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Saturation of Electromagnetic Flute Mode Instability in the Precursor Plasma of a Z-Pinch

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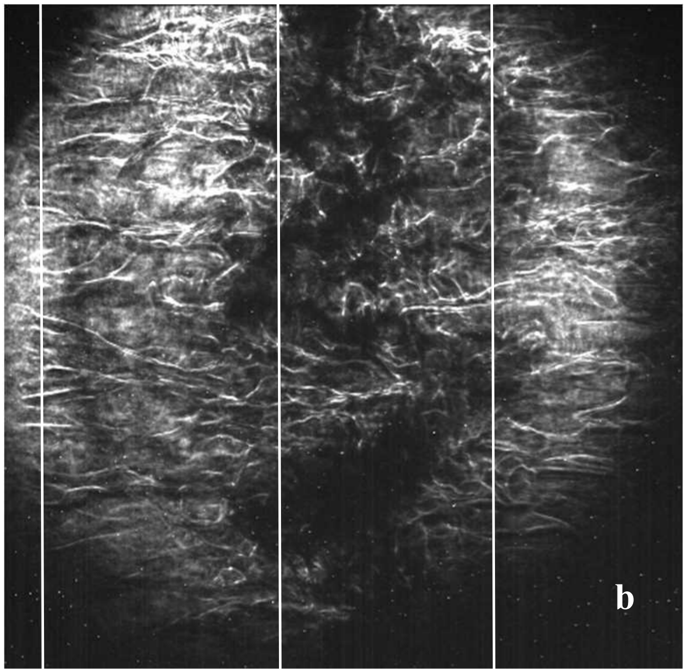
Goals

- To derive nonlinear equations for flute modes in a high beta plasma, when magnetic components of excited wave field are included into the system.
- To analyze linear stage of flute mode instability in high beta plasma with inhomogeneous magnetic field.
- To develop a code for numerical solution of the nonlinear system of equations for flute mode turbulence.
- To investigate nonlinear saturation of flute mode instability

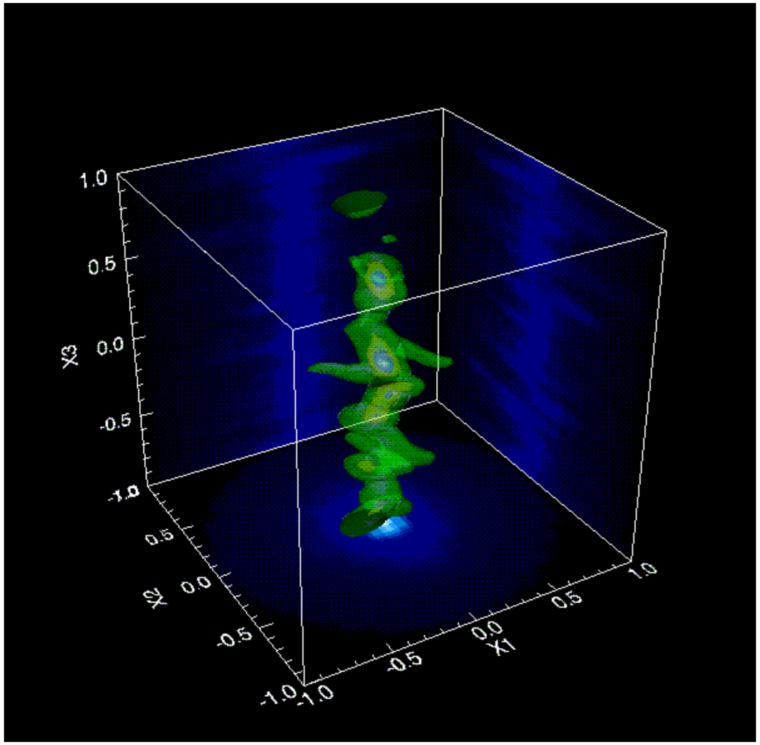
Global MHD modes in the precursor

Cathode



Anode

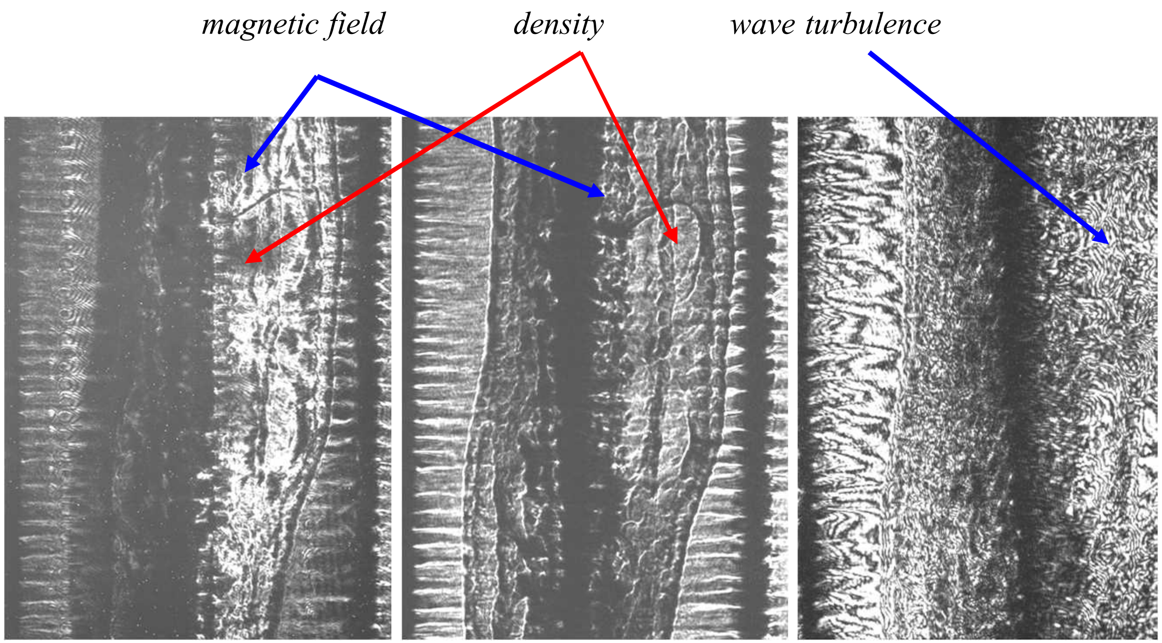
Experiment



Hybrid simulations

Laser probing and Faraday rotation experimental results

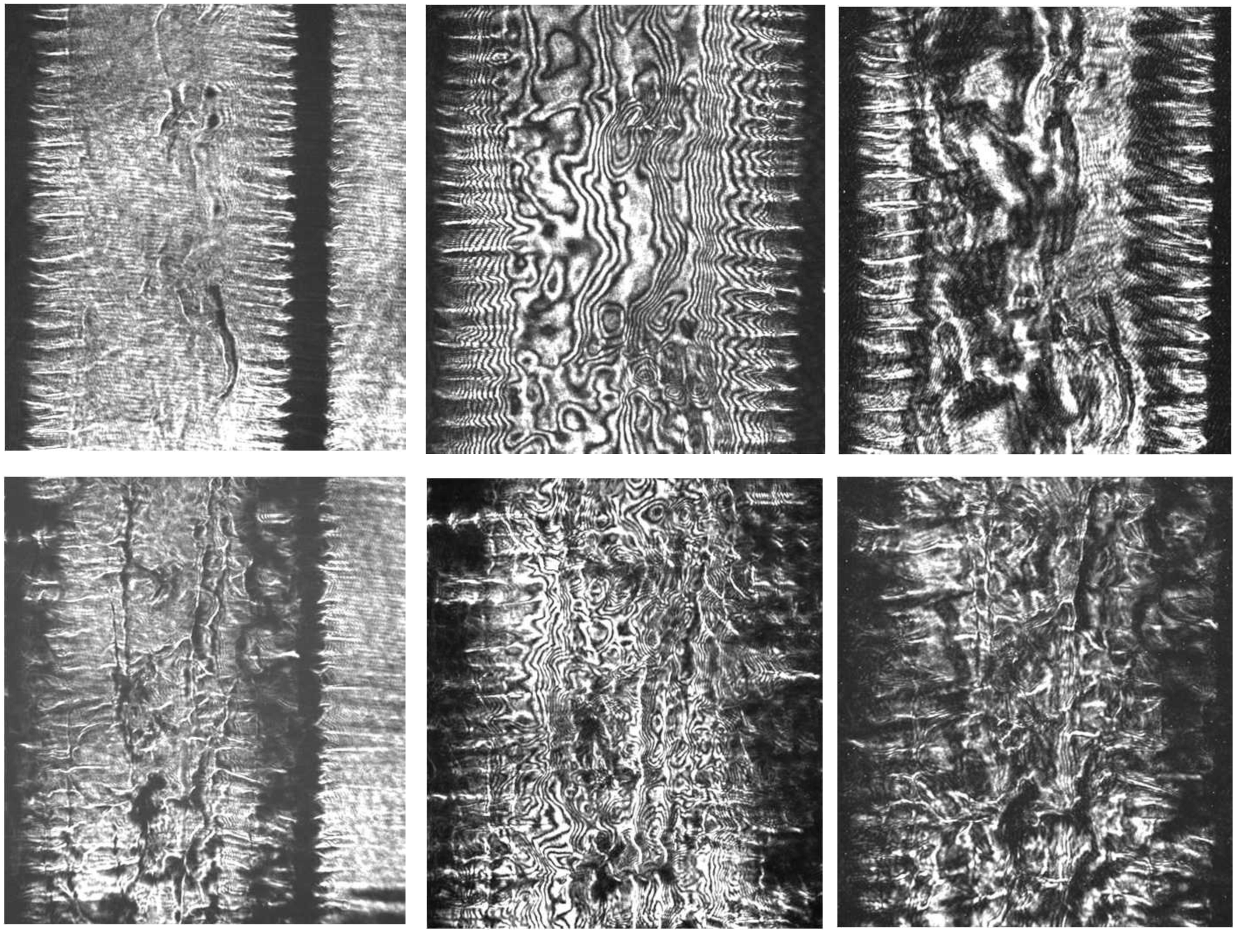
Experimental results indicate non MHD behavior of excited wave spectrum



Faraday channel   Shadowgraphy   Interferometry

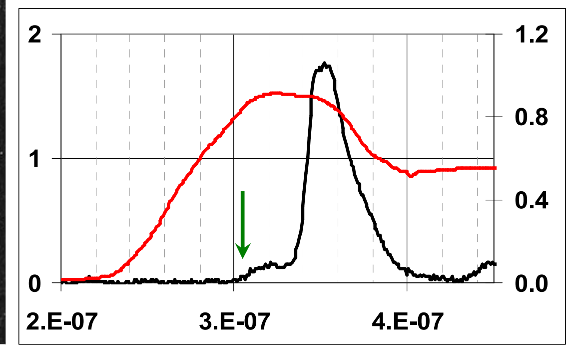
Development of plasma turbulence in the precursor

