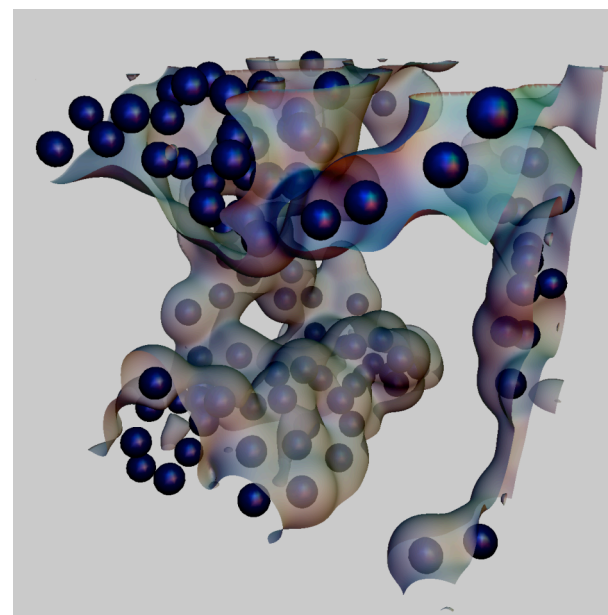
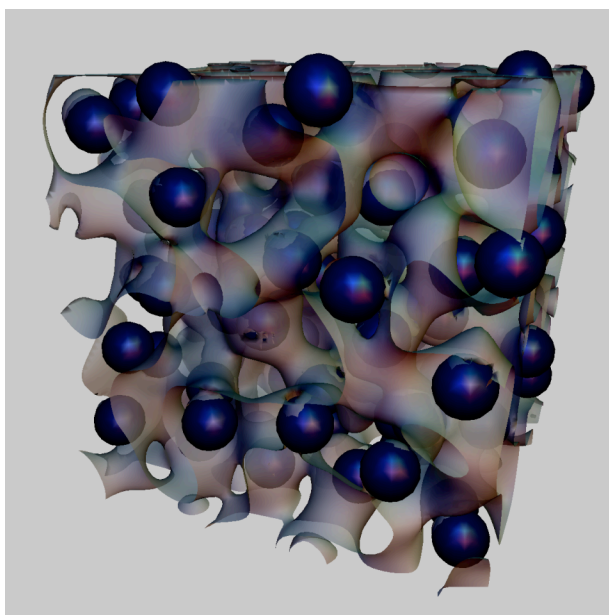


Quantum Molecular Dynamics Calculation of Electrical and Thermodynamic Properties for High Energy Density Physics



Mike Desjarlais
HEDP Theory &
ICF Target Design
Sandia National Laboratories

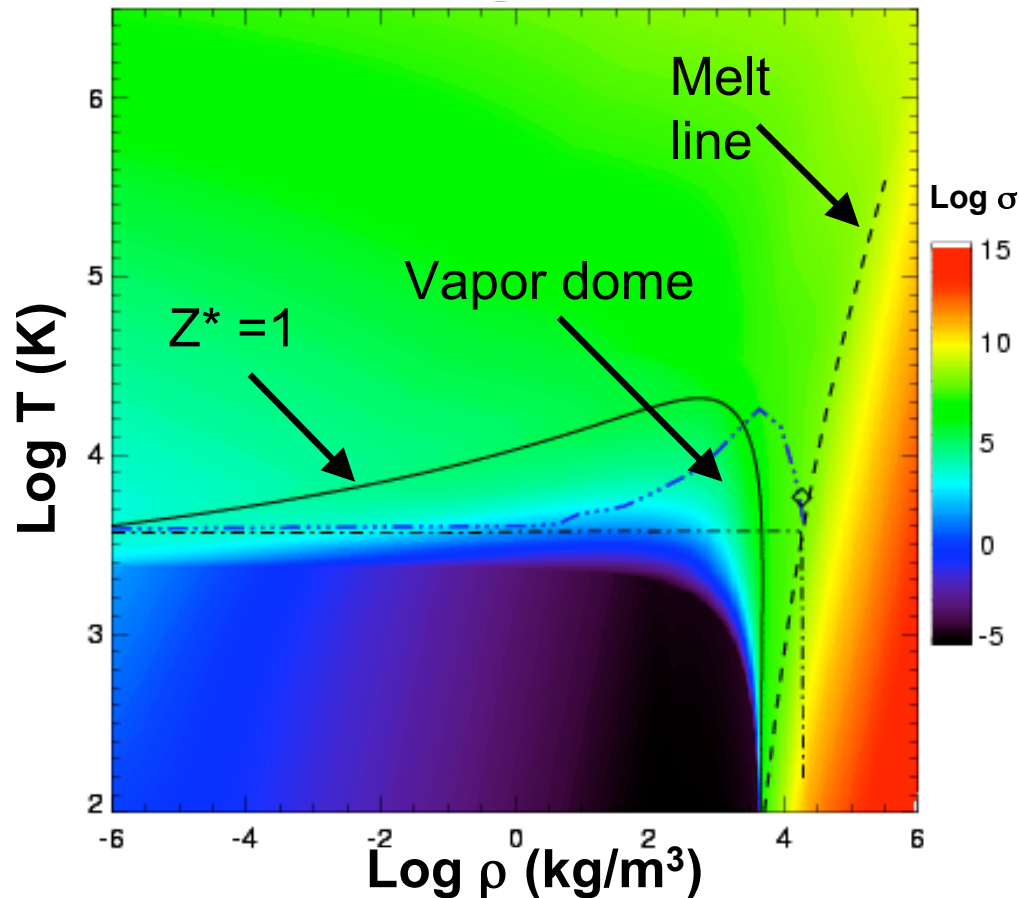


Briefing to VNIIEF Visitors
Albuquerque, New Mexico
November 14, 2006



HEDP computer simulations rely on “physics packages”: Conductivities, Equations of State, and Opacities

Tungsten Conductivity



Definitions of Warm Dense Matter are varied, but generally center around **strongly coupled ions and moderately degenerate electrons** --- many different interactions are comparable.

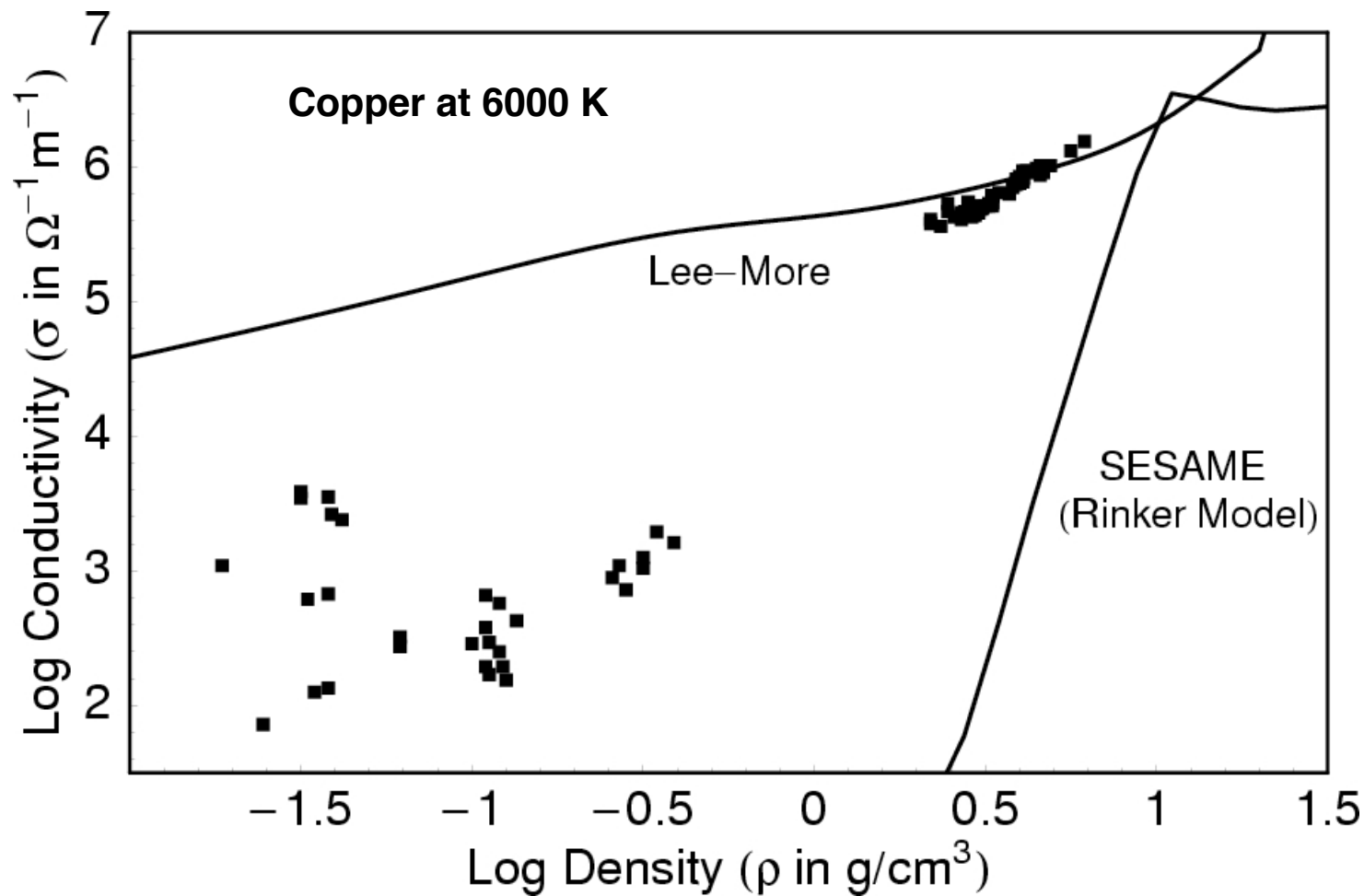
A **quantum mechanical** treatment is generally necessary.

Most wide-range equations of state interpolate through this difficult area.

This highly structured portion of phase space is
Warm Dense Matter



The Lee-More and SESAME (Rinker) models are inaccurate near the metal-insulator transition

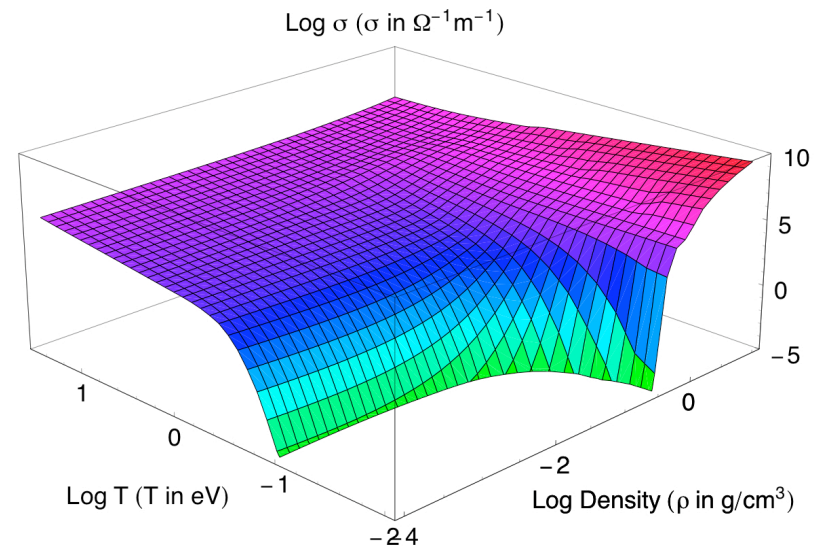
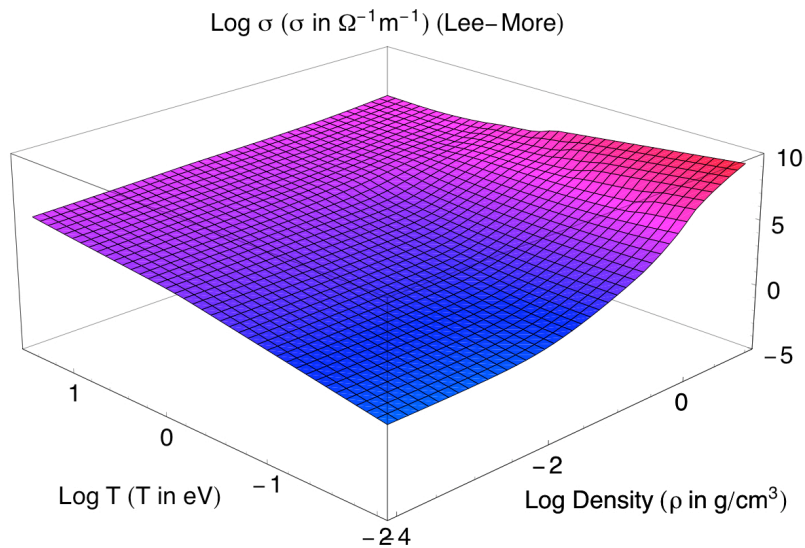


