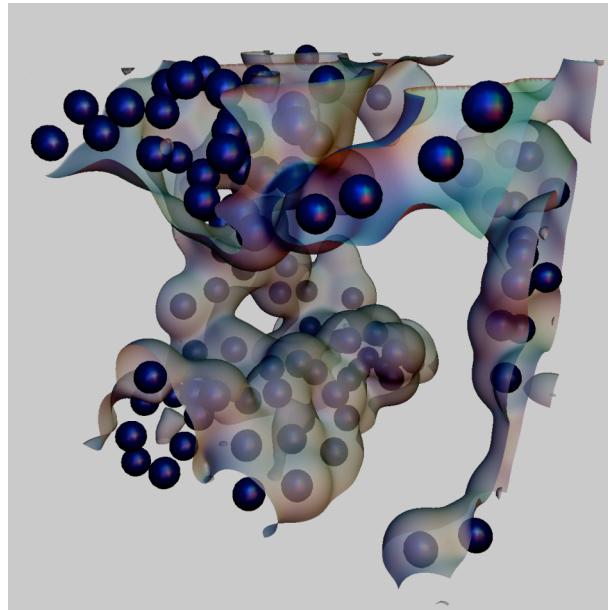
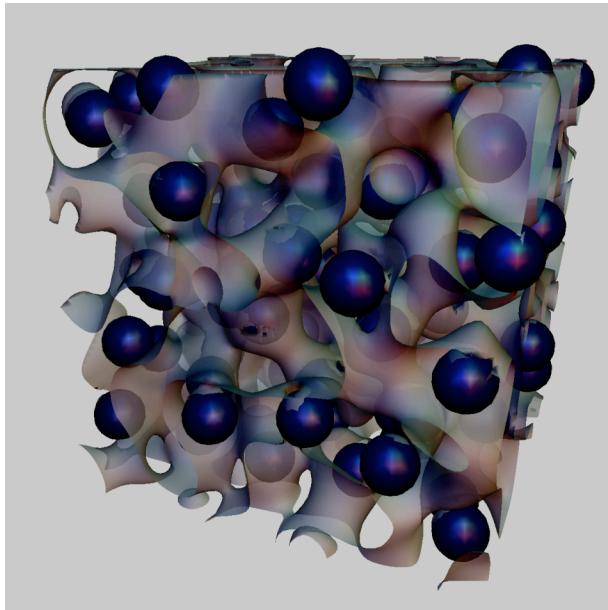


# Quantum Molecular Dynamics Calculation of Electrical and Thermodynamic Properties for High Energy Density Physics



**Mike Desjarlais**  
**HEDP Theory &**  
**ICF Target Design**  
**Sandia National Laboratories**



**Briefing to VNIIIEF Visitors**  
**Albuquerque, New Mexico**  
**November 14, 2006**



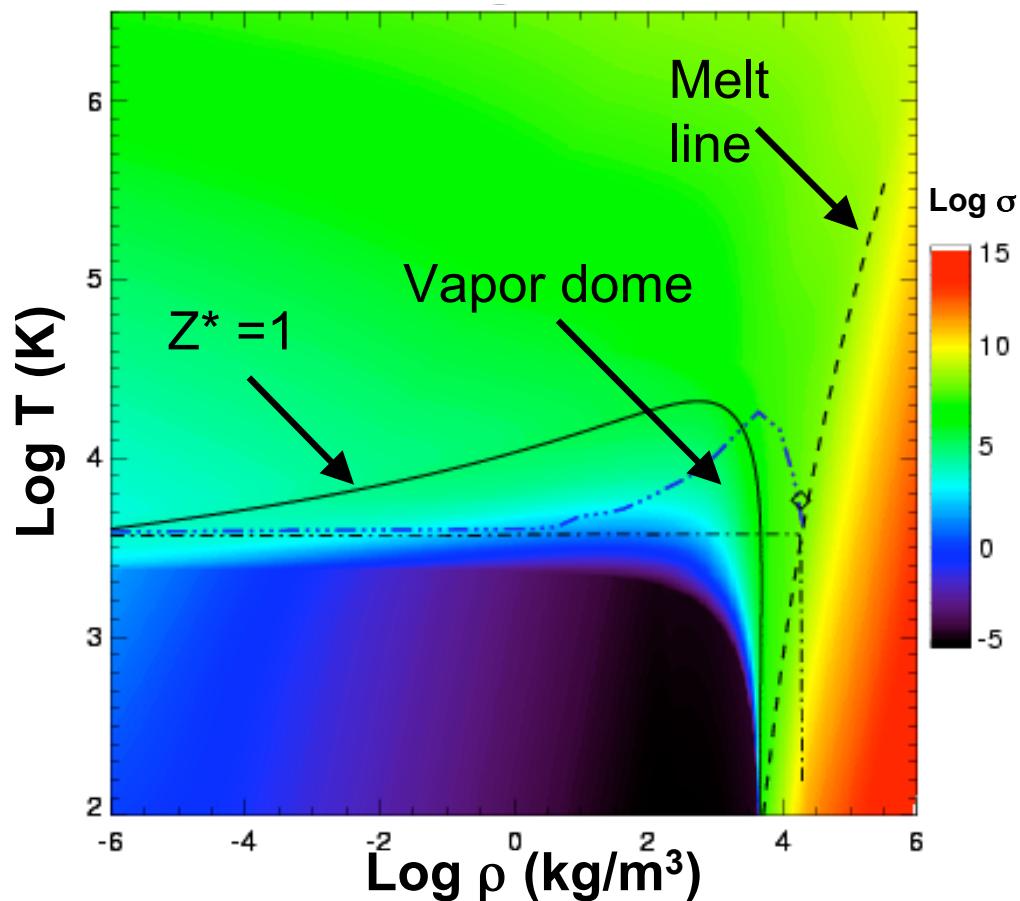
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## HEDP computer simulations rely on “physics packages”: Conductivities, Equations of State, and Opacities

Tungsten Conductivity



This highly structured portion of phase space is Warm Dense Matter

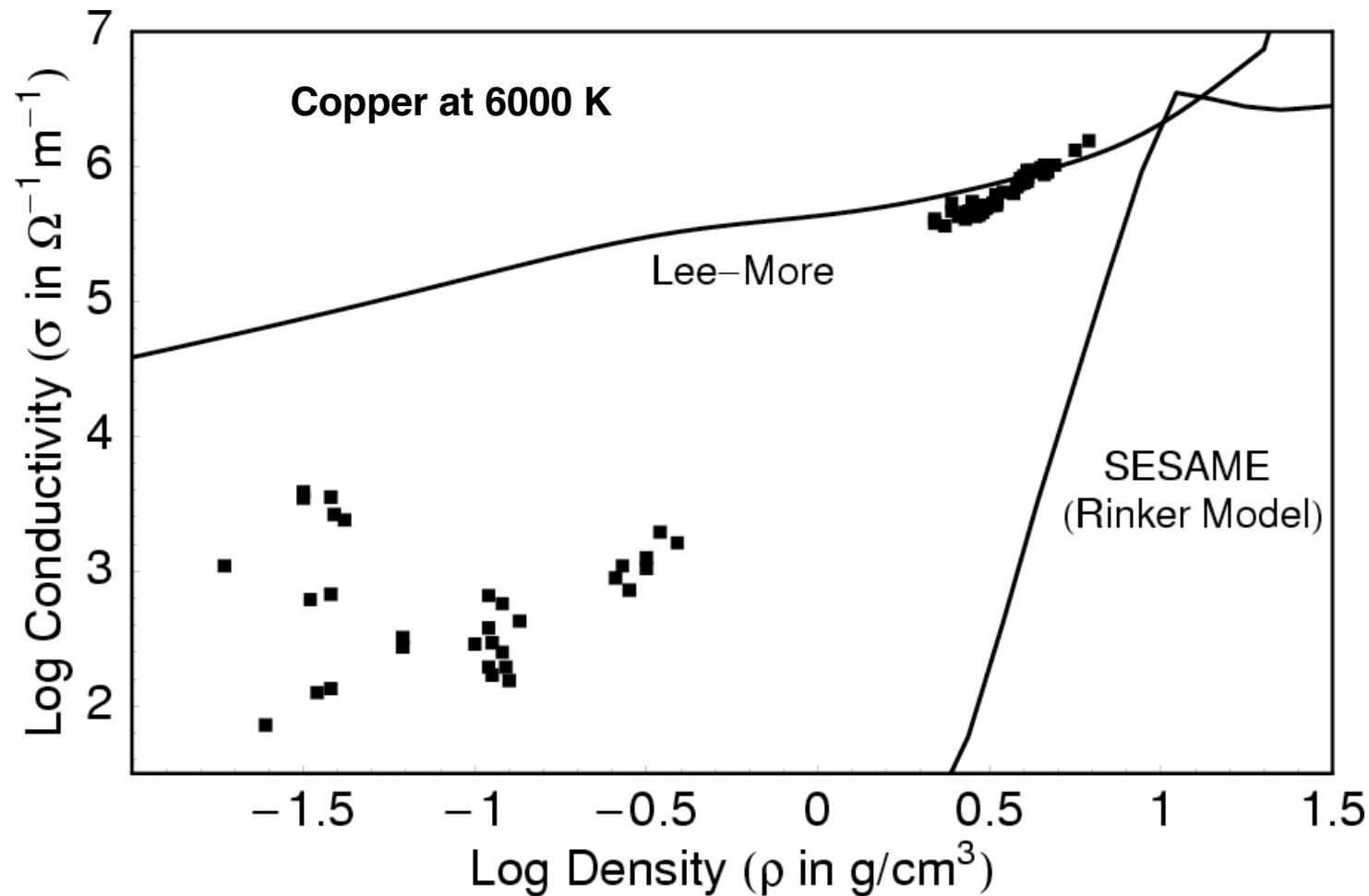
Definitions of Warm Dense Matter are varied, but generally center around **strongly coupled ions and moderately degenerate electrons** --- many different interactions are comparable.

A **quantum mechanical** treatment is generally necessary.

Most wide-range equations of state interpolate through this difficult area.



## The Lee-More and SESAME (Rinker) models are inaccurate near the metal-insulator transition

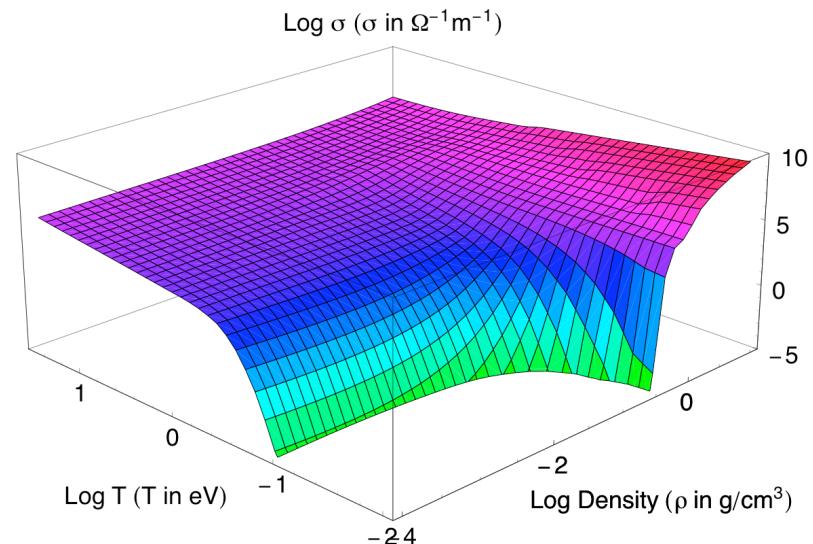
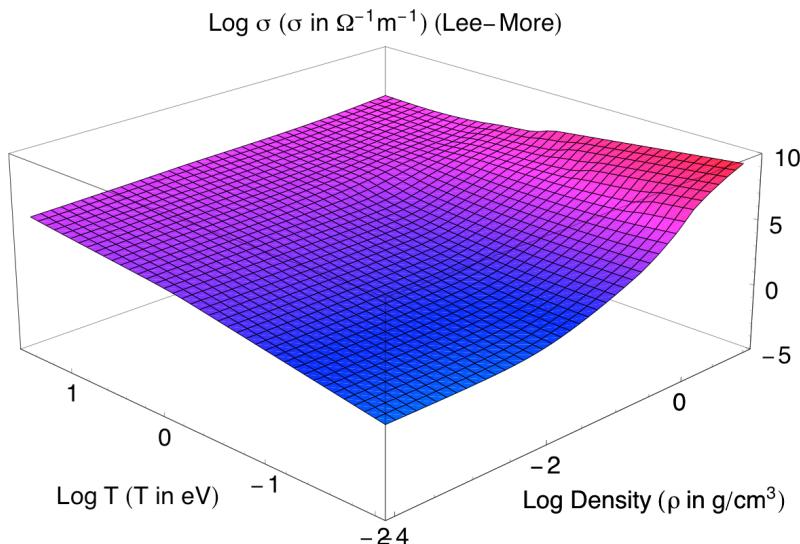


