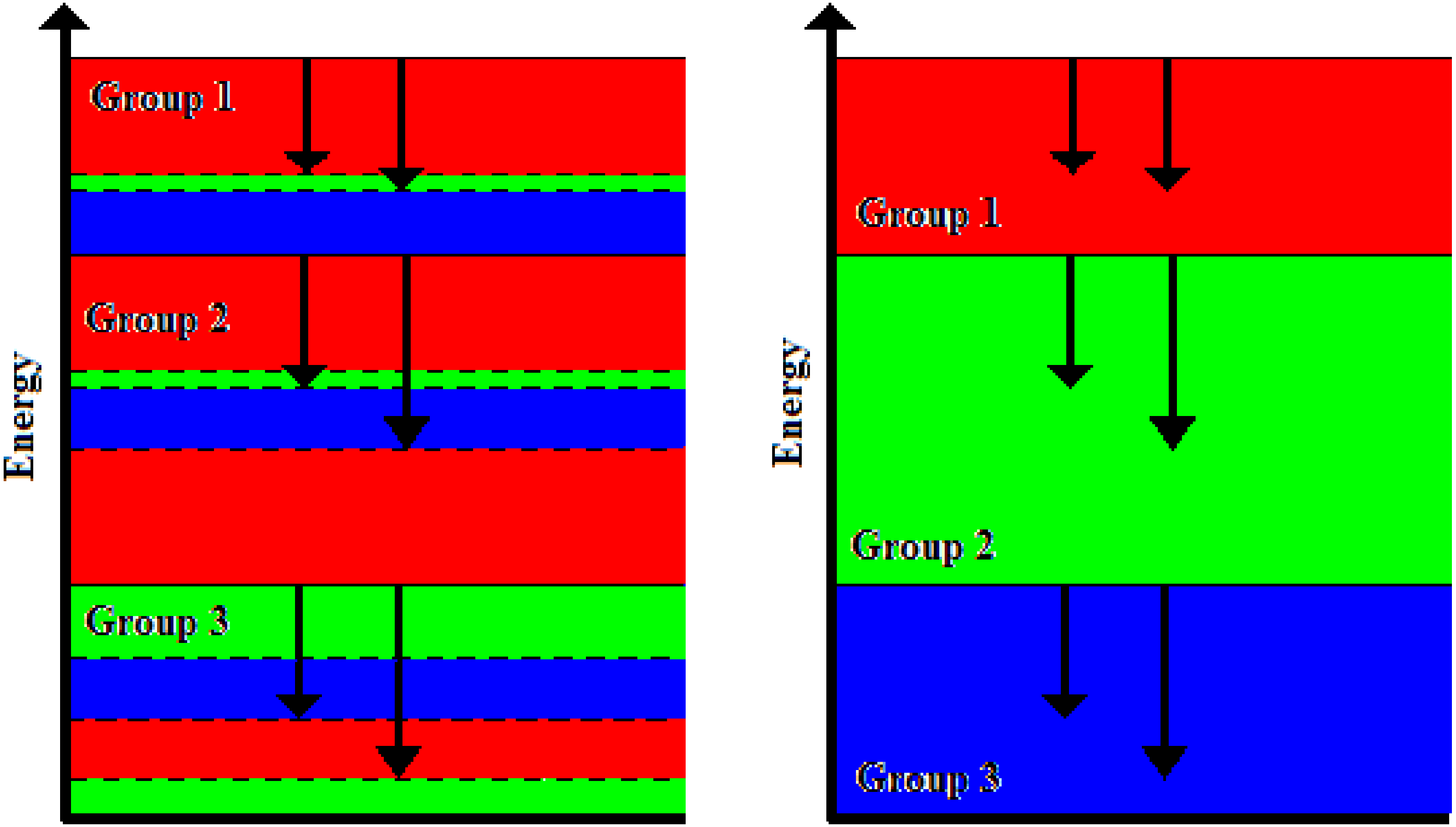
Sample Interaction

Tally this weight

# Weight Adjustments: The Key to GBFP[†]

- **Energy/angular scattering are assumed to be independent and separable; only energy affects the particle's weight**
- **Weights corrections are ratio of energetic inscatter to outscatter**
- **Regions of "multiplication" and "absorption" both occur and depend on energy group structure**
- **Weight windows, Russian roulette, and splitting are some obvious next steps**