Al 8 x 15 μm wire arrays

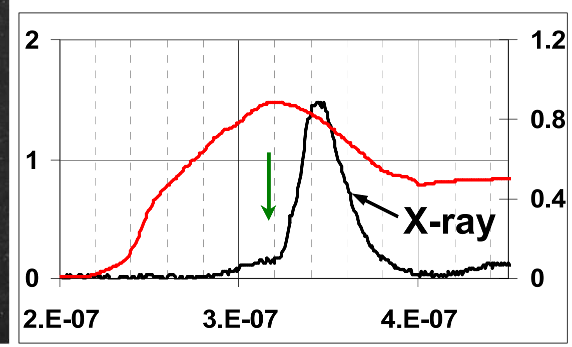


Shadowgram | Interferometry | Dark field schlieren diagnostic

Shot 454



Shot 453



Flute instability

small density perturbation $\delta n \sim \cos(\omega_k t - k_z z)$

⇓

perturbation of "gravitational" force $\delta F_r = g \delta n$

⇓

ions drift in axial z-direction

⇓

separation of charge

⇓

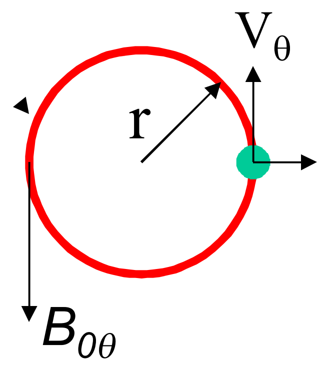
electric field

⇓

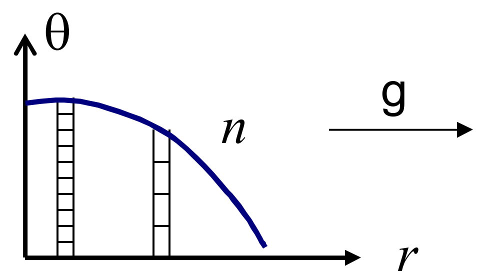
drift motion in *r*-direction brings particles from denser plasma core

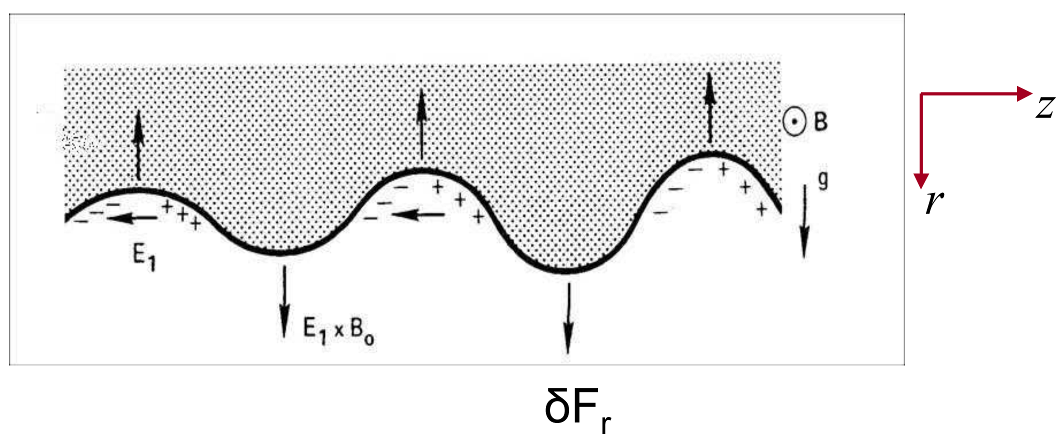
⇓

instability

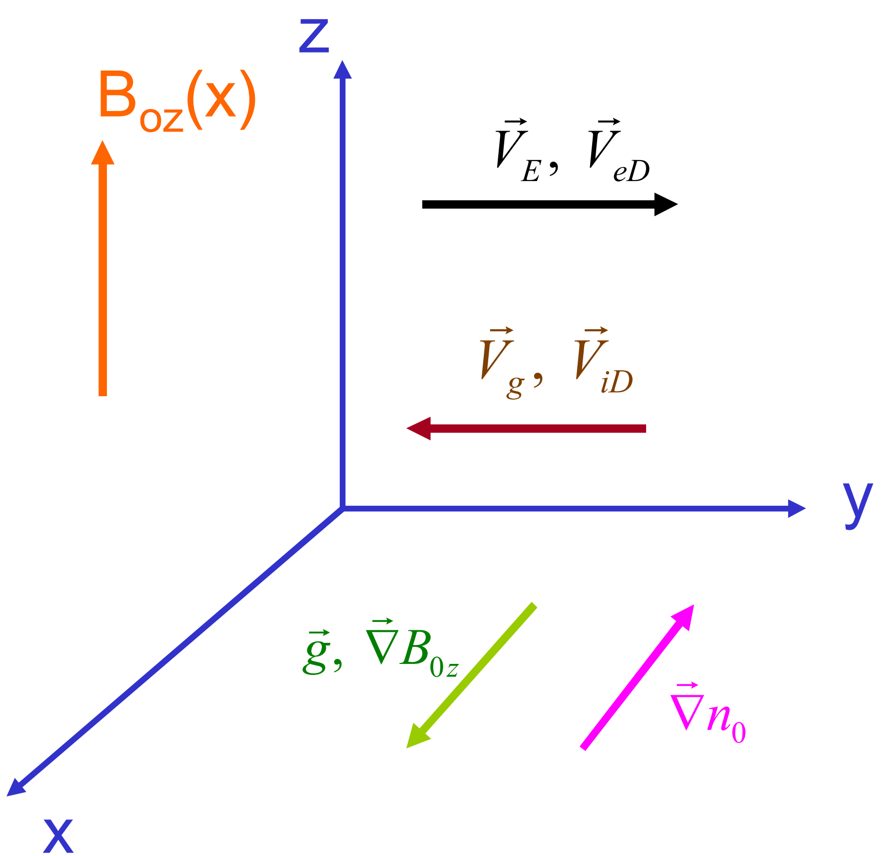


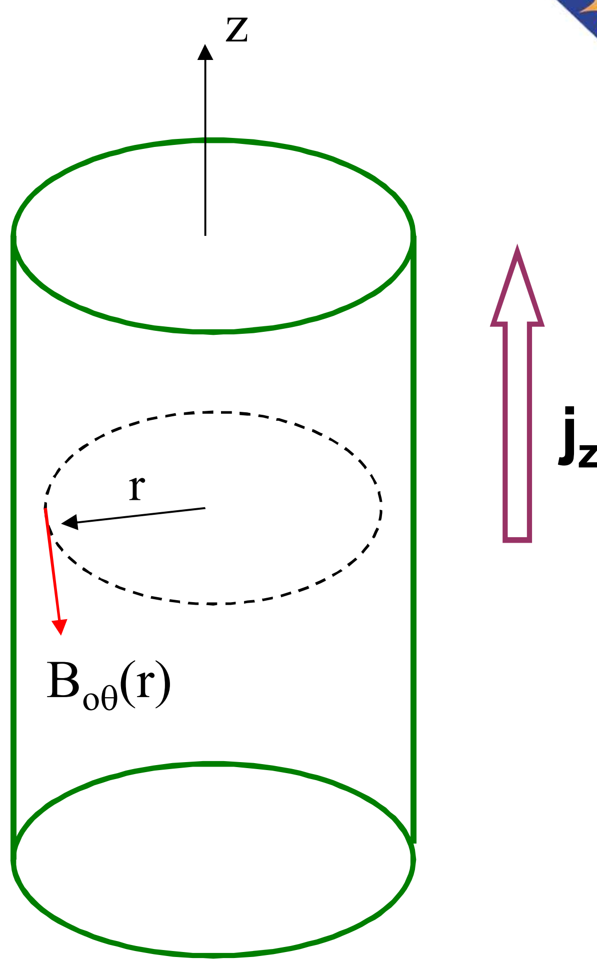
$$g = \frac{V_\theta^2}{r} \approx \frac{V_T^2}{r}$$





Geometry





$$\begin{array}{ccc} x & y & z \\ \downarrow & \downarrow & \downarrow \\ r & -z & \theta \end{array}$$

Flute mode turbulence in high beta plasma

For flute mode turbulence the contributions of density fluctuations and finite Larmor radius effects are significant.

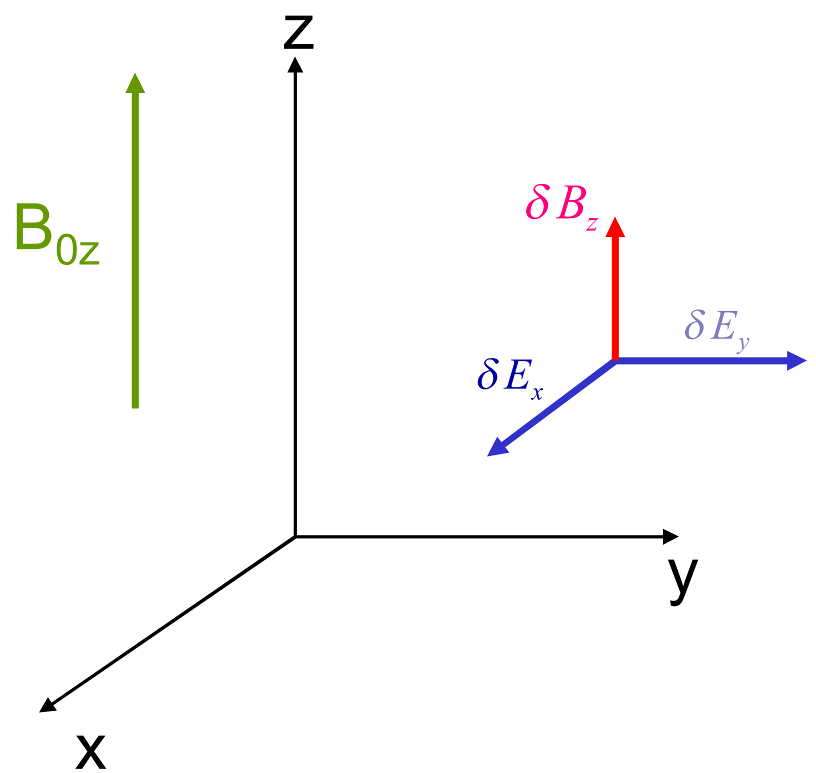
In high beta z-pinch plasma contribution due to electromagnetic effects should also be included.

$$\frac{\omega}{\Omega_i} << 1, \quad k_{\parallel} = 0, \quad \Phi(x,y,t), \quad N(x,y,t), \quad A_x(x,y,t), \quad A_y(x,y,t)$$

$$\vec{E} = -\vec{\nabla}\Phi(x,y,t) - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \qquad \vec{B} = rot\ \vec{A}$$

$$N(x,y,t) = n_0(x) + \delta n(x,y,t) \qquad \vec{B}(x,y,t) = [B_{0z}(x) + \delta B_z(x,y,t)]\ \vec{e}_z$$