Data courtesy of Alan DeSilva, University of Maryland



## Modifications of the Lee-More algorithm were made to obtain an improved wide-range model\*

\* M. P. Desjarlais, Contrib. Plasma Phys. **41** (2001) 2-3, 267-270



### Principal modifications:

$$\sigma = n_e A \left( \frac{\mu}{kT}, \omega\tau \right) \frac{e^2}{m} \tau$$

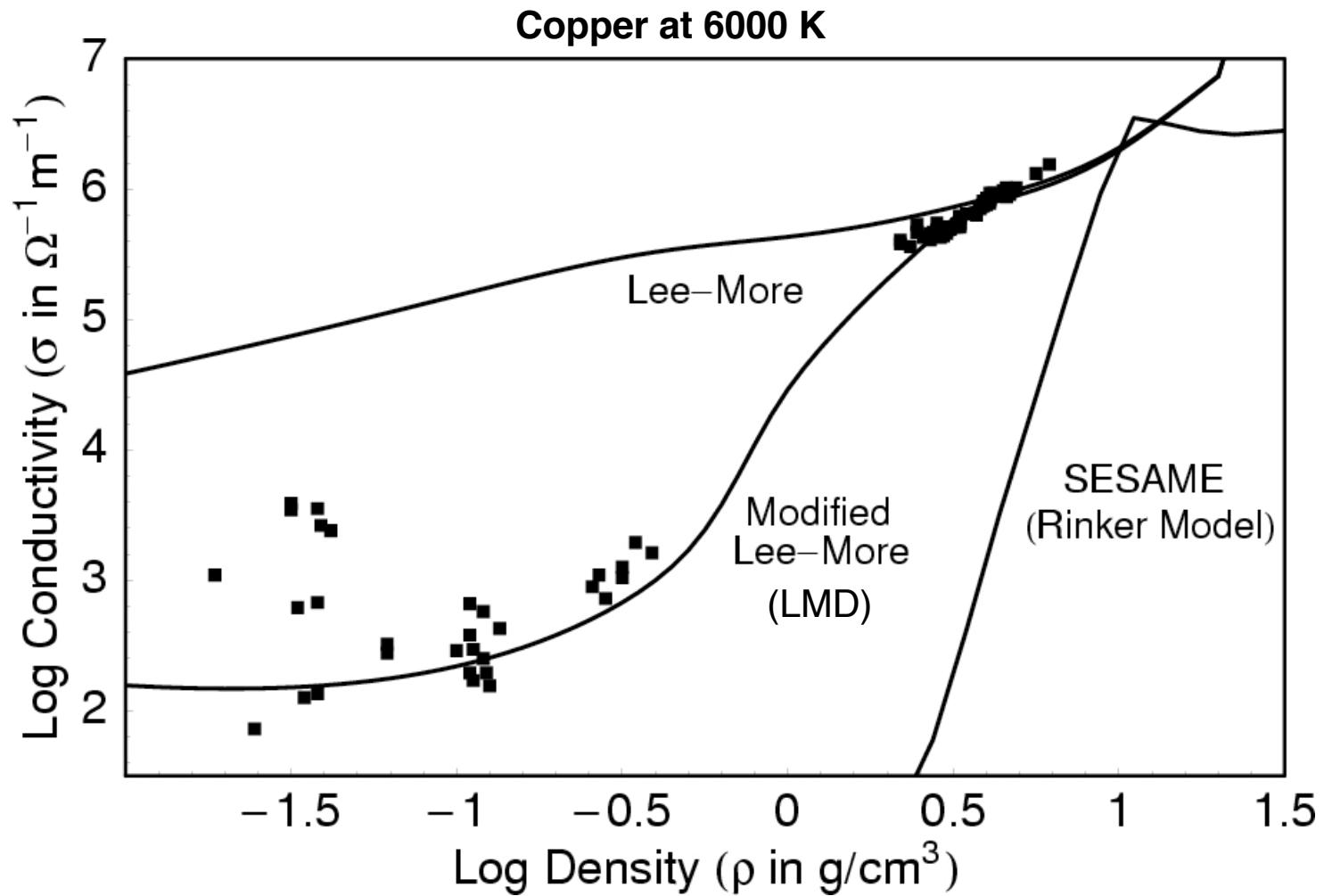
Ionization equilibrium model, pressure ionization

Electron-neutral collisions,  
Minimum collision time



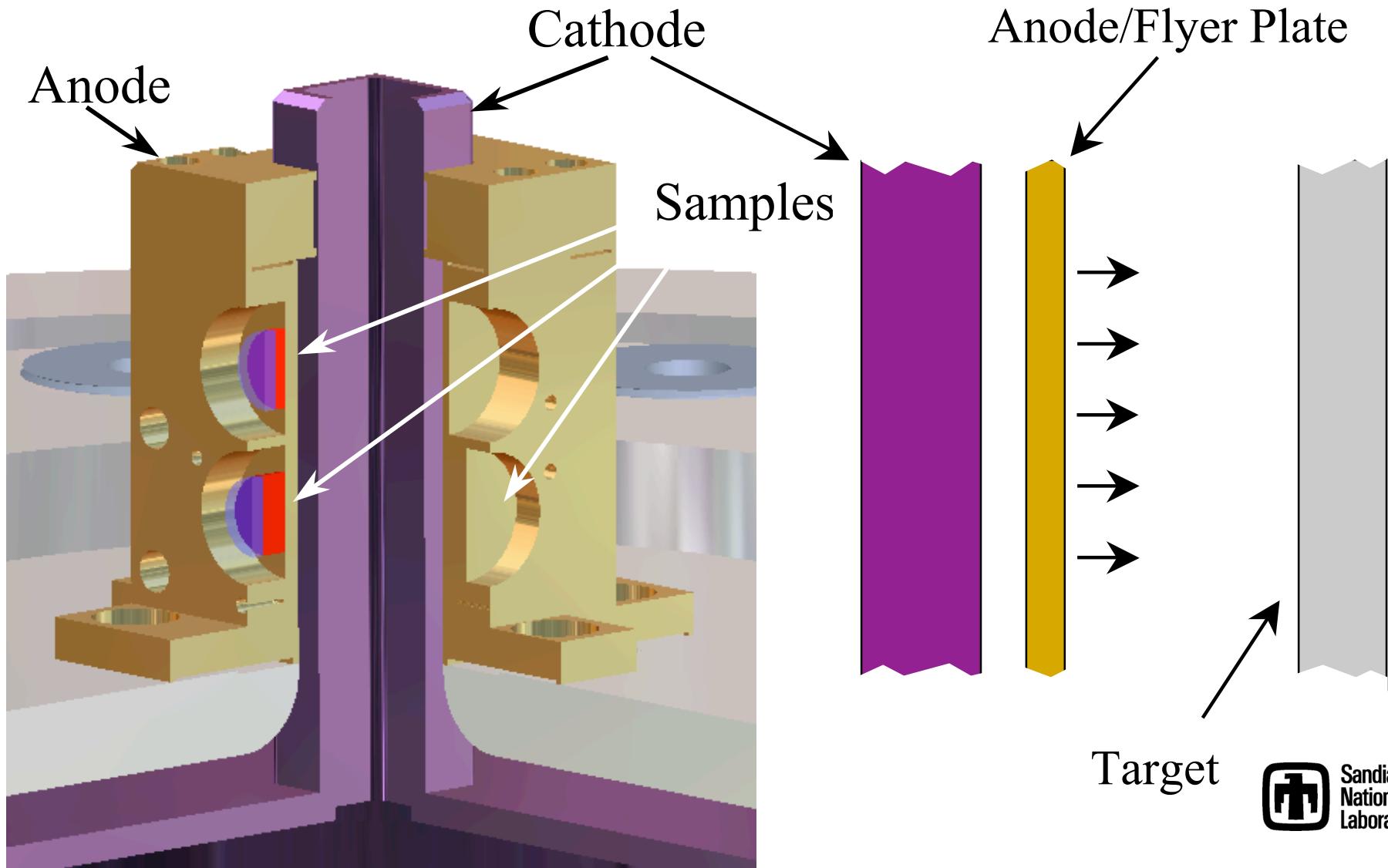
This modified Lee-More model provides much better agreement with DeSilva's copper data

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## A demanding application: Ultra-high velocity magnetically launched flyer plates (30 km/sec, Multi-Mbar pressures)

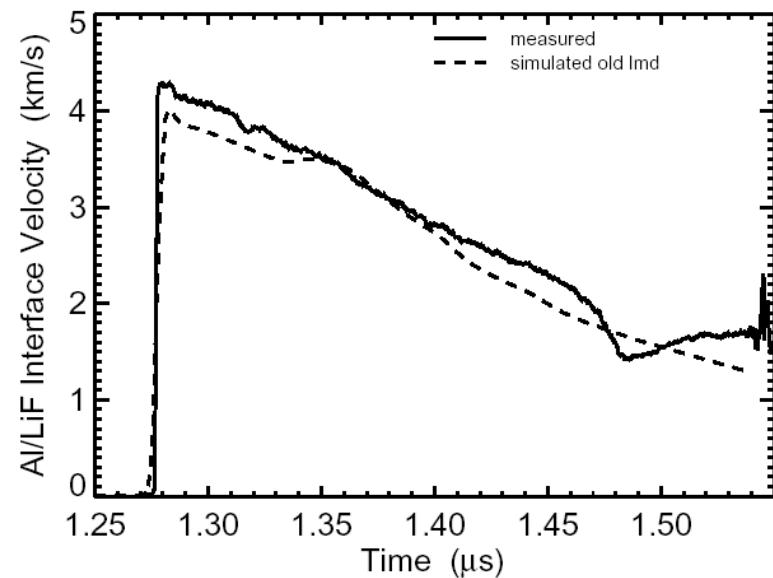
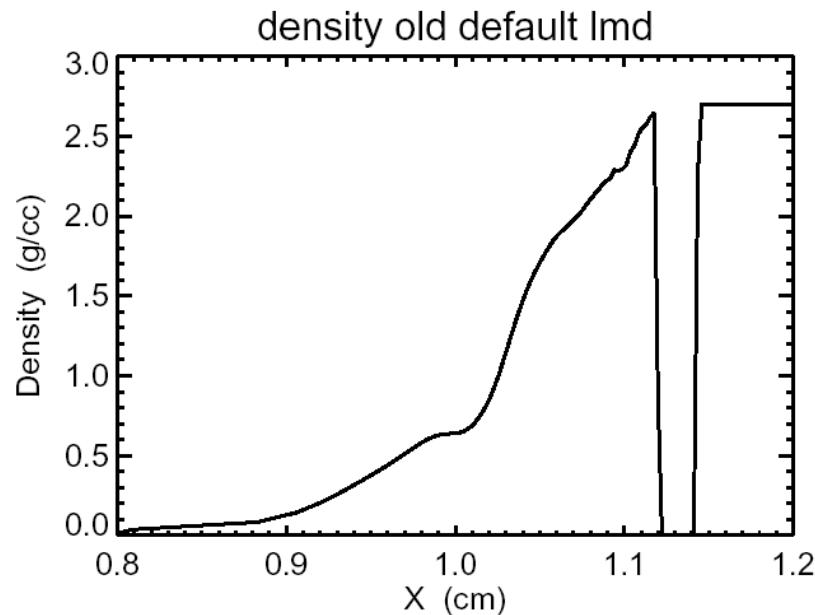




We simulated these magnetically launched flyer plates using the modified Lee-More (LMD) conductivities

Detailed comparison between simulations and experiments for magnetically launched flyer plates suggested that our *improved* conductivities were still not sufficiently accurate for the warm dense liquid aluminum.