Data courtesy of Alan DeSilva, University of Maryland



Modifications of the Lee-More algorithm were made to obtain an improved wide-range model*

* M. P. Desjarlais, Contrib. Plasma Phys. **41** (2001) 2-3, 267-270



Principal modifications:

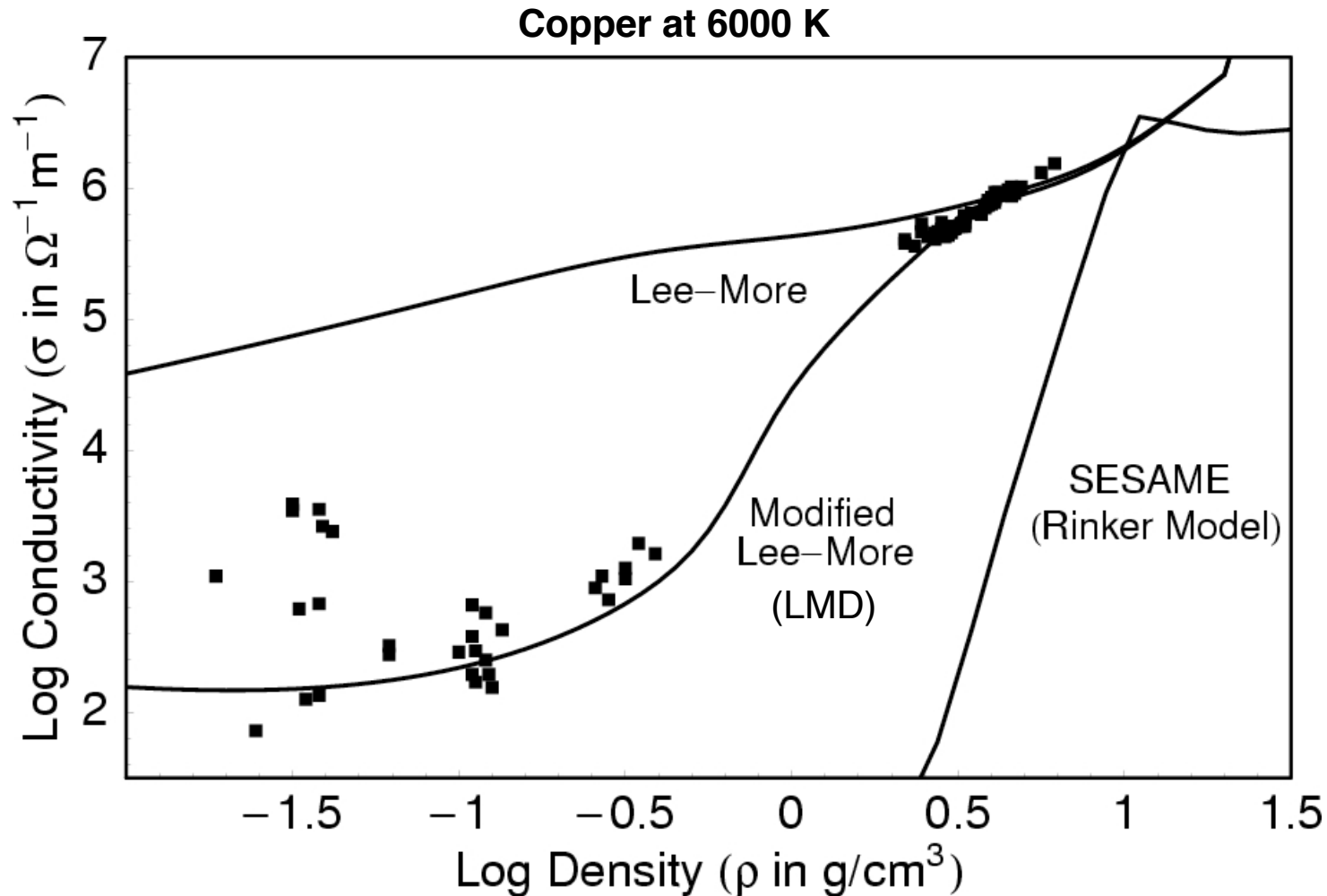
$$\sigma = n_e A\left(\frac{\mu}{kT}, \omega\tau\right) \frac{e^2}{m} \tau$$

Ionization equilibrium model, pressure ionization

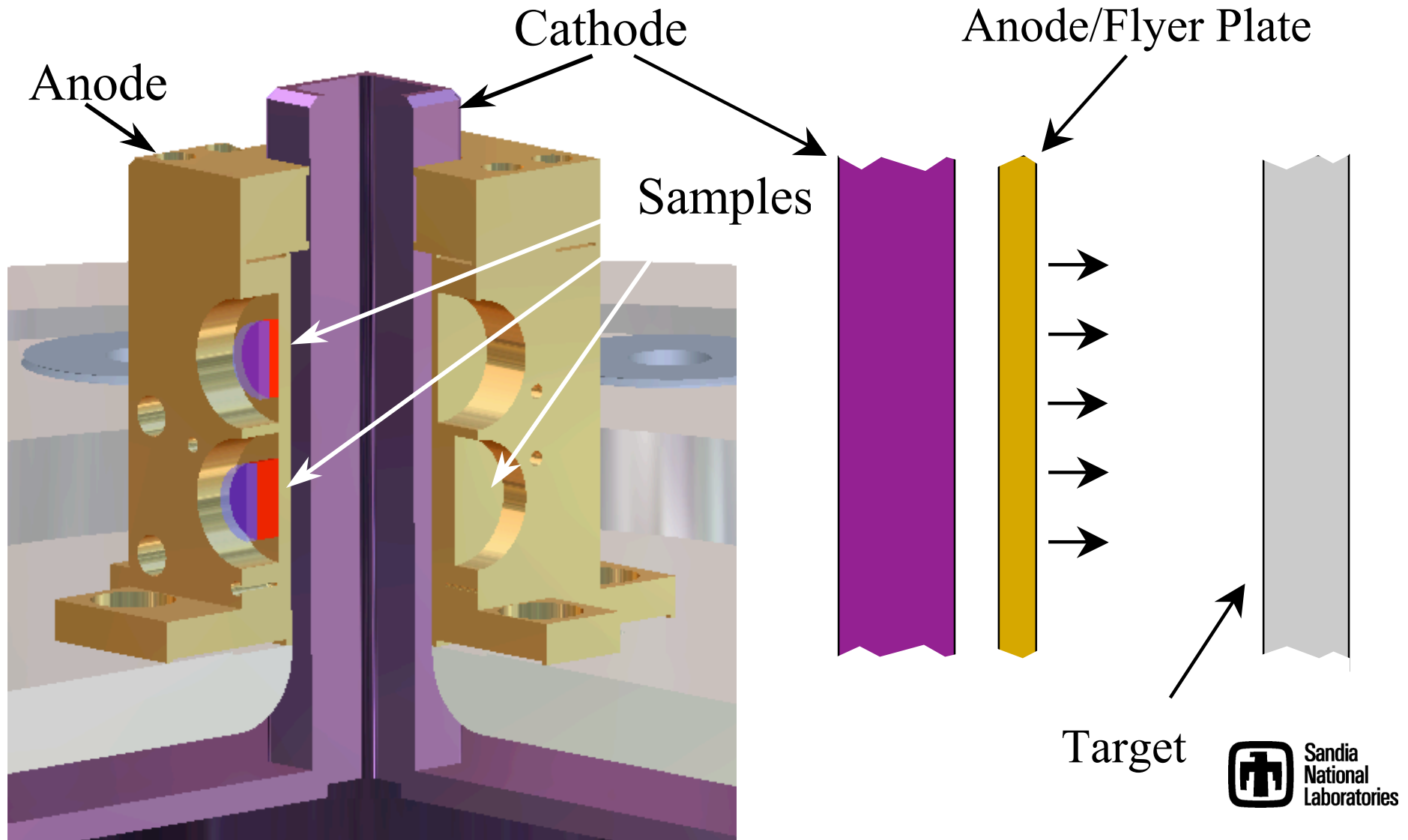
Electron-neutral collisions,
Minimum collision time



This modified Lee-More model provides much better agreement with DeSilva's copper data



A demanding application: Ultra-high velocity magnetically launched flyer plates (30 km/sec, Multi-Mbar pressures)

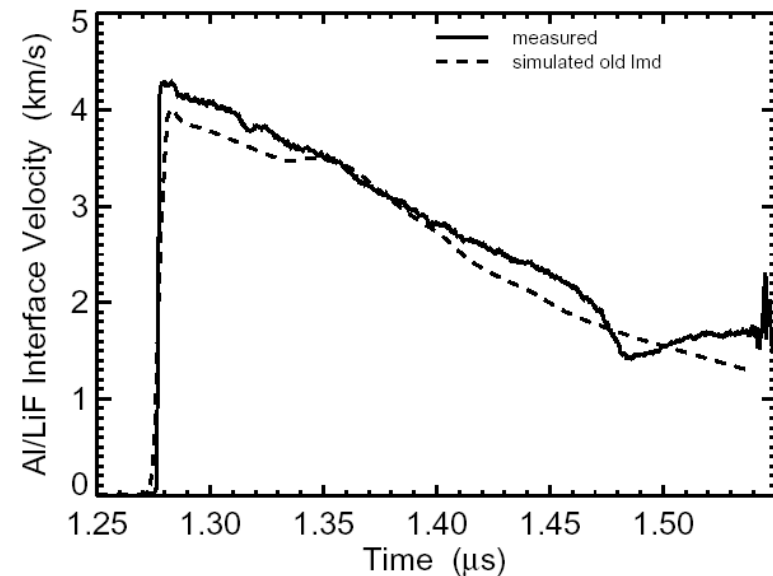
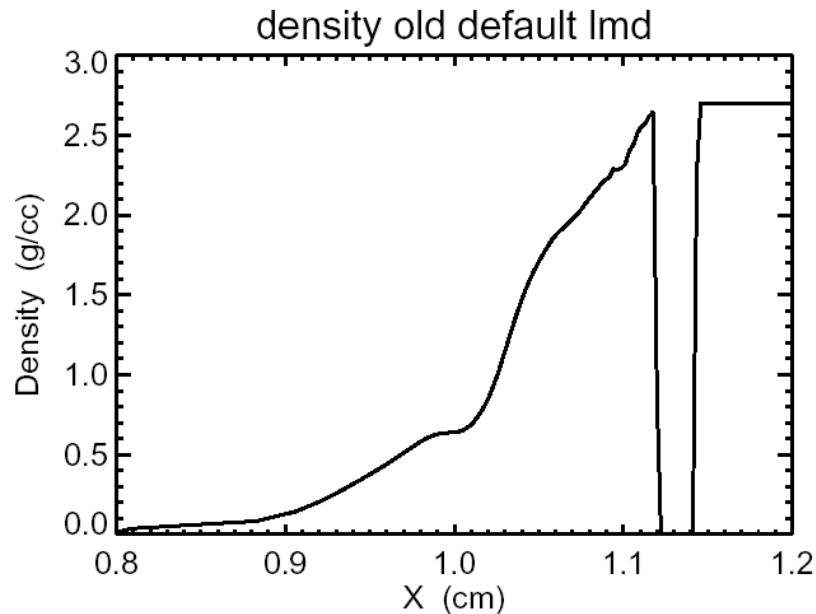




We simulated these magnetically launched flyer plates using the modified Lee-More (LMD) conductivities

Detailed comparison between simulations and experiments for magnetically launched flyer plates suggested that our *improved* conductivities were still not sufficiently accurate for the warm dense liquid aluminum.