Configuration of excited wave fields



$$\delta B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

$$\delta E_x = -\frac{\partial \Phi}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}$$

$$\delta E_y = -\frac{\partial \Phi}{\partial y} - \frac{1}{c}\frac{\partial A_y}{\partial t}$$

$$\frac{\delta B_z}{B_{0z}} = -\frac{1}{2}\beta\frac{\delta n}{n_o}$$

Basic equations

$$div\,\vec{j} = 0$$

$$\frac{\partial n_e}{\partial t} + div\; n_e \vec{v}_e = 0$$

$$\frac{\partial B_z}{\partial y} = \frac{4\pi}{c} j_x$$

$$n_e = Z n_i$$

Nonlinear equations for flute modes in high beta plasma

$$\frac{\delta B_z}{B_{0z}} = -\frac{1}{2}\beta\frac{\delta n}{n_o} \qquad (5)$$

$$\beta_i = \frac{8\pi \mathrm{n}_{0\mathrm{i}} T_i}{B_0^2} \qquad \beta = \beta_i + \beta_\mathrm{e} \qquad \beta_e = \frac{8\pi \mathrm{n}_{0\mathrm{e}} T_e}{B_0^2}$$

$$\frac{\partial^*}{\partial t} = \frac{\partial}{\partial t} + V_{iy}^{(0)}\frac{\partial}{\partial y}$$

$$V_x^{(1)} = -\frac{c}{B_z}\frac{\partial}{\partial y}(\Phi + \frac{T_i}{e}\ln N_i) - \frac{1}{B_z}\frac{\partial A_y}{\partial t} \qquad V_y^{(1)} = \frac{c}{B_z}\frac{\partial}{\partial x}(\Phi + \frac{T_i}{e}\ln N_i) - \frac{1}{B_z}\frac{\partial A_x}{\partial t}$$

$$V_{ix}^{(3)} = -\frac{1}{\Omega_i}\left\{\frac{\partial^* V_{iy}^{(1)}}{\partial t} + \left(V_{ix}^{(1)}\frac{\partial V_{iy}^{(1)}}{\partial x} + V_{iy}^{(1)}\frac{\partial V_{iy}^{(1)}}{\partial y}\right) + \frac{1}{M_i N_i}\frac{\partial}{\partial x}\left[\frac{N_i T_i}{2\Omega_{ci}}\left(\frac{\partial V_{ix}^{(1)}}{\partial x} - \frac{\partial V_{iy}^{(1)}}{\partial y}\right)\right] + \frac{1}{M_i N_i}\frac{\partial}{\partial y}\left[\frac{N_i T_i}{2\Omega_{ci}}\left(\frac{\partial V_{ix}^{(1)}}{\partial y} + \frac{\partial V_{iy}^{(1)}}{\partial x}\right)\right]\right\}$$

$$V_{iy}^{(3)} = \frac{1}{\Omega_i}\left\{\frac{\partial^* V_{ix}^{(1)}}{\partial t} + \left(V_{ix}^{(1)}\frac{\partial V_{ix}^{(1)}}{\partial x} + V_{iy}^{(1)}\frac{\partial V_{ix}^{(1)}}{\partial y}\right) + \frac{1}{M_i N_i}\frac{\partial}{\partial x}\left[-\frac{N_i T_i}{2\Omega_{ci}}\left(\frac{\partial V_{ix}^{(1)}}{\partial y} + \frac{\partial V_{iy}^{(1)}}{\partial x}\right)\right] + \frac{1}{M_i N_i}\frac{\partial}{\partial y}\left[\frac{N_i T_i}{2\Omega_{ci}}\left(\frac{\partial V_{ix}^{(1)}}{\partial x} - \frac{\partial V_{iy}^{(1)}}{\partial y}\right)\right]\right\}$$

$$\frac{\partial^*}{\partial t} = \frac{\partial}{\partial t} + V_{iy}^{(0)}\frac{\partial}{\partial y}$$

Nonlinear equations for flute modes in high beta plasma

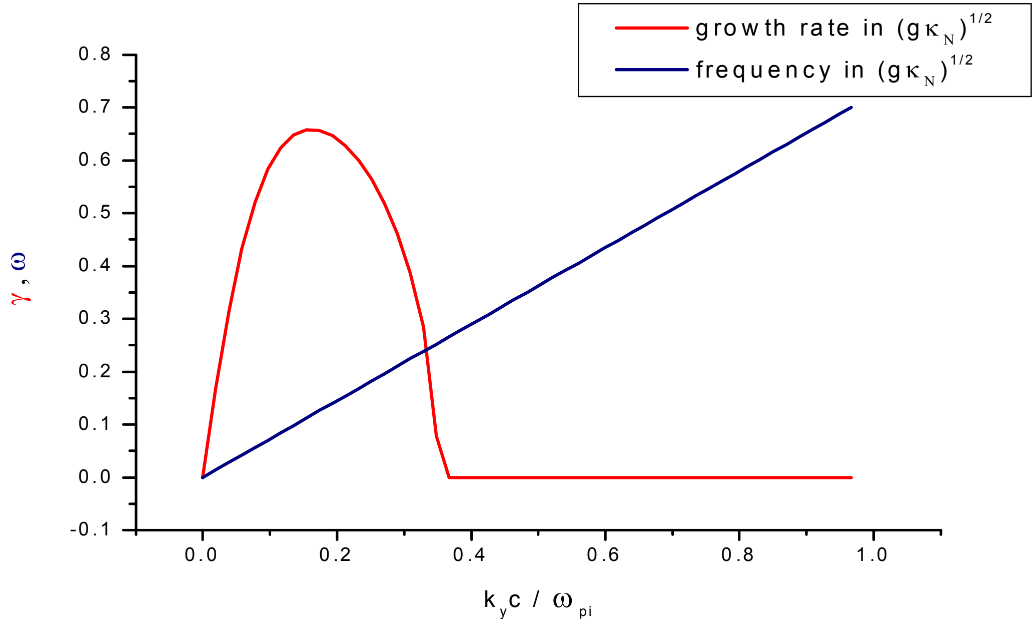
$$\frac{\partial \delta n}{\partial t}+\frac{V_{Te}^{*2}}{\Omega_{0i}}\kappa_N\frac{\partial n}{\partial y}+\kappa_N c\frac{n_0}{B_{0z}}\frac{\partial \Phi}{\partial y}-\frac{1}{B_{0z}}\frac{\partial \delta B_z}{\partial t}=\frac{c}{B_{0z}}\left[\vec{\nabla}\delta n\times\vec{\nabla}\Phi\right]_z$$

$$\frac{\partial^*}{\partial t}\Delta_\perp\Phi+\frac{T_i}{en_0}\frac{\partial^*}{\partial t}\Delta_\perp\delta n-\frac{T_i}{2eB_{0z}}\frac{\partial}{\partial t}\Delta_\perp\delta B_z+$$

$$+(1-\frac{\beta_i}{2})\frac{B_{0z}}{n_0}\frac{g}{c}\frac{\partial \delta n}{\partial y}-\frac{g}{c}\frac{\partial \delta B_z}{\partial y}=$$

$$=\frac{c}{B_{0z}}[\vec{\nabla}(\Delta_\perp\Phi)\times\vec{\nabla}\Phi]_z+\frac{V_{Ti}^2}{\Omega_{0i}}\frac{1}{n_0}div[\vec{\nabla}(\nabla_\perp\delta n)\times\vec{\nabla}\Phi]_z$$

Growth rate and frequency of flute modes



Experimentally observed plasma parameters

$$n_{0e} = 2.0 \times 10^{19}\ \mathrm{cm}^{-3},\ n_{0i} = 2.0 \times 10^{18}\ \mathrm{cm}^{-3},\ T_i \sim T_e\text{e}= 50\ \mathrm{eV},\ \ B_0 = 0.3\ \mathrm{MG}$$

$$\frac{\omega_{pAl}}{c} = 4 \times 10^2\ cm^{-1} \quad \frac{kc}{\omega_{pAl}} \approx 0.3 \quad \lambda \approx 0.5\ mm \qquad R \approx 1 \div 3\ mm \qquad \mathrm{g} \approx$$

$$\beta = 0.55 \quad \Omega_{cAl} = 1.1 \times 10^{10}\ s^{-1} \quad \omega_{\mathrm{pAL}} = 1.4 \times 10^{13}\ s^{-1}\ \gamma \approx 3 \times 10^{7}\ s^{-1}$$

Nonlinear cascades

In contrast to Hasegawa-Mima equation for electrostatic drift turbulence, for flute mode turbulence in a low beta plasma there are two perturbed quantities: potential Φ and density N.

The polarization drift nonlinearity is $[\vec{\nabla}\Phi \times \vec{\nabla}(\Delta_{\perp}\Phi)]_z$
is known to cascade power towards long scales.

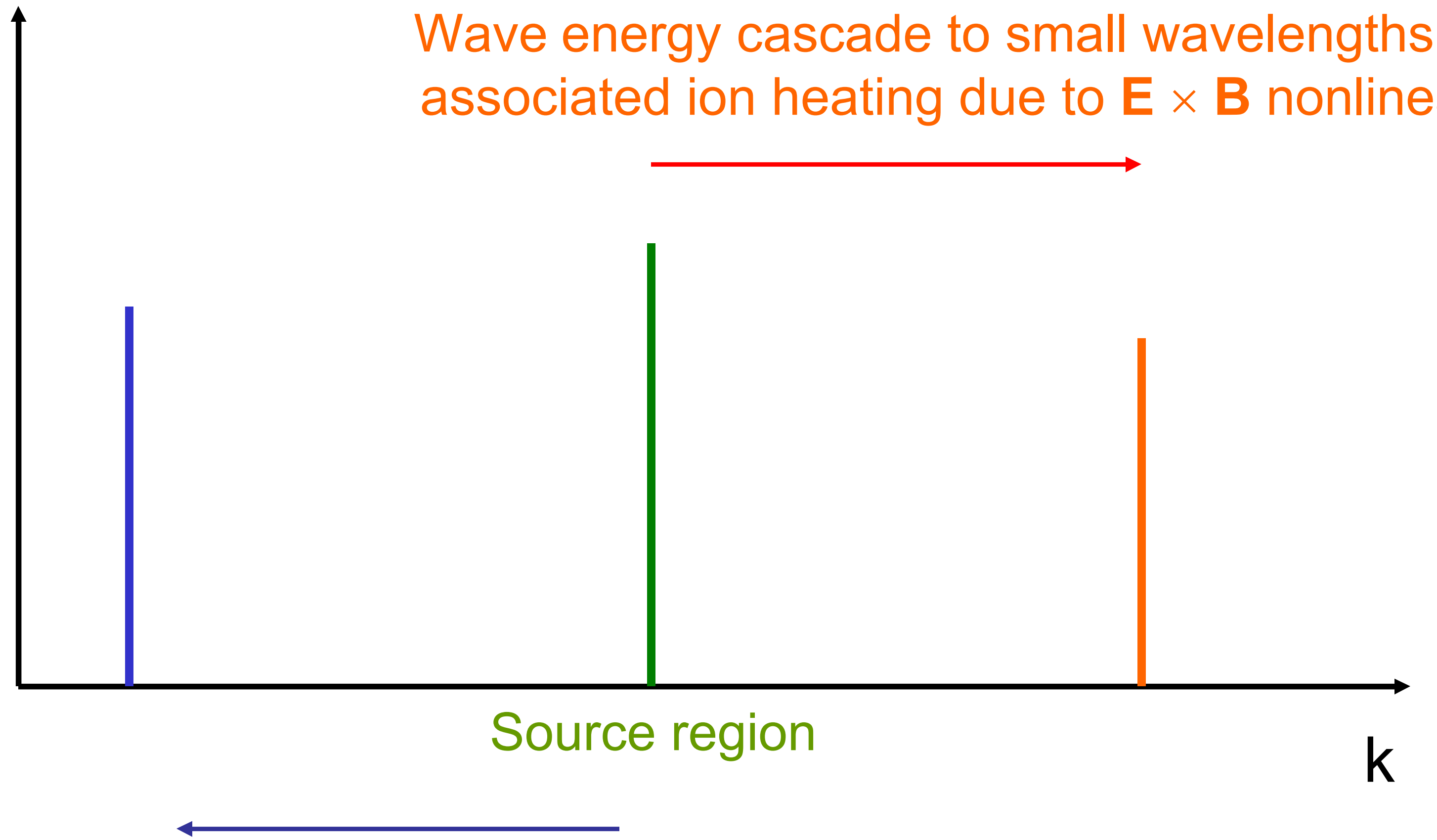
The contribution of density fluctuations to the ion polarization drift is significant.
The diamagnetic component of polarization nonlinearity