Simulations by Ray Lemke with Sandia's 3D Rad-MHD ALEGRA code

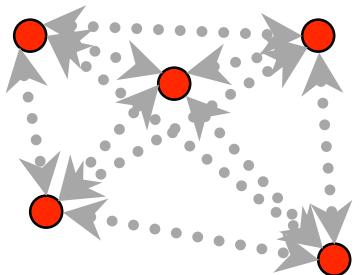


For many of our applications we require conductivities accurate to well within a factor of two.



## Density Functional Theory (DFT) is a formally exact representation of the N electron Schrödinger Equation

### Schrödinger view

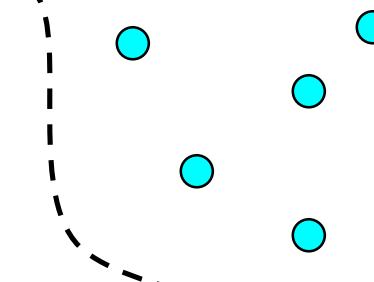


- electron interaction
- ↔ external potential

Hard problem to solve  
(scales like  $N^5$  )

Formally equivalent

### DFT view



- Kohn-Sham particle (non-interacting)
- effective potential

“Easy” problem to solve  
( scales like  $N^3$  or better)

Hohenberg and Kohn proved this (1964)

Nobel prize in Chemistry for Kohn in 1998



## We are using Density Functional Theory (DFT) to perform Quantum Molecular Dynamics (QMD) simulations of Warm Dense Matter

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Density Functional Theory is formally exact, but is in practice an accurate approximate solution to the  $N$  electron Schrödinger equations. DFT is a work-horse tool in condensed matter physics, but a relative newcomer to high energy density physics.

In addition to our Sandia effort, other groups with a large effort in this area include LANL (Collins, Kress); CEA (Clerouin, Mazevet, Recoules, Blottiau); LLNL (Galli, Gygi, Schwegler)

**QMD:** The **Kohn-Sham\*** DFT equations are solved for a given atomic configuration (fixed in the Born-Oppenheimer approximation) and the **quantum mechanical forces** on all the atoms are calculated from the wavefunctions following the Feynman-Hellmann theorem, **the atomic positions are advanced classically**, and a new solution to the DFT equations is calculated.

\*Kohn and Sham, 1965



## The Kubo-Greenwood formula is used to calculate the frequency dependent electrical conductivity

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$$\sigma_{\mathbf{k}}(\omega) = \frac{2\pi e^2 \hbar^2}{3m^2 \omega \Omega} \sum_{\alpha=1}^3 \sum_{j=1}^N \sum_{i=1}^N (F(\varepsilon_{i,\mathbf{k}}) - F(\varepsilon_{j,\mathbf{k}})) \left| \left\langle \Psi_{j,\mathbf{k}} \left| \nabla_{\alpha} \right| \Psi_{i,\mathbf{k}} \right\rangle \right|^2 \delta(\varepsilon_{j,\mathbf{k}} - \varepsilon_{i,\mathbf{k}} - \hbar\omega),$$

where  $e$  and  $m$  are the electron charge and mass. The  $i$  and  $j$  summations are over the  $N$  discrete bands of the triply periodic calculation for the cubic supercell with volume  $\Omega$ . The coordinate index is  $\alpha$  and in general we average over  $\alpha$  to improve the statistics.  $F(\varepsilon_{i,\mathbf{k}})$  is the Fermi weight corresponding to the energy for the  $i$ -th band at  $\mathbf{k}$  with wavefunction  $\Psi_{i,\mathbf{k}}$ .

We integrate over the Brillouin zone using the method of special  $\mathbf{k}$ -points

$$\sigma(\omega) = \sum_{\mathbf{k}} \sigma_{\mathbf{k}}(\omega) W(\mathbf{k}) \quad ,$$

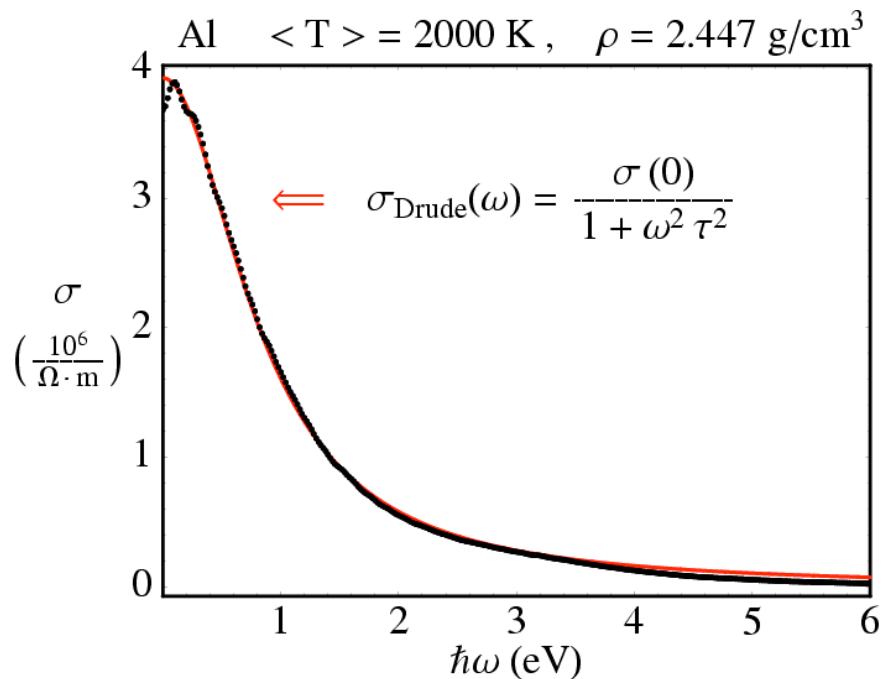
and average over 10 to 20 configurations selected from the MD run.

This is really nothing more than the quantum analog of the classical current-current correlation function representation of the conductivity

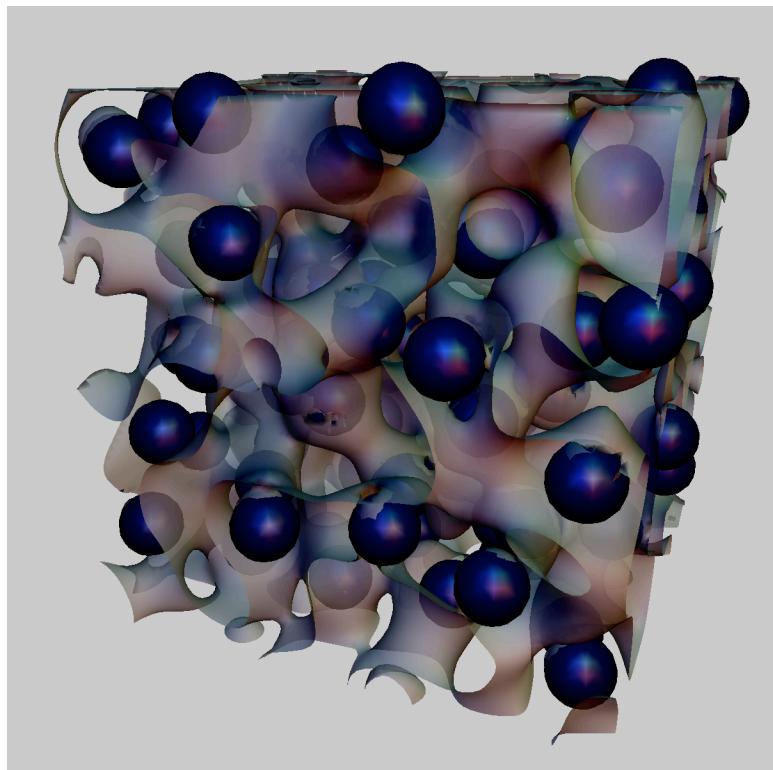


## At liquid densities just below solid, the optical conductivity is well fit by the Drude model

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The agreement with the Drude model indicates 'nearly free' electrons

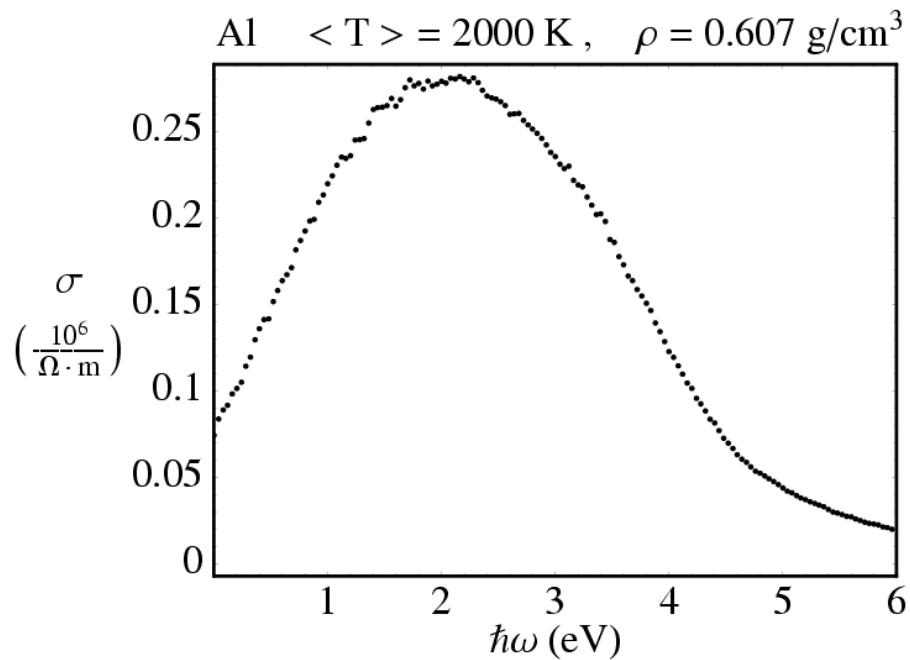


Ion cores displayed with iso-surfaces of the mean valence charge density

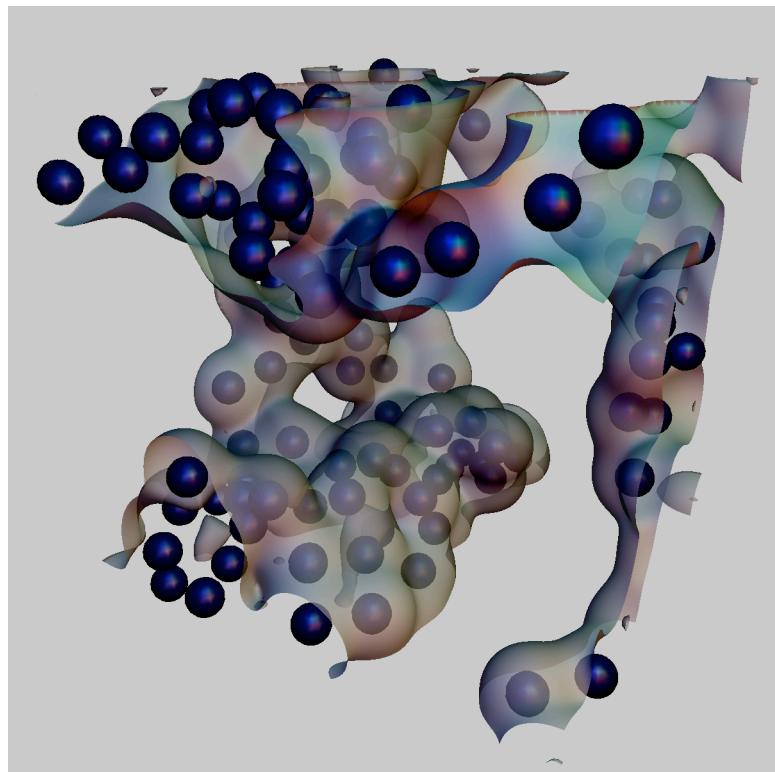


At lower density, where phase separation is pronounced, a gap begins to form at low energy