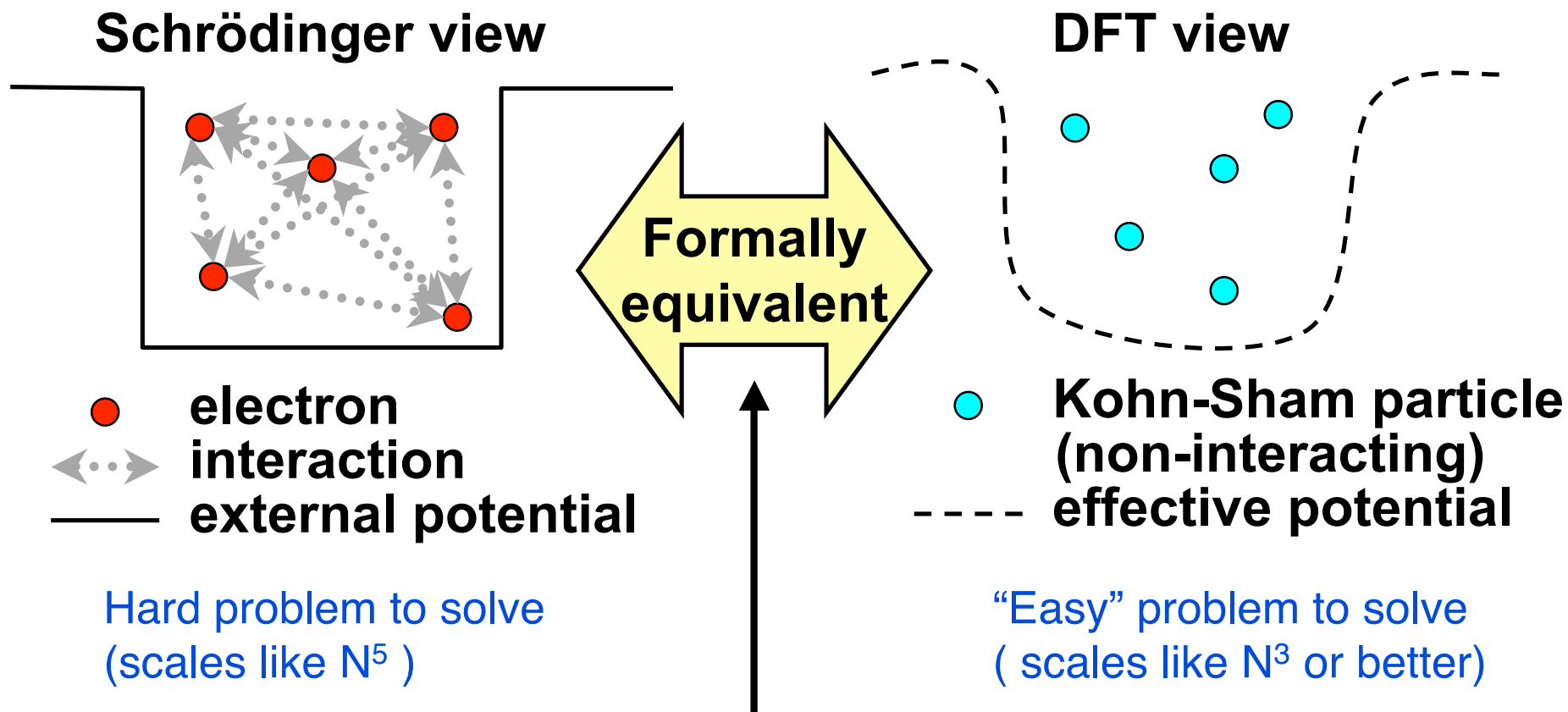
Simulations by Ray Lemke with Sandia's 3D Rad-MHD ALEGRA code



For many of our applications we require conductivities accurate to well within a factor of two.



Density Functional Theory (DFT) is a formally exact representation of the N electron Schrödinger Equation



Hohenberg and Kohn proved this (1964)

Nobel prize in Chemistry for Kohn in 1998



We are using Density Functional Theory (DFT) to perform Quantum Molecular Dynamics (QMD) simulations of Warm Dense Matter

Density Functional Theory is formally exact, but is in practice an accurate approximate solution to the N electron Schrödinger equations. DFT is a work-horse tool in condensed matter physics, but a relative newcomer to high energy density physics.

In addition to our Sandia effort, other groups with a large effort in this area include LANL (Collins, Kress); CEA (Clerouin, Mazevet, Recoules, Blottiau); LLNL (Galli, Gygi, Schwegler)

QMD: The **Kohn-Sham*** DFT equations are solved for a given atomic configuration (fixed in the Born-Oppenheimer approximation) and the **quantum mechanical forces** on all the atoms are calculated from the wavefunctions following the Feynman-Hellmann theorem, **the atomic positions are advanced classically**, and a new solution to the DFT equations is calculated.

*Kohn and Sham, 1965



The Kubo-Greenwood formula is used to calculate the frequency dependent electrical conductivity

$$\sigma_{\mathbf{k}}(\omega) = \frac{2\pi e^2 \hbar^2}{3m^2 \omega \Omega} \sum_{\alpha=1}^3 \sum_{j=1}^N \sum_{i=1}^N (F(\varepsilon_{i,\mathbf{k}}) - F(\varepsilon_{j,\mathbf{k}})) \left| \langle \Psi_{j,\mathbf{k}} | \nabla_{\alpha} | \Psi_{i,\mathbf{k}} \rangle \right|^2 \delta(\varepsilon_{j,\mathbf{k}} - \varepsilon_{i,\mathbf{k}} - \hbar\omega),$$

where e and m are the electron charge and mass. The i and j summations are over the N discrete bands of the triply periodic calculation for the cubic supercell with volume Ω . The coordinate index is α and in general we average over α to improve the statistics. $F(\varepsilon_{i,\mathbf{k}})$ is the Fermi weight corresponding to the energy for the i -th band at \mathbf{k} with wavefunction $\Psi_{i,\mathbf{k}}$.

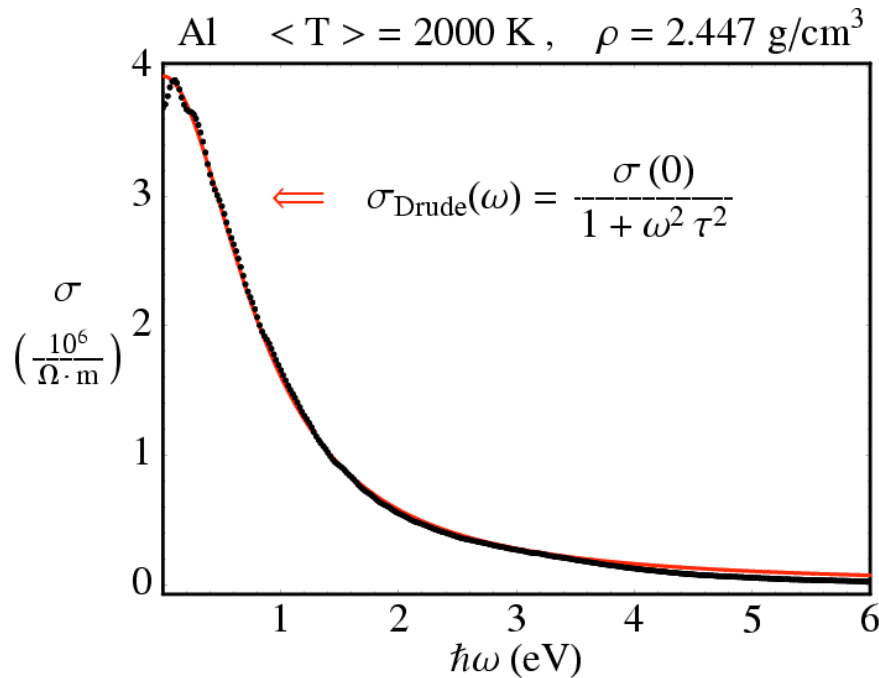
We integrate over the Brillouin zone using the method of special \mathbf{k} -points

$$\sigma(\omega) = \sum_{\mathbf{k}} \sigma_{\mathbf{k}}(\omega) W(\mathbf{k}) ,$$

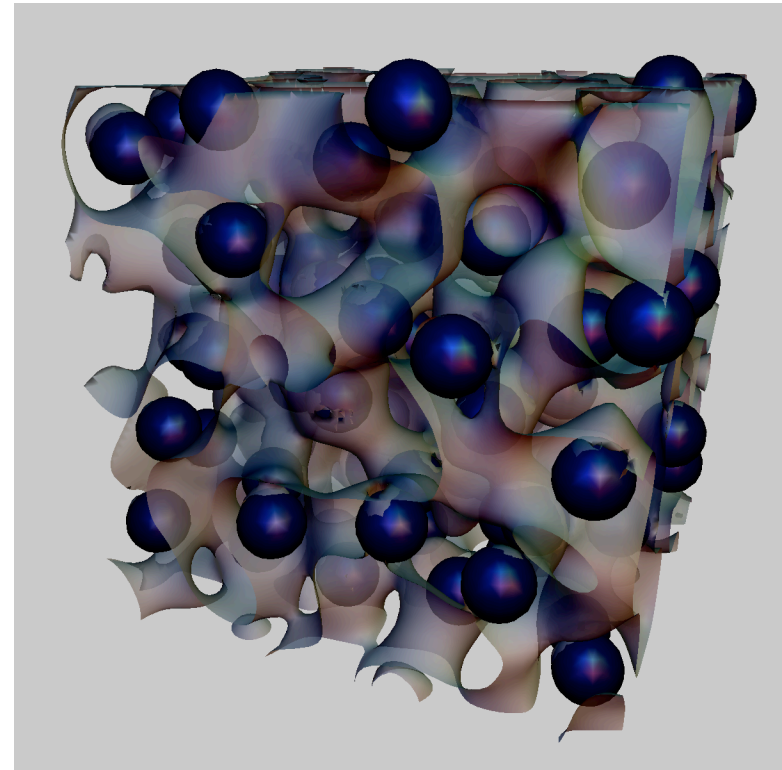
and average over 10 to 20 configurations selected from the MD run.

This is really nothing more than the quantum analog of the classical current-current correlation function representation of the conductivity

At liquid densities just below solid, the optical conductivity is well fit by the Drude model



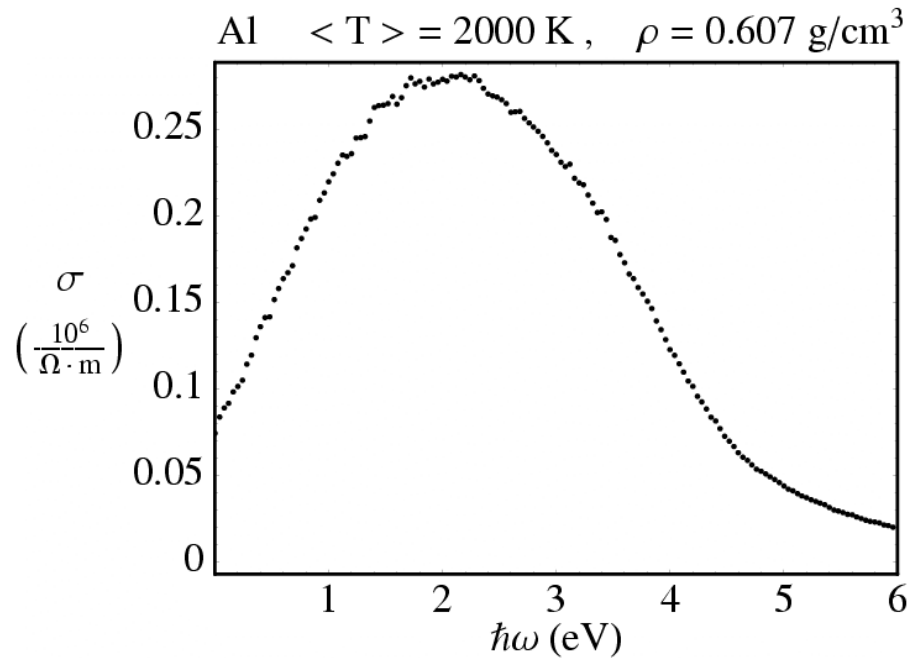
The agreement with the Drude model indicates 'nearly free' electrons