$\frac{T}{e} div\, [\vec{\nabla} n \times \vec{\nabla}(\nabla_{\perp}\Phi)]$ can significantly modify cascade properties.

The convective nonlinearity $\frac{c}{B_{0z}}\left[\vec{\nabla}\Phi \times \vec{\nabla} n\right]_z$ on the

contrary, cascades power into the shorter scale lengths.

Nonlinear wave cascades



Wave energy cascade to large wavelengths and excitation of large scale structures due to polariztion drift nonlinearity

Fluid Model of 2D Flute Turbulence

- **Numerical Solutions of Nonlinear Flute Equations.**
- **Pseudo-spectral spatial representation**
- **Two-step predictor-corrector time advance**

- Arakawa method used throughout to treat Poisson bracket nonlinear terms
- Viscous and biharmonic dissipations advanced explicitely in time

Dimensionless variables

$\tau = \Omega_{ci}\tilde{t}$ $\tilde{t}$ - in sec (has dimension), tilda ~ - means dimensional

$$\Omega_{ci} = \frac{ZeB_{oz}}{M_i c} \quad \rho_i^* = \frac{c_s}{\Omega_{ci}} \quad \delta\mathrm{n} = \frac{\delta\tilde{n}_i}{n_{0i}} = \frac{\delta\tilde{n}_e}{n_{0e}} \quad n_{0e} = Zn_{0i} \quad c_s^2 = \frac{T_e}{M_i}$$

$$\Phi = \frac{e\tilde{\Phi}}{T_e} \quad \mathrm{g} = \frac{2\mathrm{c}_\mathrm{s}^2}{\mathrm{R}} \quad x = \frac{\tilde{x}}{c_s / \Omega_{ci}}$$

$\tilde{x}$ - dimensional variable

$$\frac{\partial^*}{\partial\tau} = \frac{\partial}{\partial\tau} + \frac{V_{iy}^{(0)}}{c_s}\frac{\partial}{\partial y}$$

Nonlinear equations in dimensionless form

$$\frac{\partial}{\partial\tau}[\Delta_\perp\Phi]-[\frac{T_i}{T_e}(1+\frac{1}{4}\beta)\frac{V_n}{1+\frac{1}{2}\beta}+V_{gn}]\frac{\partial}{\partial y}\Delta_\perp\Phi+[\frac{T_i}{4ZT_e}\beta V_{gn}-\frac{T_i}{T_e}(1+\frac{1}{4}\beta)(\frac{V_n}{1+\frac{1}{2}\beta}+\frac{V_{gn}}{Z})]\frac{\partial}{\partial y}\Delta_\perp\delta n+$$

$$(1+\frac{\beta_e}{2})\frac{V_g}{Z^2}\frac{\partial\delta n}{\partial y}=Z\square\{\Delta_\perp\Phi,\Phi\}+\frac{T_i}{T_e}[(1+\frac{1}{4}\beta)\frac{1}{1+\frac{1}{2}\beta}-1]\{\Phi,\Delta_\perp\delta n\}+$$

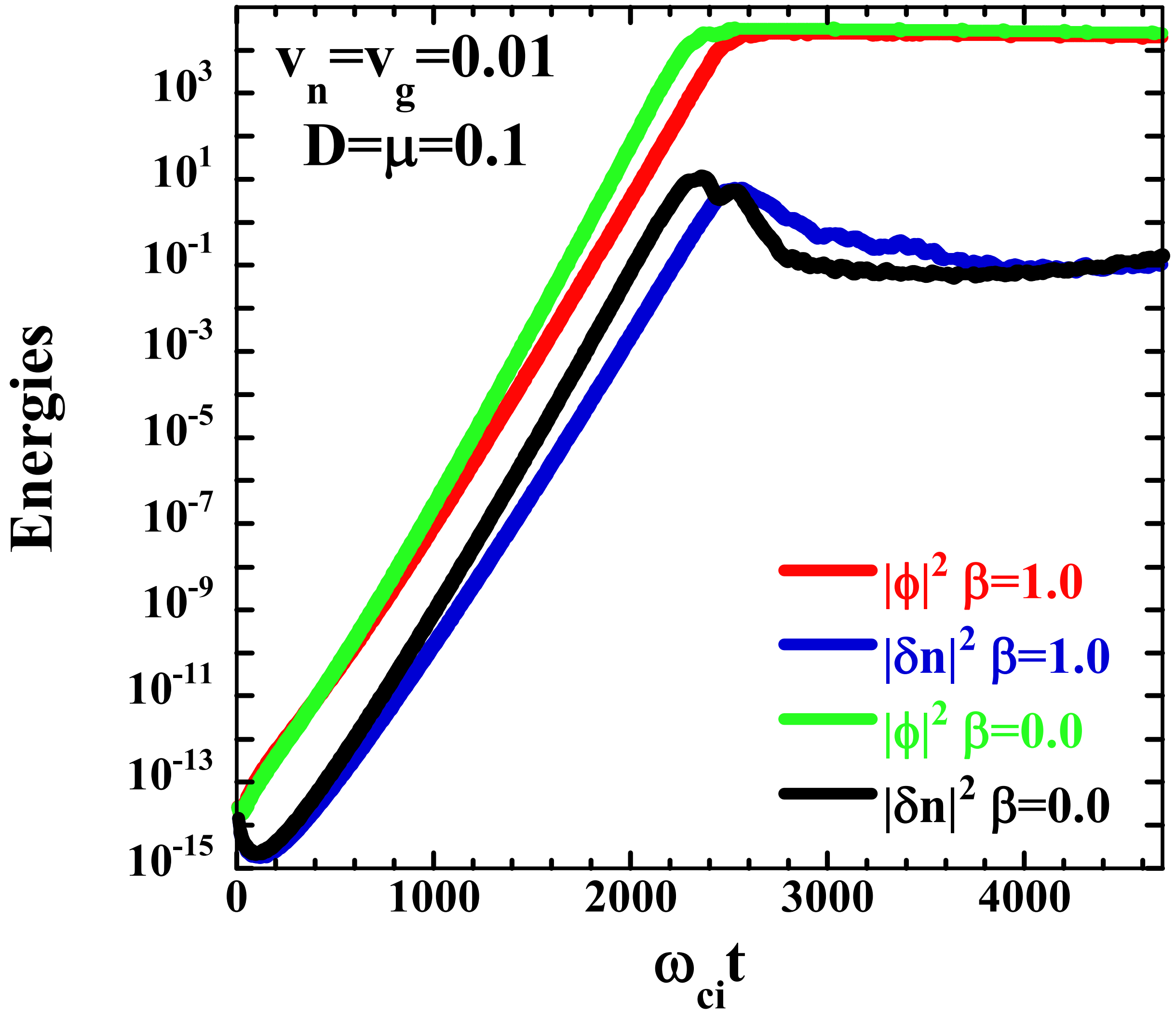
$$+\frac{T_i}{T_e}[(1+\frac{1}{4}\beta)\frac{2}{1+\frac{1}{2}\beta}-1][\{\frac{\partial\Phi}{\partial x},\frac{\partial\delta n}{\partial x}\}+\{\frac{\partial\Phi}{\partial y},\frac{\partial\delta n}{\partial y}\}]+\frac{T_i}{T_e}(1+\frac{1}{4}\beta)\frac{1}{1+\frac{1}{2}\beta}\{\Delta_\perp\Phi,\delta n\}+$$

$$+\mu\Delta_\perp(\Delta_\perp\Phi)-\frac{T_i}{T_e}(1+\frac{1}{4}\beta)D\Delta_\perp^2\delta n$$

$$[1+\frac{1}{2}(\beta_e+\beta_i)]\frac{\partial\delta n}{\partial\tau}+ZV_n\frac{\partial\delta n}{\partial y}+ZV_n\frac{\partial\Phi}{\partial y}=Z\square\{\delta n,\Phi\}+D\Delta_\perp\delta n$$

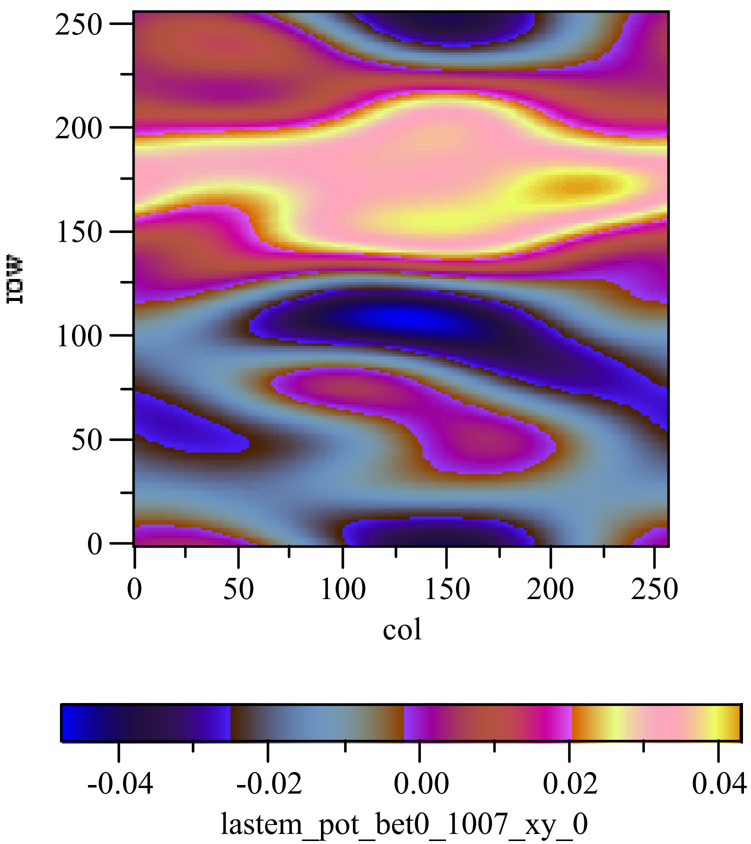
$$\mu=\frac{3}{10}\frac{T_i\nu_i}{M_i\Omega_{ci}}\qquad D=\frac{m_e}{e^2}\frac{T_e\nu_e}{B_{0z}^2}$$