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The dc conductivity has dropped by a factor of 25 for a factor of 4 drop in density

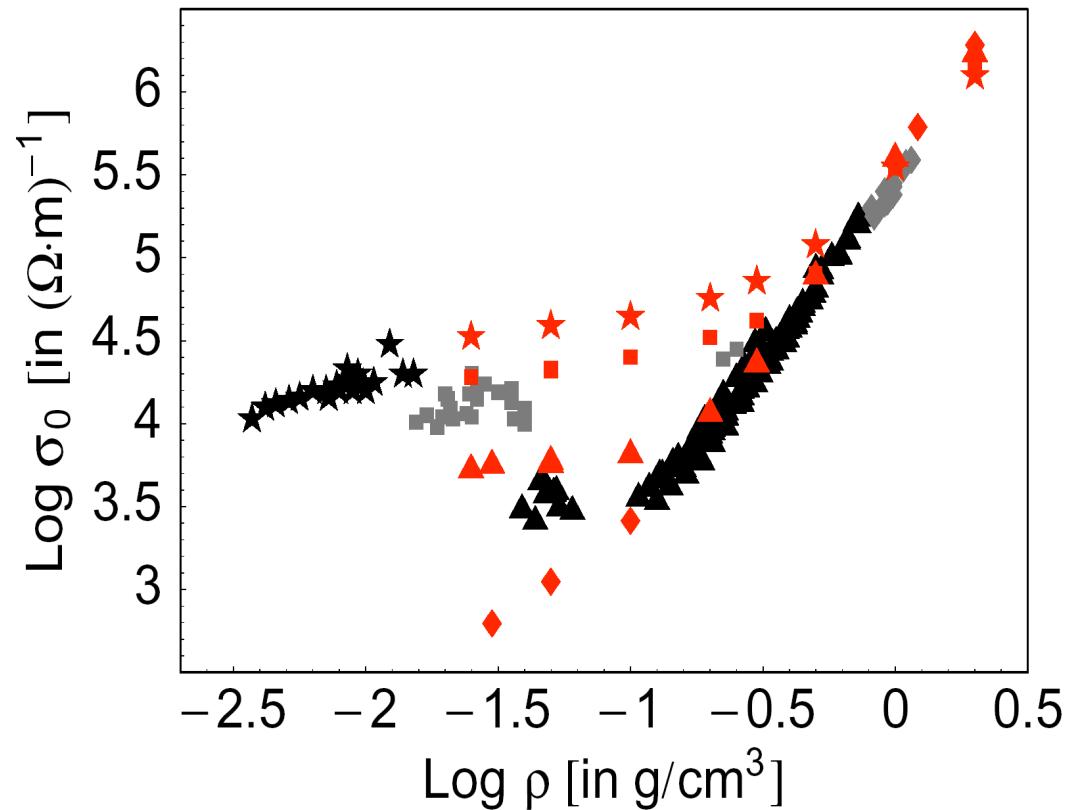


Note the pronounced separation into liquid and void (vapor) regions



The QMD-KG results are in good agreement with DeSilva's data over a two decade range of density

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DeSilva and Katsouros data in black or grey, MD-KG results in red

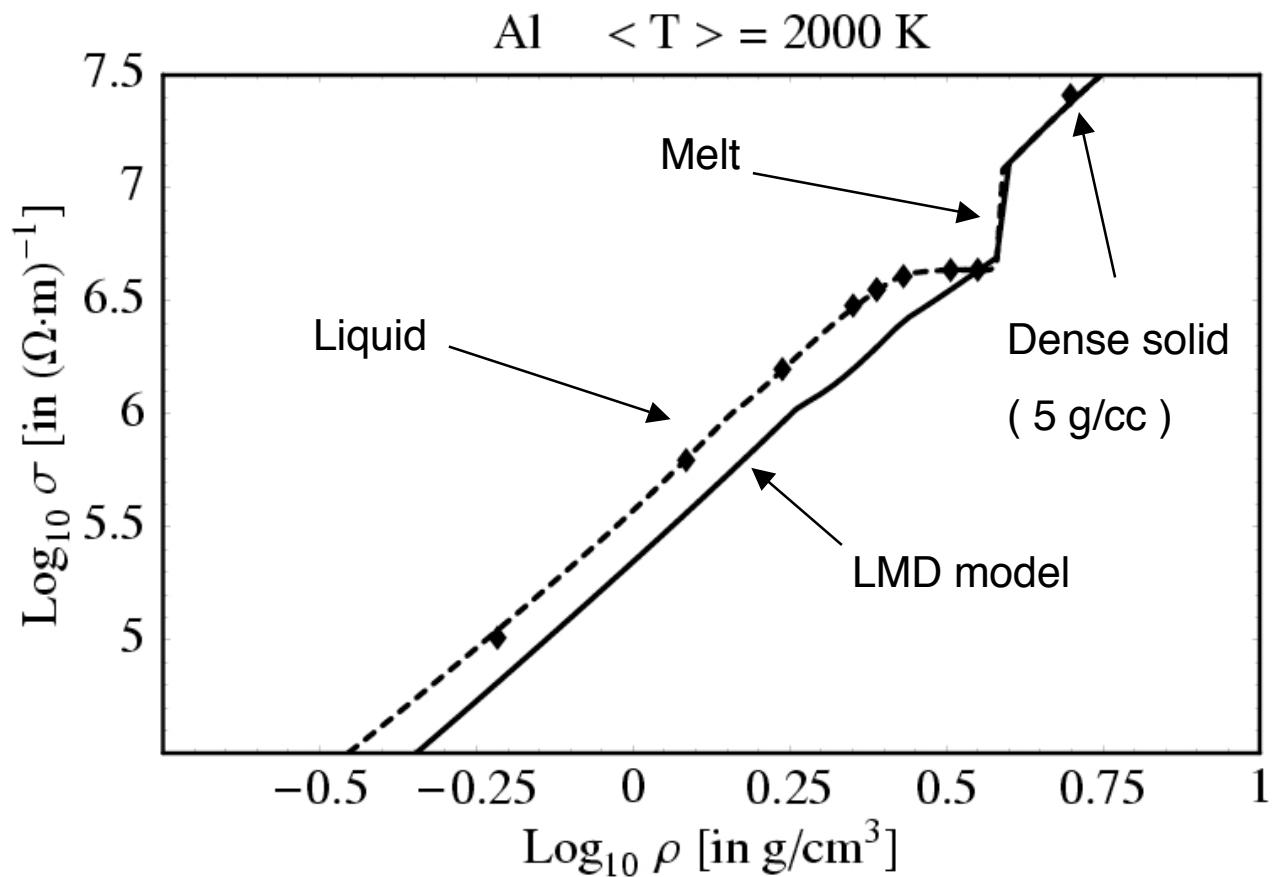
★ 30000 K, ■ 20000 K, ▲ 10000 K, ◆ 6000 K

[Desjarlais, Kress, and Collins, PRE 66, 025401(R) (2002)]



**The calculated liquid aluminum conductivities are higher than the *improved* Lee-More (LMD) model predictions**

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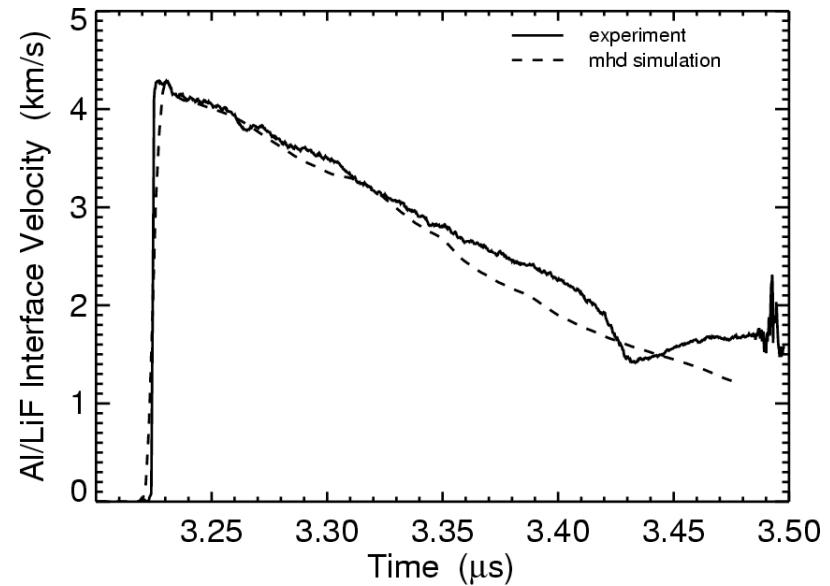
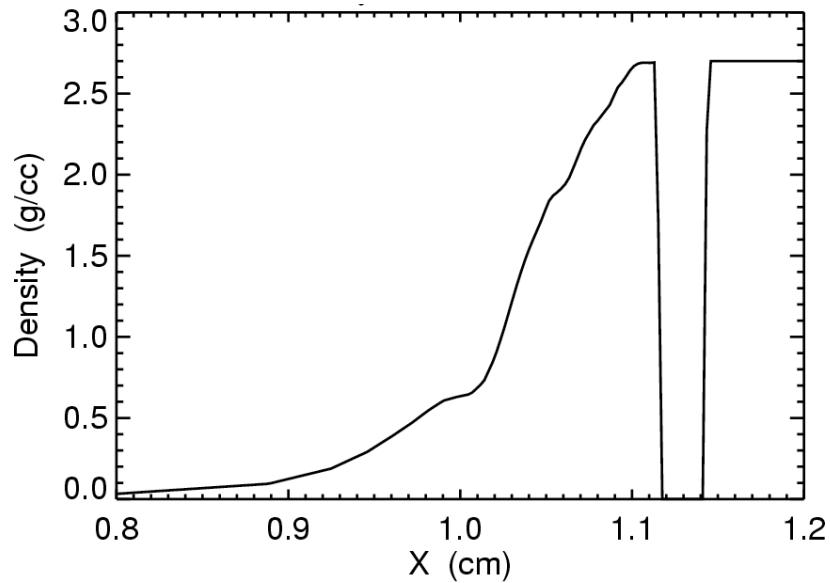
The dashed line shows the 2000 K isotherm from our QMD-tuned wide-range aluminum model



## Flyer plate simulations with the QMD based conductivities give very good agreement with experiment

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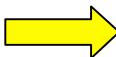
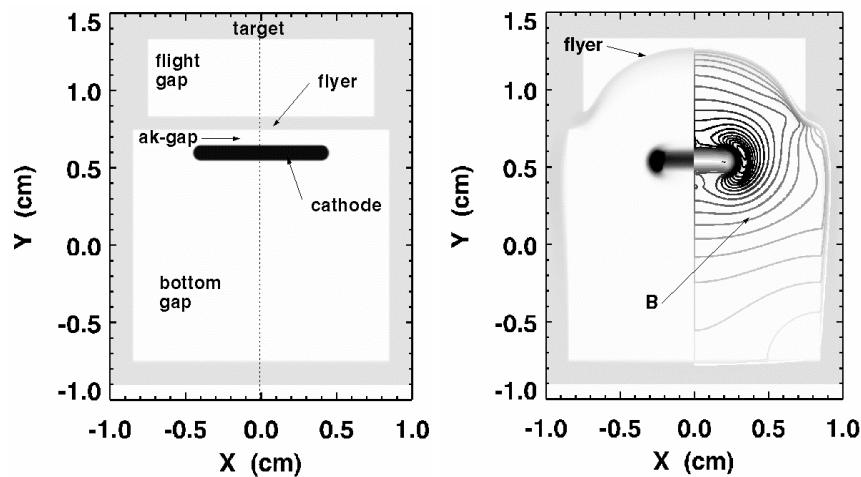
Simulations by Ray Lemke with Sandia's 3D Rad-MHD ALEGRA code