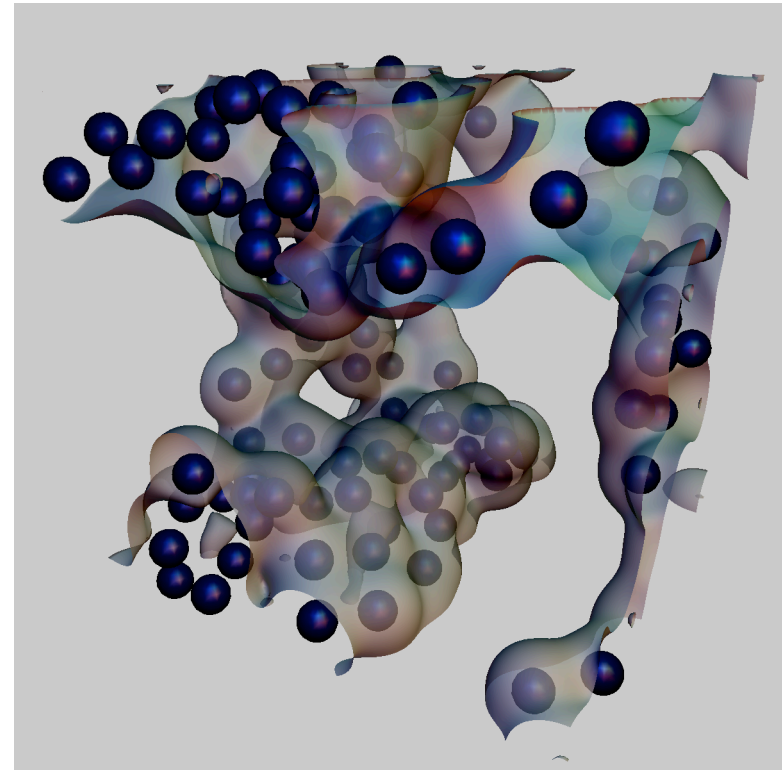
Ion cores displayed with iso-surfaces of the mean valence charge density



At lower density, where phase separation is pronounced, a gap begins to form at low energy



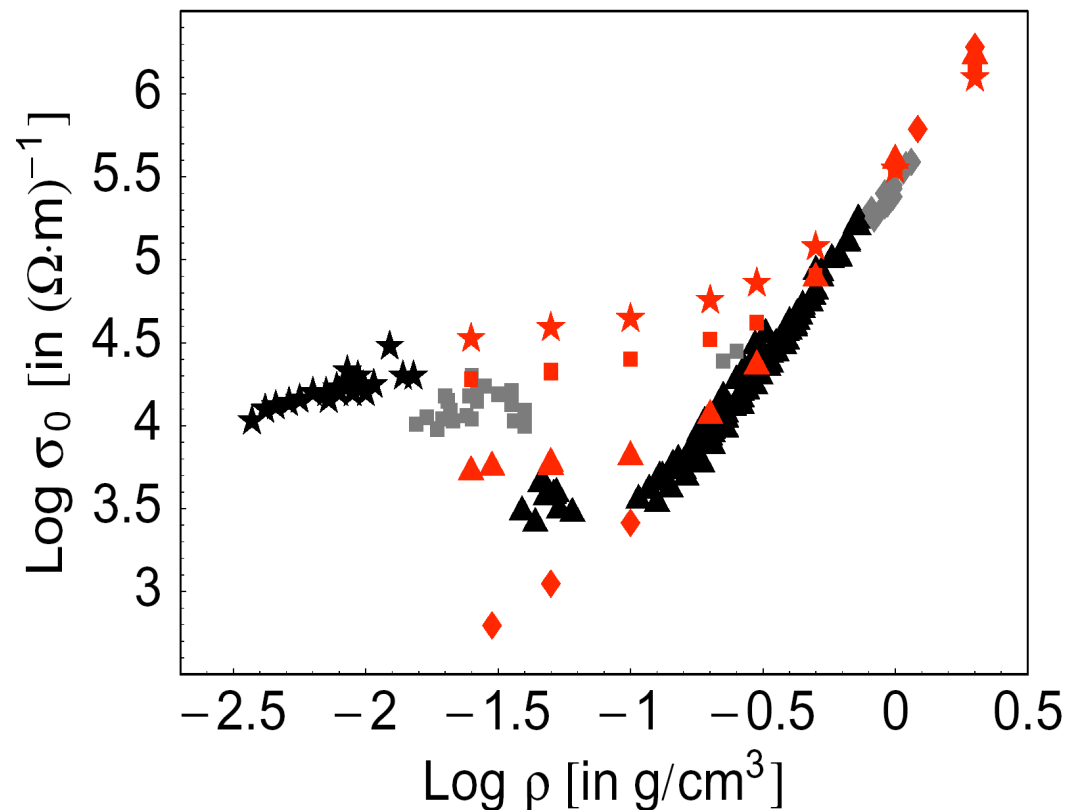
The dc conductivity has dropped by a factor of 25 for a factor of 4 drop in density



Note the pronounced separation into liquid and void (vapor) regions



The QMD-KG results are in good agreement with DeSilva's data over a two decade range of density



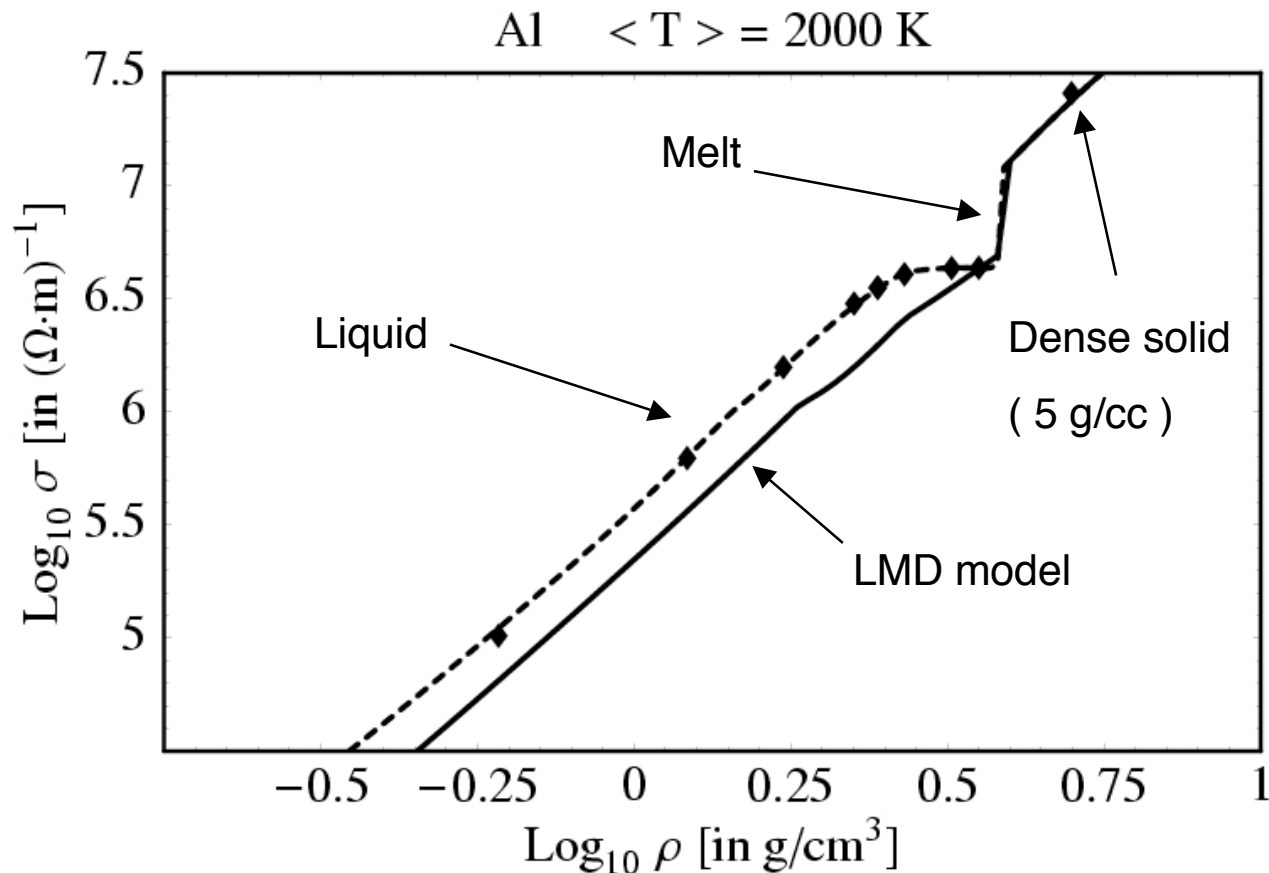
DeSilva and Katsouras data in black or grey, MD-KG results in red

★ 30000 K, ■ 20000 K, ▲ 10000 K, ◆ 6000 K

[[Desjarlais, Kress, and Collins, PRE 66, 025401\(R\) \(2002\)](#)]



The calculated liquid aluminum conductivities are higher than the *improved* Lee-More (LMD) model predictions

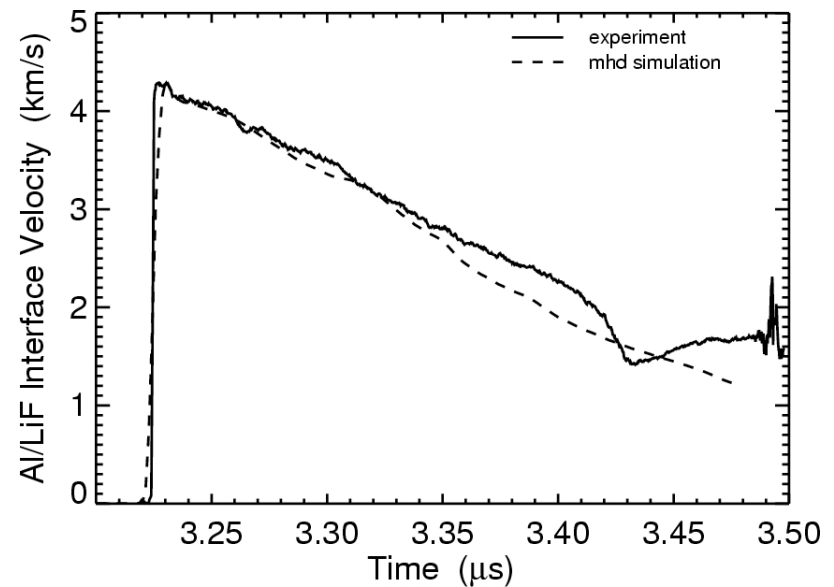
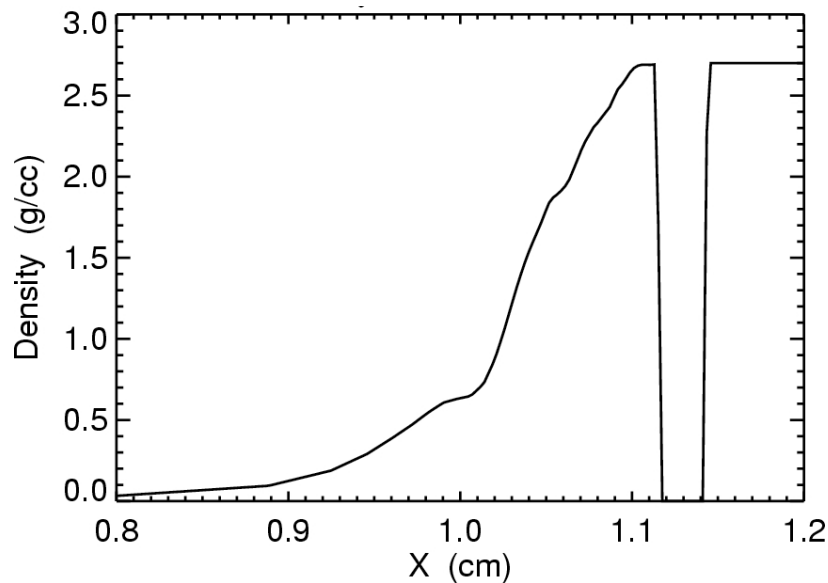