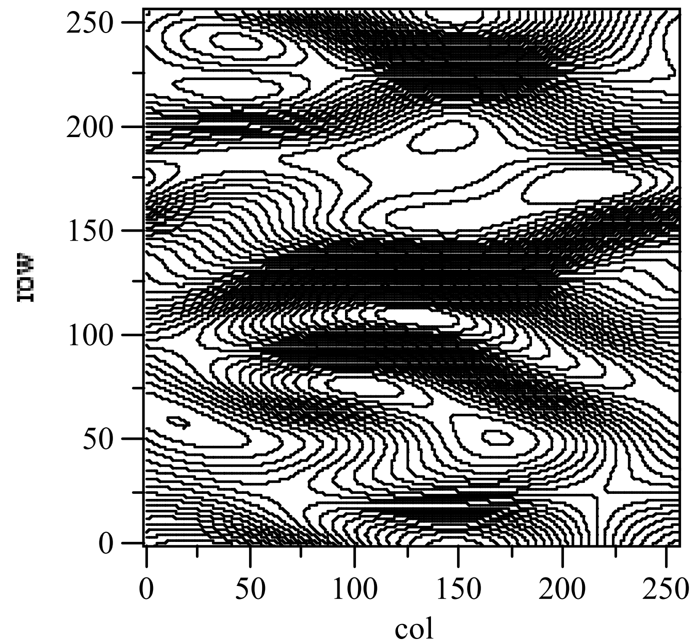
Time dependence of energy



Potential in linear stage

$$\beta = 0$$

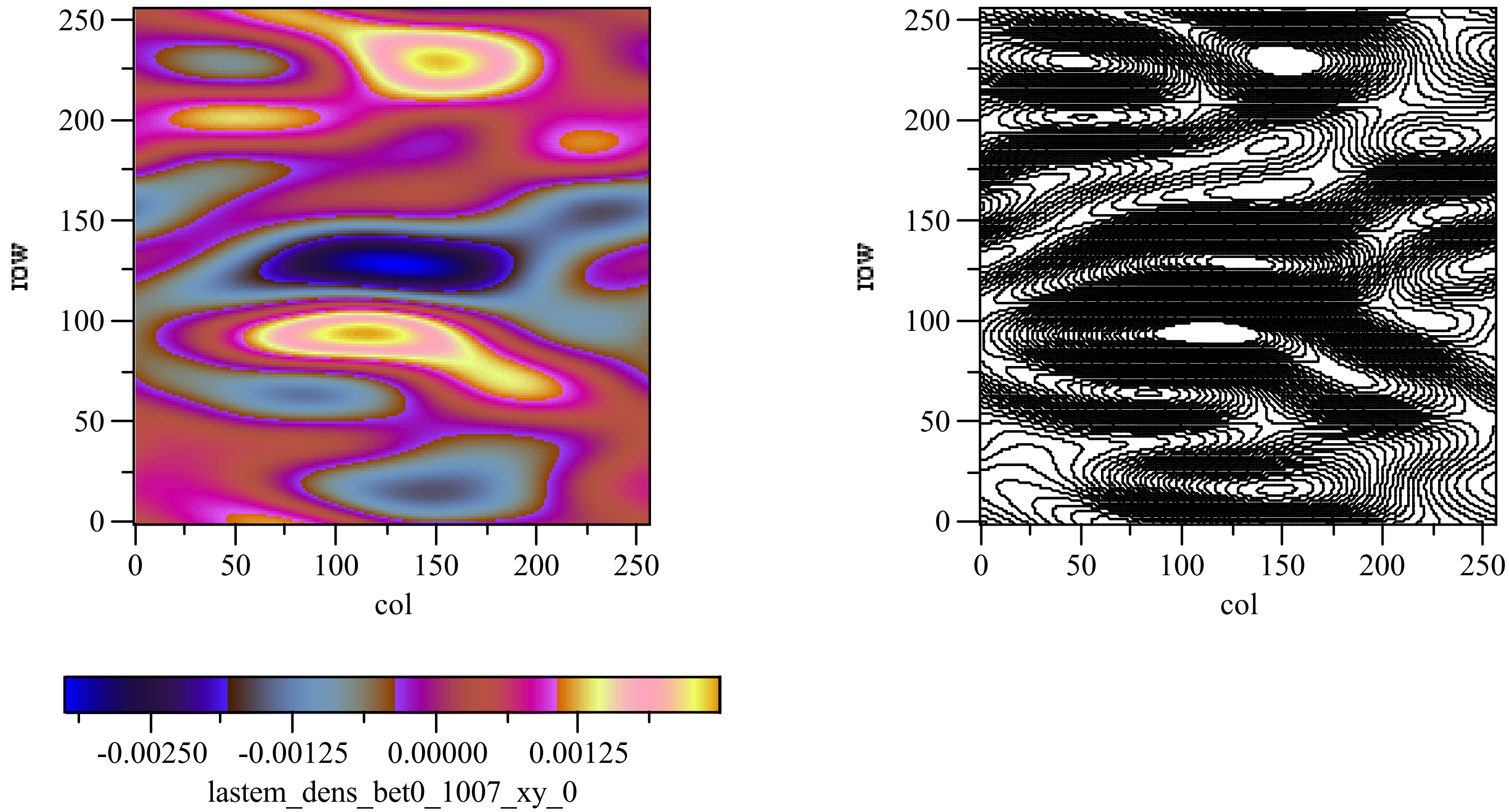




$$\Omega_{ci} t = 1400$$

Density in linear stage

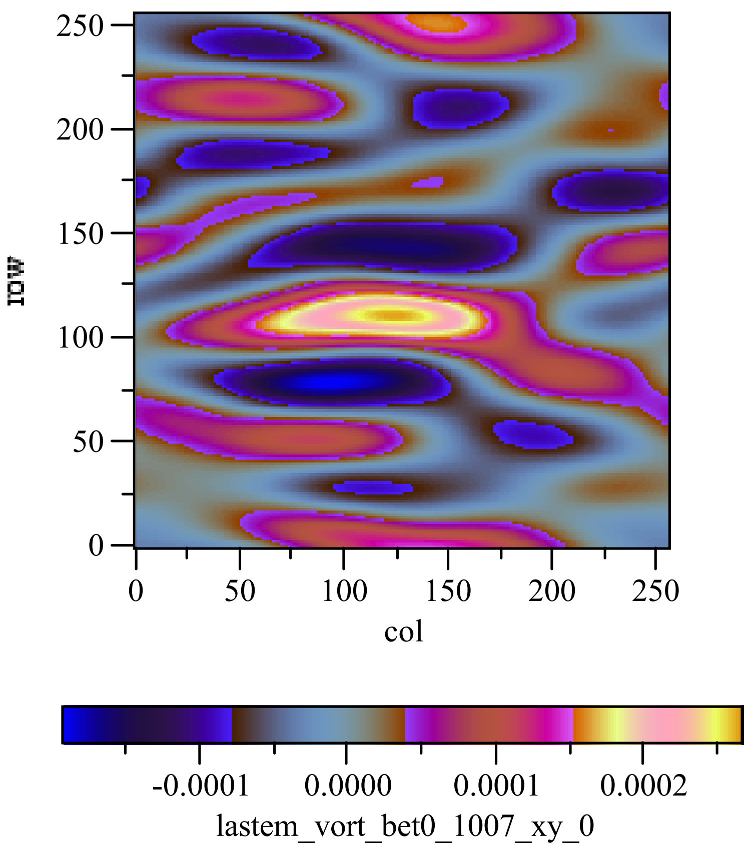
$$\beta = 0$$

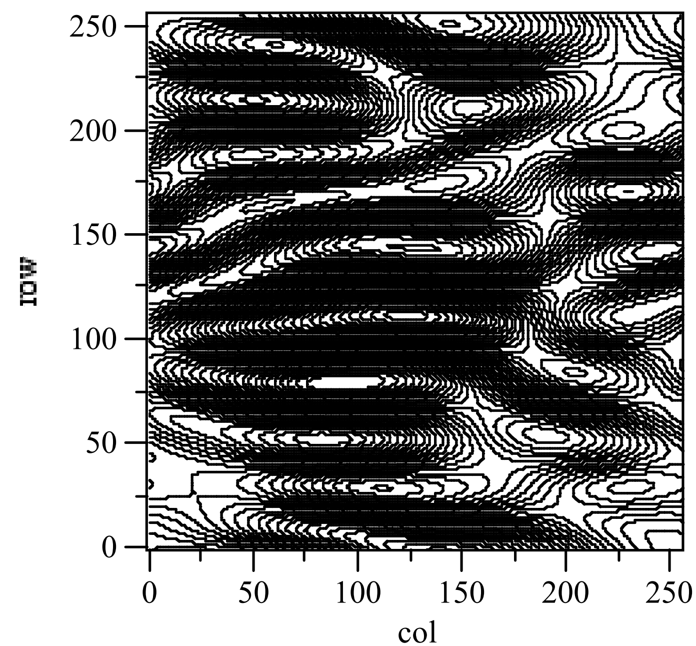


$$\Omega_{ci} t = 1400$$

Vorticity in linear stage

$$\beta = 0$$