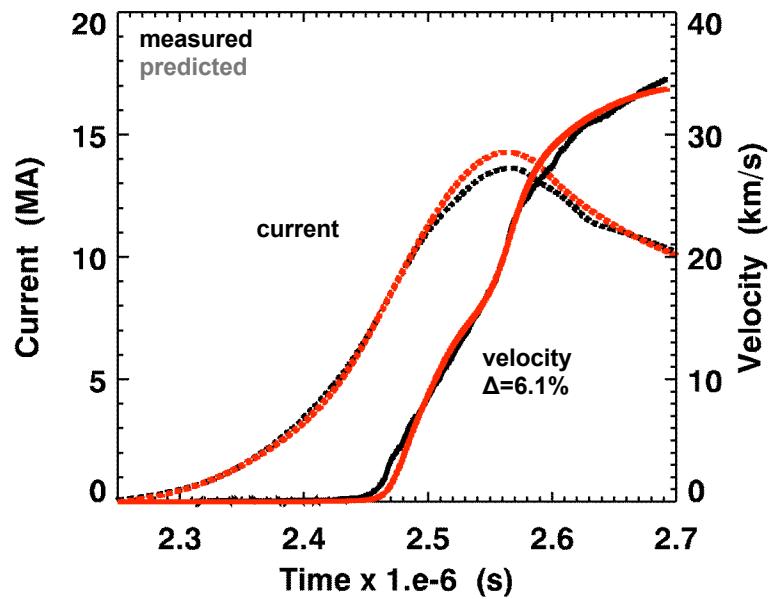
Conductivities based on the QMD calculations have given us a new predictive capability.



We have used our simulation capability with the new aluminum model to optimize flyer performance on Z



Measured / predicted current & flyer velocity (850  $\mu\text{m}$  Al)



The simulations were performed by Ray Lemke using Sandia's ALEGRA code in 2-D



## Other optical properties are obtained through the Kramers-Krönig relations for $\sigma(\omega)$

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Through the Kramers-Kronig relations we have  $\sigma_2(\omega) = -\frac{2}{\pi} P \int \frac{\sigma_1(\omega)}{v^2 - \omega^2} dv$

where P indicates the principal value of the integral. The dielectric function is then given by  $\varepsilon_1(\omega) = 1 - \frac{4\pi}{\omega} \sigma_2(\omega)$  and  $\varepsilon_2(\omega) = \frac{4\pi}{\omega} \sigma_1(\omega)$

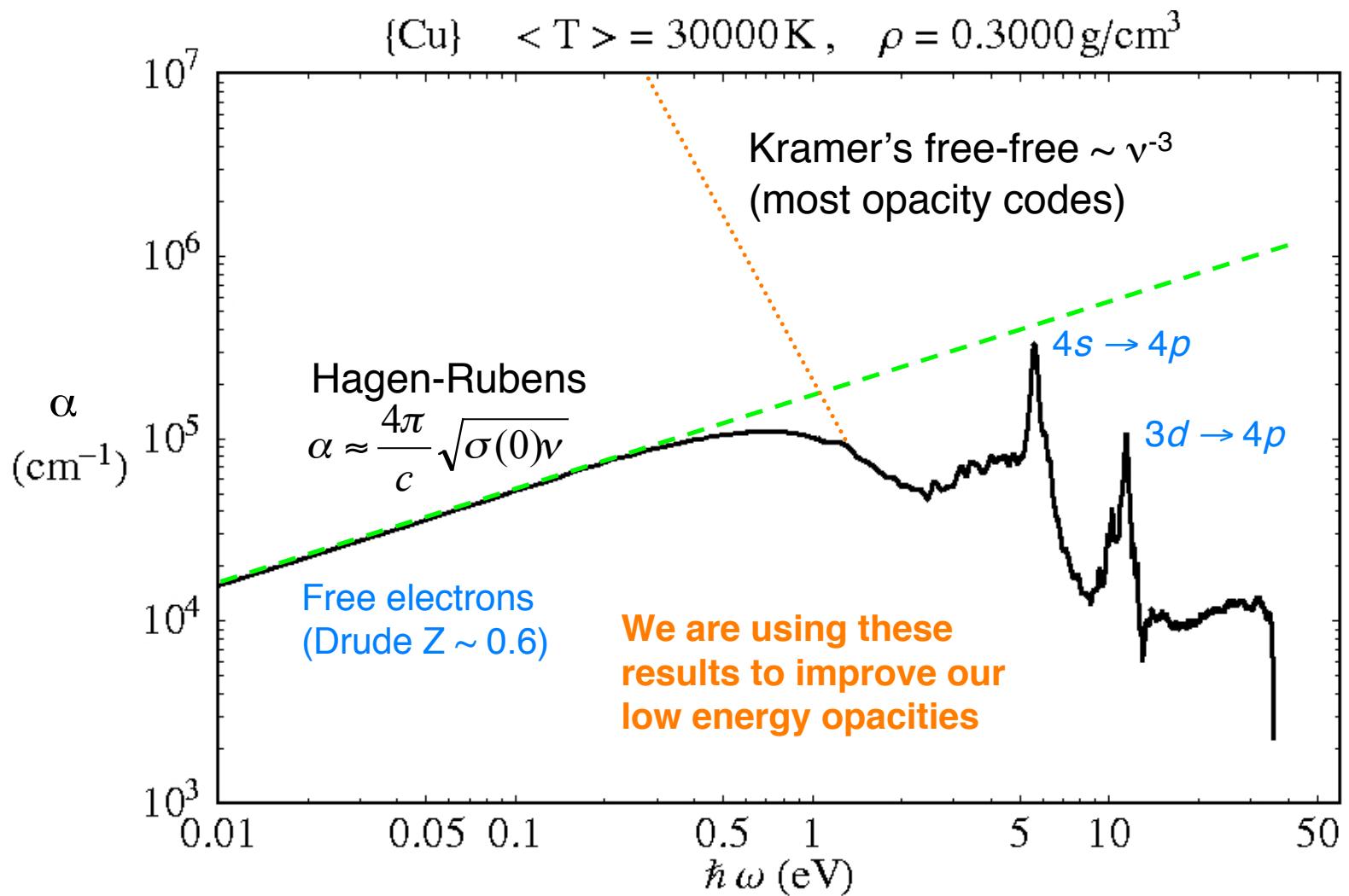
Defining  $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega) \equiv [n(\omega) + ik(\omega)]^2$  we can write the reflectivity  $r(\omega)$  and absorption coefficient  $\alpha(\omega)$ :

$$r(\omega) = \frac{[1 - n(\omega)]^2 + k(\omega)^2}{[1 + n(\omega)]^2 + k(\omega)^2} , \quad \alpha(\omega) = \frac{4\pi}{n(\omega)c} \sigma_1(\omega) .$$

We can now improve our low energy (below  $\sim 30$  eV) opacities  $\alpha(\omega)/\rho$  where  $\rho$  is the mass density.



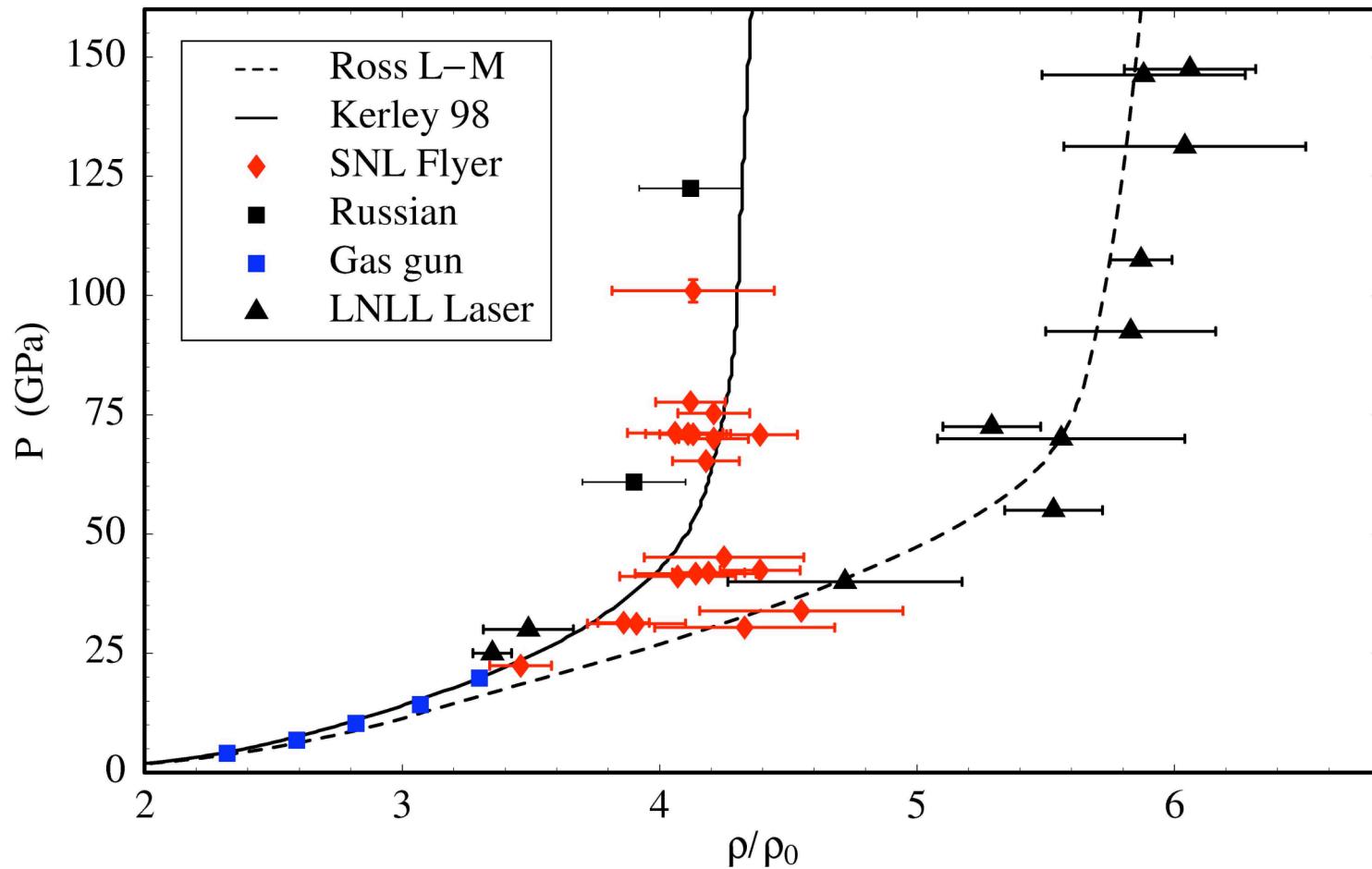
The absorption has the proper Hagen-Rubens low frequency behavior, consistent with the dc conductivity