The dashed line shows the 2000 K isotherm from our QMD-tuned wide-range aluminum model



Flyer plate simulations with the QMD based conductivities give very good agreement with experiment

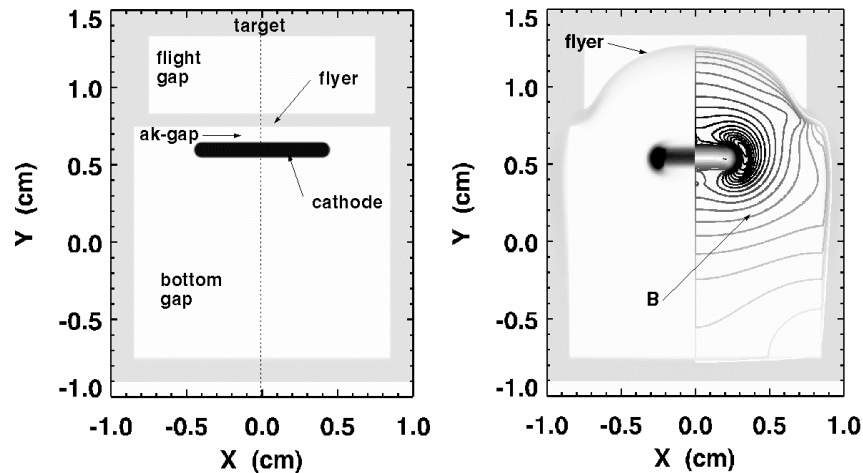
Simulations by Ray Lemke with Sandia's 3D Rad-MHD ALEGRA code



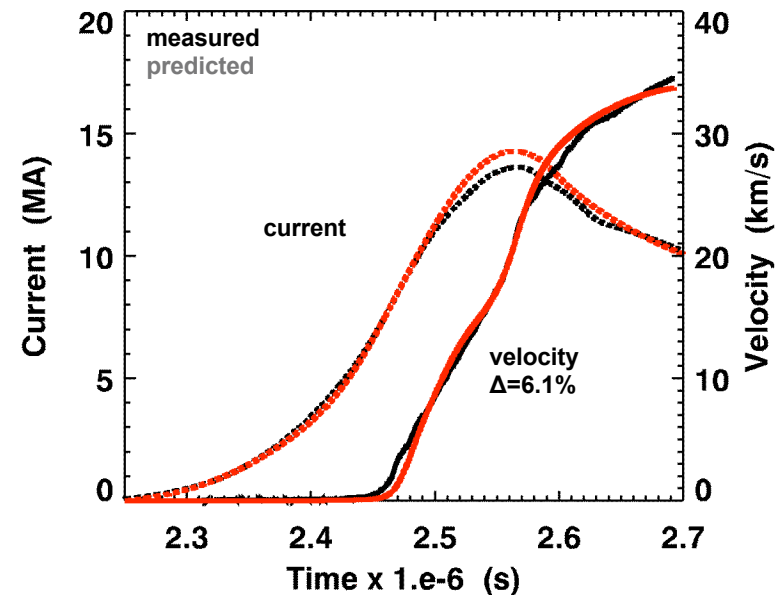
Conductivities based on the QMD calculations have given us a new predictive capability.



We have used our simulation capability with the new aluminum model to optimize flyer performance on Z



Measured / predicted current & flyer velocity (850 μm Al)



The simulations were performed by Ray Lemke using Sandia's ALEGRA code in 2-D



Other optical properties are obtained through the Kramers-Krönig relations for $\sigma(\omega)$

Through the Kramers-Kronig relations we have $\sigma_2(\omega) = -\frac{2}{\pi} P \int \frac{\sigma_1(\omega')}{\omega'^2 - \omega^2} d\omega'$

where P indicates the principal value of the integral. The dielectric function is then given by $\epsilon_1(\omega) = 1 - \frac{4\pi}{\omega} \sigma_2(\omega)$ and $\epsilon_2(\omega) = \frac{4\pi}{\omega} \sigma_1(\omega)$

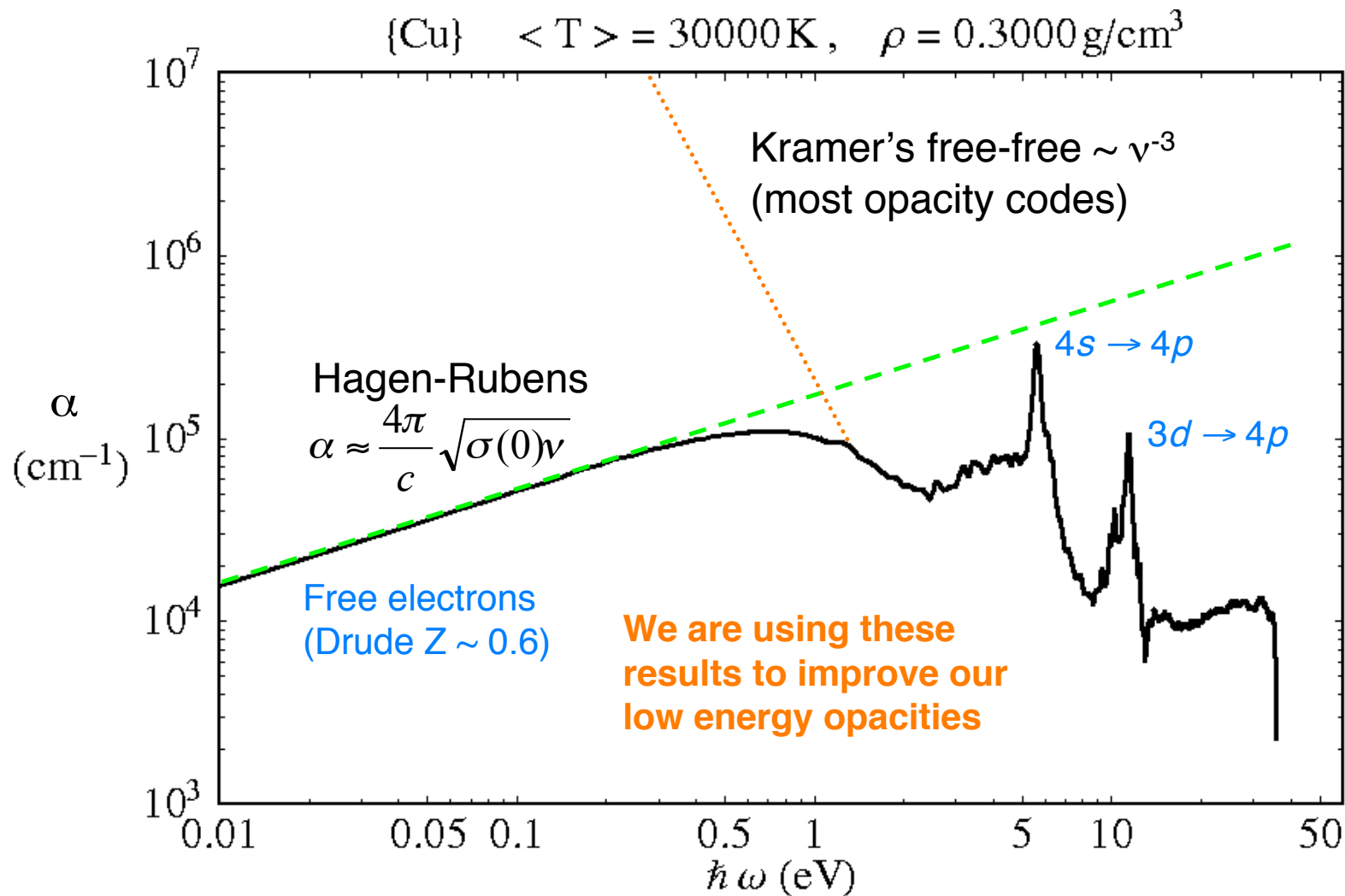
Defining $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega) \equiv [n(\omega) + ik(\omega)]^2$ we can write the reflectivity $r(\omega)$ and absorption coefficient $\alpha(\omega)$:

$$r(\omega) = \frac{[1 - n(\omega)]^2 + k(\omega)^2}{[1 + n(\omega)]^2 + k(\omega)^2}, \quad \alpha(\omega) = \frac{4\pi}{n(\omega)c} \sigma_1(\omega).$$

We can now improve our low energy (below ~ 30 eV) opacities $\alpha(\omega)/\rho$ where ρ is the mass density.

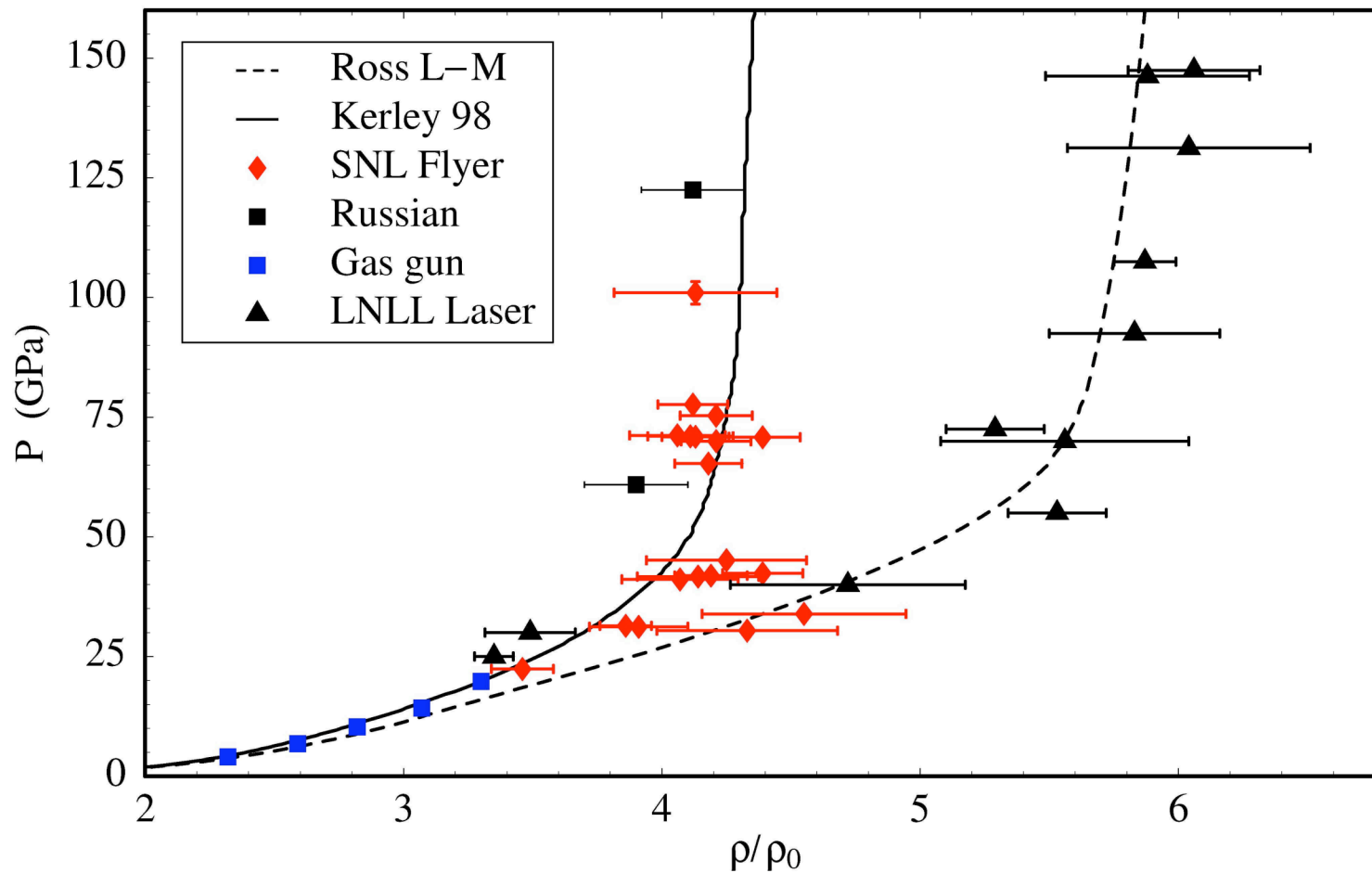


The absorption has the proper Hagen-Rubens low frequency behavior, consistent with the dc conductivity