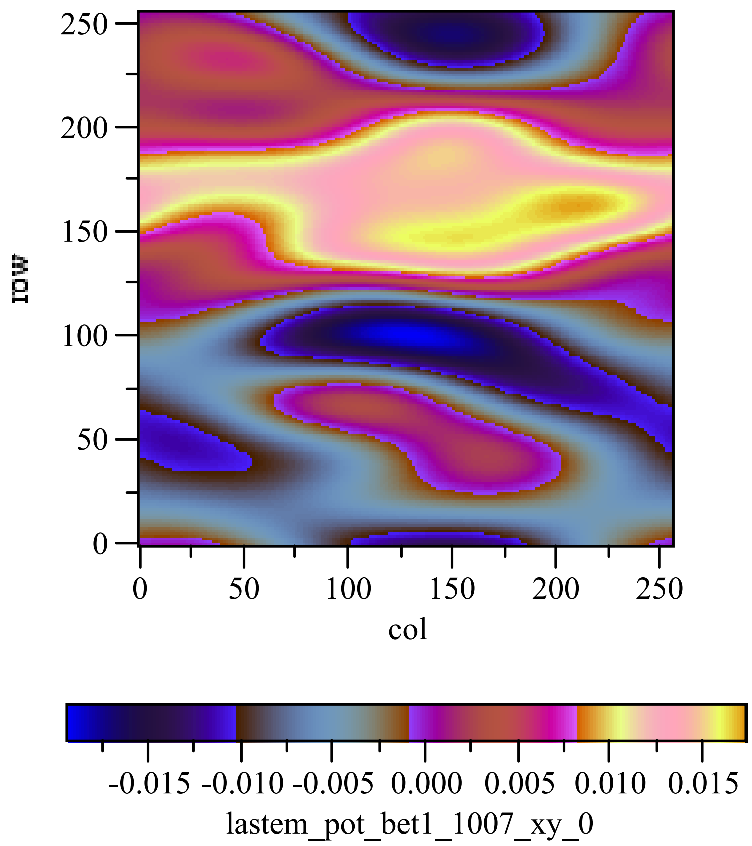


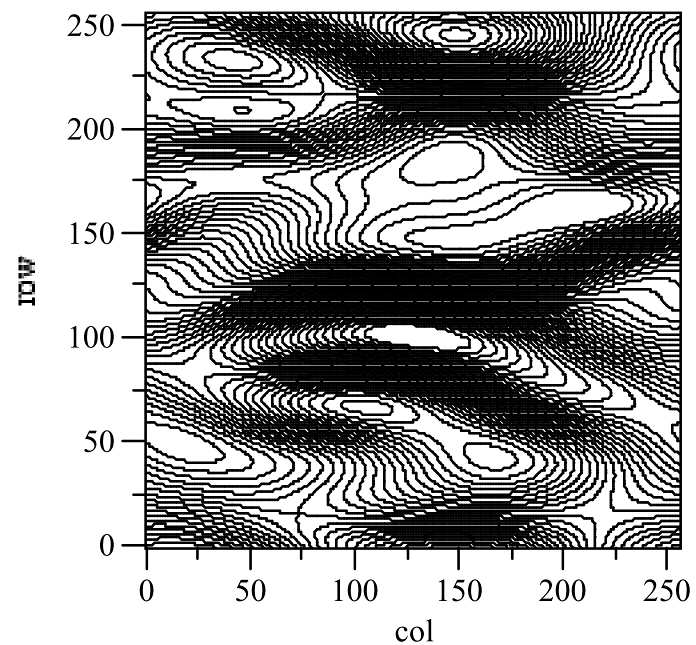


$$\Omega_{ci} t = 1400$$

Potential in linear stage

$$\beta = 1.0$$





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Density in linear stage

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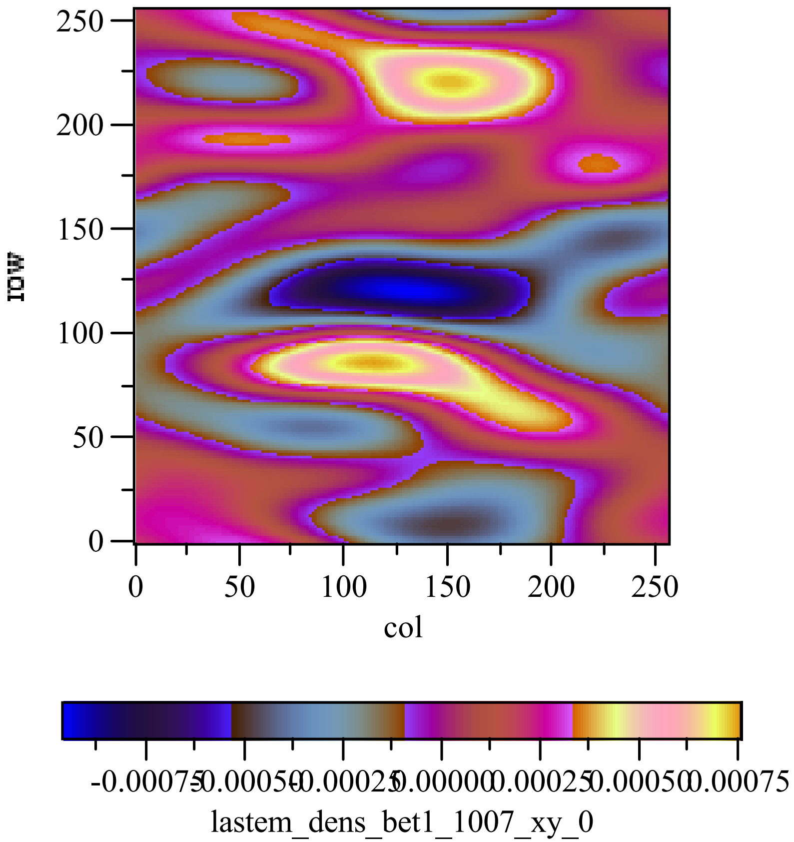
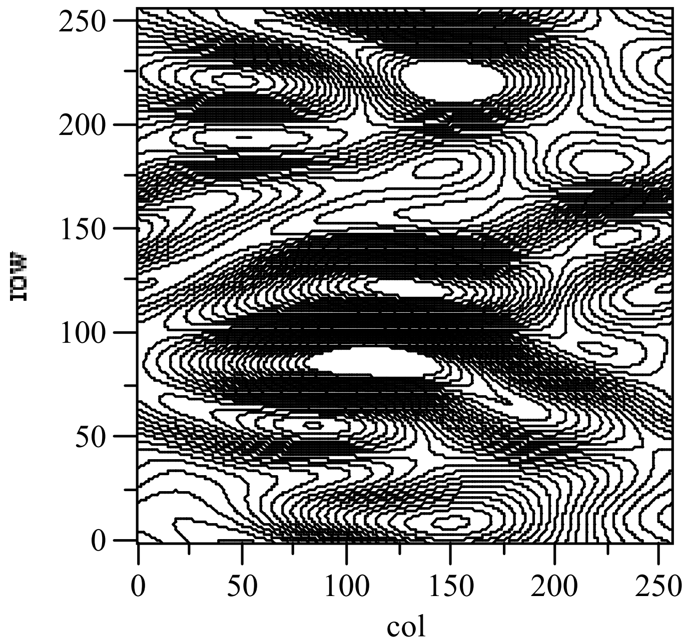
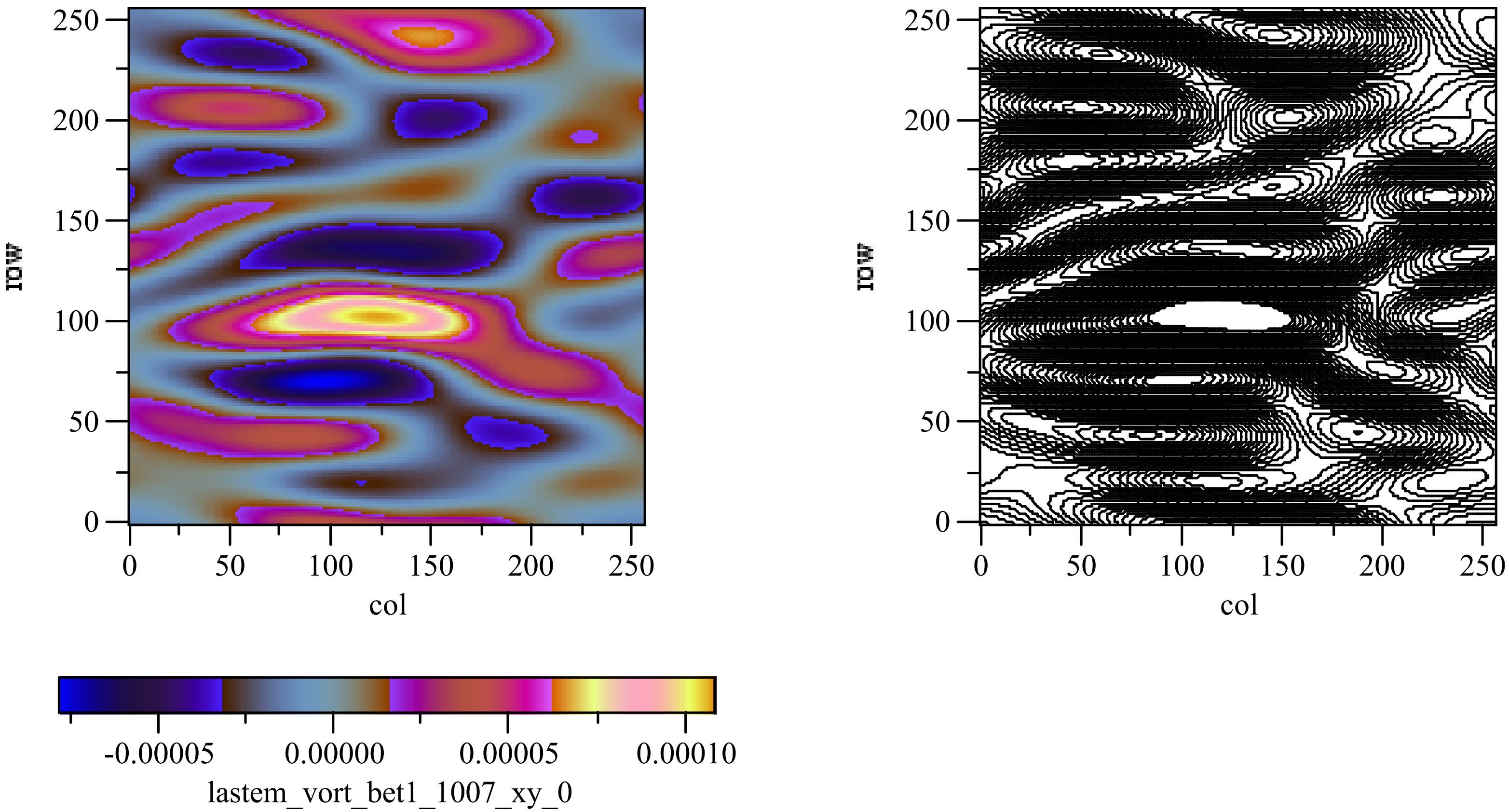