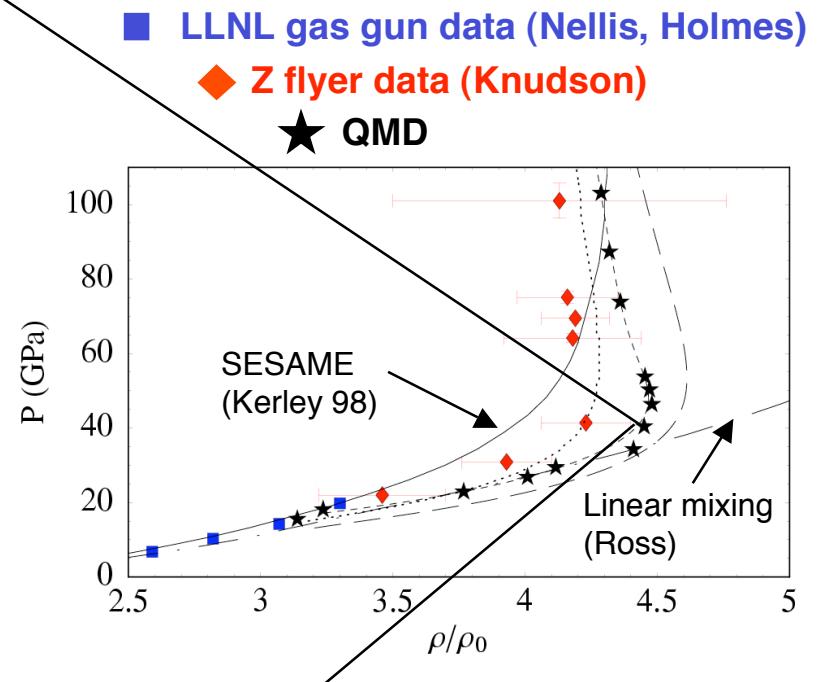
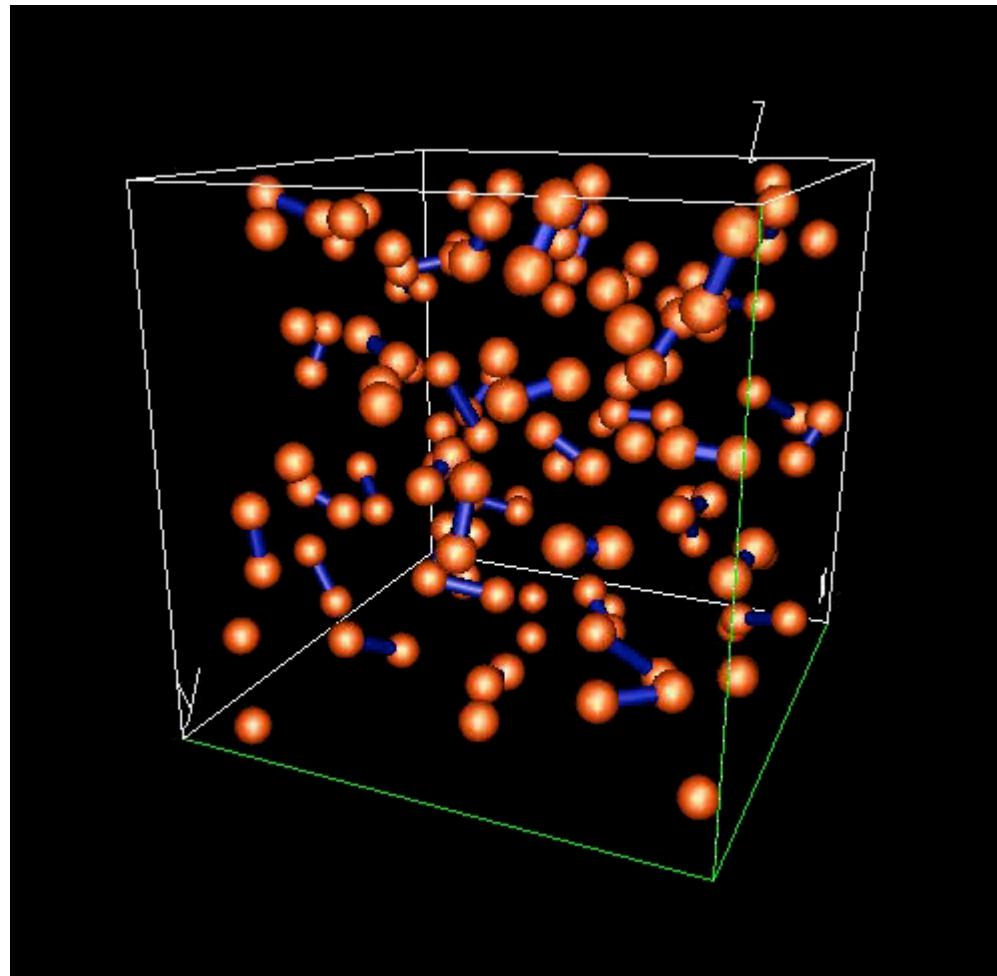
In recent years there has been great interest in the deuterium Hugoniot

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## We have used quantum molecular dynamics to study the principal Hugoniot of shocked liquid deuterium

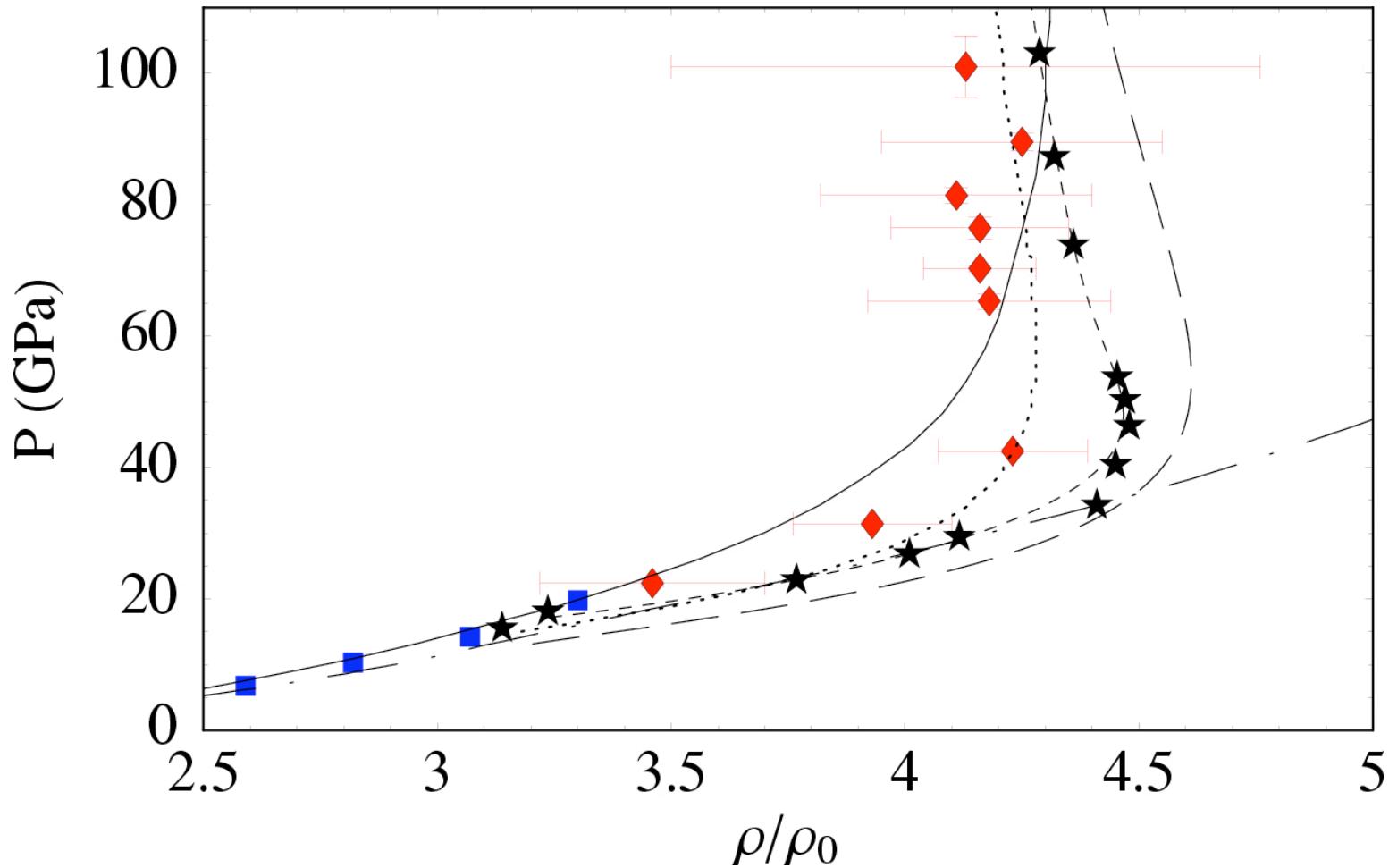


Our QMD simulations of  $D_2$  indicate a Hugoniot in very good agreement with data from Sandia's Z machine

M. P. Desjarlais, Phys. Rev. B **68**, 064204 (2003)



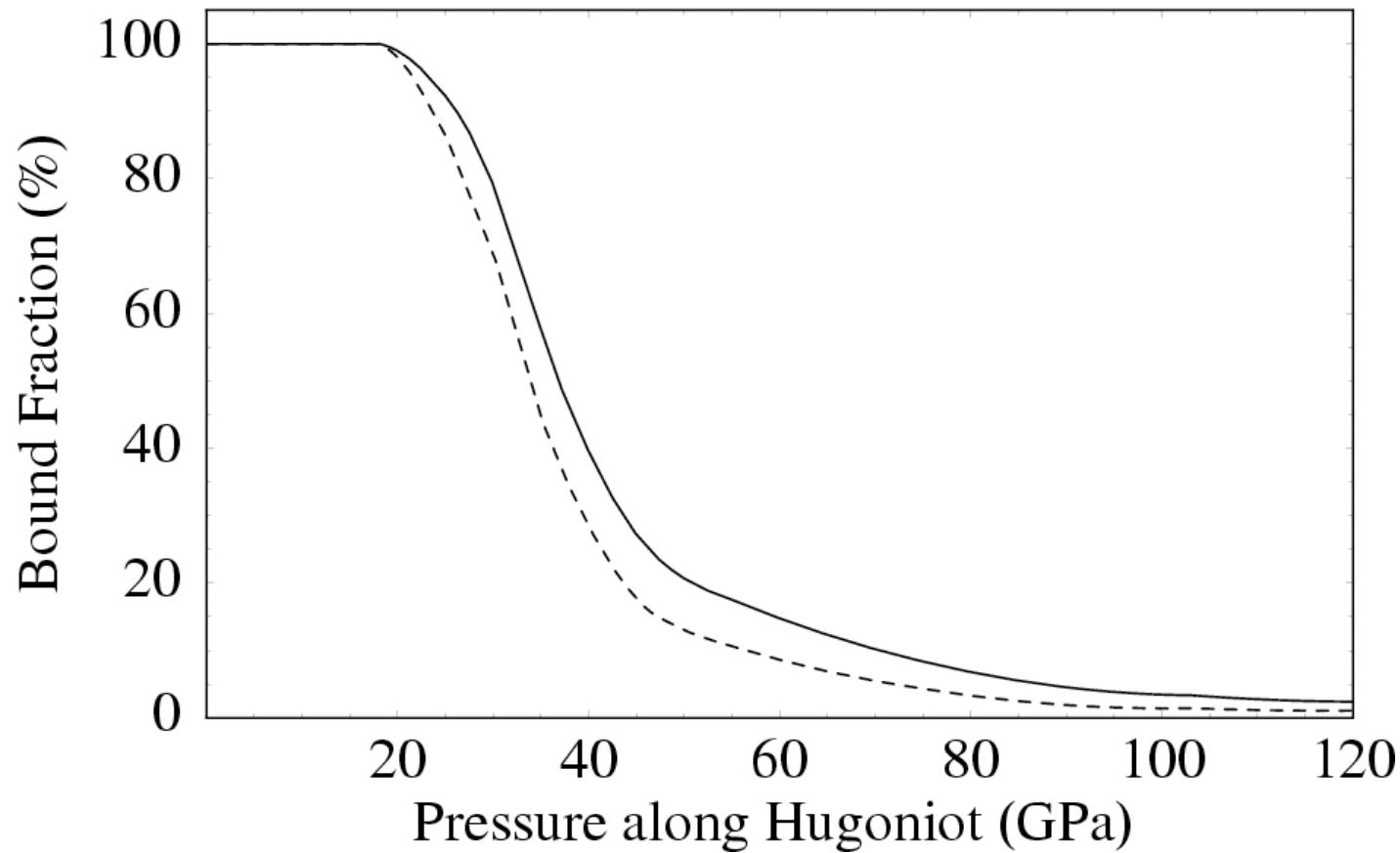
In contrast to earlier DFT/GGA results, our calculations ( $\star$ ) are consistent with the gas-gun data ( $\square$ )