# Complexity of Adjoint Energy Grid



# Test Problem

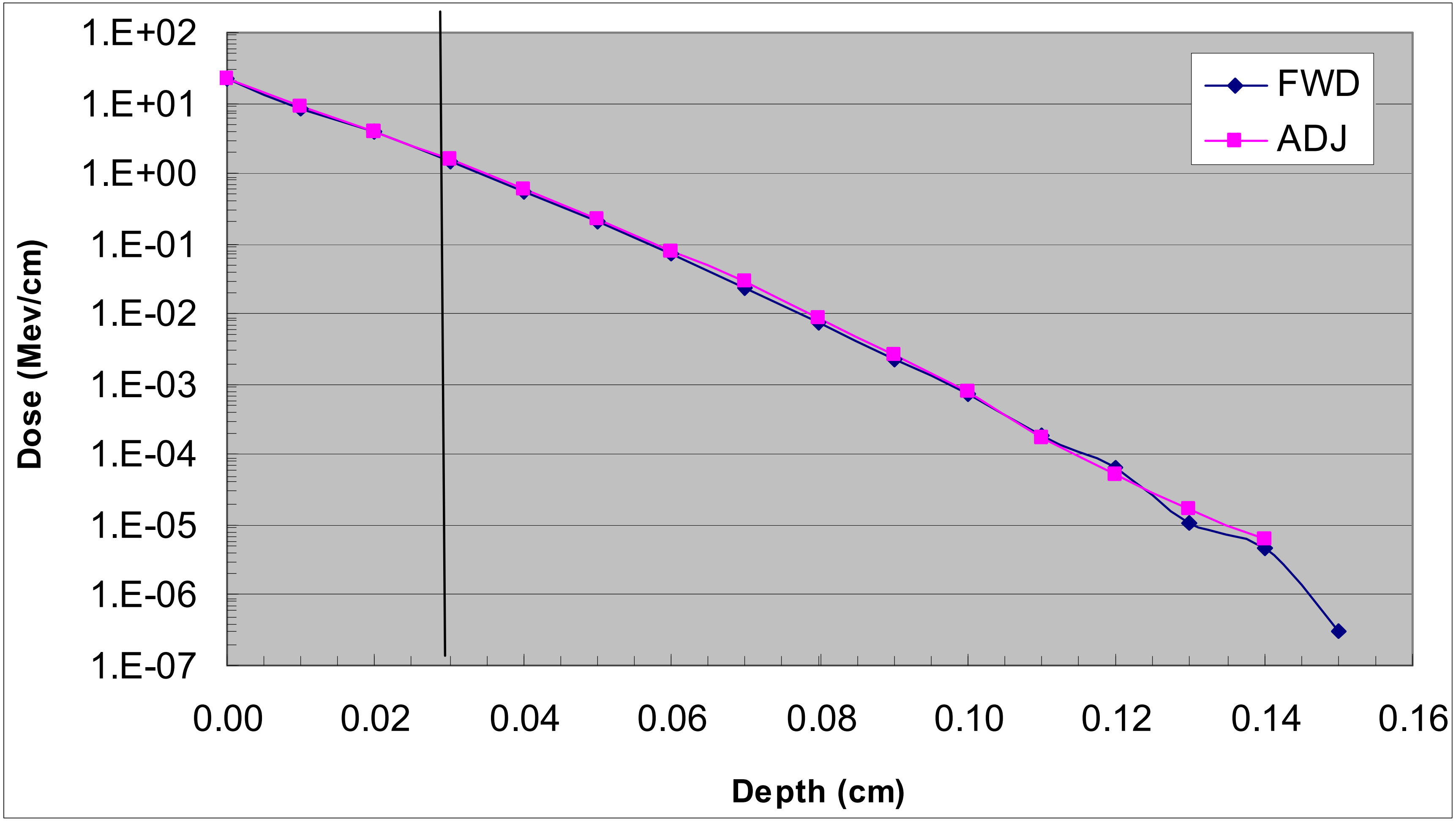
- Slab geometry with source at left interface
- Two materials: 0.03 cm of Au followed by 0.47 cm of (*essentially* infinite) Si
- Source is isotropic in angle and uniform in energy [0, 1 MeV]

(all histories in prescribed energy range are tallied equally)

- Simulation used two discrete angles and one discrete energy
- Results generated using $10^6$ histories

# Energy Deposition

**Au/Si Slabs, 2 Angles, 1 Energy, $10^6$ History, Uniform and Isotropic Source**



# Questions? Comments? Unequivocal praise?