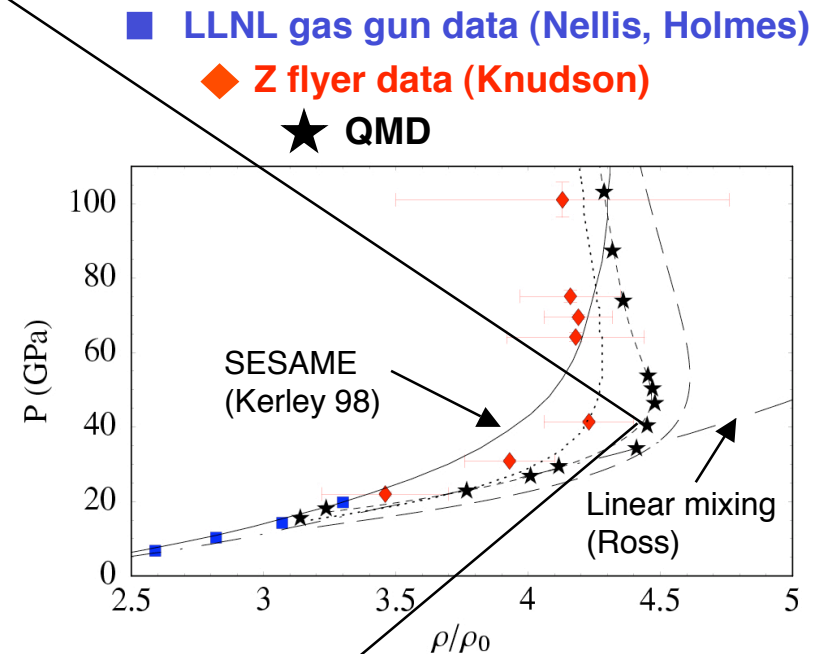
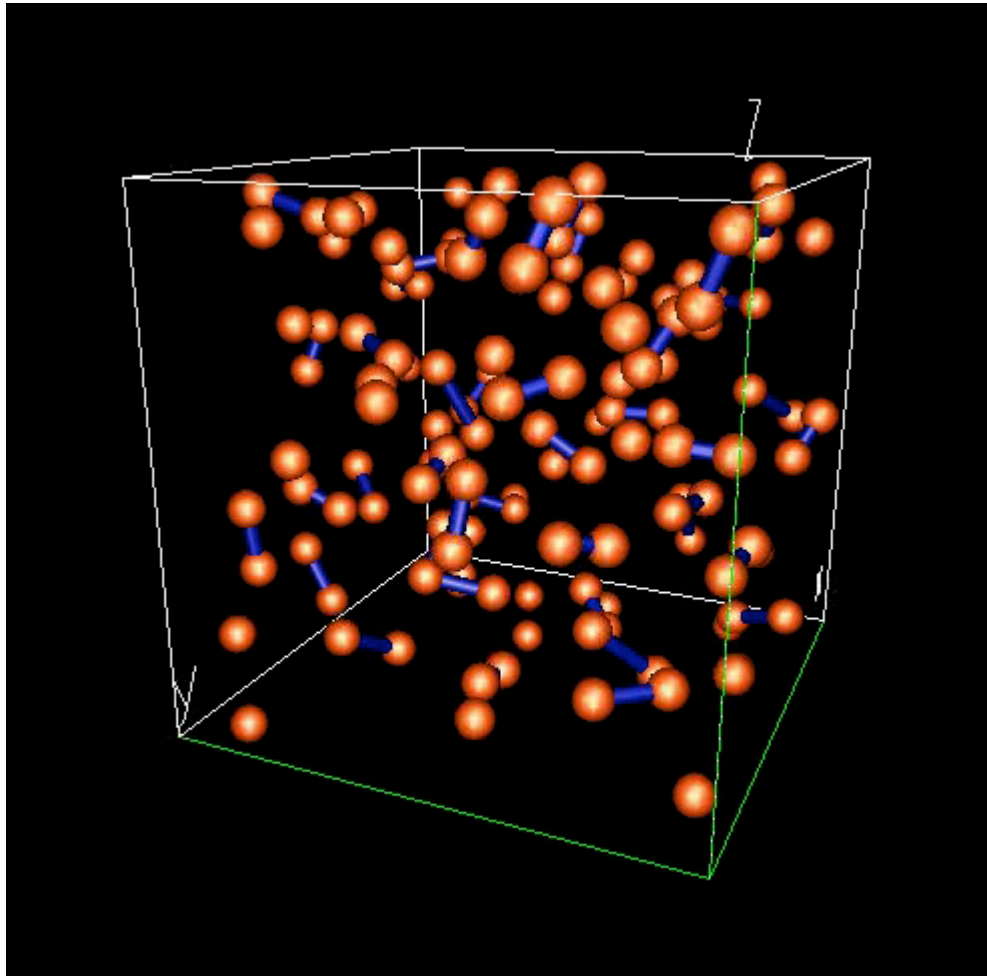


In recent years there has been great interest in the deuterium Hugoniot





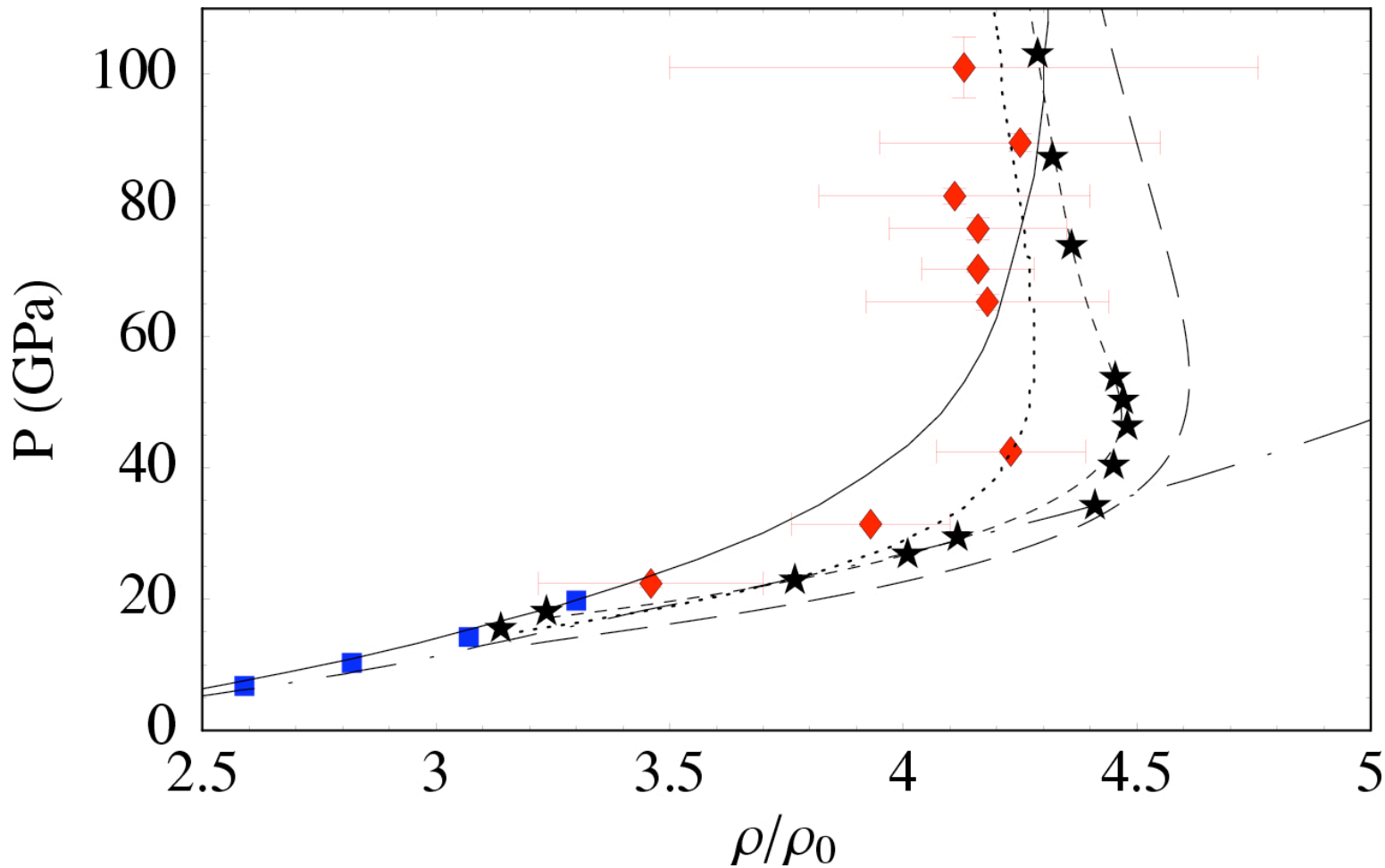
We have used quantum molecular dynamics to study the principal Hugoniot of shocked liquid deuterium



Our QMD simulations of D_2 indicate a Hugoniot in very good agreement with data from Sandia's Z machine



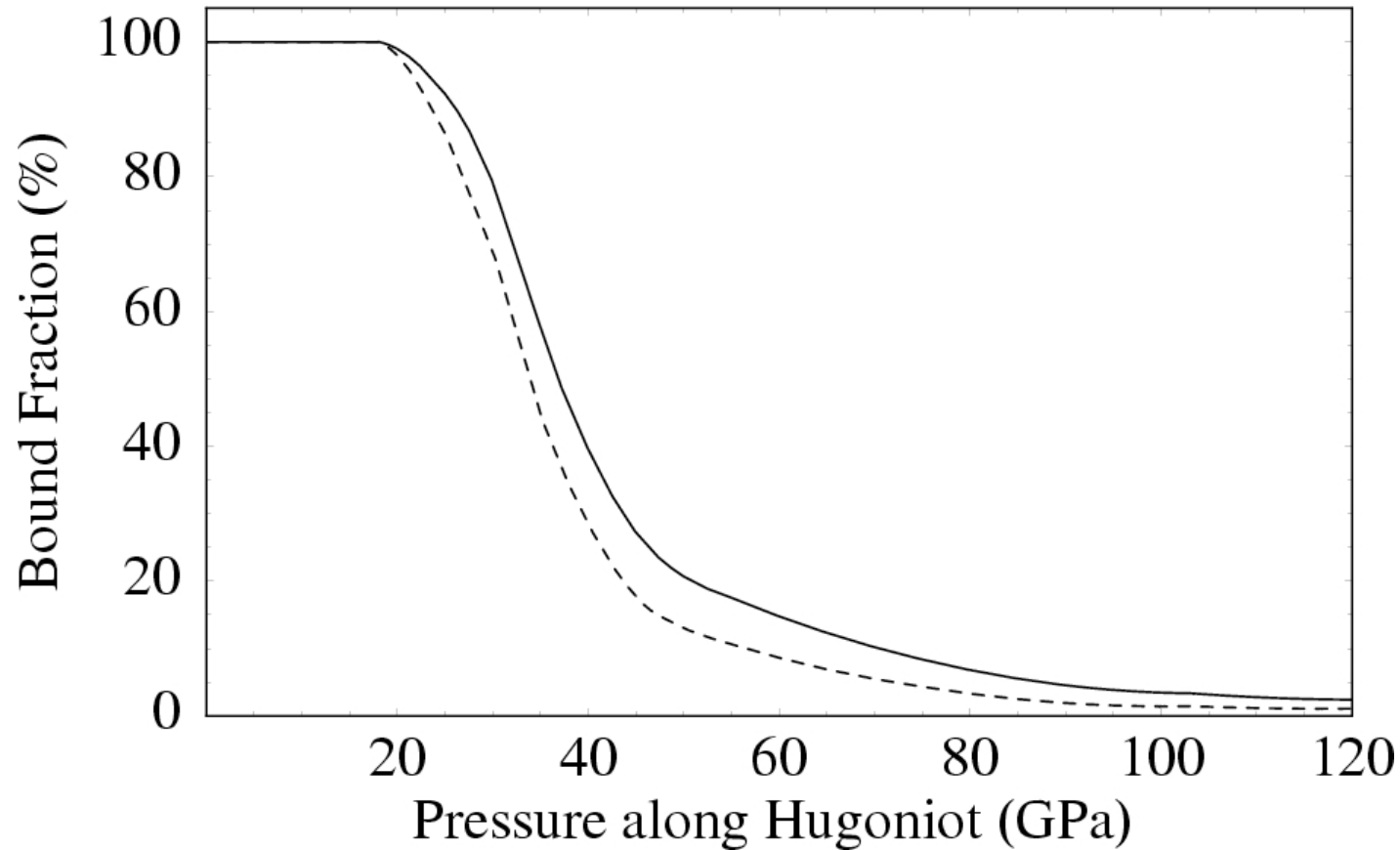
In contrast to earlier DFT/GGA results, our calculations (★) are consistent with the gas-gun data (■)