$$\Omega_{ci} t = 1400$$

Vorticity in linear stage

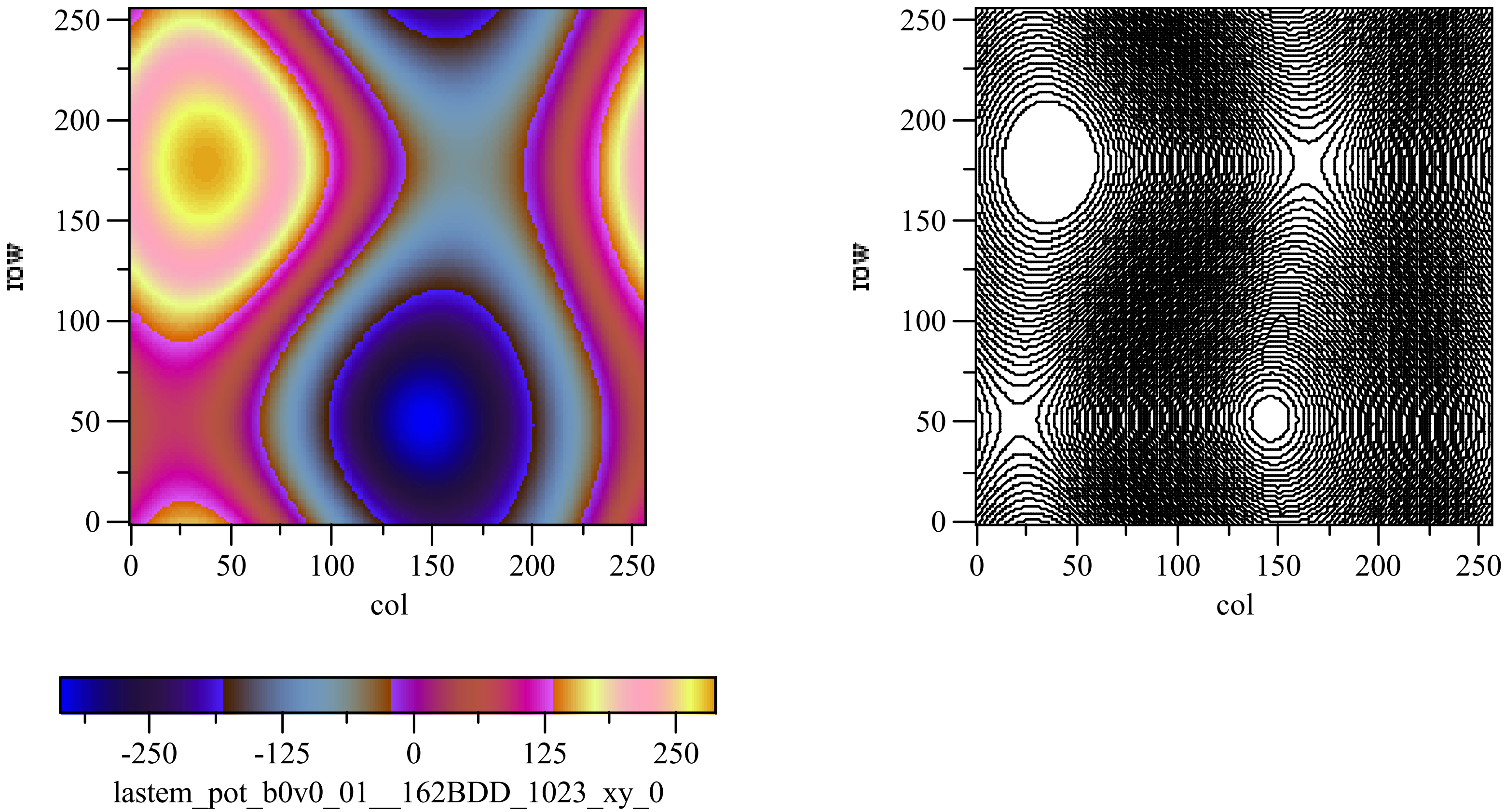
$$\beta = 1.0$$



$$\Omega_{ci} t = 1400$$

Potential in nonlinear stage

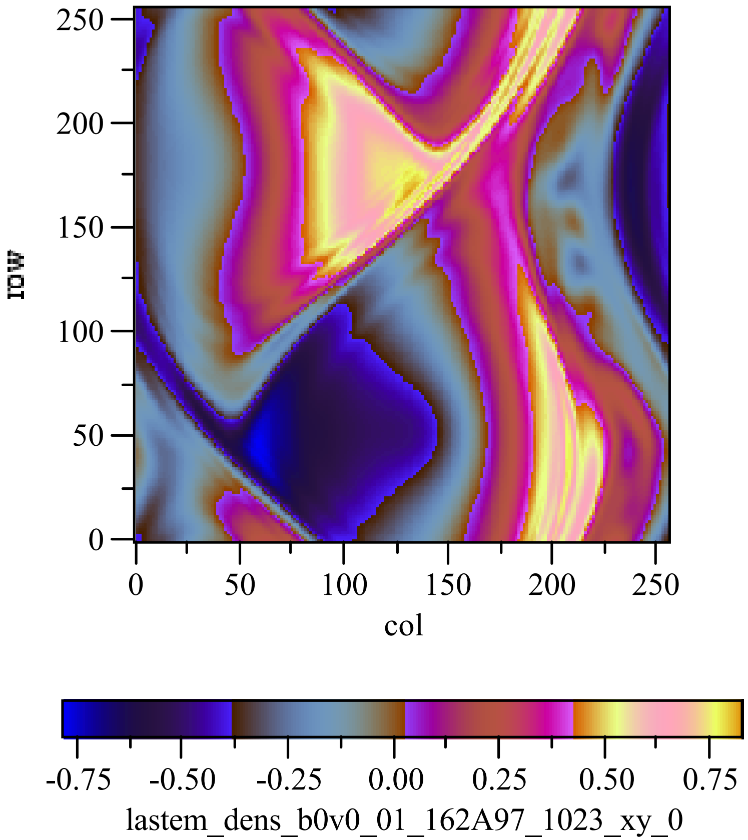
$$\beta = 0$$

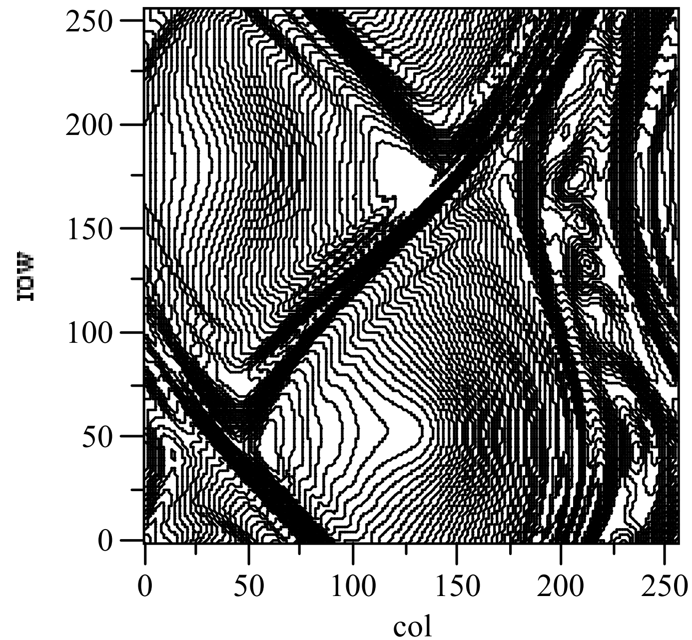


$$\Omega_{ci} t = 4600$$

Density in nonlinear stage

$$\beta = 0$$

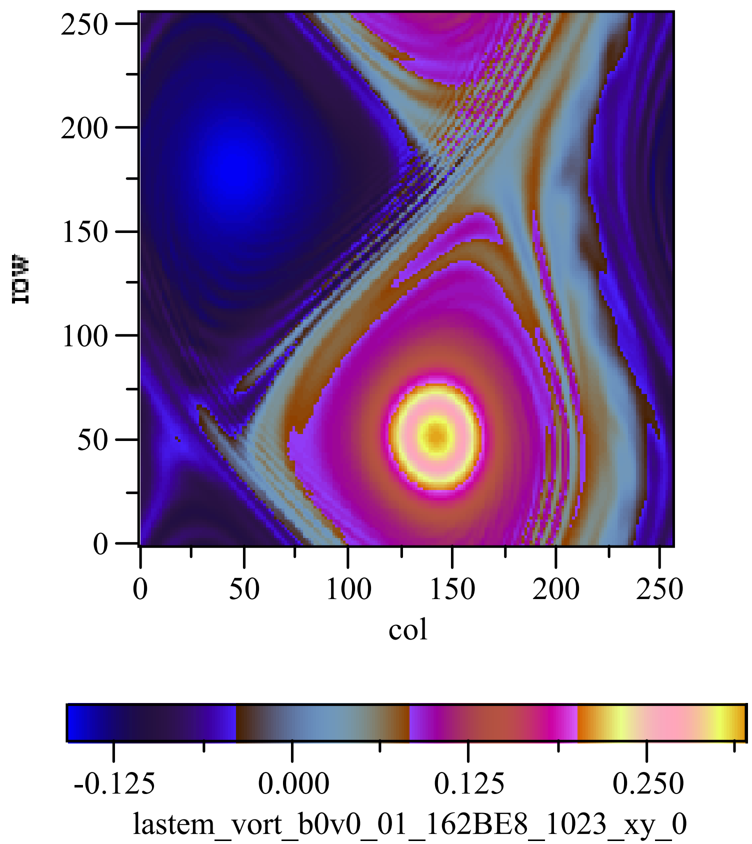


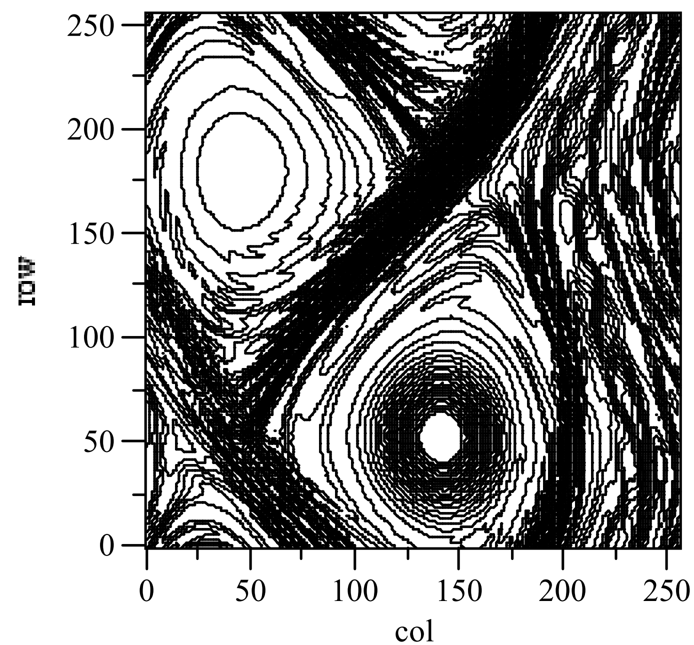


$$\Omega_{ci} t = 4600$$

Vorticity in nonlinear stage

$$\beta = 0$$

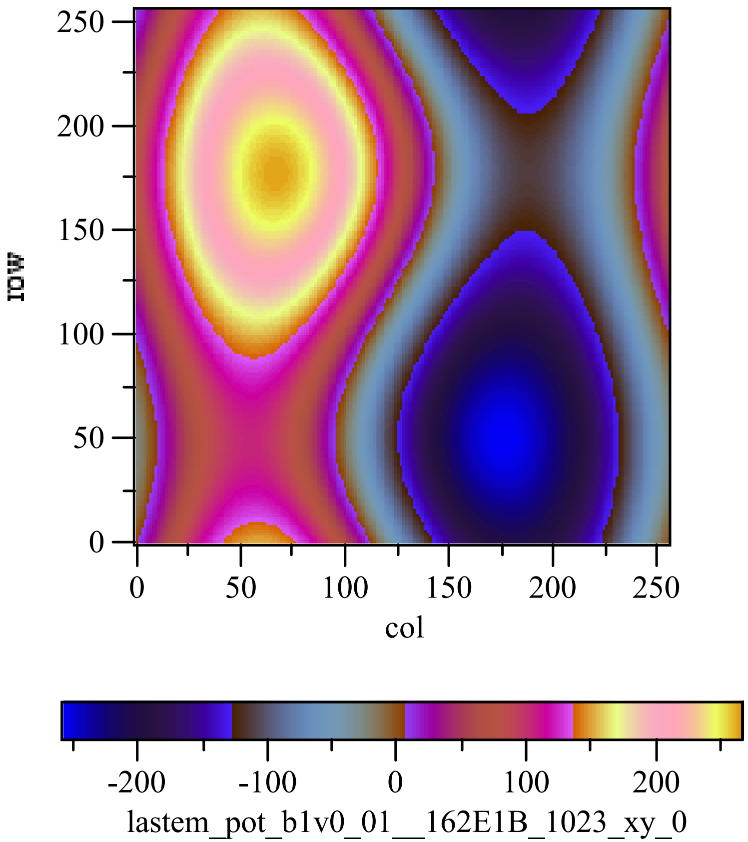


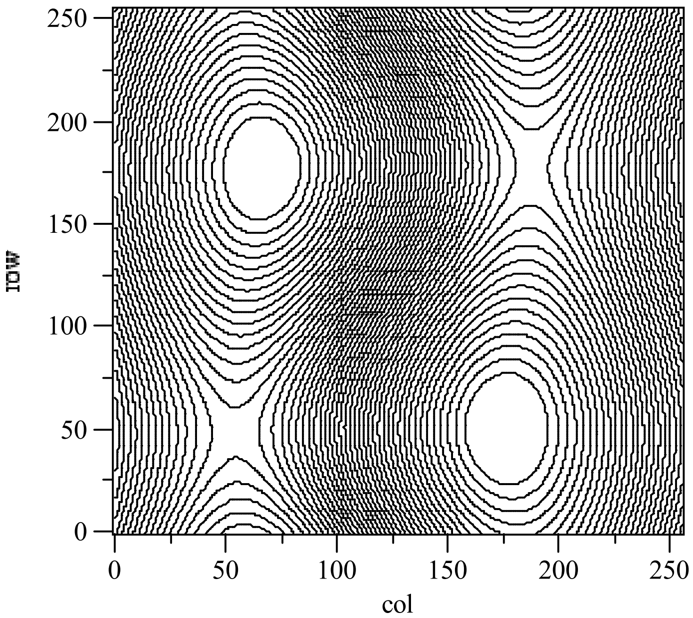


$$\Omega_{ci} t = 4600$$

Potential in nonlinear stage

$$\beta = 1.0$$

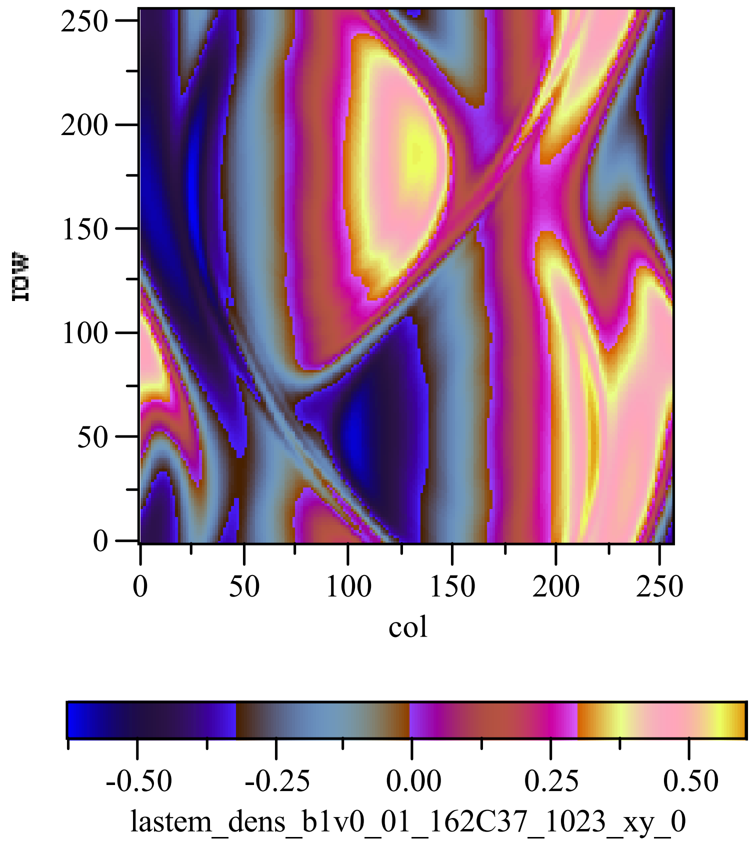


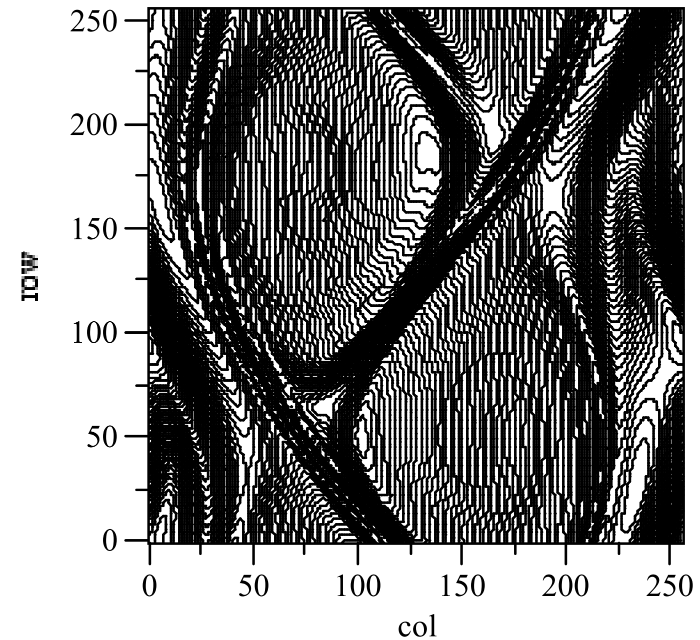


$$\Omega_{ci} t = 4600$$

Density in nonlinear stage

$$\beta = 1.0$$

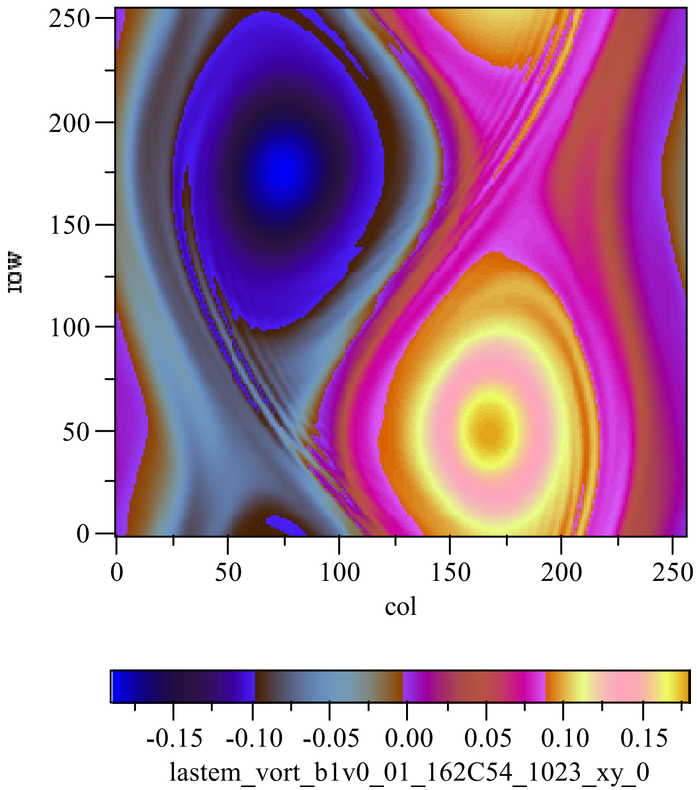