Temperatures are also in very good agreement with the gas-gun data



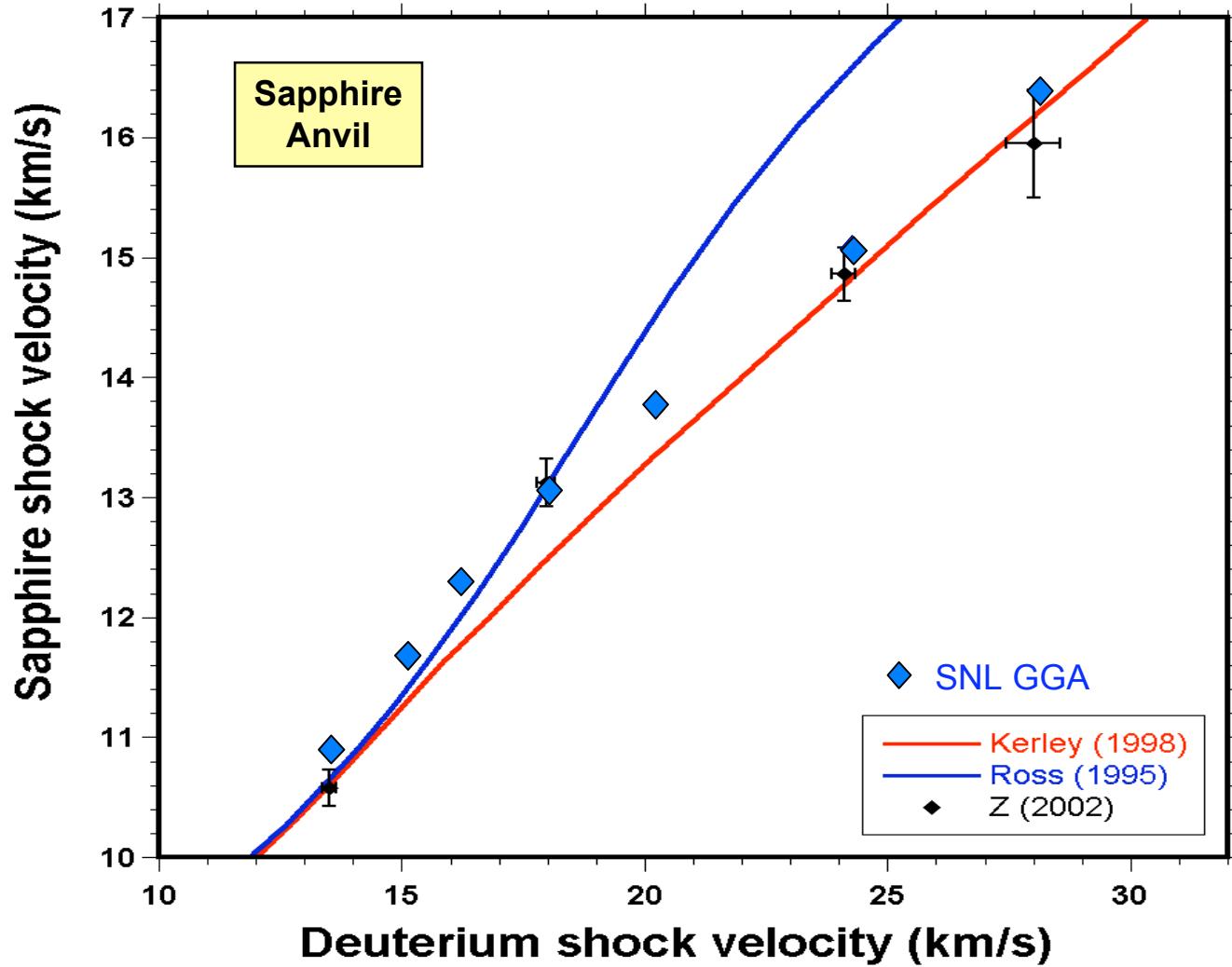
**The dissociation along the Hugoniot begins at 20 GPa and is largely over by 50 GPa**



Bonds defined for nearest neighbors or mutual nearest neighbors persisting two vibron periods or greater



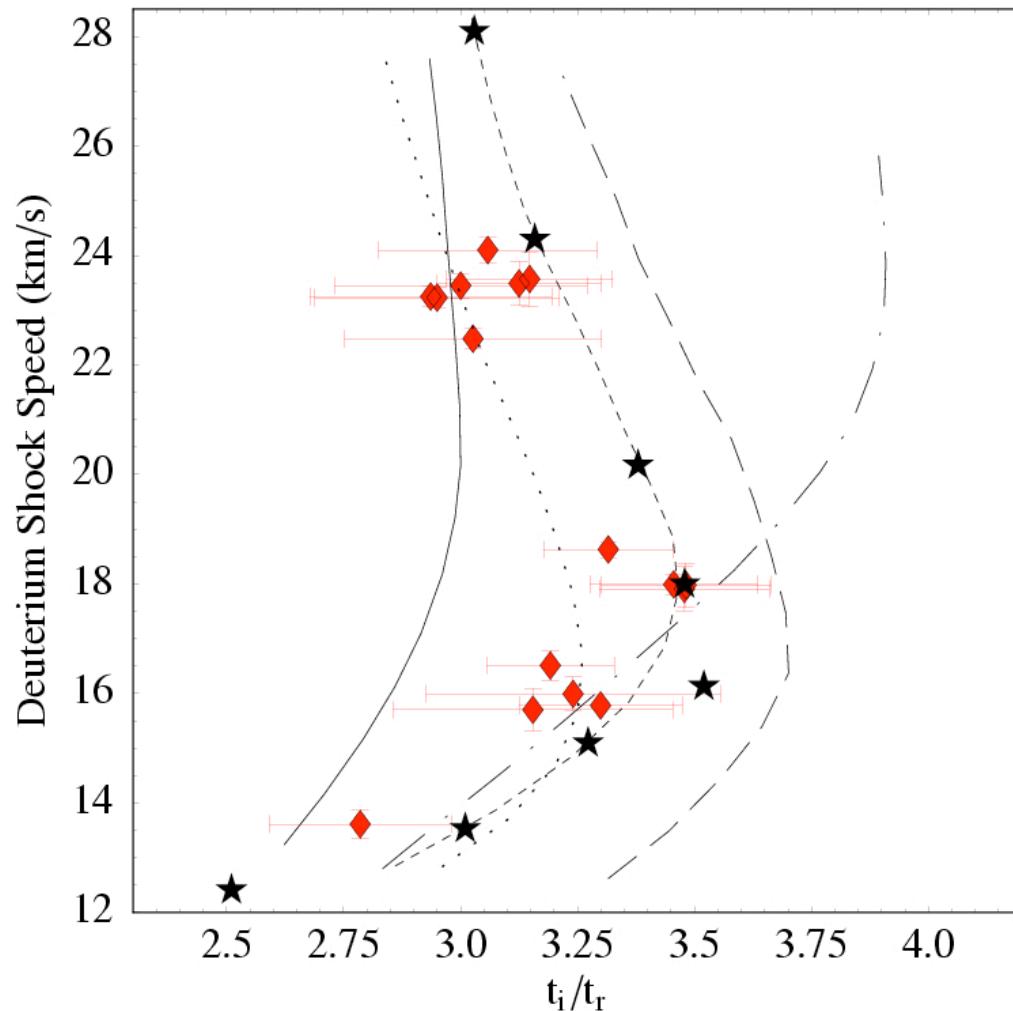
Reshock states with a sapphire anvil are in good agreement with the flyer data from Z





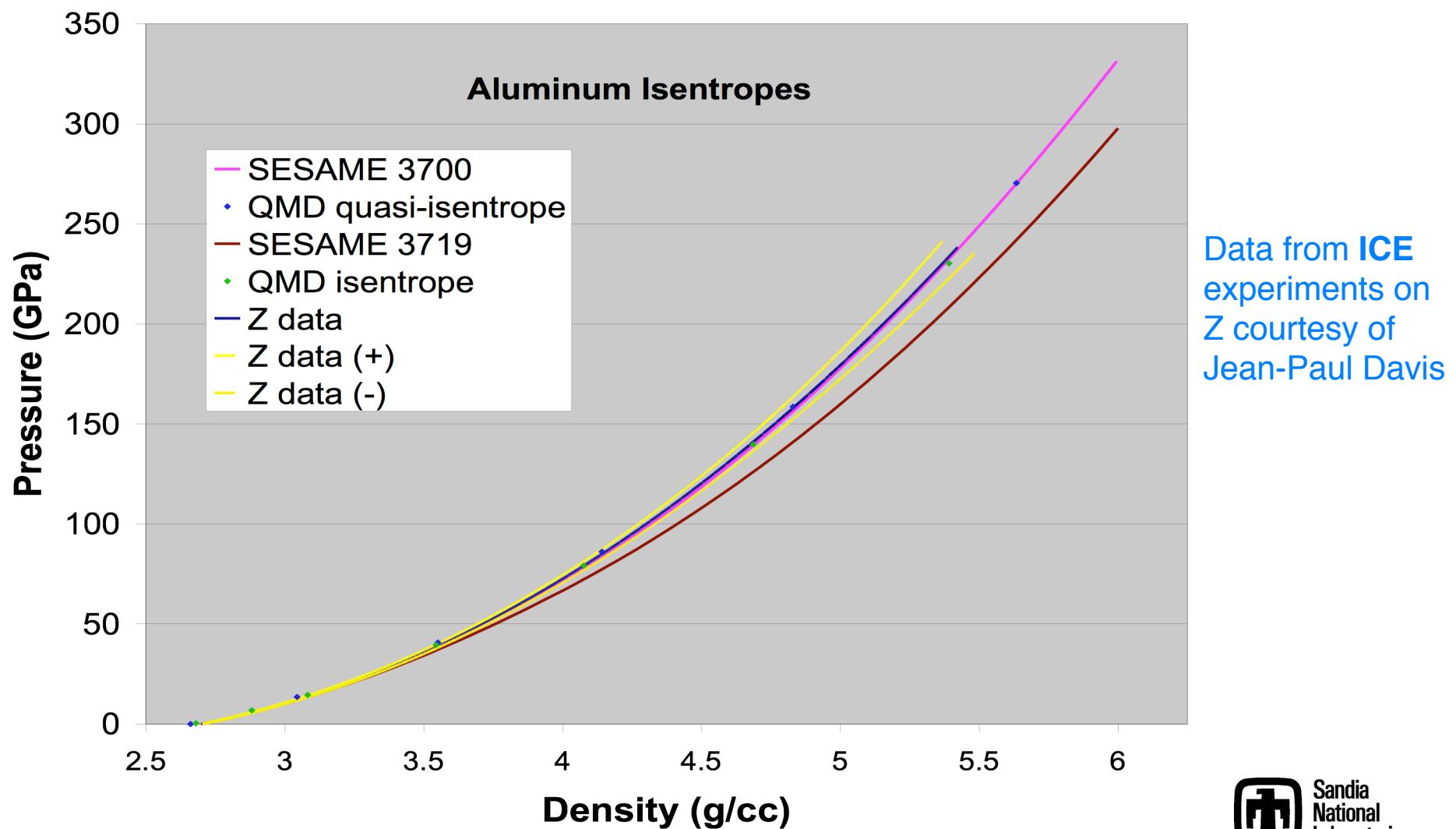
**The calculated reverberation timing ratios are in very good agreement with the Z experiments**