Temperatures are also in very good agreement with the gas-gun data



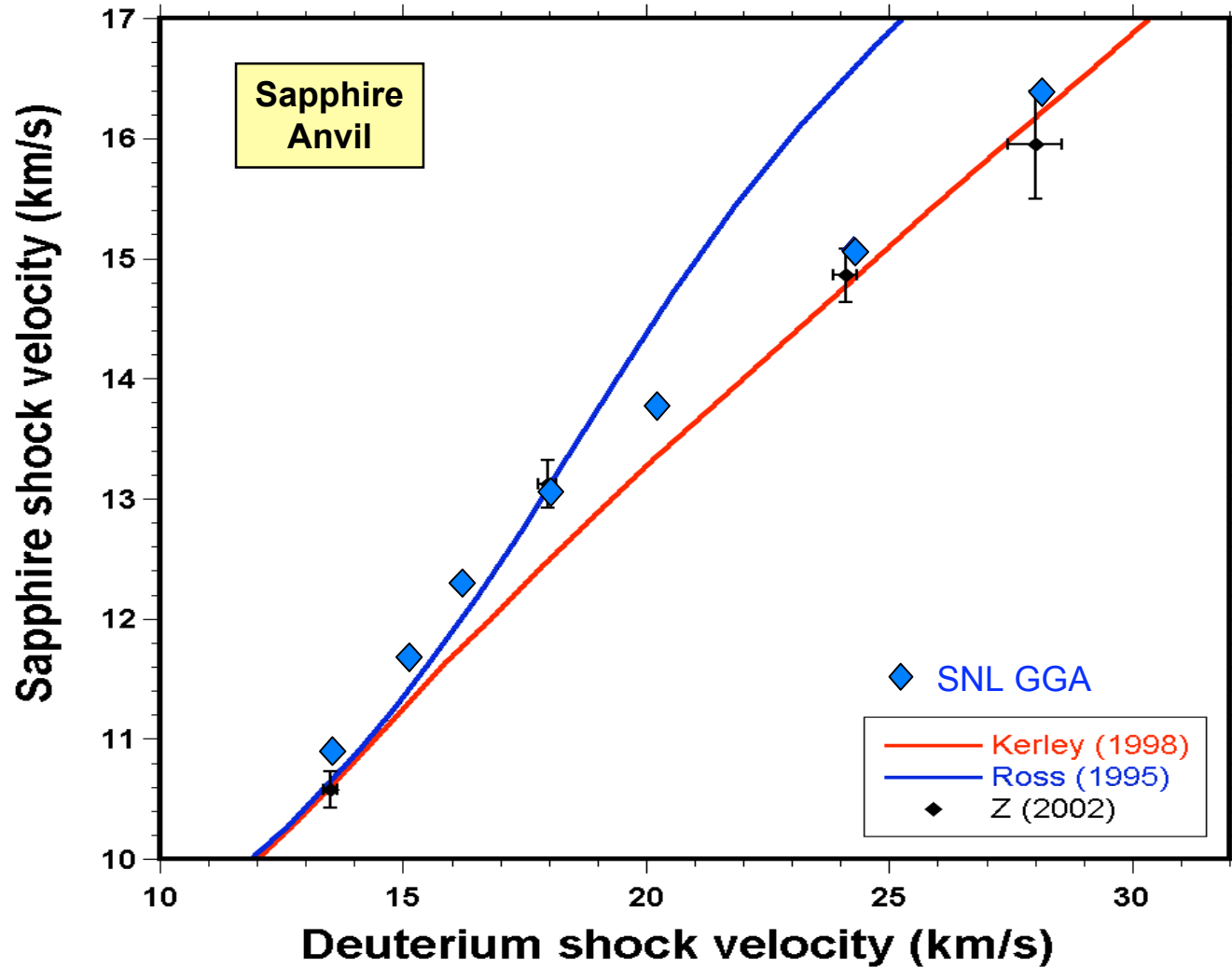
The dissociation along the Hugoniot begins at 20 GPa and is largely over by 50 GPa



Bonds defined for nearest neighbors or mutual nearest neighbors persisting two vibron periods or greater

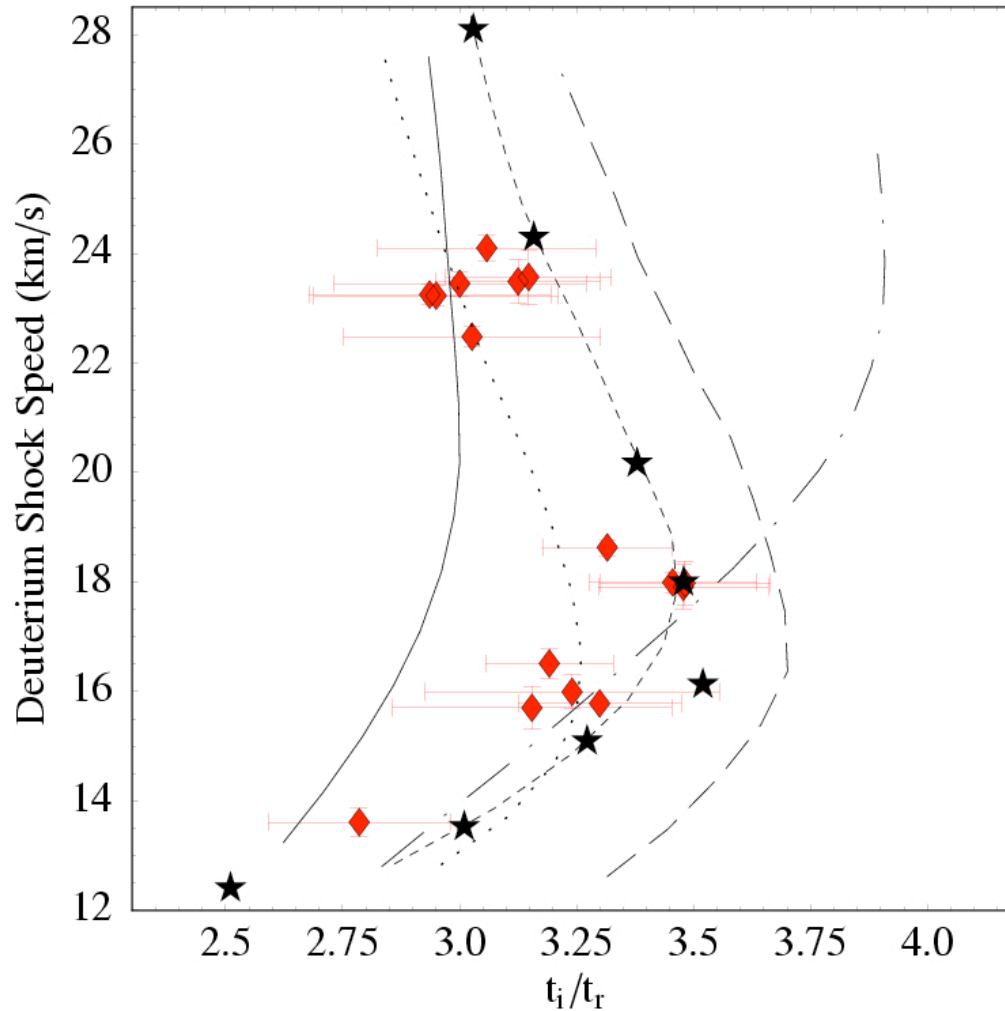


Reshock states with a sapphire anvil are in good agreement with the flyer data from Z



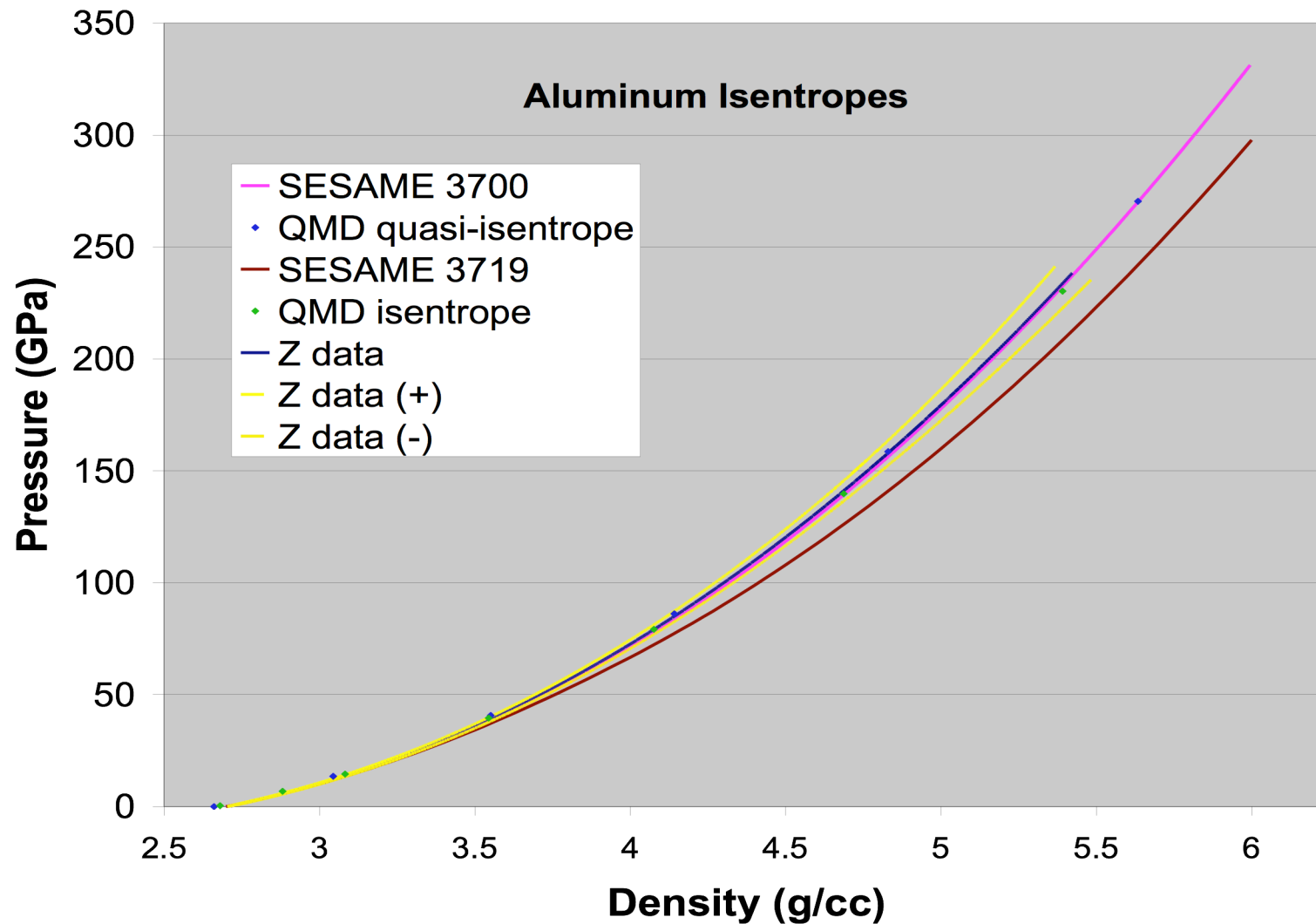


The calculated reverberation timing ratios are in very good agreement with the Z experiments