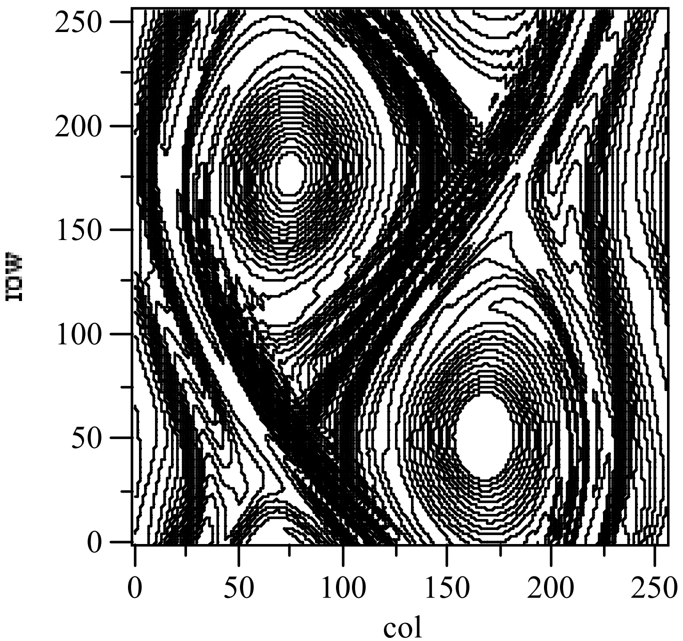


$$\Omega_{ci} t = 4600$$

Vorticity in nonlinear stage

$$\beta = 1.0$$





$$\Omega_{ci} t = 4600$$

Conclusions

- Wave activity experimentally observed in the central region of imploding wire array, where significant part of the current is concentrated, can be connected with excitation of flute-type electromagnetic oscillations in high beta plasma.

- Equations derived for electromagnetic flute-type instability contain electrostatic potential, magnetic field and density perturbations.

- Linear dispersion equation for electromagnetic flute-like mode instability has growing solution even in the limit of a high beta plasma.

- Fluid model of flute turbulence can describe nonlinear dynamics of large scale density and magnetic field structures as well as wave energy cascading towards the short scales.