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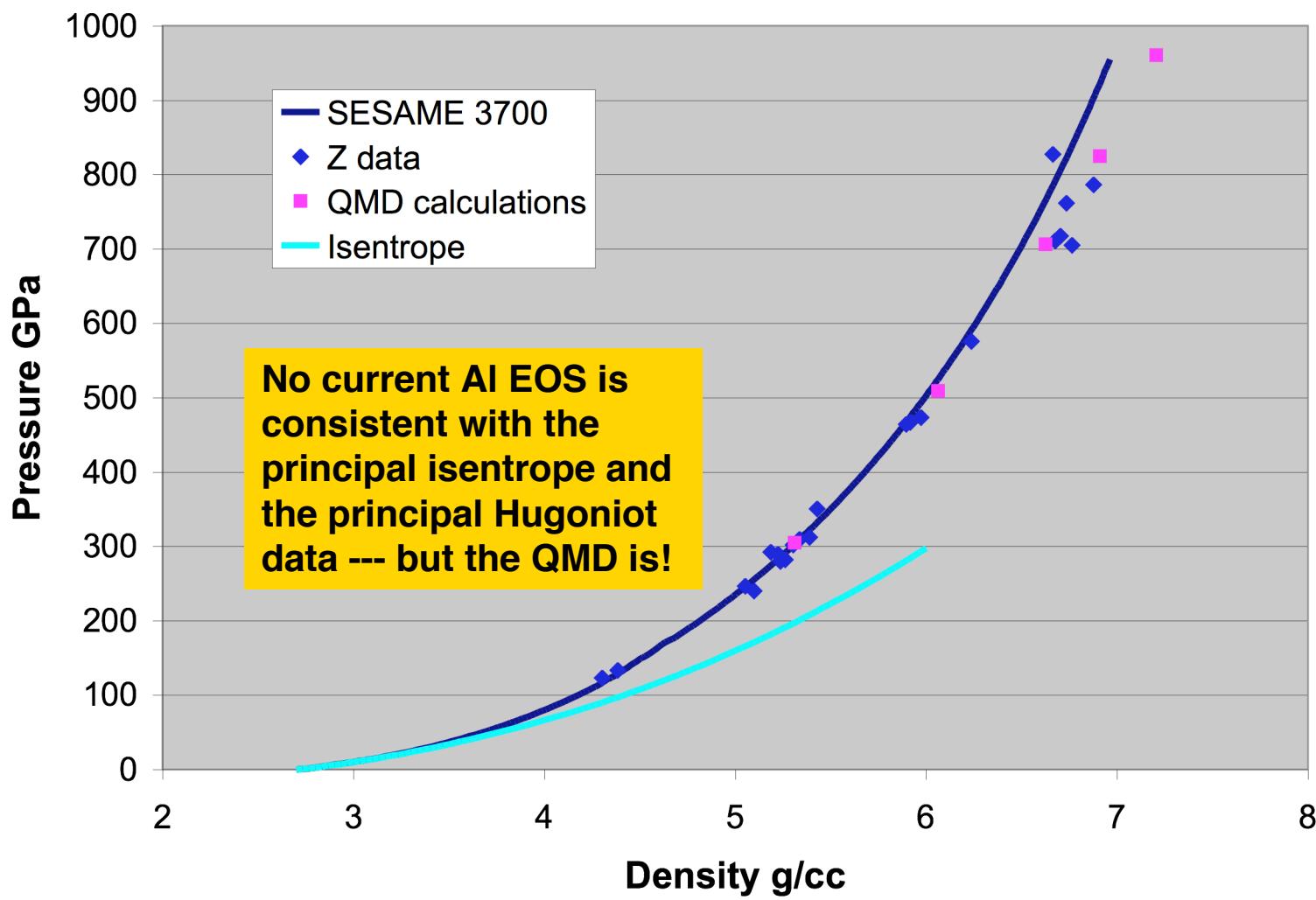


Our calculated principal isentrope for aluminum is in excellent agreement with data from Sandia's Isentropic Compression Experiments (ICE) on Z





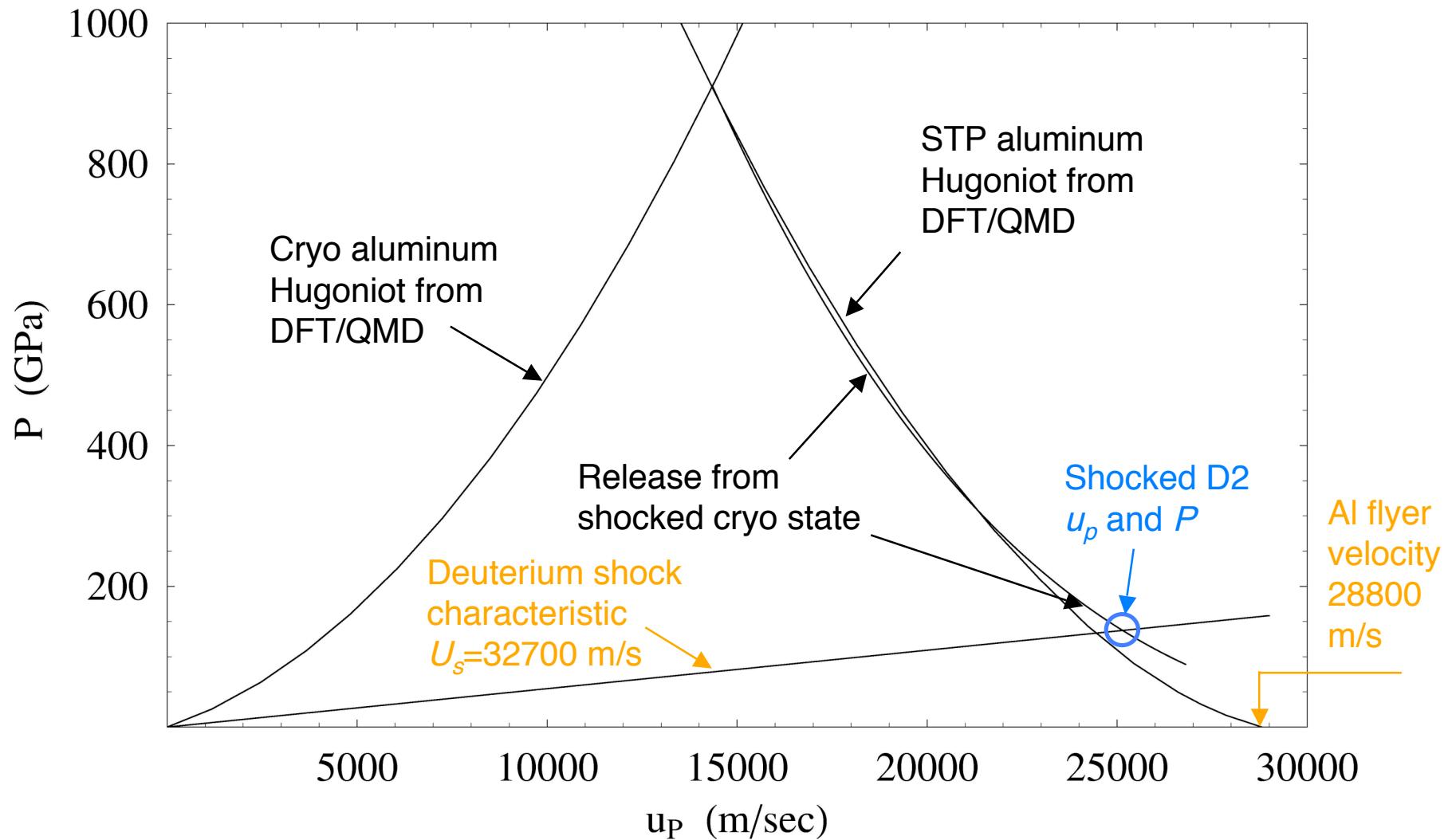
We have also computed the aluminum shock Hugoniot and compared with shock data from Z



AI Hugoniot data from Z courtesy of Marcus Knudson

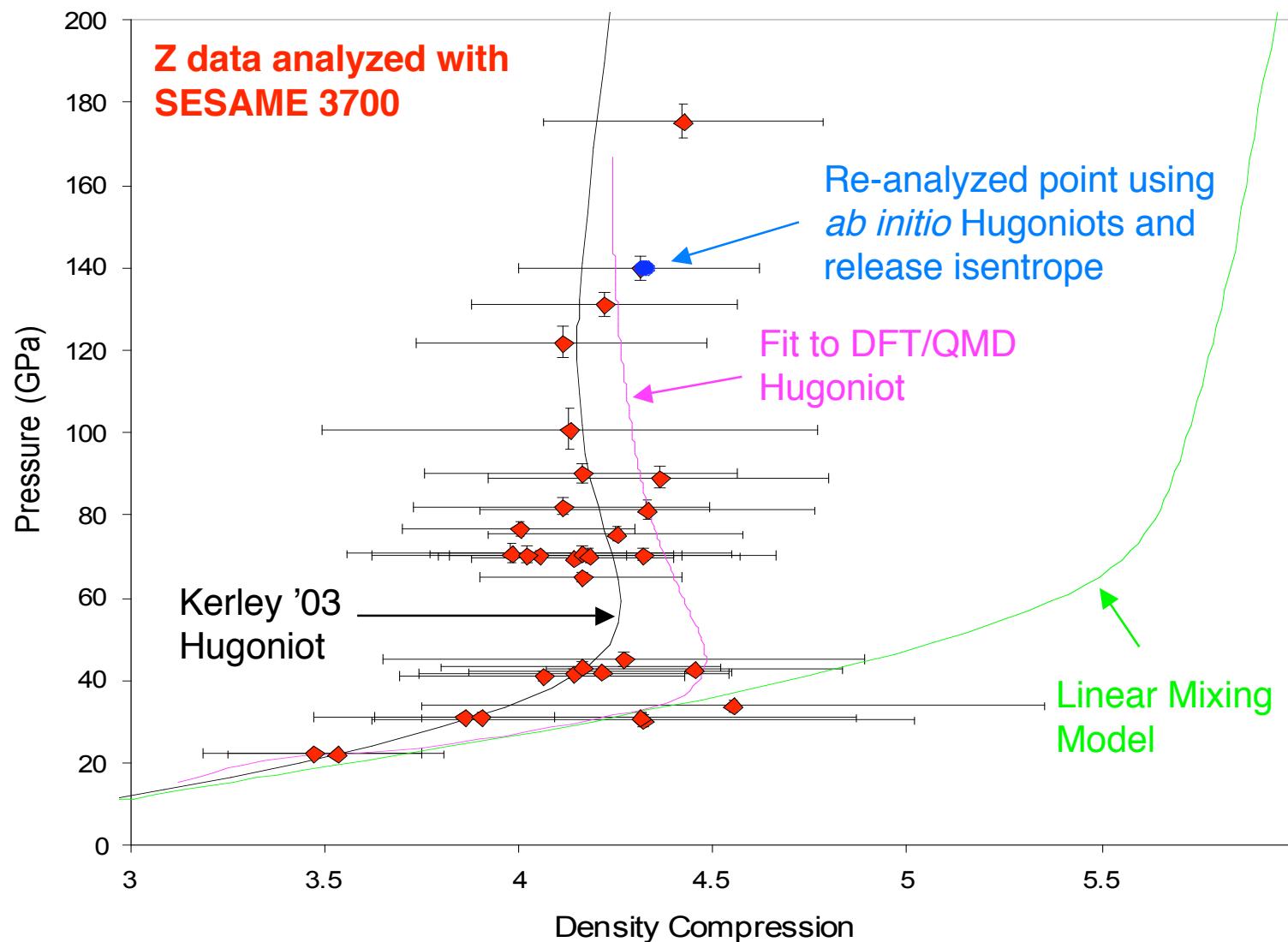


We have all the pieces to do *ab initio* impedance matching calculations





Our re-calculated impedance match point is very close to that obtained with SESAME 3700





## Recent and active research areas

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- Principal Hugoniot and reshock properties of deuterium
- QMD based conductivity models for Al, W, Be, and stainless steel
- Liquid-vapor critical points for W and Al
- Principal and release isentropes for aluminum
- Shock melting of Be and Diamond (NIF Ignition Campaign)
- Electrical and thermodynamic properties of water at high energy densities
- Research on advanced electronic structure methods for High Energy Density Physics (finite temperature Exact Exchange, finite temperature GW)