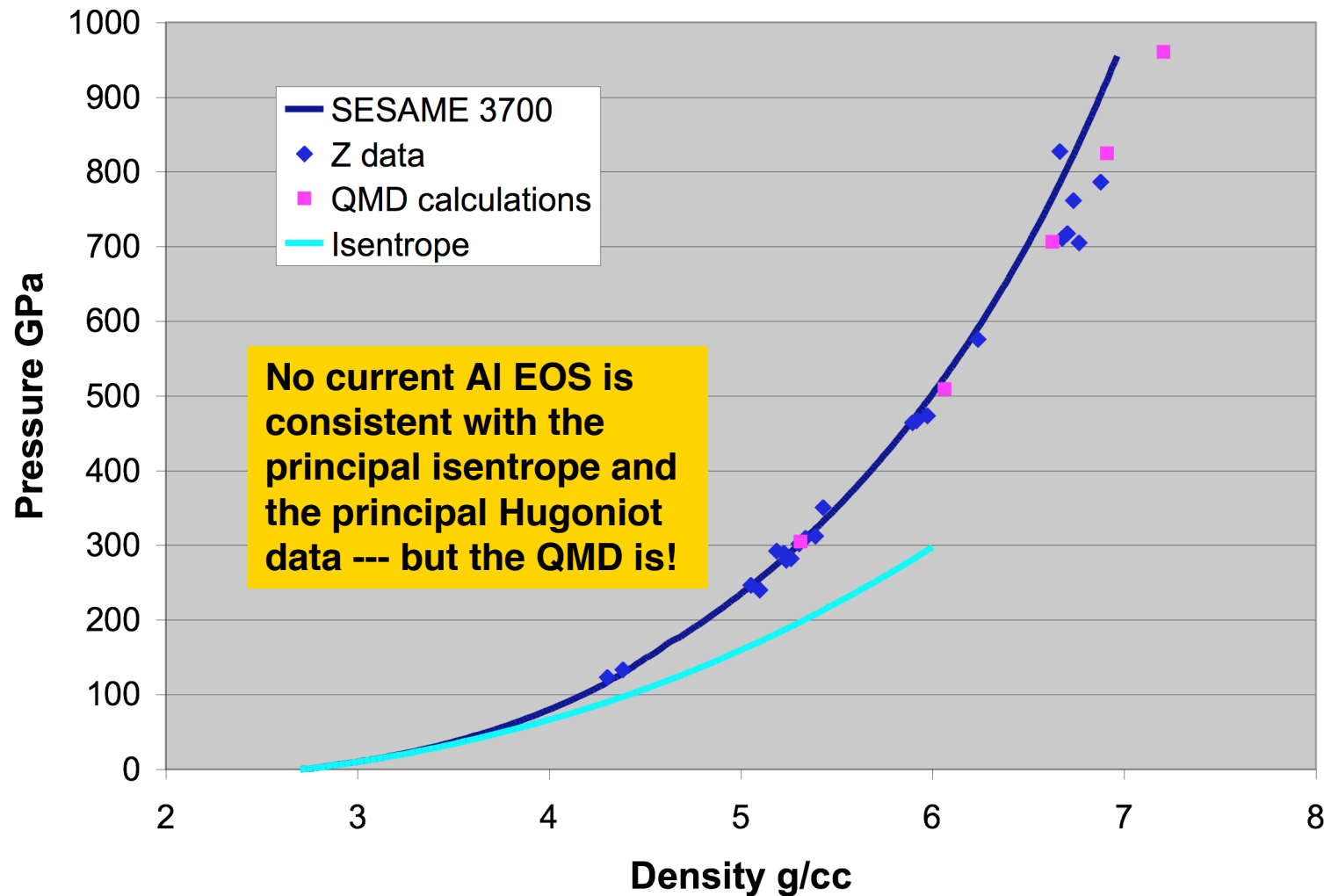
Our calculated principal isentrope for aluminum is in excellent agreement with data from Sandia's Isentropic Compression Experiments (ICE) on Z



Data from **ICE** experiments on Z courtesy of Jean-Paul Davis



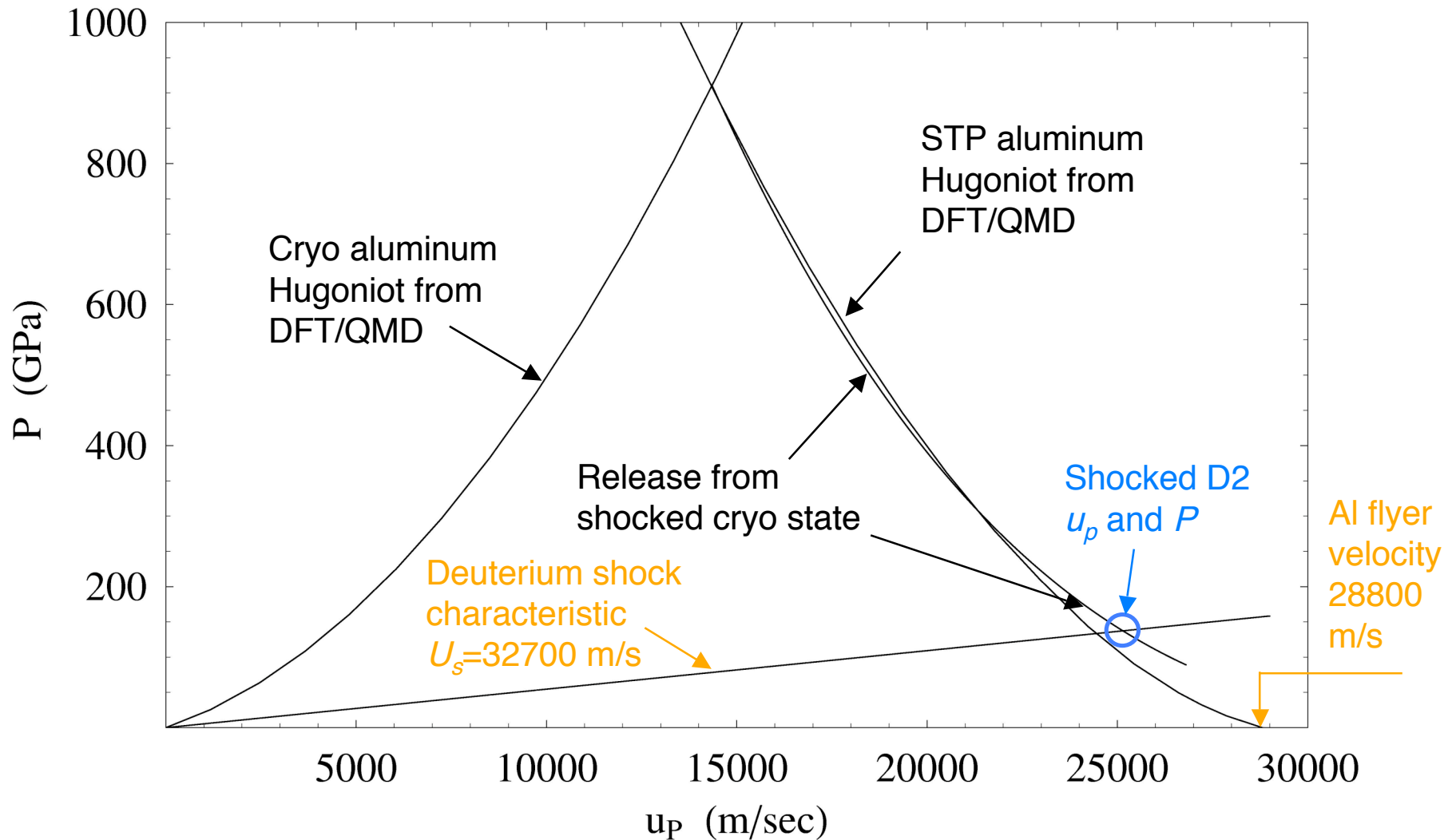
We have also computed the aluminum shock Hugoniot and compared with shock data from Z



Al Hugoniot data from Z courtesy of Marcus Knudson

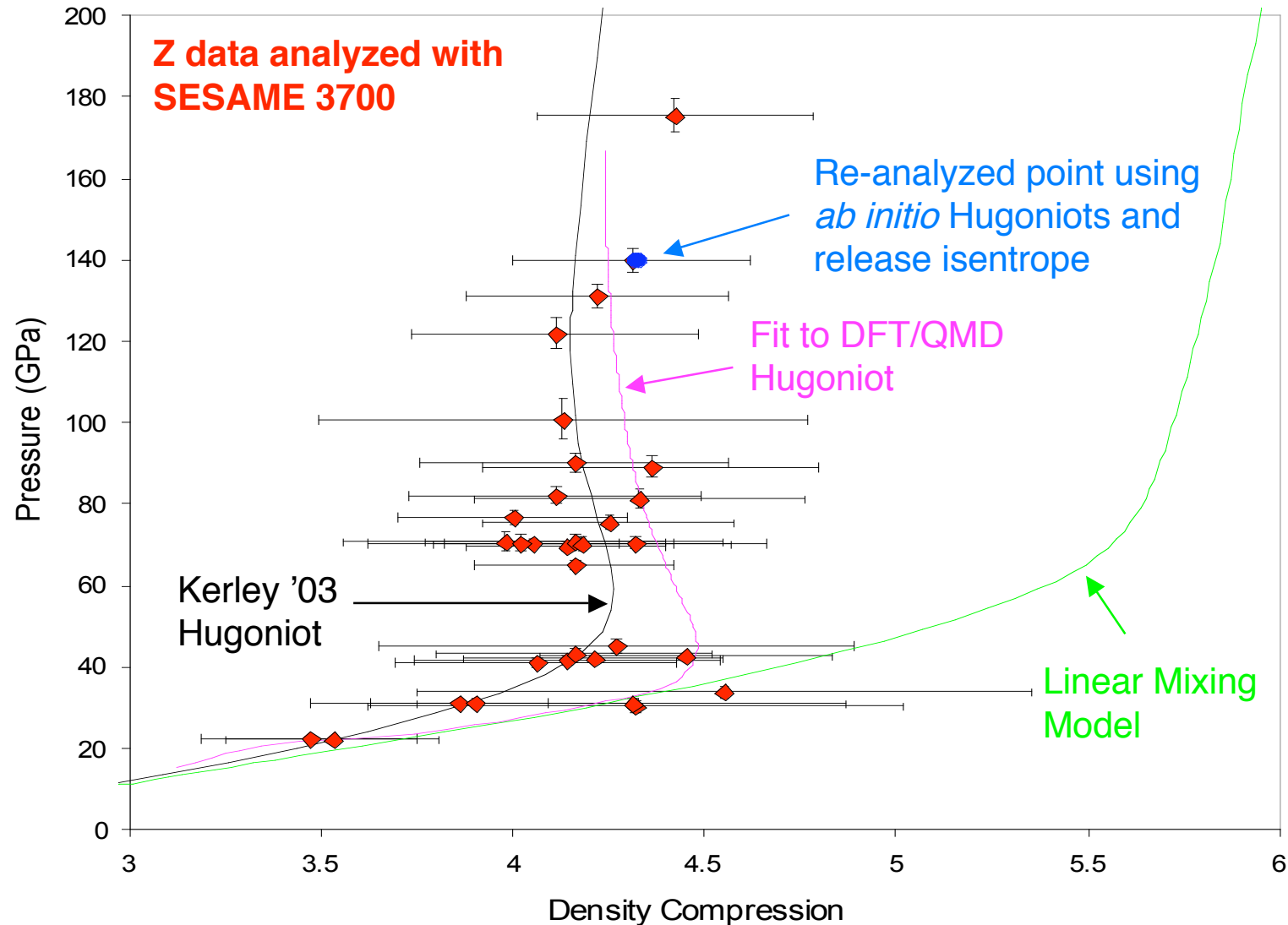


We have all the pieces to do *ab initio* impedance matching calculations





Our re-calculated impedance match point is very close to that obtained with SESAME 3700





Recent and active research areas

- Principal Hugoniot and reshock properties of deuterium
- QMD based conductivity models for Al, W, Be, and stainless steel
- Liquid-vapor critical points for W and Al
- Principal and release isentropes for aluminum
- Shock melting of Be and Diamond (NIF Ignition Campaign)
- Electrical and thermodynamic properties of water at high energy densities
- Research on advanced electronic structure methods for High Energy Density Physics (finite temperature Exact Exchange, finite temperature GW)